

Calculation of Survival Rates and
Effective Inactivation Cross
Sections of Targets for
Fast Neutrons Based on
Microdose Concept

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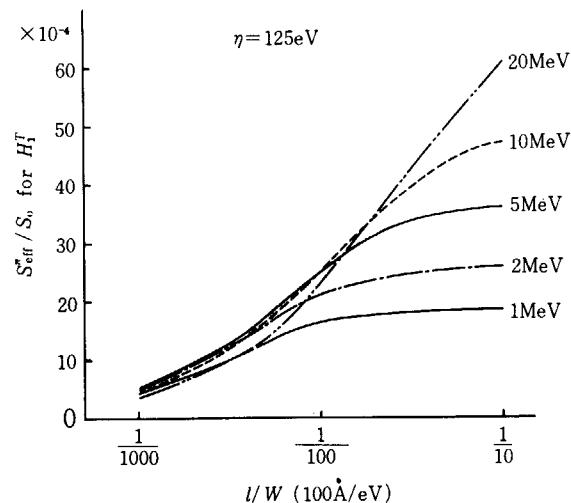
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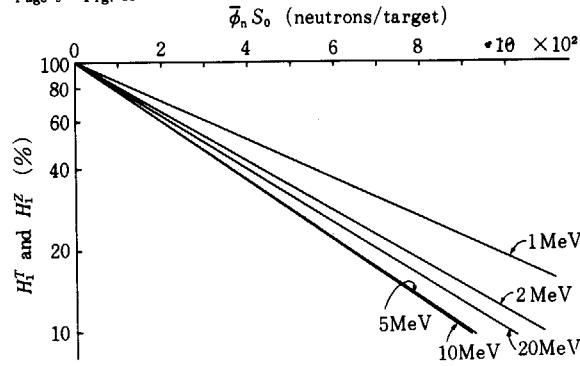
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Page 8 Fig. 8



Page 9 Fig. 11



Calculation of Survival Rates and Effective Inactivation Cross Sections of Targets for Fast Neutrons based on Microdose Concept

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Abstract

The survival rates and the effective inactivation cross sections of biological targets for neutrons of energies 1~20 MeV were calculated according to the target theory based on the microdose concept, using an IBM-7044 computer. The survival rates were calculated with one-, two- and three-events models for different combinations of the following parameters: the incident energy of fast neutrons, the cut-off energy of delta-rays and the average thickness of targets. The effective inactivation cross sections of targets were derived from the survival rates for one-target model.

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微視的線量概念にもとづいた速中性子線に対する 標的不活性化有効断面積と生存率の計算

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要　旨

放射線の線量評価法を確立するため、線量一効果の関係を微視的線量概念にもとづいて計算により求めた。計算はエネルギーが1~20 MeVの速中性子線に対する標的不活性化有効断面積と生存率について、IBM-7044を用いて行なった。生存率はone, twoおよびthree-events modelの標的について、速中性子線の入射エネルギー E_n 、デルタ線のcut-offエネルギー η 、および標的の厚み ℓ をパラメータとし、それぞれの異なる組合せをした場合について求めた。標的不活性化有効断面積はone-target modelの生存率から求めた。

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1. Introduction

The absorbed dose, D (in a unit of rad), introduced by the ICRU¹⁾ in 1953 is used as one of the most fundamental physical quantities, describing the energy absorption of medium from ionizing radiations and defined as

$$D = \frac{\Delta E}{\Delta m},$$

where ΔE is the energy (in a unit of 100 ergs) actually deposited in a volume element of mass Δm (in a unit of gram) in question by ionizing particles. However, with respect to the (input-output) relation between the dose and the effect, it is well known that an extent of effects depends not only on the absorbed dose but also on the radiation quality. Therefore, in order to describe pertinently the relation between the dose and the effect, the quality factor, QF , and the relative biological effectiveness, RBE , are used as the modifying factors for the radiation protection and for the radiation biology, respectively. For neutrons, in order to take into account the effect of the radiation quality on the radiation effects, BOAG²⁾, along the line proposed by ZIRKLE³⁾, described the energy absorption when fast neutrons bombard water as the distribution of LET values for the recoil nuclei. In this method of description, there are two ways of plotting the distribution of LET : (1) the proportion of the total absorbed energy which is dissipated in each interval of LET and (2) the proportion of the total track length belonging to each interval of LET , that is, the LET distribution of the particle fluence (or flux) of recoil nuclei. In BOAG's description, δ -rays were not regarded to give rise to separate tracks, but included in the total loss of energy of the recoil protons. This approximation may lead to incorrect results for some cases which are usually encountered in the biological effects of radiations. BOAG also suggested as a future promising step the third kind of description of the energy dissipation in his paper²⁾ which is now called the microscopic description of the energy absorption, that is, the microdosimetric energy absorption based on the microdose concept. The dissipation of energy may be considered as series of discrete events, that is, a quantum theoretical discontinuous phenomena which is in essence of the statistical character, occurring in the vicinity of the tracks of the ionizing particles. On the other hand, concerning the medium absorbing the energy from ionizing particles, the biological system of special interest here has the socalled targets which correspond to the sensitive site such as a DNA, its composite constituents and so on, and the dimension of targets is usually very small compared to the track length of most ionizing particles but still of the finite size. Therefore, it is impossible to describe relevantly the relation between the input and the output, disregarding the correlation between the target size and the amount of energy transferred into the target volume and its statistical fluctuation. For this reason, the concept of microdose was introduced and theoretical as well as experimental studies have been developed since 1959^{4),5)}. This new field of dosimetry is called the microdosimetry.

In the previous papers^{6),7)}, the primary energy transfers, the effective inactivation cross sections, and the survival rates of targets for several kinds of heavy charged particles were studied.

In this paper, following the method of the previous ones, the survival rates and the effective inactivation cross sections of targets for fast neutrons are evaluated using the target theory based on the microdose concept, where the δ -rays are regarded as separate tracks and the statistical character of the energy deposition to the target is fully taken into consideration.

2. Method of Calculation

2.1 Mathematical Expressions of Survival Rates and Effective Inactivation Cross Sections of Targets Based on Microdose Concept

First, two kinds of fundamental quantities are introduced^{5),8)} for the microscopic description of the primary energy transfer into the target volume.

T ; the transferred energy in a target per track per target. T is independent of dose and dose-rate.

Z ; the transferred energy per target. Z depends on dose, contrary to T .

For simplicity, T and Z are expressed in a unit of W , that is an average energy necessary to produce a primary event which is usually regarded as a primary ionization event but here is not restricted to this event in order to include all possible activation modes such as the superexcitation, the dissociative electron capture and so on, as a primary event. Then, T and Z are expressed as $T=jW$ and $Z=JW$, where j stands for the number of events produced by a single track in a target and J the total number of events produced in a target by all tracks traversing a target, respectively.

The statistical structure of the primary energy transfer based on the T -distribution has been for the first time formulated in Ref.⁵⁾ and this formulation has been later on applied to the cases of the ^{60}Co gamma-ray¹⁴⁾ and the heavy particle⁶⁾ irradiations. The details of the derivation of formulae may be referred to the references cited above and only the final expressions of formulae used in this paper are recapitulated.

In a case of T -input, the survival rate, H_n^T , for one-target and n -events model which means that the threshold number of events for inactivation of target is n and the system has a single target, was given as follows;

$$H_n^T = \exp -\bar{\phi}_t S_0 \left(1 - \sum_{j=0}^{n-1} f_j \right), \quad (1)$$

$$\bar{\phi}_t = \sum_{\alpha} \int \phi_{\alpha}(E_{\alpha}) dE_{\alpha}, \quad (2)$$

$$\text{and } f_j = \sum_{\alpha} \int \phi_{\alpha}(E_{\alpha}) P(E_{\alpha}, j) dE_{\alpha} / \bar{\phi}_t. \quad (3)$$

The suffix α stands for the kind of charged particles traversing the target and therefore $\phi_{\alpha}(E_{\alpha})$ means the energy spectra of the particle fluence of the α -kind particles. $P(E_{\alpha}, j)$ is defined as a probability that a track with energy E_{α} of the α -kind particle provides the j primary events in a target. S_0 is the geometrical cross section of the target.

In the case of fast neutron irradiation, ionizing particles produced by neutrons in the water medium are mostly protons and δ -rays produced by protons. Relations among the total neutron fluence, $\bar{\phi}_n$, the total proton fluence, $\bar{\phi}_p$, and the total δ -ray fluence, $\bar{\phi}_{\delta}$, can be expressed as follows;

$$\bar{\phi}_t = \bar{\phi}_p + \bar{\phi}_{\delta}, \quad (4)$$

$$\bar{\phi}_p = A \bar{\phi}_n, \quad (5a)$$

$$\bar{\phi}_{\delta} = B \bar{\phi}_n, \quad (5b)$$

where A and B are conversion factors. Using eqs. (4), (5a) and (5b), eq. (1) can be rewritten as

$$H_n^T = \exp -\bar{\phi}_n (A + B) S_0 \left(1 - \sum_{j=0}^{n-1} f_j \right), \quad (6)$$

where f_j is expressed as a sum of the contributions of protons and δ -rays as follows;

$$f_j = \frac{1}{\bar{\phi}_t} \left[\int \phi_p(E_p) P(E_p, j) dE_p + \int \phi_\delta(E_\delta) P(E_\delta, j) dE_\delta \right]. \quad (7)$$

Then, the effective inactivation cross section of targets, S_{eff}^n , is expressed by

$$S_{\text{eff}}^n = S_0(A + B)(1 - \sum_{j=0}^{n-1} f_j). \quad (8)$$

On the other hand, the probability distribution function based on Z -distribution, h_J , which means that the probability of occurrence of J total events in a target by all tracks traversing a target is h_J , is given, using a generating function and its convolution, as follows;

$$h_J = \exp - \bar{\phi}_t S_0 (1 - f_0), \quad J=0 \quad (9)$$

$$= \frac{\bar{\phi}_t S_0}{J} \sum_{i=0}^{J-1} (J-i) f_{J-i} h_i, \quad J \geq 1. \quad (10)$$

Then, the survival rate for one-target and n -events model based on Z -distribution, H_n^Z , is given as

$$H_n^Z = \sum_{J=0}^{n-1} h_J. \quad (11)$$

In the case of Z -input, the survival curves are generally not of the exponential shape, and one cannot define the quantity, S_{eff} , except for one-event model.

2.2 Procedure of Calculation

The biological system is represented by the water medium as an approximated substitute following BOAG²⁾, and in the water or tissue about 90% of the total energy transferred by fast neutrons to nuclei and considerably more than 90% of the track length is accounted for by neutron-proton collisions^{9), 10)} and in the first place, we shall consider these alone. It is assumed that the relevant dimensions of the irradiated material may be small compared with the mean free paths of neutrons in it. Therefore, the practical calculations were carried out for only the first collisions of each neutron with a proton, corresponding to the first collision dose. Input parameters used are tabulated in TABLE 1.

In the case of fast neutron irradiation, as mentioned above, there are two kinds of ionizing particles, protons and δ -rays. In the energy region under consideration (1~20 MeV), it can be shown⁹⁾ that the initial energies, E_p , of recoil protons are uniformly distributed between zero and the energy of the incident neutron, E_n , that is,

$$P(E_p) dE_p = \frac{dE_p}{E_n}. \quad (12)$$

Then, the intergrated number of protons per unit volume $I_p(E_p, E_n)$, having the initial energies greater than E_p produced by neutron flux of energy E_n , $\phi_n(E_n)$, is given by

$$\begin{aligned} I_p(E_p, E_n) &= \phi_n(E_n) \sigma(E_n) N_H \int_{E_p}^{E_n} P(E_p) dE_p, \\ &= \phi_n(E_n) \sigma(E_n) N_H \left(1 - \frac{E_p}{E_n}\right), \end{aligned} \quad (13)$$

where $\sigma(E_n)$ is the total recoil proton production cross section of neutron of energy E_n and N_H is the number of hydrogen atoms per unit volume of the medium. Then, the slowing-down spectrum of protons, $\phi_p(E_p, E_n)$, in the medium is expressed by the following equation, based on the continuous slowing-down model of protons;

$$\phi_p(E_p, E_n) = \frac{I_p(E_p, E_n)}{L_t(E_p)}, \quad (14)$$

where $L_t(E_p)$ is the total LET of proton of energy E_p .

For $E_p \geq 0.5$ MeV, $L_i(E_p)$ is given by

$$L_i(E_p) = \frac{4\pi e^4 N_e}{mc^2 \beta_p^2} \left[\ln \frac{2mc^2 \beta_p^2}{(1 - \beta_p^2) I} - \beta_p^2 \right], \quad (15)$$

where N_e is the number of electrons per unit volume, I is the mean excitation potential of the medium, and other notations are the same as the usual. For $E_p < 0.5$ MeV, the theoretical formula of $L_i(E_p)$ given by eq. (15) cannot be used, because the Born approximation used to derive eq. (15) cannot be applied for protons of energies below 0.5 MeV in water medium, and so far no exact theoretical expression has not been given. Therefore experimental values^{12), 13)} were used instead of eq. (15). In the practical calculation of $\phi_p(E_p, E_n)$, was introduced a new quantity $\phi'_p(E_p, E_n)$, instead of $\phi_p(E_p, E_n)$, which is a kind of the normalized slowing-down spectrum of protons and is defined as follows in a unit of cm/MeV;

$$\phi'_p(E_p, E_n) = \phi_p(E_p, E_n) / N_e \int \phi_n(E) \sigma(E) \delta(E - E_n) dE, \quad (16)$$

where a delta function $\delta(E - E_n)$ is introduced in the integrant to reflect the calculation condition that incident neutrons are all of a definite energy E_n .

