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Rapid Communications

Observation of $e^+e^- \to \pi^+\pi^-\pi^0\chi_{b1,2}(1P)$ and search for $e^+e^- \to \phi\chi_{b1,2}(1P)$ at $\sqrt{s} = 10.96$—11.05 GeV

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We report searches for the processes $e^+e^- \to \pi^+\pi^-\pi^0\chi_{bJ}$ and $e^+e^- \to \phi\chi_{bJ}$ (J = 1, 2) based on data samples collected by the Belle experiment at the KEKB collider. We report the first observation of the process $e^+e^- \to (\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{b1}$ and first evidence for $e^+e^- \to \omega\chi_{bJ}$ in the vicinity of the $\Upsilon(11020)$ resonance, with center-of-mass energies from 10.96 to 11.05 GeV. The significances for $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{b1}$ and $\omega\chi_{bJ}$ are greater than 5.3σ and 4.0σ, respectively. We also investigate the energy dependence of the $e^+e^- \to \pi^+\pi^-\pi^0\chi_{bJ}$ cross section, but we cannot determine whether the contributions are from the $\Upsilon(10860)$ and $\Upsilon(11020)$ resonances or nonresonant continuum processes. The signals for $e^+e^- \to \phi\chi_{bJ}$ are not significant, and the upper limits of the Born cross sections at the 90% confidence level are 0.7 and 1.0 pb for $e^+e^- \to \phi\chi_{b1}$ and $\phi\chi_{b2}$, respectively, for center-of-mass energies from 10.96 to 11.05 GeV.

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Hadronic transitions among heavy quarkonium states serve as a key source of information for better understanding the strong interaction between a quark and antiquark, and thus quantum chromodynamics (QCD). The heavy quarkonium systems, in which the speed of quarks is sufficiently small, are approximately nonrelativistic, and the hadronic transitions to lower lying states have long been described using the QCD multipole expansion [1]. However, the existence of anomalously large hadronic transition rates from the $\Upsilon(10860)$, as reported by the Belle experiment [2–9], challenges the theoretical calculations, as well as the pure bottomonium nature of the $\Upsilon(10860)$ and $\Upsilon(11020)$ [10–12].

The processes $e^+e^- \to \omega\chi_{bJ}$ were observed recently [4] using data samples taken at energies near the $\Upsilon(10860)$ peak, but the dependence of the $e^+e^- \to \omega\chi_{bJ}$ cross section versus energy was not measured. Therefore, it is unclear whether this process occurs from the $\Upsilon(10860)$ meson or continuum process. Nevertheless, the result has been investigated extensively by theorists to understand the dynamics of these transitions, producing studies of S- and D-wave mixing for the observed heavy quark spin-symmetry violation from the comparison of $\omega\chi_{b1}$ and $\omega\chi_{b2}$ [13], a possible contribution of $\Upsilon(10860) \to \pi Z_b \to \pi\rho\Upsilon(1S)$ [14], a molecular component in the $\Upsilon(10860)$ wave function [14], and hadronic-loop effects [15].

By extending the calculation in Ref. [15] to the $\Upsilon(11020)$ case, assuming the hadronic-loop effect is a universal mechanism in the higher bottomonium transitions, the authors of Ref. [16] predict the branching fractions of $\Upsilon(11020) \to \omega\chi_{bJ}$ in addition to $\Upsilon(11020) \to \phi\chi_{bJ}$, where $J = 0$, 1, and 2, as listed in Table I. Relative magnitudes of these branching fractions are also predicted (and listed in Table I), which are weakly dependent on the free parameters introduced in the theoretical calculation. An experimental measurement of these ω and ϕ transitions will give a crucial test on how well the hadronic-loop effect works in $\Upsilon(11020)$ decay, and a test of the similarity between $\Upsilon(11020)$ and $\Upsilon(10860)$.

In this paper, we report the results of a search for $\omega\chi_{bJ}$ and $\phi\chi_{bJ}$ using the $\Upsilon(10860)$ and $\Upsilon(11020)$ energy scan data collected with the Belle detector. The data that we are using consist of 22 samples of high integrated luminosity (listed in Table II), and 18 additional samples of about 50 pb^{-1} per point taken in 5 MeV steps between 10.96 and 11.05 GeV [17]. We use $\chi_{bJ} \to \gamma\Upsilon(1S)$, $\Upsilon(1S) \to \ell^+\ell^-(\ell = e, \mu)$, $\omega \to \pi^+\pi^-\pi^0$ to reconstruct the $e^+e^- \to \omega\chi_{b1,2}$ signal; for the $e^+e^- \to \phi\chi_{bJ}$ signal, we reconstruct ϕ with its decays to K^+K^- and check the production of χ_{bJ} by studying the K^+K^- recoil mass.

