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Derivation of Transfer Functions of Natural Circulation Boiling Water Reactor

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Derivation of Transfer Functions of Natural Circulation Boiling Water Reactor

Summary

The dynamic behavior of a natural circulation boiling water reactor is quite complicated, because it includes the dynamics of the void generation in the core and the hydrodynamic considerations in connection with the determination of the recirculation flow. The transfer functions of the reactors of this type are obtained under several basic assumptions. Efforts are made to cover all the important features of the boiling water reactor dynamics and also to avoid too complex expressions to simulate on an analog computer.

The basic assumptions used are

(1) The distribution of the parameters in the axial direction of the core is taken into account, while the change in radial direction is ignored.

(2) The distribution of heat flux along the channel is considered uniform.

(3) The slip between the steam and the water is ignored.

October, 1962

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自然循環式沸騰水型原子炉の伝達関数

要　　旨

自然循環式沸騰水型原子炉の動特性は，炉心内ボイド量の変化とか，自然循環に伴なう種々の水力学的特性のために，かなり複雑なものとなる．ここでは，いくつかの基本的な仮定のもとに，この型の原子炉の伝達関数を求めた．その際には，沸騰水型原子炉の動特性に重要な影響をもつと思われる要因はもれなく含めるように努力するとともに，アナログ計算機による解析を容易にするために，比較的簡単な形にまとめるように考慮を払った．

用いた主な仮定は，

(1) 炉心の軸方向には分布定数系として扱ったが，炉心半径方向については集中定数系とみなしている．

(2) 炉心の軸方向の出力分布は一様と仮定する．

(3) 蒸気と水の間のすべりを無視する．

等である．

1962 年 10 月

原子力工学部計測制御研究室
三井田 純 一，須 田 信 英

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1. Objective of Study

The mathematical model of the dynamic characteristics of the boiling water reactors is necessary for four purposes; first to evaluate the stability of the reactors, second for the physical interpretation of the measured transfer function, third to provide information for more efficient core design and finally to provide information for control system design.

This report is the first part of a series of studies along the above line on the dynamic behavior of the boiling water reactor power plants. This series of studies includes the development of a mathematical model of the boiling water reactor dynamics, the stability analysis with the aid of an analog computer and the experimental determination of the system parameters of the JPDR, a 12.5 MWe boiling water reactor plant.

The purpose of the present work is to develope a new mathematical model of the boiling water reactor dynamics, which covers all the important features of the boiling water reactor dynamics.

The mathematical models developed so far[2)3)4)5)6)7)], either overlook some of the important features to be incorporated or are too complex to simulate on an analog computer. The particular model derived here is considered to be a compromise of completeness and moderate complexity.

Since it is intended to apply this model to the analysis of the JPDR, the model is derived for a single cycle, natural circulation boiling water reactor.

The rest of this series of the studies will appear in the subsequent reports.

2. Summary of Dynamic Behavior of Boiling Water Reactor

The over-all system will be summarized in this chapter to provide a better understanding of the whole system, prior to the detailed analysis given in the following chapters.

The kinetics of a power reactor is summarized in Fig. 1, in which is shown the zero power reactor transfer function $G(s)$ modified by the power-to-reactivity feedback transfer function $F(s)$.

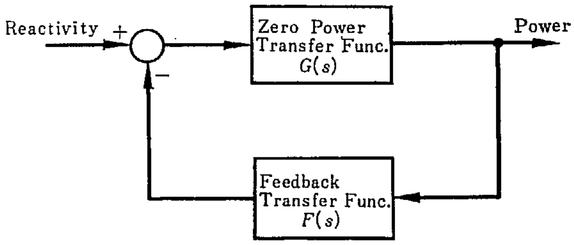


Fig. 1 Kinetics of Power Reactor

In the case of the boiling water reactor the feedback paths consists of three major parts, that is,

Fuel temperature effect

Moderator temperature effect

Void effect

The fuel temperature is chiefly determined by the reactor power. The water temperature is known to have little influence on the fuel temperature.

The moderator temperature effect is subdivided into two parts, those in the boiling and the non-boiling regions. The latter depends upon the reactor power, the flow rate and other factors, while the former, as far as the water in the boiling region is saturated, depends only upon the system pressure.

The void effect is the most important and also most complicated of all the feedback mechanisms. The void volume in the reactor core is determined by the four parameters:

Reactor power

Coolant velocity

Position of boiling boundary

System pressure

In the natural circulation boiling water reactcr, the coolant velocity is obtained from the hydrodynamic calculation of the circulating loop, consisting of the reactor core, the riser, the upper and lower plenums and the downcomer. The reactor power, the void volume in the core, the position of the boiling boundary and the system pressure are the parameters to be taken into account in this hydrodynamic calculation.

The position of the boiling boundary is determined from the dynamics of the non-boiling region of the reactor core. The parameters which influence the non-boiling region dynamics are the reactor power, the coolant velocity, the system pressure and the coolant subcooling or the enthalpy of the coolant at the core inlet.

The dynamics of the whole reactor vessel, including the reactor core, the steam dome, the downcomer and so on, must be investigated in order to determine the system pressure, the subcooling and the recirculation flow rate. The parameters of importance are the reactor power, the steam load, the feedwater flow rate, the feedwater enthalpy and others.

The above description of the kinetics of the boiling water reactor is summarized in Fig. 2. Here four parameters, namely, the reactivity, the steam load and the feedwater flow rate and the feedwater enthalpy are considered to be external disturbances to the system.

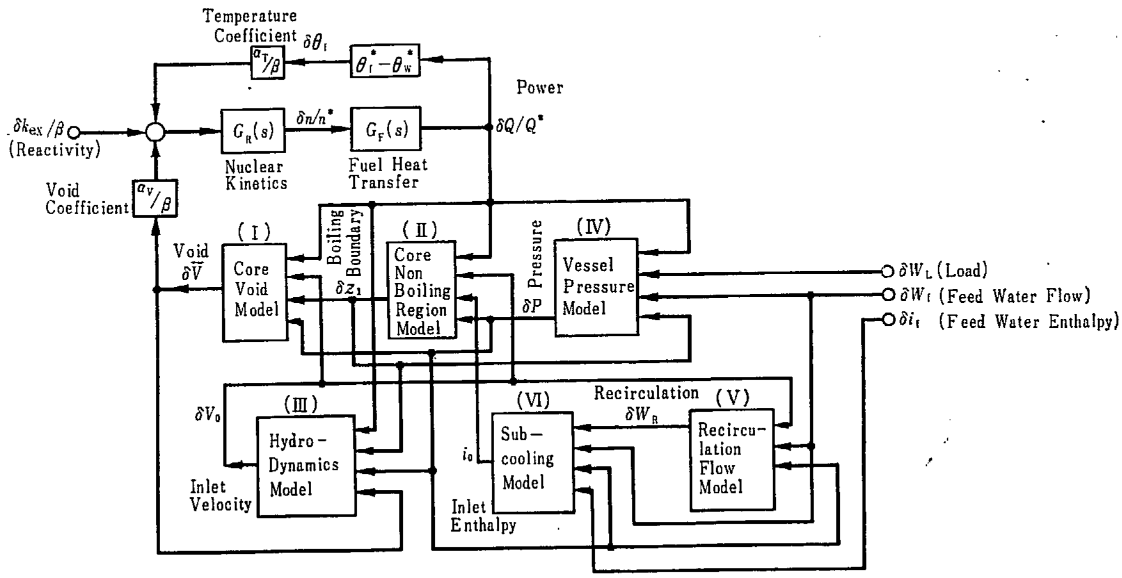


Fig. 2 Schematic Diagram of Kinetics of Boiling Water Reactor

In the following chapter the transfer functions of the natural circulation boiling water reactor are derived in accordance with Fig. 2. The moderator temperature effect, however, is ignored, because it has less importance among the feedback mechanisms, and only the fuel temperature effect and the void effect are investigated.

3. Formulation of Transfer Functions

3.0 Major Assumptions

In the course of the formulation of the transfer functions, the following assumptions are made for simplicity.

1) In the nuclear kinetics analysis (Sec. 3.1) a simple, one group, space independent, one delayed neutron group approximation is used.

2) In order to derive the core transfer functions (Secs. 3.2 through 3.5) the changes of parameters in the axial direction is taken into account. The change in the radial direction is ignored and a single channel with the averaged values of the parameters is considered to represent the whole core.

3) In Secs. 3.2 through 3.4 the change in pressure in the axial direction is ignored to simplify the analysis, while in Sec. 3.5 it is taken into account.

4) The distribution of heat flux along the channel is considered uniform.

5) The slip between the steam and the water is ignored, in other words, the slip ratio is assumed to be unity.

6) The density of subcooled water is considered to be independent of the pressure and the temperature.

7) A rather crude, lumped parameter model is used for the reactor vessel dynamics. (Secs. 3.6 through 3.8)

8) Only the small deviations from the steady state are considered. Thus the products of two or more deviation terms are ignored.

3.1 Zero Power Transfer Function

As is well known the zero power transfer function of a reactor is given by

$$G_R(s)=\frac{\delta n/n^*}{\delta k/\beta}=\frac{1}{s\left(\frac{l}{\beta}+\frac{1}{s+\lambda}\right)}=\frac{s+\lambda}{\frac{l}{\beta}s\left(s+\frac{\beta}{l}+\lambda\right)} \tag{1}$$

3.2 Fuel Heat Transfer Dynamics

The detailed analysis of the fuel heat transfer dynamics is reported in the literature[8)12)13)]. Here is used a very simple model with lumped parameters.

$$\frac{\delta Q/Q^*}{\delta n/n^*}=\frac{1}{1+T_f s} \tag{2}$$

$$T_f=\frac{C_f}{H}$$

where C_f is the heat capacity of the fuel and H is the average heat transfer coefficient between the fuel and the coolant.