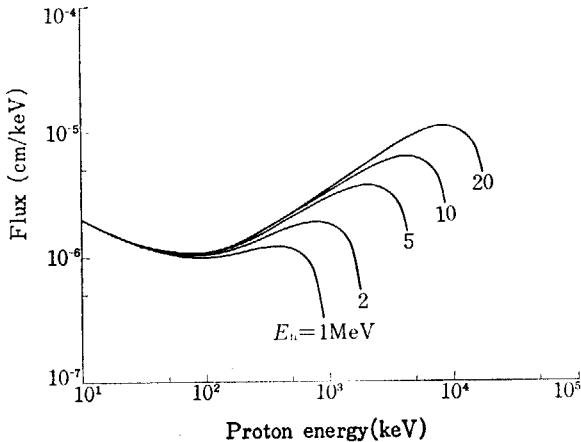


Fig. 1 The slowing-down spectrum of recoil protons $\phi'_p(E_p, E_n)$, produced by neutrons of energies $E_n = 1, 2, 5, 10$ and 20 MeV in the water medium. The ordinate is given in a unit of cm/keV.

The numerical values of $\phi'_p(E_p, E_n)$ are given in TABLE 2 and also graphically shown in Fig. 1. The number of electrons produced by protons, $\bar{\phi}_p(E_p, E_n) \Delta E_p$, having the initial energies greater than E_δ is given by

$$\begin{aligned} I_{\delta 0}(E_\delta, E_p, E_n) &= \int_{E_\delta}^{E_p \max} \frac{2\pi e^4 N_e}{mc^2 \beta^2} \bar{\phi}_p(E_p, E_n) \Delta E_p \cdot \frac{dE'_\delta}{E'_\delta \Delta E_p}, \\ &= \frac{2\pi e^4 N_e}{mc^2 \beta^2} \left[\frac{1}{E_\delta} - \frac{1}{E_p \max} \right] \bar{\phi}_p(E_p, E_n) \Delta E_p, \end{aligned} \quad (17)$$

where $\bar{\phi}_p(E_p, E_n)$ is the number of protons per cm^2 in the energy range from $(E_p - \frac{1}{2} \Delta E_p)$ to $(E_p + \frac{1}{2} \Delta E_p)^*$. Therefore, the total number of electrons, $I_{\delta 1}(E_\delta, E_n)$, having the initial energies greater than E_δ , produced by all of the protons having the slowing-down spectrum $\phi_p(E_p, E_n)$ is given by

$$I_{\delta 1}(E_\delta, E_n) = \sum_{E_p} I_{\delta 0}(E_\delta, E_p, E_n). \quad (18)$$

Then, the slowing-down spectrum of δ -rays, $\phi_\delta(E_\delta, \eta, E_n)$, in the water medium is given by

* See Appendix for the procedure of the numerical calculations of the slowing-down spectra and the LET.

the modified continuous slowing-down model¹⁴⁾ and expressed by

$$\phi_\delta(E_\delta, \eta, E_n) = L_t(E_\delta)^{-1} \left[I_{\delta 1}(E_\delta, E_n) + \int_{2E_1}^{E_\delta \max} \phi_\delta(E'_\delta, \eta, E_n) \tilde{\Sigma}(E'_\delta, E_\delta) dE'_\delta \right], \quad (19)$$

where $\tilde{\Sigma}(E'_\delta, E_\delta)$ is the integrated δ -ray production cross section and given by integrating Møller cross section¹⁴⁾, and η is the cut-off energy which corresponds to the lowest energy of δ -rays at a moment of their origination. $L_t(E_\delta)$ is the total *LET* of electrons and calculated according to the procedure described in Ref. (14). The practical calculations of $\phi_\delta(E_\delta, \eta, E_n)$ were carried out, in a similar way as for that of $\phi_p(E_p, E_n)$, introducing a new quantity $\phi'_\delta(E_\delta, \eta, E_n)$ as follows;

$$\phi'_\delta(E_\delta, \eta, E_n) = \phi_\delta(E_\delta, \eta, E_n) / N_H \int \phi_n(E) \sigma(E) \delta(E - E_n) dE. \quad (20)$$

In TABLE 3, the numerical values of $\phi'_\delta(E_\delta, \eta, E_n)$ are tabulated. In Fig. 2, $\phi'_\delta(E_\delta, \eta, E_n)$ are shown for $\eta = 0.125$ keV.

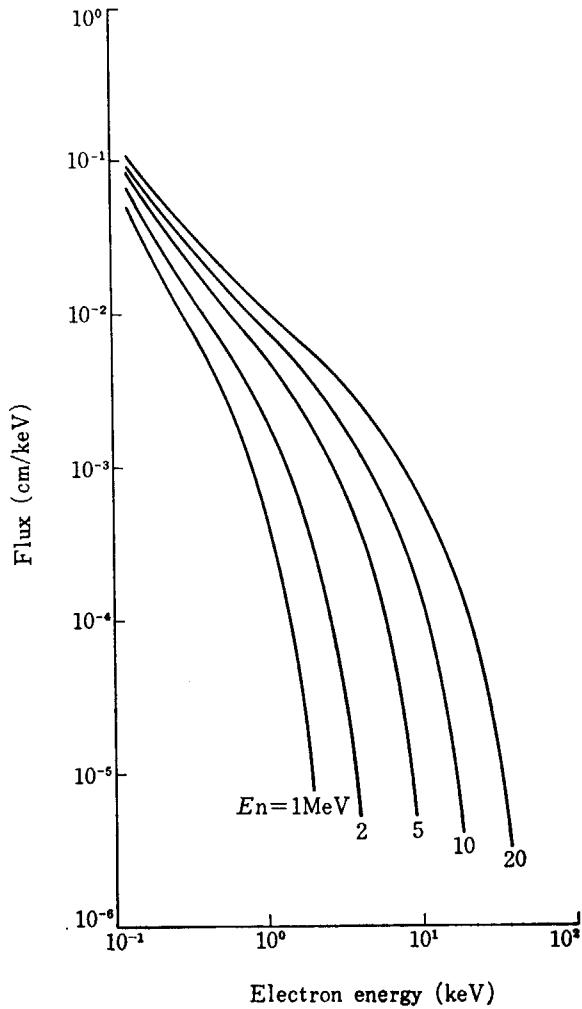


Fig. 2 The slowing-down spectrum of δ -rays $\phi'_\delta(E_\delta, \eta, E_n)$, produced by neutrons of energies $E_n = 1, 2, 5, 10$ and 20 MeV in the water medium. The ordinate is given in a unit of cm/keV and the cut-off energy of δ -rays, η , is taken as 0.125 keV.

While the actual form of $P(E, j)$ in eq. (3) would be much complicated, the Poisson distribution was here assumed as a considerably good approximation for it and given by

$$P(E, j) = \frac{\left[\left(\frac{\ell}{W} \right) L_t(E, \eta) \right]^j}{j!} \exp - \left[\left(\frac{\ell}{W} \right) L_t(E, \eta) \right], \quad (21)$$

where ℓ is the average thickness of the targets, and $L_l(E, \eta)$ is the local *LET* of a particle with energy E and means the energy transferred in the vicinity of the track, mainly coming from the glancing collisions. Hereafter, ℓ/W is expressed in a unit of 100 \AA/eV , that is, in a case of $\ell/W = 1/100$, if we assume $W = 35 \text{ eV}$, the value of ℓ is 35 \AA . $L_l(E_p, \eta)$ for proton is given by

$$L_l(E_p, \eta) = \frac{2\pi e^4 N_e}{mc^2 \beta_p^2} \left[\ln \frac{2mc^2 \beta_p^2 \eta}{(1-\beta_p^2) I^2} - \beta_p^2 \right], \text{ for } E_p \geq 0.5 \text{ MeV}, \quad (22a)$$

$$= L_t(E_p) - \frac{2\pi e^4 N_e}{mc^2 \beta_p^2} \left[\ln \frac{2mc^2 \beta_p^2}{(1-\beta^2) \eta} \right], \text{ for } E_p < 0.5 \text{ MeV}. \quad (22b)$$

In eq. (22b), empirical values^{12), 13)} were used for $L_t(E_p)$ as mentioned before.

The numerical values of $L_t(E_p)$ and $L_l(E_p, \eta)$ are tabulated in TABLE 4 and graphically shown in Fig. 3, where the dashed line is the one calculated according to Bethe's formula and solid lines are ones calculated using the experimental values for $L_t(E_p)$. Details of calculation and the numerical values for $L_t(E_\delta)$ and $L_l(E_\delta, \eta)$ were described and tabulated in Ref. (6). Among them, $L_t(E_\delta)$ and $L_l(E_\delta, \eta)$ for $E_\delta = 0.125 \text{ keV} \sim 1.0 \text{ keV}$ are shown in Fig. 4.

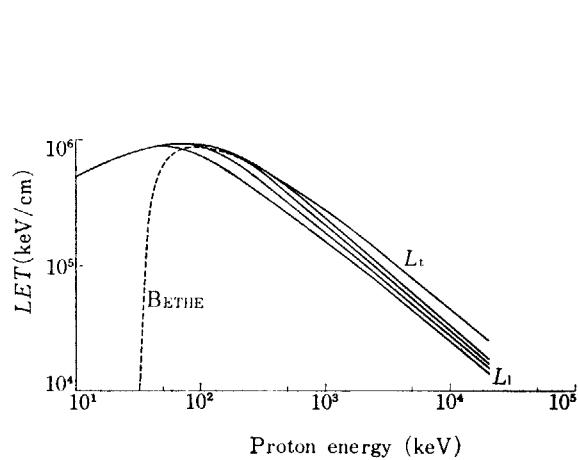


Fig. 3 The total *LET*, $L_t(E_p)$, and the local *LET*, $L_l(E_p, \eta)$, of proton in the water medium. The dashed line is the one calculated according to Bethe's formula and solid lines are calculated ones using the experimental values and varying the cut-off energy of δ -rays as $\eta = 0.125, 0.250, 0.500$ and 1.000 keV .

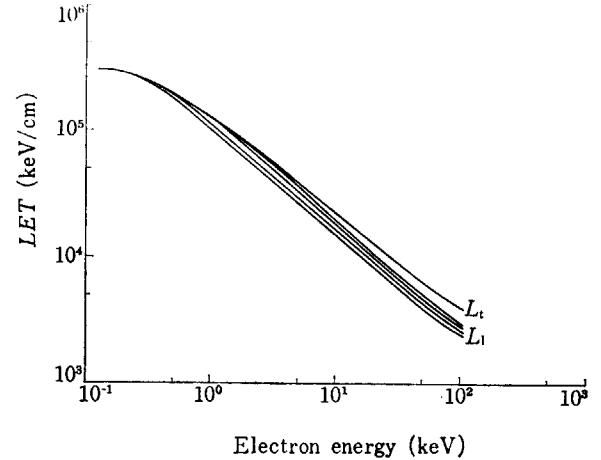


Fig. 4 The total *LET*, $L_t(E_\delta)$, and the local *LET*, $L_l(E_\delta, \eta)$, of electrons in the water medium. $L_l(E_\delta, \eta)$ are shown varying the cut-off energy of δ -rays as $\eta = 0.125, 0.250, 0.500$ and 1.000 keV .

In TABLE 6, numerical values of $P(E, j)$ for protons are tabulated and ones for electrons were also tabulated in Ref. (6).

In the case of neutron irradiation, f_j is given by

$$f_j = \frac{\int_{E_p \text{ min}}^{E_n} \phi_p(E_p, E_n) P(E_p, j) dE_p + \int_{\eta}^{E_\delta \text{ max}} \phi_\delta(E_\delta, \eta, E_n) P(E_\delta, j) dE_\delta}{\bar{\phi}_t(E_n, \eta)}, \quad (23)$$

where

$$\bar{\phi}_t(E_n, \eta) = \bar{\phi}_p(E_n) + \bar{\phi}_\delta(E_n, \eta), \quad (24)$$

$$\bar{\phi}_p(E_n) = \int_{E_p \text{ min}}^{E_n} \phi_p(E_p, E_n) dE_p, \quad (25)$$

and

$$\bar{\phi}_\delta(E_n, \eta) = \int_{\eta}^{E_\delta \text{ max}} \phi_\delta(E_\delta, \eta, E_n) dE_\delta. \quad (26)$$

The numerical values of f_j are tabulated in TABLE 7. In Fig. 5, f_0 are shown as a function of

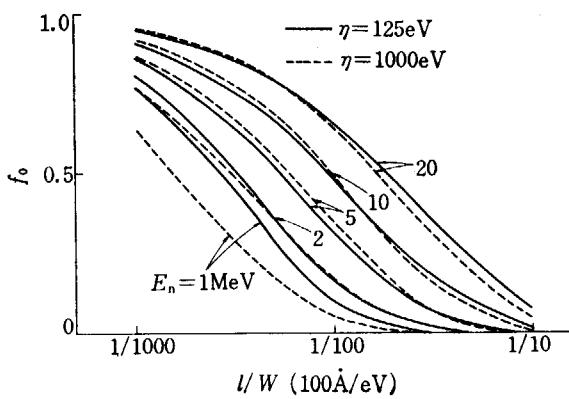


Fig. 5 The probability distribution of occurrence of no event in a target by neutrons of energies, $E_n=1, 2, 5, 10$ and 20 MeV . The solid lines and the dashed lines were calculated assuming the cut-off energy, $\eta=0.125\text{ keV}$ and $\eta=1.000\text{ keV}$, respectively.