The Belle detector, located at the KEKB asymmetric-energy e^+e^- collider [18] is described in Ref. [19]. The EVTGEN [20] generator, as well as a GEANT3 [21]-based detector simulation, is used to produce simulated events using Monte Carlo (MC) methods. The nominal parameters of the states in the decay chains are quoted from Ref. [22]. To take the initial-state radiation (ISR) into consideration, the radiator function from Ref. [23] is introduced in EVTGEN. A generic MC sample at the $\Upsilon(10860)$ peak

TABLE I. The predicted branching fractions of $\Upsilon(11020) \to \omega\chi_{bJ}$ and $\phi\chi_{bJ}$ [16], as well as the relative magnitudes, where $\mathcal{B}_j \equiv \mathcal{B}(\Upsilon(11020) \to \omega(\phi)\chi_{bj})$, $R_{ij} \equiv \frac{\mathcal{B}_i}{\mathcal{B}_j}$.

Decay mode	$\mathcal{B}_0$	$\mathcal{B}_1$	$\mathcal{B}_2$	R_{10}	R_{20}	R_{21}
$\omega\chi_{bJ}$	$(0.15\text{–}2.81) \times 10^{-3}$	$(0.63\text{–}11.68) \times 10^{-3}$	$(1.08\text{–}20.02) \times 10^{-3}$	≈ 4.11	≈ 7.06	≈ 1.72
$\phi\chi_{bJ}$	$(0.68\text{–}4.62) \times 10^{-6}$	$(0.50\text{–}3.43) \times 10^{-6}$	$(2.22\text{–}15.18) \times 10^{-6}$	≈ 0.74	≈ 3.28	≈ 4.43

including all possible decays is used to study the possible background channels and investigate the background shape.

For charged tracks, the impact parameters perpendicular to and along the beam direction with respect to the interaction point are required to be less than 1.0 and 3.5 cm, respectively. The transverse momentum is restricted to be higher than 0.1 GeV$/c$. A particle identification (PID) hypothesis [24] $\mathcal{L}(X)$ for each charged track is formed from different detector subsystems for particle $X \in e, \mu, \pi, K, p$. Tracks with a likelihood ratio $\mathcal{R}(K) = \mathcal{L}(K)/(\mathcal{L}(K) + \mathcal{L}(\pi)) < 0.4$ are identified as pions while those with $\mathcal{R}(K) > 0.6$ are identified as kaons. Similarly, we define the likelihood ratios $\mathcal{R}(e)$ and $\mathcal{R}(\mu)$ for identification of electrons and muons, respectively, with $\mathcal{R}(e) > 0.01$ and $\mathcal{R}(\mu) > 0.1$. A neutral cluster in the electromagnetic calorimeter is reconstructed as a photon if it does not match the extrapolated position of any charged track and its energy is greater than 30 MeV.

TABLE II. Integrated luminosity at different c.m. energy as well as the energy-dependent Born cross sections for $e^+e^- \to \pi^+\pi^-\pi^0\chi_{bJ}$ with statistical uncertainty only. A 11.9% common systematic uncertainty is not included.

$E_{\text{c.m.}}$ (GeV)	$\mathcal{L}$ (fb^{-1})	$\sigma^{\text{Born}}(\pi^+\pi^-\pi^0\chi_{bJ})$ (pb)
10.7711	0.955	$-1.44^{+2.62}_{-1.74}$
10.8203	1.164	$2.72^{+2.07}_{-1.43}$
10.8497	0.989	$2.70^{+2.19}_{-1.41}$
10.8589	0.989	$0.64^{+1.51}_{-0.75}$
10.8633	47.648	$0.82^{+0.10}_{-0.10}$
10.8667	45.553	$0.68^{+0.10}_{-0.10}$
10.8686	22.938	$0.89^{+0.16}_{-0.16}$
10.8695	0.978	$1.23^{+1.96}_{-1.21}$
10.8785	0.978	$1.90^{+1.90}_{-1.17}$
10.8836	1.230	$1.37^{+1.56}_{-1.01}$
10.8889	0.989	$1.20^{+1.63}_{-0.93}$
10.8985	0.983	$1.14^{+1.55}_{-0.88}$
10.9011	0.873	$-1.25^{+1.82}_{-1.06}$
10.9077	0.980	$0.51^{+1.50}_{-0.87}$
10.9275	0.667	$2.12^{+2.11}_{-1.30}$
10.9575	0.851	$0.70^{+1.67}_{-0.83}$
10.9775	0.999	$2.84^{+1.96}_{-1.32}$
10.9919	0.986	$1.10^{+1.50}_{-0.87}$
11.0068	0.976	$3.05^{+1.86}_{-1.28}$
11.0164	0.771	$3.47^{+2.11}_{-1.46}$
11.0175	0.849	$0.00^{+0.95}_{-0.32}$
11.0220	0.982	$0.84^{+1.49}_{-0.98}$

