

STAX 2 : A Computer Program
for Calculating Neutron Elastic
and Inelastic Scattering Cross
Sections by Means of the Opti-
cal Model and Moldauer's
Theory

July 1970

日本原子力研究所

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STAX 2
A Computer Program for Calculating
Neutron Elastic and Inelastic Scattering Cross Sections
by Means of
the Optical Model and Moldauer's Theory

Summary

This program computes the cross sections for neutron elastic and inelastic scattering using the optical model and Moldauer's theory, and searches for potential parameters which reproduce experimental cross sections.

Improvement is made in the treatment of the effect of resonance interference. The quantity "Q" characterizing this effect has been regarded as an arbitrary parameter in the calculations hitherto published. In the present program this quantity can be calculated considering its dependence on the strength function.

Calculation using the Hauser-Feshbach theory is also possible.

November 1969

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STAX 2
光学模型と Moldauer の理論による中性子弹性
非弹性散乱断面積計算コード

要　　旨

このコードは、光学模型と Moldauer の理論によって、中性子の弹性非弹性散乱の断面積を計算し、実験値を再現するポテンシャルパラメーターを求める。

これまでの計算では任意なパラメーターとしてあつかわれてきた、共鳴準位の干渉の効果を示す量 "Q" を、強度関数への依存性を考慮することによって計算できるよう改良した。

Hauser-Feshbach の理論による計算も可能になっている。

1969年11月

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1. Introduction

STAX2 is a computer program for calculating neutron elastic and inelastic scattering cross sections using the optical model and Moldauer's theory, and can search for potential parameters which can reproduce experimental cross sections. Search is made on all potential parameters with respect to any combination of the following cross sections:

- 1) total cross section,
- 2) elastic scattering cross section (integral or differential),
- 3) inelastic scattering cross section for the first excited level (integral or differential).

Although there exist many optical model programs^{1),2)}, none of them seems to use Moldauer's theory for the calculation of compound nuclear process. The program NEARREX³⁾, often used to compute reaction cross sections using Moldauer's theory, can not compute angular distributions and does not include an optical model routine. These are the reasons why the present program has been written.

In this program improvement is made in the treatment of the resonance interference by considering the dependence of the quantity Q^{II} , characterizing this effect, on the strength function.

2. Theory

We consider the scattering of neutrons by a target nucleus of mass M . The cross section σ_{el} for elastic scattering is composed of the shape elastic cross section σ_{se} and the compound elastic cross section σ_{co} :

$$\sigma_{el} = \sigma_{se} + \alpha\sigma_{co}. \quad (1)$$

The quantity α is a correction factor equal to or less than unity and is used to approximately correct for the effect of other open channels than neutron channels, because this program can treat only neutrons. This quantity is also used when higher energy levels are not well known. For the inelastic channels we neglect the direct process and the cross sections are given by

$$\sigma_n = \alpha\sigma_{cn}, \quad (2)$$

where n indicates the n -th excited state.

Cross sections computed in the present program are those in the center-of-mass system.

2.1 Shape Elastic Scattering—Optical Model

The shape elastic scattering is treated by the optical model. The optical potential employed in the present program is of the form

$$-Vf(r) - iWg(r) - V_{so}(\mathbf{l} \cdot \boldsymbol{\sigma})h(r), \quad (3)$$

where

$$f(r) = \frac{1}{1 + \exp[(r - r_0 A^{1/3})/a]}, \quad (4)$$

$$g(r) = \frac{4 \exp[(r - r_s A^{1/3})/b]}{(1 + \exp[(r - r_s A^{1/3})/b])^2}, \quad (5)$$

$$h(r) = -\lambda_\pi^2 \frac{1}{r} \frac{d}{dr} g(r), \quad (6)$$

$$\lambda_\pi = 2.04264 \text{ fm}. \quad (7)$$

Each of the seven parameters V , W , V_{so} , r_s , a and b may have an energy dependence

$$V = \sum_{n=0}^5 V^{(n)} E^n \quad \text{etc.}, \quad (8)$$

where E is the kinetic energy in the center-of-mass system.

The radial Schroedinger equation describing the scattering is

$$\left[-\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right) - Vf(r) - iWg(r) - C_l^{(\pm)} V_{so} h(r) - E \right] \varphi_l^{(\pm)} = 0, \quad (9)$$

where (\pm) corresponds to the value of coupled angular momentum $j=l \pm 1/2$ of \mathbf{l} and σ , and

$$C_l^{(+)} = -l, \quad C_l^{(-)} = l+1, \quad (10)$$

$$\mu = \frac{mM}{m+M}. \quad (11)$$

Here m is the neutron mass, which is 1.008665 amu-¹²C.

The scattering amplitude $\eta_l^{(\pm)}$ is obtained from the asymptotic behavior of the wave function :

$$\eta_l^{(\pm)}(r) \sim r(h_l^{(2)}(kr) + \eta_l^{(\pm)} h_l^{(1)}(kr)), \quad (12)$$

where $h_l^{(1)}$ and $h_l^{(2)}$ are the spherical Hankel functions and

$$k = \frac{1}{\lambda} = \sqrt{\frac{2\mu E}{\hbar^2}}. \quad (13)$$

The shape elastic cross section and the cross section for the formation of compound nucleus are given by

$$\sigma_{se} = \pi \lambda^2 \sum_l \{(l+1)|1-\eta_l^{(+)}|^2 + l|1-\eta_l^{(-)}|^2\}, \quad (14)$$

$$\begin{aligned} \frac{d\sigma_{se}}{d\Omega} = & \frac{\lambda^2}{4} \left[\sum_l \{(l+1)(1-\eta_l^{(+)}) + l(1-\eta_l^{(-)})\} P_l(\cos \theta) \right|^2 \\ & + \left| \sum_l (\eta_l^{(+)} - \eta_l^{(-)}) P_l^1(\cos \theta) \right|^2 \right], \end{aligned} \quad (15)$$

$$\sigma_c = \pi \lambda^2 \sum_l \{(l+1)T_l^{(+)} + lT_l^{(-)}\}, \quad (16)$$

where transmission coefficients $T_l^{(\pm)}$ are given by

$$T_l^{(\pm)} = 1 - |\eta_l^{(\pm)}|^2. \quad (17)$$

2.2 Compound Nuclear Process—Moldauer's Theory

By using Moldauer's theory⁽⁴⁾ differential cross section for the n-th excited state can be written in the following form :

$$\frac{d\sigma_{cn}}{d\Omega} = \frac{\lambda^2}{2(2I_0+1)} \sum_L B_L P_L(\cos \theta), \quad (18)$$

$$B_L = B_L^{(1)} + B_L^{(2)} + B_L^{(3)}, \quad (19)$$

$$\begin{aligned} B_L^{(1)} = & \frac{(-)^{I_n - I_0}}{4} \sum_{J\Pi(lj) \neq (l'j')} (2J+1)^2 \delta(\pi_0(-)^l, \Pi) \delta(\pi_n(-)^{l'}, \Pi) \\ & \times \bar{Z}(ljlj; \frac{1}{2}L) W(jJjJ; I_0L) \bar{Z}(l'l'l'j'; \frac{1}{2}L) W(j'Jj'J; I_nL) \\ & \times \left\langle \frac{\Theta_{\mu,0lj}\Theta_{\mu,nl'j'}}{\Theta_\mu} \right\rangle_\mu^{J\Pi}, \end{aligned} \quad (20)$$

$$\begin{aligned} B_L^{(2)} = & \frac{\delta(n, 0)}{4} \sum_{J\Pi(lj) \neq (l'j')} (2J+1)^2 \left\{ \bar{Z}(ljlj'; \frac{1}{2}L) W(jJj'J; I_0L) \right\}^2 \\ & \times \delta(\pi_0(-)^l, \Pi) \delta(\pi_0(-)^{l'}, \Pi) R_{lj;l'j'} B \left\langle \frac{\Theta_{\mu,0lj}\Theta_{\mu,0l'j'}}{\Theta_\mu} \right\rangle_\mu^{J\Pi}, \end{aligned} \quad (21)$$

$$B_L^{(3)} = \frac{-\delta(n, 0)}{4} \sum_{J\Pi l j' l' j'} (2J+1)^2 \left\{ \bar{Z}(l j l' j' ; \frac{1}{2} L) W(j J j' J; I_0 L) \right\}^2 \\ \times \delta(\pi_0(-)^l, \Pi) \delta(\pi_0(-)^{l'}, \Pi) R_{l j, l' j'} Q_\mu^{J\Pi} \langle \Theta_{\mu, n l j} \rangle_\mu^{J\Pi} \langle \Theta_{\mu, n l' j'} \rangle_\mu^{J\Pi}, \quad (22)$$

where I_n and π_n are the spin and parity of the n -th excited state (the ground state is denoted by $n=0$), J and Π are the total angular momentum and parity, and $\delta(n, m)$ is Kronecker's symbol. The coefficients \bar{Z} and W are the Blatt-Biedenharn and the Racah coefficients, and

$$Q^{J\Pi} = \frac{2B}{N^2} \left[1 - \Phi_0 \left(\frac{\langle \Theta_\mu \rangle_\mu^{J\Pi}}{2N^2} \right) \right], \quad (23)$$

$$B = |b_{nlj}|^2, \quad (24)$$

$$R_{l j, l' j'} = Re(b_{0 l j} b_{0 l' j'}^*) / B. \quad (25)$$

Here B is assumed to be independent of (nlj) .

The function Φ_0 is defined by

$$\Phi_0(x) = 1 - \frac{1}{x} \left[1 - \frac{1}{x} e^{-x} \sinh x \right] - \frac{1}{x} \text{Ei}(-x) \left[\cosh x - \frac{1}{x} \sinh x \right], \quad (26)$$

and

$$\Theta_\mu = \sum_{nlj} \Theta_{\mu, n l j}. \quad (27)$$

Other quantities $\Theta_{\mu, n l j}$, $b_{n l j}$ and N are defined in ref.⁴⁾ and the average $\langle \cdot \rangle_\mu^{J\Pi}$ is taken over the compound levels μ which have a spin and parity of $J\Pi$. The quantity $\langle \Theta_{\mu, n l j} \rangle_\mu^{J\Pi}$ is proportional to the strength function $\langle \Gamma_{\mu, n l j} / D \rangle_\mu^{J\Pi}$, and is related to the transmission coefficient $T_{n l j}$ by

$$T_{n l j} = \langle \Theta_{\mu, n l j} \rangle_\mu^{J\Pi} - \frac{Q^{J\Pi}}{4} \{ \langle \Theta_{\mu, n l j} \rangle_\mu^{J\Pi} \}^2. \quad (28)$$

Assuming a χ^2 -distribution with v degrees of freedom for $\Theta_{\mu, n l j}$ we obtain⁵⁾

$$\left\langle \frac{\Theta_{\mu, n l j} \Theta_{\mu, n' l' j'}}{\Theta_\mu} \right\rangle_\mu^{J\Pi} = \frac{\langle \Theta_{\mu, n l j} \rangle_\mu^{J\Pi} \langle \Theta_{\mu, n' l' j'} \rangle_\mu^{J\Pi}}{\langle \Theta_\mu \rangle_\mu^{J\Pi}} S_{n l j, n' l' j'}^{J\Pi}, \quad (29)$$

$$S_{n l j^{J\Pi}, n' l' j'} = \int_0^\infty dt \frac{1 + 2\delta_{nn'}\delta_{ll'}\delta_{jj'}}{f_{n l j} f_{n' l' j'} \prod_{n'' l'' j''} f_{n'' l'' j''}^{J\Pi} t^{v/2}}, \quad (30)$$

where the product in the denominator is taken over all channels which have a total angular momentum and parity of $J\Pi$, and

$$f_{n l j} = 1 + \frac{\langle \Theta_{\mu, n l j} \rangle_\mu^{J\Pi} 2t}{\langle \Theta_\mu \rangle_\mu^{J\Pi} v}. \quad (31)$$

Various quantities given above have limited ranges of magnitude:

$$1 \geq B \geq 0, \quad (32)$$

$$N \geq 1, \quad (33)$$

$$1 \geq R_{l j, l' j'} \geq -1, \quad (34)$$

$$R_{l j, l j} = 1, \quad (35)$$

$$2 \geq Q^{J\Pi} \geq 0, \quad (36)$$

and in the limit of $\Gamma/D \rightarrow 0$ they become

$$B = 1, \quad (37)$$

$$N = 1, \quad (38)$$

$$R_{l j, l' j'} = 1, \quad (39)$$

$$Q^{J\Pi} = 2. \quad (40)$$

The quantity $R_{l j, l' j'}$ for $(l j) \neq (l' j')$ is assumed to be independent of $(l j, l' j')$ in the present program. This quantity does not affect the cross sections when the spin of the target nucleus is zero.