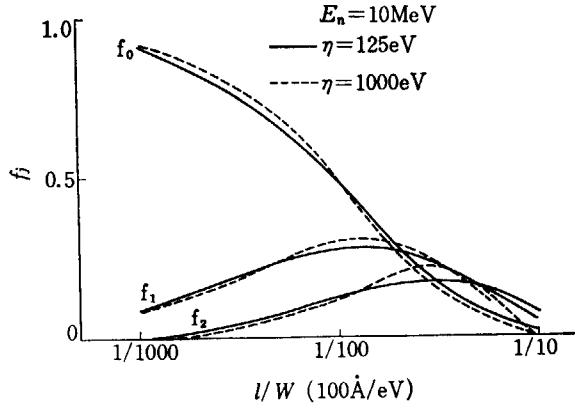


Fig. 6 The probability distribution of j primary events in a target by neutrons of energies $E_n=10\text{ MeV}$. The solid lines and the dashed lines were calculated assuming the cut-off energy, $\eta=0.125\text{ keV}$ and $\eta=1.000\text{ keV}$, respectively.

the target thickness for $\eta=0.125\text{ keV}$, and in Fig. 6, f_0 , f_1 and f_2 are shown for $E_n=10\text{ MeV}$ and $\eta=0.125\text{ keV}$ and 1 keV .

3. Results

The survival rates for n -events and one-target model based on T -distribution, H_n^T , is given by

$$H_n^T = \exp - \bar{\phi}_t(E_n, \eta) S_0 \left(1 - \sum_{j=0}^{n-1} f_j \right), \quad (27a)$$

$$= \exp - \bar{\phi}_n(E_n) \sigma(E_n) N_H S_0 \bar{\phi}_t'(E_n, \eta) \left[1 - \sum_{j=0}^{n-1} f_j \right], \quad (27b)$$

where

$$\bar{\phi}_t'(E_n, \eta) = \bar{\phi}_p'(E_n) + \bar{\phi}_\delta'(E_n, \eta), \quad (28)$$

$$\bar{\phi}_p'(E_n) = \int \phi_p'(E_p, E_n) dE_p, \quad (29)$$

$$\bar{\phi}_\delta'(E_n, \eta) = \int \phi_\delta'(E_\delta, \eta, E_n) dE_\delta. \quad (30)$$

The survival rate for T -input H_n^T is expressed, using S_{eff}^n , as follows:

$$H_n^T = \exp - \bar{\phi}_n(E_n) S_{\text{eff}}^n, \quad (31)$$

where the effective inactivation cross section of the targets for H_n^T can be expressed by

$$S_{\text{eff}}^n = S_0 \sigma(E_n) N_H \bar{\phi}_t'(E_n, \eta) \left(1 - \sum_{j=0}^{n-1} f_j \right). \quad (32)$$

S_{eff}^n is obtained as a function of E_n , η , S_0 , n and ℓ/W and the numerical values of S_{eff}^n/S_0 tabulated in TABLE 8. The numerical values of H_n^T would be readily calculated with the use of values S_{eff}^n/S_0 .

In Fig. 7, S_{eff}^n/S_0 for $H_n^T(E_n=10\text{ MeV})$ are shown as a function of ℓ/W , varying n and η . In Fig. 8, S_{eff}^n/S_0 are shown for $n=1$ as a function of the target size as well as E_n .

The survival rates for the one-target n -events model based on Z -distribution, H_n^Z , is given by eq. (12), and for $n=1$ and 2 are expressed as

$$H_1^Z = \exp - \bar{\phi}_n(E_n) S_0 \sigma(E_n) N_H \bar{\phi}_t'(E_n, \eta) (1 - f_0)$$

$$H_2^Z = [1 + \bar{\phi}_n(E_n) S_0 \sigma(E_n) N_H \bar{\phi}_t'(E_n, \eta) f_1] H_1^Z.$$

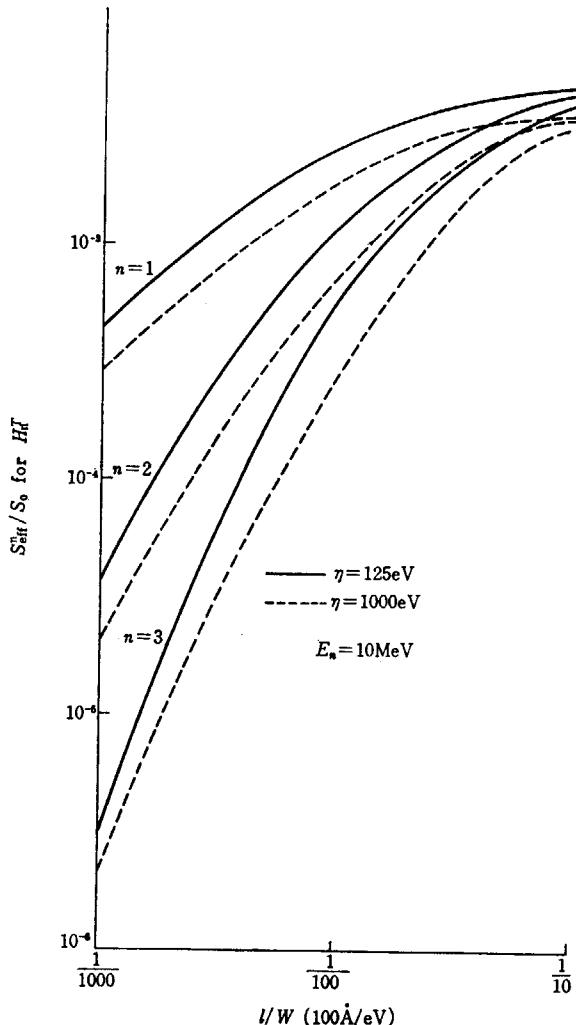


Fig. 7 The variation of the ratio of the effective inactivation cross section to the geometrical cross section of target, S_{eff}^n/S_0 , with the number of threshold events for inactivation, $n=1, 2$ and 3 , for neutrons of $E_n=10 \text{ MeV}$ and $\eta=0.125 \text{ keV}$ and 1.000 keV , based on T -distribution.

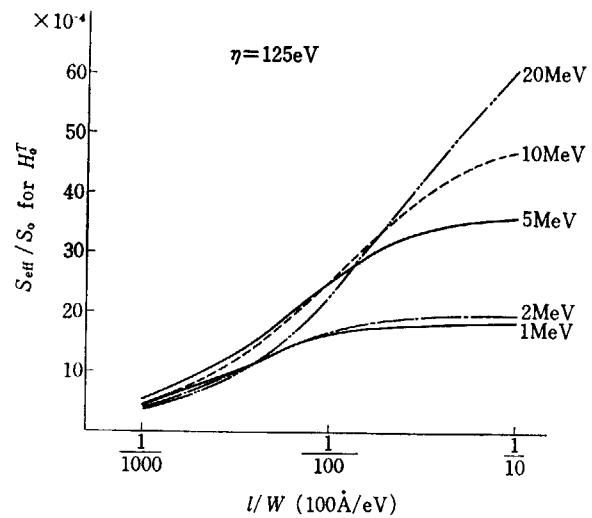


Fig. 8 The variation of the ratio of effective inactivation cross section to geometrical cross section of target S_{eff}^1/S_0 , with the neutron energy, $E_n=1, 2, 5, 10$ and 20 MeV , based on T -distribution. The cut-off energy is taken as 0.125 keV .

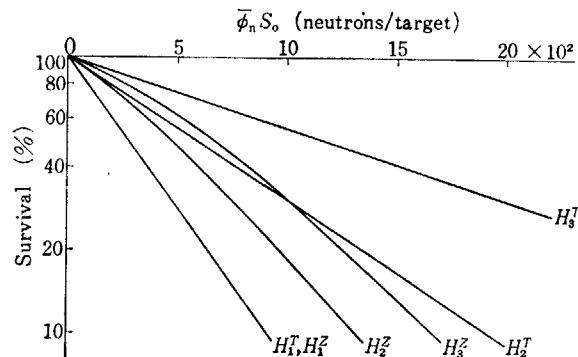


Fig. 9 The variation of the survival rates, H_n^T and H_n^Z , based on T - and Z -distribution, respectively, with the number of threshold events for inactivation, $n(n=1, 2$ and $3)$, for neutrons of $E_n=10 \text{ MeV}$. The abscissa corresponds to the average number of neutrons passing through a target.

η is taken as 0.125 keV and the target thickness ℓ/W is taken as $1/100$ in a unit of 100 Å/eV .

The practical calculations of H_n^Z are more complicated than H_n^T , and the numerical values of H_n^Z are tabulated in TABLE 5, varying E_n , η , n and ℓ/W versus a variable $\bar{\phi}_n S_0$. In the following only a few typical ones of numerical results are here illustrated. From these figures, one can see that while the survival curves for H_n^T are of the exponential type, the ones for H_n^Z are not. In Fig. 9, the survival curves corresponding to H_n^T and H_n^Z are shown, where $E_n=10 \text{ MeV}$, $\ell/W=1/100$ and $\eta=0.125 \text{ keV}$. In Fig. 10, the variation of H_1^T with the target size are shown for $E_n=10 \text{ MeV}$ and $\eta=0.125 \text{ keV}$, and in Fig. 11, the variation of H_1^T with the incident energies of neutrons, E_n , are shown for $\ell/W=1/100$ and $\eta=0.125 \text{ keV}$.

Discussions about these results, the application to practical problems and the comparison with the result obtained by Boag²⁾ will be published elsewhere together with the results obtained for X-rays and γ -rays.

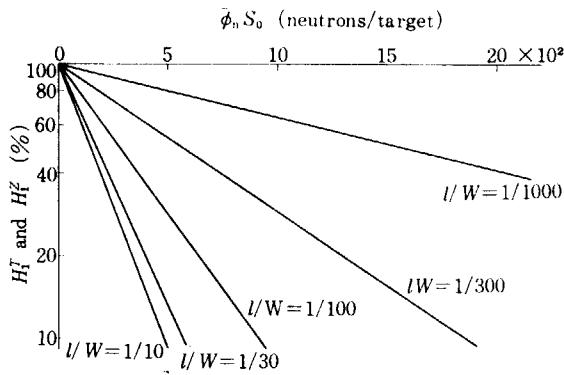


Fig. 10 The variation of the survival rates for one event model, H_1^T and H_1^Z , with the target thickness ℓ/W in a unit of 100 \AA/eV , for neutrons of $E_n = 10 \text{ MeV}$ and for $\eta = 0.125 \text{ keV}$. The abscissa corresponds to the average number of neutrons passing through a target.

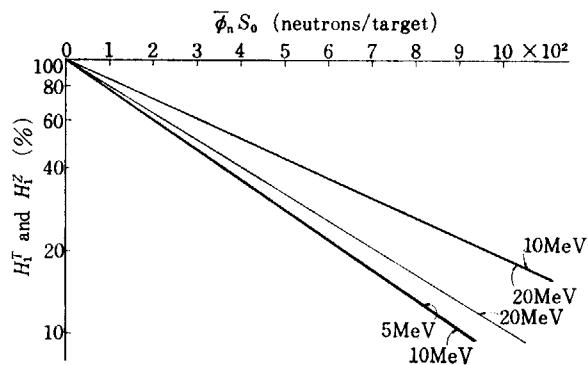


Fig. 11 The variation of the survival rates for one event model, H_1^T and H_1^Z for neutrons with $E_n = 1, 2, 5, 10$ and 20 MeV . The target thickness ℓ/W , in a unit of 100 \AA/eV is $1/100$ and η is 0.125 keV .

