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ARTICLE

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Count-loss effect in subcriticality measurement by pulsed neutron source method, (I) Investigation on count-loss effect in determination of neutron decay constant

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Abstract

The count-loss effect in determination of neutron decay constant by pulsed neutron source method was investigated. It was found that overestimation of neutron decay constant due to count-loss effect is seen while underestimation appears superiorly as the intensity of pulsed neutron source is getting higher. It was further demonstrated that the well-known count-loss correction procedures are not effective for overestimation although they suppress underestimation. Therefore, the pulsed neutron source method should be modified so as to have robustness against the count-loss effect.

KEYWORDS: pulsed neutron source method; count-loss effect; neutron decay constant; subcriticality; measurement; nuclear criticality safety

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I. Introduction

Measurement of subcriticality (i.e., negative reactivity) is one of the most important subjects in the field of reactor physics since it is closely related to the nuclear criticality safety. The pulsed neutron source method that can measure the subcriticality through determination of neutron decay constant is the most popular one[1] and has been applied to many subcritical reactor systems.[2–6] Experimental investigations based on pulsed neutron source method are still being pursued to meet a renewed necessity of subcriticality measurement (or monitoring) for accelerator-driven systems.[7–11] In future researches on criticality management of fuel debris generated in the severe accident of light water reactors,[12] it is expected that the pulsed neutron source method will play important roles.

Owing to a great amount of efforts devoted by many researchers,[13–19] the pulsed neutron source method is now regarded to be the most reliable and established one for measuring the subcriticality. Nevertheless, a further discussion on pulsed neutron source method is provided in the present paper since it is needed to investigate an unresolved issue, i.e., the count-loss effect in determination of neutron decay constant.

It is well-known that the radiation counting system (that consists of a radiation detector and instrumentation circuits) inevitably involves the count-loss process owing to the dead time, i.e., the minimum duration of time for separating a couple of detection events in radiation detector into two output pulse signals.[20] Therefore, one is not always able to extract accurate information on radiation filed of interest from the radiation counting system owing to the count-loss effect.

It is naturally expected that the count-loss effect also appears in the pulsed neutron source method that employs the neutron counting system (i.e., a neutron detector and instrumentation circuits). One hence often intends to eliminate the count-loss effect by introducing the well-known count-loss correction procedures[20] in determining the neutron decay constant. However, elimination of count-loss effect that is brought by these count-loss correction procedures has not been verified yet. Furthermore, the characteristics of count-loss effect in determination of neutron decay constant themselves have not been well understood yet. The goal of present paper is to investigate these two subjects on the basis of the neutron observation probability that is derived by explicitly considering the count-loss process in neutron counting system and the periodic burst of pulsed neutrons.

In the following section, derivation of neutron observation probability is provided. The count-loss effect in determination of neutron decay constant by pulsed neutron source method is then discussed in Section III. Verification of count-loss correction procedures for eliminating the count-loss effect is described in the same section. Finally, the conclusion is summarized in Section IV.

II. Theory of pulsed neutron source method with count-loss process

1. Degweker's formulation

It is well-known that there are two different models for count-loss process that correspond to the idealized responses of radiation counting system, i.e., the paralyzable and non-paralyzable ones.[20] In the present paper, the paralyzable count-loss process is assumed for simple discussion. We would like to note here that the real counting system involves a count-loss process that is not one of either of these two models but an intermediate one between them. However, the difference between these two models is small when the frequency of neutron capture events in neutron detector is low, so that this assumption brings few problems for this region.

In 1989, Degweker paid attention to the fact that the neutron counting system that involves the paralyzable count-loss process with dead time τ can generate an output pulse signal at time t when the neutron detector captures no neutrons within the preceding time interval $(t-\tau, t)$ as shown in **Figure 1** and developed a rigorous formulation approach for treating the count-loss process[21] on the basis of the backward master equation formalism.[22–25] According to Degweker, one derives the neutron observation probability within infinitesimal time interval dt around t (or the expectation value of number of neutrons observed within dt around t) that explicitly considers the paralyzable count-loss process in neutron counting system, i.e., $p(\tau, t)\,\mathrm{d}t$, as follows:

$$p(\tau, t)\,\mathrm{d}t = \sum_{Z_2=0}^{\infty} Z_2 \left[\lim_{T_2 \to \mathrm{d}t} \widetilde{P}(0, \tau, Z_2, T_2, t) \right]. \quad (1)$$

Here,

$$\widetilde{P}(Z_1, T_1, Z_2, T_2, t), \qquad \sum_{Z_1=0}^{\infty} \sum_{Z_2=0}^{\infty} \widetilde{P}(Z_1, T_1, Z_2, T_2, t) = 1, \quad (2)$$

is the source induced probability that Z_1 and Z_2 neutrons are captured by the neutron detector within the respective time intervals $(t-T_1-T_2, t-T_2)$ and $(t-T_2, t)$ that are mutually adjacent and non-overlapping under the condition that an extraneous neutron source is introduced at time 0. The timing diagram of $\widetilde{P}(Z_1, T_1, Z_2, T_2, t)$ is illustrated in **Figure 2**.

[Fig. 1 about here.]

[Fig. 2 about here.]

One understands from Equation (1) that $p(\tau, t)\,\mathrm{d}t$ is derived when the analytical expression of $\widetilde{P}\left(Z_1, T_1, Z_2, T_2, t\right)$ is obtained. In the Degweker's formulation, however, one obtains not $\widetilde{P}\left(Z_1, T_1, Z_2, T_2, t\right)$ but the following probability generating function (PGF) for simple mathematical manipulation, i.e.,

$$\widetilde{G}\left(x_1, T_1, x_2, T_2, t\right) \equiv \sum_{Z_1=0}^{\infty} \sum_{Z_2=0}^{\infty} x_1^{Z_1} x_2^{Z_2} \widetilde{P}\left(Z_1, T_1, Z_2, T_2, t\right). \tag{3}$$

Using this PGF, Equation (1) is re-written as follows:

$$p\left(\tau, t\right) \mathrm{d}t = \left\{ \frac{\partial}{\partial x_2} \left[\lim_{T_2 \to \mathrm{d}t} \widetilde{G}\left(x_1, \tau, x_2, T_2, t\right) \right] \right\} \Bigg|_{x_1=0,\, x_2=1}. \tag{4}$$

In the following subsections, to derive the neutron observation probability that explicitly considers the count-loss process in neutron counting system and the periodic burst of pulsed neutrons, the analytical expression of $\widetilde{G}\left(x_1, T_1, x_2, T_2, t\right)$ is obtained within the one-point reactor approximation model with one group of neutron energy and no delayed neutrons.

2. Source induced probability generating function

Let us suppose a zero-power subcritical reactor system that is coupled with a pulsed neutron source and introduce the following probability,

$$\widetilde{P}\left(Z_1, T_1, Z_2, T_2, t \,|\, t_0\right), \qquad \sum_{Z_1=0}^{\infty} \sum_{Z_2=0}^{\infty} \widetilde{P}\left(Z_1, T_1, Z_2, T_2, t \,|\, t_0\right) = 1. \tag{5}$$

This is the source induced probability that Z_1 and Z_2 neutrons are captured by the neutron detector within the respective time intervals $\left(t - T_1 - T_2, t - T_2\right)$ and $\left(t - T_2, t\right)$ under the condition that the pulsed neutron source is introduced into the subcritical reactor system at time t_0 ($\leq t$). The terminal condition of $\widetilde{P}\left(Z_1, T_1, Z_2, T_2, t \,|\, t_0\right)$ is read as

$$\widetilde{P}\left(Z_1, T_1, Z_2, T_2, t \,|\, t\right) = \delta_{Z_1,0} \delta_{Z_2,0}. \tag{6}$$

The corresponding PGF and its terminal condition are as follows:

$$\widetilde{G}\left(x_1, T_1, x_2, T_2, t \mid t_0\right) \equiv \sum_{Z_1=0}^{\infty} \sum_{Z_2=0}^{\infty} x_1^{Z_1} x_2^{Z_2} \widetilde{P}\left(Z_1, T_1, Z_2, T_2, t \mid t_0\right), \tag{7}$$

$$\widetilde{G}\left(x_1, T_1, x_2, T_2, t \mid t\right) = 1. \tag{8}$$

[Fig. 3 about here.]

The time dependence of pulsed neutron source intensity $S(t)$ is illustrated in **Figure 3** and is expressed as

$$S(t) \equiv S_{\mathrm{c}} + S_0 \sum_{j=0}^{\infty} f\left(t - jT_0\right), \qquad 0 \leq t, \tag{9}$$

$$f(t) \equiv H(t) - H(t - W), \tag{10}$$

where $H(t)$ is the Heaviside's step function, S_{c} and S_0 the respective intensity factors of constant and pulsed components, W the pulse width, and T_0 the pulse period ($0 \leq W < T_0$). In the present paper, to avoid complicated mathematical manipulation due to discontinuity of $S(t)$, the Fourier series expansion is alternatively utilized, i.e.,

$$S(t) = S_{\mathrm{DC}} + S_{\mathrm{MOD}}(t), \tag{11}$$

with the following notations,

$$S_{\mathrm{DC}} \equiv S_{\mathrm{c}} + \overline{S_0}, \tag{12}$$

$$\overline{S_0} \equiv \frac{S_0 W}{T_0}, \tag{13}$$

$$S_{\mathrm{MOD}}(t) \equiv \overline{S_0} \sum_{k=1}^{\infty} \left[\mathcal{C}_k \cos\left(\omega_k t\right) + \mathcal{S}_k \sin\left(\omega_k t\right)\right], \tag{14}$$

$$\omega_k \equiv \frac{2k\pi}{T_0}, \qquad k = 1, 2, \ldots, \tag{15}$$

$$\mathcal{C}_k \equiv \frac{2\sin\left(\omega_k W\right)}{\omega_k W}, \tag{16}$$

$$\mathcal{S}_k \equiv \frac{2\left[1-\cos\left(\omega_k W\right)\right]}{\omega_k W}. \tag{17}$$

Let us consider $\widetilde{P}\left(Z_1, T_1, Z_2, T_2, t \,|\, t_0 + \Delta t_0\right)$ where introduction of pulsed neutron source is delayed by Δt_0. Assuming that the time interval $\left(t_0, t_0 + \Delta t_0\right)$ is enough small to neglect any two-fold source events, one can establish the following probability balance equation:[24)]

$$\begin{aligned}
&\widetilde{P}\left(Z_1, T_1, Z_2, T_2, t \,|\, t_0\right) \\
&\quad = \left[1 - S\left(t_0\right)\Delta t_0\right]\widetilde{P}\left(Z_1, T_1, Z_2, T_2, t \,|\, t_0 + \Delta t_0\right) \\
&\qquad + S\left(t_0\right)\Delta t_0 \sum_{\zeta_1=0}^{Z_1}\sum_{\zeta_2=0}^{Z_2} \widetilde{P}\left(Z_1-\zeta_1, T_1, Z_2-\zeta_2, T_2, t \,|\, t_0+\Delta t_0\right) \\
&\qquad\quad \times P\left(\zeta_1, T_1, \zeta_2, T_2, t - t_0 - \Delta t_0\right),
\end{aligned} \tag{18}$$

where

$$P\left(Z_1, T_1, Z_2, T_2, t\right), \qquad \sum_{Z_1=0}^{\infty}\sum_{Z_2=0}^{\infty} P\left(Z_1, T_1, Z_2, T_2, t\right) = 1, \tag{19}$$

is the single-particle induced probability that Z_1 and Z_2 neutrons are captured by the neutron detector within the respective time intervals $\left(t - T_1 - T_2, t - T_2\right)$ and $\left(t - T_2, t\right)$ under the condition that an initial neutron is injected into the subcritical reactor system at time 0. The terminal condition of $P\left(Z_1, T_1, Z_2, T_2, t\right)$ is read as

$$P\left(Z_1, T_1, Z_2, T_2, 0\right) = \delta_{Z_1,0}\delta_{Z_2,0}. \tag{20}$$

Multiplying $x_1^{Z_1} x_2^{Z_2}$ then performing summation from zero to infinity with respect to Z_1 and Z_2, one obtains the following equation from Equation (18):

$$\begin{aligned}
\widetilde{G}\left(x_1, T_1, x_2, T_2, t \,|\, t_0\right) &= \left[1 - S\left(t_0\right)\Delta t_0\right]\widetilde{G}\left(x_1, T_1, x_2, T_2, t \,|\, t_0 + \Delta t_0\right) \\
&\quad + S\left(t_0\right)\Delta t_0\, \widetilde{G}\left(x_1, T_1, x_2, T_2, t \,|\, t_0 + \Delta t_0\right) \\
&\qquad \times G\left(x_1, T_1, x_2, T_2, t - t_0 - \Delta t_0\right),
\end{aligned} \tag{21}$$

where

$$G\left(x_1, T_1, x_2, T_2, t\right) \equiv \sum_{Z_1=0}^{\infty} \sum_{Z_2=0}^{\infty} x_1^{Z_1} x_2^{Z_2} P\left(Z_1, T_1, Z_2, T_2, t\right), \tag{22}$$

$$G\left(x_1, T_1, x_2, T_2, 0\right) = 1. \tag{23}$$

By letting $\Delta t_0 \to 0$ after some rearrangement, the following differential equation is obtained:

$$\begin{aligned} &\frac{\partial \widetilde{G}\left(x_1, T_1, x_2, T_2, t \mid t_0\right)}{\partial t_0} \\ &\qquad = S\left(t_0\right) \widetilde{G}\left(x_1, T_1, x_2, T_2, t \mid t_0\right) \left[1 - G\left(x_1, T_1, x_2, T_2, t - t_0\right)\right]. \end{aligned} \tag{24}$$

