PROCEEDINGS OF THE THIRD SYMPOSIUM ON SCIENCE OF HADRONS UNDER EXTREME CONDITIONS
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(Ed.) Satoshi CHIBA

日本原子力研究所
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The third symposium on Science of Hadrons under Extreme Conditions, organized by the Research Group for Hadron Science, Advanced Science Research Center, was held at Tokai Research Establishment of JAERI on January 29 to 31, 2001. The symposium was devoted for discussions and presentations of research results in wide variety of hadron physics such as nuclear matter, high-energy nuclear reactions, quantum chromodynamics, neutron stars, supernovae, nucleosynthesis as well as finite nuclei to understand various aspects of hadrons under extreme conditions. Twenty two papers on these topics presented at the symposium, including a special talk on the present status of JAERI-KEK joint project on high-intentisy proton accelerator, aroused lively discussions among approximately 40 participants.

Keywords: Proceedings, Hadrons under Extreme Conditions, Neutron Stars, Nuclear Matter, Supernovae, Nucleosynthesis, High-energy Nuclear Reactions, Quantum Chromodynamics, Joint Project

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第3回「極限条件におけるハドロン科学」研究会
報告集
2001年1月29日～1月31日、東海研究所、東海村

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第3回「極限条件におけるハドロン科学」研究会は、先端基礎研究センター第203回
基礎科学セミナーとして極限ハドロン科学研究グループの主催で、2001年1月29日～31
日にかけて、東海研究所先端基礎研究交流棟にて行われた。核子、ハドロン及びクォー
クから成る多体系の極限条件における様々な様相を理解する上で重要な核子物質、高エ
ネルギー核反応、中性子星、量子色力学及び超新星爆発・元素合成、有限核子系等のト
ピックスについての22件の発表が行われた。特別セッションとして、原研及び高エネルギ
ー加速器研究機構で進められている統合計画の概要についての講演が設けられた。参
加総数は約40名で、盛況のうちに全日程を終えた。本レポートはその報告集である。

本研究会の世話人は、日本原子力研究所先端基礎研究センター 千葉 敏、竹本 宏輝、
福島 昌宏、平田 雄一、筑波大学·原研物質科学研究部 近角 眞平、京都大学 堀内
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山 智幸、福岡教育大学 松崎 昌之、愛知淑徳大学 賀松 和浩が担当した。

日本原子力研究所 (東海駐在):〒319-1195 茨城県那珂郡東海村白方白根2-4
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付録：第3回「極限条件におけるハドロン科学」研究会プログラム .............. 114
1. クォーク強磁性における二重性
Duality in quark ferromagnetism

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abstract

A self-consistent treatment of quark ferromagnetism is presented within the relativistic Hartree-Fock approximation. There appear various kinds of mean fields in the system, among which the axial-vector mean field is responsible to ferromagnetism. The ferromagnetic phase is further analyzed by the use of relativistic Stoner model, where the gluon propagator is replaced by a constant. The single-particle energies for two spin states of quarks exhibit the momentum-dependent exchange splitting, which in turn induces deformation of Fermi sea. These features resemble those in the pion-condensation theory. Actually, it is shown that there is a duality in the quark ferromagnetism in this model; it can be regarded as a pion condensed phase from another viewpoint.

I. INTRODUCTION

In the recent paper [1] we have presented an idea that quark matter should be ferromagnetic at low baryon densities, using the perturbational discussion. The original discussion goes in parallel with that for the electron system [2], except relativistic effects. We have also suggested that the superstrong magnetic field, recently observed in magnetars [3], can be explained by this idea, if they are regarded as quark stars [4]. We have used the Dirac spinor of free particle and the momentum independent spin vector to estimate the polarization energy due to the Fock exchange interaction of the one-gluon-exchange (OGE), so that single particle energies become the same with each other and in turn the shape of Fermi sea remains spherical.

On the other hand it has been shown by using a variational method that the momentum dependent spin vector and deformation of Fermi sea should be more favored for the system to be ferromagnetic in the relativistic ferromagnetism [5]. These effects are nonperturbative one. If this is the case also for quark ferromagnetism, we must carefully treat quark matter in the nonperturbative way. The purpose of this paper is to give a formulation to this end within the Hartree-Fock (HF) approximation.

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II. SELF-CONSISTENT APPROACH

In the non-relativistic theories, the plane-wave function with the definite spin vector is always the solution of HF equation and their single-particle energies show the (momentum-independent) exchange splitting. On the other hand, the form of the Dirac spinor should be no longer the same as the one of free particle in the relativistic theories, because spin operator does not commute with Hamiltonian and it should be momentum dependent. So we must solve the selfconsistent HF equation in the relativistic theories.

Total energy of quark matter with OGE, $E$, is given as:

$$E = \sum_\zeta \int \frac{d^3k}{(2\pi)^3} \bar{u}(k, \zeta)(k\gamma + m_q)u(k, \zeta)$$
\[-\frac{g^2}{2} \text{tr}(\lambda_a/2\lambda_a/2) \sum_\zeta \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \bar{u}(k, \zeta)\gamma_\mu u(q, \zeta')\bar{u}(q, \zeta')\gamma_\nu u(k, \zeta)D^{\mu\nu}(k - q). \]  \hspace{1cm} (1)

with the gluon propagator, $D^{\mu\nu}(k - q) = -g^{\mu\nu}/(k - q)^2$ with the Feynman gauge. The Fock exchange interaction is manifestly chiral-invariant under the transformation:

$$u(q) \rightarrow \exp(i\gamma_5\theta)u(q). \] \hspace{1cm} (2)

