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# The KAK Program for the Numerical Solution of Few-Group Neutron Diffusion Equations in Two Dimensions

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# The KAK Program for the Numerical Solution of Few-Group Neutron Diffusion Equations in Two Dimensions


## Summary

The KAK program for the IBM 7044 is capable of solving neutron diffusion problems in cylindrical or slab geometry for one to four groups. Up to 1500 mesh points may be used. The diffusion difference equation is solved by the matrix factorization method. The source iteration is extraporated by the Tchebysheff polynomial method. The criticality search by the poison control, the adjoint flux calculation and the perturbation calculation may be performed at the user's option. Normalization of fluxes to an arbitrary input power is allowed. The regionwise-average neutron fluxes and leakages are listed as the output data. A typical running time is 18 min. for the case with 530 mesh points and three energy groups which converged with three source iterations.

June, 1966



MAKOTO AKANUMA, Computing Center

YASUSHI KUGE, JPDR-II Project Office

SHIGERU YASUKAWA, Thermal Reactor Design Office

Tokai Research Establishment, Japan Atomic Energy Research Institute


# 2次元小数群中性子拡散コード：KAK


## 要旨

IBM 7044 用中性子拡散コード，KAK，はエネルギー群数の最大が4群までの円筒状または平板状の体系内の中性子拡散問題を解く計算コードである．使用可能なメッシュ点最大数は 1,500 点である．中性子拡散の階差方程式はマトリックス因数分解法により解かれる．中性子源収束計算はチェビシェフ多項式法により加速されている．コード使用者の選択によって，吸収材の断面積変更による臨界調整，アジョイント中性子束の計算あるいは摂動計算をおこなうことができる．中性子束は，出力が与えられた値になるように規格化される．出力量として領域平均中性子束や漏洩量なども与えられる．計算時間の1例を記すと，3群で 530 メッシュ点の問題を3回の中性子源繰返し計算で収束させた場合の計算時間は 18 分である．

1966 年 6 月



日本原子力研究所　東海研究所

動力炉開発部　計算センター　赤沼　誠

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## 1. Introduction

The few group neutron diffusion equation code is one of the most frequently used digital computer codes in the reactor physics calculations. Various numerical methods for solving the twodimensi onal neutron diffusion equation were developed and programmed for use with the high speed digital computers. Among them PDQ, CURE and Twenty Grand codes are widely used in the nuclear reactor criticality calculations. These codes, however, make use of some pointwise or linewise relaxation method to solve the diffusion difference equation in one energy group. Such an inner iteration routine is one of the time consuming part in the diffusion code and many efforts were concentrated in developing the procedure for rapid convergence of the inner iteration.

The authors developed one scheme of numerically solving the two-dimensional diffusion equation without any use of the inner iteration process by the direct generalization of the method applied to the one-dimensional diffusion code. This method was originally developed by G. I. MARCHUK[1] and designated as the matrix factorization method by R. S. VALGA[2]. The important features of this method are (1) the significant reduction of computational steps by the exclusion of the inner iteration process (merit) and (2) the requirement of larger core memories for the matrix inversion (demerit). However, the recent trend of the digital computer development foresees larger core memories as well as higher computation speed. Thus, the requirement for larger core memories (or more frequent use of tapes) will not prevent the application of this method. In our institute the only available two-dimensional diffusion code applicable for more than two energy groups, at present, is the Twenty Grand code which requires fairly long computation time. The two-dimensional diffusion code, KAK, presented in this report will save the computation time in the diffusion equation calculation hereafter.

This code solves the two-dimensional diffusion equation, as well as its adjoint equation by the matrix factorization method. The criticality search by the poison concentration control and the reactivity calculation by the few group perturbation theory can also be performed. Varieties of flux averaged values are listed as the output of the edit routine. The log derivative condition can be applied on the outer boundary, but not on the normal mesh lines within the boundary. The code is programmed by the FORTRAN IV for use in the IBM 7044 computer.

## 2. Main feature of KAK code

The main features of the two-dimensional diffusion equation code, KAK, are summerized as follows.

1) Name of the code: KAK.
2) Equation to be solved: two-dimensional, few-group neutron diffusion equation.
3) Geometry: $x$–$y$ or $r$–$z$.
4) Energy group: 4 groups (max.).
5) Boundary condition: Vanishing flux, symmetrical flux, flux with logarithmic derivative.
6) Material regions: 40 regions (max.).
7) Material specification: overlapping permissible.
8) Radial mesh points: 30 points (max. for $r$ or $x$).
9) Axial mesh points: 50 points (max. for $z$ or $y$).
10) Method of calculating the pointwise flux: Matrix factorization method.
11) Method of the source iteration: Tchebysheff polynomial method.

12) Flux convergence criterion: pointwise source ratio.
13) Criticality search: performed by controlling the poison absorption cross section.
14) Adjoint flux: calculated by the user's option.
15) Reactivity change by the perturbation: calculated by the user's option.
16) Programming language: FORTRAN-IV.
17) Computer to be used: IBM-7044

## 3. Derivation of the difference equation

The few-group, two-dimensional diffusion equation is expressed by the following second order differential equation,

$$-D^i(r)\nabla^2\phi^i(r)+\Sigma_T{}^i(r)\phi^i(r)=X^iS(r)+\Sigma_r{}^{i-1}(r)\phi^{i-1}(r) \tag{1}$$

where

$$S(r)=\frac{1}{\lambda}\sum_i \nu\Sigma_f{}^i(r)\phi^i(r) \tag{2}$$

$$\Sigma_T{}^i(r)=\Sigma_a{}^i(r)+\Sigma_r{}^i(r)+D^i(r)B_z{}^2(r) \tag{3}$$

$$(i=1,\ 2,\ \cdots\cdots,\ I)$$

and

$$\Sigma_r{}^0=\Sigma_r{}^I=0$$

$$X^I=0, \qquad \sum_i X^i=1.0$$

$$B_z{}^2=0 \qquad \text{(for the cylindrical geometry).}$$

The radial co-ordinate ($r$ or $x$) axis of the two-dimensional ($r$-$z$ or $x$-$y$) system to be solved is devided into $K$ mesh points, and the axial co-ordinate ($z$ or $y$) axis into $L$ mesh points. (The mesh interval can be varied arbitrarily). An arbitrary mesh point in the system is represented by the index ($k$, $l$). The mesh interval between the mesh point ($k$, $l$) and its adjacent point is specified as $L$, $R$, $T$ or $B$ for the left-side, right-side, upper or lower direction, respectively. Each quadrant around the point ($k$, $l$) is numbered as 1, 2, 3 or 4, as shown in **Fig. 1**. The coordinates of the point ($k$, $l$) are represented as ($r_p$, $z_p$) for convenience.

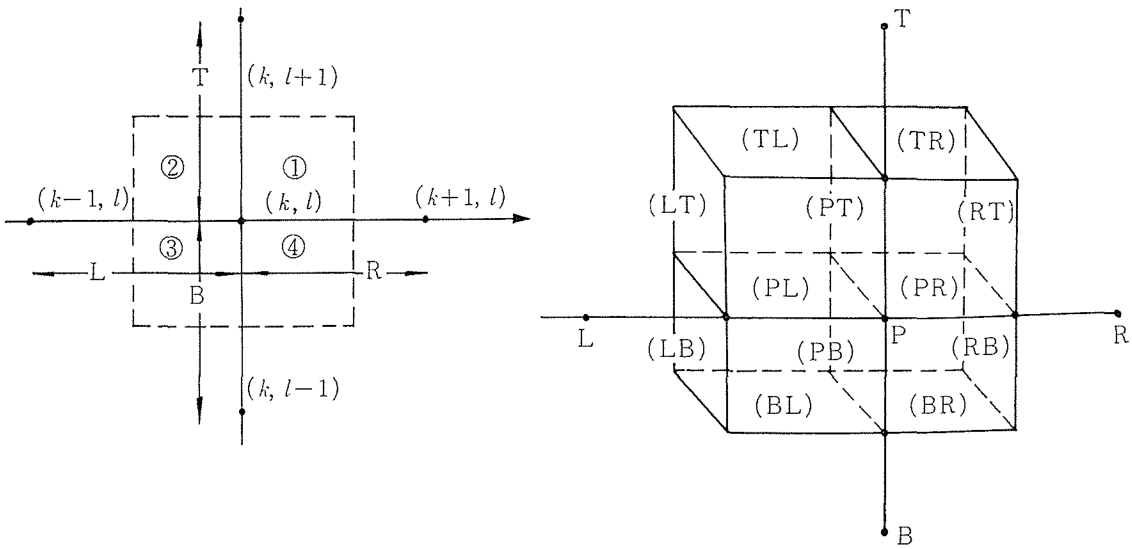


**Fig. 1** Mesh interval and area

The diffusion equation (1) is integrated between the small intervals.

$$r_p-\frac{L}{2}\leq r\leq r_p+\frac{R}{2}$$

$$z_p-\frac{B}{2}\leq z\leq z_p+\frac{T}{2}$$

to derive the difference equations.

The material composition for each quadrant around the point $(k, l)$ may be specified arbitrarily. The nuclear constant, for example, the diffusion coefficient, $D^i(r, z)$, for each quadrant is represented by $D_q$ ($q=1$, 2, 3 and 4) for simplicity.