This equation can be solved with the terminal condition given in Equation (8), i.e.,

$$\widetilde{G}\left(x_1, T_1, x_2, T_2, t \mid t_0\right) = \exp\left\{\int_{t_0}^{t} \mathrm{d}t'\, S\left(t'\right) \left[G\left(x_1, T_1, x_2, T_2, t - t'\right) - 1\right]\right\}. \tag{25}$$

When setting $t_0 = 0$, the following equations,

$$\widetilde{G}\left(x_1, T_1, x_2, T_2, t\right) \equiv \widetilde{G}\left(x_1, T_1, x_2, T_2, t \mid 0\right) \equiv \exp\left\{\Gamma\left(x_1, T_1, x_2, T_2, t\right)\right\}, \tag{26}$$

$$\Gamma\left(x_1, T_1, x_2, T_2, t\right) \equiv \Gamma_{\mathrm{DC}}\left(x_1, T_1, x_2, T_2, t\right) + \Gamma_{\mathrm{MOD}}\left(x_1, T_1, x_2, T_2, t\right), \tag{27}$$

$$\Gamma_{\mathrm{DC}}\left(x_1, T_1, x_2, T_2, t\right) \equiv -S_{\mathrm{DC}} \int_0^t \mathrm{d}t'\, g\left(x_1, T_1, x_2, T_2, t - t'\right), \tag{28}$$

$$\Gamma_{\mathrm{MOD}}\left(x_1, T_1, x_2, T_2, t\right) \equiv -\int_0^t \mathrm{d}t'\, S_{\mathrm{MOD}}\left(t'\right) g\left(x_1, T_1, x_2, T_2, t - t'\right), \tag{29}$$

are obtained. Here, $g\left(x_1, T_1, x_2, T_2, t\right)$ is defined as

$$g\left(x_1, T_1, x_2, T_2, t\right) \equiv 1 - G\left(x_1, T_1, x_2, T_2, t\right), \tag{30}$$

$$g\left(x_1, T_1, x_2, T_2, 0\right) = 0. \tag{31}$$

In the following subsection, the analytical expression of $g\left(x_1, T_1, x_2, T_2, t\right)$ is obtained by considering the elementary processes of neutron interactions in subcritical reactor system.

3. Single-particle induced probability generating function

The neutron interactions in subcritical reactor system are described by

$$\lambda_x \equiv v\Sigma_x, \qquad x = \mathrm{c}, \mathrm{f}, \mathrm{d}, \tag{32}$$

where λ_c is the probability that one neutron is captured per unit time by materials excluding neutron detector in subcritical reactor system, λ_f the probability that one neutron undergoes a fission reaction per unit time, λ_d the probability that one neutron is captured per unit time by the neutron detector, v the velocity of neutrons, and Σ_x the macroscopic cross-sections of corresponding interaction types $x = \mathrm{c}, \mathrm{f}, \mathrm{d}$.

The probability that n neutrons are generated per fission reaction is denoted by

$$p\left(n\right), \qquad \sum_{n=0}^{\infty} p\left(n\right) = 1, \tag{33}$$

and the r-th order factorial moment of number of neutrons generated per fission reaction is defined as follows:

$$\left\langle \nu\left(\nu-1\right)\left(\nu-2\right)\cdots\left(\nu-r+1\right)\right\rangle \equiv \sum_{n=0}^{\infty} n\left(n-1\right)\left(n-2\right)\cdots\left(n-r+1\right)p\left(n\right). \tag{34}$$

Here, let us consider $P\left(Z_1, T_1, Z_2, T_2, t-\Delta t\right)$ where injection of initial neutron is delayed by Δt. Neglecting any two-fold neutron interactions within infinitesimal time interval $(0, \Delta t)$, the following probability balance equation can be established:[25)]

$$
\begin{aligned}
& P\left(Z_1, T_1, Z_2, T_2, t\right) \\
& \quad = \left[1 - \left(\lambda_{\mathrm{c}} + \lambda_{\mathrm{f}} + \lambda_{\mathrm{d}}\right) \Delta t\right] P\left(Z_1, T_1, Z_2, T_2, t - \Delta t\right) + \lambda_{\mathrm{c}} \Delta t\, \delta_{Z_1,0} \delta_{Z_2,0} \\
& \qquad + \lambda_{\mathrm{f}} \Delta t \sum_{n=0}^{\infty} p\left(n\right) R_n\left(Z_1, T_1, Z_2, T_2, t - \Delta t\right) \\
& \qquad + \lambda_{\mathrm{d}} \Delta t \left[\delta_{Z_1,0} \delta_{Z_2,0} \Delta_0\left(t\right) + \delta_{Z_1,1} \delta_{Z_2,0} \Delta_1\left(t\right) + \delta_{Z_1,0} \delta_{Z_2,1} \Delta_2\left(t\right)\right].
\end{aligned}
\tag{35}
$$

The respective terms in right hand side correspond to mutually exclusive neutron interactions within $(0, \Delta t)$. Here, $R_n\left(Z_1, T_1, Z_2, T_2, t\right)$ is the multi-particle induced probability that Z_1 and Z_2 neutrons are captured by the neutron detector within the respective time intervals $(t - T_1 - T_2, t - T_2)$ and $(t - T_2, t)$ under the condition that n neutrons are simultaneously injected at time 0. This probability can be easily constructed with $P\left(Z_1, T_1, Z_2, T_2, t\right)$. For example, $R_n\left(Z_1, T_1, Z_2, T_2, t\right)$ with respect to $n = 0, 1, 2, 3$ are written as follows:

$$
R_0\left(Z_1, T_1, Z_2, T_2, t\right) = \delta_{Z_1,0} \delta_{Z_2,0}, \tag{36}
$$

$$
R_1\left(Z_1, T_1, Z_2, T_2, t\right) = P\left(Z_1, T_1, Z_2, T_2, t\right), \tag{37}
$$

$$
\begin{aligned}
& R_2\left(Z_1, T_1, Z_2, T_2, t\right) \\
& \quad = \sum_{Z_1'=0}^{Z_1} \sum_{Z_2'=0}^{Z_2} P\left(Z_1 - Z_1', T_1, Z_2 - Z_2', T_2, t\right) P\left(Z_1', T_1, Z_2', T_2, t\right),
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
& R_3\left(Z_1, T_1, Z_2, T_2, t\right) \\
& \quad = \sum_{Z_1'=0}^{Z_1} \sum_{Z_2'=0}^{Z_2} \sum_{Z_1''=0}^{Z_1'} \sum_{Z_2''=0}^{Z_2'} P\left(Z_1 - Z_1', T_1, Z_2 - Z_2', T_2, t\right) \\
& \qquad \times P\left(Z_1' - Z_1'', T_1, Z_2' - Z_2'', T_2, t\right) P\left(Z_1'', T_1, Z_2'', T_2, t\right).
\end{aligned}
\tag{39}
$$

On the other hand, by using the Heaviside's step function $H\left(t\right)$, $\Delta_1\left(t\right)$ and $\Delta_2\left(t\right)$ are defined as

$$\Delta_1(t) \equiv H(t - T_2) - H(t - T_1 - T_2), \tag{40}$$

$$\Delta_2(t) \equiv H(t) - H(t - T_2). \tag{41}$$

These functions are introduced to describe the status of counting gates by taking zero when closed or unity when opened. Furthermore, $\Delta_0(t)$ is defined as follows:

$$\Delta_0(t) \equiv 1 - \Delta_1(t) - \Delta_2(t). \tag{42}$$

Multiplying $x_1^{Z_1} x_2^{Z_2}$ then performing summation from zero to infinity with respect to Z_1 and Z_2, one obtains the following equation from Equation (35):

$$\begin{aligned} G(x_1, T_1, x_2, T_2, t) = {} & [1 - (\lambda_\mathrm{c} + \lambda_\mathrm{f} + \lambda_\mathrm{d}) \Delta t] \, G(x_1, T_1, x_2, T_2, t - \Delta t) \\ & + \lambda_\mathrm{c} \Delta t + \lambda_\mathrm{f} \Delta t \sum_{n=0}^{\infty} p(n) \, G^n(x_1, T_1, x_2, T_2, t - \Delta t) \\ & + \lambda_\mathrm{d} \Delta t \, [\Delta_0(t) + x_1 \Delta_1(t) + x_2 \Delta_2(t)]. \end{aligned} \tag{43}$$

Hence, the following backward master equation is obtained by making some rearrangement then letting $\Delta t \to 0$:

$$\begin{aligned} \frac{\partial G(x_1, T_1, x_2, T_2, t)}{\partial t} = {} & -(\lambda_\mathrm{c} + \lambda_\mathrm{f} + \lambda_\mathrm{d}) \, G(x_1, T_1, x_2, T_2, t) \\ & + (\lambda_\mathrm{c} + \lambda_\mathrm{d}) + \lambda_\mathrm{f} \sum_{n=0}^{\infty} p(n) \, G^n(x_1, T_1, x_2, T_2, t) \\ & - \lambda_\mathrm{d} \, [(1 - x_1) \Delta_1(t) + (1 - x_2) \Delta_2(t)]. \end{aligned} \tag{44}$$

Rewriting Equation (44) by using $g(x_1, T_1, x_2, T_2, t)$, the Pál–Bell equation[22,23] of two-interval version is obtained as

$$\begin{aligned}\frac{\partial g\left(x_1,T_1,x_2,T_2,t\right)}{\partial t}\\ &= -\alpha g\left(x_1,T_1,x_2,T_2,t\right)\\ &\quad -\lambda_{\mathrm{f}}\sum_{r=2}^{\infty}\frac{(-1)^r}{r!}\left\langle \nu\left(\nu-1\right)\left(\nu-2\right)\cdots\left(\nu-r+1\right)\right\rangle g^r\left(x_1,T_1,x_2,T_2,t\right)\\ &\quad +\lambda_{\mathrm{d}}\left[\left(1-x_1\right)\Delta_1\left(t\right)+\left(1-x_2\right)\Delta_2\left(t\right)\right],\end{aligned} \tag{45}$$

where α is the neutron decay constant that is defined as

$$\alpha \equiv \left(\lambda_{\mathrm{c}}+\lambda_{\mathrm{f}}+\lambda_{\mathrm{d}}\right)-\lambda_{\mathrm{f}}\left\langle \nu\right\rangle . \tag{46}$$

To solve the Pál–Bell equation of two-interval version, the two-forked approximation that neglects the higher-order components than third of second term in right hand side of Equation (45) is introduced, i.e.,

$$\begin{aligned}\frac{\partial g\left(x_1,T_1,x_2,T_2,t\right)}{\partial t} &= -\alpha g\left(x_1,T_1,x_2,T_2,t\right)-\frac{\lambda_{\mathrm{f}}\left\langle \nu\left(\nu-1\right)\right\rangle}{2}g^2\left(x_1,T_1,x_2,T_2,t\right)\\ &\quad +\lambda_{\mathrm{d}}\left[\left(1-x_1\right)\Delta_1\left(t\right)+\left(1-x_2\right)\Delta_2\left(t\right)\right].\end{aligned} \tag{47}$$

Regarding the two-forked approximation, Furuhashi and Izumi pointed out that the contributions from higher-order components than third are negligible except in reactor systems with far large subcriticalities.[26,27]