Applying the Fierz transformation for the Fock exchange interaction, we get

$$\bar{u}(k, \zeta)\gamma_\mu u(q, \zeta')\bar{u}(q, \zeta')\gamma_\mu u(k, \zeta)$$
\n$$= (1\gamma^5\gamma^5 - 1/2\gamma_\mu\gamma^\mu - \gamma_\mu\gamma^5\gamma^5\gamma^5\gamma^5). \] \hspace{1cm} (NJL type)

where $\Gamma \cdot \Gamma \equiv \bar{u}(k, \zeta)\Gamma u(k, \zeta)\bar{u}(q, \zeta')\Gamma u(q, \zeta')$. Then the HF equation for each color quark becomes

$$(\slashed{p} - m_q)u(p, \zeta) + \left[U_s(p, \zeta) + U_{ps}(p, \zeta)\gamma^5 + U_v(p, \zeta) + \gamma_5 U_a(p, \zeta)\right] u(p, \zeta) = 0. \] \hspace{1cm} (4)

Each mean field has a non-local form s.t.

$$U_\Gamma(p) \equiv \sum_\zeta \int \frac{d^3q}{(2\pi)^3} \bar{u}(q)(\Gamma u(q))D(p - q) = -i \int_C \frac{d^4q}{(2\pi)^4} \text{tr}[\Gamma S(q)]D(p - q). \] \hspace{1cm} (5)

III. RELATIVISTIC STONER MODEL

If the propagator $D(p - q)$ is replaced by the constant $1/\Lambda^2$, each mean field becomes local one and the coupling strength between quarks can be measured by the effective coupling strength, $g^2/\Lambda^2$ or $\alpha_c/\Lambda^2[\text{MeV}^{-2}]$. Then we can easily see that $U_a^0 = U_s^0 = U_{ps}^0 = 0$ are self-consistent solutions, and other mean fields are constant. We can put $U_v^0 = U_v$ in this case. The HF equation then renders

$$(\slashed{p}^* - m^* + \gamma_5 U_a^0)u(p, \zeta) = 0. \] \hspace{1cm} (6)

where $p^* = p + U_v$ and $m^* = m - U_s$.

The quark propagator is given by
\[ S(p) = \frac{1}{\vec{p}^* - m^* + \gamma_0 U_a} \equiv \frac{N}{D}. \]  

The solutions for \( D = 0 \), which is a transcendental equation, give the four energies corresponding to positive and negative solutions with two polarizations \( \zeta = \pm 1 \). When \( U_a = 0 \), the positive energy solutions are given by

\[ E^\pm_p = -U_v^0 + \sqrt{E^*_p^2 + |U_a|^2 \pm 2\sqrt{(p^* \cdot U_a)^2 + m^*2|U_a|^2}}, \]  

with \( E^*_p = (m^* + |p^*|^2)^{1/2} \). In the nonrelativistic limit,

\[ E^\pm_p \simeq -U_v^0 + m^* + \frac{p^2}{2m} \pm |U_a|, \]  

and the exchange splitting is given by \( \Delta_{exch} \equiv 2|U_a| \), which is momentum independent. On the other hand it becomes momentum dependent in the relativistic case and shows an anisotropy in the momentum space. Therapy, Fermi sea becomes deformed. In the figures we depict a rough image of these features.

**FIG. 1.** Dispersion relations for "up" and "down" states. They show the anisotropy in the momentum space.

**FIG. 2.** Shape of the Fermi seas: "prolate" for majority spins (\( \uparrow \)) and "oblate" for minority spins (\( \downarrow \)).

The total energy density is then given by

\[ \epsilon = 3 \int \frac{d^3q}{(2\pi)^3} \left[ E_q^- \theta_- + E_q^+ \theta_+ \right] + \frac{9}{8g^2} U_v^2 \Lambda^2 - \frac{9}{4g^2} U_a^2 \Lambda^2 + \frac{9}{4g^2} U_v^2 \Lambda^2, \]  

with \( \theta_\pm = \theta(E_F - E_q^\pm) \). Some numerical analysis of this model is presented in ref. [6].

**IV. DUALITY**

Pion condensatin has a long history since first suggested in early seventies. Its important feature is the \( p \)-wave Bose-Einstein condensation (BEC) due to the derivative coupling
with nucleons. Since pions are pseudo-scalar bosons, the coupling becomes spin-dependent. Hence the spin configuration of nucleons in the condensed phase shows a distinctive features; ferromagnetic or anti-ferromagnetic one [7]. Here we are interested in the latter case. We have already seen that the total energy (1) is chiral-invariant except the mass term and the Fock exchange interaction becomes NJL type within the relativistic Stoner model. In the following we show ferromagnetism becomes equivalent with pion condensation in the chiral models.

Take, e.g., the $\sigma$ model and consider the following ansatz\(^1\).

$$\sigma(r) = \bar{\sigma} \cos(q \cdot r), \pi_3(r) = \bar{\sigma} \sin(q \cdot r), \pi_1 = \pi_2 = 0.$$  \hspace{1cm} (11)

Since $\sigma + i\gamma_5 \tau \cdot \pi = \bar{\sigma} \exp(i\gamma_5 \tau_3 q \cdot r)$, the Dirac equation has the form,

$$(-i \alpha \cdot \nabla + \beta m \exp(i\gamma_5 \tau_3 q \cdot r))u(r) = Eu(r), \hspace{1cm} m = g\bar{\sigma}$$  \hspace{1cm} (12)

Using the Weinberg transformation s.t. \(u(r) = \exp(-1/2i\gamma_5 \tau_3 q \cdot r)u^R(r)\), we find that \(u^R(k) \equiv \int 1/(2\pi)^4 \exp(-ikx)u^R(x)\) satisfies the equation,

$$\{\not{\gamma} - m - 1/2\gamma_5 \gamma_3 \gamma_\tau\}u^R(k) = 0.$$  \hspace{1cm} (13)

This form is quite similar to that for ferromagnetism (see Eq.(6)), and thereby we find a correspondence, $q/2 \leftrightarrow U_a$ ("momentum"). Note that the $p$-wave nature of pions is important in this context. We can easily see that the expectation value of $\gamma_5$ vanishes in this case, \(\bar{u}^R(k)\gamma_5 u^R(k) = 0\), which correctly corresponds to $U_{ps} = 0$ in Eq. (6); in other words we can say that non-zero chirality is converted to the non-zero axial-vector field under the Weinberg transformation.