To select $e^+e^- \to \pi^+\pi^-\pi^0\chi_{bJ}$ candidates, we require that there be exactly four tracks with zero net charge, of which two are positively identified as pions and the other two as leptons. At least three photons are required in the event, and a π^0 list is created with the invariant mass of the photon pairs satisfying $M(\gamma\gamma) \in [0.12, 0.15]$ GeV$/c^2$, which covers nearly $\pm 3\sigma$ around the π^0 peak. To improve the track momentum and photon energy resolutions, and to suppress the background, a five-constraint (5C) kinematic fit is performed for the $\gamma\pi^+\pi^-\pi^0\ell^+\ell^-$ candidates enforcing energy and momentum conservation and constraining the invariant mass of π^0 candidates. The four momenta of the final-state particles after the 5C kinematic fit are kept for further analysis. The $\chi^2_{5\text{C}}/\text{ndf}$ is required to be less than 20, where $\chi^2_{5\text{C}}$ is the resulting χ^2 of the kinematic fit, and $\text{ndf} = 5$ is the number of degrees of freedom. If there are multiple π^0 candidates surviving the kinematic fit in an event, the one with the smallest $\chi^2_{5\text{C}}$ is kept. The lepton pair is taken as an $\Upsilon(1S)$ candidate if its invariant mass is in the region $[9.42, 9.60]$ GeV$/c^2$.

The χ_{bJ} candidates are reconstructed with the selected $\Upsilon(1S)$ and the photon not used to form a π^0 candidate. The invariant mass of $\pi^+\pi^-\pi^0$ ($M(\pi^+\pi^-\pi^0)$) versus the corrected invariant mass of $\gamma\Upsilon(1S)$ ($M(\gamma\Upsilon(1S)) \equiv M(\gamma\ell^+\ell^-) - M(\ell^+\ell^-) + m_{\Upsilon(1S)}$) is shown in Fig. 1 for the sum of the data samples in the $\Upsilon(11020)$ energy region, which is defined as $E_{\text{c.m.}} > 10.96$ GeV. Clusters of events for the production of χ_{bJ} can be seen both when $M(\pi^+\pi^-\pi^0)$ is in the ω mass region ($[0.75, 0.81]$ GeV$/c^2$) and at higher masses (> 0.81 GeV/c^2). For events having $M(\pi^+\pi^-\pi^0)$ in the ω mass region, the χ_{b2} signal is dominant while for signal events with higher $\pi^+\pi^-\pi^0$ masses, the χ_{b1} signal is dominant. The background in this case comes predominantly from false π^0 candidates produced by combinatorial photons.

An unbinned two-dimensional (2D) extended maximum likelihood fit to the $M(\pi^+\pi^-\pi^0)$ and $M(\gamma\Upsilon(1S))$ distributions of the candidate events is applied to determine the numbers of $\omega\chi_{bJ}$ and $\pi^+\pi^-\pi^0\chi_{bJ}$ events. In the fit, the shapes of $\omega\chi_{bJ}$ and $\pi^+\pi^-\pi^0\chi_{bJ}$ obtained from MC simulation are used to describe the signals, and a 2D function $f(x, y) = ax + by$ ($x = M(\gamma\Upsilon(1S))$ and $y = M(\pi^+\pi^-\pi^0)$) is used to fit the background. Here the $\pi^+\pi^-\pi^0\chi_{bJ}$ MC sample is generated following a four-body phase space

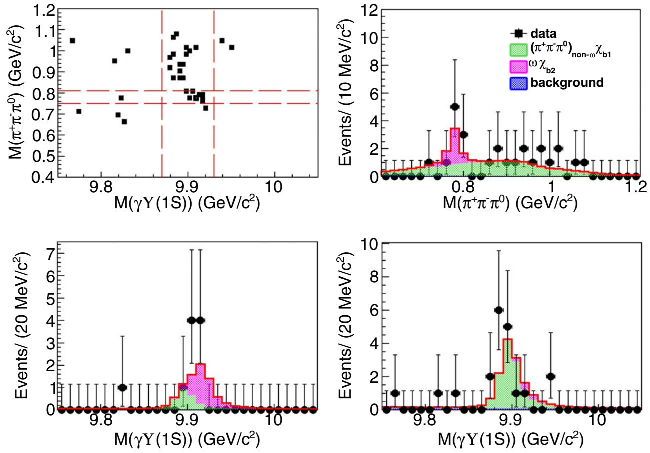

FIG. 1. A scatter plot of $M(\pi^+\pi^-\pi^0)$ versus $M(\gamma\Upsilon(1S))$ from data (top left), and the projections of the 2D fit for events in the χ_{bJ} signal region (top right), in the ω signal region (bottom left), and out of the ω signal region (bottom right).