In the calculations hitherto published, $Q^{J\Pi}$ was treated as a constant neglecting its dependence on $\langle \Theta_{\mu, n l j} \rangle_\mu^{J\Pi}$. However, this treatment sometimes results in negative values⁶⁾ for the compound

elastic cross sections for some partial waves, when many channels are open and the constant is not taken sufficiently small. This difficulty is avoided if Eq. (28) is solved for $\langle \Theta_{\mu, nlj} \rangle_{\mu}^{JII}$ by considering the functional dependence of Q^{JII} on $\langle \Theta_{\mu, nlj} \rangle_{\mu}^{JII}$ given by Eq. (23). Both treatment of Q^{JII} are possible in the present program.

The largest possible Q^{JII} is given for $B=1$ and $N=1$.

By putting b_{nlj} to zero and $S_{nlj, n'l'j'}^{JII}$ to unity, B and Q^{JII} become zero and the Hauser-Feshbach formula is obtained in Eqs. (18)-(22).

3. Fitting to Experimental Cross Sections

Fitting the calculation to experimental cross sections is made by minimizing the quantity χ^2 defined by

$$\begin{aligned} \chi^2 = & \left\{ W_t \left(\frac{\sigma_t^{cal} - \sigma_t^{exp}}{\Delta \sigma_t} \right)^2 + W_{el} \left(\frac{\sigma_{el}^{cal} - \sigma_{el}^{exp}}{\Delta \sigma_{el}} \right)^2 + W_1 \left(\frac{\sigma_1^{cal} - \sigma_1^{exp}}{\Delta \sigma_1} \right)^2 \right. \\ & + W_{ela} \sum_{\theta} \left(\frac{\left(\frac{d\sigma_{el}^{cal}(\theta)}{d\Omega} - \frac{d\sigma_{el}^{exp}(\theta)}{d\Omega} \right)^2}{\frac{\Delta d\sigma_{el}(\theta)}{d\Omega}} \right) + W_{1a} \sum_{\theta} \left(\frac{\left(\frac{d\sigma_1^{cal}(\theta)}{d\Omega} - \frac{d\sigma_1^{exp}(\theta)}{d\Omega} \right)^2}{\frac{\Delta d\sigma_1(\theta)}{d\Omega}} \right) \left. \right\} \\ & / (W_t + W_{el} + W_1 + \sum_{\theta} W_{ela} + \sum_{\theta} W_{1a}), \end{aligned} \quad (41)$$

where σ_t is the total cross section, "cal" and "exp" indicate the calculated and the experimental cross sections, Δ indicates the experimental uncertainty, and W_t etc. are weighting factors which may be arbitrarily chosen. Search for the "best" parameters is made on any number of parameters $V^{(0)}$, $W^{(0)}$, $V_{so}^{(0)}$, $r_0^{(0)}$, $r_s^{(0)}$, $\alpha^{(0)}$ and $b^{(0)}$ by means of Gauss-Newton's method.

For brevity we denote the set of parameters to be searched for by $\{x_i\}$ and the cross sections to be fitted by $\{\sigma_{\alpha}\}$. Around $\{x_i^{(n)}\}$ the cross sections are approximated by

$$\sigma_{\alpha}^{cal}(\{x_i^{(n)} + \delta x_i\}) = \sigma_{\alpha}^{cal}(\{x_i^{(n)}\}) + \sum_i \frac{\partial \sigma_{\alpha}^{cal}(\{x_i^{(n)}\})}{\partial x_i} \delta x_i, \quad (42)$$

where $\{x_i^{(n)}\}$ indicates the parameters obtained after the n -th iteration. The parameters which minimize χ^2 can be obtained by

$$\sum_{\alpha j} \frac{W_{\alpha}}{(\Delta \sigma_{\alpha})^2} \frac{\partial \sigma_{\alpha}^{cal}}{\partial x_i} \frac{\partial \sigma_{\alpha}^{cal}}{\partial x_j} \delta x_j = \sum_{\alpha} \frac{W_{\alpha} (\sigma_{\alpha}^{exp} - \sigma_{\alpha}^{cal})}{(\Delta \sigma_{\alpha})^2} \frac{\partial \sigma_{\alpha}^{cal}}{\partial x_i}. \quad (43)$$

This equation can be solved easily. However, since we neglected higher order terms of $\{\delta x_i\}$ in Eq. (42), this equation is not very accurate when some of the eigenvalues of the matrix

$$\left(\sum_{\alpha} \frac{W_{\alpha}}{(\Delta \sigma_{\alpha})^2} \frac{\partial \sigma_{\alpha}^{cal}}{\partial x_i} \frac{\partial \sigma_{\alpha}^{cal}}{\partial x_j} \right)$$

are small, and such circumstances may often happen because of the well known optical potential ambiguities. For this reason Eq. (43) is not solved as it is. We define a symmetric matrix and column vectors in the parameter space:

$$(A_{ij}) = \left(\sum_{\alpha} \frac{W_{\alpha}}{(\Delta \sigma_{\alpha})^2} \frac{\partial \sigma_{\alpha}^{cal}}{\partial x_i} \frac{\partial \sigma_{\alpha}^{cal}}{\partial x_j} \delta x_i^{max} \delta x_j^{max} \right), \quad (44)$$

$$(b_i) = \left(\sum_{\alpha} \frac{W_{\alpha} (\sigma_{\alpha}^{cal} - \sigma_{\alpha}^{exp})}{(\Delta \sigma_{\alpha})^2} \frac{\partial \sigma_{\alpha}^{cal}}{\partial x_i} \delta x_i^{max} \right), \quad (45)$$

$$(\delta \xi_i) = \left(\frac{\delta x_i}{\delta x_i^{max}} \right), \quad (46)$$

where $\{\delta x_i^{max}\}$ is introduced in order to make the magnitude of A_{ij} uniform and also to set

a limit for $\{\delta x_i\}$, and is given as follows:

$$\left. \begin{aligned} \delta V^{\max} &= \delta W^{\max} = \delta V_{so}^{\max} = 3.0 \text{ MeV}, \\ \delta r_0^{\max} &= 0.05 \text{ fm}, \\ \delta r_s^{\max} &= 0.1 \text{ fm}, \\ \delta a^{\max} &= 0.07 \text{ fm}, \\ \delta b^{\max} &= 0.1 \text{ fm}. \end{aligned} \right\} \quad (47)$$

Then Eq. (43) can be written as

$$\mathbf{A}\delta\boldsymbol{\xi} = \mathbf{b}. \quad (48)$$

The matrix \mathbf{A} is diagonalized by a orthogonal matrix \mathbf{U} :

$$\mathbf{U}\mathbf{A}\mathbf{U}^t = \mathbf{D} = \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \end{pmatrix} \quad (49)$$

Solution of Eq. (48) is given by

$$\delta\boldsymbol{\xi} = \mathbf{U}^t \delta\boldsymbol{\eta}, \quad (50)$$

$$\delta\boldsymbol{\eta} = \mathbf{D}^{-1} \mathbf{U} \mathbf{b}. \quad (51)$$

Because of the reason mentioned above, we neglect small eigenvalues and replace \mathbf{D}^{-1} by $\tilde{\mathbf{D}}^{-1}$, the elements of which are given by

$$\tilde{D}_{ij}^{-1} = \begin{cases} \frac{\delta_{ij}}{\lambda_i}, & \text{if } |\lambda_i| \geq \varepsilon \\ 0, & \text{if } |\lambda_i| < \varepsilon, \end{cases} \quad (52)$$

where ε is a small number defined below in Eq. (57). Furthermore, since the approximation (42) is valid only in the neighbourhood of $\{x_i^{(n)}\}$, $\delta\boldsymbol{\eta}$ is truncated as follows:

$$\delta\eta_i = \begin{cases} \tilde{\delta\eta}_i, & \text{if } |\tilde{\delta\eta}_i| \leq 1 \\ \frac{\tilde{\delta\eta}_i}{|\tilde{\delta\eta}_i|}, & \text{if } |\tilde{\delta\eta}_i| > 1, \end{cases} \quad (53)$$

where

$$\tilde{\delta\boldsymbol{\eta}} = \tilde{\mathbf{D}}^{-1} \mathbf{U} \mathbf{b}. \quad (54)$$

At $\{x_i^{(n)} + \delta x_i\}$, χ^2 is expected to decrease by

$$\Delta\chi^2 = \sum_{i, |\lambda_i| \geq \varepsilon} \lambda_i (\delta\eta_i)^2 \quad (55)$$

in the same approximation as above. When $\chi^2(\{x_i^{(n)}\}) - \chi^2(\{x_i^{(n)} + \delta x_i\}) \geq \frac{\Delta\chi^2}{4}$, $\{x_i^{(n)} + \delta x_i\}$ is taken as $\{x_i^{(n+1)}\}$. When $\chi^2(\{x_i^{(n)}\}) - \chi^2(\{x_i^{(n)} + \delta x_i\}) \leq \frac{\Delta\chi^2}{4}$, we consider the approximation (42) to be not good and compute χ^2 at $\{x_i^{(n)} + \frac{1}{2}\delta x_i\}$. Then $\{x_i^{(n+1)}\}$ is obtained by approximating $\chi^2(x_i^{(n)} + \alpha\delta x_i)$ by a quadratic polynomial of α .

This parameter search is stopped when one of the following conditions is encountered:

- 1) a specified number of iteration has been made,
- 2) specified calculation time has elapsed,
- 3) χ^2 has become smaller than a specified value χ^2_{\min} ,
- 4) $\chi^2(\{x_i^{(n)}\}) - \chi^2(\{x_i^{(n+1)}\}) \leq 0.01 \chi^2_{\min}$.