The continuity conditions of the neutron flux and the net neutron current are applied on the boundary surfaces including the point $(k, l)$.

$$\begin{aligned} &\phi^i_{r-}(p)=\phi^i_{r+}(p), \qquad \phi^i_{z-}(p)=\phi^i_{z+}(p) \\ &J^i_{r-}(p)=J^i_{r+}(p), \qquad J^i_{z-}(p)=J^i_{z+}(p) \end{aligned} \tag{4}$$

Integrating the equation (1), the first term of the left hand side becomes (the energy group index, $i$, is omitted for simplicity)

$$-\iiint D\nabla^2\phi dV=-\iint D\nabla\phi\cdot\boldsymbol{dS}$$

$$\begin{aligned} =&-\frac{D_1}{R}(RT)(\phi_{k+1,l}-\phi_{k,l})-\frac{D_1}{T}(TR)(\phi_{k,l+1}-\phi_{k,l})-\frac{D_2}{T}(TL)(\phi_{k,l+1}-\phi_{k,l}) \\ &+\frac{D_2}{L}(LT)(\phi_{k,l}-\phi_{k-1,l})+\frac{D_3}{L}(LB)(\phi_{k,l}-\phi_{k-1,l})+\frac{D_3}{B}(BL)(\phi_{k,l}-\phi_{k,l-1}) \\ &+\frac{D_4}{B}(BR)(\phi_{k,l}-\phi_{k,l-1})-\frac{D_4}{R}(RB)(\phi_{k+1,l}-\phi_{k,l}) \end{aligned}$$

by application of the conditions (4). Integration of the second term of the left hand side and the right hand side are approximated, respectively, by

$$\iiint \Sigma_T\phi dV\doteqdot\left\{\Sigma_1^T(PR)\frac{T}{2}+\Sigma_2^T(PL)\frac{T}{2}+\Sigma_3^T(PL)\frac{B}{2}+\Sigma_4^T(PR)\frac{B}{2}\right\}\phi_{k,l}$$

and

$$\iiint fdV\doteqdot f_1(PR)\frac{T}{2}+f_2(PL)\frac{T}{2}+f_3(PL)\frac{B}{2}+f_4(PR)\frac{B}{2}$$

where

$$f_q=X^iS_{k,l,q}+\Sigma_{rq}^{i-1}\phi_{k,l}^{i-1}$$
$$(q=1,\ 2,\ 3,\ 4)$$

$(PR)$, $(PL)$, etc. are the surface areas of the cube illustrated in **Fig. 1** and are calculated by the following formulae,

$$\begin{aligned} &(PR)=\frac{R}{2}\left(r+\frac{R}{4}\right)^a, \qquad (PL)=\frac{L}{2}\left(r-\frac{L}{4}\right)^a, \\ &(RT)=\frac{T}{2}\left(r+\frac{R}{2}\right)^a, \qquad (LT)=\frac{T}{2}\left(r-\frac{L}{2}\right)^a, \\ &(RB)=\frac{B}{2}\left(r+\frac{R}{2}\right)^a, \qquad (LB)=\frac{B}{2}\left(r-\frac{L}{2}\right)^a, \\ &(PT)=\frac{T}{2}r^a, \qquad (PB)=\frac{B}{2}r^a, \\ &(BR)=(TR)=(PR), \qquad (BL)=(TL)=(PL) \end{aligned} \tag{5}$$

where $a=0$ for $x$–$y$ geometry and
$a=1$ for $r$–$z$ geometry.

The integrated equation derived above is reduced to the five-point difference equation.

$$-a_{k,l}\phi_{k+1,l}-b_{k,l}\phi_{k,l-1}-c_{k,l}\phi_{k-1,l}-d_{k,l}\phi_{k,l+1}+p_{k,l}\phi_{k,l}=f_{k,l} \tag{6}$$

where

$$a_{k,l}=\frac{\{D_1(RT)+D_4(RB)\}}{R}$$

$$b_{k,l}=\frac{\{D_3(BL)+D_4(BR)\}}{B}$$

$$c_{k,l}=\frac{\{D_2(LT)+D_3(LB)\}}{L}$$

$$d_{k,l}=\frac{\{D_1(TR)+D_2(TL)\}}{T} \tag{7}$$

$$p_{k,l}=a_{k,l}+b_{k,l}+c_{k,l}+d_{k,l}+\gamma_{k,l}$$

$$\gamma_{k,l}=\frac{\{\Sigma_1{}^T(PR)+\Sigma_2{}^T(PL)\}T}{2}+\frac{\{\Sigma_3{}^T(PL)+\Sigma_4{}^T(PR)\}B}{2}$$

$$f_{k,l}=\frac{\{f_1(PR)+f_2(PL)\}T}{2}+\frac{\{f_3(PL)+f_4(PR)\}B}{2}$$

The five-point difference equation (6) and its coefficent formulae (7) are the fundamental equations of the two-dimensional diffusion equation code.

The equation (6) may be represented by the matrix form,

$$\boldsymbol{A\phi}=\boldsymbol{f} \tag{8}$$

where

$$\boldsymbol{A}=\begin{pmatrix} \boldsymbol{b}_1 & -\boldsymbol{a}_1 & & & \\ -\boldsymbol{c}_2 & \boldsymbol{b}_2 & -\boldsymbol{a}_2 & & \\ & \ddots & \ddots & \ddots & \\ & -\boldsymbol{c}_k & \boldsymbol{b}_k & -\boldsymbol{a}_k & \\ & & \ddots & \ddots & \ddots \\ & & & -\boldsymbol{c}_K & \boldsymbol{b}_K \end{pmatrix} \tag{9}$$

$\boldsymbol{a}_k$, $\boldsymbol{b}_k$ and $\boldsymbol{c}_k$ are submatrices having the following elements;

$$\boldsymbol{a}_k=\begin{pmatrix} a_{k1} & & \\ & a_{k2} & \\ & & \ddots \\ & & & a_{KL} \end{pmatrix}$$

$$\boldsymbol{b}_k=\begin{pmatrix} p_{k1} & -d_{k1} & & \\ -b_{k2} & p_{k2} & -d_{k2} & \\ & \ddots & \ddots & \ddots \\ & & -b_{KL} & p_{KL} \end{pmatrix}$$

$$\boldsymbol{c}_k=\begin{pmatrix} c_{k1} & & \\ & c_{k2} & \\ & & \ddots \\ & & & c_{KL} \end{pmatrix} \tag{10}$$

$\boldsymbol{\phi}$ and $\boldsymbol{f}$ are vectors with the following elements;

$$\boldsymbol{\phi}=\begin{pmatrix} \phi_{11} \\ \phi_{12} \\ \vdots \\ \phi_{kl} \\ \vdots \\ \phi_{KL} \end{pmatrix} \qquad \boldsymbol{f}=\begin{pmatrix} f_{11} \\ f_{12} \\ \vdots \\ f_{kl} \\ \vdots \\ f_{KL} \end{pmatrix} \tag{11}$$

## 4. Matrix factorization method

The five point difference equation (6) for the i-th energy group is solved by the matrix factorization method. This is the direct generalization to the matrix form of the line inversion method applied to the three-point difference equation in case of the one-dimensional diffusion problem.

The vectors, $\boldsymbol{\phi}$ and $\boldsymbol{f}$, are subdivided into $K$ subvectors, $\boldsymbol{\phi}_k$ and $\boldsymbol{f}_k$;

$$\boldsymbol{\phi}_k=\begin{pmatrix}\phi_{k1}\\ \phi_{kL}\end{pmatrix},\qquad \boldsymbol{f}_k=\begin{pmatrix}\boldsymbol{f}_{k1}\\ \boldsymbol{f}_{kL}\end{pmatrix} \tag{12}$$

The matrix equation (8) can be expressed by the simultaneous submatrix equations, as follows;

$$-\boldsymbol{a}_k\boldsymbol{\phi}_{k+1}+\boldsymbol{b}_k\boldsymbol{\phi}_k-\boldsymbol{c}_k\boldsymbol{\phi}_{k-1}=\boldsymbol{f}_k \tag{13}$$
$$(k=1,\ 2,\ \cdots\cdots,\ K)$$

where $\boldsymbol{a}_k$, $\boldsymbol{b}_k$ and $\boldsymbol{c}_k$ are the submatrices given by (10).
The equations (13) are modified to the form of

$$\boldsymbol{\phi}_{k+1}=\boldsymbol{B}_k\boldsymbol{\phi}_k-\boldsymbol{C}_k\boldsymbol{\phi}_{k-1}-\boldsymbol{F}_k \tag{14}$$

$$\boldsymbol{B}_k=\boldsymbol{a}_k^{-1}\boldsymbol{b}_k$$
$$\boldsymbol{C}_k=\boldsymbol{a}_k^{-1}\boldsymbol{c}_k$$
$$\boldsymbol{F}_k=\boldsymbol{a}_k^{-1}\boldsymbol{f}_k \tag{15}$$

We try to solve the matrix equation (14) by the backward reccurence formula,

$$\boldsymbol{\phi}_k=\boldsymbol{C}_{k+1}^{-1}(\boldsymbol{\beta}_{k+1}\boldsymbol{\phi}_{k+1}+\boldsymbol{Z}_{k+1}) \tag{16}$$

and obtain the reccurence formulae for the coefficient matrix and vector, $\boldsymbol{\beta}_k$ and $\boldsymbol{Z}_k$. Substituting the similar expression for $\boldsymbol{\phi}_{k-1}$ of the equation (16) into equation (14), the following expression of $\boldsymbol{\phi}_k$ is derived;

$$\boldsymbol{\phi}_k=(\boldsymbol{B}_k-\boldsymbol{\beta}_k)^{-1}(\boldsymbol{\phi}_{k+1}+\boldsymbol{Z}_k+\boldsymbol{F}_k) \tag{17}$$

Equating the coefficients of the equations (16) and (17), the forward reccurence formulae for the coefficient matrix and vector, $\boldsymbol{\beta}_k$ and $\boldsymbol{Z}_k$, are obtained,

$$\boldsymbol{\beta}_{k+1}=\boldsymbol{C}_{k+1}(\boldsymbol{B}_k-\boldsymbol{\beta}_k)^{-1} \tag{18}$$

$$\boldsymbol{Z}_{k+1}=\boldsymbol{\beta}_{k+1}(\boldsymbol{Z}_k+\boldsymbol{F}_k) \tag{19}$$

The initial coefficient matrix and vector, $\boldsymbol{\beta}_1$ and $\boldsymbol{Z}_1$, are given by the left-side boundary condition. Succeeding matrices and vectors, $\boldsymbol{\beta}_k$ and $\boldsymbol{Z}_k$, are calculated in the increasing order of $k$ by the reccurence formulae (18) and (19), respectively. The initial flux vector, $\boldsymbol{\phi}_{K-1}$, is given by the right-side boundary condition and succeeding flux vectors, $\boldsymbol{\phi}_k$, are obtained in the decreasing order of $k$ by the reccurence formula (16).

The inverse matrix, $(\boldsymbol{B}_k-\boldsymbol{\beta}_k)^{-1}$, is calculated by the method of inverse triangular matrices, which is in wide use (e.g. TNS code[3]) in solving the simultaneous linear algebraic equations.

More rigorous derivation of the matrix factorization method is shown in the references 1) and 2).

The merit of this method is the direct solution of the i-th group flux distribution without use of the inner iteration, which shortens the computation time. Its demerit is the requirement of larger memories for storage of the matrix elements of $\boldsymbol{\beta}_k$.