In the same way the fuel temperature transfer function is given by

$$\frac{\delta\theta_f}{\delta n/n^*}=\frac{\theta_f^*-\theta_w^*}{1+T_f s} \tag{3}$$

3.3 Core Void Transfer Function†

Under the assumptions given in Sec. 3.0, the equations of conservation of mass and energy for the steam-water mixture per unit length of the fuel channel are as follows;

$$\frac{\partial}{\partial t}\{\rho_w(1-f)+\rho_s f\}+\frac{\partial}{\partial z}\{\rho_w V_w(1-f)+\rho_s V_s f\}=0 \tag{4}$$

$$\frac{\partial}{\partial t}\{\rho_w i_w(1-f)+\rho_s i_s f\}+\frac{\partial}{\partial z}\{\rho_w V_w i_w(1-f)+\rho_s V_s i_s f\}=Q \tag{5}$$

From these two equations the following void equation is derived[7]

$$\frac{\partial f}{\partial t}+U\frac{\partial f}{\partial z}=q \tag{6}$$

where U is the velocity of void transmission and q is the external "force" to void. If the slip ratio is assumed to be unity, these are defined as

$$U=V_0+\int_{z_1}^{z}\left[\frac{\Delta v}{\Delta i}\left\{Q+F_1(f)\frac{dp}{dt}\right\}+F_2(f)\frac{dp}{dt}\right]dz \tag{7}$$

$$q=\frac{1}{\rho_s\Delta i}F_3(f)\left\{Q+F_1(f)\frac{dp}{dt}\right\}+F_4(f)\frac{dp}{dt} \tag{8}$$

where functions $F_1(f)$ through $F_4(f)$ are defined as below.

$$F_1(f)=-\left\{\rho_w(1-f)\frac{\partial i_w}{\partial p}+\rho_s f\frac{\partial i_s}{\partial p}\right\} \tag{9}$$

$$F_2(f)=-\left\{\frac{1-f}{\rho_w}\frac{\partial\rho_w}{\partial p}+\frac{f}{\rho_s}\frac{\partial\rho_s}{\partial p}\right\} \tag{10}$$

$$F_3(f)=1-f+f\frac{\rho_s}{\rho_w} \tag{11}$$

$$F_4(f)=f(1-f)\left\{\frac{1}{\rho_w}\frac{\partial\rho_w}{\partial p}-\frac{1}{\rho_s}\frac{\partial\rho_s}{\partial p}\right\} \tag{12}$$

Now the variables in Eq. (6) are divided into two parts i.e., the steady state values and the small deviations from the steady state. Then, ignoring all the products of two or more deviation terms and substituting the following steady state conditions,

$$U^*(z)=V_w^*(z)=V_S^*(z)=V_0^*+Q^*\frac{\Delta v}{\Delta i}(z-z_1)=V_0^*+\Omega(z-z_1) \tag{13}$$

$$\frac{df^*}{dz}=\frac{\frac{Q^*}{\rho_s\Delta i}\left\{1-\left(1-\frac{\rho_s}{\rho_w}\right)f^*\right\}}{U^*}=\frac{Q^*}{\rho_s\Delta i}\,\frac{1-\rho_s\Delta v f^*}{U^*} \tag{14}$$

$$\frac{df^*}{dU^*}=\frac{1}{\rho_s\Delta v}\,\frac{1-\rho_s\Delta v f^*}{U^*} \tag{15}$$

---

† The core void transfer function means the transfer function which determines the core void volume in terms of the other variables. The titles of the following sections are named in the same way.

$$f^*=\frac{1}{\rho_s\Delta v}\left(1-\frac{V_0^*}{U^*}\right)=\frac{\Omega}{\rho_s\Delta v}\,\frac{z-z_1}{U^*} \tag{16}$$

$$q^*=\frac{Q^*}{\rho_s\Delta i}(1-\rho_s\Delta v\cdot f^*)=\frac{Q^*}{\rho_s\Delta i}\,\frac{V_0^*}{U^*} \tag{17}$$

$$F_3^*=1-\left(1-\frac{\rho_s}{\rho_w}\right)f^*=1-\rho_s\Delta v f^*=\frac{V_0^*}{U^*} \tag{18}$$

where $\Omega=Q^*\dfrac{\Delta v}{\Delta i}$※

and then taking Laplace transform of the equation, Eq. (6) is solved for $\delta f(z,s)$.

$$\delta f(z,s)=U^{*^{-\frac{s}{\Omega}-1}}\left\{\int_{z_1}^{z}U^{*^{\frac{s}{\Omega}}}\varphi(z,s)dz+C'(s)\right\} \tag{19}$$

where $\varphi(z,s)$ represents

$$\varphi(z,s)=\frac{Q^*}{\rho_s\Delta i}\left(\frac{1}{y^2}\left\{\frac{\delta Q}{Q^*}-\frac{\delta V_0}{V_0^*}+\frac{\Delta v}{\Delta i}\,\frac{Q^*}{V_0^*}\delta z_1\right\}\right.$$
$$-\left[\frac{1}{y}\,\frac{s}{\Omega}\left\{\frac{v_w}{v_s}\left(\frac{D}{v_w}-\frac{C}{\Delta v}\right)+\frac{D}{v_w}\right\}+\left(1+\frac{s}{\Omega}\right)\frac{v_w}{v_s}\left(\frac{C}{\Delta v}-\frac{D}{v_w}\right)\right.$$
$$\left.\left.+\frac{1}{y^2}\left\{\frac{s}{\Omega}\left(\frac{\Delta v}{\Delta i}\,\frac{B}{v_w}-\frac{D}{v_w}\right)-\frac{C}{\Delta v}+\frac{A}{\Delta i}\right\}+E\frac{s}{\Omega}\,\frac{\ln y}{y^2}\right]\delta p\right) \tag{20}$$

and the $C'(s)$ and $y(z)$ stand for

$$\left.\begin{aligned}C'(s)&=-\frac{Q^*}{\rho_s\Delta i}V_0^{*^{\frac{s}{\Omega}}}\delta z_1\\ y(z)&=\frac{V_w(z)}{V_0^*}=\frac{V_s(z)}{V_0^*}=\frac{V_0^*+\Omega(z-z_1)}{V_0^*}\end{aligned}\right\} \tag{21}$$

Performing the integration of Eq. (19), the transfer functions relating the local void fraction to the four variables, i.e., the reactor power, the inlet water velocity, the boiling boundary shift and the system pressure, are obtained as follows,

(a) $\delta Q\rightarrow\delta f$

$$G_1(z,s)=\frac{\delta f(z,s)}{\delta Q(s)/Q^*}=\frac{v_s}{\Delta v}\,\frac{1}{y^2}\,\frac{1-ye^{-\tau_{1z}s}}{\tau_e s-1} \tag{22}$$

(b) $\delta V_0\rightarrow\delta f$

$$G_2(z,s)=\frac{\delta f(z,s)}{\delta V_0(s)/V_0^*}=-G_1(z,s) \tag{23}$$

(c) $\delta z_1\rightarrow\delta f$

$$G_3(z,s)=\frac{\delta f(z,s)}{\delta z_1(s)}=\frac{v_s}{\Delta v}\,\frac{1}{\tau_e V_0^*}\,\frac{1}{y^2}\left\{\frac{1}{\tau_e s-1}(1-ye^{-\tau_{1z}s})-ye^{-\tau_{1z}s}\right\} \tag{24}$$

(d) $\delta p\rightarrow\delta f$

※ The quantity Ω defined above may be called "the rate of steam raising", that is, the steam volume generated per unit time per unit volume of water. The reciprocal of Ω is named τ_e, or "steam raising time", during which a unit volume of steam is raised per unit volume of water. Ω and τ_e are the functions of the power and pressure.

$$G_4(z,s)=\frac{\delta f(z,s)}{\delta p(s)}=-\frac{v_s}{\Delta v}\Big[\frac{v_w}{v_s}(C'-D')\Big(1-\frac{1}{y}\Big)+D'\Big(\frac{1}{y}-\frac{1}{y^2}\Big)$$
$$+E\frac{\ln y}{y^2}+B'\Big(\frac{1}{y^2}-\frac{1}{y}e^{-\tau_{1z}s}\Big)$$
$$+\frac{1}{\tau_e s-1}\Big\{2(B'-D')\Big(\frac{1}{y^2}-\frac{1}{y}e^{-\tau_{1z}s}\Big)+E\frac{\ln y}{y^2}\Big\}$$
$$-\frac{1}{(\tau_e s-1)^2}E\Big(\frac{1}{y^2}-\frac{1}{y}e^{-\tau_{1z}s}\Big)\Big] \tag{25}$$

where $\tau_{1z}=\tau_e \ln y(z)$

It should be noted that τ_{1z} is the transit time of steam from z_1 to z, since the transit time τ_t is given by

$$\tau_t=\int_{z_1}^{z}\frac{dz}{U^*}=\frac{1}{V_0^*}\int_{z_1}^{z}\frac{dz}{y}=\tau_e \ln y(z)=\tau_{1z}$$

Integration of the local void fraction over the whole boiling region of the core gives the total void volume in the core.

$$\overline{V}=\int_{z_1}^{z_2} f(z,s)A_{co}\,dz \tag{26}$$

The transfer functions relating the total void volume to the four variables are thus obtained and given below.