The matrix \mathbf{A} is diagonalized by means of Jacobi's method and the accuracy of diagonalization is taken as

$$\varepsilon' = \frac{10^{-K}}{N_p} \sum A_{ii}, \quad (56)$$

where N_p is the number of the parameter to be searched for and K is usually set equal to five unless otherwise specified. The ε in Eq. (52) is given by

$$\varepsilon = 10 N_p \varepsilon'. \quad (57)$$

Derivative $\frac{\partial \sigma_a}{\partial x_i}$ is approximated by

$$\frac{\sigma_a(x_i^{(n)} + \Delta x_i) - \sigma_a(x_i^{(n)})}{\Delta x_i}, \quad (58)$$

where $\{\Delta x_i\}$ is given as follows:

$$\left. \begin{array}{l} \Delta V = \Delta W = \Delta V_{so} = 0.1 \text{ MeV}, \\ \Delta r_0 = r_s = \Delta a = \Delta b = 0.01 \text{ fm}. \end{array} \right\} \quad (59)$$

In computing $\sigma_a(\{x_i^{(n)} + \Delta x_i\})$ two methods are employed:

- (A) $\sigma_a(\{x_i^{(n)} + \Delta x_i\})$ is calculated rigorously,
- (B) only the shape elastic cross section and the cross section for the formation of compound nucleus are calculated, and the cross sections through compound process are approximated as follows:

$$\sigma_{cn}(x_i^{(n)} + \Delta x_i) = \frac{\sigma_c(x_i^{(n)} + \Delta x_i)}{\sigma_c(x_i^{(n)})} \sigma_{cn}(x_i^{(n)}), \quad (60)$$

$$\frac{d\sigma_{cn}(x_i^{(n)} + \Delta x_i)}{d\Omega} = \frac{\sigma_c(x_i^{(n)} + \Delta x_i)}{\sigma_c(x_i^{(n)})} \frac{d\sigma_{cn}(x_i^{(n)})}{d\Omega}. \quad (61)$$

Method (B) is inferior in accuracy to method (A), but requires much shorter calculation time especially when many channels are open. In most cases method (B) is preferred.

4. Methods of Numerical Calculation

4.1 Solution of the Schroedinger Equation

Equation (9) is solved by means of Noumerov's method⁷⁾ in the internal region. Rewriting Eq. (9) as

$$\frac{d^2}{dr^2}\varphi_l^{(\pm)} = D_l^{(\pm)}\varphi_l^{(\pm)}, \quad (62)$$

and defining the quantity

$$\Psi_l^{(\pm)} = \left(1 - \frac{h^2}{12}D_l^{(\pm)}\right)\varphi_l^{(\pm)}, \quad (63)$$

we can replace Eq. (9) by the recurrence relation

$$\Psi_l^{(\pm)}(r_n) = \left[2 + \frac{h^2 D_l^{(\pm)}(r_{n-1})}{1 - \frac{h^2}{12}D_l^{(\pm)}(r_{n-1})}\right] \Psi_l^{(\pm)}(r_{n-1}) - \Psi_l^{(\pm)}(r_{n-2}), \quad (64)$$

where

$$r_n = r_1 + (n-1)h \quad (65)$$

and $h=0.25 \text{ fm}$ unless otherwise specified. Two values $\Psi_l^{(\pm)}(r_1)$ and $\Psi_l^{(\pm)}(r_2)$ are necessary for solving Eq. (65). These values are obtained by expanding the functions in power series of r :

$$\varphi_l^{(\pm)}(r) = \sum_{n=0}^{\infty} b_{ln}^{(\pm)} r^{n+l+1}, \quad (66)$$

$$D_l^{(\pm)}(r) = \frac{l(l+1)}{r_2} - \frac{K_l^{(\pm)}}{r} - \sum_{n=0}^{\infty} a_{ln}^{(\pm)} r^n, \quad (67)$$

where

$$K_l^{(\pm)} = \frac{2\mu\lambda_\pi^2 V_{so} C_l^{(\pm)}}{\hbar^2} \frac{\exp(-r_0 A^{1/3}/a)}{\{1 + \exp(-r_0 A^{1/3}/a)\}^2}, \quad (68)$$

$$\alpha_{ln}^{(\pm)} = \frac{2\mu}{\hbar^2} [E\delta_{n0} + f_n V + 4ig_n W + (n+2)\lambda_\pi^2 V_{sc} C_l^{(\pm)} f_{n+2}]. \quad (69)$$

The coefficients f_n and g_n are the derivatives of potentials and are given by the following equations:

$$f_n = F_n(r_0, a) \quad (70)$$

$$g_n = -b(n+1)F_n(r_s, b), \quad (71)$$

$$F_0(r, c) = [1 + \exp(rA^{1/3}/c)]^{-1}, \quad (72)$$

$$F_{n+1}(r, c) = \frac{1}{c(n+1)} \left[-F_n(r, c) + \sum_{k=0}^n F_k(r, c) F_{n-k}(r, c) \right]. \quad (73)$$

The expansion coefficients of $\varphi_l^{(\pm)}$ are given by

$$b_{l0}^{(\pm)} = 1, \quad (74)$$

$$b_{l1}^{(\pm)} = -\frac{K_l^{(\pm)}}{2(l+1)}, \quad (75)$$

$$b_{ln}^{(\pm)} = \frac{-1}{n(2l+n+1)} \left[K_l^{(\pm)} b_{ln-1}^{(\pm)} + \sum_{k=0}^{n-2} b_{lk}^{(\pm)} \alpha_{ln-k-2}^{(\pm)} \right]. \quad (76)$$

Only the first eight terms are taken in the expansion of $\varphi_l^{(\pm)}$.

Because of the singularity of $l(l+1)/r^2$, Eq. (64) is not accurate in the neighbourhood of $r=0$. Therefore r_1 and r_2 should be taken as large as possible. Error of $\varphi_l^{(\pm)}$ due to the truncation of the power series (66) is of the order of

$$\frac{(Kr)^8}{9!} \varphi_l^{(\pm)}(r), \quad (77)$$

where

$$K = \sqrt{\frac{2\mu}{\hbar^2} (E + V)}. \quad (78)$$

We require $\varphi_l^{(\pm)}(r_2)$ to be accurate to 10^{-5} and take r_2 as

$$r_2 = \frac{1.2}{K}. \quad (79)$$

4.2 Determination of Scattering Amplitude

Scattering amplitude $\eta_l^{(\pm)}$ is obtained by matching the internal wave function with the external wave function (12):

$$\frac{\varphi_l^{(\pm)}(r_M)}{\varphi_l^{(\pm)}(r_{M-1})} = \frac{r_M}{r_{M-1}} \cdot \frac{h_l^{(2)}(kr_M) + \eta_l^{(\pm)} h_l^{(1)}(kr_M)}{h_l^{(2)}(kr_{M-1}) + \eta_l^{(\pm)} h_l^{(1)}(kr_{M-1})}. \quad (80)$$

Matching radius r_M is determined by the requirement that at $r > r_M$ influence of the optical potential is negligible. By rewriting Eq. (9) as

$$\frac{d^2}{dr^2} \varphi_l^{(\pm)} + \left\{ k^2 - \frac{l(l+1)}{r^2} - v(r) \right\} \varphi_l^{(\pm)} = 0, \quad (81)$$

relative error of $\varphi_l^{(\pm)}(r_M)$ introduced by neglecting $v(r)$ at $r > r_M$ can be shown to be of the order of

$$\frac{1}{k} \int_{r_M}^{\infty} dr r^2 v(r) h_l^{(1)}(kr) h_l^{(2)}(kr) \lesssim \frac{1}{k} \int_{r_M}^{\infty} dr |v(r)|. \quad (82)$$

The right hand side of the inequality becomes

$$\frac{|C|}{\alpha k} e^{-\alpha r_M} \quad (83)$$

for $v(r) = Ce^{-\alpha r}$. Therefore we require r_M to satisfy the following conditions

$$\left. \begin{array}{l} \frac{akV}{E} e^{\frac{r_0 A^{1/3} - r_M}{a}} < 10^{-6}, \\ \frac{4bkW}{E} e^{\frac{r_s A^{1/3} - r_M}{a}} < 10^{-6}, \\ \frac{10k\lambda_\pi^2 V_{so}}{a A^{1/3} E} e^{\frac{r_0 A^{1/3} - r_M}{a}} < 10^{-6}. \end{array} \right\} \quad (84)$$

4.3 Calculation of Q^{JII} and $\langle \theta_{\mu, nlj} \rangle_{\mu}^{JII}$

Equation (28) can be rewritten as

$$\Theta_c = \frac{2}{Q} \{1 - \sqrt{Q} T_c\} \quad (85)$$

and Θ_c may be regarded as a function of Q , where we denote $\langle \theta_{\mu, nlj} \rangle_{\mu}^{JII}$, T_{nlj} and Q^{JII} by Θ_c , T_c and Q for brevity. Then Eq. (23) becomes an equation with only one variable Q . This equation is solved by Newton's iteration method. Starting values are taken as

$$\Theta_c^{(0)} = T_c, \quad (86)$$

$$Q^{(0)} = \frac{2B}{N^2} \left(1 - \Phi_0 \left(\frac{\sum \Theta_c^{(0)}}{2N^2} \right) \right), \quad (87)$$

and the next value is given by

$$Q^{(n+1)} = Q^{(n)} + \frac{\frac{2B}{N^2} \left\{ 1 - \Phi_0 \left(\frac{\sum \Theta_c^{(n)}}{2N^2} \right) \right\} - Q^{(n)}}{1 + \frac{B}{4N^2} \sum_c \frac{\{\Theta_c^{(n)}\}^2}{\sqrt{1 - Q_c^{(n)}} T_c} \Phi_0' \left(\frac{\sum \Theta_c^{(n)}}{2N^2} \right)}, \quad (88)$$

where $\Phi_0'(x)$ is replaced by

$$100 \{ \Phi_0(x+0.01) - \Phi_0(x) \}. \quad (89)$$

Iteration is continued until

$$|\Theta_c^{(n+1)} - \Theta_c^{(n)}| < 10^{-6} \quad (90)$$

is reached for all c .

4.4 Calculation of Width Fluctuation Factor

For brevity we denote the integral in Eq. (30) by

$$I = \int_0^\infty \frac{dt}{\prod_k (1 + \alpha_k t)^{\nu/2}}. \quad (91)$$

By the transformation

$$y = (1 + \bar{\alpha}t)^{1-n} \quad (92)$$

the integral becomes

$$I = \frac{1}{(n-1)\bar{\alpha}} \int_0^1 \frac{dy}{\prod_k Y_k}, \quad (93)$$

where

$$n = \frac{1}{2} \sum_k \nu, \quad (94)$$

$$\bar{\alpha} = \frac{1}{n} \sum_k \alpha_k, \quad (95)$$

$$Y_k = \left(1 - \frac{\alpha_k}{\bar{\alpha}} \right) y^{n-1} + \frac{\alpha_k}{\bar{\alpha}}. \quad (96)$$

The integrand is a monotonously decreasing function of y , and when values of α_k are scattered widely around their average $\bar{\alpha}$ it has a large derivative in the neighbourhood of $y=0$. In order to avoid this rapid variation further transformation

$$y = z^5 \quad (97)$$

is made and the integral becomes

$$I = \frac{5}{(n-1)\bar{\alpha}} \int_0^1 \frac{z^4}{\prod_k Y_k} dz, \quad (98)$$

which is evaluated by means of the 16-point Legendre-Gauss formula.

5. Subroutines

The following subroutines are used in the present program.