Appendix

In the practical calculations of the slowing-down spectra and the *LET*, the total energy range is divided into logarithmic scale and the mesh point of energy, E_i , and the energy width at that point, ΔE_i , are determined by following equations,

$$E_i = a^{i-1} \eta, \\ \Delta E_i = \frac{2(a-1)}{a+1} E_i = 0.172854 E_i,$$

where

$$a = 2^{\frac{1}{4}}.$$

Although the maximum energy of recoil protons or δ -rays, which is expressed as E^{\max} , can be determined exactly by the theoretical formulae, for the convenience of calculation, the following prescriptions were used,

$$\left. \begin{array}{l} E^{\max} = E_i, \\ \Delta E_{\max} = \frac{1}{2} \Delta E_i + (E^{\max} - E_i) \end{array} \right\} \text{for } E^{\max} < E_i + \frac{1}{2} \Delta E_i, \\ \left. \begin{array}{l} E^{\max} = E_{i+1} \\ \Delta E_{\max} = E^{\max} - \left(E_i + \frac{1}{2} \Delta E_i \right), \end{array} \right\} \text{for } E^{\max} > E_i + \frac{1}{2} \Delta E_i.$$

The energy width at the cut-off energy, ΔE_{η} , is chosen as

$$\Delta E_{\eta} = \frac{1}{2} \Delta E_i = 0.0864272 E_i.$$

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Tables**TABLE 1** Input parameters

Incident neutron energy, E_n (MeV).	1, 2, 5, 10, 20
Cut-off energy for δ -rays, η (keV).	0.125, 0.250, 0.500, 1.000.
Target size, ℓ / W (100 Å/eV).	1/1000, 1/300, 1/100, 1/30, 1/10.
Number of primary events per track per target, j .	0, 1, 2.
Number of primary events per target, J .	0, 1, 2.
Number of threshold events for inactivation, n .	1, 2, 3.
Average thickness of target: ℓ (Å)	
Average energy required to produce one primary event: W (eV)	
Recoil proton energy, E_p (MeV): 0.01~20.48	
Delta-ray energy, E_δ (keV): 0.125~107.6	

TABLE 2 Slowing-down spectrum of recoil protons $\phi_p'(E_p, E_n)$, in a unit of cm/MeV, in the water medium. The values are shown as a function of the incident energy of neutrons E_n (MeV) and the energy of recoil protons E_p (MeV).

TABLE 3 Slowing-down spectrum of δ -rays $\phi'_\delta(E_\delta, \eta, E_n)$, in a unit of cm/keV in the water medium. The values are shown as a function of the incident energy of neutrons E_n (MeV), the energy of δ -rays E_δ (keV) and the cut-off energy of δ -rays η (keV).

TABLE 4 Total LET, $L_i(E_p)$, and local LET $L_i(E_p)$ of proton in the water medium. The values of $L_i(E_p)$ (MeV/cm) are shown as a function of E_p (MeV). $L_i(E_p)$ (MeV/cm) are shown as a function of E_p (MeV/cm) and the cut-off energy of δ -rays η (keV).

TABLE 5 The survival rate of target for neutrons based on Z-distribution, H_n^z . The values are shown as a function of neutron flux (neutrons/target), the cut-off energy η (keV), and the number of threshold events for inactivation n .

TABLE 6 Probability that a track of proton of energy E_p (MeV) gives j -events, $P(E_p, j)$. The values are shown as a function of E_p , the target size $l/W(100 \text{ \AA/eV})$, the cut-off energy η (keV), and the number of events j . "0" means that the value of $P(E, j)$ is less than 10^{-8} .

TABLE 7 Probability of occurrence of j primary events in a target, f_j , by neutrons of energy E_n (MeV). The values are shown as a function of E_n , the target size $l/W(100 \text{ \AA/eV})$ and the number of events j .

TABLE 8 The ratio of effective inactivation cross section to geometrical cross section of target, S_{eff}^n/S_0 , for neutrons having energy E_n based on T-distribution. The ratio is shown as a function of E_n , the target size $l/W(100 \text{ \AA/eV})$, the cut-off energy η (keV), and the number of threshold events for inactivation n .

In TABLE 2~TABLE 8, because of typographical limitations, symbols of parameters used in text are indicated as follows:

$\phi'_p(E_p, E_n)$: $\phi(P)$
E_p	: $E(P)$
$\phi'_\delta(E_\delta, \eta, E_n)$: $\phi(\delta)$
E_δ	: $E(\delta)$
E_n	: $E(N)$
f_j	: FJ
f_o	: FO
f_1	: F1
f_2	: F2
S_{eff}/S_0	: SEFF/SO

Table 2

E (F)	ENERGY PLATE OF EBSOOL PHOTON : $\Phi(F)$			
	0.0	5.0	10.0	25.0
2.0100E-01	0.	0.	0.	0.
1.7277E-01	0.	0.	0.	4.7134E-03
1.4452E-01	0.	0.	0.	1.0487E-03
1.2121E-01	0.	0.	0.	1.0487E-03
1.0000E-01	0.	0.	0.	1.0487E-03
8.0000E-02	0.	0.	0.	1.0487E-03
7.2400E-02	0.	0.	2.6832E-03	1.0487E-03
6.0887E-02	0.	0.	4.6537E-03	1.0714E-02
5.1290E-02	0.	0.	6.6242E-03	1.0714E-02
4.3000E-02	0.	0.	1.3474E-03	1.0714E-02
3.6200E-02	0.	0.	2.6748E-03	9.7385E-03
3.0444E-02	0.	0.	5.3491E-03	5.8928E-03
2.5600E-02	0.	0.	3.6181E-03	3.1974E-03
2.1377E-02	0.	0.	2.1893E-03	1.4397E-03
1.7102E-02	0.	0.	1.6944E-03	1.0714E-02
1.3522E-02	0.	0.	3.1980E-03	2.8578E-03
1.0000E-02	0.	0.	1.3997E-03	1.2905E-03
1.0782E-02	0.	0.	2.1821E-03	2.3137E-03
9.4744E-03	0.	0.	3.6531E-03	1.3180E-03
7.6109E-03	0.	0.	1.9078E-03	2.8578E-03
6.4000E-04	9.89E-03	1.8004E-03	2.3857E-03	2.6483E-03
5.3837E-04	0.1129E-03	1.1973E-03	2.1821E-03	2.7959E-03
4.4670E-04	0.1129E-03	1.1973E-03	1.9078E-03	2.3137E-03
3.6055E-04	0.1129E-03	1.3764E-03	1.7978E-03	1.8727E-03
3.0000E-04	0.1129E-03	1.4944E-03	1.6373E-03	1.6493E-03
2.6790E-04	0.1129E-03	1.7116E-03	1.4993E-03	1.1943E-03
2.4040E-04	0.1129E-03	1.7116E-03	1.3642E-03	1.0432E-03
2.0215E-04	0.1129E-03	1.7052E-03	1.2812E-03	1.0307E-03
1.6000E-04	0.1129E-03	1.7424E-03	1.2020E-03	1.2311E-03
1.3494E-04	0.1129E-03	1.9973E-03	1.1446E-03	1.1494E-03
1.1319E-04	0.1129E-03	1.9973E-03	1.0707E-03	1.0707E-03
9.3544E-05	0.1129E-03	1.9973E-03	1.0707E-03	1.0707E-03
8.0000E-05	0.1129E-03	1.9973E-03	1.0707E-03	1.0707E-03
6.7777E-05	0.1129E-03	1.9936E-03	1.0581E-03	1.0581E-03
5.5468E-05	0.1129E-03	1.9936E-03	1.0581E-03	1.0581E-03
4.3946E-05	0.1129E-03	1.9936E-03	1.0581E-03	1.0581E-03
4.0000E-05	0.1129E-03	1.3114E-03	1.1150E-03	1.1150E-03
3.6534E-05	0.1129E-03	1.1933E-03	1.2054E-03	1.2054E-03
3.2894E-05	0.1129E-03	1.1933E-03	1.2702E-03	1.2702E-03
2.7884E-05	0.1129E-03	1.2392E-03	1.3497E-03	1.3497E-03
2.0000E-05	0.1129E-03	1.4481E-03	1.4481E-03	1.4481E-03
1.6700E-05	0.1129E-03	1.5647E-03	1.5647E-03	1.5647E-03
1.4342E-05	0.1129E-03	1.7036E-03	1.7109E-03	1.7132E-03
1.1894E-05	0.1129E-03	1.8537E-03	1.8488E-03	1.8574E-03
1.0000E-02	1.9800E-03	1.9900E-03	1.9960E-03	1.9980E-03

Table 4

E (P)	TOTAL LSF	LOCAL LSF
	$\eta = 0.125$	$\eta = 0.250$
	$\eta = 0.500$	$\eta = 1.000$
2.0480E-01	2.2057E-01	1.3848E-01
1.7272E-01	7.7477E-02	1.7678E-01
1.4464E-01	2.0791E-02	2.1210E-01
1.2373E-01	3.9561E-02	2.3940E-01
1.0246E-01	6.9949E-02	2.7307E-01
8.1908E-02	1.1774E-02	3.4399E-01
6.3092E-02	1.8406E-02	4.2232E-01
5.1200E-02	7.3523E-02	4.8320E-01
4.3059E-02	9.6054E-02	4.7685E-01
3.6204E-02	1.3337E-02	5.7364E-01
3.0240E-02	1.6484E-02	6.5032E-01
2.5460E-02	1.3488E-02	7.3607E-01
2.1597E-02	1.3388E-02	8.4669E-01
1.8192E-02	1.7577E-02	9.7137E-01
1.5320E-02	1.9377E-02	1.0447E-01
1.2990E-02	1.9377E-02	1.0506E-01
1.0246E-02	7.1410E-02	1.2096E-01
8.0748E-03	2.5595E-02	1.4737E-02
6.9510E-03	7.9488E-02	1.6787E-02
7.1800E-03	1.0180E-02	1.7500E-02
6.4000E-03	1.6531E-02	2.1515E-02
5.3875E-03	4.8582E-02	2.3221E-02
4.3235E-03	6.1630E-02	2.7959E-02
3.8053E-03	5.3466E-02	3.5500E-02
3.2000E-03	9.7108E-02	3.7664E-02
2.8400E-03	8.1000E-02	4.0393E-02
2.6267E-03	6.4949E-02	4.5070E-02
1.9027E-03	7.5092E-02	4.9979E-02
1.4000E-03	8.5932E-02	5.4995E-02
1.1314E-03	8.6872E-02	6.4771E-02
9.3157E-04	9.1200E-02	7.0318E-02
8.0000E-03	9.1900E-02	7.5349E-02
7.1775E-03	8.7100E-02	8.1919E-02
6.5159E-03	8.1949E-02	8.1919E-02
4.7584E-03	8.7933E-02	8.7933E-02
4.0000E-03	8.8400E-02	8.6600E-02
3.3648E-03	8.7238E-02	8.6600E-02
2.8825E-03	8.7238E-02	8.6268E-02
2.3784E-03	7.3784E-02	7.3784E-02
2.0000E-03	7.1000E-02	7.0000E-02
1.6881E-03	6.3456E-02	6.3456E-02
1.4588E-03	5.3784E-02	5.3784E-02
1.1892E-03	5.3784E-02	5.3784E-02
1.0000E-03	5.0000E-02	5.0000E-02

Table 3

$x(f)$		ENERGY	FLUX	OF δ - RATE	$\Phi(f)$
		1.0	2.0	5.0	10.0
					20.0
4.5253e-01	01	0.	0.	0.	0.
3.6055e-01	01	0.	0.	0.	3.2468e-03
3.2890e-01	01	0.	0.	0.	1.3533e-03
3.0690e-01	01	0.	0.	0.	3.2430e-03
2.8490e-01	01	0.	0.	0.	7.3238e-03
2.6290e-01	01	0.	0.	0.	1.3533e-03
1.9927e-01	01	0.	0.	6.1049e-04	2.0378e-03
1.6000e-01	01	0.	0.	6.1234e-03	2.2436e-03
1.3454e-01	01	0.	0.	4.6127e-03	3.4436e-03
1.1317e-01	01	0.	0.	4.6127e-03	3.4436e-03
9.1371e-02	01	0.	0.	5.9207e-06	1.7376e-03
8.0000e-02	01	0.	0.	2.1327e-03	2.9524e-03
6.7737e-02	00	0.	0.	5.8657e-03	4.5728e-03
5.6586e-02	00	0.	0.	5.8657e-03	4.5728e-03
4.5337e-02	00	0.	0.	5.3784e-03	2.0172e-03
4.0000e-02	00	0.	0.	2.7352e-03	2.5253e-03
3.5364e-02	00	0.	2.4146e-03	6.2473e-04	3.0816e-03
2.8284e-02	00	0.	7.2328e-03	9.2764e-04	2.1387e-03
1.9192e-02	00	0.	1.2084e-03	1.2084e-03	3.2352e-03
7.0000e-03	00	7.0718e-04	1.7331e-04	1.7915e-03	5.4263e-03
1.4813e-03	00	6.3284e-05	5.6380e-05	2.3940e-03	4.3324e-03
1.1412e-03	01	1.0740e-03	9.9469e-05	3.1631e-03	7.6355e-03
1.1192e-03	01	1.0740e-03	9.9469e-05	1.0000e-03	3.2352e-03
1.0000e-03	01	9.7547e-04	2.0094e-03	5.1400e-03	7.1919e-03
8.4906e-04	-01	7.9246e-04	3.8748e-03	1.6804e-03	9.5846e-03
7.0711e-04	-01	1.4240e-03	5.9713e-03	8.0993e-03	1.1562e-02
2.9468e-04	-01	2.2095e-03	5.9101e-03	1.0000e-02	1.0000e-02
5.0000e-04	-01	1.0000e-03	1.0000e-03	1.0000e-02	1.0000e-02
6.2048e-04	-01	4.4991e-03	9.0001e-03	1.3374e-02	2.0200e-02
7.0711e-04	-01	1.7115e-03	1.7115e-03	1.9983e-02	2.4404e-02
2.9468e-04	-01	1.9911e-03	1.2938e-03	2.3946e-02	2.9644e-02
2.9709e-04	-01	1.9911e-03	1.2938e-03	2.3946e-02	3.0000e-02
2.7500e-04	-01	1.7266e-03	1.7949e-03	2.5942e-02	3.3805e-02
2.1024e-04	-01	1.7639e-02	4.2346e-02	3.7772e-02	4.9175e-02
1.1192e-04	-01	3.2332e-02	4.9495e-02	5.9447e-02	7.6464e-02
1.4852e-04	-01	3.2332e-02	4.9495e-02	7.6499e-02	9.6464e-02
1.2500e-04	-01	5.1196e-02	6.8235e-02	8.8938e-02	1.0339e-01
1.1666e-04	-01	5.1196e-02	6.8235e-02	8.8938e-02	1.1666e-01

