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Momentum Transfer through Landau Damping and Radio Frequency Current Drive

Kazuya UEHARA

Division of Advanced Plasma Research
Fusion Research and Development Directorate

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Kazuya UEHARA

Division of Advanced Plasma Research, Fusion Research and Development Directorate,
Japan Atomic Energy Agency, Naka-shi, Ibaraki-ken

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Summary - The current drive generated by radio frequency waves is regarded as the momentum transfer from waves to resonant electrons through the Landau damping process. The momentum transfer is evaluated using a well-defined model of Landau damping and the plasma current is formulated from the computation between the acceleration by the effective force acting on electrons and the collision. A deformed distribution function to yield the plasma current is formulated with a Boltzmann equation including a simple collision term. This is the basis of the first order current drive theory at the linear Landau damping.

Keywords: Current Drive, Radio Frequency Waves, Momentum Transfer, Landau Damping, Effective Force, Boltzmann Equation, Collision Term

ランダウ減衰の過程における運動量変換と高周波電流駆動

日本原子力研究開発機構
核融合研究開発部門　先進プラズマ研究開発ユニット
上原　和也

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高周波によって駆動される電流駆動はランダウ減衰を通して波から共鳴粒子に運動量が変換されるものと見なす事が出来る。運動量の変換がよく知られたランダウ減衰のモデルによって評価され、電子に働く等価的な力による電子の加速と衝突との競合から電流値が定式化される。簡単な衝突項を含んだボルツマン方程式から変形された分布関数が評価され、電流駆動に於ける電流値が定式化される。これは線形ランダウ減衰における電流駆動の理論的基礎を与える。

那珂核融合研究所（駐在）：〒３１１－０１９３　茨城県那珂市向山８０１－１

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1. INTRODUCTION

A non-inductive current drive is fully expected for the stationary fusion reactor in large tokamaks. In particular, a lower-hybrid wave can drive a more substantial current[1] than other non-inductive current drive methods such as the electron cyclotron wave[2] and the neutral beam[3] in tokamaks, and furthermore, the current drive has been recently performed for several hours in the super-conducting tokamak of TRIAM-1M[4]. This idea of the Direct Current Tokamak (DCT) was first proposed by Wort for the Alfven wave[5]. The theoretical basis of this phenomena has been discussed in articles by Kima[6)-7] and Fisch[8], making, a quasi-linear approximation, and in those by Mizuno[9] regarding the collisionless case. The current drive phenomena is regarded as the momentum transfer from waves to resonant electrons via the Landau damping process[10]. However, in the framework of quasi-linear theory, which is often used for describing phenomena of the current drive due to rf waves, the transfer of momentum is not included. It should be recognized that a process of Landau damping is basically not only the energy transfer but also the momentum transfer to resonant particles, causing the plasma heating as well as the current drive, respectively. Nevertheless, the quasilinear scheme describing the current drive phenomenon is not clear whether the momentum is really transfered to particles. As is indicated by Stix if the momentum transfer is completed in the quasi-linear scheme the electromagnetic component of the wave is necessary which does not exist[11]. Regarding this problem, Berndtson derived the Vlasov-Poisson equation, including a collision term which is applicable to the current drive phenomena[11], as an alternative to the quasi-linear scheme. The diffusion of electrons in the velocity space induced by a spectrum wave is numerically calculated, and results show strong disagreement with the quasi-linear scheme in the higher rf electric field regime[12)-13]. Furthermore, it is reported that the current flow found in experiments does not agree with the conventional theoretical understanding based on the quasi-linear scheme. On the other hand, we have already reported on a scheme to describe the current drive phenomena without using any quasi-linear scheme elsewhere[14]. There we formulated a dynamical picture to describe the current drive on the basis of a well-defined treatment of Landau damping. This theory may be regarded as a zeroth order linear theory for the current drive.

In this paper, we attempt to give the detailed physical background of this picture, that is, to exactly evaluate the momentum transfer from rf wave to particles. In this evaluation, the effective DC electric field which is actually felt by resonant electrons is derived, and the deformed distribution function which yields the current is evaluated according to the simple Lorentz gas model.