## 5. Boundary conditions

The outer boundary conditions applicable to the code, KAK, are

BC (1) : $\phi=0$

BC (2) : $\partial\phi/\partial z=0$ or $\partial\phi/\partial r=0$

BC (3) : $\partial\phi/\partial z=-\phi/\gamma$ or $\partial\phi/\partial r=-\phi/\gamma$

at the boundary. When the condition BC (1) is applied, the boundary exists on the mesh line of $l$ (or $k$)$=0$ or $l$ (or $k$)$=L$ (or $K$).
when the condition BC (2) or BC (3) is applied, the boundary exists on the line in the middle of $l$ (or $k$)$=0$ and 1 or $l$ (or $k$)$=L-1$ (or $K-1$) and $L$ (or $K$).

### 5. 1 Vertical boundary conditions

The bottom and top boundary conditions are included in the upper left corner element, $(b_k)_{11}$, and lower right corner element, $(b_k)_{LL}$, respectively, of the coefficient matrix, $\boldsymbol{b}_k$. These elements are given by the following formulae according to the boundary condition applied.

(1) $\phi=0$

$$(b_k)_{11}=p_{k1} \qquad \text{(bottom)}$$
$$(b_k)_{LL}=p_{kL} \qquad \text{(top)}$$

(2) $\partial\phi/\partial z=0$

$$(b_k)_{11}=p_{k1}-b_{k1} \qquad \text{(bottom)}$$
$$(b_k)_{LL}=p_{kL}-d_{kL} \qquad \text{(top)}$$

(3) $\partial\phi/\partial z=-\phi/\gamma$

$$(b_k)_{11}=p_{k1}-b_{k1}\frac{1+(\Delta z_1/2\gamma)}{1-(\Delta z_1/2\gamma)} \qquad \text{(bottom)}$$
$$(b_l)_{LL}=p_{kL}-d_{kL}\frac{1-(\Delta z_L/2\gamma)}{1+(\Delta z_L/2\gamma)} \qquad \text{(top)}$$

The coefficients, $p_{kl}$, $b_{kl}$ and $d_{kl}$, are given by the equation ( 7 ).

### 5. 2 Lateral boundary conditions

The left-side boundary condition defines the coefficient matrix and vector, $\boldsymbol{\beta}_1$ and $\boldsymbol{Z}_1$, for the forward reccurence formulae (18) and (19). The right-side boundary condition defines the flux vector, $\boldsymbol{\phi}_{K-1}$, for the backward reccurence formula (16). $\boldsymbol{\beta}_1$, $\boldsymbol{Z}_1$ and $\boldsymbol{\phi}_{K-1}$ are given by the following formulae according to the boundary condition applied.

(1) $\phi=0$

$$\left.\begin{array}{l}\boldsymbol{\beta}_1=0\\ \boldsymbol{Z}_1=0\end{array}\right\} \qquad \text{(left)}$$
$$\boldsymbol{\phi}_{K-1}=\boldsymbol{C}_K^{-1}\boldsymbol{Z}_K \qquad \text{(right)}$$

(2) $\partial\phi/\partial r=0$

$$\left.\begin{array}{l}\boldsymbol{\beta}_1=\boldsymbol{C}_1\\ \boldsymbol{Z}_1=0\end{array}\right\} \qquad \text{(left)}$$
$$\boldsymbol{\phi}_{L-1}=(\boldsymbol{C}_K-\boldsymbol{\beta}_K)^{-1}\boldsymbol{Z}_K \qquad \text{(right)}$$

(3) $\partial\phi/\partial r=-\phi/\gamma$

$$\left.\begin{array}{l}\boldsymbol{\beta}_1=\dfrac{1+(\Delta r_1/2\gamma)}{1-(\Delta r_1/2\gamma)}\boldsymbol{C}_1\\ \boldsymbol{Z}_1=0\end{array}\right\} \qquad \text{(left)}$$
$$\boldsymbol{\phi}_{K-1}=\left\{\frac{1+(\Delta r_K/2\gamma)}{1-(\Delta r_K/2\gamma)}\boldsymbol{C}_K-\boldsymbol{\beta}_K\right\}^{-1}\boldsymbol{Z}_K \qquad \text{(right)}$$

## 6. Source iteration

After the neutron flux distributions, $\phi^i_{k,l}$ $(i=1, 2, \cdots\cdots, I)$, are calculated successively from the first to the I-th energy group, convergence of the neutron source distribution is tested. If the convergence criterion is unsatisfied, the source distribution is extrapolated to minimize the deference between the estimated and the converged eigenvalues, after which the neutron flux distributions are recalculated. This iteration process is designated as the source iteration.

The neutron source, $\psi^{(m)}_{k,l}$, at the point $(k, l)$ after the m-th source iteration is given by

$$\psi^{(m)}_{k,l}=\frac{\sum_i \phi^i_{k,l}(\sum_q \nu\Sigma^i_{fq}V_q\delta_q)}{\sum_q V_q\delta_q} \tag{20}$$

where $V_q$ is the volume of the $q$-th quadrant around the point $(k, l)$, i.e.

$$V_1=(PR)T/2, \qquad V_2=(PL)T/2,$$
$$V_3=(PL)B/2, \qquad V_4=(PR)B/2$$

and

$$\delta_q\begin{cases}=1.0 & (\text{if } \sum_i \nu\Sigma^i_{fq}\neq 0)\\ =0.0 & (\text{if } \sum_i \nu\Sigma^i_{fq}=0)\end{cases}$$

The convergence criterion and the source extrapolation technique used in KAK code are similar with those used in PDQ code[4]. The eigenvalue and its upper and lower bound at the m-th iteration, $\lambda^{(m)}$, $\bar{\lambda}^{(m)}$, $\underline{\lambda}^{(m)}$ are defined by

$$\bar{\lambda}^{(m)}=\lambda^{(m-1)}\underset{k,l}{\mathrm{Max}}\left\{\frac{\psi^{(m)}_{k,l}}{\psi^{*(m-1)}_{k,l}}\right\} \tag{21}$$

$$\underline{\lambda}^{(m)}=\lambda^{(m-1)}\underset{k,l}{\mathrm{Min}}\left\{\frac{\psi^{(m)}_{k,l}}{\psi^{*(m-1)}_{k,l}}\right\} \tag{22}$$

$$\lambda^{(m)}=\lambda^{(m-1)}\frac{\boldsymbol{\psi}^{(m)}\cdot\boldsymbol{\psi}^{(m)}}{\boldsymbol{\psi}^{*(m-1)}\cdot\boldsymbol{\psi}^{(m)}} \tag{23}$$

If the convergence criterion

$$\frac{\bar{\lambda}^{(m)}-\underline{\lambda}^{(m)}}{2\lambda^{(m)}}\leq\varepsilon_1 \tag{24}$$

where $\varepsilon_1$ is an input parameter, is satisfied, the problem is considered to be converged. If the inequality (24) is unsatisfied, the extrapolated neutron source, $\psi^{*(m)}_{k,l}$, after the m-th iteration is calculated by the formula,

$$\psi^{*(m)}_{k,l}=K^{(m)}\{(1+\theta^{(m)})\psi^{(m)}_{k,l}-\theta^{(m)}\psi^{*(m-1)}_{k,l}\} \tag{25}$$

where

$$K^{(m)}=\frac{\frac{\lambda^{(m)}}{\lambda^{(m-1)}}\|\boldsymbol{\psi}^{*(m-1)}\|}{(1+\theta^{(m)})\|\boldsymbol{\psi}^{(m)}\|-\theta^{(m)}\|\boldsymbol{\psi}^{*(m-1)}\|} \tag{26}$$

and the (m+1) -th source iteration is performed. The sequence of the extrapolation factors $\theta^{(m)}=\theta_j(l)\{j=0, 1, 2, \cdots\cdots, l-1\}$ is given by

$$\theta_j(l)=\frac{\bar{\delta}\left(1+\cos\frac{2j+1}{2}\pi\right)}{2-\bar{\delta}\left(1+\cos\frac{2j+1}{2l}\pi\right)} \quad (j=0, 1, 2, \cdots\cdots l-1) \tag{27}$$

The underlying theory of the source extrapolation technique, based on TCHEBYSHEFF polynomials, is described in the reference[5]. The dominance ratio, $\bar{\delta}$, defined by the ratio of the second largest eigenvalue to the largest eigenvalue of the multi-group diffusion difference equation, is approximated[6] by

$$\bar{\delta}=\frac{1}{I}-\sum_i \frac{\|\boldsymbol{R}_i^{(m)}\|}{\|\boldsymbol{R}_i^{(m-1)}\|} \tag{28}$$

where

$$\|\boldsymbol{R}_i^{(m)}\| = \|\boldsymbol{\phi}_i^{(m)}-\boldsymbol{\phi}_i^{(m-1)}\|$$

is the residual of the flux distribution of the i-th group. More detail procedure of the source iteration is described in the reference[4].

## 7. Criticality search

In KAK code the criticality search is performed by the poison absorption control. The control rod regions (or controlled regions) are specified by the factor, $W_n$ ($n$ is the region index). The $n$-th region is uncontrolled if $W_n=0$, and controlled if $W\neq 0$. The value (arbitrary input) of $W_n$ is the weight of the poisoning in the $n$-th region. Only the thermal absorption cross section is controlled in the criticality search.

The thermal absorption cross section of the $n$-th region, $\Sigma^I_{an}$, is the sum of the uncontrolled and controlled absorption cross sections, i.e.,

$$\Sigma^I_{an}=\Sigma^I_{aun}+\Sigma_{ap}\cdot W_n t \tag{29}$$

where

$\Sigma^I_{ua}$; uncontrolled absorption cross section

$\Sigma_{ap}$; controlled absorption cross section

$t$; criticality search parameter.

When the criticality search option is chosen, the criticality search iteration is performed to ensure the eigenvalue $\lambda$ (or $K_{eff}$)$=1.0$. After the source iteration of the $l$-th criticality search iteration is converged in the sense of the ineguality (24), the convergence of the criticality search iteration is tested by the criterion,

$$|\lambda_l-1.0|\leq\varepsilon_2 \tag{30}$$

where $\varepsilon_2$ is an arbitrary convergence parameter. If the inequality (30) is satisfied, the criticality search iteration is considered to be converged. If the inequality (30) is unsatisfied the criticality search parameter for the $(l+1)$-th iteration, $t_{l+1}$, is linearly interpolated,

$$t_{l+1}=t_l+\Delta\lambda_l\frac{t_l-t_{l-1}}{\lambda_l-\lambda_{l-1}} \tag{31}$$

where $\Delta\lambda_l=1.0-\lambda_l$. The parameter, $t_{l+1}$, is substituted into the equation (29) and the eigenvalue $\lambda_{l+1}$ is recalculated by the source iteration procedure described in Section 6.