$\sum \Psi_0(\theta) \Delta R(\theta)$ 5.8054e-03 11.0301e-02 21.0301e-01 37.0301e-01

T (K)	INTERF. PLATE OF β - RAT. I. $\phi(\beta)$				
	2.0	5.0	10.0	20.0	
1.0	0.	0.	0.	0.	U.
4.5955E-01	0.	0.	0.	0.	1.1889E-05
3.0255E-01	0.	0.	0.	0.	1.2754E-05
3.2020E-01	0.	0.	0.	0.	1.2754E-05
3.6804E-01	0.	0.	0.	0.	3.4229E-05
2.5377E-01	0.	0.	0.	0.	7.3349E-05
1.9021E-01	0.	0.	0.	4.1023E-04	1.3522E-05
1.6000E-01	0.	0.	0.	1.4823E-04	1.3522E-05
1.3537E-01	0.	0.	0.	1.7632E-04	7.0266E-05
9.3327E-02	0.	0.	0.	2.4687E-04	4.7134E-05
6.0000E-02	0.	0.	0.	4.9336E-05	3.4681E-05
1.3534E-01	0.	0.	0.	9.3017E-05	5.3030E-05
1.1111E-01	0.	0.	0.	1.2301E-04	5.3030E-05
9.3327E-02	0.	0.	0.	1.7632E-04	7.0266E-05
6.0000E-02	0.	0.	0.	2.4687E-04	4.7134E-05
6.7722E-00	0.	0.	0.	5.9336E-05	4.1632E-04
1.6866E-00	0.	0.	0.	9.1763E-05	2.7107E-04
2.5377E-00	0.	0.	0.	2.3712E-05	9.3327E-05
4.0000E-00	0.	0.	0.	5.9870E-05	1.7757E-04
3.0000E-00	0.	0.	0.	1.7757E-03	1.7757E-03
3.6364E-00	0.	0.	4.2430E-03	4.2186E-04	1.7757E-03
2.8284E-00	0.	0.	1.1638E-03	9.1763E-04	2.7107E-04
2.1322E-00	0.	0.	2.8712E-03	2.3712E-04	9.3327E-05
2.0000E-00	0.	0.	3.0771E-03	5.9870E-05	1.7757E-04
1.4818E-00	0.	5.9336E-05	5.9336E-04	2.3641E-03	4.2430E-03
1.3428E-00	0.	1.0594E-00	9.1763E-04	5.9336E-03	1.5892E-03
1.2919E-00	0.	1.5234E-00	2.3712E-04	1.0594E-02	2.9209E-03
1.0000E-00	0.	1.9692E-01	5.9870E-03	3.0503E-02	7.7596E-03
8.0908E-01	0.	5.1174E-01	7.7637E-03	3.1591E-03	9.3454E-03
7.0713E-01	0.	3.1360E-03	7.6793E-03	7.8409E-03	1.1228E-02
5.9464E-01	0.	2.1230E-03	1.9534E-03	2.0501E-02	2.4228E-02
5.0000E-01	0.	1.4902E-03	1.9534E-03	2.0501E-02	1.2329E-02
4.1510E-01	0.	1.3707E-03	8.8903E-04	1.8845E-02	2.4228E-02
3.5524E-01	0.	6.6537E-03	1.1939E-02	1.8845E-02	2.4010E-02
2.7530E-01	0.	1.0594E-03	5.1534E-03	2.3958E-02	2.9466E-02
2.0000E-01	0.	1.7663E-02	1.9904E-02	7.9522E-02	3.6512E-02
1.4581E-01	0.	1.0594E-02	1.9904E-02	7.9522E-02	3.6314E-02
1.0000E-01	0.	1.0594E-02	1.9904E-02	7.9522E-02	3.6314E-02
7.0713E-01	0.	3.1360E-03	7.6793E-03	7.8409E-03	1.1228E-02
5.9464E-01	0.	2.1230E-03	1.9534E-03	2.0501E-02	2.4228E-02
5.0000E-01	0.	1.4902E-03	1.9534E-03	2.0501E-02	1.2329E-02
4.1510E-01	0.	1.3707E-03	8.8903E-04	1.8845E-02	2.4228E-02
3.5524E-01	0.	6.6537E-03	1.1939E-02	1.8845E-02	2.4010E-02
2.7530E-01	0.	1.0594E-03	5.1534E-03	2.3958E-02	2.9466E-02
2.0000E-01	0.	1.7663E-02	1.9904E-02	7.9522E-02	3.6512E-02

$$\sum \Phi(\delta) \Delta \Phi(\delta) = 2.42948 \times 10^{-3} - 5.74146 \times 10^{-5} + 1.57036 \times 10^{-2} - 2.41544 \times 10^{-2} + 4.03388 \times 10^{-2}$$

Table 5-1

Table 5-2

Table 5 - 3

E (S) = 5.0

	/ V + 1 / 1000	/ V + 1 / 300	/ V + 1 / 100	/ V + 1 / 30	/ V + 1 / 30						
E (S,1)	E (S,2)	E (S,3)	E (S,1)	E (S,2)	E (S,3)	E (S,1)	E (S,2)	E (S,3)	E (S,1)	E (S,2)	E (S,3)
0	1.0000 00	1.0000 00	1.0000 00	0	1.0000 00	1.0000 00	1.0000 00	1.0000 00	1.0000 00	1.0000 00	1.0000 00
0.4340E 02	7.4747E-03	9.5844E-03	9.1259E-03	1.7200E 02	7.4937E-03	9.3776E-03	9.1336E-03	1.6900E 02	6.3115E-02	7.3098E-02	8.3098E-02
0.4340E 02	6.3116E-03	8.1202E-03	9.0402E-03	3.1200E 02	6.1816E-03	7.6736E-03	8.2292E-03	2.7300E 02	5.0319E-02	6.1304E-02	7.1336E-02
1.1070E 02	6.3114E-03	8.1201E-03	9.0401E-03	6.8300E 02	5.9494E-03	6.3874E-03	7.2264E-03	3.1200E 02	5.0318E-02	6.1303E-02	7.1335E-02
1.1070E 02	1.2648E-03	1.9367E-03	1.9367E-03	6.8300E 02	5.9494E-03	6.3874E-03	7.2264E-03	3.1200E 02	5.0318E-02	6.1303E-02	7.1335E-02
1.8200E 03	1.9367E-03	1.9367E-03	1.9367E-03	1.7000E 02	1.6137E-01	2.0275E-01	2.0275E-01	1.6000E 02	5.1464E-01	4.8622E-01	3.8776E-01
2.2400E 03	1.1170E-01	6.4336E-01	6.4336E-01	1.5400E 02	1.1486E-01	5.7474E-01	7.0300E-01	5.2000E 02	2.5152E-01	5.1939E-01	5.1939E-01
2.7400E 03	7.2314E-01	5.6426E-01	5.6426E-01	1.1900E 02	7.0121E-01	4.3026E-01	5.3026E-01	6.4100E 02	1.9946E-01	3.3228E-01	4.4312E-01
3.1900E 03	7.2313E-01	5.6425E-01	5.6425E-01	1.1900E 02	7.0120E-01	4.3025E-01	5.3025E-01	6.4100E 02	1.9945E-01	3.3227E-01	4.4311E-01
3.6300E 03	1.1937E-01	4.2121E-01	4.2120E-01	1.2400E 02	1.0400E-01	3.4882E-01	5.0200E-01	7.0000E 02	2.0138E-01	3.4797E-01	4.0300E-01
4.1100E 03	1.1936E-01	4.2120E-01	4.2120E-01	1.2400E 02	1.0400E-01	3.4882E-01	5.0200E-01	7.0000E 02	2.0137E-01	3.4796E-01	4.0300E-01
4.5400E 03	1.1045E-01	5.0745E-01	5.0745E-01	1.7000E 02	1.0117E-01	2.0275E-01	2.0275E-01	1.3000E 02	1.9205E-01	1.2987E-01	1.2987E-01
5.1400E 03	2.1928E-02	2.2341E-02	1.7200E 02	1.7200E 02	1.0000E 03	2.2442E-01	5.3700E-01	1.0000E 03	6.1146E-01	2.4211E-01	7.9700E-01
5.1400E 03	7.0495E-02	2.1948E-02	4.3015E-02	2.0500E 02	6.2135E-02	1.870F-01	3.0400E-01	1.1000E 03	2.0010E-01	2.0010E-01	2.0010E-01
5.1400E 03	4.1181E-02	2.1947E-02	4.3014E-02	2.0500E 02	6.2134E-02	1.870F-01	3.0400E-01	1.1000E 03	2.0009E-01	2.0009E-01	2.0009E-01
5.1400E 03	1.1337E-02	1.8335E-02	3.3101E-02	2.0500E 02	1.5970E-02	6.2886E-02	1.2120E-01	1.1000E 03	1.9367E-02	2.0008E-01	2.0008E-01
5.1400E 03	1.1336E-02	1.8334E-02	3.3100E-02	2.0500E 02	1.5969E-02	6.2885E-02	1.2120E-01	1.1000E 03	1.9366E-02	2.0007E-01	2.0007E-01
5.1400E 03	1.1335E-02	1.8333E-02	3.3100E-02	2.0500E 02	1.5968E-02	6.2884E-02	1.2120E-01	1.1000E 03	1.9365E-02	2.0006E-01	2.0006E-01
5.1400E 03	1.1334E-02	1.8332E-02	3.3099E-02	2.0500E 02	1.5967E-02	6.2883E-02	1.2120E-01	1.1000E 03	1.9364E-02	2.0005E-01	2.0005E-01
5.1400E 03	1.1333E-02	1.8331E-02	3.3098E-02	2.0500E 02	1.5966E-02	6.2882E-02	1.2120E-01	1.1000E 03	1.9363E-02	2.0004E-01	2.0004E-01
5.1400E 03	1.1332E-02	1.8330E-02	3.3097E-02	2.0500E 02	1.5965E-02	6.2881E-02	1.2120E-01	1.1000E 03	1.9362E-02	2.0003E-01	2.0003E-01
5.1400E 03	1.1331E-02	1.8329E-02	3.3096E-02	2.0500E 02	1.5964E-02	6.2880E-02	1.2120E-01	1.1000E 03	1.9361E-02	2.0002E-01	2.0002E-01
5.1400E 03	1.1330E-02	1.8328E-02	3.3095E-02	2.0500E 02	1.5963E-02	6.2879E-02	1.2120E-01	1.1000E 03	1.9360E-02	2.0001E-01	2.0001E-01
5.1400E 03	1.1329E-02	1.8327E-02	3.3094E-02	2.0500E 02	1.5962E-02	6.2878E-02	1.2120E-01	1.1000E 03	1.9359E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1328E-02	1.8326E-02	3.3093E-02	2.0500E 02	1.5961E-02	6.2877E-02	1.2120E-01	1.1000E 03	1.9358E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1327E-02	1.8325E-02	3.3092E-02	2.0500E 02	1.5960E-02	6.2876E-02	1.2120E-01	1.1000E 03	1.9357E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1326E-02	1.8324E-02	3.3091E-02	2.0500E 02	1.5959E-02	6.2875E-02	1.2120E-01	1.1000E 03	1.9356E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1325E-02	1.8323E-02	3.3090E-02	2.0500E 02	1.5958E-02	6.2874E-02	1.2120E-01	1.1000E 03	1.9355E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1324E-02	1.8322E-02	3.3089E-02	2.0500E 02	1.5957E-02	6.2873E-02	1.2120E-01	1.1000E 03	1.9354E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1323E-02	1.8321E-02	3.3088E-02	2.0500E 02	1.5956E-02	6.2872E-02	1.2120E-01	1.1000E 03	1.9353E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1322E-02	1.8320E-02	3.3087E-02	2.0500E 02	1.5955E-02	6.2871E-02	1.2120E-01	1.1000E 03	1.9352E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1321E-02	1.8319E-02	3.3086E-02	2.0500E 02	1.5954E-02	6.2870E-02	1.2120E-01	1.1000E 03	1.9351E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1320E-02	1.8318E-02	3.3085E-02	2.0500E 02	1.5953E-02	6.2869E-02	1.2120E-01	1.1000E 03	1.9350E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1319E-02	1.8317E-02	3.3084E-02	2.0500E 02	1.5952E-02	6.2868E-02	1.2120E-01	1.1000E 03	1.9349E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1318E-02	1.8316E-02	3.3083E-02	2.0500E 02	1.5951E-02	6.2867E-02	1.2120E-01	1.1000E 03	1.9348E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1317E-02	1.8315E-02	3.3082E-02	2.0500E 02	1.5950E-02	6.2866E-02	1.2120E-01	1.1000E 03	1.9347E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1316E-02	1.8314E-02	3.3081E-02	2.0500E 02	1.5949E-02	6.2865E-02	1.2120E-01	1.1000E 03	1.9346E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1315E-02	1.8313E-02	3.3080E-02	2.0500E 02	1.5948E-02	6.2864E-02	1.2120E-01	1.1000E 03	1.9345E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1314E-02	1.8312E-02	3.3079E-02	2.0500E 02	1.5947E-02	6.2863E-02	1.2120E-01	1.1000E 03	1.9344E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1313E-02	1.8311E-02	3.3078E-02	2.0500E 02	1.5946E-02	6.2862E-02	1.2120E-01	1.1000E 03	1.9343E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1312E-02	1.8310E-02	3.3077E-02	2.0500E 02	1.5945E-02	6.2861E-02	1.2120E-01	1.1000E 03	1.9342E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1311E-02	1.8309E-02	3.3076E-02	2.0500E 02	1.5944E-02	6.2860E-02	1.2120E-01	1.1000E 03	1.9341E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1310E-02	1.8308E-02	3.3075E-02	2.0500E 02	1.5943E-02	6.2859E-02	1.2120E-01	1.1000E 03	1.9340E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1309E-02	1.8307E-02	3.3074E-02	2.0500E 02	1.5942E-02	6.2858E-02	1.2120E-01	1.1000E 03	1.9339E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1308E-02	1.8306E-02	3.3073E-02	2.0500E 02	1.5941E-02	6.2857E-02	1.2120E-01	1.1000E 03	1.9338E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1307E-02	1.8305E-02	3.3072E-02	2.0500E 02	1.5940E-02	6.2856E-02	1.2120E-01	1.1000E 03	1.9337E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1306E-02	1.8304E-02	3.3071E-02	2.0500E 02	1.5939E-02	6.2855E-02	1.2120E-01	1.1000E 03	1.9336E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1305E-02	1.8303E-02	3.3070E-02	2.0500E 02	1.5938E-02	6.2854E-02	1.2120E-01	1.1000E 03	1.9335E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1304E-02	1.8302E-02	3.3069E-02	2.0500E 02	1.5937E-02	6.2853E-02	1.2120E-01	1.1000E 03	1.9334E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1303E-02	1.8301E-02	3.3068E-02	2.0500E 02	1.5936E-02	6.2852E-02	1.2120E-01	1.1000E 03	1.9333E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1302E-02	1.8300E-02	3.3067E-02	2.0500E 02	1.5935E-02	6.2851E-02	1.2120E-01	1.1000E 03	1.9332E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1301E-02	1.8299E-02	3.3066E-02	2.0500E 02	1.5934E-02	6.2850E-02	1.2120E-01	1.1000E 03	1.9331E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1300E-02	1.8298E-02	3.3065E-02	2.0500E 02	1.5933E-02	6.2849E-02	1.2120E-01	1.1000E 03	1.9330E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1299E-02	1.8297E-02	3.3064E-02	2.0500E 02	1.5932E-02	6.2848E-02	1.2120E-01	1.1000E 03	1.9329E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1298E-02	1.8296E-02	3.3063E-02	2.0500E 02	1.5931E-02	6.2847E-02	1.2120E-01	1.1000E 03	1.9328E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1297E-02	1.8295E-02	3.3062E-02	2.0500E 02	1.5930E-02	6.2846E-02	1.2120E-01	1.1000E 03	1.9327E-02	2.0000E-01	2.0000E-01
5.1400E 03	1.1296E-02	1.8294E-02	3.3061E-02	2.0500E 02	1.5929E-02	6.2845E-02	1.2120E-01	1.1000E 03	1.9326E-02	2.0000E-01	2.0000E-01
5.140											