(a) $\delta Q\rightarrow\delta V$

$$G_1(s)=\frac{\delta\overline{V}(s)}{\delta Q(s)/Q^*}=\frac{v_s}{\Delta v}\tau_{12}V_0^*A_{co}\frac{1}{\tau_e s-1}\left\{\frac{1-\frac{1}{y_2}}{\ln y_2}-\frac{1-e^{-\tau_{12}s}}{\tau_{12}s}\right\} \tag{27}$$

where $y_2=y(z_2)$, $\tau_{12}=\tau_e\ln y_2$

(b) $\delta V_0\rightarrow\delta\overline{V}$

$$G_2(s)=\frac{\delta\overline{V}(s)}{\delta V_0/V_0^*}=-G_1(s) \tag{28}$$

(c) $\delta z_1\rightarrow\delta\overline{V}$

$$G_3(s)=\frac{\delta\overline{V}(s)}{\delta z_1(s)}=\frac{v_s}{\Delta v}\frac{A_{co}}{y_2}\frac{-1+y_2e^{-\tau_{12}s}}{\tau_e s-1} \tag{29}$$

(d) $\delta p\rightarrow\delta\overline{V}$

$$G_4(s)=\frac{\delta\overline{V}(s)}{\delta p}=-\frac{v_s}{\Delta v}\tau_e V_0^*A_{co}\Big[\frac{v_w}{v_s}(C'-D')(y_2-1-\ln y_2)+D'\ln y_2$$
$$+(A'-C')\Big(1-\frac{1}{y_2}\Big)-E\frac{\ln y_2}{y_2}+(A'-C'-D')\frac{\tau_{12}}{\tau_e}\frac{(1-e^{-\tau_{12}s})}{\tau_{12}s}$$
$$+\frac{1}{\tau_e s-1}\Big\{F\Big(\frac{1}{y_2}-e^{-\tau_{12}s}\Big)-E\frac{\ln y_2}{y_2}\Big\}+\frac{1}{(\tau_e s-1)^2}E\Big(\frac{1}{y_2}-e^{-\tau_{12}s}\Big)\Big] \tag{30}$$

3.4 Core Boiling Boundary Transfer Function

The fundamental equation of the water temperature in the subcooled region of the reactor core is as follows:

$$\rho_w c_w\left(\frac{\partial\theta}{\partial t}+V_0\frac{\partial\theta}{\partial z}\right)=Q \tag{31}$$

where θ is the water temperature and c_w is water heat capacity.

Considering small variations around a steady state, and taking the Laplace transforms,

$$\frac{d\delta\theta}{dz}+\frac{s}{V_0{}^*}\delta\theta=\frac{Q^*}{\rho_w c_w V_0{}^*}\left\{\frac{\delta Q}{Q^*}-\frac{\delta V_0}{V_0{}^*}-\frac{1}{\rho_w}\frac{\partial\rho_w}{\partial p}\delta p\right\} \tag{32}$$

Solving Eq. (32) with respect to z, with the initial condition of $[\delta\theta(z,s)]_{z=0}=\delta\theta_0(s)$

$$\delta\theta(z,s)=\frac{1-e^{-\frac{z}{V_0}s}}{s}\frac{Q^*}{\rho_w c_w}\left\{\frac{\delta Q}{Q^*}-\frac{\delta V_0}{V_0{}^*}-\frac{1}{\rho_w}\frac{\partial\rho_w}{\partial p}\delta p\right\}+\delta\theta_0(s)e^{-\frac{z}{V_0}s} \tag{33}$$

The temperature at the boiling boundary is always the saturation temperature,

$$\theta(z_1+\delta z_1,t)=\theta_{sat}(t) \tag{34}$$

Again considering small variations around a steady state,

$$\theta(z_1,o)+\frac{\partial\theta(z_1,o)}{\partial z_1}\delta z_1+\delta\theta(z_1,t)=\theta_{sat}(o)+\delta\theta_{sat}(t) \tag{35}$$

Performing the Laplace transforms and considering the fact that $\theta(z_1,o)$ is equal to $\theta_{sat}(o)$ and the saturation temperature depends only upon the system pressure,

$$\frac{\partial\theta(z_1,o)}{\partial z_1}\delta z_1(s)=\frac{\partial\theta_{sat}}{\partial p}\partial p(s)-\delta\theta(z_1,s) \tag{36}$$

Substituting the steady state condition

$$\frac{\partial\theta(z,o)}{\partial z}=\frac{Q^*}{\rho_w c_w V_0{}^*} \tag{37}$$

it gives

$$\delta z_1(s)=\frac{\rho_w c_w V_0{}^*}{Q^*}\left\{\frac{\partial\theta_{sat}}{\partial p}\delta p-\delta\theta(z_1,s)\right\} \tag{38}$$

Combining Eq. (33) and (38),

$$\delta z_1(s)=-(z_1-z_0)\left\{\frac{\delta Q}{Q^*}-\frac{\delta V_0}{V_0{}^*}\right\}\frac{1-e^{-\tau_{01}s}}{\tau_{01}s}-\frac{\rho_w c_w V_0{}^*}{Q^*}e^{-\tau_{01}s}\,\delta\theta_0(s)$$
$$+\left\{\frac{\rho_w c_w V_0{}^*}{Q^*}\frac{\partial\theta_{sat}}{\partial p}+\left(z_1-z_0\right)\frac{1}{\rho_w}\frac{\partial\rho_w}{\partial p}\frac{1-e^{-\tau_{01}s}}{\tau_{01}s}\right\}\delta p \tag{39}$$

3.5 Inlet Velocity Transfer Function (Hydrodynamics)

In order to derive the hydrodynamic transfer functions of the natural circulation loop, the reactor vessel is divided into six parts; the core non-boiling region, the core boiling region, the riser, the upper reflector on top of the core, the downcomer and the lower plenum, as shown in Fig. 3.

In addition to the mass balance and energy balance equations, derived and solved in Sec. 3.3, the fundamental equation used is the momentum equation given below.

$$\frac{\partial}{\partial t}[\text{Momentum}]=[\text{M. Tr.}]_1-[\text{M.Tr.}]_2-g\int_1^2\rho\,dz+(p_1-p_2)A$$
$$-[\text{Frictional Pressure Drop}]_1^2 A \tag{40}$$

where [M. Tr.] denotes "Rate of Momentum Transport", and the subscripts 1 and 2 denote the inlet and outlet of the part of the loop under consideration, respectively, and A denotes the flow area.

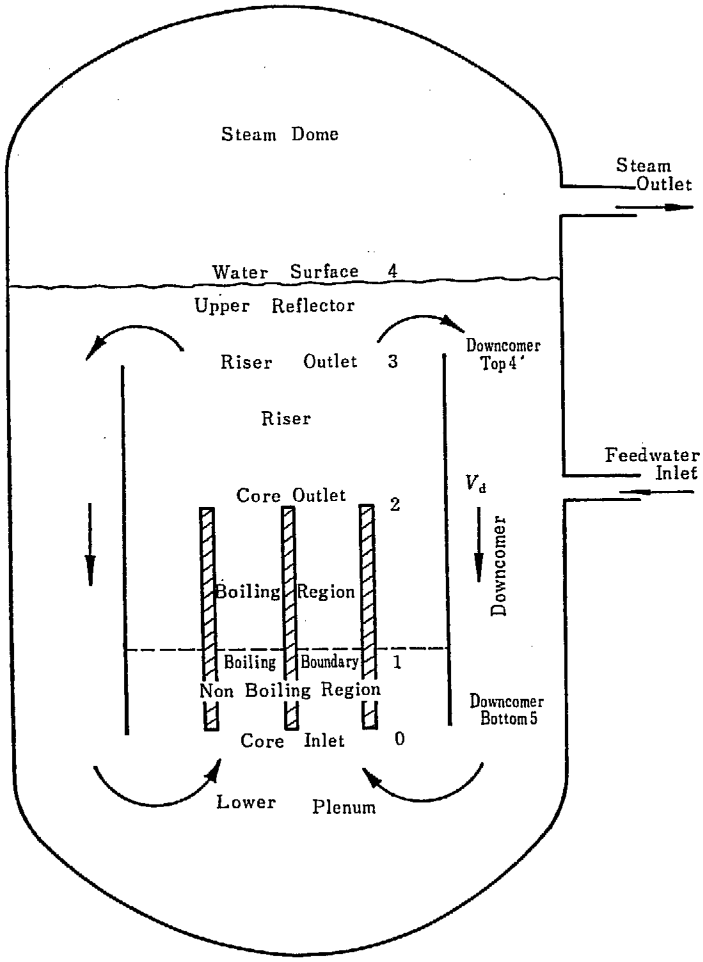


Fig. 3. Schematic Diagram of Reactor Vessel

The equation is simplified by considering small deviations from the steady state and ignoring all the higher order terms.

Then it is solved, making use of the results of Sec. 3. 3, for the non-boiling region,

$$\frac{z_1-z_0}{A_{co}}\rho_w s\,\delta V_0+\frac{\partial[\mathrm{FPD}]_0^1}{\partial V_0}\delta V_0$$
$$=-\frac{\rho_w}{A_{co}}(V_0{}^*s+g)\delta z_1+\frac{(z_1-z_0)}{v_w{}'}\frac{D}{v_w}(V_0{}^*s+g)\delta p+\delta(p_0-p_1) \tag{41}$$

and for the boiling region,

$$\rho_w V_0{}^{*2}\left\{\frac{\tau_e{}^2s^2}{(\tau_e s-1)}\ln y_2-\frac{1}{(\tau_e s-1)^2}(1-y_2e^{-\tau_{12}s})\right\}\frac{\delta V_0}{V_0^*}+\frac{\partial}{\partial V_0}[\mathrm{FPD}]_1^2\,\delta V_0\cdot A_{co}$$
$$=g(\rho_w-\rho_s)\frac{1}{A_{co}}\delta\overline{V}$$
$$+\rho_w V_0{}^{*2}\left[\frac{\tau_e{}^2s^2}{(\tau_e s-1)}\ln y_2-\frac{1}{(\tau_e s-1)^2}(1-y_2e^{-\tau_{12}s})-(\tau_e s+2)(y_2-1)\right]\frac{\delta Q}{Q^*}$$

$$+\frac{\rho_w V_0^*}{\tau_e}\left[\frac{\tau_e\tau_{12}s^2}{\tau_e s-1}+1-\frac{\tau_e s}{(\tau_e s-1)^2}(1-y_2e^{-\tau_{12}s})+\frac{\tau_e g}{V_0^*}\right]\delta z_1$$