When the initial criticality search is performed ($l=0$), $t_1$ is guessed by

$$t_1=t_0(1+C\Delta\lambda_0)$$

where $C$ is an initial guess of the gradient $(1/t)$ $(dt/d\lambda)$.

## 8. Calculation of the adjoint flux

The multi-group neutron diffusion equations are of the form:

$$-\nabla D^i\nabla\phi^i+\Sigma^i_T\phi^i=\Sigma^{i-1}_r\phi^{i-1}+\frac{x^i}{\lambda}\sum_i \nu\Sigma^i_f\phi^i \tag{32}$$

$$(i=1, 2, \cdots\cdots, I)$$

where $X^I=0$ and $\Sigma_r=\Sigma_r^I=0$. The equation (32) is represented in the matrix form:

$$\boldsymbol{L}\cdot\boldsymbol{\phi}=0 \tag{33}$$

where

$$\boldsymbol{\phi}=\begin{pmatrix}\phi_1\\ \phi_2\\ \vdots\\ \phi^I\end{pmatrix}$$

and the matrix elements of the operator $L$ are

$$L_{i,i}=-\nabla D^i\nabla+\Sigma_{\mathrm{T}}^i-\frac{x^i}{\lambda}\nu\Sigma_{\mathrm{f}}^i$$

$$L_{i,i-1}=-\Sigma_r^{i-1}-\frac{x^i}{\lambda}\nu\Sigma_{\mathrm{f}}^{i-1}$$

$$L_{i,j}=-\frac{x^i}{\lambda}\nu\Sigma_{\mathrm{f}}^j \qquad (j\neq i \text{ or } i-1)$$

$$(i, j=1, 2, \cdots\cdots, I)$$

The adjoint operator, $\boldsymbol{L}^*$, to the equation (33) and its solution, i.e. the adjoin flux, $\boldsymbol{\phi}^*$, must satisfy the condition;

$$[\boldsymbol{\phi}^*\cdot\boldsymbol{L}\boldsymbol{\phi}]=[\boldsymbol{\phi}\cdot\boldsymbol{L}^*\boldsymbol{\phi}^*] \tag{34}$$

Therefore, the adjoint flux equations are of the form:

$$-\nabla D^i\nabla\phi^{*i}+\Sigma_{\mathrm{T}}^i\phi^{*i}=\Sigma_r^i\phi^{*i+1}+\frac{\nu\Sigma_{\mathrm{f}}^i}{\lambda}\sum_i X^i\phi^{*i} \tag{35}$$

$$(i=1, 2, \cdots\cdots, I)$$

or in the matrix representation;

$$\boldsymbol{L}^*\cdot\boldsymbol{\phi}^*=0 \tag{36}$$

where

$$\boldsymbol{\phi}^*=\begin{pmatrix}\phi^{*1}\\ \phi^{*2}\\ \vdots\\ \phi^{*I}\end{pmatrix}$$

and the operator $\boldsymbol{L}^*$ with the elements of the form;

$$L_{i,i}=-\nabla D^i\nabla+\Sigma_{\mathrm{T}}^i-\frac{x^i}{\lambda}\nu\Sigma_{\mathrm{f}}^i$$

$$L_{i,i+1}=-\Sigma_r^i-\frac{\nu\Sigma_{\mathrm{f}}^i}{\lambda}x^{i+1}$$

TABLE 1 Nuclear constant interchange table

| Diffusion eq. | Adjoint eq. | Diffusion eq. | Adjoint eq. |
|---|---|---|---|
| $\phi^1$ | $\phi^{*4}$ | $\Sigma_{\mathrm{T}}^1$ | $\Sigma_{\mathrm{T}}^4$ ($\Sigma_r=0$) |
| $\phi^2$ | $\phi^{*3}$ | $\Sigma_{\mathrm{T}2}$ | $\Sigma_{\mathrm{T}}^3$ |
| $\phi^3$ | $\phi^{*2}$ | $\Sigma_{\mathrm{T}}^3$ | $\Sigma_{\mathrm{T}}^2$ |
| $\phi^4$ | $\phi^{*4}$ | $\Sigma_{\mathrm{T}4}$ ($\Sigma_r^4=0$) | $\Sigma_{\mathrm{T}}^1$ |
| $X^1$ | $\nu\Sigma_{\mathrm{f}}^4$ | $\Sigma_r^1$ | $\Sigma_r^3$ |
| $X^2$ | $\nu\Sigma_{\mathrm{f}}^3$ | $\Sigma_r^2$ | $\Sigma_{r2}$ |
| $X^3$ | $\nu\Sigma_{\mathrm{f}}^2$ | $\Sigma_r^3$ | $\Sigma_r^1$ |
| $X^4$ | $\nu\Sigma_{\mathrm{f}}^1$ | $\nu\Sigma_{\mathrm{f}}^1$ | $X^4=0$ |
| $D_1$ | $D^4$ | $\nu\Sigma_{\mathrm{f}}^2$ | $X^3$ |
| $D_2$ | $D^3$ | $\nu\Sigma_{\mathrm{f}}^3$ | $X^2$ |
| $D_3$ | $D^2$ | $\nu\Sigma_{\mathrm{f}}^4$ | $X^1$ |
| $D_4$ | $D^1$ | | |

$$L_{i,j}=-\frac{\nu\Sigma_f^i}{\lambda}x^j \qquad (j\neq i \text{ or } i+1)$$

$$(i, j=1, 2, \cdots\cdots, I)$$

is the transposed operator of $\boldsymbol{L}$.

The adjoint flux equation (35) can be solved by the same method as that used for the solution of the diffusion equation (32) with the suitable interchange of nuclear constants. The interchange of constants is illustrated in TABLE 1 in the case of four energy groups. The calculation of the adjoint flux distribution is performed by the user's option. Note that the adjoint fluxes, $\phi^{*i}$, are calculated in the descending order of $i$ ($i=I, I-1, \cdots\cdots, 1$).

## 9. Reactivity change by perturbation

The reactivity change caused by small variation in some of the nuclear constants in the diffusion equation (32) is estimated by perturbation theory. The perturbed diffusion equation is represented in the matrix form:

$$\boldsymbol{L}'\boldsymbol{\phi}'=0$$

where (37)

$$\boldsymbol{L}'=\boldsymbol{L}+\boldsymbol{\delta L} \tag{38}$$

$\boldsymbol{\phi}'$ : perturbed flux

$\boldsymbol{\delta L}$ : perturbed part of the operator, $L$.

By the theory of perturbation[1] the functional equation

$$[\boldsymbol{\phi}^*\cdot\boldsymbol{\delta L}\boldsymbol{\phi}']=0 \tag{39}$$

is deduced, from which the reactivity change (or change in the eigenvalue), $\delta\lambda/\lambda$, is estimated.

The explicit form of the reactivity change based on the perturbation theory is given bellow:

$$\frac{\delta\lambda}{\lambda}=-\lambda\delta\left(\frac{1}{\lambda}\right) \tag{40}$$

where $\delta(1/\lambda)$ is calculated by the volume integrals;

$$\delta\left(\frac{1}{\lambda}\right)=\frac{1}{F}\int_G dV\sum_i\left\{\delta\Sigma_T^i\phi^i\phi^{*i}+\delta D^i(\nabla\phi^i\cdot\nabla\phi^{*i})-\delta\Sigma_r^{i-1}\phi^{i-1}\phi^{*i}-\frac{x_i}{\lambda}\delta S\phi^{*i}\right\} \tag{41}$$

where

$$F=\int_G dV\sum_i X_i S\phi^{*i} \tag{42}$$

$$S=\sum_i\nu\Sigma_f^i\phi^i, \qquad \delta S=\sum_i\delta(\nu\Sigma_f^i)\phi^i,$$

$$\phi^o=0, \qquad \Sigma_r^o=\Sigma_r^I=0, \qquad x^I=0$$

and the domain of the integration, $G$, is the reactor system under consideration.

## 10. Edit of output data

Varieties of integrated and region-averaged values are edited as the output of the code for convenience of the user. The following is the list of output quantities.

(1) Eigenvalue

$m, \quad \bar{\lambda}^{(m)}, \quad \lambda^{(m)}, \quad \underline{\lambda}^{(m)}, \quad \theta^{(m)}$

(2) Criticality search parameter

$l, \quad t_l, \quad \lambda_l, \quad \Delta\lambda_l$

(3) Neutron flux

(a) Pointwise flux: $\phi^i_{k,l}$

Renormalized by the formula:

$$\phi^i_{k,l}=\beta\phi'^i_{k,l} \tag{43}$$

$$\beta^{-1}=\sum_i\int_{V_c}\frac{K^i}{\nu^i}\nu\Sigma^i_f\phi'^i dV/P_T \tag{44}$$

(b) Regionwise flux: $\phi^i_n$

$$\phi^i_n=\int_{V_n}\phi^i dV/V_n \tag{45}$$

(c) Core average: $\phi^i_c$

$$\phi^i_c=\sum_{n=KC}\phi^i_n V_n/V_c \tag{46}$$

(d) Reflector average: $\phi^i_r$

Similar with eq. (46) for $n\neq KC$

(e) Groupwise flux ratio: $\alpha^i_n$

$$\alpha^i_n=\phi^i_n/\phi^i_n \tag{47}$$

(4) Neutron absorption

(a) Regionwise absorption cross section: $\Sigma^i_{an}$

When the criticality search is performed, $\Sigma^I_{an}$ is given by eq. (29).

(b) Regionwise absorption: $A_n$

$$A_n=\sum_i\Sigma^i_{an}\phi^i_n V_n \tag{48}$$

(c) Core average: $\Sigma^i_{ac}$

$$\Sigma^i_{ac}=\sum_{n=KC}\Sigma^i_{an}\phi^i_n V_n/\phi^i_c V_c \tag{49}$$

(d) Reflector average: $\Sigma^i_{ar}$

Similar with eq. (49) for $n\neq KC$.