1. SUBROUTINE SEARCH searches for the "best" potential parameters.
2. SUBROUTINE EIGEN diagonalizes the matrix A in Eq. (49).
3. SUBROUTINE XSECT computes various cross sections.
4. SUBROUTINE DIFEQ solves the Schroedinger equation.
5. SUBROUTINE BESSEL AND BESSE2 compute the spherical Bessel and Neumann functions.
6. SUBROUTINE SHAPE computes the shape elastic cross section.
7. SUBROUTINE LEGND1 and LEGND2 compute the Legendre functions.
8. SUBROUTINE TRANS computes Q^{JII} and $\langle \Theta_{\mu, nlj} \rangle_{\mu}^{JII}$.
9. SUBROUTINE PHAI computes the function Φ_0 .
10. SUBROUTINE EI computes the exponential integral.
11. SUBROUTINE BLL computes B_L in Eq. (18).
12. FUNCTION S computes the width fluctuation factor.
13. SUBROUTINE FACTRL, FUNCTION DELT and EXDELT are used for computing the Clebsch-Gordan coefficient and the Racah coefficient.
14. FUNCTION CG0 computes the Clebsch-Gordan coefficient.
15. FUNCTION WRAC computes the Racah coefficient.

6. Input

The meaning and FORMAT of input cards are described below. Units of energy and length are MeV and fm, respectively.

CARD 1

(DATE(N), N=1, 3), (NAME(N), N=1, 3)

FORMAT (6 A 4)

Date and name.

CARD 2

(TITLE(N), N=1, 18)

FORMAT (18 A 4)

Title of the calculation.

CARD 3

FMASST, NOMASS, NLEV, NLOUT, ELAB, PHIPRE, DR, ITEST, IFDPR, NPRE.

FORMAT (F 12. 0, 3 I 6, F 6. 0, E 12. 3, F 6. 0, 3 I 6)

FMASST : Mass of the target nucleus in amu-C¹².

NOMASS : Mass number of the target nucleus.

NLEV : Number of levels. (≤ 25)

NLOUT : Cross sections are calculated only for the lowest NLOUT levels.

ELAB : Energy of the incident neutrons in the laboratory system.

PHIPRE : The factor 10^{-6} in (84) is replaced by PHIPRE, if PHIPRE $\neq 0$.

DR : If ITEST $\neq 0$, matrices A , b , D and U in Eqs. (44)-(49) are printed as output.

IFDPR : If IFDPR=1, $\{\Delta x_i\}$ in Eqs. (59) are replaced by the values on CARD 4.

If IFDPR=2, $\{\delta x_i^{\max}\}$ in Eqs. (47) are replaced by the values on CARD 5.

If IFDPR=3, both quatities are replaced.

NPRE : If NPRE $\neq 0$, the factor K in Eq. (56). is replaced by NPRE.

CARD 4 (Necessary only when IFDPR=1 or 3.)

(DPARA(N), N=1, 7)

FORMAT (7 F 6. 0)

: ΔV , ΔW , ΔV_{so} , Δr_0 , Δr_s , Δa and Δb in Eqs. (59).

CARD 5 (Necessary only when IFDPR=2 or 3.)

(DRPMAX(N), N=1, 7)

FORMAT (7 F 6. 0)

: δV^{\max} , δW^{\max} , δV_{so}^{\max} , δr_0^{\max} , δr_s^{\max} , δa^{\max} and δb^{\max} in Eqs. (47).

CARD 6

(EX(N), N=1, NLEV)

FORMAT (12 F 6. 0)

: Excitation energy of each levels in ascending order.

CARD 7

(SPIN(N), PARITY(N), N=1, NLEV)

FORMAT (12(F 5. 0, A 1))

: Spin and parity of each levels. Parities are punched as + or -.

CARD 8

NANGL, IFCM, IFCOS

FORMAT (3 I 6)

NANGL : Number of angles for which differential cross sections are calculated. (≤ 20)

IFCM : If IFCM=1, the values of angles are those of in the center-of-mass system, and if IFCM=0, they are in the laboratory system.

IFCOS : If IFCOS=1, angles are given by cosine, and if IFCOS=0, they are given in degrees.

CARD 9

(ANG(N) or COSIN(N), N=1-NANGL)

FORMAT (12 F 6. 0)

: Values of angles.

CARD 10

NSRCH0, NSRCH, CHIMIN, TIME, WGTOT, WGEL, WG 1, WGELA, WG 1 A.

FORMAT (2 I 6, 7 F 6. 0)

NSRCH0 and NSRCH: Number of iteration of the parameter search by method (B) and by method (A). Usually method (B) is tried first and then method (A), but if NSRCH<0, method (A) is tried first. Either may be zero.

CHIMIN : Search ends when χ^2 has become smaller than CHIMIN.

TIME : Search is interrupted when calculation time has exceeded TIME.
 TIME is given in minutes.

WGTOT, WGEL, WG 1, WGELA and WG 1 A:

Weighting factors W_t , W_{el} , W_1 , W_{ela} and W_{1a} in Eq. (41).

CARD 11

IFV, VO, (VE(N), N=1, 5)

FORMAT (A 1, F 11. 0, 5 F 12. 0)

IFV : When search is made for the depth V of real potential, * is punched for IFV.
 VO : $V^{(0)}$ in Eq. (8).
 VE(N) : $V^{(N)}$ in Eq. (8).

CARD 12

IFW, WO (WE(N) N=1, 5)

FORMAT (A 1, F 11. 0, 5 F 12. 0)

: Same as CARD 11 for the depth W of imaginary potential.

CARD 13

IFVSO, VSOO, (VSOE(N) N=1, 5)

FORMAT (A 1, F 11. 0, 5 F 12. 0)

: Same as CARD 11 for the depth V_{so} of spin-orbit potential.

CARD 14

IFRR, RRO, (RRE(N), N=1, 5)

FORMAT (A 1, F 11. 0, 5 F 12. 0)

: Same as CARD 11 for the radius parameter r_0 of real potential

CARD 15

IFRS, RSO, (RSE(N), N=1, 5)

FORMAT (A 1, F 11. 0, 5 F 12. 0)

: Same as CARD 11 for the radius parameter r_s of imaginary potential.

CARD 16

IFA, AO, (AE(N), N=1, 5)

FORMAT (A 1, F 11. 0, 5 F 12. 0)

: Same as CARD 11 for the diffuseness α of real potential.

CARD 17

IFB, BO, (BE(N), N=1, 5)

FORMAT (A 1, F 11. 0, 5 F 12. 0)

: Same as Card 11 for the diffuseness b of imaginary potential.

CARD 18 (Necessary only when at least one of WGTOT, WGEL and WG 1 is not zero.)

STOTEX, ERTOT, SELEX, EREL, S1STEX, ERIST

FORMAT (6 F 6. 0)

STOTEX and ERTOT: Experimental total cross section and its uncertainty.

SELEX and EREL : Same as above for the elastic scattering cross section.

S1STEX and ER1ST : Same as above for the first level cross section.

(CARD 19-21 are for the experimental values of differential elastic scattering cross sections, and are necessary only when WGELA \neq 0.)

CARD 19

ERRAT

FORMAT (F 6. 0)

: Uncertainty in cross section in percent. When ratios of uncertainty to cross section

are different for various angles, ERRAT is set equal to zero and uncertainties are given on CARD 21.

CARD 20

(SELAEX(N), N=1, NANGL)

FORMAT (12 F 6. 0)

: Values of the differential cross section in mb/sr. For angles where experimental values are lacking, SELAEX (N) are set equal to zero. They are not included in the χ^2 calculation.

CARD 21 (Necessary only when ERRAT=0.)

(EREELA(N), N=1, NANGL)

FORMAT (12 F 6. 0)

: Uncertainty in the above values in mb/sr.

(CARD 22-24 are for the experimental values of differential inelastic scattering cross sections leading to the first excited state, and are necessary only when WG 1 A \neq 0. The meaning of these cards are same as CARD 19-21.)

CARD 22

ERRAT

FORMAT (F 6. 0)

CARD 23

(S 1 AEX(N), N=1, NANGL)

FORMAT (12 F 6. 0)

CARD 24 (Necessary only when ERRAT=0.)

(ER 1 A(N), N=1, NANGL)

FORMAT (12 F 6. 0)

CARD 25

FCHI, IQCONS, QCONST, BC, REBB, FNR, COMP

FORMAT (F 6. 0, I 6, 5 F 6. 0)

FCHI : Degrees of freedom of the χ^2 -distribution for $\theta_{\mu, nlj}$. IF FCHI=0, cross sections are calculated by means of the Hauser-Feshbach theory.

IQCONS and QCONST: If IQCONS=0, Q^{II} is calculated as described in Sec. 2. 2. If IQCONS=1, Q^{II} is set equal to QCONST.

BC : B in Eq. (24).

REBB : $R_{lj; l'j'}$ for $(l'j) \neq (l'j')$ in Eq. (25).

FNR : N in Eq. (23).

COMP : α in Eqs. (1) and (2).

7. Requirement for Machine

This program is coded in FORTAN for a FACOM-230/60 computer which has a library subroutine CLOCK which gives the elapsed time in units of Second. Memory size required for this program is about 40K words.

The auther is indebted to S. IGARASHI for making available the code ELIESE-2 and for advices. Comparison with the calculation by ELIESE-2 greatly facilitated the error checking of STAX 2. He also wishes to express his thanks to K. TSUKADA, K. OKAMOTO and M. MARUYAMA for careful reading of the manuscript.

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Appendix A Sample Calculation

For illustration, the input and output of a calculation is shown below. In this calculation fitting is tried to the data of Zn at $E_n=2.24$ MeV by using the largest possible values of Q .

| 氏名 所属 | 日付 年月 電話 | プログラム名 研究テーマ番号 | JOB NO. | | | | | | | | カード色指定 | | PUNCH 73-80 YES <input type="checkbox"/> NO <input type="checkbox"/> | | |
|------------|----------------|-------------------|-------------------|-------------|-------------|-------------|-------------|-------------|-------------|------------|-------------|-------------|---|-------------|---|
| | | | IBJOB DECKNAME | | | | 備考 | | | | | | | | |
| 123456789 | 10123456789 | 20123456789 | 30123456789 | 40123456789 | 50123456789 | 60123456789 | 70123456789 | 80123456789 | 90123456789 | 0123456789 | 10123456789 | 20123456789 | 30123456789 | 40123456789 | |
| 1969-10-20 | Y.TOMITA | ZN-2.24MEY | Q-MAX | | | | | | | | | | | | |
| 65.289 | 65 | 4 | 4 | 2.24 | | | | | | | | | | | |
| 0.0 | 1.02 | 1.835 | 1.90 | | | | | | | | | | | | |
| 0+ | 2+ | 2+ | 0+ | | | | | | | | | | | | |
| 1.9 | | | | | | | | | | | | | | | |
| 0. | 1.0. | 2.0. | 3.0. | 4.0. | 5.0. | 6.0. | 7.0. | 8.0. | 9.0. | 10.0. | 11.0. | 12.0. | 13.0. | 14.0. | |
| 1.20. | 1.30. | 1.40. | 1.50. | 1.60. | 1.70. | 1.80. | | | | | | | | | |
| 2.0 | 2.25 | 5. | | | 1. | 1. | | | | | | | | | |
| * | 5.0. | | | | | | | | | | | | | | |
| * | 1.0. | | | | | | | | | | | | | | |
| * | 1.0. | | | | | | | | | | | | | | |
| * | 1.2 | | | | | | | | | | | | | | |
| * | 1.2 | | | | | | | | | | | | | | |
| * | .65 | | | | | | | | | | | | | | |
| * | .50 | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | |
| 0. | | | | | | | | | | | | | | | |
| 1.12.0 | 9.5.3 | 9.2.0 | 1.0.2 | 2 | 5.5.0.0 | 3.6.8.1 | 2.3.0.1 | 1.3.7.2 | 8.8.0 | 8.2.0 | 9.0.5 | 1.0.4.8 | 1.1.0.6 | 1 | 1 |
| 6.0 | 5.7 | 5.4 | 6.0 | | 1.6.0 | 1.3.0 | 1.0.0 | 1.7.8 | 5.4 | 5.1 | 5.4 | 6.0 | 6.0 | | |
| 1.0 | 0 | | | | 1. | 1. | 1. | 1. | | | | | | | |