To solve Equation (47) efficiently, let us divide $g\left(x_1,T_1,x_2,T_2,t\right)$ into three regions as

$$g\left(x_1,T_1,x_2,T_2,t\right)=\begin{cases} g_0\left(x_1,T_1,x_2,T_2,t\right), & T_1+T_2\le t,\\ g_1\left(x_1,T_1,x_2,T_2,t\right), & T_2\le t<T_1+T_2,\\ g_2\left(x_1,T_1,x_2,T_2,t\right), & 0\le t<T_2,\end{cases} \tag{48}$$

$$\frac{\partial g_m\left(x_1, T_1, x_2, T_2, t\right)}{\partial t}$$
$$= -\alpha g_m\left(x_1, T_1, x_2, T_2, t\right) - \frac{\lambda_\mathrm{f}\left\langle \nu\left(\nu-1\right)\right\rangle}{2} g_m^2\left(x_1, T_1, x_2, T_2, t\right) \tag{49}$$
$$+ \lambda_\mathrm{d}\left(1 - x_m\right), \qquad m = 0, 1, 2,$$

$$x_0 \equiv 1, \tag{50}$$

with the following boundary conditions,

$$g_0\left(x_1, T_1, x_2, T_2, T_1 + T_2\right) = g_1\left(x_1, T_1, x_2, T_2, T_1 + T_2\right), \tag{51}$$

$$g_1\left(x_1, T_1, x_2, T_2, T_2\right) = g_2\left(x_1, T_1, x_2, T_2, T_2\right), \tag{52}$$

$$g_2\left(x_1, T_1, x_2, T_2, 0\right) = 0. \tag{53}$$

Although Equation (49) is non-linear, it is of the Ricatti-type that can be solved analytically. One hence obtains the solution as follows:

$$g_m\left(x_1, T_1, x_2, T_2, t\right) = -\frac{\lambda_\mathrm{d}}{\alpha Y_\infty}\left[1 + \gamma_m\left(1 - \frac{2}{1 + \eta_m \mathrm{e}^{-\alpha\gamma_m t}}\right)\right], \tag{54}$$

$$\gamma_m \equiv \sqrt{1 + 2Y_\infty\left(1 - x_m\right)}, \tag{55}$$

$$Y_\infty \equiv \frac{\lambda_\mathrm{d}\lambda_\mathrm{f}\left\langle \nu\left(\nu-1\right)\right\rangle}{\alpha^2}, \tag{56}$$

$$\eta_0 = \frac{\left(\gamma_0 - \gamma_1\right) + \left(\gamma_0 + \gamma_1\right)\eta_1 \mathrm{e}^{-\alpha\gamma_1\left(T_1 + T_2\right)}}{\left(\gamma_0 + \gamma_1\right) + \left(\gamma_0 - \gamma_1\right)\eta_1 \mathrm{e}^{-\alpha\gamma_1\left(T_1 + T_2\right)}} \frac{1}{\mathrm{e}^{-\alpha\gamma_0\left(T_1 + T_2\right)}}, \tag{57}$$

$$\eta_1 = \frac{\left(\gamma_1 - \gamma_2\right) + \left(\gamma_1 + \gamma_2\right)\eta_2 \mathrm{e}^{-\alpha\gamma_2 T_2}}{\left(\gamma_1 + \gamma_2\right) + \left(\gamma_1 - \gamma_2\right)\eta_2 \mathrm{e}^{-\alpha\gamma_2 T_2}} \frac{1}{\mathrm{e}^{-\alpha\gamma_1 T_2}}, \tag{58}$$

$$\eta_2 = \frac{\gamma_2 - 1}{\gamma_2 + 1}. \tag{59}$$

4. Neutron observation probability

When $T_1 + T_2 < t$, Equations (28) and (29) are calculated as

$$\begin{aligned}\Gamma_{\mathrm{DC}}\left(x_1, T_1, x_2, T_2, t\right) = & -S_{\mathrm{DC}} \int_0^{t-T_1-T_2} \mathrm{d}t'\, g_0\left(x_1, T_1, x_2, T_2, t-t'\right) \\ & - S_{\mathrm{DC}} \int_{t-T_1-T_2}^{t-T_2} \mathrm{d}t'\, g_1\left(x_1, T_1, x_2, T_2, t-t'\right) \\ & - S_{\mathrm{DC}} \int_{t-T_2}^{t} \mathrm{d}t'\, g_2\left(x_1, T_1, x_2, T_2, t-t'\right),\end{aligned} \tag{60}$$

$$\begin{aligned}\Gamma_{\mathrm{MOD}}\left(x_1, T_1, x_2, T_2, t\right) = & -\int_0^{t-T_1-T_2} \mathrm{d}t'\, S_{\mathrm{MOD}}\left(t'\right) g_0\left(x_1, T_1, x_2, T_2, t-t'\right) \\ & - \int_{t-T_1-T_2}^{t-T_2} \mathrm{d}t'\, S_{\mathrm{MOD}}\left(t'\right) g_1\left(x_1, T_1, x_2, T_2, t-t'\right) \\ & - \int_{t-T_2}^{t} \mathrm{d}t'\, S_{\mathrm{MOD}}\left(t'\right) g_2\left(x_1, T_1, x_2, T_2, t-t'\right).\end{aligned} \tag{61}$$

Hence, by using a function $\theta_k(u, t)$ that is defined as

$$\theta_k(u, t) \equiv \frac{\left(u\mathcal{C}_k - \omega_k \mathcal{S}_k\right)\cos\left(\omega_k t\right) + \left(\omega_k \mathcal{C}_k + u\mathcal{S}_k\right)\sin\left(\omega_k t\right)}{u^2 + \omega_k^2}, \tag{62}$$

the concrete expressions of $\Gamma_{\mathrm{DC}}\left(x_1, T_1, x_2, T_2, t\right)$ and $\Gamma_{\mathrm{MOD}}\left(x_1, T_1, x_2, T_2, t\right)$ for $T_1 + T_2 < t$ are obtained as follows:

$$\begin{aligned}&\Gamma_{\mathrm{DC}}\left(x_1, T_1, x_2, T_2, t\right) \\ &= -\frac{2\lambda_{\mathrm{d}}\left(S_{\mathrm{c}} + \overline{S_0}\right)}{\alpha^2 Y_\infty} \ln\left[\begin{array}{l}\dfrac{1+\eta_0 \mathrm{e}^{-\alpha t}}{1+\eta_0 \mathrm{e}^{-\alpha\left(T_1+T_2\right)}} \\ \qquad \times \dfrac{1+\eta_1 \mathrm{e}^{-\alpha\gamma_1\left(T_1+T_2\right)}}{1+\eta_1 \mathrm{e}^{-\alpha\gamma_1 T_2}} \mathrm{e}^{\frac{1}{2}\alpha\left(\gamma_1-1\right)T_1} \\ \qquad \times \dfrac{1+\eta_2 \mathrm{e}^{-\alpha\gamma_2 T_2}}{1+\eta_2} \mathrm{e}^{\frac{1}{2}\alpha\left(\gamma_2-1\right)T_2}\end{array}\right],\end{aligned} \tag{63}$$

$$
\begin{aligned}
&\Gamma_{\mathrm{MOD}}\left(x_1, T_1, x_2, T_2, t\right) \\
&= -\frac{2\lambda_{\mathrm{d}}\overline{S_0}}{\alpha Y_\infty}\sum_{k=1}^{\infty}
\left[
\begin{array}{l}
\frac{1}{2}\left(\gamma_1 - 1\right)\left[\theta_k\left(0, t - T_2\right) - \theta_k\left(0, t - T_1 - T_2\right)\right] \\
\quad + \frac{1}{2}\left(\gamma_2 - 1\right)\left[\theta_k\left(0, t\right) - \theta_k\left(0, t - T_2\right)\right] \\
\quad + \sum_{\xi=1}^{\infty}\left\{
\begin{array}{l}
\left[-\eta_0 \mathrm{e}^{-\alpha\left(T_1 + T_2\right)}\right]^{\xi} \\
\qquad \times \theta_k\left(\alpha\xi, t - T_1 - T_2\right) \\
\quad - \left(-\eta_0 \mathrm{e}^{-\alpha t}\right)^{\xi}\theta_k\left(\alpha\xi, 0\right)
\end{array}
\right\} \\
\quad + \gamma_1\sum_{\xi=1}^{\infty}\left\{
\begin{array}{l}
\left(-\eta_1 \mathrm{e}^{-\alpha\gamma_1 T_2}\right)^{\xi}\theta_k\left(\alpha\gamma_1\xi, t - T_2\right) \\
\quad - \left[-\eta_1 \mathrm{e}^{-\alpha\gamma_1\left(T_1 + T_2\right)}\right]^{\xi} \\
\qquad \times \theta_k\left(\alpha\gamma_1\xi, t - T_1 - T_2\right)
\end{array}
\right\} \\
\quad + \gamma_2\sum_{\xi=1}^{\infty}\left[
\begin{array}{l}
\left(-\eta_2\right)^{\xi}\theta_k\left(\alpha\gamma_2\xi, t\right) \\
\quad - \left(-\eta_2 \mathrm{e}^{-\alpha\gamma_2 T_2}\right)^{\xi} \\
\qquad \times \theta_k\left(\alpha\gamma_2\xi, t - T_2\right)
\end{array}
\right]
\end{array}
\right].
\end{aligned}
\tag{64}
$$

Therefore, by substituting Equations (63) and (64) into Equations (26) and (27), one obtains the analytical expression of $\widetilde{G}\left(x_1, T_1, x_2, T_2, t\right)$ for $T_1 + T_2 < t$.

[Fig. 4 about here.]

Here, some arrangement with respect to $\widetilde{G}\left(x_1, T_1, x_2, T_2, t\right)$ is made to take actual experimental conditions of pulsed neutron source method into account. First, in actual experiments, the neutron counting rate is measured as a function of time after periodic burst of pulsed neutrons. Let us hence introduce a new parameter Δ that is the time after j-th burst of pulsed neutrons and set $t - T_2 = jT_0 + \Delta$ as shown in **Figure 4**. Second, in actual experiments, measurement is started when the steady state is realized by the pulsed neutron source after a long waiting time. Therefore, let us take limitation $j \to \infty$ as follows:

$$\begin{aligned}\widetilde{G}\left(x_1, T_1, x_2, T_2, \Delta\right) &\equiv \lim_{j\to\infty} \widetilde{G}\left(x_1, T_1, x_2, T_2, jT_0 + \Delta + T_2\right) \\ &\equiv \exp\left\{\Gamma_{\mathrm{DC}}\left(x_1, T_1, x_2, T_2, \Delta\right) + \Gamma_{\mathrm{MOD}}\left(x_1, T_1, x_2, T_2, \Delta\right)\right\},\end{aligned} \tag{65}$$

where

$$\begin{aligned}&\Gamma_{\mathrm{DC}}\left(x_1, T_1, x_2, T_2, \Delta\right) \\ &\quad\equiv \lim_{j\to\infty} \Gamma_{\mathrm{DC}}\left(x_1, T_1, x_2, T_2, jT_0 + \Delta + T_2\right) \\ &\quad= -\frac{2\lambda_{\mathrm{d}}\left(S_{\mathrm{c}} + \overline{S_0}\right)}{\alpha^2 Y_\infty} \ln\left[\begin{array}{r}\dfrac{1}{1 + \eta_0 \mathrm{e}^{-\alpha(T_1+T_2)}} \\ \times \dfrac{1 + \eta_1 \mathrm{e}^{-\alpha\gamma_1(T_1+T_2)}}{1 + \eta_1 \mathrm{e}^{-\alpha\gamma_1 T_2}} \mathrm{e}^{\frac{1}{2}\alpha(\gamma_1 - 1)T_1} \\ \times \dfrac{1 + \eta_2 \mathrm{e}^{-\alpha\gamma_2 T_2}}{1 + \eta_2} \mathrm{e}^{\frac{1}{2}\alpha(\gamma_2 - 1)T_2}\end{array}\right],\end{aligned} \tag{66}$$

$$
\begin{aligned}
&\Gamma_{\mathrm{MOD}}\left(x_1, T_1, x_2, T_2, \Delta\right) \\
&\quad \equiv \lim_{j \to \infty} \Gamma_{\mathrm{MOD}}\left(x_1, T_1, x_2, T_2, jT_0 + \Delta + T_2\right) \\
&\quad = -\frac{2\lambda_{\mathrm{d}}\overline{S_0}}{\alpha Y_\infty} \sum_{k=1}^{\infty}
\left[\begin{array}{l}
\frac{1}{2}\left(\gamma_1 - 1\right)\left[\theta_k\left(0, \Delta\right) - \theta_k\left(0, \Delta - T_1\right)\right] \\
\quad + \frac{1}{2}\left(\gamma_2 - 1\right)\left[\theta_k\left(0, \Delta + T_2\right) - \theta_k\left(0, \Delta\right)\right] \\
\quad + \sum_{\xi=1}^{\infty}\left[-\eta_0 \mathrm{e}^{-\alpha\left(T_1 + T_2\right)}\right]^{\xi} \theta_k\left(\alpha\xi, \Delta - T_1\right) \\
\quad + \gamma_1 \sum_{\xi=1}^{\infty}\left\{\begin{array}{c}
\left(-\eta_1 \mathrm{e}^{-\alpha\gamma_1 T_2}\right)^{\xi} \theta_k\left(\alpha\gamma_1\xi, \Delta\right) \\
-\left[-\eta_1 \mathrm{e}^{-\alpha\gamma_1\left(T_1 + T_2\right)}\right]^{\xi} \\
\times \theta_k\left(\alpha\gamma_1\xi, \Delta - T_1\right)
\end{array}\right\} \\
\quad + \gamma_2 \sum_{\xi=1}^{\infty}\left[\begin{array}{c}
\left(-\eta_2\right)^{\xi} \theta_k\left(\alpha\gamma_2\xi, \Delta + T_2\right) \\
-\left(-\eta_2 \mathrm{e}^{-\alpha\gamma_2 T_2}\right)^{\xi} \\
\times \theta_k\left(\alpha\gamma_2\xi, \Delta\right)
\end{array}\right]
\end{array}\right].
\end{aligned}
\tag{67}
$$