(5) Neutron emission

(a) Regionwise fission cross section $x$ neu.: $\nu\Sigma^i_{fn}$

(b) Regionwise neutron emission: $F_n$

$$F_n=\sum_i\nu\Sigma^i_{fn}\phi^i_n V_n \tag{50}$$

(c) Core average: $\nu\Sigma^i_{fc}$

$$\nu\Sigma^i_{fc}=\sum_{n=KC}\nu\Sigma^i_{fn}\phi^i_n V_n/\phi^i_c V_c \tag{51}$$

(6) Neutron removal

(a) Regionwise removal cross section: $\Sigma^i_r$

(b) Groupwise removal in core: $R^i_c$

$$R^i_c=\sum_{n=KC}\Sigma^i_r\phi^i_n V_n \tag{52}$$

(c) Core average: $\Sigma^i_{rc}$

$$\Sigma^i_{rc}=R^i_c/\phi^i_c V_c \tag{53}$$

(7) Neutron leakage

(a) Regionwise leakage: $L^i_n$

$$L^i_n=-\int_n D^i_n\nabla\phi^i\cdot dS \tag{54}$$

(b) Regionwise buckling: $(DB^2)^i_n$

$$(DB^2)^i_n = L^i_n / \phi^i_n V_n \tag{55}$$

(c) Core average: $(DB^2)^i_c$

$$(DB^2)^i_c = \sum_{n=\mathrm{KC}} L^i_n / \phi^i_c V_c \tag{56}$$

(d) Core average diffusion coefficient: $D^i_c$

$$D^i_c = \sum_{n=\mathrm{KC}} D^i_n \phi^i_n V_n / \phi^i_c V_c \tag{57}$$

(8) Neutron source

(a) Pointwise source: $S_{k,l}$

$$S_{k,l} = \gamma \phi_{k,l} \tag{58}$$

$$\gamma^{-1} = \int_{V_c} \phi dV \tag{59}$$

(b) Regionwise average source

$$S_n = \int_{V_n} S dV / V_n \tag{60}$$

(9) Power

(a) Pointwise power: $P_{k,l}$

$$P_{k,l} = \sum_i \frac{K^i}{\nu^i} \nu \Sigma^i_f \phi^i_{k,l} \tag{61}$$

(b) Region average power: $P_n$

$$P_n = \int_{V_n} P dV / V_n \tag{62}$$

(c) Core average power: $P_c$

$$P_c = \sum_{n=\mathrm{KC}} P_n V_n / V_c \tag{63}$$

(10) Flux and Power ratio

(a) Flux ration: $d^i_{\max}$, $d^i_{\min}$

$$d^i_{\max} = \mathrm{Max}\{\phi^i_{k,l} / \phi^i_c\} \tag{64}$$

$$d^i_{\min} = \mathrm{Min}\{\phi^i_{k,l} / \phi^i_c\} \tag{65}$$

(b) Power ratio: $P_{\max}$, $P_{\min}$

$$P_{\max} = \mathrm{Max}\{P_{k,l} / P_c\} \tag{66}$$

$$P_{\min} = \mathrm{Min}\{P_{k,l} / P_c\} \tag{67}$$

(11) Volume

(a) Regionwise volume: $V_n$

$$V_n = \int_{\mathrm{reg},\, n} dV \tag{68}$$

(b) Total, core and refector volumes: $V_t$, $V_c$, $V_r$

$$V_t = \sum_n V_n$$

$$V_c = \sum_{n=\mathrm{KC}} V_n$$

$$V_r = \sum_{n \neq \mathrm{KC}} V_n \tag{69}$$

(12) Normalization factors

(a) Flux normalization factor: $\beta$

(b) Source normalization factor: $\gamma$

(13) Core average neutron multiplication (The following output data are given only by the small KAK code)

(a) Non-absorption probability: $p_i$

$$p_i=\Sigma_c^i/(\Sigma_{ac}^i+\Sigma_{rc}^i+(DB^2)_c^i) \qquad (70)$$

$$\Sigma_c^i=\Sigma_r^i \qquad (i\neq I)$$

$$\Sigma_c^i=\Sigma_a^i \qquad (i\neq I)$$

(b) Neutron emission probability: $(\eta f)_i$

$$(\eta f)_i=\nu\Sigma_{fc}^i/\Sigma_c^i \qquad (71)$$

(c) Groupwise neutron multiplication: $k_i$

$$k_i=(\eta f)_i p_1\cdots\cdots p_i$$

(d) Two-group model

$$\Sigma_{af}=\sum_{i\neq I}\Sigma_{ac}^i\phi_c^i/\sum_{i\neq I}\phi_c^i$$

$$\Sigma_{rf}=\Sigma_{rc}^{I-1}\phi_c^{I-1}/\sum_{i\neq I}\phi_c^i$$

$$\nu\Sigma_{ff}=\sum_{i=I}\nu\Sigma_{fc}^i\phi_c^i/\sum_{i\neq I}\phi_c^i$$

$$D_f=\tau\cdot(\Sigma_{rf}+\Sigma_{af})$$

$$\tau=\sum_{i\neq I}\tau_i$$

$$\tau_i=D_c^i/(\Sigma_{ac}^i+\Sigma_{rc}^i)$$

(e) One-group model

$$k_\infty=\sum_i k_i$$

$$M^2=\sum_i \tau_i$$

## 11. Program links

KAK is written entirely in FORTRAN IV and is a chain program which consists of 6 dependent links. Each stage of the program fit into a 32K core storage, and uses 5 scratch tapes. All input-output operation are done with tapes. No sense switches or lights are used. Fig. 2 is the flow chart of main link, where LZ is a indicator for poison or rod search and KZ for adjoint and KP for perturbation calculation. TABLE 2 below lists the function of each dependent link.

TABLE 3 Gives the logical tape numbers (with the actual tape unit) that are reffered to in the code, with the function of each tape.

TABLE 2 Program chain links

| Chain link | Function |
|---|---|
| CHAIN (1) | Reads and writes input parameters etc. and sets up initial conditions. |
| CHAIN (2) | Calculates the coefficients of difference form of diffusion equation including the modification for power or rod search. |
| CHAIN (3) | Does the source iteration. (The flux is calculated by direct methods.) |
| CHAIN (4) | Does the source normalization, and if needed, does also the preparation for poison or power search.<br>The adjoint flux is also printed out if it is already calculated. |
| CHAIN (5) | Calculates the region integrated quantity etc. and if needed prepares for adjoint calculation. |
| CHAIN (6) | Link for perturbation calculation. |