ZN=2.24MEV Q-MAX

1969-10-20 Y. TOMITA

MASS OF TARGET NUCLEUS..... 65.2890
 MASS NUMBER..... 65
 ENERGY IN LAB. SYSTEM..... 2.2400 MEV
 ENERGY IN C.M. SYSTEM..... 2.2059 MEV

| | GROUND | 1ST | 2ND | 3RD |
|------|--------|-------|-------|-------|
| E | 0.000 | 1.020 | 1.835 | 1.900 |
| SPIN | 0 + | 2 + | 2 + | 0 + |

INITIAL POTENTIAL PARAMETERS

| POWERS OF E | | 0 | 1 | 2 | 3 | 4 | 5 |
|--|--|--------|--------|--------|--------|--------|--------|
| * V | | 50.000 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| * W | | 10.000 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| VSO | | 10.000 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| RR | | 1.200 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| RS | | 1.200 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| * A | | 0.650 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| * B | | 0.500 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| *'S INDICATE THE PARAMETERS TO BE SEARCHED FOR | | | | | | | |

*'S INDICATE THE PARAMETERS TO BE SEARCHED FOR

EXPERIMENTAL CROSS SECTIONS (AND ERRORS)

1ST LEVEL CROSS SECTION... 0.690(0.013) BARN WEIGHT..... 1.000

| ELASTIC ANGULAR DISTRIBUTION (MILLIBARN/STERAD.) | | | | WEIGHT..... 1.000 | | | | | | | |
|--|---------------|-------|---------------|-------------------|---------------|-------|---------------|------|-------|-------|---------|
| ANGLE | CROSS SECTION | ANGLE | CROSS SECTION | ANGLE | CROSS SECTION | ANGLE | CROSS SECTION | | | | |
| 0.0 | -0.0 | -0.0 | 10.0 | -0.0 | -0.0 | 20.0 | -0.0 | -0.0 | 30.0 | 550.0 | (16.0) |
| 40.0 | 368.1 | 13.0 | 50.0 | 230.1 | 10.0 | 60.0 | 137.2 | 7.8 | 70.0 | 88.0 | (5.4) |
| 80.0 | 82.0 | 5.1 | 90.0 | 90.5 | 5.4 | 100.0 | 104.8 | 6.0 | 110.0 | 110.6 | (6.0) |
| 120.0 | 112.0 | 6.3 | 130.0 | 95.3 | 5.7 | 140.0 | 92.0 | 5.4 | 150.0 | 102.2 | (6.0) |
| 160.0 | -0.0 | -0.0 | 170.0 | -0.0 | -0.0 | 180.0 | -0.0 | -0.0 | | | |

MQ UNDERFLOW AT 060067

Zn-2.24MeV Q-MAX

1969-10-20 Y.TOMITA

PARAMETER SEARCH

ZN=2.24MEV Q-MAX

1969-10-20 Y.TOMITA

FINAL POTENTIAL PARAMETERS

| | 0 | 1 | 2 | 3 | 4 | 5 |
|-----|--------|--------|--------|--------|--------|--------|
| * V | 54.801 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| * W | 9.299 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| VSO | 10.000 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| HR | 1.200 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| RS | 1.200 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| * A | 0.596 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |
| * R | 0.595 | -0.000 | -0.000 | -0.000 | -0.000 | -0.000 |

*'S INDICATE THE PARAMETERS TO BE SEARCHED FOR

TOTAL CROSS SECTION..... 3.912 BARN
 ABSORPTION CROSS SECTION... 1.863 BARN
 SHAPE ELASTIC CROSS SECTION.. 1.449 BARN
 CHI-SQUARE VALUE..... 0.609

TRANSMISSION COEFFICIENTS

| LEVEL | GROUND | 1ST | 2ND | 3RD |
|---------------|------------|------------|------------|------------|
| ECM (EXIT) | 1.206 | 1.186 | 0.371 | 0.306 |
| MESH | 0.250 | 0.250 | 0.250 | 0.250 |
| MATCH. RADIUS | 14.979 | 15.296 | 15.991 | 16.441 |
| L L-S | | | | |
| 0 + | 7.2579F-01 | 6.7528E-01 | 5.1629E-01 | 4.8683E-01 |
| 1 - | 6.3922E-01 | 4.4889E-01 | 1.3751E-01 | 1.2459E-01 |
| 1 + | 7.8605E-01 | 5.8511E-01 | 2.1475E-01 | 1.7277E-01 |
| 2 - | 5.5007E-01 | 2.9226E-01 | 3.1704E-02 | 2.0422E-02 |
| 2 + | 3.4701E-01 | 1.6961E-01 | 1.9508E-02 | 1.2748E-02 |
| 3 - | 8.5040E-02 | 1.2634E-02 | 2.6281E-04 | 1.3400E-04 |
| 3 + | 2.0112E-01 | 2.7640E-02 | 5.1739E-04 | 2.6552E-04 |
| 4 - | 9.7385E-03 | 7.5084E-04 | 0. | 0. |
| 4 + | 7.0109E-03 | 5.5110E-04 | 0. | 0. |
| 5 - | 3.0413E-04 | 1.2353E-05 | 0. | 0. |
| 5 + | 6.7815E-04 | 2.6867E-05 | 0. | 0. |
| 6 - | 1.0997E-05 | 0. | 0. | 0. |
| 6 + | 1.5602E-05 | 0. | 0. | 0. |

ZN=2.24MEV Q-MAX

1969-10-20 Y.TOMITA

S-MATRIX ELEMENTS FOR THE ELASTIC CHANNELS

| L | L-S - | | L-S + | |
|---|-------------|----------------|--------------|----------------|
| | REAL PART | IMAGINARY PART | REAL PART | IMAGINARY PART |
| 0 | | | -2.18351E-01 | 4.75951E-01 |
| 1 | 3.19318E-01 | -5.08737E-01 | 3.17079E-01 | -3.36771E-01 |
| 2 | 5.84469E-01 | -3.29129E-01 | 7.22493E-01 | -3.62065E-01 |
| 3 | 9.56496E-01 | 8.69566E-03 | 8.93224E-01 | 3.20154E-02 |
| 4 | 9.93119E-01 | 1.76826E-04 | 9.96486E-01 | 2.08639E-03 |
| 5 | 9.99848E-01 | 1.01511E-04 | 9.99641E-01 | 1.73409E-04 |
| 6 | 9.99994E-01 | 4.33369E-06 | 9.99992E-01 | 1.34409E-05 |

VARIOUS STATISTICAL QUANTITIES

CALCULATION OF COMPOUND PROCESS WAS CARRIED OUT BY USING MOLDAUER'S THEORY

N = 1.000

BC = 1.000

REAL PART OF THE RELATIVE PHASE OF SQUARES OF REDUCED WIDTH AMPLITUDES BETWEEN DIFFERENT CHANNELS.... 1.000
DEGREE OF FREEDOM OF CHI-SQUARE DISTRIBUTION FOR SQUARES OF REDUCED WIDTH AMPLITUDE..... 1.000

| TOTAL SPIN | TOTAL STRENGTH | COMPOUND FORMATION | |
|------------|----------------|--------------------|-------|
| 1/2- | 1.8829 | 0.192 | 1.002 |
| 1/2+ | 2.0458 | 0.217 | 0.963 |
| 3/2- | 2.7752 | 0.471 | 0.822 |
| 3/2+ | 2.6063 | 0.330 | 0.851 |
| 5/2- | 1.7579 | 0.076 | 1.033 |
| 5/2+ | 2.3814 | 0.312 | 0.897 |
| 7/2- | 1.2413 | 0.241 | 1.189 |
| 7/2+ | 0.5782 | 0.012 | 1.483 |
| 9/2- | 0.0419 | 0.000 | 1.926 |
| 9/2+ | 0.2124 | 0.011 | 1.740 |
| 11/2- | 0.0293 | 0.001 | 1.945 |
| 11/2+ | 0.0013 | 0.000 | 1.996 |
| 13/2- | 0.0000 | 0.000 | 2.000 |
| 13/2+ | 0.0006 | 0.000 | 1.998 |

ZN=2.24MEV

Ω-MAX

1969-10-20 Y.TOMITA

GROUND 0.000 MEV. 0 +

| LAB. ANGLE | LAB. COSINE | C.M. ANGLE | C.M. COSINE | COMPOUND ELASTIC | SHAPE ELASTIC | TOTAL ELASTIC |
|---------------|----------------|---------------|----------------|---------------------|------------------|------------------|
| 0.00 | 1.000 | 0.00 | 1.000 | 135.10 | 852.59 | 987.69 |
| 10.00 | 0.985 | 10.15 | 0.984 | 120.40 | 800.28 | 928.68 |
| 20.00 | 0.940 | 20.30 | 0.936 | 111.45 | 660.24 | 771.69 |
| 30.00 | 0.866 | 30.44 | 0.862 | 91.37 | 475.11 | 566.48 |
| 40.00 | 0.766 | 40.57 | 0.760 | 74.36 | 294.34 | 368.70 |
| 50.00 | 0.643 | 50.68 | 0.634 | 62.56 | 155.05 | 217.61 |
| 60.00 | 0.500 | 60.77 | 0.488 | 54.77 | 72.00 | 126.77 |
| 70.00 | 0.342 | 70.83 | 0.326 | 49.18 | 39.08 | 88.25 |
| 80.00 | 0.174 | 80.87 | 0.159 | 45.38 | 38.12 | 83.50 |
| 90.00 | 0.000 | 90.89 | -0.015 | 44.15 | 49.10 | 93.25 |
| 100.00 | -0.174 | 100.87 | -0.189 | 45.88 | 57.26 | 103.14 |
| 110.00 | -0.342 | 110.83 | -0.356 | 49.99 | 55.88 | 105.87 |
| 120.00 | -0.500 | 120.77 | -0.512 | 55.78 | 45.45 | 101.23 |
| 130.00 | -0.643 | 130.68 | -0.654 | 63.87 | 30.70 | 94.57 |
| 140.00 | -0.766 | 140.57 | -0.772 | 76.01 | 17.09 | 93.10 |
| 150.00 | -0.866 | 150.44 | -0.870 | 93.07 | 8.11 | 101.18 |
| 160.00 | -0.940 | 160.30 | -0.941 | 112.63 | 4.20 | 116.83 |
| 170.00 | -0.985 | 170.15 | -0.985 | 128.78 | 3.49 | 132.27 |
| 180.00 | -1.000 | 180.00 | -1.000 | 135.10 | 3.60 | 138.70 |