As seen in Equation (4), the neutron observation probability that explicitly considers the count-loss process in neutron counting system and the periodic burst of pulsed neutrons, i.e., $p\left(\tau, \Delta\right) \mathrm{d}\Delta$, is derived by using $\widetilde{G}\left(x_1, T_1, x_2, T_2, \Delta\right)$ as

$$
p\left(\tau, \Delta\right) \mathrm{d}\Delta = \left.\left\{\frac{\partial}{\partial x_2}\left[\lim_{T_2 \to \mathrm{d}\Delta} \widetilde{G}\left(x_1, \tau, x_2, T_2, \Delta\right)\right]\right\}\right|_{x_1 = 0,\, x_2 = 1}. \tag{68}
$$

Let us hence calculate the $T_2 \to \mathrm{d}\Delta$ limitation of $\widetilde{G}\left(x_1, T_1, x_2, T_2, \Delta\right)$ as the Maclaurin expansion with elimination of higher-order terms of $\mathrm{d}\Delta$ as

$$\begin{aligned}\lim_{T_2 \to \mathrm{d}\Delta} \widetilde{G}\left(x_1, \tau, x_2, T_2, \Delta\right) &\simeq \widetilde{G}\left(x_1, \tau, x_2, 0, \Delta\right) + \left.\frac{\partial \widetilde{G}\left(x_1, \tau, x_2, T_2, \Delta\right)}{\partial T_2}\right|_{T_2=0} \mathrm{d}\Delta \\ &= \widetilde{G}\left(x_1, \tau, x_2, 0, \Delta\right)\left[1 + \left.\frac{\partial \Gamma\left(x_1, \tau, x_2, T_2, \Delta\right)}{\partial T_2}\right|_{T_2=0} \mathrm{d}\Delta\right] \\ &\equiv \widetilde{G}\left(x_1, \tau, x_2, \Delta\right),\end{aligned} \tag{69}$$

where

$$\widetilde{G}\left(x_1, \tau, x_2, 0, \Delta\right) \equiv \exp\left\{\Gamma_{\mathrm{DC}}\left(x_1, \tau, x_2, 0, \Delta\right) + \Gamma_{\mathrm{MOD}}\left(x_1, \tau, x_2, 0, \Delta\right)\right\}, \tag{70}$$

$$\begin{aligned}&\left.\frac{\partial \Gamma\left(x_1, \tau, x_2, T_2, \Delta\right)}{\partial T_2}\right|_{T_2=0} \\ &\quad \equiv \left.\frac{\partial \Gamma_{\mathrm{DC}}\left(x_1, \tau, x_2, T_2, \Delta\right)}{\partial T_2}\right|_{T_2=0} + \left.\frac{\partial \Gamma_{\mathrm{MOD}}\left(x_1, \tau, x_2, T_2, \Delta\right)}{\partial T_2}\right|_{T_2=0}.\end{aligned} \tag{71}$$

Their concrete expressions are written as follows:

$$\begin{aligned}&\Gamma_{\mathrm{DC}}\left(x_1, \tau, x_2, 0, \Delta\right) \\ &\quad = -\frac{2\lambda_{\mathrm{d}}\left(S_{\mathrm{c}} + \overline{S_0}\right)}{\alpha^2 Y_\infty} \ln\left[\frac{\left(\gamma_1 + 1\right)^2}{4\gamma_1} \mathrm{e}^{\frac{1}{2}\alpha\left(\gamma_1 - 1\right)\tau} - \frac{\left(\gamma_1 - 1\right)^2}{4\gamma_1} \mathrm{e}^{-\frac{1}{2}\alpha\left(\gamma_1 + 1\right)\tau}\right],\end{aligned} \tag{72}$$

$$\begin{aligned}&\Gamma_{\mathrm{MOD}}\left(x_1, \tau, x_2, 0, \Delta\right) \\ &\quad = -\frac{2\lambda_{\mathrm{d}}\overline{S_0}}{\alpha Y_\infty} \sum_{k=1}^{\infty} \left\{\begin{aligned}&\frac{1}{2}\left(\gamma_1 - 1\right)\left[\theta_k\left(0, \Delta\right) - \theta_k\left(0, \Delta - \tau\right)\right] \\ &\quad + \sum_{\xi=1}^{\infty}\left[\frac{\left(\gamma_1^2 - 1\right)\left(1 - \mathrm{e}^{-\alpha\gamma_1\tau}\right)}{\left(\gamma_1 + 1\right)^2 - \left(\gamma_1 - 1\right)^2 \mathrm{e}^{-\alpha\gamma_1\tau}}\right]^{\xi} \\ &\qquad \times \theta_k\left(\alpha\xi, \Delta - \tau\right) \\ &\quad + \gamma_1 \sum_{\xi=1}^{\infty}\left(-\frac{\gamma_1 - 1}{\gamma_1 + 1}\right)^{\xi} \theta_k\left(\alpha\gamma_1\xi, \Delta\right) \\ &\quad - \gamma_1 \sum_{\xi=1}^{\infty}\left(-\frac{\gamma_1 - 1}{\gamma_1 + 1}\mathrm{e}^{-\alpha\gamma_1\tau}\right)^{\xi} \theta_k\left(\alpha\gamma_1\xi, \Delta - \tau\right)\end{aligned}\right\},\end{aligned} \tag{73}$$

$$\left.\frac{\Gamma_{\mathrm{DC}}\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{T_2=0} = -\frac{\lambda_{\mathrm{d}}\left(S_{\mathrm{c}}+\overline{S_0}\right)}{\alpha Y_\infty}\left(\gamma_2^2-1\right)\frac{\left(\gamma_1+1\right)+\left(\gamma_1-1\right)\mathrm{e}^{-\alpha\gamma_1\tau}}{\left(\gamma_1+1\right)^2-\left(\gamma_1-1\right)^2\mathrm{e}^{-\alpha\gamma_1\tau}}, \tag{74}$$

$$\begin{aligned}
&\left.\frac{\Gamma_{\mathrm{MOD}}\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{T_2=0} \\
&= -\frac{2\lambda_{\mathrm{d}}\overline{S_0}}{Y_\infty}\left(\gamma_2^2-1\right) \\
&\times\sum_{k=1}^{\infty}\sum_{\xi=1}^{\infty}\left\{\begin{aligned}
&\frac{4\gamma_1^2\xi\mathrm{e}^{-\alpha\gamma_1\tau}}{\left[\left(\gamma_1+1\right)^2-\left(\gamma_1-1\right)^2\mathrm{e}^{-\alpha\gamma_1\tau}\right]^2} \\
&\qquad\times\left[\frac{\left(\gamma_1^2-1\right)\left(1-\mathrm{e}^{-\alpha\gamma_1\tau}\right)}{\left(\gamma_1+1\right)^2-\left(\gamma_1-1\right)^2\mathrm{e}^{-\alpha\gamma_1\tau}}\right]^{\xi-1} \\
&\qquad\times\theta_k\left(\alpha\xi,\Delta-\tau\right) \\
&+\frac{\gamma_1^2\xi}{\left(\gamma_1+1\right)^2}\left(-\frac{\gamma_1-1}{\gamma_1+1}\right)^{\xi-1}\theta_k\left(\alpha\gamma_1\xi,\Delta\right) \\
&-\frac{\gamma_1^2\xi\mathrm{e}^{-\alpha\gamma_1\tau}}{\left(\gamma_1+1\right)^2}\left(-\frac{\gamma_1-1}{\gamma_1+1}\mathrm{e}^{-\alpha\gamma_1\tau}\right)^{\xi-1} \\
&\qquad\times\theta_k\left(\alpha\gamma_1\xi,\Delta-\tau\right)
\end{aligned}\right\}.
\end{aligned} \tag{75}$$

The analytical expression of $p\left(\tau,\Delta\right)\mathrm{d}\Delta$ can now be derived from Equation (68) as

$$\begin{aligned}
&p\left(\tau,\Delta\right)\mathrm{d}\Delta \\
&= \left.\frac{\partial\widetilde{G}\left(x_1,\tau,x_2,\Delta\right)}{\partial x_2}\right|_{x_1=0,\,x_2=1} \\
&= \widetilde{G}\left(0,\tau,1,0,\Delta\right) \\
&\times\left[\begin{aligned}
&\left.\frac{\partial\Gamma\left(x_1,\tau,x_2,0,\Delta\right)}{\partial x_2}\right|_{x_1=0,\,x_2=1} \\
&\qquad\times\left[1+\left.\frac{\partial\Gamma\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{x_1=0,\,x_2=1,\,T_2=0}\mathrm{d}\Delta\right] \\
&+\left.\left\{\frac{\partial}{\partial x_2}\left[\left.\frac{\partial\Gamma\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{T_2=0}\right]\right\}\right|_{x_1=0,\,x_2=1}\mathrm{d}\Delta
\end{aligned}\right],
\end{aligned} \tag{76}$$

where

$$\widetilde{G}\left(0,\tau,1,0,\Delta\right) \equiv \exp\left\{\Gamma_{\mathrm{DC}}\left(0,\tau,1,0,\Delta\right) + \Gamma_{\mathrm{MOD}}\left(0,\tau,1,0,\Delta\right)\right\}, \tag{77}$$

$$\begin{aligned} &\left.\frac{\partial\Gamma\left(x_1,\tau,x_2,0,\Delta\right)}{\partial x_2}\right|_{x_1=0,\,x_2=1} \\ &\quad\equiv \left.\frac{\partial\Gamma_{\mathrm{DC}}\left(x_1,\tau,x_2,0,\Delta\right)}{\partial x_2}\right|_{x_1=0,\,x_2=1} + \left.\frac{\partial\Gamma_{\mathrm{MOD}}\left(x_1,\tau,x_2,0,\Delta\right)}{\partial x_2}\right|_{x_1=0,\,x_2=1}, \end{aligned} \tag{78}$$

$$\begin{aligned} &\left.\frac{\partial\Gamma\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{x_1=0,\,x_2=1,\,T_2=0} \\ &\quad\equiv \left.\frac{\partial\Gamma_{\mathrm{DC}}\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{x_1=0,\,x_2=1,\,T_2=0} \\ &\qquad + \left.\frac{\partial\Gamma_{\mathrm{MOD}}\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{x_1=0,\,x_2=1,\,T_2=0}, \end{aligned} \tag{79}$$

$$\begin{aligned} &\left.\left\{\frac{\partial}{\partial x_2}\left[\left.\frac{\partial\Gamma\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{T_2=0}\right]\right\}\right|_{x_1=0,\,x_2=1} \\ &\quad\equiv \left.\left\{\frac{\partial}{\partial x_2}\left[\left.\frac{\partial\Gamma_{\mathrm{DC}}\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{T_2=0}\right]\right\}\right|_{x_1=0,\,x_2=1} \\ &\qquad + \left.\left\{\frac{\partial}{\partial x_2}\left[\left.\frac{\partial\Gamma_{\mathrm{MOD}}\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{T_2=0}\right]\right\}\right|_{x_1=0,\,x_2=1}. \end{aligned} \tag{80}$$