We summarize the duality relation:

\[
\begin{align*}
\text{FM} & \quad \leftrightarrow \quad \text{BEC} \\
\text{M}(\text{magnetization}) & \quad \leftrightarrow \quad A \times \nabla \phi \\
\text{Spin wave} & \quad \leftrightarrow \quad \text{Boson}
\end{align*}
\]

Fierz trans.

\[\]

\[1\]This may remind you of the spin density wave by Overhauser [8], where spin up and down sates oscillate in space, keeping the total density uniform. In this sense, the configuration (11) may be called chiral density wave.
V. SUMMARY AND CONCLUDING REMARKS

We have presented a self-consistent formulation for quark ferromagnetism and analyzed it within the relativistic Stoner model, which is similar to the Nambu-Jona-Lasinio model. We have seen the momentum-dependent exchange splitting for single-particle energies and deformation of Fermi sea. It may be natural to expect such a symmetry breaking in the momentum space, since spin and momentum are coupled with each other in the relativistic theories.

We have indicated a duality hidden in the relativistic ferromagnetism; the similarity of the HF equation to the Dirac equation in the presence of classical p-wave pion field in the chiral theories. We need further study to figure out the full picture of the duality. If this is the case, the spin-wave, expected as a low-lying excitation mode in the ferromagnetic phase, should have a nature of pions.

It may be interesting to explore coexistence or competition of the ferromagnetism (FM) and the superconductivity (SC) in quark matter. It should be worth noting, in this context, that their coexistence has been recently discovered in UGe$_2$ [9]. We need to treat these phenomena in a selfconsistent way to this end. We have not solved yet the full HF equations for FM phase due to the non-locality of the mean-fields, but it should be rather easy to discuss this subject within the relativistic Stoner model. Duality relation also suggested the interplay of FM and SC in the chiral symmetric model. The coexistence of SC and FM in the context of condensed matter physics has been recently discussed [10].

Finally it should be worth mentioning that there are some applications of our idea in relativistic heavy-ion physics or astrophysics and cosmology. In particular, the origin of the primordial magnetic field may be interesting subject.

2. Spontaneous Spin-Polarization and Phase Transition in the Relativistic Approach

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Abstract

We study the spin-polarization mechanism in the highly dense nuclear matter with the relativistic mean-field approach. In the relativistic Hartree-Fock framework we find that there are two kinds of spin-spin interaction channels, which are the axial-vector and tensor exchange ones. If each interaction is strong and different sign, the system loses the spherical symmetry and holds the spin-polarization in the high-density region. When the axial-vector interaction is negative enough, the system holds ferromagnetism.

1. Introduction

Recent discovery of "Magnetar" [1], which is a neutron star with super strong magnetic field, seems to revive a big question on the origin of the strong magnetic field. Since there is spread bulk hadronic matter beyond the nuclear density inside neutron stars, it should be interesting to consider its origin in the context of dynamics of hadronic matter; e.g., if the spin-polarization of baryons are realized in nuclear matter, ferromagnetism may occur in neutron stars.

For quark matter, one of the authors (T.T.) has recently indicated a possibility of spin-polarization of quarks interacting with one-gluon-exchange (OGE) interaction [2]. There relativistic effects in the Fock exchange interaction are found to give rise to a new mechanism for ferromagnetism, which is never appeared in the nonrelativistic case.

As for the normal nuclear matter Niembro et al. [3, 4] have suggested a possibility of spin-polarized nuclear matter using the relativistic Hartree-Fock (RHF) approach [5], though spontaneous spin-polarization occurs at too high density. The results suggest that the relativistic framework may be more favorite for spin-polarization than the nonrelativistic one.

Checking their framework, however, we find some problems about the calculation. Anyway there has been no systematic and sufficient discussions on this topic in the relativistic many-body approach [6], particularly in view of the relativistic effects. In this work we examine the spin-polarization of nucleon matter within the RHF approach, focusing on the breaking down of spherical symmetry and importance of the relativistic effects.
2. Formalism

In this section we briefly explain our formulation to describe the spin-polarized system. There should appear a special direction along the spin-polarization; it is defined to be oriented to the positive $z$-direction. Such a system breaks spherical symmetry while the axial symmetry around the $z$-axis is preserved.

In the RHF framework the interaction energy density in the isospin saturated system is generally written as

$$
\epsilon_{\text{int}} = \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \int \left[ T \{ iS(p) \} D_S(p-k) T \{ iS(k) \} \\ + T \{ iS(p) \gamma_\mu \} D_V(p-k) T \{ iS(k) \gamma^\mu \} + T \{ iS(p) \gamma_5 \} D_F(p-k) T \{ iS(k) \gamma_5 \} \\ + T \{ iS(p) \gamma_\mu \gamma_\nu \} D_A(p-k) T \{ iS(k) \gamma_\nu \gamma^\mu \} + T \{ iS(p) \sigma_{\mu\nu} \} D_T(p-k) T \{ iS(k) \sigma^{\mu\nu} \} \right] 
$$

(1)

for the one-boson exchange type interaction, assuming no derivative coupling. Here $S(p)$ is the nucleon propagator with momentum $p$, $D_\alpha (\alpha = S, V, P, A, T)$ is the linear combination of meson-propagators with the nucleon-meson couplings.

In the relativistic mean-field (RMF) approach we usually neglect the momentum dependence of the propagator because the nucleon-nucleon interaction can be effectively treated as the zero-range one in low energy phenomena as far as the typical energy and/or momentum scale is much less than the meson masses. Actually only very small momentum dependence has appeared in the full RHF calculation [5, 7]. Here, we take the zero-range approximation for two nucleon interaction which can be described as follows:

$$
D_\alpha = \frac{\tilde{C}_\alpha}{2M^2} \quad (\alpha = S, V, P, A, T).
$$

(2)

Even with this approximation the RHF calculation is still complicated because there appear the axial-vector and tensor mean fields. Instead of solving the exact self-consistent RHF equation, we take a variational approach in the RHF framework.