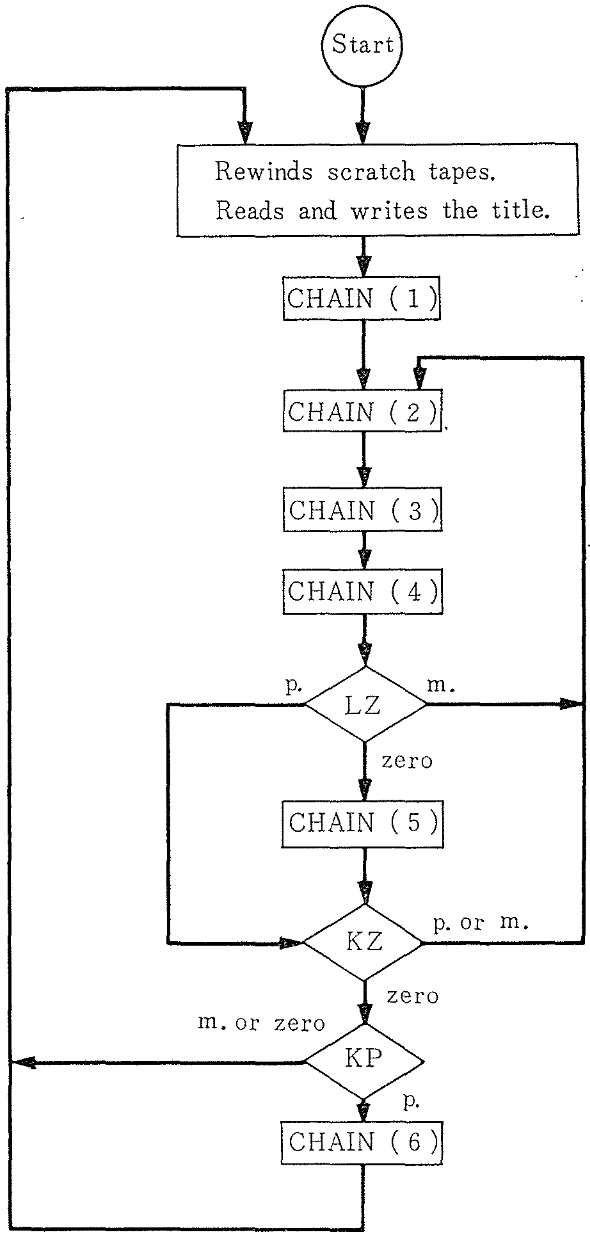


**Fig. 2** Flow chart of the main program

TABLE 3 Tapes required for KAK

| Logical tape no. | Actual tape unit | Function |
|---|---|---|
| SYSLB | C 1 | System tape |
| SYSIN | C 2 | Input tape |
| SYSOU | C 3 | Output tape |
| FTCO 2 | C 5 | Scratch tape |
| FTCO 3 | B 2 | Scratch tape |
| FTCO 4 | B 4 | Scratch tape |
| FTCO 8 | C 4 | Scratch tape |
| FTCO 9 | B 3 | Scratch tape |
| UO 6 | C 6 | Program tape |

```
12345678901234567890123456789012345678901234567890123456789012345678901234567890
    KAK SAMPLE PRōBLEM    PōISōN SEARCH
     1     1     0     0     1     1     0     1     0
     2     2    14     8     4     2
 1.0       0.0       0.001     0.005     1.0       4.82   +8
 1.14      0.0       0.0       0.008476  0.0       0.0       1.0
 0.883     0.0       0.002     0.0       0.0032541 2.46      1.0
 1.14      0.0       0.0       0.008476  0.0       0.0       1.0
 0.883     0.0       0.6565 -4 0.0       0.0       0 0       1.0
 2.0       1 9.0       2 10.0     11 10.0     14
 12.0      1 17.0      8
   2 014 0 8   1 011 0 8 999
     1     1
 3.1108-11 0.0
 0.2660-02 0.2      -1.0
 1.0       0.0
12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

Fig. 3 KAK sample problem input data

## 12. Input data card

Fig. 3 shows the input data forms filled out for the sample problem. There are 10 types of data cards required for input to KAK: (1) title card, (2) control cards, (3) composition specification cards, (4) mesh specification cards, (5) region specification cards, (6) core region specification card, (7) logarithmic derivative data cards, (8) poison control data cards, (9) rod search parameter cards and (10) input cards for perturbation. Given below are the instructions for writing KAK input. Note that the number formats are described in FORTRAN nomenclature and are given in parentheses immediately following the input number symbol.

### Title card

Columns 1 through 72 may contain any desired information and are printed on a cover of the output.

### Control card 1

Columns 1 through 6, kG (I 6): Geometry indicator. If KG is 0, rectangular $x$–$y$ geometry is specified; if this number is +1, cylindrical $r$–$z$ geometry is specified.

Columns 7 through 12, KS (I 6): Search indicator. If KS is 0, $k_{eff}$ is calculated; if KS is 1, poison search is performed; if KS is 2 rod search is executed.

Columns 13 through 18, KA (I 6): Adjoint indicator. If KA is 1, the adjoint fluxes will be computed following the flux calculation; if this number is 0, the adjoint flux calculation is skipped.

Columns 19 through 24, KP (I 6): Perturbation indicator. If KP is 1, reactivity change by perturbation is calculated; if KP is 0, perturbation calculation is skipped.

Columns 25 through 30, KF (I 6): Flux-guess indicator. If KF is 0, the initial flux guess is supplied by the code; if this number is 1, the flux guess is given regionwise by input card; if this number is 2, the flux distribution from the proceeding case is used as the initial guess.

Columns 31 through 36, KL (I 6): Left side boundary indicator. If KL is 0, flux on the left side (column 0) is assumed to be zero; if this number is 1, a symmetry boundary is assumed to exist midway between columns 0 and 1. If this number is 2, the logarithmic derivative is given on the same boundary as KL=1. The value of logarithmic derivative is given by an input card (see below).

Columns 37 through 42 KR (I 6): Right side boundary indicator. If KR is 0, flux on the right side (Column IMAX) is assumed to be zero; if this number is 1, a symmetry boundary is assumed to exist midway between columns IMAX−1 and IMAX. If this number is 2 the logarithmic derivative is given on the same boundary as KR=1. The value of logarithmic derivative is given by an input card.

Columns 43 through 48, KT (I 6): Top boundary indicator. If this number is 0, flux on the top (row 0) is assumed to be zero; if this number is 1, a symmetry boundary is assumed to exist midway between rows 0 and 1. If this number is 2, the logarithmic derivative is given on the same boundary as KT=1.

Columns 49 through 54, KB (I 6): Bottom boundary indicator. If this number is 0, flux on the bottom (row JMAX) is assumed to be zero; if this number is 1, a symmetry boundary is assumed to exist midway between rows JMAX-1 and JMAX. If this number is 2, the logarithmic derivative is given on the same boundary as KB=1.

Control card 2

Columns 1 through 6, NGMAX (I 6): Total number of energy groups $\leq 4$.

Columns 7 through 12, NRMAX (I 6): Total number of compositions $\leq 40$.

Columns 13 through 18, IMAX (I 6): Total number of columns (X or R direction) $\leq 30$.

Columns 19 through 24, JMAX (I 6): Total number of rows (Y or Z direction) $\leq 50$.

Columns 25 through 30, MMAX (I 6): Total number of mesh regions (X or R direction) $\leq 30$, in each of which mesh size is set to be equal.

Columns 31 through 36, NMAX (I 6): Total number of mesh region (Y or Z direction) $\leq 50$.

Control card 3

Columns 1 through NGMAX×10, (YK (I), I=1, NGMAX) (8E 10. 7): The fraction of neutrons produced from fission that are born in group I. Note that $\sum_{I=1}^{NGMAX} YK(I)=1.0$.

Columns (1+NGMAX×10) through (NGMAX+1)×10, EPS 1 (E 10. 7): Convergence criterion for source iteration. A value of $10^{-3}$ for this number will usually assure reasonable convergence.

Columns 11+NGMAX×10 through (NGMAX+2)×10, EPS 2 (E 10. 7): Convergence criterion for the criticality search option.

Columns 21+NGMAX×10 through (NGMAX+3)×10, EIGEN (E 10. 7): Initial guess of $k_{eff}$.

Columns 31+NGMAX×10 through (NGMAX+4)×10, PT (E 10. 7): Power of reactor under consideration. The neutron flux is normalized to attain this value. The unit of power is in watt.

Composition specification cards

Columns 1 through 10, D (E 10. 7): Diffusion coefficient.

Columns 11 through 20, $B^2$ (E 10. 7): Composition-group dependent buckling.

Columns 21 through 30, $\Sigma_A$ (E 10. 7): Macroscopic absorption cross section.

Columns 31 through 40, $\Sigma_R$ (E 10. 7): Macroscopic removal cross section.

Columns 41 through 50, $\nu\Sigma_f$ (E 10. 7): $\nu$ times macroscopic fission cross section.

Columns 51 through 60. $\nu$ (E 10. 7): The average number of neutrons produced per fission.

Columns 61 through 70, F (E 10. 7): Composition-group dependent initial flux guess that is used when KF is 1.

Columns 71 through 80. Any number for your identification.

These items on one card are repeated firstly for each group. These items on NGMAX cards are repeated secondly for each composition. So NGMAX×NRMAX cards are necessary for the composition specification.

### Mesh specification cards

Columns 1 through 10, DM (E 10. 5) : Mesh width of 1st mesh region (X or R Direction). Even if KL is 1, the width between columns 0 and 1 must be specified.

Columns 11 through 12 MK (I 2) : The column number of last column which has the same mesh width.

Up to six of these data could be specified on a card. $\left[\frac{\text{MMAX}}{6}\right]+1$ cards are necessary for this specification.

Columns 1 through 10, DN (E 10. 5) : Mesh width for 1st mesh region (Y or Z direction). Even if KT is 1, the width between rows 0 and 1 must be specified.

Columns 11 through 12, NK (I 2) : The row number of last row that has the same mesh width.

Up to six of these data could be specified on a card. $\left[\frac{\text{NMAX}}{6}\right]+1$ cards are necessary for this specification.

### Region specification cards

The regions of the reactor are specified as rectangles.

Columns 1 through 5, NA (I 4) : Composition number to be specified.

Columns 5 through 6, NL (I 2) : Left column number of the region (including 0).

Columns 7 through 8, NR (I 2) : Right column number of the region.

Columns 9 through 10, NT (I 2) : Top row number of the region.

Columns 11 through 12, NB (I 2) : Bottom row number of the region.

The compositions are numbered beginning with 1 ; however, more than one region may have the same composition.

Up to six sets of these data could be specified on a card.

The composition number which is lastly specified is stored in the memory. (One can overlay composition numbers on the same sub-regions.)

Composition specification is terminated when NA is set to 999.

### Core region specfication cards

Columns 1 through 6, KCMAX (I 6) : Total number of core regions.

Columns 7 through 12, $KC_i$ i=1, KCMAX : The composition number of the core region. The format of the first card, (11 I 6) ; the following cards, if any, (12 I 6).

Columns 1 through NRMAX×10, $K_i$ i=1, . . . NRMAX, (8 E 10. 7) : Power conversion factor in each region. Watt per fission per sec.

The following cards must be skipped if they are not needed.

### Logarithmic derivative data card

The following card is necessary only when KL=2.

Columns 1 through 10×NGMAX, $\gamma_{Li}$ i=1, . . . NGMAX, (4 E 10. 7) : $\gamma_{Li}$ is the logarithmic derivative on the left side boundary for group i.

The following card is necessary only when KR=2.

Columns 1 through 10×NGMAX, $\gamma_{Ri}$ i=1, NGMAX (4 E 10. 7) : $\gamma_{Ri}$ is the logarithmic derivative on the right side boundary for group i.

The following card is necessary only when KT=2.

Columns 1 through 10×NGMAX, $\gamma_{Ti}$ i=1, . . . NGMAX, (4 E 10. 7) : $\gamma_{Ti}$ is the logarithmic derivative on the top boundary for group i.

The following card is necessary only when KB=2.

Columns 1 through 10×NGMAX, ($\gamma_{Bi}$ i=1, . . . NGMAX) (4 E 10. 7) : $\gamma_{Bi}$ is the logarithmic derivative on the bottom boundary for group i.

### Poison control data cards

The following cards are necessary only when KS=1.

Columns 1 through 10, $\Sigma_{ap}$, (E 10. 7): Poison cross section for the thermal group.

Columns 11 through 20, $t_0$ (E 10. 7): Initial guess of the poison parameter.

Columns 21 through 30, C (E 10. 7): Parameter for the second search.

The second poison parameter is calculated by

$$T_1 = t_0(1 + C\Delta\lambda_0)$$

where

$$\Delta\lambda_0 = 1 - \lambda_0$$

Changing the card,

Columns 1 through 10×NRMAX, $W_i$ i=1, ... NRMAX (8 E 10. 7): Region specification of poisoning. The thermal absorption cross section of region i is calculated by

$$\Sigma_{api}^{NGMAX} = \Sigma_{ai}^{NGMAX} + W_i t \Sigma_{ap}$$

### Perturbation calculated data cards

The following cards are necessary only when KP=1.

Columns 1 through 10, $\delta D$ (E 10. 7): The change of diffusion coefficient in group 1 and region 1.

Columns 11 through 20, $\delta\Sigma_a$, (E 10. 7): The change of absorption cross section in group 1 and region 1.

Columns 21 through 30, $\delta\Sigma_R$, (E 10. 7): The change of removal cross section in group 1 and region 1.

Columns 31 through 40, $\delta\nu\Sigma_f$, (E 10. 7): The change of fission cross section multiplied by $\nu$ in group 1 and region 1.

These of four data are repeated continuously for each region. Namely $\left[\frac{NRMAX+1}{2}\right]$ cards arenecessary for one fixed energy group. These $\left[\frac{NRMAX+1}{2}\right]$ cards are repeated for each group, so that. NGMAX× $\left[\frac{NRMAX+1}{2}\right]$ cards are necessary for a perturbation calculation.

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## References


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KAK SAMPLE PROBLEM    POISON SEARCH

POISON SEARCH CALCULATIONS ARE INCLUDED.

SYSTEM

GEOMETRY ...................... RZ
NUMBER OF GROUPS ............... 2
NUMBER OF REGIONS ............. 2
NUMBER OF SUBREGIONS ...... 4X 2
NUMBER OF MESH POINTS ..... 14X 8

BOUNDARY CONDITION

LEFT SIDE .......... ZERO DERIVATIVE
RIGHT SIDE ......... ZERO FLUX
UP SIDE ............ ZERO DERIVATIVE
DOWN SIDE .......... ZERO FLUX

CONVERGENCE CRITERION

FOR SOURCE ITERATION ..... 0.100E-02
FOR SEARCH ITERATION ..... 0.200E-02

NORMALIZATION CONSTANT

TOTAL POWER ........ 0.4820000E 09

GROUP CONSTANTS

FISSION SPECTRUM    0.10000000E 01    0.

REGION 1

| GROUP | D | BUCKLING | SIGMA-A | SIGMA-R | NU SIGMA-F | NU | FLUX GUESS |
|---|---|---|---|---|---|---|---|
| 1 | 0.1140000E 01 | 0. | 0. | 0.8476000E-02 | 0. | 0. | 0.1000000E 01 |
| 2 | 0.8830000E 00 | 0. | 0.2000000E-02 | 0. | 0.3254100E-02 | 0.2460000E 01 | 0.1000000E 01 |

REGION 2

| GROUP | D | BUCKLING | SIGMA-A | SIGMA-R | NU SIGMA-F | NU | FLUX GUESS |
|---|---|---|---|---|---|---|---|
| 1 | 0.1140000E 01 | 0. | 0. | 0.8476000E-02 | 0. | 0. | 0.1000000E 01 |
| 2 | 0.8830000E 00 | 0. | 0.6565000E-04 | 0. | 0. | 0. | 0.1000000E 01 |

REGION CONSTANTS

NUMBERS OF FISSION PER WATT    0.31108000E-10    0.

CONTROL CONSTANTS
POISON PARAMETER    0.20000000E-00
POISONING DIRECTION    -0.10000000E 01
POISONING CROSS SECTION    0.26600000E-02
OMEGA FOR EACH REGION    0.10000000E 01    0.

MATERIAL MAP

MESH SPECIFICATION
R DELTA COL    2.000  1  9.000  2 10.000 11 10.000 14

Z DELTA ROW    12.000  1 17.000  8

CORE REGION NO. ..... 1