Using the following notation,

$$\gamma = \sqrt{1+2Y_\infty}, \tag{81}$$

their concrete expressions are obtained as follows:

$$\begin{aligned} &\Gamma_{\mathrm{DC}}\left(0,\tau,1,0,\Delta\right) \\ &\quad= -\frac{2\lambda_{\mathrm{d}}\left(S_{\mathrm{c}}+\overline{S_0}\right)}{\alpha^2 Y_\infty}\ln\left[\frac{(\gamma+1)^2}{4\gamma}\mathrm{e}^{\frac{1}{2}\alpha(\gamma-1)\tau} - \frac{(\gamma-1)^2}{4\gamma}\mathrm{e}^{-\frac{1}{2}\alpha(\gamma+1)\tau}\right], \end{aligned} \tag{82}$$

$$
\begin{aligned}
&\Gamma_{\mathrm{MOD}}\left(0, \tau, 1, 0, \Delta\right) \\
&\quad = -\frac{2\lambda_{\mathrm{d}}\overline{S_0}}{\alpha Y_\infty} \sum_{k=1}^{\infty} \left\{ \begin{aligned}
&\frac{1}{2}\left(\gamma-1\right)\left[\theta_k\left(0,\Delta\right)-\theta_k\left(0,\Delta-\tau\right)\right] \\
&\qquad + \sum_{\xi=1}^{\infty}\left[\frac{\left(\gamma^2-1\right)\left(1-\mathrm{e}^{-\alpha\gamma\tau}\right)}{\left(\gamma+1\right)^2-\left(\gamma-1\right)^2\mathrm{e}^{-\alpha\gamma\tau}}\right]^{\xi} \\
&\qquad\qquad \times \theta_k\left(\alpha\xi, \Delta-\tau\right) \\
&\qquad + \gamma\sum_{\xi=1}^{\infty}\left(-\frac{\gamma-1}{\gamma+1}\right)^{\xi}\theta_k\left(\alpha\gamma\xi,\Delta\right) \\
&\qquad - \gamma\sum_{\xi=1}^{\infty}\left(-\frac{\gamma-1}{\gamma+1}\mathrm{e}^{-\alpha\gamma\tau}\right)^{\xi}\theta_k\left(\alpha\gamma\xi,\Delta-\tau\right)
\end{aligned} \right\},
\end{aligned} \tag{83}
$$

$$
\left.\frac{\partial\Gamma_{\mathrm{DC}}\left(x_1,\tau,x_2,0,\Delta\right)}{\partial x_2}\right|_{x_1=0,\,x_2=1} = 0, \tag{84}
$$

$$
\left.\frac{\partial\Gamma_{\mathrm{MOD}}\left(x_1,\tau,x_2,0,\Delta\right)}{\partial x_2}\right|_{x_1=0,\,x_2=1} = 0, \tag{85}
$$

$$
\left.\frac{\partial\Gamma_{\mathrm{DC}}\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{x_1=0,\,x_2=1,\,T_2=0} = 0, \tag{86}
$$

$$
\left.\frac{\partial\Gamma_{\mathrm{MOD}}\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{x_1=0,\,x_2=1,\,T_2=0} = 0, \tag{87}
$$

$$
\begin{aligned}
&\left.\left\{\frac{\partial}{\partial x_2}\left[\left.\frac{\partial\Gamma_{\mathrm{DC}}\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{T_2=0}\right]\right\}\right|_{x_1=0,\,x_2=1} \\
&\quad = \frac{\lambda_{\mathrm{d}}\left(S_{\mathrm{c}}+\overline{S_0}\right)}{\alpha}\left[1-\frac{\left(\gamma^2-1\right)\left(1-\mathrm{e}^{-\alpha\gamma\tau}\right)}{\left(\gamma+1\right)^2-\left(\gamma-1\right)^2\mathrm{e}^{-\alpha\gamma\tau}}\right],
\end{aligned} \tag{88}
$$

$$
\begin{aligned}
&\left\{\frac{\partial}{\partial x_2}\left[\left.\frac{\partial\Gamma_{\mathrm{MOD}}\left(x_1,\tau,x_2,T_2,\Delta\right)}{\partial T_2}\right|_{T_2=0}\right]\right\}\Bigg|_{x_1=0,\,x_2=1} \\
&= 4\lambda_{\mathrm{d}}\overline{S_0}\sum_{k=1}^{\infty}\sum_{\xi=1}^{\infty}\left\{\begin{array}{c}
\dfrac{4\gamma^2\xi\mathrm{e}^{-\alpha\gamma\tau}}{\left[(\gamma+1)^2-(\gamma-1)^2\mathrm{e}^{-\alpha\gamma\tau}\right]^2} \\
\times\left[\dfrac{\left(\gamma^2-1\right)\left(1-\mathrm{e}^{-\alpha\gamma\tau}\right)}{(\gamma+1)^2-(\gamma-1)^2\mathrm{e}^{-\alpha\gamma\tau}}\right]^{\xi-1} \\
\times\,\theta_k\left(\alpha\xi,\Delta-\tau\right) \\
+\dfrac{\gamma^2\xi}{(\gamma+1)^2}\left(-\dfrac{\gamma-1}{\gamma+1}\right)^{\xi-1}\theta_k\left(\alpha\gamma\xi,\Delta\right) \\
-\dfrac{\gamma^2\xi\mathrm{e}^{-\alpha\gamma\tau}}{(\gamma+1)^2}\left(-\dfrac{\gamma-1}{\gamma+1}\mathrm{e}^{-\alpha\gamma\tau}\right)^{\xi-1} \\
\times\,\theta_k\left(\alpha\gamma\xi,\Delta-\tau\right)
\end{array}\right\}.
\end{aligned}
\tag{89}
$$

Therefore, the analytical expression of $p\left(\tau,\Delta\right)\mathrm{d}\Delta$ is derived as follows:

$$
p\left(\tau,\Delta\right)\mathrm{d}\Delta \equiv \left[S_{\mathrm{c}}A\left(\tau\right)+S_0B\left(\tau,\Delta\right)\right]\exp\left[-S_{\mathrm{c}}X\left(\tau\right)\right]\exp\left[-S_0Y\left(\tau,\Delta\right)\right]\mathrm{d}\Delta, \tag{90}
$$

$$
A\left(\tau\right) \equiv \frac{\lambda_{\mathrm{d}}}{\alpha}\left[1-\frac{\left(\gamma^2-1\right)\left(1-\mathrm{e}^{-\alpha\gamma\tau}\right)}{(\gamma+1)^2-(\gamma-1)^2\mathrm{e}^{-\alpha\gamma\tau}}\right], \tag{91}
$$

$$\begin{aligned}
&B(\tau,\Delta)\\
&\quad\equiv \frac{W}{T_0}A(\tau)\\
&\qquad+\frac{4\lambda_\mathrm{d}W}{T_0}\sum_{k=1}^{\infty}\sum_{\xi=1}^{\infty}\left\{\begin{aligned}
&\frac{4\gamma^2\xi\mathrm{e}^{-\alpha\gamma\tau}}{\left[(\gamma+1)^2-(\gamma-1)^2\mathrm{e}^{-\alpha\gamma\tau}\right]^2}\\
&\qquad\times\left[\frac{\left(\gamma^2-1\right)\left(1-\mathrm{e}^{-\alpha\gamma\tau}\right)}{(\gamma+1)^2-(\gamma-1)^2\mathrm{e}^{-\alpha\gamma\tau}}\right]^{\xi-1}\\
&\qquad\times\theta_k(\alpha\xi,\Delta-\tau)\\
&\quad+\frac{\gamma^2\xi}{(\gamma+1)^2}\left(-\frac{\gamma-1}{\gamma+1}\right)^{\xi-1}\theta_k(\alpha\gamma\xi,\Delta)\\
&\quad-\frac{\gamma^2\xi\mathrm{e}^{-\alpha\gamma\tau}}{(\gamma+1)^2}\left(-\frac{\gamma-1}{\gamma+1}\mathrm{e}^{-\alpha\gamma\tau}\right)^{\xi-1}\\
&\qquad\times\theta_k(\alpha\gamma\xi,\Delta-\tau)
\end{aligned}\right\},
\end{aligned}\tag{92}$$

$$X(\tau)\equiv\frac{2\lambda_\mathrm{d}}{\alpha^2Y_\infty}\ln\left[\frac{(\gamma+1)^2}{4\gamma}\mathrm{e}^{\frac{1}{2}\alpha(\gamma-1)\tau}-\frac{(\gamma-1)^2}{4\gamma}\mathrm{e}^{-\frac{1}{2}\alpha(\gamma+1)\tau}\right],\tag{93}$$

$$\begin{aligned}
&Y(\tau,\Delta)\\
&\quad\equiv\frac{W}{T_0}X(\tau)\\
&\qquad+\frac{2\lambda_\mathrm{d}W}{\alpha T_0Y_\infty}\sum_{k=1}^{\infty}\left\{\begin{aligned}
&\frac{1}{2}(\gamma-1)\left[\theta_k(0,\Delta)-\theta_k(0,\Delta-\tau)\right]\\
&\quad+\sum_{\xi=1}^{\infty}\left[\frac{\left(\gamma^2-1\right)\left(1-\mathrm{e}^{-\alpha\gamma\tau}\right)}{(\gamma+1)^2-(\gamma-1)^2\mathrm{e}^{-\alpha\gamma\tau}}\right]^{\xi}\\
&\qquad\times\theta_k(\alpha\xi,\Delta-\tau)\\
&\quad+\gamma\sum_{\xi=1}^{\infty}\left(-\frac{\gamma-1}{\gamma+1}\right)^{\xi}\theta_k(\alpha\gamma\xi,\Delta)\\
&\quad-\gamma\sum_{\xi=1}^{\infty}\left(-\frac{\gamma-1}{\gamma+1}\mathrm{e}^{-\alpha\gamma\tau}\right)^{\xi}\theta_k(\alpha\gamma\xi,\Delta-\tau)
\end{aligned}\right\}.
\end{aligned}\tag{94}$$

Here, let us substitute $\tau = 0$ into Equation (90). As a result, one obtains the following expression,

$$p\left(0,\Delta\right)\mathrm{d}\Delta = \frac{\lambda_{\mathrm{d}}\left(S_{\mathrm{c}}+\overline{S_0}\right)}{\alpha}\mathrm{d}\Delta + \lambda_{\mathrm{d}}\overline{S_0}\sum_{k=1}^{\infty}\theta_k\left(\alpha,\Delta\right)\mathrm{d}\Delta. \tag{95}$$

On the other hand, the conventional analytical expression of pulsed neutron source method that does not include the count-loss process is written as follows:

$$p\left(\Delta\right)\mathrm{d}\Delta = \begin{cases} \dfrac{\lambda_{\mathrm{d}}S_{\mathrm{c}}}{\alpha}\mathrm{d}\Delta + \dfrac{\lambda_{\mathrm{d}}S_0}{\alpha}\left[1-\dfrac{1-\mathrm{e}^{-\alpha(T_0-W)}}{1-\mathrm{e}^{-\alpha T_0}}\mathrm{e}^{-\alpha\Delta}\right]\mathrm{d}\Delta, & 0 \le \Delta < W, \\ \dfrac{\lambda_{\mathrm{d}}S_{\mathrm{c}}}{\alpha}\mathrm{d}\Delta + \dfrac{\lambda_{\mathrm{d}}S_0}{\alpha}\dfrac{\mathrm{e}^{\alpha W}-1}{1-\mathrm{e}^{-\alpha T_0}}\mathrm{e}^{-\alpha\Delta}\mathrm{d}\Delta, & W \le \Delta < T_0. \end{cases} \tag{96}$$

One confirms that Equation (95) is the Fourier series expansion of Equation (96).

III. Discussions

1. Determination of neutron decay constant

Let us suppose that there exists no constant component of pulsed neutron source (i.e., $S_{\mathrm{c}} = 0$) for simple discussion. Under such a condition, $p\left(\tau,\Delta\right)$ given in Equation (90) is re-written as follows:

$$p\left(\tau,\Delta\right) = S_0 B\left(\tau,\Delta\right)\exp\left[-S_0 Y\left(\tau,\Delta\right)\right]. \tag{97}$$

[Table 1 about here.]