2. MOMENTUM TRANSFER DURING LANDAU DAMPING

Simple Landau damping describes the wave energy transfered to particles in a collisionless state. The plasma wave has momentum as well as energy, so that the wave can transfer momentum to particles. If the wave is traveling, the wave can give momentum continuously in one direction to form a current. This momentum transfer is performed through the force acting on electrons. This force , F = d(mv)/dt , pushes electrons and so drives an electron current. The problem here is how the rf wave can transfer the momentum to particles to form a DC current although rf wave is oscillating. By a perturbation technique, we can calculate the absorption of momentum by particles from an rf wave. It is known that the absorption of momentum is large when there are a large number of particles with streaming velocities equal to the phase velocity. In first, we formulate the effective force acting on resonant electron and we derive the momentum transfer form waves driven landau damping process. We describe the motion of a single particle in one dimension in a the traveling type rf electric field by the, $E=E_{rf}\cos(k_z z-\omega t)$ so that[15)]

$$m\frac{dv}{dt} = eE_{rf}\cos(k_z z - \omega t) \tag{2.1}$$

The zeroth order motion corresponds to the free streaming, setting R.H.S.=0($v=v_0$, $z=z_0$ at t=0 in equation (2.1))

$$z=v_0 t+z_0 \tag{2.2}$$

The first order motion is obtained by substituting equation (2.2) into equation (2.1), and setting $v_1=0$ at t=0, the Lamdau damping being an initial value problem, resulting

$$v_1 = \frac{eE_{rf}}{m\alpha}[\sin(k_z z_0 + \alpha t) - \sin(k_z z_0)] \tag{2.3}$$

For the second term, we obtain

$$z_1 = \int_0^t v_1 dt = \frac{eE_{rf}}{m\alpha}\frac{\cos(k_z z_0 + \alpha t) - \cos(k_z z_0)}{\alpha} - t sin(k_z z_0) \qquad (2.4)$$

where

$$\alpha = k_z v_0 - \omega \qquad (2.5)$$

We substitute $z=z_0+v_0t+z_1$ into equation (2.1) and average on z_0, considering that $k_z z_1<<1$ and $\cos k_z z_1 \sim 1$, $\sin k_z z_1 = k_z z_1$, $<\cos(k_z z_0)\sin(k_z z_0+\alpha t)>=(1/2)\sin(\alpha t)$, and $<\sin(k_z z_0)\sin(k_z z_0+\alpha t)>=(1/2)\cos(\alpha t)$. After some trigonometric expansions, we can average over the initial position z_0 and we thus estimate the momentum transfer, which is directry the effective force $<F>$ acting on electrons as

$$\langle F\rangle = \left\langle \frac{d}{dt}(mv)\right\rangle_{z_0} = k_z \frac{e^2E_{rf}^2}{2m}\left(-\frac{sin\alpha t}{\alpha^2} + \frac{t\cos\alpha t}{\alpha}\right) = k_z \frac{e^2E_{rf}^2\pi}{2m}\delta'(\alpha) \qquad (2.6)$$

where $\delta'(\alpha)$ is the derivative of the Dirac's delta function :

$$\delta'(\alpha) = \lim_{t\to\infty}\frac{1}{\pi}\left(-\frac{\sin\alpha t}{\alpha^2} + \frac{t\cos\alpha t}{\alpha}\right) \qquad (2.7)$$

, with respect to α and ω is the injected wave frequency. The equation (2.6) indicates the force and/or the momentum transfer from waves which are acting on electrons. The rf wave interacts with almost all particles only slightly ; however, particles around the phase velocity strongly interact with waves to received their force and/or momentum since the value of $\delta'(a)$ is a finite value only at $\alpha=0$, that is , where $v_0=\omega/k_z$. Equation (2.6) is graphed against v in Fig.1, where we can see that this value is positive only where $v_0< \omega/k_z$ and negative at $v_0>\omega/k$, and almost zero where v is far from ω/k_z. We integrate equation (2.6) substituting the velocity distribution function f(v) for the velocity v and we get

$$P = \int_{-\infty}^{\infty} \left\langle \frac{d}{dt}(mv) \right\rangle_{z_0} f(v)dv = \int_{-\infty}^{\infty} k_z \frac{e^2 E_{rf}^2 \pi}{2m} \delta'(\alpha) f(v)dv = -\frac{e^2 E_{rf}^2 \pi}{2mk_z} \frac{df}{dv_z}\bigg|_{v_z = \frac{\omega}{k_z}} \qquad (2.8)$$