```
R 0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
Z
0  ******************************************
   * 1  1  1  1  1  1  1  1  1  1  1* 2  2  2*
1  *                                *        *
   * 1  1  1  1  1  1  1  1  1  1  1* 2  2  2*
2  *                                *        *
   * 1  1  1  1  1  1  1  1  1  1  1* 2  2  2*
3  *                                *        *
   * 1  1  1  1  1  1  1  1  1  1  1* 2  2  2*
4  *                                *        *
   * 1  1  1  1  1  1  1  1  1  1  1* 2  2  2*
5  *                                *        *
   * 1  1  1  1  1  1  1  1  1  1  1* 2  2  2*
6  *                                *        *
   * 1  1  1  1  1  1  1  1  1  1  1* 2  2  2*
7  *                                *        *
   * 1  1  1  1  1  1  1  1  1  1  1* 2  2  2*
8  ******************************************
```

CRITICAL IT.   0   POISONING PARAMETER   0.20000000E-00   LAMBDA   0.10245104E 01   DIF.   0.

| SOURCE IT. | MAX LAMBDA | LAMBDA | MIN LAMBDA | ACC. PARAMETER USED |
|---|---|---|---|---|
| 1 | 0.12512685E 01 | 0.10245104E 01 | 0.33374937E-00 | 0. |
| 2 | 0.11837011E 01 | 0.10457414E 01 | 0.64443813E 00 | 0. |
| 3 | 0.11137668E 01 | 0.10374615E 01 | 0.86216255E 00 | 0.17910121E-00 |
| 4 | 0.10715085E 01 | 0.10288480E 01 | 0.92548607E 00 | 0.88614324E-01 |
| 5 | 0.10489939E 01 | 0.10239056E 01 | 0.96111194E 00 | 0.11025915E-01 |
| 6 | 0.10238858E 01 | 0.10180219E 01 | 0.10063057E 01 | 0.96871430E 00 |
| 7 | 0.10200455E 01 | 0.10174805E 01 | 0.10111130E 01 | 0.35812563E-00 |
| 8 | 0.10188524E 01 | 0.10173250E 01 | 0.10133609E 01 | 0.36621675E-01 |
| 9 | 0.10180303E 01 | 0.10172227E 01 | 0.10150488E 01 | 0.22667925E-00 |
| 10 | 0.10174872E 01 | 0.10171606E 01 | 0.10162511E 01 | 0.57092771E 00 |

CRITICAL IT.   1   POISONING PARAMETER   0.20343212E-00   LAMBDA   0.10140664E 01   DIF.   -0.17160594E-01

| SOURCE IT. | MAX LAMBDA | LAMBDA | MIN LAMBDA | ACC. PARAMETER USED |
|---|---|---|---|---|
| 1 | 0.10143438E 01 | 0.10140664E 01 | 0.10140534E 01 | 0.33914405E-20 |
| 2 | 0.10140664E 01 | 0.10140664E 01 | 0.10140664E 01 | 0. |

CRITICAL IT.   2   POISONING PARAMETER   0.21903445E-00   LAMBDA   0.10002345E 01   DIF.   -0.14066353E-01

| SOURCE IT. | MAX LAMBDA | LAMBDA | MIN LAMBDA | ACC. PARAMETER USED |
|---|---|---|---|---|
| 1 | 0.10014741E 01 | 0.10002345E 01 | 0.10001762E 01 | 0.33914405E-20 |
| 2 | 0.10002344E 01 | 0.10002344E 01 | 0.10002344E 01 | 0. |

CRITICAL IT.   3   POISONING PARAMETER   0.21903445E-00   LAMBDA   0.10002344E 01   DIF.   -0.23443997E-03

GROUP 1 FLUX

| | R | 1.000E 00 | 1.000E 01 | 2.000E 01 | 3.000E 01 | 4.000E 01 | 5.000E 01 | 6.000E 01 | 7.000E 01 | 8.000E 01 | 9.000E 01 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Z | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 6.000E 00 | 1 | 2.730E 15 | 2.706E 15 | 2.632E 15 | 2.511E 15 | 2.347E 15 | 2.143E 15 | 1.903E 15 | 1.631E 15 | 1.327E 15 | 9.770E 14 |
| 2.300E 01 | 2 | 2.625E 15 | 2.601E 15 | 2.530E 15 | 2.415E 15 | 2.257E 15 | 2.060E 15 | 1.830E 15 | 1.568E 15 | 1.276E 15 | 9.393E 14 |
| 4.000E 01 | 3 | 2.401E 15 | 2.379E 15 | 2.314E 15 | 2.208E 15 | 2.064E 15 | 1.884E 15 | 1.674E 15 | 1.435E 15 | 1.167E 15 | 8.590E 14 |
| 5.700E 01 | 4 | 2.067E 15 | 2.049E 15 | 1.993E 15 | 1.902E 15 | 1.777E 15 | 1.623E 15 | 1.441E 15 | 1.235E 15 | 1.005E 15 | 7.398E 14 |
| 7.400E 01 | 5 | 1.640E 15 | 1.625E 15 | 1.581E 15 | 1.509E 15 | 1.410E 15 | 1.287E 15 | 1.143E 15 | 9.799E 14 | 7.969E 14 | 5.868E 14 |
| 9.100E 01 | 6 | 1.137E 15 | 1.127E 15 | 1.096E 15 | 1.046E 15 | 9.776E 14 | 8.926E 14 | 7.927E 14 | 6.794E 14 | 5.525E 14 | 4.069E 14 |
| 1.080E 02 | 7 | 5.820E 14 | 5.768E 14 | 5.611E 14 | 5.354E 14 | 5.004E 14 | 4.568E 14 | 4.057E 14 | 3.477E 14 | 2.828E 14 | 2.082E 14 |
| 1.250E 02 | 8 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |

GROUP 1 FLUX

| | R | 1.000E 02 | 1.100E 02 | 1.200E 02 | 1.300E 02 |
|---|---|---|---|---|---|
| Z | | 11 | 12 | 13 | 14 |
| 6.000E 00 | 1 | 5.381E 14 | 2.144E 14 | 7.446E 13 | 0. |
| 2.300E 01 | 2 | 5.174E 14 | 2.061E 14 | 7.159E 13 | 0. |
| 4.000E 01 | 3 | 4.732E 14 | 1.885E 14 | 6.547E 13 | 0. |
| 5.700E 01 | 4 | 4.075E 14 | 1.623E 14 | 5.638E 13 | 0. |
| 7.400E 01 | 5 | 3.232E 14 | 1.287E 14 | 4.471E 13 | 0. |
| 9.100E 01 | 6 | 2.241E 14 | 8.926E 13 | 3.100E 13 | 0. |
| 1.080E 02 | 7 | 1.147E 14 | 4.568E 13 | 1.587E 13 | 0. |
| 1.250E 02 | 8 | 0. | 0. | 0. | 0. |

GROUP 2 FLUX

| | R | 1.000E 00 | 1.000E 01 | 2.000E 01 | 3.000E 01 | 4.000E 01 | 5.000E 01 | 6.000E 01 | 7.000E 01 | 8.000E 01 | 9.000E 01 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Z | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 6.000E 00 | 1 | 7.611E 15 | 7.543E 15 | 7.338E 15 | 7.003E 15 | 6.546E 15 | 5.982E 15 | 5.326E 15 | 4.600E 15 | 3.831E 15 | 3.066E 15 |
| 2.300E 01 | 2 | 7.317E 15 | 7.251E 15 | 7.054E 15 | 6.732E 15 | 6.294E 15 | 5.751E 15 | 5.120E 15 | 4.422E 15 | 3.683E 15 | 2.947E 15 |
| 4.000E 01 | 3 | 6.691E 15 | 6.631E 15 | 6.451E 15 | 6.157E 15 | 5.756E 15 | 5.259E 15 | 4.683E 15 | 4.044E 15 | 3.368E 15 | 2.695E 15 |
| 5.700E 01 | 4 | 5.762E 15 | 5.710E 15 | 5.555E 15 | 5.301E 15 | 4.956E 15 | 4.529E 15 | 4.032E 15 | 3.482E 15 | 2.900E 15 | 2.321E 15 |
| 7.400E 01 | 5 | 4.569E 15 | 4.529E 15 | 4.406E 15 | 4.204E 15 | 3.930E 15 | 3.592E 15 | 3.198E 15 | 2.761E 15 | 2.300E 15 | 1.840E 15 |
| 9.100E 01 | 6 | 3.168E 15 | 3.140E 15 | 3.054E 15 | 2.915E 15 | 2.725E 15 | 2.490E 15 | 2.217E 15 | 1.914E 15 | 1.595E 15 | 1.276E 15 |
| 1.080E 02 | 7 | 1.621E 15 | 1.607E 15 | 1.563E 15 | 1.492E 15 | 1.395E 15 | 1.274E 15 | 1.135E 15 | 9.798E 14 | 8.161E 14 | 6.530E 14 |
| 1.250E 02 | 8 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |

GROUP 2 FLUX

| | R | 1.000E 02 | 1.100E 02 | 1.200E 02 | 1.300E 02 |
|---|---|---|---|---|---|
| Z | | 11 | 12 | 13 | 14 |
| 6.000E 00 | 1 | 2.388E 15 | 1.651E 15 | 8.172E 14 | 0. |
| 2.300E 01 | 2 | 2.295E 15 | 1.587E 15 | 7.856E 14 | 0. |
| 4.000E 01 | 3 | 2.099E 15 | 1.451E 15 | 7.184E 14 | 0. |
| 5.700E 01 | 4 | 1.807E 15 | 1.249E 15 | 6.185E 14 | 0. |
| 7.400E 01 | 5 | 1.433E 15 | 9.907E 14 | 4.905E 14 | 0. |
| 9.100E 01 | 6 | 9.936E 14 | 6.868E 14 | 3.400E 14 | 0. |
| 1.080E 02 | 7 | 5.065E 14 | 3.515E 14 | 1.740E 14 | 0. |
| 1.250E 02 | 8 | 0. | 0. | 0. | 0. |

SOURCE

| | R | 1.000E 00 | 1.000E 01 | 2.000E 01 | 3.000E 01 | 4.000E 01 | 5.000E 01 | 6.000E 01 | 7.000E 01 | 8.000E 01 | 9.000E 01 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Z | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 6.000E 00 | 1 | 6.498E-07 | 6.439E-07 | 6.265E-07 | 5.978E-07 | 5.589E-07 | 5.107E-07 | 4.547E-07 | 3.927E-07 | 3.271E-07 | 2.617E-07 |
| 2.300E 01 | 2 | 6.247E-07 | 6.191E-07 | 6.023E-07 | 5.748E-07 | 5.373E-07 | 4.910E-07 | 4.371E-07 | 3.775E-07 | 3.145E-07 | 2.516E-07 |
| 4.000E 01 | 3 | 5.713E-07 | 5.662E-07 | 5.508E-07 | 5.256E-07 | 4.914E-07 | 4.490E-07 | 3.998E-07 | 3.452E-07 | 2.876E-07 | 2.301E-07 |
| 5.700E 01 | 4 | 4.919E-07 | 4.875E-07 | 4.743E-07 | 4.526E-07 | 4.231E-07 | 3.866E-07 | 3.442E-07 | 2.973E-07 | 2.476E-07 | 1.981E-07 |
| 7.400E 01 | 5 | 3.901E-07 | 3.866E-07 | 3.761E-07 | 3.589E-07 | 3.356E-07 | 3.066E-07 | 2.730E-07 | 2.357E-07 | 1.964E-07 | 1.571E-07 |
| 9.100E 01 | 6 | 2.705E-07 | 2.681E-07 | 2.608E-07 | 2.489E-07 | 2.326E-07 | 2.126E-07 | 1.893E-07 | 1.634E-07 | 1.361E-07 | 1.089E-07 |
| 1.080E 02 | 7 | 1.384E-07 | 1.372E-07 | 1.335E-07 | 1.274E-07 | 1.191E-07 | 1.088E-07 | 9.686E-08 | 8.364E-08 | 6.967E-08 | 5.575E-08 |
| 1.250E 02 | 8 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |

SOURCE

| | R | 1.000E 02 | 1.100E 02 | 1.200E 02 | 1.300E 02 |
|---|---|---|---|---|---|
| Z | | 11 | 12 | 13 | 14 |
| 6.000E 00 | 1 | 2.038E-07 | 0. | 0. | 0. |
| 2.300E 01 | 2 | 1.960E-07 | 0. | 0. | 0. |
| 4.000E 01 | 3 | 1.792E-07 | 0. | 0. | 0. |
| 5.700E 01 | 4 | 1.543E-07 | 0. | 0. | 0. |
| 7.400E 01 | 5 | 1.224E-07 | 0. | 0. | 0. |
| 9.100E 01 | 6 | 8.483E-08 | 0. | 0. | 0. |
| 1.080E 02 | 7 | 4.341E-08 | 0. | 0. | 0. |
| 1.250E 02 | 8 | 0. | 0. | 0. | 0. |

POWER

| | R | 1.000E 00 | 1.000E 01 | 2.000E 01 | 3.000E 01 | 4.000E 01 | 5.000E 01 | 6.000E 01 | 7.000E 01 | 8.000E 01 | 9.000E 01 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Z | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 6.000E 00 | 1 | 3.132E 02 | 3.104E 02 | 3.019E 02 | 2.882E 02 | 2.694E 02 | 2.462E 02 | 2.192E 02 | 1.893E 02 | 1.577E 02 | 1.262E 02 |
| 2.300E 01 | 2 | 3.011E 02 | 2.984E 02 | 2.903E 02 | 2.770E 02 | 2.590E 02 | 2.367E 02 | 2.107E 02 | 1.820E 02 | 1.516E 02 | 1.213E 02 |
| 4.000E 01 | 3 | 2.753E 02 | 2.729E 02 | 2.655E 02 | 2.533E 02 | 2.368E 02 | 2.164E 02 | 1.927E 02 | 1.664E 02 | 1.386E 02 | 1.109E 02 |
| 5.700E 01 | 4 | 2.371E 02 | 2.350E 02 | 2.286E 02 | 2.182E 02 | 2.039E 02 | 1.864E 02 | 1.659E 02 | 1.433E 02 | 1.193E 02 | 9.550E 01 |
| 7.400E 01 | 5 | 1.880E 02 | 1.864E 02 | 1.813E 02 | 1.730E 02 | 1.617E 02 | 1.478E 02 | 1.316E 02 | 1.136E 02 | 9.465E 01 | 7.573E 01 |
| 9.100E 01 | 6 | 1.304E 02 | 1.292E 02 | 1.257E 02 | 1.199E 02 | 1.121E 02 | 1.025E 02 | 9.123E 01 | 7.878E 01 | 6.562E 01 | 5.251E 01 |
| 1.080E 02 | 7 | 6.672E 01 | 6.612E 01 | 6.432E 01 | 6.139E 01 | 5.739E 01 | 5.244E 01 | 4.669E 01 | 4.032E 01 | 3.358E 01 | 2.687E 01 |
| 1.250E 02 | 8 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |

POWER

| Z | | R 1.000E 02 (11) | 1.100E 02 (12) | 1.200E 02 (13) | 1.300E 02 (14) |
|---|---|---|---|---|---|
| 6.000E 00 | 1 | 9.825E 01 | 0. | 0. | 0. |
| 2.300E 01 | 2 | 9.445E 01 | 0. | 0. | 0. |
| 4.000E 01 | 3 | 8.637E 01 | 0. | 0. | 0. |
| 5.700E 01 | 4 | 7.437E 01 | 0. | 0. | 0. |
| 7.400E 01 | 5 | 5.898E 01 | 0. | 0. | 0. |
| 9.100E 01 | 6 | 4.089E 01 | 0. | 0. | 0. |
| 1.080E 02 | 7 | 2.092E 01 | 0. | 0. | 0. |
| 1.250E 02 | 8 | 0. | 0. | 0. | 0. |