[Fig. 5 about here.]

[Fig. 6 about here.]

[Fig. 7 about here.]

In order to grasp the count-loss effect in determination of neutron decay constant α by conventional formula of pulsed neutron source method $\Omega e^{-\alpha\Delta}$, different $p(\tau, \Delta)$ curves were plotted by substituting the parameters listed in **Table 1**, where a typical thermal subcritical reactor system for pulsed neutron source method with subcriticality of 3 $\%\Delta k/k$ and neutron decay constant of 1566.3 s^{-1} is supposed, into Equation (97) (see **Figure 5**), then α was determined by the least square fitting technique using conventional formula for $2W \leq \Delta < T_0$ (see **Figure 6**). Thus obtained α was plotted in **Figure 7**. Here, we note that the fitting range $2W \leq \Delta < T_0$ is a typical one in actual experiments of pulsed neutron source method. One finds that the conventional formula overestimates α when the intensity factor of pulsed component S_0 is not high and underestimates α when S_0 is high.

[Fig. 8 about here.]

[Fig. 9 about here.]

[Fig. 10 about here.]

To acquire further understanding on count-loss effect in determination of α, $B(\tau, \Delta)$ normalized by $B(\tau, W)$, $Y(\tau, \Delta)$, and $\exp[-S_0 Y(\tau, \Delta)]$ for $\tau = 4$ μs were plotted in **Figures 8 to 10**, respectively. In Figure 8, one observes faster decaying behaviors of $B(\tau, \Delta)$ for $W \leq \Delta$ as τ is getting larger. On the other hand, although $Y(\tau, \Delta)$ itself has an obvious dependency on Δ (see Figure 9), it is found in Figure 10 that $\exp[-S_0 Y(\tau, \Delta)]$ is almost unity when S_0 is not high. Therefore, with regard to not high S_0, the faster decaying behavior of $B(\tau, \Delta)$ is responsible for overestimation of α, since the decaying behavior of $p(\tau, \Delta)$ is almost identical to that of $B(\tau, \Delta)$.

On the other hand, one finds in Figure 10 that $\exp[-S_0 Y(\tau, \Delta)]$ has a clearer dependency on Δ as S_0 is getting higher. Hence, with regard to high S_0, $p(\tau, \Delta)$ shows slower decaying behaviors that result in underestimation of α since it is proportional to the product of $B(\tau, \Delta)$ and $\exp[-S_0 Y(\tau, \Delta)]$. Furthermore, underestimation of α appears superiorly as S_0 is getting higher.

2. Verification of count-loss correction procedures

It is well-known that the relations between true counting rate n and observed counting rate m for paralyzable and non-paralyzable count-loss processes are expressed as follows:[20)]

$$m = n\mathrm{e}^{-n\tau}, \qquad \text{for paralyzable count-loss process,} \tag{98}$$

$$m = \frac{n}{1 + n\tau}, \qquad \text{for non-paralyzable count-loss process.} \tag{99}$$

By solving them with respect to n, one gets

$$n \simeq m\left(1 + m\tau + \frac{3}{2}m^2\tau^2 + \frac{8}{3}m^3\tau^3 + \frac{125}{24}m^4\tau^4 + \frac{54}{5}m^5\tau^5\right), \tag{100}$$

for paralyzable count-loss process,

$$n = \frac{m}{1 - m\tau}, \qquad \text{for non-paralyzable count-loss process.} \tag{101}$$

[Fig. 11 about here.]

[Fig. 12 about here.]

In the pulsed neutron source method, the correction procedures based on Equations (100) and (101) are often introduced to eliminate the count-loss effect. To verify these count-loss correction procedures, the following corrected $p(\tau, \Delta)$ curves, i.e.,

$p^{\mathrm{corr,p}}(\tau,\Delta)$ and $p^{\mathrm{corr,np}}(\tau,\Delta)$, were plotted by using the parameters listed in Table 1:

$$p^{\mathrm{corr,p}}(\tau,\Delta) \equiv p(\tau,\Delta)\left[\begin{array}{c} 1+p(\tau,\Delta)\tau+\frac{3}{2}p^2(\tau,\Delta)\tau^2+\frac{8}{3}p^3(\tau,\Delta)\tau^3 \\ +\frac{125}{24}p^4(\tau,\Delta)\tau^4+\frac{54}{5}p^5(\tau,\Delta)\tau^5 \end{array}\right], \tag{102}$$

for paralyzable count-loss process,

$$p^{\mathrm{corr,np}}(\tau,\Delta) \equiv \frac{p(\tau,\Delta)}{1-p(\tau,\Delta)\tau}, \quad \text{for non-paralyzable count-loss process.} \tag{103}$$

Regarding the $p^{\mathrm{corr,p}}(\tau,\Delta)$ and $p^{\mathrm{corr,np}}(\tau,\Delta)$ curves thus obtained, α was determined by the least square fitting technique using conventional formula $\Omega e^{-\alpha\Delta}$ for $2W \leq \Delta < T_0$ as plotted in **Figures 11 and 12**. It is clearly seen in these figures that the both count-loss correction procedures suppress underestimation of α. However, it is confirmed that these correction procedures are not effective for overestimation.

IV. Conclusion

The count-loss effect in determination of neutron decay constant by pulsed neutron source method was investigated on the basis of the neutron observation probability that was derived by explicitly considering the count-loss process in neutron counting system and the periodic burst of pulsed neutrons. It was found that overestimation of neutron decay constant due to count-loss effect is seen while underestimation appears superiorly as the intensity of pulsed neutron source is getting higher. It was further demonstrated that the well-known count-loss correction procedures that are often introduced to eliminate the count-loss effect are not effective for overestimation although they successfully suppress underestimation. Therefore, the pulsed neutron source method should be modified so as to have robustness against the count-loss effect.

On the basis of the conclusion provided in the present paper, a modification of pulsed neutron source method will be proposed and discussed in a forthcoming paper.

Acknowledgement

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Table 1 Parameters for plotting neutron observation probability $p(\tau, \Delta)$.

Parameter	Value
λ_c [s^{-1}]	33196.2315
λ_f [s^{-1}]	21685.7490
λ_d [s^{-1}]	334.8504
$\langle \nu \rangle$ [-]	2.4740
$\langle \nu(\nu-1) \rangle$ [-]	4.8965
T_0 [s]	0.005
W [s]	$0.1 \times T_0$
S_0 [s^{-1}]	$1 \times 10^2 \sim 1 \times 10^6$
τ [μs]	$0, 2, 4, 6, 8, 10$

Figure Captions

Fig. 1. Illustration of paralyzable count-loss process.

Fig. 2. Timing diagram of source induced probability $\widetilde{P}(Z_1, T_1, Z_2, T_2, t)$.

Fig. 3. Time dependence of pulsed neutron source intensity $S(t)$.

Fig. 4. Timing diagram of neutron observation probability $p(\tau, \Delta)$.

Fig. 5. Examples of neutron observation probability $p(\tau, \Delta)$ (for $S_\mathrm{c} = 0$ and $S_0 = 5 \times 10^4$).

Fig. 6. Examples of fitted curve (for $S_\mathrm{c} = 0$ and $\tau = 4\ \mu$s).

Fig. 7. Neutron decay constant α determined by conventional formula.

Fig. 8. Examples of function $B(\tau, \Delta)$ (normalized by $B(\tau, W)$).

Fig. 9. Examples of function $Y(\tau, \Delta)$.

Fig. 10. Examples of function $\exp[-S_0 Y(\tau, \Delta)]$ (for $\tau = 4\ \mu$s).

Fig. 11. Neutron decay constant α determined by conventional formula with paralyzable count-loss correction.

Fig. 12. Neutron decay constant α determined by conventional formula with non-paralyzable count-loss correction.

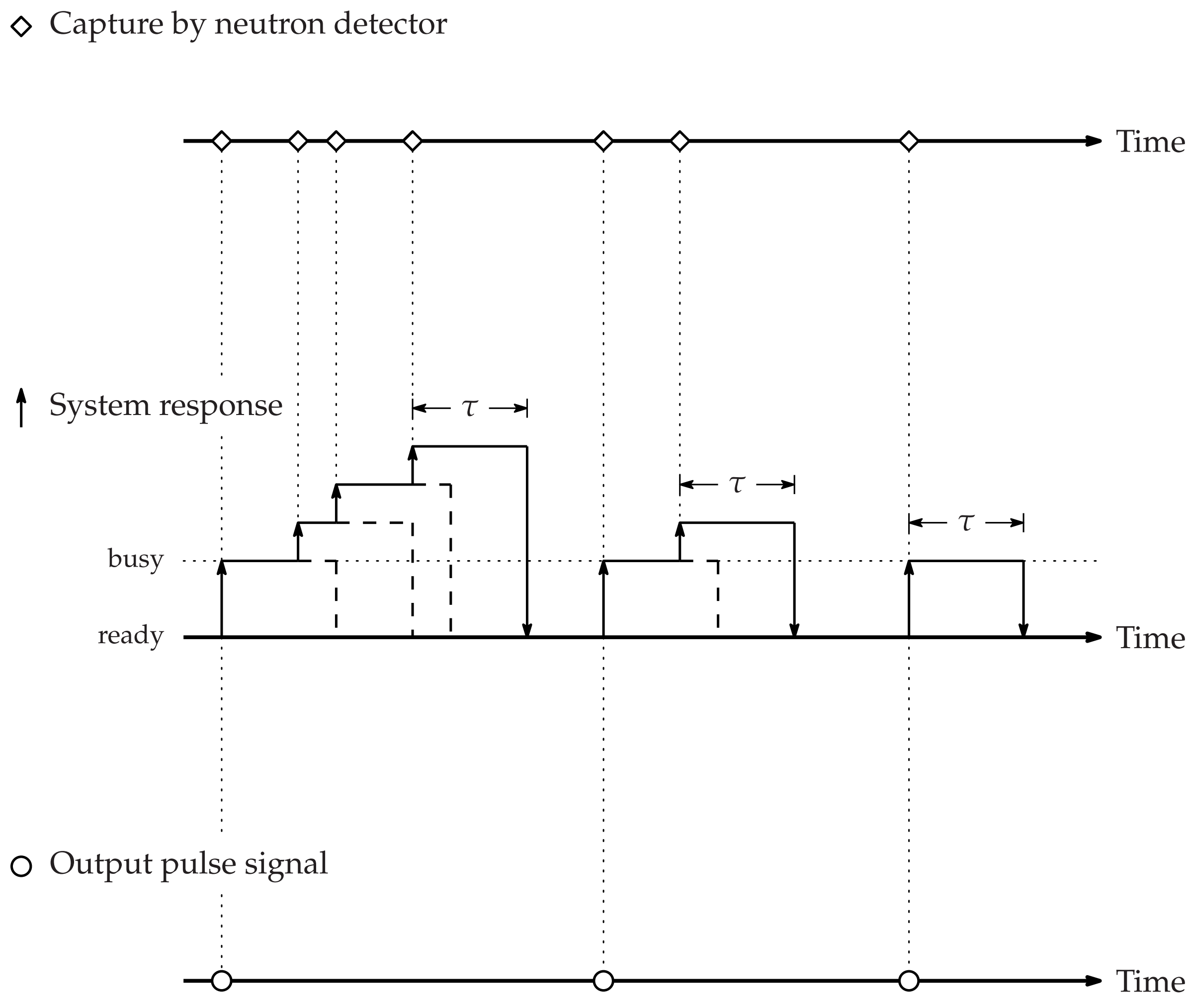

Fig. 1 Illustration of paralyzable count-loss process.