, which is the whole momentum transfer from waves to particles. Equation (2.8) gives us an answer for the question posed at the beginning of this section: absorption of momentum occurs if there are many particles streaming infinitesimally slower than the phase velocity of the electric field and relatively fewer particles streaming fatser than the phase velocity. A negative sign for $f'(\omega/k_z)$ means that momentum can be transferred from the waves which are velocities close to those for Maxwellian plasma, and a positive sign for $f'(\omega/k_z)$ means that momentum can be transferred from particles to waves. Considering the energy transfer in Landau damping[16),

$$W = \int_{-\infty}^{\infty} \left\langle \frac{d}{dt}\left(\frac{1}{2}mv^2\right) \right\rangle_{z_0} f(v)dv = -\frac{\pi\omega e^2 E_{rf}^2}{2mk_z^2} \frac{df}{dv_z}\bigg|_{v_z = \frac{\omega}{k_z}} \qquad (2.9)$$

, and the ratio of the momentum transfer to the energy transfer is well-known relation

$$\frac{P}{W} = \frac{k}{\omega} \qquad (2.10)$$

3. FORMULATION OF CURRENT DRIVE WITH A MODEL COLLISION TERM

The equation (2.8) expreses the force acting on electrons, and considering the average electric field P/(-e), and the particle accelerated by this electric field, and balancing this with collisions, we can form the current j

$$j = \frac{eP}{m\nu_d} = \frac{e}{m\nu_d} \int_{-\infty}^{\infty} \left\langle \frac{d}{dt}(mv) \right\rangle_{z_0} f(v)dv = -\frac{e^3 E_{rf}^2 \pi}{2m^2 k_z \nu_d} \frac{df}{dv_z}\bigg|_{v_z = \frac{\omega}{k_z}} \qquad (3.1)$$

where ν_d is the collision frequency. In this formulation, the driving efficiency becomes

$$\frac{j}{W} = \frac{e}{mv_d}\frac{k_z}{\omega} \qquad (3.2)$$

Equation (3.2) is a well-known expression of driving efficiency of the current drive.

The Vlasov equation having a collisional term approximated by $(f-f_0)/\tau$ is given by

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m}\cdot\frac{\partial f}{\partial \vec{v}} = -\frac{f-f_0}{\tau} \qquad (3.3)$$

where f_0 is Maxwellian distribution , which is transformed by f applying to the force $\vec{F}$. The electrons receiving force $\vec{F}$ accelerate to one direction, and the system is brought into a non-equilibrium state. Since the time derivative is small for small $\vec{F}$ the stationary state ($\partial/\partial t=0$) is considered. The f must change depending on the location, but we assume that the deviation from the thermal equilibrium state is small and the spatial displacement can be neglected ($\partial/\partial\vec{r} = 0$) . The value of τ is the relaxation time, when the collisionality tends to restore f to the Maxwellian f_0. The problem here is how the distribution function is deformed by the constraint of the force $\vec{F}$. The Boltzmann equation (3.3) is a linear differential and integral equation determining the deviation from the Maxwellian distribution. This equation is solved by assuming that the solution is given by $f = f_0 + \vec{v}\cdot\vec{c}\frac{\partial f_0}{\partial \varepsilon}$ exactly, where $\varepsilon(=mv^2/2)$ is the kinetic energy. The vector $\vec{c}$ is estimated as , $\vec{c} = -\vec{F}/v_d$, following Grechiko[17] and using the Lorentz gas model, where $v_d(=\tau^{-1})$ is the collision frequency. The distribution function reaches a thermodynamical state of equilibrium between the acceleration due to $\vec{F}$ and collisional relaxation. If the force $\vec{F}$ remains at a finite value continuously to one direction, then f becomes non-Maxwellian and forms a net current in the toroidal direction. The distribution function f is more relaxed than f_0 if the constraint on $\vec{F}$ is removed. The current j as a function of f is given by

$$j = -e\int_{-\infty}^{\infty} vf(v)dv \qquad (3.4)$$

When $\vec{F}=0$, then f becomes f_0 and no net current flows, that is, j=0. When $\vec{F} = e\vec{E}$, which is the case where there is the inductive current drive, then f becomes

$$f=f_0(1+eE/kT_e)\tau v) \tag{3.5}$$

, where for the second term in equation (3.3) the Maxwellian f_0 is assumed. We can obtain equation (3.4) by substituting in equation (3.5) the well-known relation $j=\sigma E$ for constant τ, where $\sigma(=en^2/m\nu_d)$ is the electric conductivity. It is well-known that σ generally can be evaluated when τ has dependence on v^3 since for fully ionized plasma, τ is proportional to v^3, as explained by Spitzer[18]. Here, we combine equation (2.6) with $F=\langle d(mv)/dt\rangle_{z0}=eE_{eff}$ so that the effective DC electric field E_{eff} for resonant electrons can be expressed as