The two degrees of freedom of the spin polarization for each nucleon is denoted by $\zeta = 1$ and $\zeta = -1$, which we call spin-up and -down, respectively. Then we take the nucleon propagator with four-momentum $p$ in the following form;

$$
S(p, \zeta) = S_F(p, \zeta) + S_D(p, \zeta)
$$

(3)

with the propagators of a vacuum piece ($S_F$) and a density-dependent piece ($S_D$),

$$
S_F(p, \zeta) = \frac{(\gamma_\mu p^{\mu} + M^*) (1 + \gamma_5 \delta(p^0, \zeta))}{p^2 - M^{*2}},
$$

(4)

$$
S_D(p, \zeta) = \frac{(\gamma_\mu p^{\mu} + M^*) (1 + \gamma_5 \delta(p^0, \zeta))}{2} \frac{i\pi}{E_p^*} n(p, \zeta) \delta(p_0 - \epsilon_p),
$$

(5)

where $p^{\mu} \equiv p^\mu - U^\mu$, $M^* = M - U_s$ and $E_p^* = \sqrt{p^2 + M^{*2}}$. In these equations $U_s$ and $U_\mu$ are the scalar and vector mean fields, and $\epsilon_p$ is the single particle energy of the nucleon with
momentum $p$, $\varepsilon_p = E_p^* + U_0$. $a(p^*, \zeta)$ is the spin-vector of the nucleon with momentum $p$ which satisfies the following conditions:

$$a_\mu a^\mu = -1, \quad a_\mu p^{*\mu} = 0.$$  \hspace{1cm} (6)

In the following we only keep the density-dependent piece $S_D$ with the momentum distribution function $n(p, \zeta)$ to be determined, for which we assume the axial symmetry along the spin-polarization.

Then the total energy density $\varepsilon_T$ is separated into two parts: the spin-independent part $\varepsilon_{SID}$ and the spin-dependent part $\varepsilon_{SD}$ as

$$\varepsilon_T = \varepsilon_{SID} + \varepsilon_{SD}. \hspace{1cm} (7)$$

The axial-vector and tensor exchange channels contribute to the spin-dependent part $\varepsilon_{SD}$ while the kinetic energy and contributions from the scalar and vector channels are involved in the spin-independent one $\varepsilon_{SID}$.

Under the zero-range approximation the spin-independent part of the Fock contribution can be incorporated into the Hartree one. Then it is possible to redefine the two-body scalar interaction by taking into account the Fock terms, which corresponds to the usual relativistic Hartree (RH) approximation.

Then the spin-independent part of the energy density $\varepsilon_{SID}$ becomes

$$\varepsilon_{SID} = 2 \sum_\zeta \int \frac{d^3 p}{(2\pi)^3} n(p; \zeta) E_p^* + \tilde{U} + \frac{g^2}{2m^2} \rho_B^2,$$  \hspace{1cm} (8)

where $\rho_B$ indicates the baryon density.

On the other hand the spin-dependent energy density $\varepsilon_{SD}$ is calculated to be,

$$\varepsilon_{SD} = \frac{\bar{C}_A}{2M^2} \rho_A^2 + \frac{\bar{C}_T}{2M^2} \rho_T^2.$$  \hspace{1cm} (9)

with the axial-vector and tensor densities,

$$\rho_A = \int \frac{d^4 p}{(2\pi)^4} Tr\{S_D(p)\gamma^3\} = \rho_B < \Sigma_3 > = \sum_\zeta \int \frac{d^3 p}{(2\pi)^3} n(p; \zeta) \frac{M^*}{E_p} a_z$$  \hspace{1cm} (10)

$$\rho_T = \int \frac{d^4 p}{(2\pi)^4} Tr\{S_D(p)\sigma_{12}\} = \rho_B < \Sigma > = \sum_\zeta \int \frac{d^3 p}{(2\pi)^3} n(p; \zeta) (a_z - \frac{p_z}{E_p} a_0)$$  \hspace{1cm} (11)

Other components of the axial-vector and tensor densities are vanished because of the axial symmetry of the momentum distribution.

In order to figure out the properties of the spin-polarized matter, we solve RH equation and calculate the energy-density by fixing the baryon density $\rho_B$ and the spin-polarization parameter $x_s$ defined by

$$x_s \equiv (\rho_+ - \rho_-)/\rho_B.$$  \hspace{1cm} (12)
where \( \rho_{\uparrow} \) and \( \rho_{\downarrow} \) are density contributed by spin-up and spin-down nucleons, respectively. (For convenience the spin-up and spin-down states are indicated by the symbols \( \uparrow \) and \( \downarrow \), respectively.)

The total spin-polarization is directed to the positive direction of the z-axis; here we define a unit vector \( \zeta_{m} = (0, 0, \zeta) \). Usually the spin-vector \( a_{\mu} \) is chosen as \( (0, \zeta_{m}) \) at the rest frame of the nucleon, we should call this choice as Choice-1(Ch1). Then the spin-vector with momentum \( \mathbf{p} \) becomes

\[
\mathbf{a} = \left[ \zeta_{m} + \frac{(\zeta_{m} \cdot \mathbf{p})\mathbf{p}}{M^* (E_p^* + M^*)} \right], \quad a_0 = \frac{\zeta_{m} \cdot \mathbf{p}}{M^*};
\]

(13)

by way of the Lorentz transformation. In this choice \( \rho_A \) and \( \rho_T \) can be written as

\[
\rho_A = \frac{1}{3} \sum_{\zeta} \zeta \{ 2\rho_s(\zeta) + \rho_B(\zeta) \}, \quad \rho_T = \frac{1}{3} \sum_{\zeta} \zeta \{ \rho_s(\zeta) + 2\rho_B(\zeta) \}.
\]

(14)

Then the interaction-energy density can be written only in terms of the scalar \( \rho_s(\zeta) \) and vector densities \( \rho_B(\zeta) \) of nucleons with the spin \( \zeta \), and the expression still holds spherical symmetry.