(PHSP) distribution, and this process is denoted as $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{bJ}$. The projections of the fit results for events in the χ_{bJ} signal region ($M(\gamma\Upsilon(1S)) \in [9.87, 9.93]$ GeV/c^2), in the ω signal region, and in the region above the ω mass are also shown in Fig. 1. The statistical significances for $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{b1}$, $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{b2}$, $\omega\chi_{b1}$ and $\omega\chi_{b2}$ are 5.3σ, 0.0σ, 0.0σ and 2.5σ, respectively. The significances are calculated based on the change in likelihood when the signal yield is set to zero in the fit [25]. The signal yields for $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{b1}$ and $\omega\chi_{b2}$ are 19.6 ± 5.3 and 7.8 ± 3.2, respectively, and the signal yields for $\omega\chi_{b1}$ and $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{b2}$ are consistent with zero. Then we assume that either the processes $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{b1}$ and $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{b2}$ exist at the same time, or the processes $\omega\chi_{b1}$ and $\omega\chi_{b2}$ exist at the same time, and the fit is repeated. The statistical significances for $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{bJ}$ and $\omega\chi_{bJ}$ are 6.1σ and 4.0σ, respectively. The changes on the significances arise from the similarity in signal shapes between $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{b1}$ and $(\pi^+\pi^-\pi^0)_{\text{non}-\omega}\chi_{b2}$, and between $\omega\chi_{b1}$ and $\omega\chi_{b2}$. Thus, evidence for $\omega\chi_{bJ}$ has been found, but we cannot determine whether the events are from $\omega\chi_{b1}$ or $\omega\chi_{b2}$. We also use other forms of background descriptions as systematics. Changes in the signal yields and significances are negligible.

In order to study the energy dependent cross section of $\pi^+\pi^-\pi^0\chi_{b1}$ and $\pi^+\pi^-\pi^0\chi_{b2}$ events, we extract the observed signal yields N_{obs} with data samples listed in Table II. Because of the limited statistics for most energy points, we do not perform a 2D fit as for the summed sample, nor do we separate $\pi^+\pi^-\pi^0$ into ω and non-ω, nor $\gamma\Upsilon(1S)$ into χ_{b1} and χ_{b2}. The number of χ_{bJ} signal events in each sample is computed using the formula: $N_{\text{obs}} = N_{\text{sig}} - N_{\text{side}}$, where N_{sig} is the number of events in the χ_{bJ} signal region and N_{side} is that in the sideband region. Here the signal region is defined as $M(\gamma\Upsilon(1S)) \in [9.852, 9.952]$ GeV/c^2, while the sideband region is [9.77, 9.82] and [9.98, 10.03] GeV/c^2.

The Born cross sections are calculated with

$$\sigma^{\text{Born}} = \frac{N_{\text{obs}}}{\epsilon\mathcal{B}_{\text{inter}}\mathcal{L}(1+\delta)/|1-\Pi|^2}, \tag{1}$$

where ϵ is the reconstructed efficiency, $\mathcal{B}_{\text{inter}}$ is the corresponding product of intermediate decay branching fractions, $\mathcal{L}$ is the integrated luminosity, $(1+\delta)$ is the ISR correction factor, and $(1/|1-\Pi|^2)$ is the vacuum polarization factor [26]. We use the weighted branching fraction $\mathcal{B}_{\text{inter}} = \mathcal{B}(\chi_{b1} \to \gamma\Upsilon(1S)) \cdot f + \mathcal{B}(\chi_{b2} \to \gamma\Upsilon(1S)) \cdot (1-f)$, where $f = N_1/(N_1+N_2) = 0.74 \pm 0.06$ is the fraction of χ_{b1} in the process $e^+e^- \to \pi^+\pi^-\pi^0\chi_{bJ}$ near the $\Upsilon(10860)$ peak [4]. In order to estimate the ISR correction factors, we use

$$1+\delta = \frac{\int_0^{1-\frac{m_0^2}{s}} G_{\text{BW}}(s(1-x))F(x,s)dx}{G_{\text{BW}}(s)}, \tag{2}$$

where m_0 is the mass threshold of $\pi^+\pi^-\pi^0\chi_{bJ}$, $F(x,s)$ is the radiative function [23] and $G_{\text{BW}}(s)$ is the Breit-Wigner (BW) function,