$$E_{eff}=\frac{F}{e}=k_z\frac{eE_{rf}^2\pi}{2m}\delta'(\alpha) \tag{3.6}$$

E_{eff} has significant value only when α is close to zero since E_{eff} is almost zero far from $\alpha=0$, which indicates that only electrons having a velocity close to the wave phase velocity feel the DC electric field from waves. An asymmetric distribution function to derive the rf current is created by substituting equation (3.6) into equation (3.5), so that f becomes

$$f=f_0(1+k_z\frac{e^2E_{rf}^2\pi}{2mkT_e}\delta'(\alpha)\tau v) \tag{3.7}$$

It should be noted that the value of $k_z\frac{e^2E_{rf}^2\pi}{2mkT_e}\delta'(\alpha)\tau v f_0$ is relatively small compared with f_0. However, the equation (3.7) express the asymmetric distribution generating the current. The term f_0 and τv ($\propto v^4$) in the equation (3.7) are both even functions; however, $\delta'(\alpha)$ is an odd function with respect to $\alpha=0$, whose behaviour is shown graphically as shown in Fig. 2 against the velocity. For the value of $k_z\frac{e^2E_{rf}^2\pi}{2mkT_e}\delta'(\alpha)\tau v f_0$, it should be noted that we can see a small finite value near $v_0=-\omega/k_z$ as well as $v_0=+\omega/k_z$, which is just the asymmetric distribution that we seek. As the case of Landau damping of the energy, the momentum transfer occurs around the phase velocity. The larger asymmetry is brought about in cases where there is higher rf power (large E_{rf}) and/or lower collision case (low ν_d) small asymmetry is found where there is lower (small E_{rf}) and/or higher collision case (ν_d). It should be noted that relatively small asymmetry is caused at the velocities whose negative velocitiy is a case of larger asymmetry, which may be caused by the nonlinear product between the derivative of the delta function and the term of v^4 caused by collision at negative phase velocity region ; however, this cancels out when the equation (3.4) applies. This always occurs when the collision frequency causes σ dependence on v^3. The rf driven current is obtained substituting equation (3.7) into (3.4), considering that τ has dependence on v^3 ($\tau=\tau_v v^3/V^3$)

$$j = -e\,\varepsilon_0 \pi \frac{\omega_p^2 E_{rf}^2}{2mk_z} \frac{mV^2}{kT_e} \tau_v (f_0' + \frac{5f_0}{V}) \tag{3.8}$$

where $\tau_v = 2\pi n V^3/\omega_p^4 \ln\Lambda$, $V=\omega/k_z$ and $\ln\Lambda$ is the Coulomb logarithm[19]. This equation is very similar to equation (31) in the formulation by Yoshikawa and Yamato, however, this formulation is deduced from a quasi-linear theory[20].

4. DISCUSSION AND CONCLUSIONS

As early as 1962, Stix derived an average force for resonant electrons in the rf trajectory in the toroidal direction[21]. This formation may be connected to the description of momentum transfer of resonant electrons discussed here. In our previous report, we gave a fundamental picture of the beam perturbation method, and under some condition the current was estimated by this scheme. However, the distribution function was not discussed any more in that report. In this paper, we have tried to clarify the shape of the distribution function based on the simple Krooks collisional operator model as the zeroth order approximation. The current indeed flows when there is asymmetry of the distribution function as predicted in our scheme. It is correct also in the inductive current drive, where that the distribution function has deformation due to the constraint of the force due to the DC electric field, which is shown in the previous section. This situation does not change even when there is rf induced current drive. The strangest feature of current drive in the quasi-linear scheme is the shape of rf spectrum when the optimum current is flowing. If the rf spectrum has wide range of wave numbers there may be a large asymmetry plateau distribution function which causes much rf current. However, experimental results indicate that relatively sharp spectrums cause much rf current[22].

The equations (2.1)-(2.8) are derived by the same method as that for deriving energy in

Landau damping, described in many textbooks on plasma physics (for example STIX[16]). This formulation may correspond to the equation derived by Midzuno[23], however, he neglects the balance achieved with collision.

There may be some problems in using the simple Krook collisional operator rather than the full Fokker-Planck Coulomb operator, however, it has been pointed out that the Krook model may serve well to incorporate collisions in any model, though only the principal effects of collisions on wave damping are modeled by Livi and Marsch[12].