Here we consider another choice, Choice-2(Ch2) for the vector \( a_{\mu} \) to give the maximum for \( |a_z| \) within our framework. In this new choice the spin-vector becomes

\[
\mathbf{a} = \frac{M^* \zeta_{m} + (\zeta_{m} \cdot \mathbf{p})\mathbf{p}}{M^* \sqrt{(\zeta_{m} \cdot \mathbf{p})^2 + M^*^2}}, \quad a_0 = \frac{E_p^* (\zeta_{m} \cdot \mathbf{p})}{M^* \sqrt{(\zeta_{m} \cdot \mathbf{p})^2 + M^*^2}};
\]

(15)

Substituting the above form into eqs.(10) and (11), we get

\[
\rho_A = 2 \sum_{\zeta} \zeta \int \frac{d^3 p}{(2\pi)^3} n(p; \zeta) \frac{\sqrt{p_z^2 + M^*^2}}{E_p^*} \]

(16)

\[
\rho_T = 2 \sum_{\zeta} \zeta \int \frac{d^3 p}{(2\pi)^3} n(p; \zeta) \frac{M^*}{\sqrt{p_z^2 + M^*^2}}
\]

(17)

Furthermore we should note that the above expressions of eq.(16) and eq.(17) do not preserve spherical symmetry. The relativistic effects automatically give rise to the spherical symmetry breaking. From this fact we can naturally expect that the momentum distribution \( n(p; \zeta) \) is allowed to be distorted while keeping the axial-symmetry. In order to estimate the effects of distortion of the momentum distribution, we introduce the quadrupole-distorted distribution function \( n(p) \) as

\[
n(p; \zeta) = n_0(e^{\lambda(\zeta)} p_z, e^{\lambda(\zeta)} p_y, e^{-2\lambda(\zeta)} p_z; \zeta),
\]

(18)

where \( n_0(p; \zeta) = \theta(p_F(\zeta) - |p|) \) with the Fermi-momentum \( p_F \). The parameter \( \lambda(\zeta) \) is determined to give the energy minimum of the spin-polarized system.
3. Results and Discussions

The two contributions from the axial-vector and tensor channels are derived by the Fierz transformation from the Fock exchange interactions in other channels. The values of two coupling strengths, $\tilde{C}_A$ and $\tilde{C}_T$, are still very ambiguous, and they cannot be individually determined at present both in theoretical and experimental ways. Then, we investigate the spin-polarization of nuclear matter by varying the values of $\tilde{C}_A$ and $\tilde{C}_T$ in this work.

The parameter-sets PM1 [8] are used for the RH calculation; We here define the spin-symmetry energy, inversely proportional to the magnetic susceptibility by

$$\varepsilon_{sp-sym} = \frac{\partial^2 E_T/A}{\partial <\Sigma z/A_x^2>^2} |_{x_0=0}. \quad (19)$$

In this work we take its value as $\varepsilon_{sp-sym} = 25$ (MeV), while it is not clearly determined from experimental information and we use three kinds of the parameter-set: $\tilde{C}_A = 0$ (SD1), $\tilde{C}_A = -50$ (SD2) and $\tilde{C}_A = -100$ (SD3). Here we restrict ourselves to the cases with $\tilde{C}_A < 0$, since matter would be ferromagnetic only in this case.

In Fig. 1 we show the density-dependence of the total energy per nucleon ($E_T/A$), the nucleon effective mass normalized by the bare mass ($M^*/M$), and the spin-symmetry energies $\varepsilon_{sp-sym}$. If this value becomes negative, the spin-saturated system becomes unstable and the spin-polarized one is favored. The long-dashed, dashed and solid lines indicate results for SD1, SD2 and SD3, respectively. For all the parameter-sets the spin-symmetry energy increases in the low density region as baryon density becomes larger. While the spin-
symmetry energy monotonously increases in the case of $\tilde{C}_A = 0$ (SD1), it decreases and becomes minus above a critical density $\rho_c$ for the cases of negative $\tilde{C}_A$ (SD2 and SD3): $\rho_c/\rho_0 = 8.74$ for SD2 and $\rho_c/\rho_0 = 4.28$ for SD3.

In the infinite density limit the effective mass $M^*$ goes to zero (Fig. 1b) and the scalar-density $\rho_s$ converge to the finite value in the RMF theory. Thus $\rho_A$ and $\rho_T$ must have density-dependence similar to $\rho_B$ and $\rho_S$, respectively; namely, when $\rho_B \to \infty$, $\rho_T$ converges to a finite value and $\rho_A$ is proportional to the baryon density $\rho_B$. In this limit, then, only the kinetic energy and the axial vector exchange channels contribute to the spin-symmetry energy $\varepsilon_{sp-sym}$. Since $<\Sigma_z/A> \sim x_s$ around the spin-saturated matter, the contribution from the kinetic energy is proportional to $\rho_B^{2/3}$ in the low density limit and to $\rho_B^{1/3}$ in the high-density limit. On the other hand, the contribution from the axial-vector exchange channels is proportional to $\tilde{C}_A \rho_B$ in the high-density limit; this behavior should be the same as the density dependence of the (isospin-) symmetry energy. From this fact we can easily see that, if $\tilde{C}_A < 0$, the spin-symmetry energy becomes negative and the spin-polarization spontaneously occurs above a certain critical density.