INTEGRATED CROSS SECTION..... 0.805 1.449 2.254

1ST 1.020 MEV. 2 +

| LAB. ANGLE | LAB. COSINE | C.M. ANGLE | C.M. COSINE | CROSS SECTION | LAB. ANGLE | LAB. COSINE | C.M. ANGLE | C.M. COSINE | CROSS SECTION |
|---------------|----------------|---------------|----------------|------------------|---------------|----------------|---------------|----------------|------------------|
| 0.00 | 1.000 | 0.00 | 1.000 | 54.17 | 100.00 | -0.174 | 101.19 | -0.194 | 54.13 |
| 10.00 | 0.985 | 10.21 | 0.984 | 54.33 | 110.00 | -0.342 | 111.13 | -0.361 | 54.46 |
| 20.00 | 0.940 | 20.41 | 0.937 | 54.73 | 120.00 | -0.500 | 121.05 | -0.516 | 54.87 |
| 30.00 | 0.866 | 30.60 | 0.861 | 55.12 | 130.00 | -0.643 | 130.92 | -0.655 | 55.19 |
| 40.00 | 0.766 | 40.78 | 0.757 | 55.28 | 140.00 | -0.766 | 140.78 | -0.775 | 55.27 |
| 50.00 | 0.643 | 50.92 | 0.639 | 55.14 | 150.00 | -0.866 | 150.60 | -0.871 | 55.08 |
| 60.00 | 0.500 | 61.05 | 0.484 | 54.78 | 160.00 | -0.940 | 160.41 | -0.942 | 54.69 |
| 70.00 | 0.342 | 71.13 | 0.323 | 54.37 | 170.00 | -0.985 | 170.21 | -0.985 | 54.32 |
| 80.00 | 0.174 | 81.19 | 0.153 | 54.07 | 180.00 | -1.000 | 180.00 | -1.000 | 54.17 |
| 90.00 | 0.000 | 91.21 | -0.021 | 53.98 | | | | | |

INTEGRATED CROSS SECTION..... 0.687

ZN=2.24MEV Ω-MAX

1969-10-20 Y.TOMITA

2ND 1.835 MEV. 2 +

| LAB. ANGLE | LAB. COSINE | C.M. ANGLE | C.M. COSINE | CROSS SECTION | LAB. ANGLE | LAB. COSINE | C.M. ANGLE | C.M. COSINE | CROSS SECTION |
|---------------|----------------|---------------|----------------|------------------|---------------|----------------|---------------|----------------|------------------|
| 0.00 | 1.000 | 0.00 | 1.000 | 23.01 | 100.00 | -0.174 | 102.13 | -0.210 | 22.26 |
| 10.00 | 0.985 | 10.37 | 0.984 | 22.99 | 110.00 | -0.342 | 112.03 | -0.375 | 22.36 |
| 20.00 | 0.940 | 20.74 | 0.935 | 22.94 | 120.00 | -0.500 | 121.87 | -0.528 | 22.50 |
| 30.00 | 0.866 | 31.08 | 0.856 | 22.86 | 130.00 | -0.643 | 131.65 | -0.665 | 22.64 |
| 40.00 | 0.766 | 41.39 | 0.750 | 22.74 | 140.00 | -0.766 | 141.39 | -0.781 | 22.77 |
| 50.00 | 0.643 | 51.62 | 0.620 | 22.59 | 150.00 | -0.866 | 151.08 | -0.875 | 22.88 |
| 60.00 | 0.500 | 61.87 | 0.471 | 22.44 | 160.00 | -0.940 | 160.74 | -0.944 | 22.95 |
| 70.00 | 0.342 | 72.03 | 0.309 | 22.32 | 170.00 | -0.985 | 170.37 | -0.986 | 22.99 |
| 80.00 | 0.174 | 82.13 | 0.137 | 22.24 | 180.00 | -1.000 | 180.00 | -1.000 | 23.01 |
| 90.00 | 0.000 | 92.16 | -0.038 | 22.22 | | | | | |

INTEGRATED CROSS SECTION..... 0.283

3RD 1.900 MEV. 0 +

| LAB. ANGLE | LAB. COSINE | C.M. ANGLE | C.M. COSINE | CROSS SECTION | LAB. ANGLE | LAB. COSINE | C.M. ANGLE | C.M. COSINE | CROSS SECTION |
|---------------|----------------|---------------|----------------|------------------|---------------|----------------|---------------|----------------|------------------|
| 0.00 | 1.000 | 0.00 | 1.000 | 9.83 | 100.00 | -0.174 | 102.34 | -0.214 | 5.93 |
| 10.00 | 0.985 | 10.41 | 0.984 | 9.68 | 110.00 | -0.342 | 112.23 | -0.378 | 6.28 |
| 20.00 | 0.940 | 20.81 | 0.935 | 9.25 | 120.00 | -0.500 | 122.06 | -0.531 | 6.79 |
| 30.00 | 0.866 | 31.19 | 0.855 | 8.62 | 130.00 | -0.643 | 131.82 | -0.667 | 7.43 |
| 40.00 | 0.766 | 41.53 | 0.749 | 7.90 | 140.00 | -0.766 | 141.53 | -0.783 | 8.11 |
| 50.00 | 0.643 | 51.82 | 0.618 | 7.18 | 150.00 | -0.866 | 151.19 | -0.876 | 8.78 |
| 60.00 | 0.500 | 62.06 | 0.469 | 6.56 | 160.00 | -0.940 | 160.81 | -0.944 | 9.33 |
| 70.00 | 0.342 | 72.23 | 0.305 | 6.10 | 170.00 | -0.985 | 170.41 | -0.986 | 9.70 |
| 80.00 | 0.174 | 82.34 | 0.133 | 5.83 | 180.00 | -1.000 | 180.00 | -1.000 | 9.83 |
| 90.00 | 0.000 | 92.38 | -0.041 | 5.78 | | | | | |

INTEGRATED CROSS SECTION..... 0.089

TIME REQUIRED 176.5 SEC.

END-OF-DATA ENCOUNTERED ON SYSTEM INPUT FILE.

Appendix B Symbolic listing of STAX 2


```

LINE=LINE+1
IF (LINE>LE-50) GO TO 1698
WHITE (6+600) TDN
LINE=N7
1698 WHITE (6+1700) EX58
1700 FORMAT (9X,39H EXPECTED VALUE OF CHI-SQUARE .....,F12.1)
C   CALL XSECT(NLO,NLD,PARA1,S0C,1+2)
1910 NG0=2
1920 MIN=2
IF (S0C.LE.S0CH10) MIN=1
X=MIN-1
IDS=10
GO TO 700
C
2010 IF (MIN.E0.1) GO TO 1010
2020 GO TO (2050,2080),MIN
2050 S0=S0C
S02=S0CH10
2060 G=0.5
GO TO 2100
2080 S0=S0CH10
S02=S0C
G=0.5
2100 LT=MN=1
DO 2200 N=1+NPARA
 1-IDPARC)
2110 PARA1(1)=PARA1(1)-0.5*DRP(N)
CALL XSECT(NLO,NLD,PARA1+S0C,1+2)
NG0=3
MIN=2
IF (S0C.LE.S0) MIN=1
X=MIN-1
IDS=3
GO TO 700
2150 IF (MIN.E0.1) GO TO 1010
C
2160 S01=S0C
A=0.2*(S02+S0U)-S01
B=0.5*(S02-S0U)
H=G
IF (S0C.LE.A) H=(1.-0.5*B)*G
DO 2200 N=1+NPARA
 1-IDPARC)
2200 PARA1(1)=PARA1(1)-(G-H)*DRP(N)
CALL XSECT(NLO,NLD,PARA1+S0C,1+2)
IF (ITEST.E0.0) GO TO 2270
LINE=LINE+1
IF (LINE>LE-50) GO TO 2250
WHITE (6+600) TDN
GO TO 700
2250 WRITE (*,2260) A+B+H
2260 FORMAT (9X,2HA+1PE11.3,5X+2HB=(E11.3+5X+2HB=E11.3)
2270 NG0=4
IDS=2
IF (A-E0.0) IDS=1
MIN=2
IF (S0C.LE.GH) MN=1
X=MIN-1
IDS=1
GO TO 2300
2300 IF (MIN.E0.1) GO TO 2350
GO TO 1010
C
2350 S0C=MN*(S00+S01)
2360 DO 2370 N=1+NPARA
 1-IDPARC)
2370 PARA1(1)=PARA1(1)-X*DPR(N)
C
4000 IF ((S0C.LE.-CHIMIN)+OR-(S0CH10=S0C.LE.CHIMIN+.01)) RETURN
  CALL CLOCK(1)
  IF (T1>E0.10) RETURN
4010 CONTINUE
C
4100 IF (NEND.E0.1) RETURN
  NEND=1
  GO TO (4210+*10),IGO
4210 IGO=2
  MN=ABS(NSRCH)
  GO TO 1130
4310 IGO=1
  MN=ABS(NSRCH0)
  GO TO 1130
C
END
EIGENVALUE
SUBROUTINE EIGEN(M+A,D+U+F+NDIM)
C
A..... MATRIX TO BE DIAGONALIZED (SYMMETRIC)
M..... ACTUAL DIMENSION OF THE MATRIX
D..... DIAGONAL MATRIX (COMPUTED ELEMENTS ARE EIGENVALUES)
U(N,:)..... ORTHONORMAL MATRIX OF TRANSFORMATION (U*DU)
U(N,:)'..... N-TH COMPONENT OF THE N-TH EIGENVECTOR
C..... UPPER LIMIT FOR THE OFF-DIAGONAL ELEMENTS
NDIM..... MAXIMUM DIMENSION TAKEN IN THE MAIN PROGRAM
C
DIMENSION A(NDIM,NDIM),D(NDIM,NDIM),U(NDIM,NDIM)
C
G=0.
MN=M-1
DO 1010 I=1,M
DO 1010 J=1,M
D(I,J)=A(I,J)
U(I,J)=0.
IF (I.EQ.J) U(I,J)=1.
1010 CONTINUE
IF (M.E0.1) GO TO 3000
C
2000 G=0.
DO 2010 I=1,M1
 1=I+1
  DO 2010 J=1,I
    G1=ABS(S0(I,J))
    IF (G.GE.G1) GO TO 2010
    K=1
    L=J
    GRS1
2010 CONTINUE
IF (G.LE.F) GO TO 3000
C
  DD=(L+1)-D(K,K)
  F=(2.*G1*ARS(DD)) GO TO 2110
  T=2.*D(K,L)/DD
  C=-T*SRT(L1.+T**2)
  GO TO 2120
2110 T=DD/(2.*D(K,L))
  C=SRT(L1.+T**2)*SIGN(1.,T)
  2120 P=1./SRT(1.+C**2)
  *C*P
C
  DO 2210 I=1,M
  IF (I.EQ.K) OR(1,E0.1) GO TO 2310
  DK1=-P*D(K,I)+P*D(L,I)
  DL1=-P*D(K,I)+P*D(L,I)
  D(K,I)=DK1
  D(L,I)=DL1
  D(L,I)=DL1
  D(L,I)=0.
2310 CONTINUE
  DKK=D(K,L)*D(K,L)*C
  DLL=D(L,L)*D(K,L)*C
  D(K,K)=DKK
  DLL=L1*L1
  D(K,L)=0.
  D(L,K)=0.
  D(L,L)=0.
  D(L,K)=0.
  G=0.
C
  GO TO 2000
C
3000 F=G
  RETURN
END
C
CROSS SECTION
C
SUBROUTINE XSECT(NLO+NLD+PAR+S0K+IX+IFC)
C
  DIMENSION BL(25,11),V(25),VS(25),RR(25),RS(25)+A(25),
  1 B(25)+PAR(7),PL(11,19)+PL(11,19)+PL(2,11,19)+PL(2,11,19)+3,
  3 TRMAT(25)+RPWR(25)
C
  COMPLEX WR,PHAIR(300)+DPHAI+DL,YETA(11,2)+YETAN+HI+HR
  X 7
  X 8
  X 9
COMMON /CLEV/NLEV,EX(25)+SPIN(25)+NPAR(25)+NLMAX(25)+DJ+NLMAX
  X 2
  X 3
  X 4
  X 5
  X 6
  X 7
  X 8
  X 9