$$G_{\text{BW}}(s) = \frac{12\pi\Gamma_{ee}\cdot\mathcal{B}\cdot\Gamma_{\text{tot}}}{(s-M^2)^2 + M^2\Gamma_{\text{tot}}^2} \times \frac{\Phi(s)}{\Phi(M)}, \tag{3}$$

where M is the nominal mass of $\Upsilon(10860)$ or $\Upsilon(11020)$, Γ_{tot} is the total width, Γ_{ee} is the partial decay width of e^+e^- channel, $\mathcal{B}$ is the branching fractions of $\pi^+\pi^-\pi^0\chi_{bJ}$, and Φ is given by considering $\pi^+\pi^-\pi^0$ as a wide resonance with mass distribution generated in four-body $e^+e^- \to \pi^+\pi^-\pi^0\chi_{bJ}$ phase space.

The energy-dependent cross sections for $e^+e^- \to \pi^+\pi^-\pi^0\chi_{bJ}$ are listed in Table II and plotted in Fig. 2. A maximum likelihood fit of the cross sections is performed.

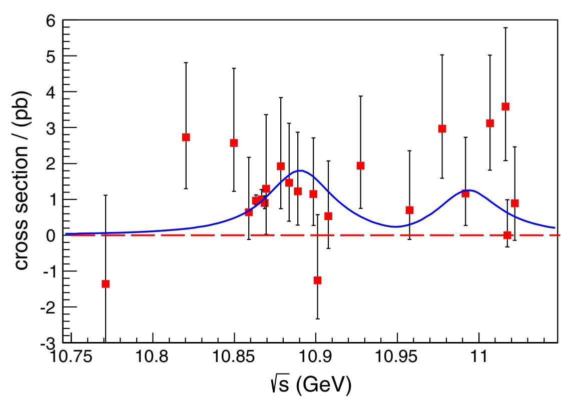

FIG. 2. Fit to the cross sections of $e^+e^- \to \pi^+\pi^-\pi^0\chi_{bJ}$ as described in the text. The red boxes with error bars are the cross sections of $e^+e^- \to \pi^+\pi^-\pi^0\chi_{bJ}$ and the solid blue curve is the fit.

The likelihood for the three data samples of larger integrated luminosity around 10.865 GeV is calculated assuming the number of signal events follows the Gaussian distribution:

$$L(\mu_{\rm sig}; N_{\rm obs}, \sigma) = \frac{1}{\sqrt{2\pi}\sigma'} e^{-\frac{(\mu_{\rm sig}-N_{\rm obs})^2}{2\sigma'^2}}, \quad (4)$$

where $\mu_{\rm sig}$ is the number of expected signal events, and σ' is the statistical uncertainty of $N_{\rm obs}$. For the other samples, the likelihood is calculated assuming the number of signal events follows the Poisson distribution:

$$L(\mu_{\rm sig}; N_{\rm sig}, N_{\rm side}) = \int_0^\infty P(N_{\rm sig}; \mu_{\rm sig} + \mu_{\rm bkg}) P(N_{\rm side}; \mu_{\rm bkg}) d\mu_{\rm bkg}, \quad (5)$$

where $P(N;\mu) = \frac{1}{N!}\mu^N e^{-\mu}$ is the probability density function of the Poisson distribution, and $\mu_{\rm bkg}$ is the number of expected background events. Since the known cross section energy dependences, i.e., those of $\pi\pi\Upsilon(nS)$ [8] and $\pi\pi h_b(mP)$ [9], exhibit $\Upsilon(10860)$ and $\Upsilon(11020)$ peaks but no nonresonant contributions. The fit function here is also a coherent sum of two BW amplitudes in the form of Eq. (3) for $\Upsilon(10860)$ and $\Upsilon(11020)$, and the masses and widths are fixed to their world average values [22] while the corresponding products $\Gamma_{ee} \cdot \mathcal{B}$ are left free. The fit results are shown in Fig. 2. Two solutions are found that differ in phase, but the resulting $\Gamma_{ee} \cdot \mathcal{B}$ are consistent with each other. The obtained product branching fractions are $\mathcal{B}(\Upsilon(10860) \to e^+e^-) \cdot \mathcal{B}(\Upsilon(10860) \to \pi^+\pi^-\pi^0\chi_{bJ}) = (15.3 \pm 3.7) \times 10^{-9}$, $\mathcal{B}(\Upsilon(11020) \to e^+e^-) \cdot \mathcal{B}(\Upsilon(11020) \to \pi^+\pi^-\pi^0\chi_{bJ}) = (18.3 \pm 9.0) \times 10^{-9}$, where the errors are statistical. We also try to introduce a coherent continuum component into the fit, but the significance of this hypothesis is only 1.4σ. The introduction of the continuum term results in a change of the $\Upsilon(10860)$ product branching fraction of 12.6×10^{-9} and that of the $\Upsilon(11020)$ product branching fraction of 12.8×10^{-9}, which are taken as systematic uncertainty due to "continuum contribution."