In Fig. 2 we give the spin-saturated matter $\Delta\varepsilon_T/A = (\varepsilon_T(x_s) - \varepsilon_T(x_s = 0))/A$ as functions of the spin-polarization parameter $x_s$ at $\rho_B = \rho_0$ (dotted line), $3\rho_0$ (dashed line), $5\rho_0$ (solid line) and $6\rho_0$ (chain-dotted line). It can be seen that above $\rho_B > 5\rho_0$ the value of the spin-polarization parameter at the energy-minimum moves from $x_s = 0$ to a finite value, whose value becomes larger as baryon density increases. There is a single local energy-minimum at the fixed density. Thus the phase transition from normal matter to the spin-polarized one is of the second order.

In order to examine effects of the spherical symmetry breaking, in Fig. 3, we show the spin-
symmetry energy $\epsilon_{sp-sym}$ using three kinds of choices (Ch1, Ch2-S, Ch2-Q). Ch2-S and -Q are the two versions of Ch2: the spin-vector of Ch2 (eq.(15)) with the spherical (Ch2-S) and the quadrupole-deformed momentum distribution (Ch2-Q).

In Ch1 the spin-symmetry energy always monotonously increases when density becomes larger; the reason has been given in the previous section. In Ch2-S the qualitative behavior is almost the same as that for Ch2-Q though the value of $\epsilon_{sp-sym}$ and the critical density are always larger than those for Ch2-Q. When $\tilde{C}_A < 0$, the spherical symmetry breaking for the spin-vector makes a critical effect for the spin-polarization. In addition such effects from the choice of the spin-vector are enhanced by the deformation of the momentum distribution. Hence the axial-vector correlation between two nucleons rather easily gives rise to the ferromagnetic state through the spherical symmetry breaking.

4. Summary

In this work we have examined a possible mechanism of the spin-polarization of nucleons and discussed magnetic properties of the system. In the relativistic framework there are two kinds of spin-spin interaction channels, the axial-vector and tensor ones, which are reduced to the same interaction channels in the nonrelativistic framework. If the interaction energies from two channels have opposite signs, there is a second-order phase transition to a spin-polarized state; the channel with negative sign becomes dominant, suppresses the spin-symmetry energy with increase of density and eventually induces a phase-transition at a certain critical density $\rho_c$, while the effects from two channels are counterbalanced with each other around the normal nuclear density. In this mechanism, the spherical symmetry breaking through the configuration of the spin vector and the deformation of the Fermi sea in the momentum space plays a significant role.; Such symmetry breaking is originated from the relativistic effect: the orientation of the spin changes as nucleon moves and the interaction energy includes an explicit momentum dependence.

Acknowledgement

The details discussed in this work will appear in Ref. [9]

References


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3. Magnetodynamics of Crusty Nuclear Matter

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Abstract: The induced by quantization of nucleon levels sharp change of spins of nuclei in strong magnetic fields is demonstrated to give rise to erratic jumps in magnetodynamics of 'magnetar' crusts. Such a noise is originating from magnetic avalanches and exhibits intensity and statistical properties which are favorably compared to the burst activity of soft gamma repeaters.

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Systematic observations of Soft Gamma Repeaters (SGRs) have been started with the discovery [1,2] of superintense gamma-ray outburst (giant flare) from SGR 0566-66 on 1979 March 5. Almost a copy of such superhigh luminosity, \( L_{\gamma} \sim 10^{44.5} \text{ erg} \), flare has been seen on 1998 August 27 from SGR 1900+14 [3]. Many properties of both events, like sharp change in the persistent X-ray flux during giant flare [4] and multi-peaked pulse profile [5], point out magnetic origin of the energy source due to very large magnetic fields with multipole components exceeding considerably the dipole fields (\( B_{\text{dipole}} \sim 10^{15} \text{ G} \) as estimated from magnetic-braking spin-down mechanism).

The observations have been intensified during past decade with the launch of RXTE, BATSE, ASCA etc. missions (see, e.g., [6-9] and refs. therein) and reveal that in addition to persistent X-ray luminosities \( L_{\gamma} \sim 10^{34.5} - 10^{36} \text{ erg/s} \) (similar with Anomalous X-ray Pulsars, AXPs) SGRs commonly emit short (\( \sim 0.1 \text{ s} \)) outbursts with \( L_{\gamma} \sim 10^{35} - 10^{36} L_{\text{Edd}} \) far above the Eddington limit \( L_{\text{Edd}} \) [10]. Such burst emissions are concentrated into short intervals (weeks to months) of intense activity separated by relatively long and quasi-regular periods (years) of quiescence [6]. Three relatively short active phases of SGR 1900+14 have been met during 1979 [1,2], June – August 1992 [12], and May 1998 – January 1999 [8,9]. This indicates close to six years period for SGR 1900+14 activity. About half a year activity periods have been reported [11] for SGR 0526-66. The plausible nearly two years breaks between intensifying burst sets have been suggested for SGR 1806-20 from an analysis of its timing residual [13]. The burst statistics during active phases display features of self-organized criticality, e.g., power law dependence of burst number on the intensity, lognormal distribution of waiting times between the bursts [8,9,14,15].

Within the concept of ultramagnetized neutron stars ('magnetars') [16] SGR-bursts are originating from caused by crust seismic activities magnetic field fluctuations and, consequent, magneto-plasma excitations. Such a model provides proper explanation of many SGR’s features. However, some properties, like quasi-periodicity of active phases and rather stable without noticeable glitches spinning down, do not corroborate with star-quake triggering mechanism.

We argue in this paper that such characteristics can be still understood within 'magnetar' concept by exploring triggering mechanism associated with a release of magnetic energy stored in the nuclear degrees of freedom of neutron star crusts. The intervals of intense activity are related to sharp, step-like change of the magnetization because of inhomogeneous crust structure [17,18]. At such conditions the demagnetization proceeds as erratic jumps associated with magnetic avalanches, similar to the Barkhausen effect (see, e.g., [19]), and sharp energy release to the magneto-sphere.