```

```

CALL NOVEL(N)
IF ((NOVFL,=0,1) WRITE (6,98767)
98766 FORMAT (////5x,15H*****)(SECT 6//)
3610 CALL BLL(NL0,NU,L,N,J,M,L)
C
DO 3670 N=1,NLOUT
  K=M0+N-3
  SGN=(PLAM1#NL*(N-1)+COMP
  IF (N,GT,NL) GO TO 3670
  NL2=M0+NLMAX(N)+NL1)
  NL1=1
  IF ((N,LE,3) GO TO 3620
  IF (NL2,LT,MAX(N)) GO TO 3640
  NL1=NLMAX(N)+1
  IF ((N,LE,2) NLMAX(2)*NL2
  3620 CALL LEGND2(NL1,NL2,NANGL,PL2,COSCM,N,[IFFL)
  CALL OVERFL(NOVFL)
  IF ((NOVFL,=0,1) WRITE (6,98707)
  3640 DO 3650 NA=1,NANGL
    SGA(NA,N)=0
    DO 3650 NL=1,NL2
      SGA(NA,N)=SGA(NA,N)+BL(N,NA)*PL2(NL,NA+K)
    3650 SGA(NA,N)=SGA(NA,N)+PLAM#COMP
    3670 CONTINUE
C
  3700 IF ((NLMAX(1)-LE,NLMAX(1)) GO TO 3701
    CALL LFON1(NLMAX(1)-1,NA1,MAX(1),NA1L,PL,PL1,COSH)
    CALL OVERFL(NOVFL)
    IF ((NOVFL,=0,1) WRITE (6,98708)
    3701 FORMAT (////5x,15H*****)(SECT 8//)
    3701 CALL LFON1(NLMAX(1)-1,NA1L,PL,PL1,PLAM,PLAM0)
    IF ((NOVFL,=0,1) WRITE (6,98709)
    3701 FORMAT (////5x,15H*****)(SECT 9//)
    3710 DO 3720 NA=1,NANGL
      IF ((COMP,=F0,0)) SGA(NA,1)=0
      SEL=SEL#SELAC(NA)+SGA(NA,1)
      IF ((COMP,=F0,0)) SG(1)=0
      SEL=SEL+SG(1)
      STOT=SARS+SSEL
C
      IF ((IPEM,=E0,0) RETURN
      S#WGT0
      IF ((WGT0,=F0,0)) GO TO 3610
      S#R#WGT0=((STOT-STOT)/EROT)*#2
  3810 IF ((WGL,=F0,0)) GO TO 3830
      S#R=S#R+WGL*((SEL#SELAC)+(EREL)*#2
  3820 IF ((WGL,=F0,0)) GO TO 3830
      S#R=S#R+WG1*((SG(2)-S1TEX)/ER1ST)*#2
  3830 IF ((WGL,=F0,0)) GO TO 3830
      S#R=S#R+WG2*((SG(3)-S2TEX)/ER2ST)*#2
      DO 3840 NA=1,NANGL
      IF ((SELAFX(NA)-LE,0)) GO TO 3840
      S#R=S#R+((SELAC(NA)-SELAFX(NA))/FRA(NA))*#2
  3840 CONTINUE
  3850 IF ((WGIA,=E,0,0)) GO TO 3870
    S#R=S#R+WGIA*S#A
    DO 3860 NA=1,NANGL
      IF ((S1AFX(NA)-LE,0)) GO TO 3860
      S#R=S#R+((SG(NA-2)-S1AFX(NA))/FRA(NA))*#2
  3860 CONTINUE
  3870 S#R=S#R+WF1*W#A
  3870 IF ((NOVFL,=0,1) WRITE (6,98710)
  3871 FORMAT (////5x,15H*****)(SECT 10//)
  4100 RETURN
C
  6010 RATS=SARS/SARST
  6010 SEL=SARS#S1TEX#RATS
  6010 SGA(NA,1)=SCFELT(NA)*RATS
  6030 IF ((WGIA,=F0,0)) GO TO 3700
  6030 DO 6040 NA=1,NANGL
  6040 SGA(NA,2)=SG#AT(NA)*RATS
  6040 GO TO 3700
C
  END
C
  SOLUTION OF SCHROEDINGER EQUATION
C
  SUBROUTINE DIFEW(L,LS,PHAIR)
C
  COMPLEX PHAIR(300),WR,P(8),UO(B),FO(300),F(300),(C,B)
C
  COMMON /CWEL/WR,VR,VSDR,RHRS,AR,B,EH,DR,MN,RPWR,RPWR1
C
  COMMON /COUT/P
  DIMENSION FSO(300),FR(10),G(8),FR93,H(8)
  DOUBLE PRECISION FR+G(FR)E,VW+EW+EA+EB+DE1+DE2
C
  CALL OVERFL(NOVFL)
  IF ((NOVFL,=0,1) WRITE (6,98765)
  98765 FORMAT (////5x,15H*****)(FE0 1//)
  GO TO (1010,1020),LS
1010  F=1
  RST1=RST1#RPWR1
  RST2=RST2#RPWR
  FZL2=FZL2#2
  GO TO 1030
1020  FACT=L
  FLL=FACT*(FACT+1.)
  DR12=DR#P#Z/12.
  FACT=FACT#VSOR
C
  STARTING VALUES
  IF ((L,LE,0)) GO TO 1200
  FV=DEXP(DRL*(HR/AH))
  F#=D#PKV(DRL*(HR/AH))
  RST1=RPWR1
  RST2=RPWR
  FZL2=2.
  FR(1)=1./((1.+EV)
  FB(1)=1./((1.+EV)
  FN=1.
  DO 1120 N=2,9
  FN=FN*1.
  NI=N-1
  FR(N)=-(FR(N-1)
  G(N)=FB(N)
  DO 1110 M=1,NI
  NM=N-M
  FN=M
  FR(N)=FR(N)+FR(M)*FR(NM)
  1110 G(N)=G(N)+FB(M)*FR(NM)
  FR(N)=FR(N)/(AH*FN)
  1120 FB(N)=G(N)/(M*FN)
  FR(10)=-(FR(10)+FR(M)*FR(M))
  DO 1130 M=1,9
  FR(10)=FR(10)/(9.+#AR)
  FN=1.
  DO 1140 N=1,8
  FN=FN*1.
  C#N=(1-NGL*(FR(N)))*WH#SNGL(G(N))
  1140 CALL OVERFL(NOVFL)
  IF ((NOVFL,=0,1) WRITE (6,98766)
  98766 FORMAT (////5x,15H*****)(FE0 2//)
  C1=C1)*#R
  1200 RES1D=FACT#FR(2)*#AR
  DO 1210 N=1,8
  1210 UO(N)=C(N)*#C(N)*FACT
C
  P(1)=1
  P(2)=RES1D#FZL2
  FN=1
  DO 1320 N=3,8
  P(N)=RES1D#P(N-1)
  FN=FN*1.
  N2=N#2
  DO 1310 M=1,N2
  NM=M-1
  1310 P(N)=P(N)-P(M)*UO(NM)
  1320 P(N)=P(N)/C(FN*(FZL2+FN-1))
C
  PHAIR(1)=P(1)*#RST1
  PHAIR(2)=P(1)*#RST2
  ST1=RST1
  ST2=RST2
  DO 1330 N=2,8
  ST1=ST1#RPWR1
  ST2=ST2#RPWR1
  END