Table 5 - 4

S (W) = 10.0

Table 5-5

x (s) = 20.0

/ V = 1 / 1000			/ V = 1 / 500			/ V = 1 / 100			/ V = 1 / 50			/ V = 1 / 10				
Φ (s) no	x (s,1)	x (s,2)	Φ (s) no	x (s,1)	x (s,2)	Φ (s) no	x (s,1)	x (s,2)	Φ (s) no	x (s,1)	x (s,2)	Φ (s) no	x (s,1)	x (s,2)	x (s,3)	
0	1.000E 00	1.000E 00	1.000E 00	0	1.000E 00	1.000E 00	1.000E 00	1.000E 00	1.000E 00	0	1.000E 00	1.000E 00	0	1.000E 00	1.000E 00	1.000E 00
1.140F 02	7.949E-01	9.351E-01	9.346E-01	2.140F 02	7.944E-01	9.343E-01	9.340E-01	1.140F 02	7.945E-01	9.342E-01	9.339E-01	5.140F 01	7.944E-01	9.341E-01	9.338E-01	
1.140F 03	6.324E-01	8.140E-01	8.134E-01	2.140F 03	6.319E-01	8.133E-01	8.130E-01	1.140F 03	6.314E-01	8.132E-01	8.129E-01	5.140F 02	6.313E-01	8.131E-01	8.128E-01	
1.140F 04	5.114E-01	7.135E-01	7.129E-01	2.140F 04	5.109E-01	7.128E-01	7.125E-01	1.140F 04	5.104E-01	7.127E-01	7.124E-01	5.140F 03	5.103E-01	7.126E-01	7.123E-01	
1.140F 05	4.114E-01	5.194E-01	5.189E-01	2.140F 05	4.109E-01	5.188E-01	5.185E-01	1.140F 05	4.104E-01	5.187E-01	5.184E-01	5.140F 04	4.103E-01	5.186E-01	5.183E-01	
1.140F 06	3.214E-01	4.264E-01	4.259E-01	2.140F 06	3.209E-01	4.258E-01	4.253E-01	1.140F 06	3.204E-01	4.257E-01	4.252E-01	5.140F 05	3.203E-01	4.256E-01	4.251E-01	
1.140F 07	2.424E-01	3.354E-01	3.349E-01	2.140F 07	2.419E-01	3.348E-01	3.343E-01	1.140F 07	2.414E-01	3.347E-01	3.342E-01	5.140F 06	2.413E-01	3.346E-01	3.341E-01	
1.140F 08	1.754E-01	2.684E-01	2.679E-01	2.140F 08	1.749E-01	2.678E-01	2.673E-01	1.140F 08	1.744E-01	2.677E-01	2.672E-01	5.140F 07	1.743E-01	2.676E-01	2.671E-01	
1.140F 09	1.204E-01	1.994E-01	1.989E-01	2.140F 09	1.199E-01	1.988E-01	1.983E-01	1.140F 09	1.194E-01	1.987E-01	1.982E-01	5.140F 08	1.193E-01	1.986E-01	1.981E-01	
1.140F 10	7.949E-02	1.334E-01	1.329E-01	2.140F 10	7.944E-02	1.333E-01	1.328E-01	1.140F 10	7.939E-02	1.332E-01	1.327E-01	5.140F 09	7.934E-02	1.331E-01	1.326E-01	
1.140F 11	5.324E-02	8.140E-02	8.135E-02	2.140F 11	5.319E-02	8.134E-02	8.129E-02	1.140F 11	5.314E-02	8.133E-02	8.128E-02	5.140F 10	5.309E-02	8.132E-02	8.127E-02	
1.140F 12	3.517E-02	4.264E-02	4.259E-02	2.140F 12	3.512E-02	4.258E-02	4.253E-02	1.140F 12	3.507E-02	4.257E-02	4.252E-02	5.140F 11	3.502E-02	4.256E-02	4.251E-02	
1.140F 13	2.207E-02	3.354E-02	3.349E-02	2.140F 13	2.202E-02	3.348E-02	3.343E-02	1.140F 13	2.197E-02	3.347E-02	3.342E-02	5.140F 12	2.192E-02	3.346E-02	3.341E-02	
1.140F 14	1.207E-02	1.994E-02	1.989E-02	2.140F 14	1.202E-02	1.988E-02	1.983E-02	1.140F 14	1.197E-02	1.987E-02	1.982E-02	5.140F 13	1.192E-02	1.986E-02	1.981E-02	
1.140F 15	6.324E-03	1.334E-01	1.329E-01	2.140F 15	6.319E-03	1.333E-01	1.328E-01	1.140F 15	6.314E-03	1.332E-01	1.327E-01	5.140F 14	6.309E-03	1.331E-01	1.326E-01	
1.140F 16	3.214E-03	4.264E-01	4.259E-01	2.140F 16	3.209E-03	4.258E-01	4.253E-01	1.140F 16	3.204E-03	4.257E-01	4.252E-01	5.140F 15	3.203E-03	4.256E-01	4.251E-01	
1.140F 17	1.754E-03	2.684E-01	2.679E-01	2.140F 17	1.749E-03	2.678E-01	2.673E-01	1.140F 17	1.744E-03	2.677E-01	2.672E-01	5.140F 16	1.739E-03	2.676E-01	2.671E-01	
1.140F 18	1.204E-03	1.994E-01	1.989E-01	2.140F 18	1.199E-03	1.988E-01	1.983E-01	1.140F 18	1.194E-03	1.987E-01	1.982E-01	5.140F 17	1.193E-03	1.986E-01	1.981E-01	
1.140F 19	7.949E-04	1.334E-02	1.329E-02	2.140F 19	7.944E-04	1.333E-02	1.328E-02	1.140F 19	7.939E-04	1.332E-02	1.327E-02	5.140F 18	7.934E-04	1.331E-02	1.326E-02	
1.140F 20	5.324E-04	4.264E-02	4.259E-02	2.140F 20	5.319E-04	4.258E-02	4.253E-02	1.140F 20	5.314E-04	4.257E-02	4.252E-02	5.140F 19	5.309E-04	4.256E-02	4.251E-02	
1.140F 21	3.214E-04	1.994E-02	1.989E-02	2.140F 21	3.209E-04	1.988E-02	1.983E-02	1.140F 21	3.204E-04	1.987E-02	1.982E-02	5.140F 20	3.203E-04	1.986E-02	1.981E-02	
1.140F 22	1.754E-04	1.334E-02	1.329E-02	2.140F 22	1.749E-04	1.333E-02	1.328E-02	1.140F 22	1.744E-04	1.332E-02	1.327E-02	5.140F 21	1.739E-04	1.331E-02	1.326E-02	
1.140F 23	1.204E-04	4.264E-02	4.259E-02	2.140F 23	1.202E-04	4.258E-02	4.253E-02	1.140F 23	1.197E-04	4.257E-02	4.252E-02	5.140F 22	1.193E-04	4.256E-02	4.251E-02	
1.140F 24	7.949E-05	1.994E-02	1.989E-02	2.140F 24	7.944E-05	1.988E-02	1.983E-02	1.140F 24	7.939E-05	1.987E-02	1.982E-02	5.140F 23	7.934E-05	1.986E-02	1.981E-02	
1.140F 25	5.324E-05	1.334E-02	1.329E-02	2.140F 25	5.319E-05	1.333E-02	1.328E-02	1.140F 25	5.314E-05	1.332E-02	1.327E-02	5.140F 24	5.309E-05	1.331E-02	1.326E-02	
1.140F 26	3.214E-05	4.264E-02	4.259E-02	2.140F 26	3.209E-05	4.258E-02	4.253E-02	1.140F 26	3.204E-05	4.257E-02	4.252E-02	5.140F 25	3.203E-05	4.256E-02	4.251E-02	
1.140F 27	1.754E-05	1.994E-02	1.989E-02	2.140F 27	1.749E-05	1.988E-02	1.983E-02	1.140F 27	1.744E-05	1.987E-02	1.982E-02	5.140F 26	1.739E-05	1.986E-02	1.981E-02	
1.140F 28	1.204E-05	1.334E-02	1.329E-02	2.140F 28	1.202E-05	1.333E-02	1.328E-02	1.140F 28	1.197E-05	1.332E-02	1.327E-02	5.140F 27	1.193E-05	1.331E-02	1.326E-02	
1.140F 29	7.949E-06	4.264E-02	4.259E-02	2.140F 29	7.944E-06	4.258E-02	4.253E-02	1.140F 29	7.939E-06	4.257E-02	4.252E-02	5.140F 28	7.934E-06	4.256E-02	4.251E-02	
1.140F 30	5.324E-06	1.994E-02	1.989E-02	2.140F 30	5.319E-06	1.988E-02	1.983E-02	1.140F 30	5.314E-06	1.987E-02	1.982E-02	5.140F 29	5.309E-06	1.986E-02	1.981E-02	
1.140F 31	3.214E-06	1.334E-02	1.329E-02	2.140F 31	3.209E-06	1.333E-02	1.328E-02	1.140F 31	3.204E-06	1.332E-02	1.327E-02	5.140F 30	3.203E-06	1.331E-02	1.326E-02	
1.140F 32	1.754E-06	4.264E-02	4.259E-02	2.140F 32	1.749E-06	4.258E-02	4.253E-02	1.140F 32	1.744E-06	4.257E-02	4.252E-02	5.140F 31	1.739E-06	4.256E-02	4.251E-02	
1.140F 33	1.204E-06	1.994E-02	1.989E-02	2.140F 33	1.202E-06	1.988E-02	1.983E-02	1.140F 33	1.197E-06	1.987E-02	1.982E-02	5.140F 32	1.193E-06	1.986E-02	1.981E-02	
1.140F 34	7.949E-07	1.334E-02	1.329E-02	2.140F 34	7.944E-07	1.333E-02	1.328E-02	1.140F 34	7.939E-07	1.332E-02	1.327E-02	5.140F 33	7.934E-07	1.331E-02	1.326E-02	
1.140F 35	5.324E-07	4.264E-02	4.259E-02	2.140F 35	5.319E-07	4.258E-02	4.253E-02	1.140F 35	5.314E-07	4.257E-02	4.252E-02	5.140F 34	5.309E-07	4.256E-02	4.251E-02	
1.140F 36	3.214E-07	1.994E-02	1.989E-02	2.140F 36	3.209E-07	1.988E-02	1.983E-02	1.140F 36	3.204E-07	1.987E-02	1.982E-02	5.140F 35	3.203E-07	1.986E-02	1.981E-02	
1.140F 37	1.754E-07	1.334E-02	1.329E-02	2.140F 37	1.749E-07	1.333E-02	1.328E-02	1.140F 37	1.744E-07	1.332E-02	1.327E-02	5.140F 36	1.739E-07	1.331E-02	1.326E-02	
1.140F 38	1.204E-07	4.264E-02	4.259E-02	2.140F 38	1.202E-07	4.258E-02	4.253E-02	1.140F 38	1.197E-07	4.257E-02	4.252E-02	5.140F 37	1.193E-07	4.256E-02	4.251E-02	
1.140F 39	7.949E-08	1.994E-02	1.989E-02	2.140F 39	7.944E-08	1.988E-02	1.983E-02	1.140F 39	7.939E-08	1.987E-02	1.982E-02	5.140F 38	7.934E-08	1.986E-02	1.981E-02	
1.140F 40	5.324E-08	1.334E-02	1.329E-02	2.140F 40	5.319E-08	1.333E-02	1.328E-02	1.140F 40	5.314E-08	1.332E-02	1.327E-02	5.140F 39	5.309E-08	1.331E-02	1.326E-02	
1.140F 41	3.214E-08	4.264E-02	4.259E-02	2.140F 41	3.209E-08	4.258E-02	4.253E-02	1.140F 41	3.204E-08	4.257E-02	4.252E-02	5.140F 40	3.203E-08	4.256E-02	4.251E-02	
1.140F 42	1.754E-08	1.994E-02	1.989E-02	2.140F 42	1.749E-08	1.988E-02	1.983E-02	1.140F 42	1.744E-08	1.987E-02	1.982E-02	5.140F 41	1.739E-08	1.986E-02	1.981E-02	
1.140F 43	1.204E-08	1.334E-02	1.329E-02	2.140F 43	1.202E-08	1.333E-02	1.328E-02	1.140F 43	1.197E-08	1.332E-02	1.327E-02	5.140F 42	1.193E-08	1.331E-02	1.326E-02	
1.140F 44	7.949E-09	4.264E-02	4.259E-02	2.140F 44	7.944E-09	4.258E-02	4.253E-02	1.140F 44	7.939E-09	4.257E-02	4.252E-02	5.140F 43	7.934E-09	4.256E-02	4.251E-02	
1.140F 45	5.324E-09	1.994E-02	1.989E-02	2.140F 45	5.319E-09	1.988E-02	1.983E-02	1.140F 45	5.314E-09	1.987E-02	1.982E-02	5.140F 44	5.309E-09	1.986E-02	1.981E-02	