The present study focuses on outer crusts which consist of nearly spherical nuclides (see, e.g., [20] and refs. therein). The induced by the field magnetic moment of a nucleus

\[
m = \mu_N \sum_n \nu_n \theta(b - b_n)
\]  

(1)

experiences jumps of a height \( \nu_n \) (measured in the nucleon magnetons \( \mu_N \)) at field strengths \( b_n \) corresponding to level crossings [17,18], and contributes to the neutron star crust magnetization.
\[ P = \frac{m}{V_{WS}} \approx \frac{m}{\mu_n} \frac{\mathcal{D}_{Nb}}{\mathcal{D}_0} 10^{15} \text{G}. \] (2)

Here an average density of bound nucleons \( \mathcal{D}_{Nb} = A/V_{WS} \) is related to the Wigner-Zeits volume \( V_{WS} \) and estimated as a tenth of normal nuclear density \( \mathcal{D}_0 \).

We consider outer crusts as a polycrystalline structure with nuclei arranged in a closed packed (plausibly bcc [20]) lattice and assume the dipolar interaction between magnetic moments. Since such a system shows the ferromagnetic ordering (cf., e.g., [21]) the crusts can be viewed as a collection of \( \Pi \) domains on a hypercubic lattice and described by the Hamiltonian

\[ \mathcal{H} = - \sum_{ij \in \Pi} J_{ij} m_i m_j - \sum_{i} (H(t) - \eta P + f_i) m_i \] (3)

where originating from the core region magnetic field \( H \) changes slowly with time \( t \), the term containing an overall magnetization

\[ P = \sum_{i=1}^{\Pi} m_i \] (4)

and parameter \( \eta \) accounts for demagnetizing effect in a global form, the interaction between domains is equal to \( J \) for nearest neighbors (nn) \( J_{ij} = J \) and zero otherwise. The random field \( f_i \) is distributed according the gaussian distribution function

\[ W(f_i) = \exp\{-f_i^2/2R \}/\sqrt{2\pi R} \] (5)

and simulates random crystalline anisotropies and varying interaction strengths caused by inhomogeneities and disorder in the form of defects, grain boundaries, impurities in the crystalline structure.

The crust magnetodynamics is determined by the spin \( m_i \), jumps arising at sign change of a difference between its local effective field \( b_i^{eff} = J \sum_{nn} m_j + H - \eta P + f_i \) and some of the quantities \( b_n \) (i.e. the value \( \delta b_n = b_i^{eff} - b_n \), see Eq. (1)). We refer below for such a model as randomly jumping spins (RJS) model.

At the condition \( \delta b_{i+1} - \delta b_i \gg \delta b_n \gg R \) almost all the spins equal to the value \( v_n = \sum_{i=1}^{n} v_n \) and point along the field \( H \) direction (i.e. \( m_i = v_n \) for all \( i \)). As the field adiabatically decreases the spin-domains progressively jump to \( v_{n-1}, v_{n-2}, \ldots \). Because of the nearest neighbor interaction the jumping spin can result in the jump of a neighbor domain which in turn might lead to the reducing spin of another neighbor, and so on, generating thereby an avalanche of spin jumps. The adiabatically changing external field \( H(t) \) remains almost constant during an avalanche, while the magnetization varies sharply according to the mean-field definition Eq. (4). The corresponding energy release to the magnetosphere is determined by the avalanche size. Assuming the field strength \( H \sim 10^{15.5} \text{ G} \) in the outer crust of a linear size \( l_{crust} \sim 100 \text{ m} \) and employing an estimate Eq. (2) the energy upper limit is evaluated

\[ E_{max}^{SGR} \approx H \mu_i^3 l_{crust} \sim 10^{41} \text{ erg} \] (6)

to be in a good agreement with SGR-burst observations. When the magnetization involves the inner crust as well, the linear size is an order of magnitude larger and the energy release extends up to \( 10^{44} \text{ erg} \), a value corresponding to giant flare events.

---

FIG. 1. The mean-field phase diagram for the RJS model Eq. (7). The upper \( H_c \) (solid line) and low \( H_c \) (dashed line) coercive fields meet at the critical point \( H_c(R_c) = b_c - I_{eff} v_c/2 \).

Within the mean-field approximation the interaction of a spin with the neighbor is replaced by an interaction with the overall magnetization \( P \) of the system yielding the simplified Hamiltonian

\[ \mathcal{H} = - \sum_{i} (J_{eff} P + H + f_i) m_i \] (7)

with an effectively reduced coupling constant \( J_{eff} = J - \eta \) because of the demagnetizing effect. Considering the ensemble average \( \mathcal{P} \) of the magnetization defined by Eq. (4) gives the self-consistency equality
\[ \mathcal{P} = \int W(f)m_d df. \]  

(8)

For simplicity we assume a single jump in step-wise \( m_d \) of Eq. (1) with \( \nu_1 = 1 \). Then using the functional form Eq. (5) we derive the magnetic Equation of State (MMeoS)

\[ \varpi = \text{erf} \left[ \frac{\varpi + B}{\varrho} \right] \]  

(9)

expressed in terms of reduced magnetization \( \varpi = 2\mathcal{P} - 1 \), disorder \( \varrho \equiv 2\sqrt{2R}/J_{\text{eff}} \), and external field \( B = 1 + 2(H - b_1)/J_{\text{eff}} \).