```

```

/F (L1+GT-1) GO TO 8
PL(1)=1
PL(2)=N
PL(3)=N
8 L1=L2-2, GO TO 11
L3=MNUC(L-1)
DO 10 NL=L, L2
10 PL(NL,N)=(FL23(NL)*X+PI(NL-1,N)-FL2(NL)*PL(NL-2,N))/FL1(NL)
11 CONTINUE
DO 20 NL=1,NANGL
  X=COSIN(N)
  Y=SINT(N)
  WO,
  IF (Y-NE.0.) ==1./Y
  DO 20 NL=1,L2
20 PL1(NL,N)= FL1(NL)*(PL(NL-1,N)*X+PL(NL,N))*W
  END
C LFFNDRE FUNCTION FOR COMPOUND PROCFS
SUBROUTINE LEGND2(L1+2,NANGL,PL,COSCM,N,IFFL)
DIMENSION PL(L1+19,3),COSCM(19,2),PI(19,3),
1   FL47(11),FL24(11),F(22(11),FL23(11))
K=MIND(N,3)
IF (IFFL,GE,1) GO TO 110
L3=IFFL-LW+2
IF (IFFL-LW+2) FLW=
DO 100 LW=L, L2
FL22(L)=FL
FL23(L)=FL-2,
FL24(L)=FL-1,
FL45(L)=FL23(L)+FL22(L)
FL47(L)=FL45(L)+2
100 IF (FLW==L2) GO TO 110
110 DO 1010 NA1=NANGL
  NA1=NA1+NANGL
  IF (NA1-NE.1) GO TO 200
  PL(1,NA1)=1,
  P1(NA1)=0,
200 IF (L2-E0,1) GO TO 1010
  T=MAX(0,2,L1)
  DO 1010 J=L1+1,L2
    PI(NA1)=(FL47(L)*X+PL(L-1,NA1)-F(2A(L)*PI(NA1))/FL23(L)
    PL(L,NA1)=(FL45(L) *X+PI(NA1)-FL23(L))/FL22(L)
1000 CONTINUE
1010 CONTINUE
  RETURN
END
C CALCULATE GAMMA/D FROM T
C SUBROUTINE TRANS(1MAGRY,NJ,NNNNNN)
COMMON /CLEV/NLEV,EX(25),SPIN(25),NPAR(25),NLMAX(25)+DJ+NLMAXA
1  /CSTA/T(1+2+25),TH(1+2+25),THDT(21+2)*B(21+2)+BCONS,
2  /CONST,FNR,RC,REHA,FEAT,COMP
3  /CETC/MLLO(2+2,25),NLUPR(2+2+25),FJ+NPAR
IMAGRY=0
IF (FK1,NE.0.) C=2,BBC/FNR
FJ=FLOAT(NL)-Dj
DO 2300 NOPAR=1,2
NJPAR=2+NPAPR-3
THTOT(NJ,NOPAR)=0
DO 2400 N=1,NLEV
  NLMIN=1
  IF (NJPAR,NE,NOPAR(N)) NLMIN=2
  DO 1120 J=1,2
    FLS=2.6*FLOAT(J)
    NL1=ABS(FJ-SPIN(N))+FLS
    NLLOW=(NL1+1)+(NL1-MOD(NI+1,NL1+2))
    NLH=SPIN(N)+FLS
    NLUUU=MIND(NL2,NLMAX(N))
    NLUPR(NOPAR,J)=NLUUU-NLUU-MOD(NLUU+NL1+2)
    IF (NLUU(NOPAR,J),LE,NLUU(NOPAR,J)) GO TO 1010
    NLUU(NOPAR,J)=0
    GO TO 1120
  1120 NL=NL1,NL2,2
    THTOT(NJ,NOPAR)=HTTOT(NJ,NOPAR)+T(NL,J+N)
  1110 TH(NL,J+N)+T(NL,J+N)
  1120 CONTINUE
  IF (FK1,NE.0.) GO TO 2300
  IF (FK1,NE.0.) GO TO 2130
  NPAPCON=0
  GO TO 2142
2130 IF(J<1,-PHAI(HTTOT(NJ,NOPAR)/(2.*FNR)))
  2142 SUM=0.
  D#0=0
  INDEX=0
  I#0=0
  DO 2200 N=1,NLEV
    DO 2200 J=1,2
      NL=MNUC(NOPAR,J)
      IF (NL2,EB,0) GO TO 2200
      NL=MNUC(NOPAR,J)
      DO 2200 NI=1,NL2,2
        DT=1.-B(NI,J)
        TO=TH(NL,J)
        TO=TH(NL,J)
  2160 IF (DT,GT,0,0) GO TO 2170
        IMNUJ=IMNUJ+1
        #JP=0.999*UP/(JL-DT)
        DT=0.999*UP
        GO TO 2170
  2170 SUM=0.
  2171 IF (ABS(CTN-TO)>GT,1.E-6,AND,1+BCONS,E#0) INDEX=1
    TH(NL,J)=TN
    SUM=SUM+TN
    D#0#0=TN#2/S#DT
  2190 CONTINUE
  2200 CONTINUE
  IF ((BCONS,E#0,1.0N-1+INDEX,E#0,0) GO TO 2250
  S#1=SUM/(2.*FNR)
  PH1=PHAI(S#1)

  D#H=100.*(PHAI(S#1+0.01)-PHI)
  #N=abs(D#H)
  #P=JP/(1.+D#H*D#H*C(8,*FNR))
  GO TO 2142
  2250 #MAGRy=IMAGRy+IMNUJ
  @NJ,NOPAR)=#JP
  2290 HTTOT(NJ,NOPAR)=SUM
  2300 CONTINUE
C   RETURN
END
C PHAI0
FUNCTION PHAI(Y)
IF (Y,LE,0.01) GO TO 100
  Y-L-E-0.01)*EXP(-2.*Y)*.5
  PHAI=1.-X*(1.-X*Z-E1(Y)*(1.-Z-X*Z))
  RETURN
100 IF (Y,LE,0.01) GO TO 300
  A=-1.000617
  B=E1(Y)*1.000617
  V#2=1-Y
  PHAI=0.0
  F#2=2,
  DO 200 I=1,6
    F1=F#2
    F#2=F#1+1,
    A=A/2,F#2
    R#=B*Y/2,F1
  200 PHAI=PHAI+A-B*(1.-#2./F#2)
  RETURN
  300 PHAI=0.
  RETURN
C   INTEGRAL OF EXP(-T)/T (T=X,INFINITY) MULTIPLIED BY EXP(X)
C   DIMENSION A(20)
COMMON /CINT/X(18),W(18),X(18),X(2)(8),XL1(8),XL2(8),XD2(8)
C   DATA A/-125.,-1111111,-8333333E-1,-5353333E-1,
1   ,2962430E-1,-14512472E-1,-6349209E-2,-2505242E-1
2   ,20299982E-3,-2881264E-3,-9120334E-4,-2509465E-1
3   ,E-A,-6878028E-5,-1712810E-5,-396881E-6,-87089E-7,E1
4   ,20424E-7,-4451E-8,416E-9,C/57721566/
C   IF (X,GF,A,) GO TO 3000
  E1=0
  X#1=1,
  X#2=X#1,
  DO 2010 N=1,20
    X#1=X#1+N
    E1#1=X#1A(N)
    IF ((ABS(CIN),LT,1,E-16) GO TO 2020
  2010 E1=E1+E1

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C      COMMON /CINT/T(10),X(10),XH(10),XL1(8),XL2(8),XN1(8),XN2(8)      S   60
1      /LEV/NLEV,F(10),NP(NP),NLMAX(NLMAX),NLMAX(255),DJ,NLMAXA      S   70
2      /ESTA/T(11:2,25),TH(11:2,25),TH(12:2,25),TH(12:2,1),ISCONS      S   80
3      *CONST*,NR=RCRMAX(FKA)*COMP      S   90
4      /CDEG/DEGU      S  100
5      /CFTC/NLLW(2+2,25),NLUPP(2+2,25),FJ,NOPAR      S  110
C      S  120
S  130
C      1000 SDO.
S  140
1  AL=TH(NL1,J(1))
1020 IF (AL<LT,1,E-6) RETURN
ALPHA=A1
AZ=TH(NL1,FJ,NF)
1040 IF (AZ<LT,1,E-6) RETURN
ALPHA=ALPHA+A2
C      S  150
C      E=FKA*I+.5
I=0
DO 1142 N=1+NLEV
DO 1144 J=1+2
NL2=NLUPL(NOPAR,J,N)
IF (NL2>1000) GO TO 1143
NL1=NLLOW(NOPAR,J,N)
DU 1140 NL=NL1+NL2+2
AO=TH(NL,J,N)
I=(AO-LT,1,E-6) GO TO 1140
I=1+1
AL=AU
ALPHA=ALPHA+AO*FP
1140 CONTINUE
1141 CONTINUE
1142 CONTINUE
IMAX=1
1  IF (IMAX>FJ,0) RETURN
DEG=2*FLOAT(I+MAX)*FP
ALPHA+ALPHA*DEG
DEG*FGO/(DEG-1.)
A1=A1/ALPHA
A2=A2/ALPHA
DO 1150 I=1,IMAX
1150 A(I)=A(I)/ALPHA
C      S  160
C      DO 2020 K=1,8
X1=X(K)*+DFG
X2=X(K)*+DEG
DS1*X01(K)           /((1,-A1)*X1+A1)*((1,-A2)*X1+A2)
DS2*X02(K)           /((1,-A1)*X2+A1)*((1,-A2)*X2+A2)
DO 2020 I=1,1+IMAX
DS1=DS1/(CL,-A1)) *X1+A1)*+EP
DS2=DS2/(CL,-A1)) *X2+A1)*+EP
2020 S=W(JK)*(DS1-DS2)/2,+S
S=S*DEG*PWTHTOT(NJ,NOPAR)/ALPHA
IF (NF<0,1,AND,NL1,E0,NLF,AND,J1,E0,JF) S=S*(1.+2./FKA)
RETURN
END
C FACTORIAL
FAC = 2
C      SUBROUTINE  FACTML
COMMON /FCAC/FAC0,FAC(100),EFAC0,FFAC(100)
FAC0=1.0
FAC=1.0
EFAC=1.0
EFACN=1.0
DU 10 N=1,100
FACN=FACN*FLOAT(N)
1  IF (FACN.LE.3.16) GO TO 5
FACN=FACN/10.
EFACN=EFACN*10.
GO TO 5
5  FACN=FACN
10 EFAC(N)=EFACN
RETURN
END
C COUPLING CONDITIONS ARE ASSUMED TO BE SATISFIED
FUNCTION  DELTA(A+B+C)
COMMON /FCAC/FAC0,FAC(100),EFAC0,FFAC(100)
1  A=B=C+1
2=B+C-A+1
3=C+A-B+1
4=A+B+C+1
DELT=FAC(1)+FAC(2)+FAC(3)-FFAC(4)
RETURN
END
C      FUNCTION  EXDFLT(A+B+C)
C      COMMON /FCAC/FAC0,FAC(100),EFAC0,FFAC(100)
C      1=A+B+C+1
1  2=B+C-A+1
3=C+A-B+1
4=A+B+C+1
EXDEL=FAC(1)+FAC(2)+FAC(3)-FFAC(4)
EXDEL=EXDEL/2.
RETURN
END
C      CLERASCH-GORDAN COEFFICIENT
C COUPLING CONDITIONS ARE ASSUMED TO BE SATISFIED
FUNCTION  CG0(A,B,C)
C      CLEM 20
CLEM=20
C      OVERFL(NOVFL)
IF (NOVFL,F0,1) WRITE (6,98765)
98765 FORMAT (////5X,15H*****CG0      1///)
D*(A+B+C)/2.
CG0=FLOAT((-1)**INT(D-C+1))
1  *SQRT((C2,-C+1)*DELT(A+B+C))/(D+1)*DELT(A/2.,B/2.,C/2.)      CLEM 140
2  *1.**(-1)*(EXDEL(A/2.,B/2.-2.+EXDEL(A/2.,B/2.+C/2.))      CLEM 150
CALL  OVERFL(NOVFL)
IF (NOVFL,F0,1) WRITE (6,98765)
98766 FORMAT (////5X,15H*****CG0      2///)
RETURN
END
C      RACAH COEFFICIENT
RAC = 20
C COUPLING CONDITIONS ARE ASSUMED TO BE SATISFIED
FUNCTION  WRAC(A,B,C,D,E,F)
C      RAC 30
RAC = 30
COMMON /FCAC/FAC0,FAC(100),EFAC0,FFAC(100)
C      CALL  OVERFL(NOVFL)
IF (NOVFL,F0,1) WRITE (6,98765)
98765 FORMAT (////5X,15H*****WRAC      1///)
W1=DELT(A,B,E)*DELT(C,D,F)*DELT(A,C,F)*DELT(B,D,F)
1  =A+B+E+1.1
2=B+C+F+1.1
3=C+A-E+1.1
4=A+B+F+1.1
J1=A+B+C+1.1
J2=A+C+E+1.1
J3=B+C+F+1.1
NL=NOFL(1,12+13,14)
N2=NOFL(J1,J2+J3)
W2=0.
FW=EXDEL(A,B+E)+EXDEL(C,D,E)+EXDFLT(A,C,F)+EXDFLT(B,D,F)
DU 1000 N=1,N2
K1=N+11
K2=N+12
K3=N+13
K4=N+14
L1=J1-N
L2=J2-N
L3=J3-N
1000 W=FLOAT((-1)**L1)*FAC(N)/(FAC(L1)*FAC(K2)*FAC(K3)*FAC(K4)*
1  *FAC(L1)*FAC(L2)*FAC(L3))*10.**FW*FAC(N)-EFAC(K1)-FFAC(K2)-
2  *EFAC(K3)-EFAC(K4)-EFAC(L1)-EFAC(L2)-EFAC(L3)
WHAC=2*SQRT(C1)
CALL  OVERFL(NOVFL)
IF (NOVFL,F0,1) WRITE (6,98766)
98766 FORMAT (////5X,15H*****WHAC      2///)
RETURN
END

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