Recently, the role of collsions in the quasi-linear theory of heating and current drive is discussed by Bilato[25]. This issue is reasonable as far as the heating scheme, however, the item must be limited to the heating regime and the item of the current drive should be removed since the momentum transfer is not included in this issue. The item of momentum is necessary as eq. (2.8) in this issue in order to discuss the current drive. The analysis based on the beam perturbation methid are agree with the experimental results, which will be given a another report[26].

In conclusion, there is momentum transfer from waves to resonant electrons in the Landau damping process. In this process, the time derivative of momentum is evaluated according to a well-defined model of Landau damping, and then current due to the radio frequency current drive is evaluated. The deformation distribution function to yield the net rf current is formulated with a Boltzmann equation including a simple collision term.

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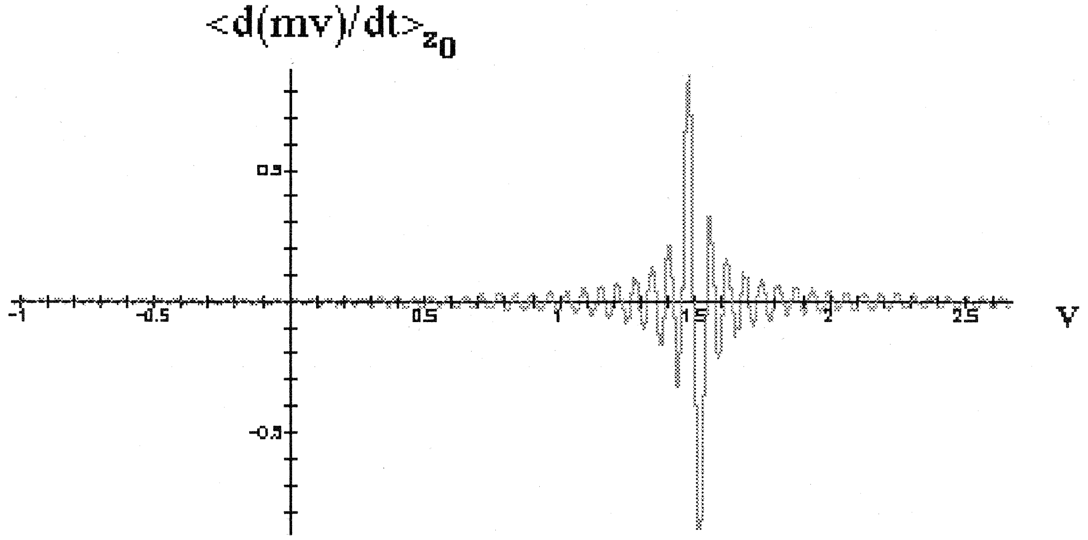

Fig.1

Fig. 1 Equation (2.6) is shown graphically as the enegy transfer of Landau damping.