$$+\rho_w V_0^{*2}\Big[(A'-C')(y_2-1)(\tau_e s+1)(\tau_e s+2)+\Big\{F+\tau_e s(F-2E)$$

$$+F\frac{1}{\tau_e s-1}+E\frac{\tau_e s}{(\tau_e s-1)^2}+(C'-A')\tau_e^2 s^2\Big\}\ln y_2$$

$$+\{-(A'-C')-D'(2\tau_e s-1)+B'\tau_e^2 s^2\}\frac{1-y_2e^{-\tau_{12}s}}{(\tau_e s-1)^3}$$

$$-\frac{E}{2}\frac{\tau_{12}^2\tau_e s^3}{(\tau_e s-1)}$$

$$-\frac{\tau_e g}{V_0^*}\cdot\frac{v_w}{v_s}\Big\{(D'-C')(y_2-1)-\Big(\frac{v_s+v_w}{v_w}D'-C'\Big)\ln y_2\Big\}\Big]\delta p$$

$$+A_{co}(\delta p_1-\delta p_2) \qquad (42)$$

where $E\equiv A'-B'-C'+D'$

$F\equiv -A'-B'+C'+D'$

A similar analysis can also be applied to the riser. The heat flux Q is, however, zero in the riser and the equation of the energy balance should be modified. The cross-sectional area of the riser A_r is larger than that of the core A_{co}, so the continuity condition at the top of the core is

$$\widetilde{V}_2=\mu V_2$$

where $\widetilde{V}_2$ is the riser inlet velocity and V_2 is the core outlet velocity and μ is the ratio of area A_{co}/A_r

The fundamental equations are solved, with the above mentioned modifications, for the riser,

$$\rho_w V_0^{*2}\Big(\mu\tau_{23}s+\frac{g}{V_0^* y_2}\frac{1-y_2e^{-\tau_{12}s}}{\tau_e s-1}\cdot\frac{1-e^{-\tau_{23}s}}{s}\Big)\frac{\delta V_0}{V_0^*}+A_r\frac{\partial[\text{FPD}]_2^3}{\partial V_r}\mu\delta V_0$$

$$=-\rho_w V_0^{*2}\Big\{\mu\tau_{23}(y_2-1)s-\frac{g}{V_0^* y_2}\frac{1-y_2e^{-\tau_{12}s}}{\tau_e s-1}\cdot\frac{1-e^{-\tau_{23}s}}{s}\Big\}\frac{\delta Q}{Q^*}$$

$$+\rho_w V_0^*\Omega\Big[\mu\tau_{23}s+\frac{g}{V_0^* y_2}\Big\{\frac{1}{\tau_e s-1}(1-y_2e^{-\tau_{12}s})-y_2e^{-\tau_{12}s}\Big\}\frac{1-e^{-\tau_{23}s}}{s}\Big]\delta z_1$$

$$+\rho_w V_0^{*2}\Big[\{(A'-C')(y_2-1)(\tau_e s+2)+(B'-D')-\tau_e sE\ln y_2\}\mu\tau_{23}s$$

$$+\frac{1}{2}\{(A'-C')(y_2-1)+B'-D'\}\mu\tau_{23}^2 s^2$$

$$-\frac{g\tau_{23}}{y_2 V_0^*}\{(A'-C')(y_2-1)+B'-D'\}$$

$$+\frac{g}{V_0^*}\frac{1-e^{-\tau_{23}s}}{s}\Big[(A'-C')\Big(1-\frac{1}{y_2}\Big)-E\frac{\ln y_2}{y_2}+B'e^{-\tau_{12}s}$$

$$-\frac{1}{\tau_e s-1}\Big\{2(B'-D')\Big(\frac{1}{y_2}-e^{-\tau_{12}s}\Big)+E\frac{\ln y_2}{y_2}\Big\}$$

$$+\frac{1}{(\tau_e s-1)^2}E\left(\frac{1}{y_2}-e^{-\tau_{12}s}\right)\Big]\Big]\delta p$$

$$+A_r\delta(p_2-p_3) \tag{43}$$

The same equation of momentum balance is applied to the hydrodynamics of the upper reflector. However here is needed an additional assumption about the behavior of the free surface. Suppose the flow rate of water coming out from the riser increases very suddenly, the recirculation flow rate does not change appreciably and instead the free surface of the reflector rises. If the change in flow rate is slow, the recirculation flow rate varies in the same way as the incoming flow changes and thus the position of the free surface is substantially constant. Thus the upward velocity δV_4 of the free surface is assumed to have the following relationship with the incoming flow rate δV_3, that is, the riser outlet velocity of the water.

$$\delta V_4=\frac{\tau_r s}{1+\tau_r s}\delta V_3 \tag{44}$$

And the time constant τ_r is roughly assumed to be the order of $\frac{T}{2\pi}$, where T is the period of the standing wave between two parallel plates separated by a distance corresponding to the diameter of the reactor vessel.

With the above assumption the momentum equation is solved, the result being as follows,

$$\frac{\mu}{A_{co}}\rho_w V_0^*\Big[\frac{1}{y_2}\Big(\frac{h}{V_0^*}s+\frac{g\tau_r/V_0^*}{1+\tau_r s}-2\mu y_2\Big)-\mu y_2^2\frac{\Delta v}{v_s}e^{-\tau_{23}s}G_1(z_2,s)\Big]\delta V_0$$

$$=-\frac{\mu}{A_{co}}\rho_w V_0^{*2}\Big\{\frac{y_2-1}{y_2}\Big(\frac{h}{V_0^*}s+\frac{g\tau_r/V_0^*}{1+\tau_r s}-2\mu y_2\Big)+\mu y_2^2\frac{\Delta v}{v_s}e^{-\tau_{23}s}G_1(z_2,s)\Big\}\frac{\delta Q}{Q^*}$$

$$+\frac{\mu}{A_{co}}\rho_w V_0^*\Big\{\frac{\Omega}{y_2}\Big(\frac{h}{V_0^*}s+\frac{g\tau_r/V_0^*}{1+\tau_r s}-2\mu y_2\Big)-\mu y_2^2 V_0^*\frac{\Delta v}{v_s}e^{-\tau_{23}s}G_3(z_2,s)\Big\}\delta z_1$$

$$+\frac{\mu}{A_{co}}\rho_w V_0^{*2}\Big[\frac{1}{y_2}\Big(\frac{h}{V_0^*}s+\frac{g\tau_r/V_0^*}{1+\tau_r s}-2\mu y_2\Big)\Big[\Big\{(A'-C')\Big(\frac{y_2-1}{\Omega}+\tau_{23}y_2\Big)$$

$$-(\tau_{12}+\tau_{23})E\Big\}s+(y_2-1)(A'-C')\Big]$$

$$-\mu y_2^2\Big[-\mu\kappa v_w+\frac{\Delta v}{v_s}\{\alpha(1-e^{-\tau_{23}s})+e^{-\tau_{23}s}G_4(z_2,s)\}\Big]\Big]\delta p$$

$$+\delta(p_3-p_4) \tag{45}$$

where $\alpha=-\frac{v_s}{\Delta v}\frac{1}{y_2^2}\Big[A'(y_2-1)+B'+(C'-D')(y_2-1)\Big(\frac{v_w}{v_s}y_2-1\Big)\Big]$

$$\kappa=\frac{1}{\mu}\Big\{-\frac{1}{y_2}\frac{D'}{v_w}+\frac{y_2-1}{y_2}\frac{D'-C'}{v_s}\Big\}$$

Finally, the momentum equations for the downcomer and the lower plenum are derived in the same way. The solutions are

$$\frac{\rho_w}{A_d}\Big\{(z_3-z_0+h)s+2\frac{A_{co}}{A_d}V_0^*\Big\}\delta V_0+\frac{\partial[\mathrm{FPD}]_4^5}{\partial V_d}\frac{A_{co}}{A_d}\cdot\delta V_0$$

$$=\left\{(z_3-z_0+h)\frac{A_{co}}{A_d}V_0{}^*s+\frac{A_{co}{}^2}{A_d{}^2}V_0{}^{*2}-g(z_3-z_0+h)\right\}\frac{1}{v_w{}'}\cdot\frac{D}{v_w}\delta p$$

$$+\delta(p_4-p_5) \tag{46}$$

for the downcomer and

$$M_{LP}s\,\delta V_0+\frac{\partial[\mathrm{FPD}]_5^0}{\partial V_0}\delta V_0=\delta(p_5-p_0) \tag{47}$$

for the lower plenum, where M_{LP} is the equivalent mass along the streamline in the lower plenum.