Table 6

X (P)	P (x, 0)			P (x, 1)			P (x, 2)		
	1/1000	1/500	1/100	1/50	1/10	1/500	1/100	1/50	1/10
2.0494e-01	9.4852e-01	9.4707e-01	9.4503e-01	9.4037e-01	9.3508e-01	9.3048e-01	9.2574e-01	9.2115e-01	9.1756e-01
1.7229e-01	9.4245e-01	9.4135e-01	9.4025e-01	9.3923e-01	9.3808e-01	9.3724e-01	9.3625e-01	9.3549e-01	9.3484e-01
1.4914e-01	9.3794e-01	9.3701e-01	9.3632e-01	9.3549e-01	9.3482e-01	9.3421e-01	9.3362e-01	9.3313e-01	9.3271e-01
1.2777e-01	9.3491e-01	9.3422e-01	9.3367e-01	9.3294e-01	9.3227e-01	9.3171e-01	9.3124e-01	9.3081e-01	9.3045e-01
1.0624e-01	9.2713e-01	9.2617e-01	9.2518e-01	9.2421e-01	9.2324e-01	9.2227e-01	9.2131e-01	9.2035e-01	9.1941e-01
8.4985e-02	9.1978e-01	9.1898e-01	9.1818e-01	9.1739e-01	9.1658e-01	9.1578e-01	9.1498e-01	9.1421e-01	9.1348e-01
6.3204e-02	9.1193e-01	9.1113e-01	9.1034e-01	9.0954e-01	9.0874e-01	9.0794e-01	9.0714e-01	9.0635e-01	9.0561e-01
4.1612e-02	9.0395e-01	9.0315e-01	9.0236e-01	9.0156e-01	9.0076e-01	9.0006e-01	9.0036e-01	9.0006e-01	9.0000e-01
2.0000e-02	8.9597e-01	8.9517e-01	8.9437e-01	8.9357e-01	8.9277e-01	8.9197e-01	8.9117e-01	8.9037e-01	8.8957e-01
9.5131e-03	8.8797e-01	8.8717e-01	8.8637e-01	8.8557e-01	8.8477e-01	8.8397e-01	8.8317e-01	8.8237e-01	8.8157e-01
7.1319e-03	8.7987e-01	8.7907e-01	8.7827e-01	8.7747e-01	8.7667e-01	8.7587e-01	8.7507e-01	8.7427e-01	8.7347e-01
5.0000e-03	8.7197e-01	8.7117e-01	8.7037e-01	8.6957e-01	8.6877e-01	8.6797e-01	8.6717e-01	8.6637e-01	8.6557e-01
3.0000e-03	8.6407e-01	8.6327e-01	8.6247e-01	8.6167e-01	8.6087e-01	8.5997e-01	8.5917e-01	8.5837e-01	8.5757e-01
1.0000e-03	8.5617e-01	8.5537e-01	8.5457e-01	8.5377e-01	8.5297e-01	8.5217e-01	8.5137e-01	8.5057e-01	8.4977e-01
7.0000e-04	8.5027e-01	8.4947e-01	8.4867e-01	8.4787e-01	8.4707e-01	8.4627e-01	8.4547e-01	8.4467e-01	8.4387e-01
5.0000e-04	8.4437e-01	8.4357e-01	8.4277e-01	8.4197e-01	8.4117e-01	8.4037e-01	8.3957e-01	8.3877e-01	8.3797e-01
3.0000e-04	8.3847e-01	8.3767e-01	8.3687e-01	8.3607e-01	8.3527e-01	8.3447e-01	8.3367e-01	8.3287e-01	8.3207e-01
1.0000e-04	8.3247e-01	8.3167e-01	8.3087e-01	8.2997e-01	8.2917e-01	8.2837e-01	8.2757e-01	8.2677e-01	8.2597e-01
7.0000e-05	8.3147e-01	8.3067e-01	8.2987e-01	8.2897e-01	8.2817e-01	8.2737e-01	8.2657e-01	8.2577e-01	8.2497e-01
5.0000e-05	8.3047e-01	8.2957e-01	8.2867e-01	8.2777e-01	8.2687e-01	8.2597e-01	8.2507e-01	8.2417e-01	8.2327e-01
3.0000e-05	8.2947e-01	8.2857e-01	8.2767e-01	8.2677e-01	8.2587e-01	8.2497e-01	8.2407e-01	8.2317e-01	8.2227e-01
1.0000e-05	8.2847e-01	8.2757e-01	8.2667e-01	8.2577e-01	8.2487e-01	8.2397e-01	8.2307e-01	8.2217e-01	8.2127e-01
7.0000e-06	8.2747e-01	8.2657e-01	8.2567e-01	8.2477e-01	8.2387e-01	8.2297e-01	8.2207e-01	8.2117e-01	8.2027e-01
5.0000e-06	8.2647e-01	8.2557e-01	8.2467e-01	8.2377e-01	8.2287e-01	8.2197e-01	8.2107e-01	8.2017e-01	8.1927e-01
3.0000e-06	8.2547e-01	8.2457e-01	8.2367e-01	8.2277e-01	8.2187e-01	8.2097e-01	8.2007e-01	8.1917e-01	8.1827e-01
1.0000e-06	8.2447e-01	8.2357e-01	8.2267e-01	8.2177e-01	8.2087e-01	8.1997e-01	8.1907e-01	8.1817e-01	8.1727e-01
7.0000e-07	8.2347e-01	8.2257e-01	8.2167e-01	8.2077e-01	8.1987e-01	8.1897e-01	8.1807e-01	8.1717e-01	8.1627e-01
5.0000e-07	8.2247e-01	8.2157e-01	8.2067e-01	8.1977e-01	8.1887e-01	8.1797e-01	8.1707e-01	8.1617e-01	8.1527e-01
3.0000e-07	8.2147e-01	8.2057e-01	8.1967e-01	8.1877e-01	8.1787e-01	8.1697e-01	8.1607e-01	8.1517e-01	8.1427e-01
1.0000e-07	8.2047e-01	8.1957e-01	8.1867e-01	8.1777e-01	8.1687e-01	8.1597e-01	8.1507e-01	8.1417e-01	8.1327e-01
7.0000e-08	8.1947e-01	8.1857e-01	8.1767e-01	8.1677e-01	8.1587e-01	8.1497e-01	8.1407e-01	8.1317e-01	8.1227e-01
5.0000e-08	8.1847e-01	8.1757e-01	8.1667e-01	8.1577e-01	8.1487e-01	8.1397e-01	8.1307e-01	8.1217e-01	8.1127e-01
3.0000e-08	8.1747e-01	8.1657e-01	8.1567e-01	8.1477e-01	8.1387e-01	8.1297e-01	8.1207e-01	8.1117e-01	8.1027e-01
1.0000e-08	8.1647e-01	8.1557e-01	8.1467e-01	8.1377e-01	8.1287e-01	8.1197e-01	8.1107e-01	8.1017e-01	8.0927e-01
7.0000e-09	8.1547e-01	8.1457e-01	8.1367e-01	8.1277e-01	8.1187e-01	8.1097e-01	8.1007e-01	8.0917e-01	8.0827e-01
5.0000e-09	8.1447e-01	8.1357e-01	8.1267e-01	8.1177e-01	8.1087e-01	8.0997e-01	8.0907e-01	8.0817e-01	8.0727e-01
3.0000e-09	8.1347e-01	8.1257e-01	8.1167e-01	8.1077e-01	8.0987e-01	8.0897e-01	8.0807e-01	8.0717e-01	8.0627e-01
1.0000e-09	8.1247e-01	8.1157e-01	8.1067e-01	8.0977e-01	8.0887e-01	8.0797e-01	8.0707e-01	8.0617e-01	8.0527e-01
7.0000e-10	8.1147e-01	8.1057e-01	8.0967e-01	8.0877e-01	8.0787e-01	8.0697e-01	8.0607e-01	8.0517e-01	8.0427e-01
5.0000e-10	8.1047e-01	8.0957e-01	8.0867e-01	8.0777e-01	8.0687e-01	8.0597e-01	8.0507e-01	8.0417e-01	8.0327e-01
3.0000e-10	8.0947e-01	8.0857e-01	8.0767e-01	8.0677e-01	8.0587e-01	8.0497e-01	8.0407e-01	8.0317e-01	8.0227e-01
1.0000e-10	8.0847e-01	8.0757e-01	8.0667e-01	8.0577e-01	8.0487e-01	8.0397e-01	8.0307e-01	8.0217e-01	8.0127e-01
7.0000e-11	8.0747e-01	8.0657e-01	8.0567e-01	8.0477e-01	8.0387e-01	8.0297e-01	8.0207e-01	8.0117e-01	8.0027e-01
5.0000e-11	8.0647e-01	8.0557e-01	8.0467e-01	8.0377e-01	8.0287e-01	8.0197e-01	8.0107e-01	8.0017e-01	8.0000e-01
3.0000e-11	8.0547e-01	8.0457e-01	8.0367e-01	8.0277e-01	8.0187e-01	8.0097e-01	8.0007e-01	8.0000e-01	8.0000e-01
1.0000e-11	8.0447e-01	8.0357e-01	8.0267e-01	8.0177e-01	8.0087e-01	8.0000e-01	8.0000e-01	8.0000e-01	8.0000e-01
7.0000e-12	8.0347e-01	8.0257e-01	8.0167e-01	8.0077e-01	8.0000e-01	8.0000e-01	8.0000e-01	8.0000e-01	8.0000e-01
5.0000e-12	8.0247e-01	8.0157e-01	8.0067e-01	8.0000e-01	8.0000e-01	8.0000e-01	8.0000e-01	8.0000e-01	8.0000e-01
3.0000e-12	8.0147e-01	8.0057e-01	8.0000e-01						
1.0000e-12	8.0047e-01	8.0000e-01							
7.0000e-13	7.9947e-01	8.0000e-01							
5.0000e-13	7.9847e-01	8.0000e-01							
3.0000e-13	7.9747e-01	8.0000e-01							
1.0000e-13	7.9647e-01	8.0000e-01							
7.0000e-14	7.9547e-01	8.0000e-01							
5.0000e-14	7.9447e-01	8.0000e-01							
3.0000e-14	7.9347e-01	8.0000e-01							
1.0000e-14	7.9247e-01	8.0000e-01							
7.0000e-15	7.9147e-01	8.0000e-01							
5.0000e-15	7.9047e-01	8.0000e-01							
3.0000e-15	7.8947e-01	8.0000e-01							
1.0000e-15	7.8847e-01	8.0000e-01							
7.0000e-16	7.8747e-01	8.0000e-01							
5.0000e-16	7.8647e-01	8.0000e-01							
3.0000e-16	7.8547e-01	8.0000e-01							
1.0000e-16	7.8447e-01	8.0000e-01							
7.0000e-17	7.8347e-01	8.0000e-01							
5.0000e-17	7.8247e-01	8.0000e-01							
3.0000e-17	7.8147e-01	8.							