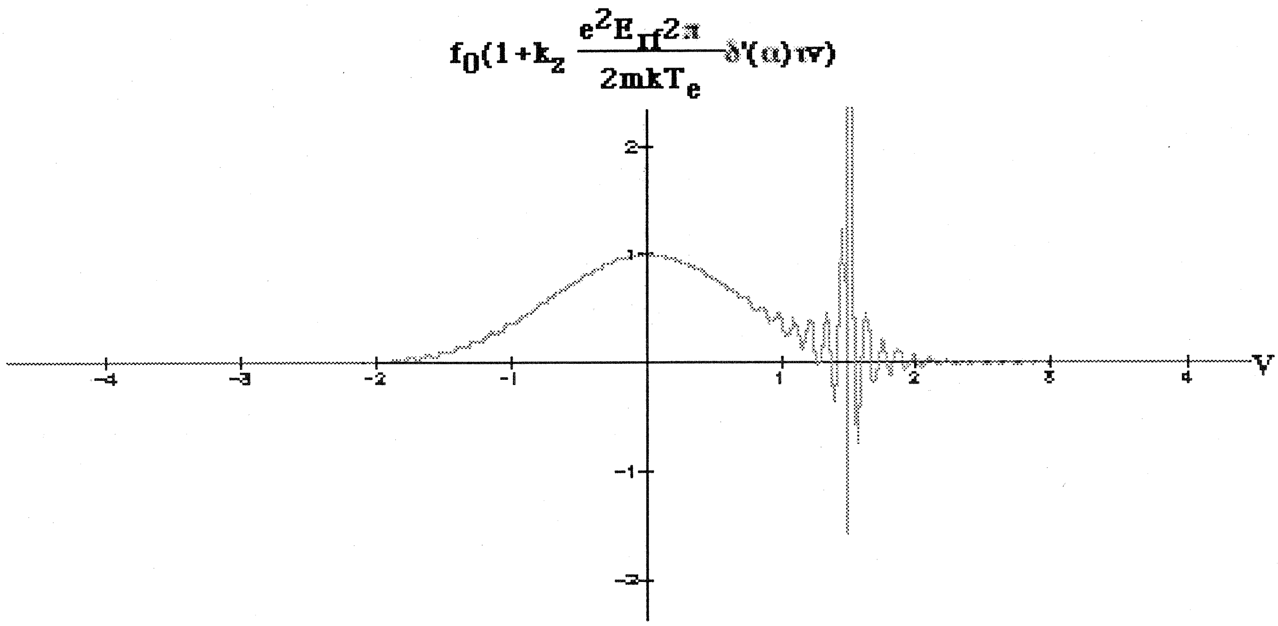

Fig.2

Fig. 2 The value of $f_0(1+k_z e^2 E_{rf} 2\pi\delta'(\alpha)\tau_v/2mkT_e)$ in equation (3.7) is shown against the velocity.

国際単位系（SI）

表１．SI 基本単位

基本量	SI 基本単位	
	名称	記号
長さ	メートル	m
質量	キログラム	kg
時間	秒	s
電流	アンペア	A
熱力学温度	ケルビン	K
物質量	モル	mol
光度	カンデラ	cd

表２．基本単位を用いて表されるSI組立単位の例

組立量	SI 基本単位	
	名称	記号
面積	平方メートル	m^2
体積	立法メートル	m^3
速さ，速度	メートル毎秒	m/s
加速度	メートル毎秒毎秒	m/s^2
波数	毎メートル	m^{-1}
密度，質量密度	キログラム毎立方メートル	kg/m^3
面積密度	キログラム毎平方メートル	kg/m^2
比体積	立方メートル毎キログラム	m^3/kg
電流密度	アンペア毎平方メートル	A/m^2
磁界の強さ	アンペア毎メートル	A/m
量濃度[a]，濃度	モル毎立方メートル	mol/m^3
質量濃度	キログラム毎立法メートル	kg/m^3
輝度	カンデラ毎平方メートル	cd/m^2
屈折率[b]	（数字の） 1	1
比透磁率[b]	（数字の） 1	1

(a) 量濃度（amount concentration）は臨床化学の分野では物質濃度（substance concentration）ともよばれる。
(b) これらは無次元量あるいは次元１をもつ量であるが、そのことを表す単位記号である数字の１は通常は表記しない。

表３．固有の名称と記号で表されるSI組立単位

組立量	SI 組立単位			
	名称	記号	他のSI単位による表し方	SI基本単位による表し方
平面角	ラジアン[b]	rad	1 [b]	m/m
立体角	ステラジアン[b]	sr[c]	1 [b]	m^2/m^2
周波数	ヘルツ[d]	Hz		s^{-1}
力	ニュートン	N		$m\ kg\ s^{-2}$
圧力，応力	パスカル	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
エネルギー，仕事，熱量	ジュール	J	N m	$m^2\ kg\ s^{-2}$
仕事率，工率，放射束	ワット	W	J/s	$m^2\ kg\ s^{-3}$
電荷，電気量	クーロン	C		s A
電位差（電圧），起電力	ボルト	V	W/A	$m^2\ kg\ s^{-3}\ A^{-1}$
静電容量	ファラド	F	C/V	$m^{-2}\ kg^{-1}\ s^4\ A^2$
電気抵抗	オーム	Ω	V/A	$m^2\ kg\ s^{-3}\ A^{-2}$
コンダクタンス	ジーメンス	S	A/V	$m^{-2}\ kg^{-1}\ s^3\ A^2$
磁束	ウエーバ	Wb	Vs	$m^2\ kg\ s^{-2}\ A^{-1}$
磁束密度	テスラ	T	Wb/m^2	$kg\ s^{-2}\ A^{-1}$
インダクタンス	ヘンリー	H	Wb/A	$m^2\ kg\ s^{-2}\ A^{-2}$
セルシウス温度	セルシウス度[e]	℃		K
光束	ルーメン	lm	cd sr[c]	cd
照度	ルクス	lx	lm/m^2	m^{-2} cd
放射性核種の放射能[f]	ベクレル[d]	Bq		s^{-1}
吸収線量, 比エネルギー分与, カーマ	グレイ	Gy	J/kg	$m^2\ s^{-2}$
線量当量, 周辺線量当量, 方向性線量当量, 個人線量当量	シーベルト[g]	Sv	J/kg	$m^2\ s^{-2}$
酸素活性	カタール	kat		s^{-1} mol