There are several sources of systematic error in the cross section measurements, and most of the uncertainties are similar to the previous work [4], including tracking efficiency (1.0% per pion and kaon track and 0.35% per lepton), PID efficiency (1.3% per pion and 1.6% per lepton), photon energy resolution calibration (1.1%), π^0 selection (2.2%), 5C kinematic fit (4.2%), and trigger simulation (3.0%). The uncertainty from luminosity is 1.5% [9]. Comparing the reconstruction efficiency with the ISR process in EVTGEN with the efficiency without the ISR process added to EVTGEN, but still corrected for with the ISR correction factor, yields an uncertainty of 1.0%. The corresponding uncertainty from the branching fractions of $\chi_{bJ} \to \gamma\Upsilon(1S)$, $\Upsilon(1S) \to \ell^+\ell^-$ is 8.2% [22]. The total systematic uncertainty, 11.9%, is obtained by adding all the above results in quadrature.

The systematic uncertainty in the measured branching fractions rises from the cross section measurements and the fit to those cross sections. The systematic uncertainties in the fit to the cross sections mainly come from the parametrization of the BW function, PHSP factor, resonance parameters, and the possible continuum contribution. The first is estimated by replacing the constant width with an energy dependent width $\Gamma_{\rm tot} = \Gamma^0_{\rm tot} \cdot \Phi(\sqrt{s})/\Phi(M)$. The second source is estimated by replacing the PHSP factor of $\pi^+\pi^-\pi^0\chi_{bJ}$ with the two-body PHSP factor of $\omega\chi_{bJ}$. The third source is estimated by varying the resonance parameters $\Upsilon(10860)$ and $\Upsilon(11020)$ within $\pm 1\sigma$. The final systematic uncertainty is estimated by adding a coherent continuum contribution to the fit function. The changes of the branching fractions are taken as the symmetrized systematic uncertainty. The details are listed in Table III.

By using $\mathcal{B}(\Upsilon(10860) \to e^+e^-) = (6.1 \pm 1.6) \times 10^{-6}$ and $\mathcal{B}(\Upsilon(11020) \to e^+e^-) = (2.1^{+1.1}_{-0.6}) \times 10^{-6}$ [22], we obtain $\mathcal{B}(\Upsilon(10860) \to \pi^+\pi^-\pi^0\chi_{bJ}) = (2.5 \pm 0.6 \pm 2.1 \pm 0.7) \times 10^{-3}$, $\mathcal{B}(\Upsilon(11020) \to \pi^+\pi^-\pi^0\chi_{bJ}) = (8.7 \pm 4.3 \pm 6.1^{+4.5}_{-2.5}) \times 10^{-3}$, where the first errors are statistical, the second are systematic errors combined from the cross section measurements and line shape fit, and the third result from the branching fractions of $\Upsilon(10860)$ and $\Upsilon(11020) \to e^+e^-$ [22].

To reconstruct $e^+e^- \to \phi\chi_{bJ}$, we require at least two kaons in one event. There is no requirement on the number of photons, but a list of photon candidates is created in one event satisfying $|M(\gamma\gamma_2) - m_{\pi^0}| > 13\ {\rm MeV}/c^2$, where γ_2 is any other photon in the event with $E_{\gamma_2} > 0.1$ GeV, and m_{π^0} is the nominal mass of the π^0. The data are divided into two categories. One includes events when one of the photons in the above list satisfies $M(\gamma K^+K^-)_{\rm recoil} \equiv \sqrt{(\sqrt{s} - E_{\gamma K^+K^-})^2 - (\vec{p}_{\gamma K^+K^-})^2} \in [9.42, 9.50]\ {\rm GeV/c^2}$, i.e., in the $\Upsilon(1S)$ mass region, to tag $\chi_{bJ} \to \gamma\Upsilon(1S)$ events; the other includes all other events, to tag $\chi_{bJ} \to {\rm non}-\gamma\Upsilon(1S)$ events. Here $(\vec{p}_{\gamma K^+K^-}, E_{\gamma K^+K^-})$ is the four momenta of γK^+K^- system in c.m. frame.