```
REGION AVERAGE FLUX
     GROUP    1
          0.10290669E 16        0.11089154E 15
     GROUP    2
          0.29827665E 16        0.74153757E 15

AVERAGE FLUX RATIO  -REGION WISE-
     GROUP    1
          0.34500416E-00        0.14954271E-00
     GROUP    2
          0.10000000E 01        0.10000000E 01

AVERAGE ABSORPTION CROSS-SECTION  -REGION WISE-
     GROUP    1
          0.                    0.
     GROUP    2
          0.25826316E-02        0.65650000E-04

TOTAL ABSORPTION  -REGION WISE-
          0.30251131E 20        0.13190974E 18

AVERAGE ABSORPTION CROSS-SECTION OF REFLECTOR  -GROUP WISE-
          0.                    0.65649999E-04

AVERAGE NU SIGF CROSS-SECTION  -REGION WISE -
     GROUP    1
          0.                    0.
     GROUP    2
          0.32541000E-02        0.

TOTAL FISSION  -REGION WISE-
          0.15494406E 20        0.

AVERAGE REMOVAL CROSS-SECTION  -REGION WISE-
     GROUP    1
          0.84760000E-02        0.84760000E-02
     GROUP    2
          0.                    0.

TOTAL REMOVAL OF CORE  -GROUP WISE-
          0.34252670E 20        0.

TOTAL LEAKAGE  -REGION WISE-
     GROUP    1
          0.28992876E 19       -0.15677330E 19
     GROUP    2
          0.47527193E 19        0.18440650E 19

BUCKLING  -REGION WISE-
     GROUP    1
          0.71744369E-03       -0.52175269E-02
     GROUP    2
          0.40575420E-03        0.91777049E-03

AVERAGE SOURCE  -REGION WISE-
          0.25464790E-06        0.

AVERAGE POWER  -REGION WISE-
          0.12274029E 03        0.

FLUX NORMALIZATION FACTOR  -BETA-
          0.77593036E 15
```

```
SOURCE NORMALIZATION FACTOR  -GAMMA-
        0.32399090E-05

FLUX RATIO  -GROUP WISE-
    D-MAX
        0.26528774E 01        0.25515895E 01
    D-MIN
        0.15417383E-01        0.58333221E-01

AVERAGE FLUX OF REFLECTOR  -GROUP WISE-
        0.11089154E 15        0.74153757E 15

POWER RATIO
    P-MAX
        0.25515894E 01
    P-MIN
        0.17046768E-00

REGION VOLUME
        0.39269909E 07        0.27096237E 07

VOLUME
    TOTAL VOLUME              0.66366146E 07
    REFLECTOR                 0.27096237E 07
    CORE                      0.39269909E 07

AVERAGE DIFFUSION COEFF. OF CORE  -GROUP WISE-
        0.11400000E 01        0.88299999E 00

AVERAGE ABSORPTION CROSS-SECTION OF CORE  -GROUP WISE-
        0.                    0.25826316E-02

AVERAGE NU SIGF CROSS-SECTION OF CORE  -GROUP WISE-
        0.                    0.32541000E-02

AVERAGE REMOVAL CROSS-SECTION OF CORE  -GROUP WISE-
        0.84759999E-02        0.

AVERAGE BUCKLING OF CORE  -GROUP WISE-
        0.71744369E-03        0.40575420E-03

AVERAGE POWER OF CORE
        0.12274029E 03

AVERAGE FLUX OF CORE  -GROUP WISE-
        0.10290669E 16        0.29827665E 16
```

KAK /M.AKANUMA /2711



| START/ | END/TOTAL TIME(1/1000)/EXECUTION | TIME(1/1000)/PAGES PRINTED BY SYSTEM | /LINES PRINTED BY OBJECT PROG. | /CARDS PUNCHED | |
|---|---|---|---|---|---|
| 18.17 18.38 | 00354 | 00141 | 000016 | 000300 | 000079 |