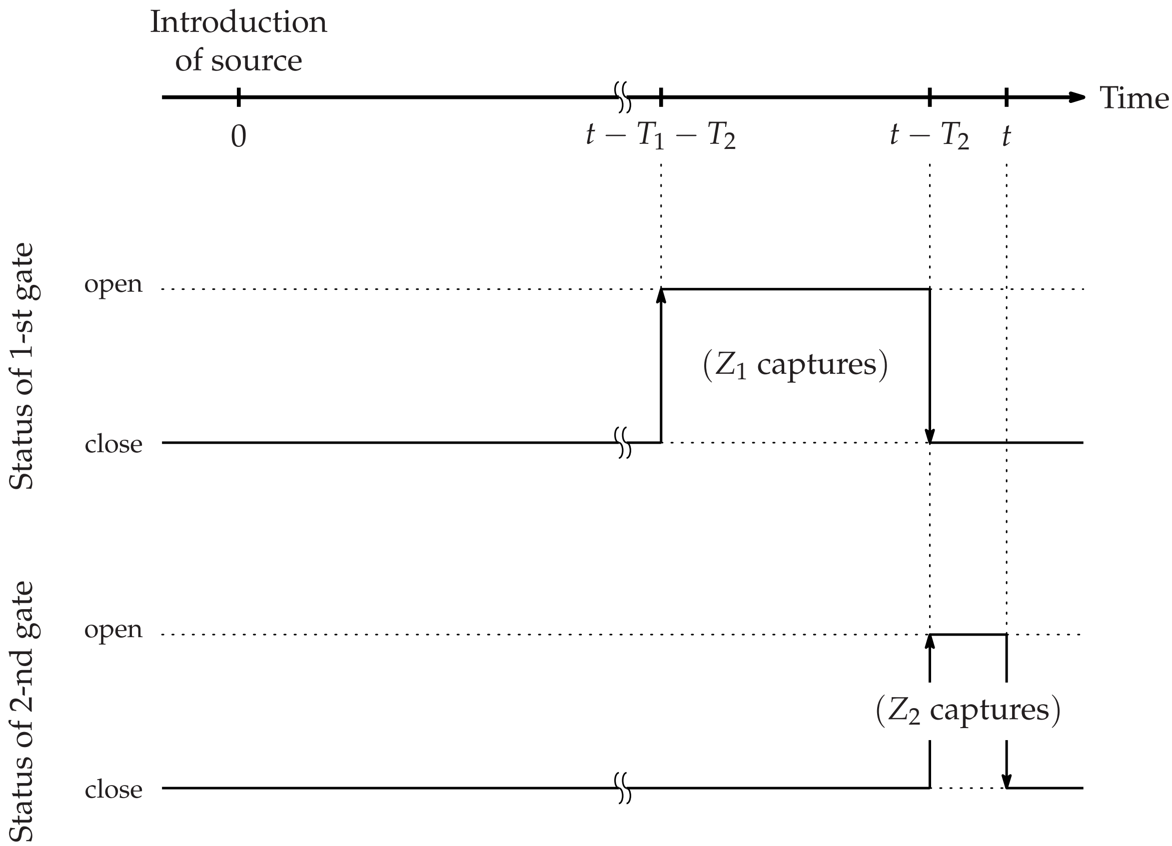

Fig. 2 Timing diagram of source induced probability $\widetilde{P}\,(Z_1, T_1, Z_2, T_2, t)$.

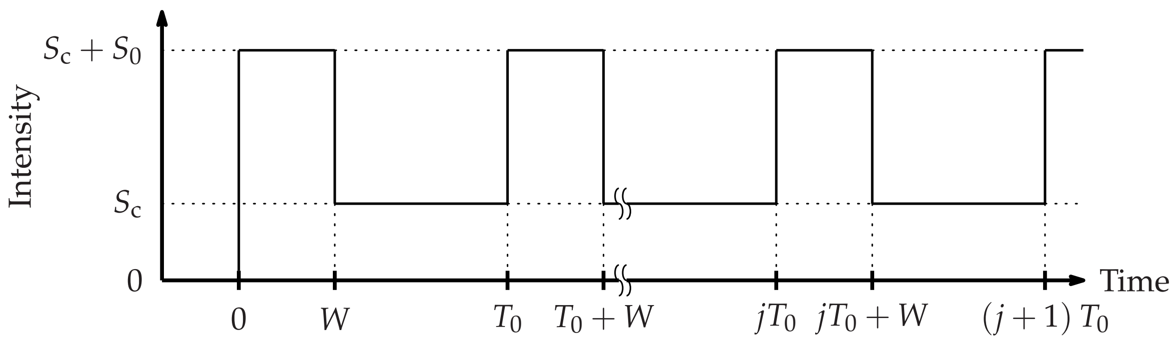

Fig. 3 Time dependence of pulsed neutron source intensity $S\,(t)$.

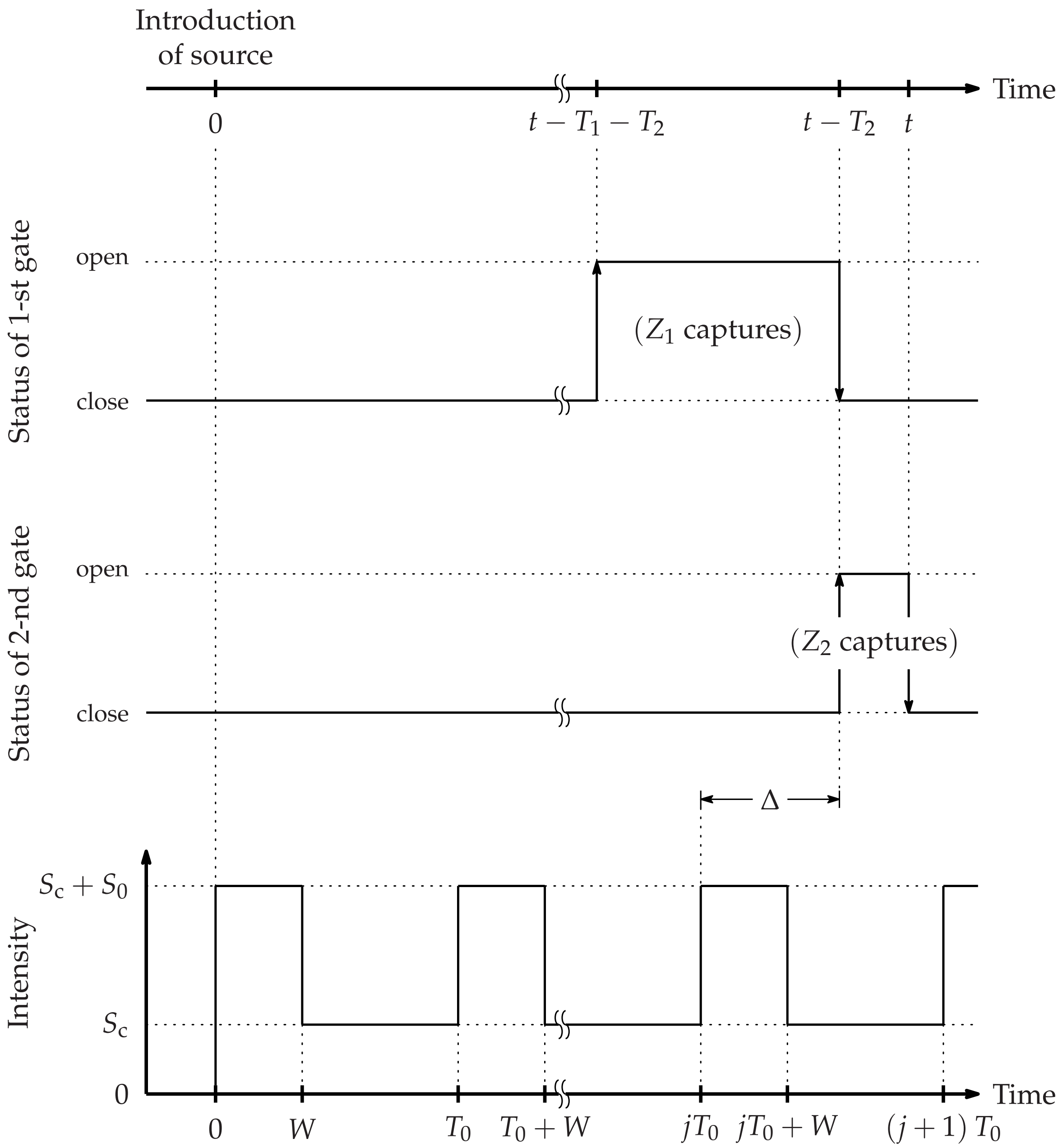

Fig. 4 Timing diagram of neutron observation probability $p\,(\tau, \Delta)$.

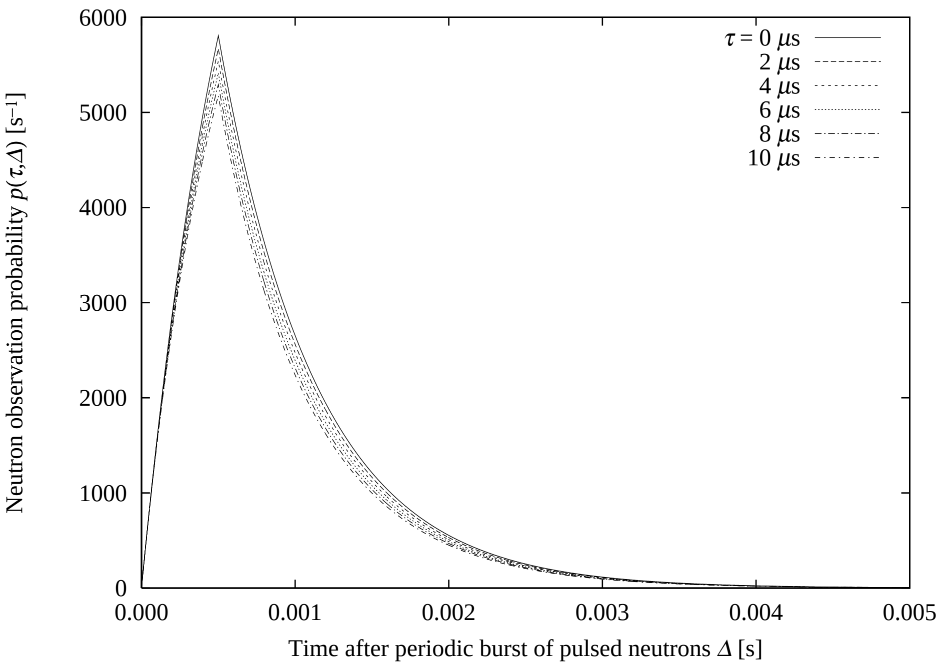

Fig. 5 Examples of neutron observation probability $p(\tau, \Delta)$ (for $S_c = 0$ and $S_0 = 5 \times 10^4$).

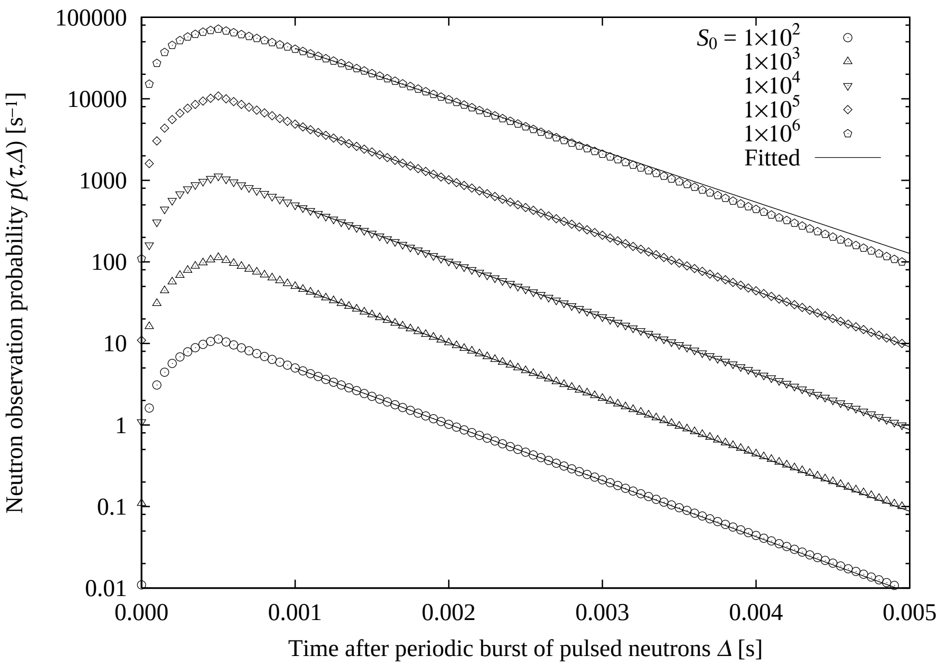

Fig. 6 Examples of fitted curve (for $S_c = 0$ and $\tau = 4$ μs).