The results obtained for each of the six parts of the loop are summarized as follows; For the non-boiling region, from Eq. (41),

$$G_v{}^{01}\delta V_0+\frac{\partial[\mathrm{FPD}]_0^1}{\partial V_0}\delta V_0$$

$$=G_{z1}{}^{01}\delta z_1+G_p{}^{01}\delta p+\delta(p_0-p_1) \tag{48}$$

For the boiling region, from Eq. (42),

$$G_v{}^{12}\delta V_0+\frac{\partial}{\partial V_0}[\mathrm{FPD}]_1^2\,\delta V_0$$

$$=G_v\delta\bar{V}+G_q{}^{12}\frac{\delta Q}{Q^*}+G_{z1}{}^{12}\delta z_1+G_p{}^{12}\delta p+\delta(p_1-p_2) \tag{49}$$

For the riser, from Eq. (43),

$$G_v{}^{23}\delta V_0+\frac{\partial}{\partial V_r}[\mathrm{FPD}]_2^3\frac{A_{co}}{A_r}\delta V_0$$

$$=G_q{}^{23}\frac{\delta Q}{Q^*}+G_{z1}{}^{23}\delta z_1+G_p{}^{23}\delta p+\delta(p_2-p_3) \tag{50}$$

For the reflector, from Eq. (45),

$$G_v{}^{34}\delta V_0=G_q{}^{34}\frac{\delta Q}{Q^*}+G_{z1}{}^{34}\delta z_1+G_p{}^{34}\delta p+\delta(p_3-p_4) \tag{51}$$

For the downcomer, from Eq. (46),

$$G_v{}^{45}\delta V_0+\frac{\partial}{\partial V_d}[\mathrm{FPD}]_4^5\frac{A_{co}}{A_d}\delta V_0$$

$$=G_p{}^{45}\delta p+\delta(p_4-p_5) \tag{52}$$

For the lower plenum, from Eq. (47),

$$G_v{}^{50}\delta V_0+\frac{\partial[\mathrm{FPD}]_5^0}{\partial V_0}\delta V_0=\delta(p_5-p_0) \tag{53}$$

Summing Eqs. (48) through (53), we obtain

$$\{G_v{}^{01}+G_v{}^{12}+G_v{}^{23}+G_v{}^{34}+G_v{}^{45}+G_v{}^{50}\}\delta V_0$$

$$+\left\{\frac{\partial[\mathrm{FPD}]_0^1}{\partial V_0}+\frac{\partial[\mathrm{FPD}]_1^2}{\partial V_0}\;\frac{\partial[\mathrm{FPD}]_2^3}{\partial V_r}\;\frac{A_{co}}{A_r}+\frac{\partial[\mathrm{FPD}]_4^5}{\partial V_d}\;\frac{A_{co}}{A_d}+\frac{\partial[\mathrm{FPD}]_5^0}{\partial V_0}\right\}\delta V_0$$

$$=G_v\delta\bar{V}+(G_q{}^{12}+G_q{}^{23}+G_q{}^{34})\frac{\delta Q}{Q^*}$$

$$+\{G_{z1}^{01}+G_{z1}^{12}+G_{z1}^{23}+G_{z1}^{34}\}\delta z_1$$
$$+\{G_p^{01}+G_p^{12}+G_p^{23}+G_p^{34}+G_p^{45}\}\delta p \qquad (54)$$

The above equation is used to determine δV_0 as a function of $\delta\bar{V}$, δQ, δz_1 and δp.

3.6 Vessel Pressure Transfer Function

The reactor vessel, excluding the core and the riser, is divided into two zones; the upper part of the vessel, which is occupied by the saturated steam and water, and the lower part, which is occupied by the subcooled water. As shown in Fig. 4, the boundary of the

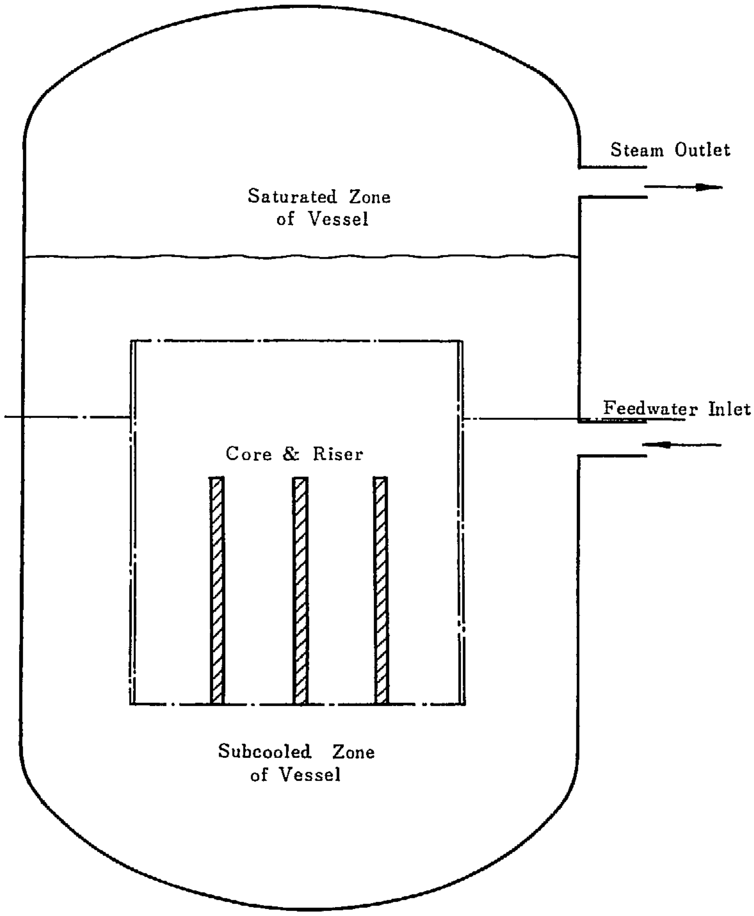


Fig. 4. The Saturated and the Subcooled Zone of Reactor Vessel

two zones is the feedwater spurger ring, where the subcooled feedwater is injected. The volume of the each zone is assumed to be constant. Then the fundamental equations of the system are as follows,

Mass balance:

$$\frac{dM_s}{dt}=W_s-W_L-u \qquad (55)$$

$$\frac{dM_w}{dt}=u+W_w-W_R \qquad (56)$$

$$\frac{dM_{sub}}{dt}=W_R+W_f-W_0 \tag{57}$$

where u stands for the rate of steam condensation due to pressure change.

Energy balance:

$$W_s i_s - W_L i_s + W_w i_w - W_R i_w = \frac{d}{dt}(M_w i_w) + \frac{d}{dt}(M_s i_s) - \frac{\overline{V}_{sat}}{J}\frac{dp}{dt} \tag{58}$$

$$W_R i_w + W_f\, i_f - W_0 i_0 = \frac{d}{dt}(M_{sub}\, i_{sub}) - \frac{\overline{V}_{sub}}{J}\frac{dp}{dt} \tag{59}$$

Constant volume:

$$\frac{d(v_s'M_s)}{dt}+\frac{d(v_w'M_w)}{dt}=0 \tag{60}$$

$$\frac{d(v'_{sub}\,M_{sub})}{dt}=0 \tag{61}$$

Combining the above equations and the core and riser flow rate transfer functions relating W_s and W_w to W_0, considering a small deviation from the steady state and taking the Laplace transform, the following solution is obtained.

$$(A_{pr}s+B_{pr})\delta p = v_w \delta W_f - v_s \delta W_L + V_0^*(y_2-1)\frac{\delta Q}{Q^*} - \Omega \delta z_1 \tag{62}$$

where

$$A_{pr}=M_w^*\left(\frac{\Delta v}{\Delta i}B-D\right)+M_s^*\left\{\frac{\Delta v}{\Delta i}(A+B)-(C+D)\right\}$$

$$-\frac{\Delta v}{\Delta i}\frac{\overline{V}_{sat}}{J}-V_0^*(C'-A')\{\tau_e(y_2-1)+\tau_{23}y_2\}+V_0^*E(\tau_{23}-\tau_{12})$$

$$B_{pr}=A'V_0^*(y_2-1)$$

3.7 Recirculation Flow Transfer Function

The transfer functions for the recirculation flow are also obtained from the fundamental equations of Sec. 3.6. Solving the Eqs. (57) and (61), the recirculation flow is given by the following equation

$$\delta W_R=\frac{1}{v_w}\delta V_0 - V_0^*\frac{1}{v_w^2}\frac{\partial v_w}{\partial p}\delta p - \frac{M_{sub}^*}{v_{sub}}\frac{\partial v_{sub}}{\partial t} - \delta W_f \tag{63}$$

3.8 Inlet Water Enthalpy Transfer Function

For the purpose of deriving the inlet water enthalpy transfer functions, the downcomer is regarded as a single tube. Let x denote the distance measured downstream along the axis of the tube, then the enthalpy of subcooled water in the downcomer is given by

$$i(x,s)=i(o,s)e^{-\frac{x}{V_d}s} \tag{64}$$

where $i(o,s)$ is the enthalpy at the top of subcooled region and is given by

$$i(o,s)=\frac{W_R i_w + W_f i_f}{W_R+W_f} \tag{65}$$

assuming the complete mixing of feedwater flow and the recirculation flow.
The total enthalpy in the subcooled region is

$$M_{sub}{}^{*}i_{sub}(s)=\int_{0}^{x_d}\frac{A_d\,i(x,s)}{v'_{sub}}dx \tag{66}$$

where x_d is the total length of the tube up to the inlet of core and v_{sub} is an average specific volume of water in the subcooled region.
Neglecting the change in water density,

$$M_{sub}{}^{*}i_{sub}=\frac{A_d}{v'_{sub}}\int_{0}^{x_d}i\,dx=\frac{A_d}{v'_{sub}}\frac{V_d}{s}i(o,s)(1-e^{-\frac{x_d}{V_d}s})=\frac{W_R{}^{*}+W_f{}^{*}}{s}(1-e^{-\frac{x_d}{V_d}s})i(o,s) \tag{67}$$

Thus,

$$M_{sub}{}^{*}s\,i_{sub}=(W_R{}^{*}+W_f{}^{*})(1-e^{-\frac{x_d}{V_d}s})i(o,s) \tag{68}$$

Combining Eqs. (59) and (68),

$$W_0{}^{*}\delta i_0=\frac{W_R{}^{*}W_f{}^{*}}{W_R{}^{*}+W_f{}^{*}}(i_w-i_f)\left(\frac{\delta W_R}{W_R{}^{*}}-\frac{\delta W_f}{W_f{}^{*}}\right)e^{-\tau_d s}$$

$$+\left(W_R{}^{*}\frac{\partial i_w}{\partial p}e^{-\tau_d s}+\frac{\bar{V}_{sub}}{J}s\right)\delta p+W_f{}^{*}\delta i_f\,e^{-\tau_d s} \tag{69}$$

where $\tau_d=x_d/V_d$

3.9 Summary of Transfer Functions

The transfer functions derived so far are summarized and given below. The interrelation of these transfer functions are shown in **Fig. 5.**