(a)SI接頭語は固有の名称と記号を持つ組立単位と組み合わせても使用できる。しかし接頭語を付した単位はもはやコヒーレントではない。
(b)ラジアンとステラジアンは数字の１に対する単位の特別な名称で、量についての情報をつたえるために使われる。実際には、使用する時には記号rad及びsrが用いられるが、習慣として組立単位としての記号である数字の１は明示されない。
(c)測光学ではステラジアンという名称と記号srを単位の表し方の中に、そのまま維持している。
(d)ヘルツは周期現象についてのみ、ベクレルは放射性核種の統計的過程についてのみ使用される。
(e)セルシウス度はケルビンの特別な名称で、セルシウス温度を表すために使用される。セルシウス度とケルビンの単位の大きさは同一である。したがって、温度差や温度間隔を表す数値はどちらの単位で表しても同じである。
(f)放射性核種の放射能（activity referred to a radionuclide）は、しばしば誤った用語で"radioactivity"と記される。
(g)単位シーベルト（PV,2002,70,205）についてはCIPM勧告2（CI-2002）を参照。

表４．単位の中に固有の名称と記号を含むSI組立単位の例

組立量	SI 組立単位		
	名称	記号	SI 基本単位による表し方
粘度	パスカル秒	Pa s	$m^{-1}\ kg\ s^{-1}$
力のモーメント	ニュートンメートル	N m	$m^2\ kg\ s^{-2}$
表面張力	ニュートン毎メートル	N/m	$kg\ s^{-2}$
角速度	ラジアン毎秒	rad/s	$m\ m^{-1}\ s^{-1}=s^{-1}$
角加速度	ラジアン毎秒毎秒	rad/s^2	$m\ m^{-1}\ s^{-2}=s^{-2}$
熱流密度，放射照度	ワット毎平方メートル	W/m^2	$kg\ s^{-3}$
熱容量，エントロピー	ジュール毎ケルビン	J/K	$m^2\ kg\ s^{-2}\ K^{-1}$
比熱容量，比エントロピー	ジュール毎キログラム毎ケルビン	J/(kg K)	$m^2\ s^{-2}\ K^{-1}$
比エネルギー	ジュール毎キログラム	J/kg	$m^2\ s^{-2}$
熱伝導率	ワット毎メートル毎ケルビン	W/(m K)	$m\ kg\ s^{-3}\ K^{-1}$
体積エネルギー	ジュール毎立方メートル	J/m^3	$m^{-1}\ kg\ s^{-2}$
電界の強さ	ボルト毎メートル	V/m	$m\ kg\ s^{-3}\ A^{-1}$
電荷密度	クーロン毎立方メートル	C/m^3	m^{-3} sA
表面電荷	クーロン毎平方メートル	C/m^2	m^{-2} sA
電束密度，電気変位	クーロン毎平方メートル	C/m^2	m^{-2} sA
誘電率	ファラド毎メートル	F/m	$m^{-3}\ kg^{-1}\ s^4\ A^2$
透磁率	ヘンリー毎メートル	H/m	$m\ kg\ s^{-2}\ A^{-2}$
モルエネルギー	ジュール毎モル	J/mol	$m^2\ kg\ s^{-2}\ mol^{-1}$
モルエントロピー, モル熱容量	ジュール毎モル毎ケルビン	J/(mol K)	$m^2\ kg\ s^{-2}\ K^{-1}\ mol^{-1}$
照射線量（Ｘ線及びγ線）	クーロン毎キログラム	C/kg	kg^{-1} sA
吸収線量率	グレイ毎秒	Gy/s	$m^2\ s^{-3}$
放射強度	ワット毎ステラジアン	W/sr	$m^4\ m^{-2}\ kg\ s^{-3}=m^2\ kg\ s^{-3}$
放射輝度	ワット毎平方メートル毎ステラジアン	$W/(m^2\ sr)$	$m^2\ m^{-2}\ kg\ s^{-3}=kg\ s^{-3}$
酵素活性濃度	カタール毎立方メートル	kat/m^3	$m^{-3}\ s^{-1}$ mol