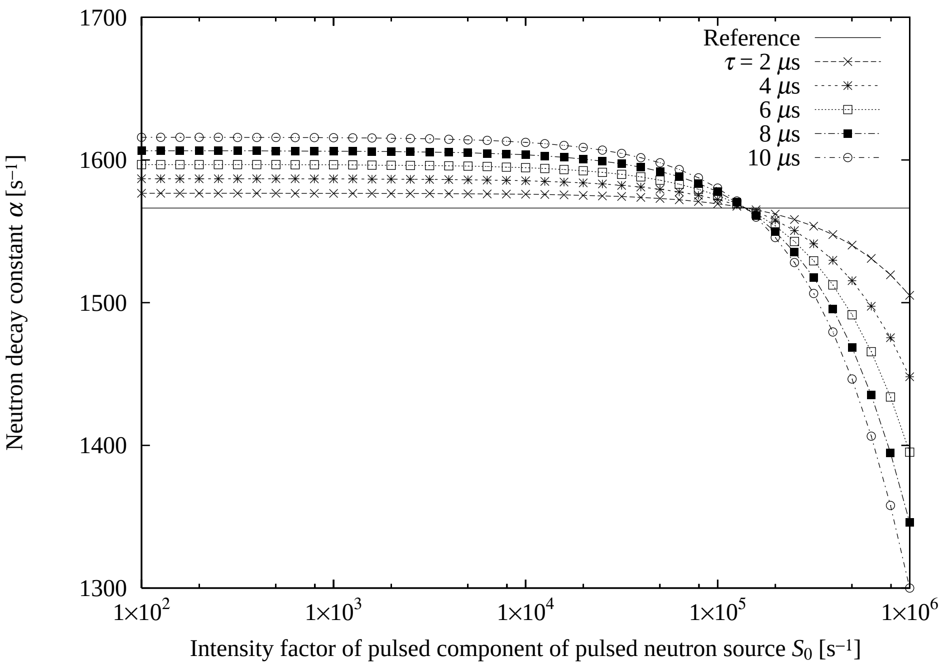

Fig. 7 Neutron decay constant α determined by conventional formula.

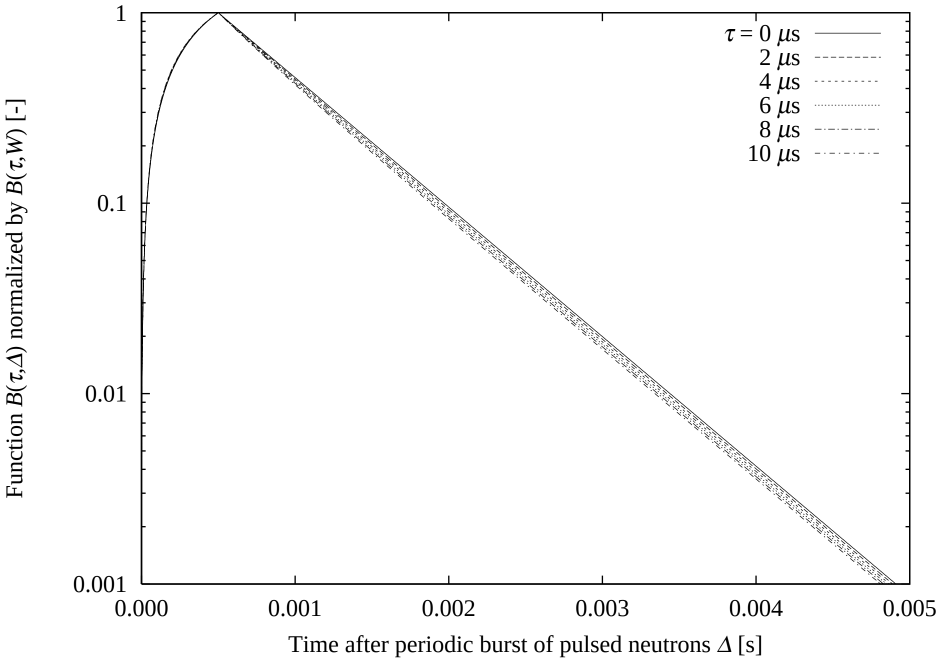

Fig. 8 Examples of function $B(\tau, \Delta)$ (normalized by $B(\tau, W)$).

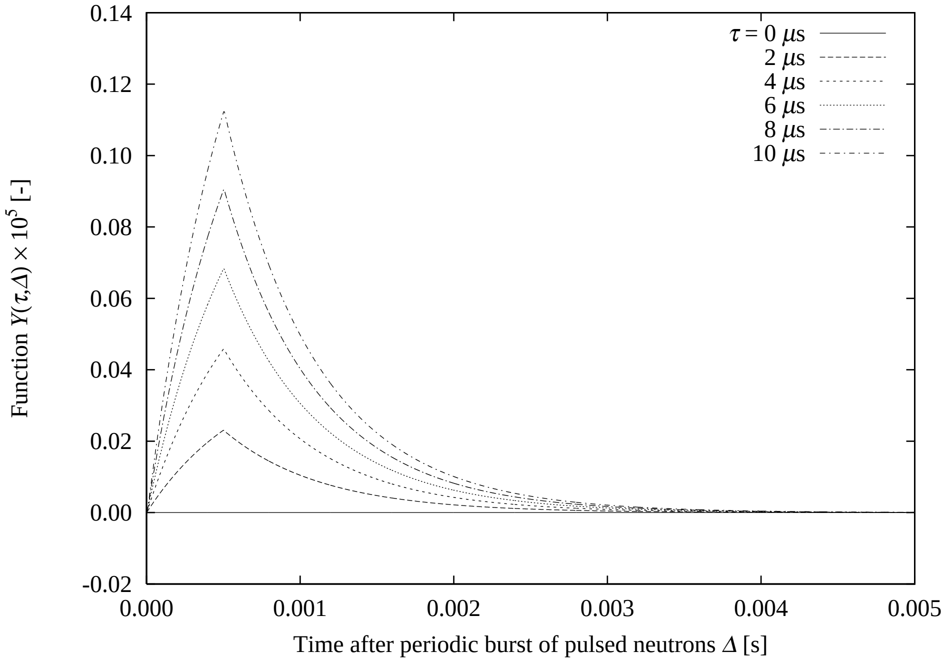

Fig. 9 Examples of function $Y(\tau, \Delta)$.

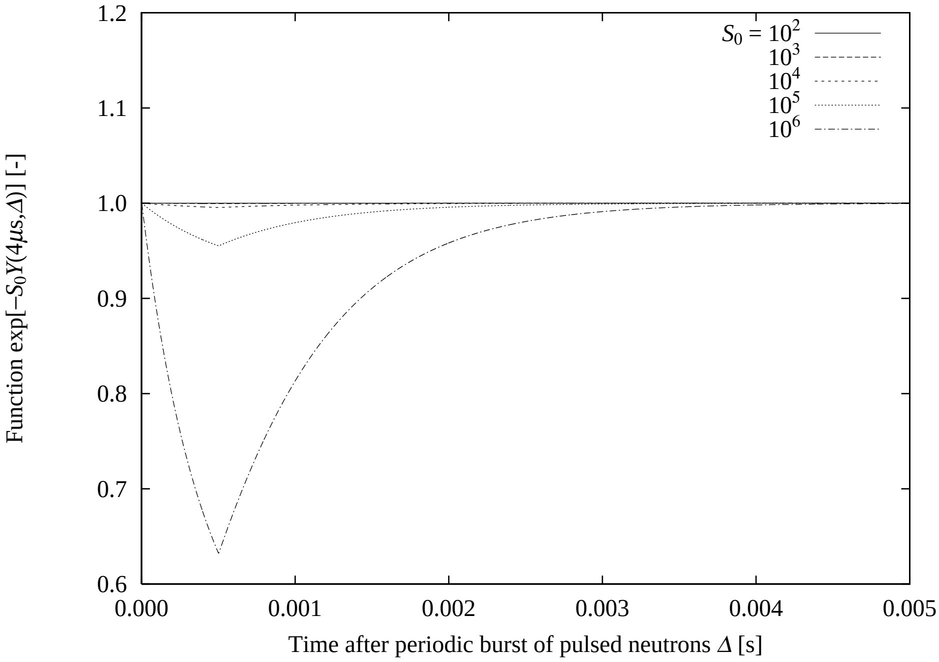

Fig. 10 Examples of function $\exp\left[-S_0 Y\left(\tau, \Delta\right)\right]$ (for $\tau = 4\ \mu$s).

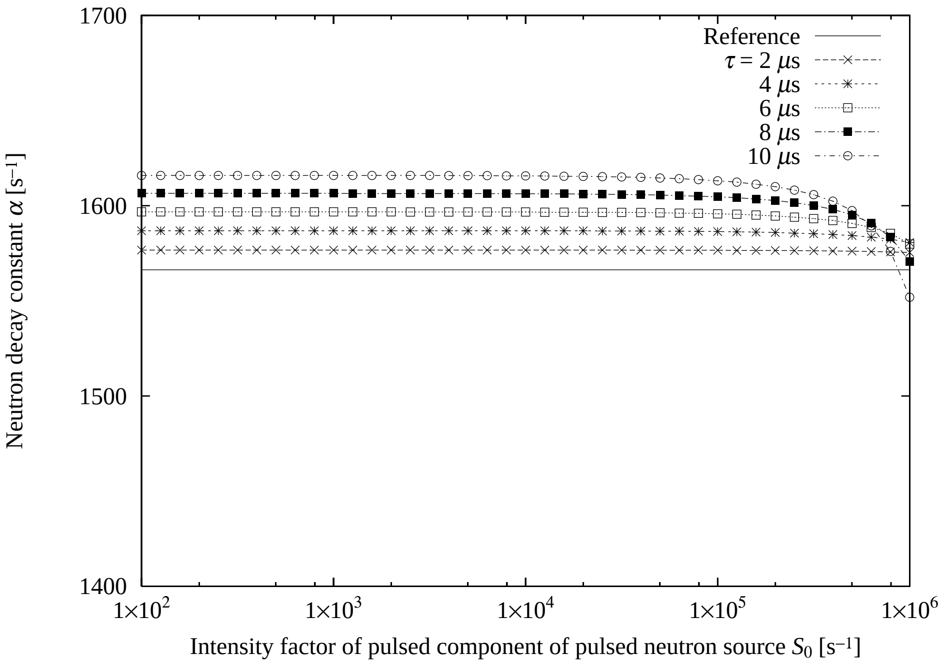

Fig. 11 Neutron decay constant α determined by conventional formula with paralyzable count-loss correction.

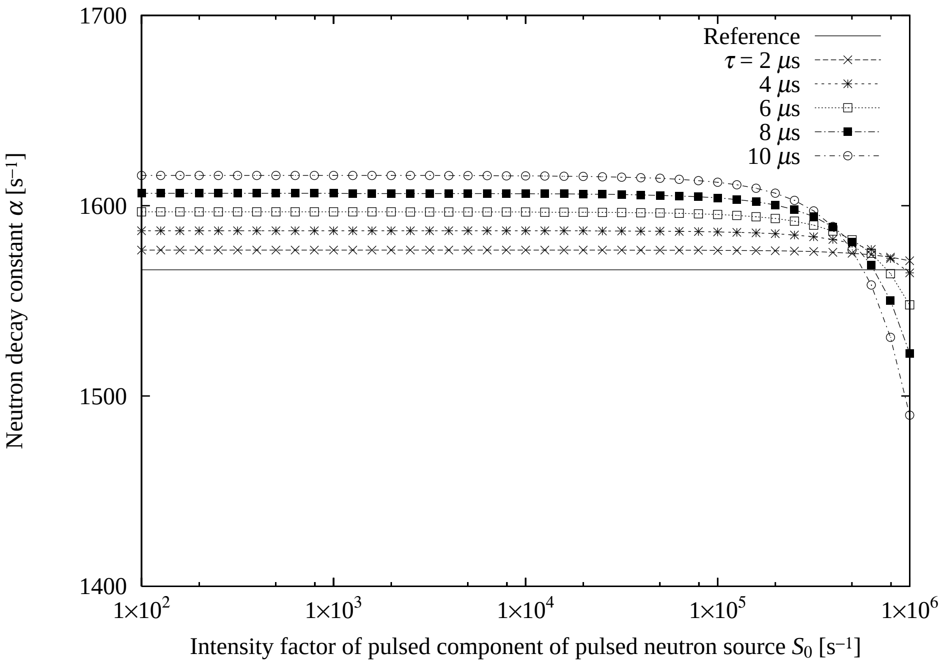

Fig. 12 Neutron decay constant α determined by conventional formula with non-paralyzable count-loss correction.