〔1〕 Zero Power Transfer Function

$$G_R(s)=\frac{\delta n/n^{*}}{\delta k/\beta}=\frac{s+\lambda}{\frac{l}{\beta}s\left(s+\frac{\beta}{l}+\lambda\right)} \tag{1}$$

〔2〕 Fuel Heat Transfer Dynamics

$$G_{f_1}(s)=\frac{\delta Q/Q^{*}}{\delta n/n^{*}}=\frac{1}{1+T_f s} \tag{2}$$

$$G_{f_2}(s)=\frac{\delta\theta_f}{\delta n/n^{*}}=\frac{\theta_f{}^{*}-\theta_w{}^{*}}{1+T_f s} \tag{3}$$

〔3〕 Void Transfer Function

$$G_1(s)=\frac{\delta\bar{V}}{\delta Q/Q^{*}}=\frac{v_s}{\Delta v}\tau_{12}V_0{}^{*}A_{co}\frac{1}{\tau_e s-1}\left[\frac{1-\frac{1}{y_2}}{\ln y_2}-\frac{1-e^{-\tau_{12}s}}{\tau_{12}s}\right] \tag{27}$$

$$G_2(s)=\frac{\delta\bar{V}}{\delta V_0/V_0{}^{*}}=-G_1(s) \tag{28}$$

$$G_3(s)=\frac{\delta\bar{V}}{\delta z_1}=\left[\frac{v_s}{\Delta v}\frac{A_{co}}{y_2}\right]\frac{-1+y_2e^{-\tau_{12}s}}{\tau_e s-1} \tag{29}$$

$$G_4(s)=\frac{\delta\bar{V}}{\delta p}=-\frac{v_s}{\Delta v}\tau_e V_0{}^* A_{co}\Big[\frac{v_w}{v_s}(C'-D')(y_2-1-\ln y_2)+D'\ln y_2$$

$$+(A'-C')\Big(1-\frac{1}{y_2}\Big)-E\frac{\ln y_2}{y_2}+(A'-C'-D')\frac{\tau_{12}}{\tau_e}\frac{1-e^{-\tau_{12}s}}{\tau_{12}s}$$

$$+\frac{1}{\tau_e s-1}\Big\{F\Big(\frac{1}{y_2}-e^{-\tau_{12}s}\Big)-E\frac{\ln y_2}{y_2}\Big\}+\frac{E}{(\tau_e s-1)^2}\Big(\frac{1}{y_2}-e^{-\tau_{12}s}\Big)\Big] \tag{30}$$

〔4〕 Boiling Boundary Transfer Function

$$G_5(s)=\frac{\delta z_1}{\delta Q/Q^*}=-(z_1-z_0)\frac{1-e^{-\tau_{01}s}}{\tau_{01}s} \tag{39a}$$

$$G_6(s)=\frac{\delta z_1}{\delta V_0/V_0{}^*}=-G_5(s) \tag{39b}$$

$$G_7(s)=\frac{\delta z_1}{\delta i_0}=-(z_1-z_0)\frac{\rho_w}{\tau_{01}Q^*}e^{-\tau_{01}s} \tag{39c}$$

$$G_8(s)=\frac{\delta z_1}{\delta p}=(z_1-z_0)\frac{\tau_e}{\tau_{01}}B' \quad \text{(second term is neglected)} \tag{39d}$$

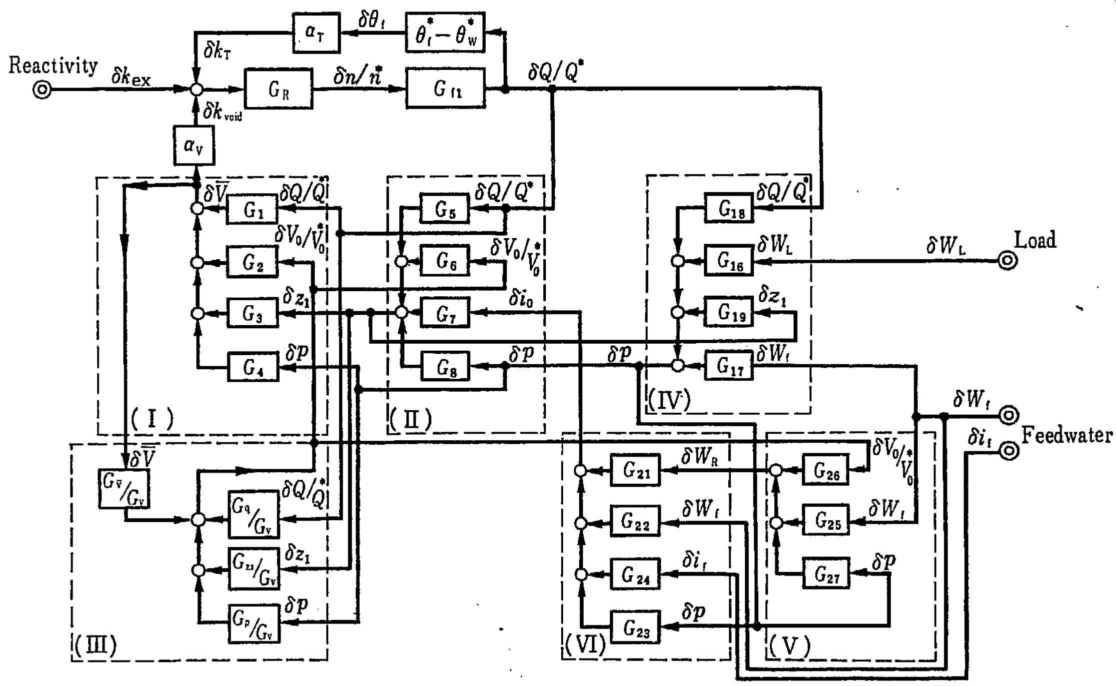
〔5〕 Inlet Velocity Transfer Function (Hydrodynamics)

$$G_v(s)\delta V_0=G_{\bar{v}}\delta\bar{V}+G_q(s)\frac{\delta Q}{Q^*}+G_{z1}(s)\delta z_1+G_p(s)\delta p \tag{54a}$$

(Dimension of Eq. (54a) is pressure, i.e. $\frac{(\mathrm{kg})}{\mathrm{cm}^2}\frac{\mathrm{cm}}{\mathrm{sec}^2}$)

where

$$G_v(s)=G_v{}^{01}+G_v{}^{12}+G_v{}^{23}+G_v{}^{34}+G_v{}^{45}+G_v{}^{50}+[\mathrm{FPD}]' \tag{54b}$$



(I) Core Void Transfer Function
(II) Boiling Boundary Transfer Function
(III) Inlet Velocity Transfer Function (Hydrodynamics)
(IV) Vessel Pressure Transfer Function
(V) Recirculation Flow Transfer Function
(VI) Inlet Water Enthalpy Transfer Function

Fig. 5. Transfer Funoticns of Natural Circulation Boiling Water Reactor

$$G_v^{01}=\left(\frac{\rho_w}{A_{co}}V_0^*\right)\tau_{01}s \tag{41a}$$

$$G_v^{12}=\left(\frac{\rho_w}{A_{co}}V_0^*\right)\left\{\frac{\tau_e\tau_{12}s^2}{\tau_e s-1}-\frac{1}{(\tau_e s-1)^2}(1-y_2e^{-\tau_{12}s})\right\} \tag{42a}$$

$$G_v^{23}=\mu\left(\frac{\rho_w}{A_{co}}V_0^*\right)\left\{\mu\tau_{23}s+\frac{\tau_{23}g}{y_2V_0^*}\cdot\frac{1-y_2e^{-\tau_{12}s}}{\tau_e s-1}\cdot\frac{1-e^{-\tau_{23}s}}{\tau_{23}s}\right\} \tag{43a}$$

$$G_v^{34}=\mu\left(\frac{\rho_w}{A_{co}}V_0^*\right)\left\{\mu\tau_{34}s+\frac{g\tau_r}{y_2V_0^*}\cdot\frac{1}{1+\tau_r s}-2\mu-\mu e^{-\tau_{23}s}\cdot\frac{1-y_2e^{-\tau_{12}s}}{\tau_e s-1}\right\} \tag{45a}$$

$$G_v^{45}=\mu_d^2\left(\frac{\rho_w}{A_{co}}V_0^*\right)(\tau_{45}s+2) \tag{46a}$$

$$G_v^{50}=M_{Lp}s=\left(\frac{\rho_w}{A_{co}}V_0^*\right)\tau_{50}s \tag{47a}$$

$$[\mathrm{FPD}]'=\frac{\partial[\mathrm{FPD}]_0^1}{\partial V_0}+\frac{\partial[\mathrm{FPD}]_1^2}{\partial V_0}+\frac{\partial[\mathrm{FPD}]_2^3}{\partial V_2}\cdot\frac{A_{co}}{A_r}$$
$$+\frac{\partial[\mathrm{FPD}]_4^5}{\partial V_d}\,\frac{A_{co}}{A_d}+\frac{\partial[\mathrm{FPD}]_5^0}{\partial V_0} \tag{54c}$$

$$G_{\bar{V}}=g(\rho_w-\rho_s)\frac{1}{A_{co}^2} \tag{42b}$$

$$G_q(s)=G_q^{12}+G_q^{23}+G_q^{34} \tag{54d}$$

$$G_q^{12}=\left(\frac{\rho_w}{A_{co}}V_0^{*2}\right)\left\{\frac{\tau_e\tau_{12}s^2}{\tau_e s-1}-\frac{1-y_2e^{-\tau_{12}s}}{(\tau_e s-1)^2}-(\tau_e s+2)(y_2-1)\right\} \tag{42c}$$

$$G_q^{23}=-\mu\left(\frac{\rho_w}{A_{co}}V_0^{*2}\right)\left\{\mu\tau_{23}(y_2-1)s-\frac{\tau_{23}g}{y_2V_0^*}\,\frac{1-y_2e^{-\tau_{12}s}}{\tau_e s-1}\cdot\frac{1-e^{-\tau_{23}s}}{\tau_{23}s}\right\} \tag{43b}$$