Table 7

$\frac{\partial^2}{\partial x^2}$						
$\frac{\partial^2}{\partial y^2}$						
τ	$1/1000$	$1/500$	$\frac{1}{\sqrt{100}}$	$1/100$	$1/50$	$1/10$
P0	0.125	1.7093e-01	4.3247e-01	1.0349e-01	1.0349e-01	1.0349e-01
	0.250	1.7093e-01	4.3247e-01	1.0349e-01	1.0349e-01	1.0349e-01
	0.500	7.5112e-01	1.0412e-01	3.1415e-01	3.1415e-01	3.1415e-01
	1.000	2.5314e-01	2.6760e-01	5.1653e-02	5.1653e-02	5.1653e-02
P1	0.125	1.7093e-01	3.4124e-01	2.0302e-01	1.0349e-01	1.0349e-01
	0.250	1.7093e-01	3.4124e-01	2.0302e-01	1.0349e-01	1.0349e-01
	0.500	2.1177e-01	3.1350e-01	1.9213e-01	1.9213e-01	1.9213e-01
	1.000	2.7154e-01	2.6760e-01	9.5453e-02	1.1875e-02	3.1929e-02
P2	0.125	2.8954e-00	1.2522e-01	2.0302e-01	2.0302e-01	2.0302e-01
	0.250	2.1053e-01	1.4543e-01	2.1973e-01	3.1415e-01	2.0063e-01
	0.500	4.5579e-02	1.3663e-01	1.1467e-01	2.0461e-01	1.6775e-01
	1.000	1.5419e-02	2.1177e-01	1.1883e-01	2.0302e-01	1.6441e-01
P3	0.125	8.0454e-01	5.0119e-01	1.0000e-01	1.0000e-01	1.0000e-01
	0.250	1.1876e-01	2.5046e-01	1.0349e-01	1.0349e-01	1.0349e-01
	0.500	1.7715e-01	2.1521e-01	1.0349e-01	1.0349e-01	1.0349e-01
	1.000	1.7715e-01	2.1521e-01	1.0349e-01	1.0349e-01	1.0349e-01
P4	0.125	1.7118e-01	5.1549e-01	2.4137e-01	3.4934e-01	5.0464e-01
	0.250	1.7118e-01	5.1549e-01	2.4137e-01	3.4934e-01	5.0464e-01
	0.500	1.6233e-01	5.0125e-01	2.6669e-01	3.0593e-01	2.7156e-01
	1.000	1.6233e-01	5.0125e-01	2.6669e-01	3.0593e-01	2.7156e-01
P5	0.125	2.1048e-01	1.2522e-01	2.0302e-01	2.0302e-01	2.0302e-01
	0.250	2.0578e-02	1.1504e-01	2.0302e-01	2.0302e-01	2.0302e-01
	0.500	2.3718e-02	1.1504e-01	2.1586e-01	7.4296e-02	2.0461e-01
	1.000	3.5418e-02	1.1504e-01	1.9114e-01	6.8487e-02	1.9696e-01
P6	0.125	6.6318e-01	6.2995e-01	3.1023e-01	5.4944e-01	2.7008e-01
	0.250	6.7593e-01	6.5464e-01	3.1722e-01	5.4944e-01	2.1759e-01
	0.500	6.7678e-01	6.6403e-01	3.1647e-01	5.4944e-01	2.1759e-01
	1.000	6.7678e-01	6.6403e-01	3.1647e-01	5.4944e-01	2.1759e-01
P7	0.125	1.2938e-02	7.9055e-02	1.8213e-01	1.3607e-01	2.1582e-02
	0.250	1.0718e-02	6.1970e-02	1.8213e-01	1.3607e-01	2.1582e-02
	0.500	1.0718e-02	6.1970e-02	1.8213e-01	1.3607e-01	2.1582e-02
	1.000	1.1644e-02	6.4232e-02	1.7759e-01	1.4116e-01	1.3718e-02
P8	0.125	6.4193e-02	2.6284e-02	2.0302e-01	1.1795e-01	1.0093e-02
	0.250	1.1445e-01	2.5547e-02	5.0820e-01	6.2995e-01	1.0093e-01
	0.500	1.1445e-01	2.5547e-02	5.0820e-01	6.2995e-01	1.0093e-01
	1.000	1.1445e-01	2.7759e-02	5.1609e-01	1.1396e-01	5.9967e-02
P9	0.125	7.1957e-02	1.3959e-02	2.7416e-01	8.1381e-01	5.0134e-00
	0.250	1.0708e-02	1.9079e-02	2.9313e-01	8.2176e-01	5.2176e-01
	0.500	1.0708e-02	1.9079e-02	3.0501e-01	8.2176e-01	5.2176e-01
	1.000	1.0708e-02	1.9079e-02	3.0501e-01	8.2176e-01	5.2176e-01
P10	0.125	9.4654e-01	7.4419e-01	1.4713e-01	1.5111e-01	9.4654e-01
	0.250	9.3598e-01	7.4619e-01	1.4772e-01	1.5148e-01	9.4654e-01
	0.500	9.2655e-01	7.4917e-01	1.4923e-01	1.5154e-01	9.4654e-01
	1.000	9.1474e-01	7.4964e-01	1.4944e-01	1.5154e-01	9.4654e-01
P11	0.125	6.4171e-02	6.2995e-02	2.0302e-01	1.0093e-01	6.4171e-02
	0.250	6.4171e-02	6.2995e-02	2.0302e-01	1.0093e-01	6.4171e-02
	0.500	6.4171e-02	6.2995e-02	2.0302e-01	1.0093e-01	6.4171e-02
	1.000	6.4171e-02	6.2995e-02	2.0302e-01	1.0093e-01	6.4171e-02
P12	0.125	4.4995e-02	1.2653e-01	2.0277e-01	2.0295e-01	1.5744e-01
	0.250	4.7630e-02	1.2653e-01	2.0277e-01	2.0295e-01	1.5744e-01
	0.500	5.1351e-02	1.2653e-01	2.0277e-01	2.0295e-01	1.5744e-01
	1.000	5.1351e-02	5.1523e-02	1.2653e-01	2.1236e-01	1.9261e-02
P13	0.125	4.4995e-02	1.2653e-01	2.0277e-01	2.0295e-01	1.5744e-01
	0.250	4.7630e-02	1.2653e-01	2.0277e-01	2.0295e-01	1.5744e-01
	0.500	5.1351e-02	1.2653e-01	2.0277e-01	2.0295e-01	1.5744e-01
	1.000	5.1351e-02	5.1523e-02	1.2653e-01	2.1236e-01	1.9261e-02
P14	0.125	4.4995e-02	1.2653e-01	2.0277e-01	2.0295e-01	1.5744e-01
	0.250	4.7630e-02	1.2653e-01	2.0277e-01	2.0295e-01	1.5744e-01
	0.500	5.1351e-02	1.2653e-01	2.0277e-01	2.0295e-01	1.5744e-01
	1.000	5.1351e-02	5.1523e-02	1.2653e-01	2.1236e-01	1.9261e-02
P15	0.125	3.2909e-03	2.0040e-02	6.3204e-02	1.3139e-01	1.7230e-01
	0.250	3.2454e-03	1.6216e-02	6.3003e-02	1.3139e-01	1.6796e-01
	0.500	3.2454e-03	1.5986e-02	6.3204e-02	1.3139e-01	1.6796e-01
	1.000	3.2454e-03	1.5986e-02	6.3204e-02	1.3139e-01	1.6796e-01
P16	0.125	3.2909e-03	2.0040e-02	6.3204e-02	1.3139e-01	1.7230e-01
	0.250	3.2454e-03	1.6216e-02	6.3003e-02	1.3139e-01	1.6796e-01
	0.500	3.2454e-03	1.5986e-02	6.3204e-02	1.3139e-01	1.6796e-01
	1.000	3.2454e-03	1.5986e-02	6.3204e-02	1.3139e-01	1.6796e-01

Table 8

$\frac{\partial^2}{\partial x^2}$						
$\frac{\partial^2}{\partial y^2}$						
τ	$1/1000$	$1/500$	$\frac{1}{\sqrt{100}}$	$1/100$	$1/50$	$1/10$
R ($\tau, 1$)	0.125	4.2358e-04	1.0468e-03	1.6648e-03	1.8418e-03	1.8479e-03
	0.250	4.2358e-04	1.0468e-03	1.6648e-03	1.8418e-03	1.8479e-03
	0.500	3.7178e-04	2.9598e-04	2.9598e-04	2.9598e-04	2.9598e-04
	1.000	1.0795e-04	1.0795e-04	2.7958e-04	2.7958e-04	2.7958e-04
R ($\tau, 2$)	0.125	4.2358e-04	4.1016e-04	1.0468e-03	1.8418e-03	1.8479e-03
	0.250	4.2358e-04	4.1016e-04	1.0468e-03	1.8418e-03	1.8479e-03
	0.500	3.7178e-04	2.9598e-04	2.9598e-04	2.9598e-04	2.9598e-04
	1.000	1.0795e-04	1.0795e-04	2.7958e-04	2.7958e-04	2.7958e-04
R ($\tau, 3$)	0.125	6.3116e-05	1.2778e-04	1.8418e-03	1.8479e-03	1.8479e-03
	0.250	6.3116e-05	1.2778e-04	1.8418e-03	1.8479e-03	1.8479e-03
	0.500	5.1815e-05	1.2778e-04	1.8418e-03	1.8479e-03	1.8479e-03
	1.000	1.0795e-05	1.0795e-04	2.7958e-04	2.7958e-04	2.7958e-04
R ($\tau, 4$)	0.125	6.3116e-05	2.3178e-05	1.8418e-03	1.8479e-03	1.8479e-03
	0.250	6.3116e-05	2.3178e-05	1.8418e-03	1.8479e-03	1.8479e-03
	0.500	5.1815e-05	2.3178e-05	1.8418e-03	1.8479e-03	1.8479e-03
	1.000	1.0795e-05	1.0795e-04	2.7958e-04	2.7958e-04	2.7958e-04
R ($\tau, 5$)	0.125	6.3116e-05	3.3578e-05	1.8418e-03	1.8479e-03	1.8479e-03
	0.250	6.3116e-05	3.3578e-05	1.8418e-03	1.8479e-03	1.8479e-03
	0.500	5.1815e-05	3.3578e-05	1.8418e-03	1.8479e-03	1.8479e-03
	1.000	1.0795e-05	1.0795e-04	2.7958e-04	2.7958e-04	2.7958e-04
R ($\tau, 6$)	0.125	6.3116e-05	4.3978e-05	1.8418e-03	1.8479e-03	1.8479e-03
	0.250	6.3116e-05	4.3978e-05	1.8418e-03	1.8479e-03	1.8479e-03