表５．SI 接頭語

乗数	接頭語	記号	乗数	接頭語	記号
10^{24}	ヨタ	Y	10^{-1}	デシ	d
10^{21}	ゼタ	Z	10^{-2}	センチ	c
10^{18}	エクサ	E	10^{-3}	ミリ	m
10^{15}	ペタ	P	10^{-6}	マイクロ	µ
10^{12}	テラ	T	10^{-9}	ナノ	n
10^{9}	ギガ	G	10^{-12}	ピコ	p
10^{6}	メガ	M	10^{-15}	フェムト	f
10^{3}	キロ	k	10^{-18}	アト	a
10^{2}	ヘクト	h	10^{-21}	ゼプト	z
10^{1}	デカ	da	10^{-24}	ヨクト	y

表６．SIに属さないが、SIと併用される単位

名称	記号	SI 単位による値
分	min	1 min=60s
時	h	1h =60 min=3600 s
日	d	1 d=24 h=86 400 s
度	°	1°=(π/180) rad
分	’	1’=(1/60)°=(π/10800) rad
秒	”	1”=(1/60)’=(π/648000) rad
ヘクタール	ha	$1ha=1hm^2=10^4m^2$
リットル	L，l	$1L=11=1dm^3=10^3cm^3=10^{-3}m^3$
トン	t	$1t=10^3$ kg

表７．SIに属さないが、SIと併用される単位で、SI単位で表される数値が実験的に得られるもの

名称	記号	SI 単位で表される数値
電子ボルト	eV	$1eV=1.602\ 176\ 53(14)\times10^{-19}J$
ダルトン	Da	$1Da=1.660\ 538\ 86(28)\times10^{-27}kg$
統一原子質量単位	u	1u=1 Da
天文単位	ua	$1ua=1.495\ 978\ 706\ 91(6)\times10^{11}m$

表８．SIに属さないが、SIと併用されるその他の単位

名称	記号	SI 単位で表される数値
バール	bar	$1\ bar=0.1MPa=100kPa=10^5Pa$
水銀柱ミリメートル	mmHg	1mmHg=133.322Pa
オングストローム	Å	$1\ Å=0.1nm=100pm=10^{-10}m$
海里	M	1 M=1852m
バーン	b	$1\ b=100fm^2=(10^{-12}cm)2=10^{-28}m^2$
ノット	kn	1 kn=(1852/3600)m/s
ネーパ	Np	SI単位との数値的な関係は、対数量の定義に依存。
ベル	B	
デジベル	dB	

表９．固有の名称をもつCGS組立単位

名称	記号	SI 単位で表される数値
エルグ	erg	$1\ erg=10^{-7}$ J
ダイン	dyn	$1\ dyn=10^{-5}N$
ポアズ	P	$1\ P=1\ dyn\ s\ cm^{-2}=0.1Pa\ s$
ストークス	St	$1\ St\ =1cm^2\ s^{-1}=10^{-4}m^2\ s^{-1}$
スチルブ	sb	$1\ sb\ =1cd\ cm^{-2}=10^4cd\ m^{-2}$
フォト	ph	$1\ ph=1cd\ sr\ cm^{-2}\ 10^4lx$
ガル	Gal	$1\ Gal\ =1cm\ s^{-2}=10^{-2}ms^{-2}$
マクスウェル	Mx	$1\ Mx = 1G\ cm^2=10^{-8}Wb$
ガウス	G	$1\ G\ =1Mx\ cm^{-2}\ =10^{-4}T$
エルステッド[c]	Oe	$1\ Oe \triangleq (10^3/4\pi)A\ m^{-1}$

(c) ３元系のCGS単位系とSIでは直接比較できないため、等号「 ≙ 」は対応関係を示すものである。

表10．SIに属さないその他の単位の例

名称	記号	SI 単位で表される数値
キュリー	Ci	$1\ Ci=3.7\times10^{10}Bq$
レントゲン	R	$1\ R = 2.58\times10^{-4}C/kg$
ラド	rad	$1\ rad=1cGy=10^{-2}Gy$
レム	rem	$1\ rem=1\ cSv=10^{-2}Sv$
ガンマ	γ	1 γ=1 nT=10-9T
フェルミ		1フェルミ=1 fm=10-15m
メートル系カラット		1メートル系カラット = 200 mg = 2×10-4kg
トル	Torr	1 Torr = (101 325/760) Pa
標準大気圧	atm	1 atm = 101 325 Pa
カロリー	cal	1cal=4.1858J（「15℃」カロリー），4.1868J（「IT」カロリー）4.184J（「熱化学」カロリー）
ミクロン	µ	$1\ \mu=1\mu m=10^{-6}m$

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