$$G_q^{34}=-\mu\left(\frac{\rho_w}{A_{co}}V_0^{*2}\right)\left\{(y_2-1)\left(\mu\tau_{34}s+\frac{g\tau_r}{y_2V_0^*}\cdot\frac{1}{1+\tau_r s}-2\mu\right)\right.$$
$$\left.+\mu e^{-\tau_{23}s}\frac{1-y_2e^{-\tau_{12}s}}{\tau_e s-1}\right\} \tag{45b}$$

$$G_{z1}(s)=G_{z1}^{01}+G_{z1}^{12}+G_{z1}^{23}+G_{z1}^{34} \tag{54e}$$

$$G_{z1}^{01}=-\left(\frac{\rho_w}{A_{co}}V_0^*\right)\frac{1}{\tau_e}\left(\tau_e s+\frac{\tau_e g}{V_0^*}\right) \tag{41b}$$

$$G_{z1}^{12}=\left(\frac{\rho_w}{A_{co}}V_0^*\right)\frac{1}{\tau_e}\left\{\frac{\tau_e^2s^2}{\tau_e s-1}\ln y_2+1+\frac{\tau_e g}{V_0^*}-\frac{\tau_e s(1-y_2e^{-\tau_{12}s})}{(\tau_e s-1)^2}\right\} \tag{42d}$$

$$G_{z1}^{23}=\mu\left(\frac{\rho_w}{A_{co}}V_0^*\right)\frac{1}{\tau_e}\left[\mu\tau_{23}s+\frac{g\tau_{23}}{y_2V_0^*}\left\{\frac{1-y_2e^{-\tau_{12}s}}{\tau_e s-1}-y_2e^{-\tau_{12}s}\right\}\frac{1-e^{-\tau_{23}s}}{\tau_{23}s}\right] \tag{43c}$$

$$G_{z1}^{34}=\mu\left(\frac{\rho_w}{A_{co}}V_0^*\right)\frac{1}{\tau_e}\left[\mu\tau_{34}s+\frac{g\tau_r}{y_2V_0^*}\,\frac{1}{1+\tau_r s}-2\mu\right.$$
$$\left.-\mu e^{-\tau_{23}s}\left\{\frac{1-y_2e^{-\tau_{12}s}}{\tau_e s-1}-y_2e^{-\tau_{12}s}\right\}\right] \tag{45c}$$

$$G_p(s)=G_p^{01}+G_p^{12}+G_p^{23}+G_p^{34}+G_p^{45} \tag{54f}$$

$$G_p^{01}=\left(\frac{\rho_w}{A_{co}}V_0^{*2}\right)D'\tau_{01}\left(s+\frac{g}{V_0^*}\right) \tag{41c}$$

$$G_p^{12}=\left(\frac{\rho_w}{A_{co}}V_0{}^{*2}\right)\Big[(A'-C')(y_2-1)(\tau_e s+1)(\tau_e s+2)$$

$$+\left\{F+\tau_e s(F-2E)+\frac{F}{\tau_e s-1}+E\frac{\tau_e s}{(\tau_e s-1)^2}+(C'-A')\tau_e{}^2 s^2\right\}\ln y_2$$

$$+\{(C'-A')-D'(2\tau_e s-1)+B'\tau_e{}^2 s^2\}\frac{1-y_2 e^{-\tau_{12}s}}{(\tau_e s-1)^3}$$

$$-\frac{E}{2}\frac{\tau_0\tau_{12}{}^2 s^3}{\tau_e s-1}$$

$$-\frac{g\tau_e}{V_0{}^*}\frac{v_w}{v_s}\left\{(D'-C')(y_2-1)-\left(\frac{v_s+v_w}{v_w}D'-C'\right)\frac{\tau_{12}}{\tau_e}\right\}\Big] \quad (42e)$$

$$G_p^{23}=\mu\left(\frac{\rho_w}{A_{co}}V_0{}^{*2}\right)\Big[\{(A'-C')(y_2-1)(\tau_e s+2)+B'-D'-E\tau_{12}s\}\mu\tau_{23}s$$

$$+\frac{1}{2}\{(A'-C')(y_2-1)+B'-D'\}\mu\tau_{23}{}^2 s^2$$

$$-\frac{g\tau_{23}}{y_2 V_0{}^*}\{(A'-C')(y_2-1)+B'-D'\}$$

$$+\frac{\tau_{23}g}{y_2V_0{}^*}\frac{1-e^{-\tau_{23}s}}{\tau_{23}s}\Big[(A'-C')(y_2-1)-E\ln y_2+B'y_2e^{-\tau_{12}s}$$

$$-\frac{1}{\tau_e s-1}\{2(B'-D')(1-y_2e^{-\tau_{12}s})+E\ln y_2\}$$

$$+E\frac{1-y_2e^{-\tau_{12}s}}{(\tau_e s-1)^2}\Big]\Big] \quad (43d)$$

$$G_p^{34}=\mu\left(\frac{\rho_w}{A_{co}}V_0{}^{*2}\right)\Big[\left(\mu\tau_{34}s+\frac{g\tau_r}{y_2V_0{}^*}\frac{1}{1+\tau_r s}-2\mu\right)\Big[(A'-C')\{\tau_e(y_2-1)+\tau_{23}y_2\}s$$

$$-(\tau_{12}+\tau_{23})Es+(A'-C')(y_2-1)\Big]$$

$$-\mu y_2{}^2\left\{-\mu\kappa v_w+\frac{\Delta v}{v_s}\alpha(1-e^{-\tau_{23}s})\right\}$$

$$+\mu e^{-\tau_{23}s}\Big[\frac{v_w}{v_s}(C'-D')y_2(y_2-1)+D'(y_2-1)+E\ln y_2$$

$$+B'(1-y_2e^{-\tau_{12}s})+\frac{1}{\tau_e s-1}\{2(B'-D')(1-y_2e^{-\tau_{12}s})$$

$$+E\ln y_2\}-E\frac{1-y_2e^{-\tau_{12}s}}{(\tau_e s-1)^2}\Big]\Big] \quad (45d)$$

$$G_p^{45}=\mu_d\left(\frac{\rho_w}{A_{co}}V_0{}^{*2}\right)D'\left(\mu_d\tau_{45}s+\mu_d-\frac{\tau_{45}g}{V_0{}^*}\right) \quad (46b)$$

[6] Vessel Pressure Transfer Function

$$G_{16}=\frac{\delta p}{\delta W_L}=-v_s G_{pr} \quad (62a)$$

$$G_{17}=\frac{\delta p}{\delta W_f}=v_w G_{pr} \quad (62b)$$

$$G_{18}=\frac{\delta p}{\delta Q/Q^*}=V_0^*(y_2-1)G_{pr} \tag{62c}$$

$$G_{19}=\frac{\delta p}{\delta z_1}=-\frac{1}{\tau_e}G_{pr} \tag{62d}$$

where

$$G_{pr}=\frac{1}{A_{pr}s+B_{pr}} \tag{62e}$$

$$A_{pr}=M_w^*\left(B\frac{\Delta v}{\Delta i}-D\right)+M_s^*\left\{(A+B)\frac{\Delta v}{\Delta i}-(C+D)\right\}$$

$$-\frac{\Delta v}{\Delta i}\frac{\overline{V}_{sat}}{J}-V_0^*(C'-A')\{\tau_e(y_2-1)+\tau_{23}y_2\}+V_0^*E(\tau_{23}-\tau_{12})$$

$$B_{pr}=V_0^*A'(y_2-1)$$

〔7〕 Recirculation Flow Transfer Function

$$G_{25}=\frac{\delta W_R}{\delta W_f}=-1 \tag{63a}$$

$$G_{26}=\frac{\delta W_R}{\delta V_0/V_0^*}=\frac{V_0^*}{v_w} \tag{63b}$$

$$G_{27}=\frac{\delta W_R}{\delta p}=-V_0^*\frac{D}{v_w^2}-\frac{M_{sub}^*}{v_{sub}}\cdot\frac{\partial v_{sub}}{\partial p}\cdot s \tag{63c}$$

〔8〕 Inlet Water Enthalpy Transfer Function

$$G_{21}=\frac{\delta i_0}{\delta W_R}=\frac{W_f^*}{W_0^*}\frac{i_w-i_f}{W_R^*+W_f^*}e^{-\tau_d s} \tag{69a}$$

$$G_{22}=\frac{\delta i_0}{\delta W_f}=-\frac{W_R^*}{W_0^*}\frac{i_w-i_f}{W_R^*+W_f^*}e^{-\tau_d s} \tag{69b}$$

$$G_{23}=\frac{\delta i_0}{\delta p}=\frac{W_R^*}{W_0^*}B\,e^{-\tau_d s}+\frac{1}{W_0^*}\frac{\overline{V}_{sub}}{J}s \tag{69c}$$

$$G_{24}=\frac{\delta i_0}{\delta i_f}=\frac{W_f^*}{W_0^*}e^{-\tau_d s} \tag{69d}$$

4. Conclusion

The complete set of transfer functions pertinent to the dynamic analysis of natural circulation boiling water reactor is derived. It includes the zero power transfer function, the fuel heat transfer dynamics, and the power to void transfer function, which consists of the core void transfer function, the core boiling boundary transfer function, the inlet velocity transfer function, the vessel pressure transfer function, the recirculation flow transfer function and the inlet water enthalpy transfer function. The interrelation of these groups of transfer functions is summarized in Fig. 2.

The derivation of each transfer function is described in Secs. 3. 1 through 3. 8. The obtained transfer functions are summarized in Sec. 3. 9. It is quite easy to substitute the numerical values of the design parameters into these equations and obtain the numerical values of the transfer functions. It is difficult, however, to simulate some of these transfer functions on the analog computer as they are, and also to obtain their numerical values for different frequencies for the purpose of stability study. The modification of the equations for the simulation and the stability study will be discussed in the subsequent reports.