TABLE III. Summary of the absolute systematic uncertainties in product branching fractions ($\times 10^{-9}$), where $\mathcal{B}(10860, 11020)$ represent $\mathcal{B}(\Upsilon(10860, 11020) \to e^+e^-) \cdot \mathcal{B}(\Upsilon(10860, 11020) \to \pi^+\pi^-\pi^0\chi_{bJ})$.

$\pi^+\pi^-\pi^0\chi_{bJ}$	$\mathcal{B}(10860)$	$\mathcal{B}(11020)$
Cross sections	1.8	2.1
BW parametrization	0.6	0.4
PHSP factor	0.6	0.2
Resonance parameters	2.4	1.6
Continuum contribution	12.2	12.6
Sum	12.6	12.8

We use the figure of merit, $\mathcal{S}/\sqrt{\mathcal{S}+\mathcal{B}}$, to optimize the K^+K^- invariant mass window requirement. Here $\mathcal{S}$ is the reconstructed number of signal events obtained from MC simulation of the signal process, $\Upsilon(11020) \to \phi\chi_{bJ}$ with $\phi \to K^+K^-$, $\chi_{bJ} \to$ anything, in the signal region, $[9.88, 9.93]$ GeV$/c^2$. The number is normalized according to the theoretical calculation of the branching fraction of $\Upsilon(11020) \to \phi\chi_{bJ}$ [16] and the total $\Upsilon(11020)$ events in our data sample. $\mathcal{B}$ is the number of background events in the signal region in the generic MC sample with the c.m. energy shifted to 11.022 GeV. We require $M(K^+K^-)$ to be within $m_\phi \pm 7.5(7.0)$ MeV/c^2 for category one (two), where m_ϕ is the nominal mass of ϕ [22]. The ϕ mass sideband region is defined as $M(K^+K^-) \in [1.000, 1.005]$ or $[1.035, 1.040]$ GeV$/c^2$. There is no evidence for the χ_{bJ} signal in the ϕ mass sideband events, nor in the generic MC sample (significance is less than 0.1σ from the fit) mentioned above.

After applying all the selection criteria, the recoil mass spectra of ϕ as a function of the initial beam four momenta from both data categories are shown in Fig. 3 for the sum of data in the energy region $\sqrt{s} = 10.96$–11.05 GeV. We perform a simultaneous unbinned maximum likelihood fit to the ϕ recoil mass spectra with the signal shapes from the simulated signal MC shapes, and a background shape obtained from data with the following procedure: a series of shapes are obtained from $\Upsilon(5S)$ data, where, in calculating the ϕ recoil mass, the c.m. energy is changed to that of each individual data point, and summing up the shapes according to the luminosity. The ratios of the numbers of χ_{b1} or χ_{b2} events in the two categories are fixed according to the expected branching fractions of χ_{b1} or $\chi_{b2} \to \gamma\Upsilon(1S)$ [22] and the efficiencies. The fit results, which yield $\chi^2/\mathrm{ndf} = 104.2/55 = 1.9$, are shown in Fig. 3. According to the fit, $(1.5 \pm 0.5) \times 10^3$ χ_{b1} and $(2.4 \pm 0.5) \times 10^3$ χ_{b2} events are produced. The statistical significances are found to be 3.3σ and 4.8σ for χ_{b1} and χ_{b2}, respectively.

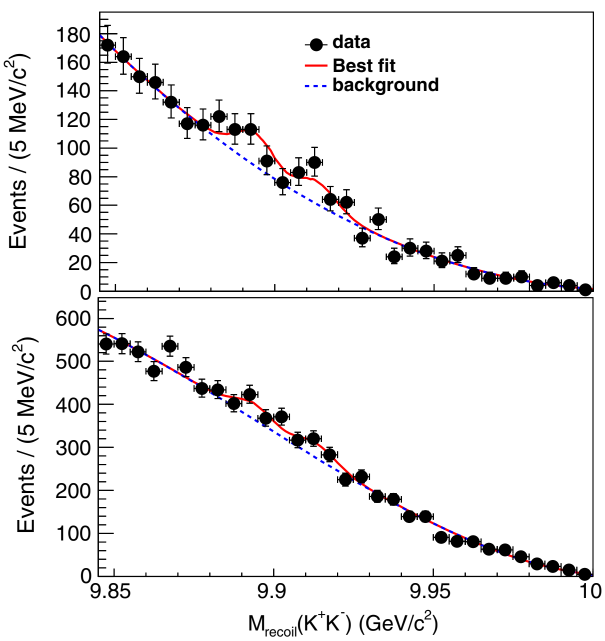

FIG. 3. The simultaneous fit results for data having $M_{\mathrm{recoil}}(\gamma K^+K^-)$, with the recoiling mass of γK^+K^-, in the $\Upsilon(1S)$ mass window (up) and out of the $\Upsilon(1S)$ mass window (down). Dots with error bars are data, the red solid lines are the best fit, and blue dashed lines are backgrounds.