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Nomenclature

Symbol		Unit
Q	heat flux per unit length	kcal/sec·cm
W_L	steam flow to load	(kg)*/sec
W_s	steam flow from riser	〃
W_w	water flow from riser	〃
W_0	water flow at core inlet	〃
W_R	recirculation flow	〃
W_f	feed water flow	〃
A_{co}	flow area of core	cm^2
A_r	flow area of riser	〃
A_d	flow area of downcomer	〃
$A_v=A_r+A_d$	total sectional area of vessel	〃
z	position in axial direction (the origin at the inlet of core)	cm
x_d	equivalent flow length from feedwater inlet to core inlet	〃
z_{Lp}	equivalent flow length in lower plenum (from z_5 to z_0)	〃
h	reflecter height above the riser	〃
μ	A_{co}/A_r	—
μ_d	A_{co}/A_d	—
V	flow velocity	cm/sec
$y(z)$	defined as $\left(V_0^*+\frac{z-z_1}{\tau_e}\right)\Big/V_0^*=\frac{\rho_w}{\rho_w(1-f)+\rho_s f}$	—
y_2, y_3	$y_2=y(z_2)$, $y_3=y(z_3)$	—
f	void fraction	—
r	slip ratio	—
θ	temperature	°C
θ_f	fuel average temperature	°C
p	pressure	kg/cm^2
u	condensation rate of steam	(kg)/sec
U	steam void wave velocity	cm/sec
$G(s)$	transfer function	
G_{pr}	defined in Eq. (62e)	$\frac{kg\cdot sec}{cm^3}$
A_{pr}	〃	cm^3/kg
B_{pr}	〃	$cm^3/(kg\cdot sec)$
α	defined as $-\frac{v_s}{\Delta v}\frac{1}{y_2^2}\left\{A'(y_2-1)+B'+(C'-D')(y_2-1)\right.$	

* (kg) denotes the unit of mass.
kg. denotes the unit of force

	$\cdot\left(\frac{v_w}{v_s}y_2-1\right)\Big\}$	cm^2/kg
κ	defined as $\frac{1}{\mu}\left\{-\frac{1}{y_2}\frac{D'}{v_w}+\frac{y_2-1}{y_2}\frac{D'-C'}{v_s}\right\}$	cm(kg)/kg
$\bar{V}$	void volume in core	cm^3
$\bar{V}_{sat\ co}$	saturated steam and water volume in core	〃
$\bar{V}_{sub\ co}$	subcooled water volume in core	〃
$\bar{V}_{sat}$	saturated steam and water volume in vessel (excluding core and riser)	〃
$\bar{V}_{sub}$	subcooled water in volume vessel (excluding core and riser)	〃
$M_{s\ co}$	saturated steam mass in core	(kg)
M_{wco}	saturated water mass in core	〃
$M_{sub\ co}$	subcooled water mass in core	〃
M_s	saturated steam mass in vessel (excluding core and riser)	〃
M_w	saturated water mass in vessel (excluding core and riser)	〃
M_{sub}	subcooled water mass in vessel (excluding core and riser)	〃
M_{Lp}	equivalent mass along flow in lower plenum	$(kg)/cm^2$
i	specific enthalpy	kcal/(kg)
i_s	〃 of saturated steam	〃
i_w	〃 of saturated water	〃
Δi	$=i_s-i_w$	kcal/(kg)
i_{sub}	average enthalpy of water in subcooled region	〃
i_f	specific enthalpy of feedwater	〃
$\rho=\rho' A_{co}$	density per unit length	(kg)/cm
ρ'	density	$(kg)/cm^3$
ρ_s, ρ_w	density per unit length of saturated steam and water, respectively	(kg)/cm
ρ_s', ρ_w'	density of saturated steam and water, respectively	$(kg)/cm^3$
$\Delta\rho$	$=\rho_s-\rho_w$	(kg)/cm
$\Delta\rho'$	$=\rho_s'-\rho_w'$	$(kg)/cm^3$
ρ_{sub}	average water density per unit length in subcooled region	(kg)/cm
ρ'_{sub}	average water density in subcooled region	$(kg)/cm^3$
v	$=1/\rho$	cm/(kg)
v'	$=1/\rho'$ specific volume	cm/(kg)
v_s, v_w	$v_s=1/\rho_s,\ v_w=1/\rho_w$	cm/(kg)
v_s', v_w'	$v_s'=1/\rho_s',\ v_w'=1/\rho_w'$	$cm^3/(kg)$
Δv	$=v_s-v_w$	cm/(kg)
$\Delta v'$	$=v_s'-v_w'$	$cm^3/(kg)$
v_{sub}	$=1/\rho_{sub}$	cm/(kg)
v'_{sub}	$=1/\rho'_{sub}$ average specific volume of water in subcooled region	$cm^3/(kg)$

A	$=\partial\Delta i/\partial p$	$\frac{\text{kcal}}{(\text{kg})}\Big/\frac{\text{kg}}{\text{cm}^2}$
B	$=\partial i_w/\partial p$	〃
C	$=\partial\Delta v/\partial p$	$\frac{\text{cm}}{(\text{kg})}\Big/\frac{\text{kg}}{\text{cm}^2}$
D	$=\partial v_w/\partial p$	〃
E	$=A'-B'-C'+D'$	cm^2/kg
F	$=-A'-B'+C'+D'$	〃
A'	$=A/\Delta i$	cm^2/kg
B'	$=\frac{\Delta v}{\Delta i}\frac{B}{v_w}$	〃
C'	$=C/\Delta v$	〃
D'	$=D/v_w$	〃
τ_e	$=\frac{\Delta i}{Q^*\Delta v}$: steam raising time, i. e. time during which the unit volume of steam is raised per unit volume of water	sec
τ_{01}	$=(z_1-z_0)/V_0^*$	〃
τ_{12}	$=\tau_e \ln y_2$: void transit time in boiling region	〃
τ_{23}	$=(z_3-z_2)/(\mu V_2^*)$	〃
τ_{34}	$=h/(\mu V_2^*)$	〃
τ_{45}	$=(z_3-z_0+h)/(\mu_d V_0^*)$	〃
τ_{50}	$=z_{Lp}/V_0^*$	〃
τ_d	time taken by flow from f. w. inlet to core inlet	〃
τ_r	time constant associated with the natural frequency of reflector free surface	〃
T_f	fuel time constant, defined in Eq. (2)	〃
Ω	$=\tau_e^{-1}=Q^*\frac{\Delta v}{\Delta i}$ rate of steam raising	sec^{-1}
[FPD]	frictional pressure drop	$(\text{kg})\frac{\text{cm}}{\text{sec}^2}\cdot\frac{1}{\text{cm}^2}$
[FPD]′	$=\frac{\partial[\text{FPD}]}{\partial V_0}$	$(\text{kg})/\text{cm}^2\cdot\text{sec}$

Subscript

w	water
s	steam
d	downcomer
r	riser
0	core inlet
1	boiling boundary
2	top of core

Subscript (cont'd)

3	top of riser
4	top of reflector above the riser
5	the same level in downcomer as core inlet
sub	subcooled
sat	saturated
avg	average

Superscript

$*$	steady state value
$\sim$	variable associated with riser necessary to distinguish from those with core due to the difference flow area in riser and core

Some Relations for two phase flow for uniform heat flux and unity slip ratio

$\tau_e = \dfrac{\Delta i}{Q^* \Delta v}$: steam raising time

$\tau_{12} = \tau_e \ln y_2$: void transit time in boiling region

$$V^*(z) = U^*(z) = V_0^* + \frac{1}{\tau_e}(z - z_1) = V_0^* \cdot y$$

$$y^*(z) = \frac{U^*(z)}{V_0^*} = \frac{V_0^* + \frac{1}{\tau_e}(z-z_1)}{V_0^*}$$

$$y^*(z) = \frac{\rho_w}{\rho_w(1-f^*) + \rho_s f^*} = \frac{\rho_w}{\bar{\rho}(z)}, \quad y^* - 1 = \frac{z - z_1}{\tau_e V_0^*}, \quad z - z_1 = \tau_e V_0^*(y^* - 1)$$

$$f^*(z) = \frac{v_s}{\Delta v} \frac{y^* - 1}{y^*}$$

$q^*(z) = \dfrac{1}{\tau_e} \dfrac{v_s}{\Delta v} \dfrac{1}{y^*}$: variable defined in Eq. (7)

Some relations for steam and water properties

$$A = \partial \Delta i / \partial p, \quad B = \frac{\partial i_w}{\partial p}, \quad C = \partial \Delta v / \partial p, \quad D = \frac{\partial v_w}{\partial p}$$

$$E = A' - B' - C' + D', \quad F = -A' - B' + C' + D'$$

$$A' = A/\Delta i, \quad B' = \frac{\Delta v}{\Delta i} \frac{B}{v_w}, \quad C' = C/\Delta v, \quad D' = D/v_w$$

$$\frac{\partial \rho_s}{\partial p} = -\frac{1}{v_s^2}(C + D), \quad \frac{\partial \rho_w}{\partial p} = -\frac{1}{v_w^2} D$$

$$\frac{\partial \rho_s}{\partial p} - \frac{\partial \rho_w}{\partial p} = \frac{\Delta v}{v_s^2}\left(\frac{v_s + v_w}{v_w} D' - C'\right)$$

$$\frac{\rho_w - \rho_s}{\rho_w \rho_s} = \Delta v, \quad \rho_w - \rho_s = \frac{\Delta v}{v_w v_s}$$

$$\frac{\partial}{\partial p}\left(\frac{\Delta v}{\Delta i}\right) = \frac{\Delta v}{\Delta i}(C' - A')$$

References

The derivation of the transfer functions outlined in Secs. 3.1 through 3.8 is presented in more detail in the following internal memoes.

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