When we vary the background shape by multiplying the nominal background shape with a first-, second-, or third-order polynomial, the smallest significances of the χ_{b1} and χ_{b2} signals are found to be 2.6σ and 2.1σ, respectively (multiplying by the third-order polynomial), yielding $\chi^2/\mathrm{ndf} = 43.6/49 = 0.89$. The most conservative upper limits on the numbers of produced signal events in all the above tests are reported. After considering the systematic uncertainty which we discuss later, the upper limits for the produced numbers of $\phi\chi_{b1}$ and $\phi\chi_{b2}$ signal events are determined to be 2.2×10^3 and 3.1×10^3 at 90% confidence level (C.L.), respectively. The upper limits on the Born cross sections of $e^+e^- \to \phi\chi_{b1}$ and $\phi\chi_{b2}$ are 0.7 and 1.0 pb, respectively, averaged over the $\Upsilon(11020)$ region, specifically $\sqrt{s} = 10.96$–11.05 GeV. The calculation is based on Eq. (1), where the reconstruction efficiency, ISR correction factor, and vacuum polarization factor are averaged with weights according to the luminosity of each sample.

The sources of systematic uncertainties in the $\phi\chi_{bJ}$ cross section measurement are similar to those of the $\pi^+\pi^-\pi^0\chi_{bJ}$ modes, including the tracking efficiency, PID, photon detection, luminosity, trigger simulation, ISR correction, ϕ mass window, and intermediate branching fraction. Most of these have been discussed in the $\pi^+\pi^-\pi^0\chi_{bJ}$ analysis. The uncertainty from the ϕ mass window requirement is found to be negligible by studying the consistency of the K^+K^- invariant mass between data and MC simulation. The uncertainty from the branching fraction of $\phi \to K^+K^-$ is 1.0% [22]. The total systematic uncertainty for the cross section measurement is thus, combining all uncertainties in quadrature, 5.5% for either $e^+e^- \to \phi\chi_{b1}$ or $\phi\chi_{b2}$.

In summary, using the energy scan data in the vicinity of the $\Upsilon(11020)$ resonance, we observe the $e^+e^- \to (\pi^+\pi^-\pi^0)_{\mathrm{non}-\omega}\chi_{b1}$ process with significance of 5.3σ. Evidence for $\omega\chi_{bJ}$ is found but we cannot tell whether it is $\omega\chi_{b1}$ or $\omega\chi_{b2}$. The limited statistics prevents us from drawing a conclusion concerning the origin of the signal events, that is, whether they arise from bottomonium decay, continuum production, or both. Since no continuum production of a multibody final state with a bottomonium is known, it is natural to assume that the origin of the signal is bottomonium. Under this assumption, the branching fractions are $\mathcal{B}(\Upsilon(10860) \to \pi^+\pi^-\pi^0\chi_{bJ}) = (2.5 \pm 0.6 \pm 2.1 \pm 0.7) \times 10^{-3}$, which is compatible with the previous measurement [4], and $\mathcal{B}(\Upsilon(11020) \to \pi^+\pi^-\pi^0\chi_{bJ}) = (8.7 \pm 4.3 \pm 6.1^{+4.5}_{-2.5}) \times 10^{-3}$, which is compatible with the theoretical predictions [16]. Based on the 2D fit with summed data, the relative magnitude $R_{21}(\omega) \equiv \frac{\mathcal{B}(\Upsilon(11020)\to\omega\chi_{b2})}{\mathcal{B}(\Upsilon(11020)\to\omega\chi_{b1})}$ can be estimated to be 0.4 ± 0.2, where the common systematic uncertainties cancel.

The processes $e^+e^- \to \phi\chi_{bJ}$ are also searched for in data within $\sqrt{s} = 10.96$–11.05 GeV, with no significant signals being observed. We report upper limits on the Born cross sections of $e^+e^- \to \phi\chi_{b1}$ and $\phi\chi_{b2}$ as 0.7 and 1.0 pb at 90% C.L., respectively. Compared with the total cross section of $e^+e^- \to \Upsilon(11020)$, these upper limits correspond to $\Upsilon(11020)$ decay branching fractions of order 10^{-3}, well above the theoretical predictions of order 10^{-6} [16].

Our measurement of the transition rate agrees with the expectation of Ref. [16], but the measured relative magnitudes $R_{21}(\omega)$ are significantly less than the theoretical predictions, which should be more reliable than the branching fraction predictions. This may inspire theorists to further investigate the discrepancy between the experimental measurement and the theoretical calculation.

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