The obtained from mean-field MMeoS Eq. (9) phase diagram is shown in fig. 1. The spinodal region is located at the field strengths \( H \sim b_1 - (J_{\text{eff}}\nu_1/2) \) corresponding to a jump of the nuclear momentum. We note that in realistic case such regions arise at values \( b_n \), defined by level crossings of crust nuclei. For disorders \( R \geq J_{\text{eff}}/\sqrt{2R} \equiv R_c \) the solution \( \mathcal{P}(H) \) of Eq. (9) is analytic at all values of \( H \). At such large disorders with a wide distribution of random fields the spins tend to jump independently of each other. Small avalanches give rise to the smooth (on a macroscopic scale) magnetization curve. At critical values of disorder \( R = R_c \) and magnetic field \( B^c(R_c) = 0 \) the magnetization curve \( \mathcal{P}(H) \) shows diverging slope. In vicinity of such transitional region in \( \{R, H\} \)-plane the system shows critical scaling behavior and the widest distribution of the avalanche sizes. For \( R < R_c \) the solution \( \mathcal{P}(H) \) is unique only for \( H \) outside of a certain interval \( [H^c_l(R), H^c_u(R)] \). In the range between the two 'coercive fields' \( H^c_l(R) \) and \( H^c_u(R) \), the equation has three solutions, two are stable and one is unstable. The small disorder with a narrow random field distribution allows for large avalanches. This leads to noticeable discontinuities in the magnetization curves similarly to what is found for the Ising model at zero temperature [22]. However, unlike equilibrium systems which in zero temperature limit always occupy the state with the lowest overall free energy, the considered nonequilibrium system is forced by the local dynamics to stay in the current metastable state (i.e. local energy minimum) until it is destabilized by the external magnetic field. For decreasing fields this implies that the system always remains in the metastable state with the largest possible magnetization.

The cumulative avalanche size distribution in the vicinity of the critical point is compared in fig. 2 with the normalized cumulative fluence distribution. The observations by various missions are in a good agreement with simulations when accounting for the scale Eq. (6) of the energy release. Such an event number dependence is well fitted by the power law with an exponent 0.66, which corresponds to the value 1.66 for the differential distribution and provides a signal of self-organized criticality in the burst statistics.

![FIG. 2. Cumulative fluence distribution of SGR-bursts. The results of the RXTE and BATSE observations for SGR 1900-14 from [8] are shown by squares and circles, respectively. RXTE (diamonds), BATSE (up-triangles), and ICE (down-triangles) data for SGR 1806-20 are from [9]. The full line represents the avalanche size distribution from RJS for the cubic lattice of a size \( (150)^3 \). The dashed line denotes the power law distribution.](image)

As seen in fig. 3 the waiting time distribution as a function of the reduced time, i.e. the time divided by the time at the maximum, displays universal function for different SGRs. The data are well reproduced by simulations and fitted by lognormal function. Such a property points out the single time scale for SGR-burst triggering processes. Within RJS such a time-scale is determined by the ratio of the disorder parameter \( R \) and the speed of external field change.
\( \tau = \frac{R}{H} \). Therefore, the scaling with respective time leads to an universal function.

![Diagram of reduced waiting time distribution](image)

**FIG. 3.** The reduced waiting time distribution between the successive RXTE/PCA bursts from SGR 1900-14 (squares) [8] and SGR 1806-20 (diamonds) [9] are compared with the waiting time distribution between avalanches (solid curve). The dashed line represents the fit to the log-normal distribution of the width 3.6.

In summary, the effects of super-strong magnetic fields on nuclear structure have been shown to result in irregular time evolution of magnetic fields in neutron star crusts. In particular, at conditions of nuclear level crossings the demagnetization proceeds as sharp steps due to avalanche propagations. As a consequence, the sudden energy releases trigger SGR-bursts. The quasi-periodic (with magnetic field) appearance of level crossings is consistent with some regularities in the burst emissions (see above). As demonstrated the model predicts some scaling properties for, e.g., the burst intensity and waiting time distributions which are in a good agreement with SGR observations. Finally, we note that quantum dot arrays (see, e.g., [23]) might provide some possibilities for laboratory tests of considered above model.

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4. ハイベロン物質中のK中間子凝縮
Kaon condensation in hyperonic matter

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In-medium kaon properties are studied in hyperonic matter, where hyperons are mixed in the ground state of neutron-star matter. Kaon dispersion relations and dependence of kaonic modes on the baryon number density are obtained. $P$-wave kaon-baryon interactions as well as the $s$-wave interactions are taken into account within chiral effective Lagrangian, and the nonrelativistic effective baryon-baryon interactions are incorporated. It is shown that the system becomes unstable with respect to a spontaneous creation of a pair of the particle-hole modes with $K^+$ and $K^-$ quantum numbers, stemming from the $p$-wave kaon-baryon interaction. The onset density of this $p$-wave kaon condensation may be lower than that of the $s$-wave $K^-$ condensation.

I. INTRODUCTION

Various hadronic phases in neutron stars have been considered both experimentally and theoretically. Possibility of kaon condensation is a long standing problem concerning a phase transition of high-density matter [1,2]. Motivated by kaon condensation, in-medium kaon properties have been also elaborated through kaon-baryon scattering, kaonic atoms, and subthreshold $K^\pm$ production in heavy-ion collisions. On the other hand, it has been discussed based on the recent development of hypernuclear experiments that hyperons ($\Lambda$, $\Xi^-$, $\Sigma^-$, \ldots) may be mixed as well as neutrons, protons, and leptons in neutron-star matter. The kaon condensation and hyperonic matter have been investigated separately from theoretical points of view. But the possible existence region may overlap each other, so that the competition or coexistence problem of these phases have to be elaborated.

Recently, we have pointed out a new mechanism of kaon condensation realized in hyperonic matter stemming from the $p$-wave kaon-baryon interaction [3]. In this talk, we have discussed in detail the interplay between kaon condensation and hyperonic matter by considering the kaon dispersion relations and dependence of kaonic modes on the baryon number density.

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II. FORMULATION

A. Kaon-baryon interaction

We take into account the kaon-baryon interaction by the use of the effective chiral Lagrangian by Kaplan and Nelson [1], which includes both s-wave and p-wave interactions. For simplicity, we take only the proton ($p$), neutron ($n$), $\Lambda$, $\Sigma^-$, and $\Xi^-$ for the octet baryons, and consider the chemically-equilibrated matter including the electron. The effective Hamiltonian $H_{\text{eff}}$ is obtained with the introduction of the charge chemical potential $\mu(=\mu_K = \mu_e)$ from the charge neutrality condition. After reducing to the nonrelativistic form for the baryonic part of $H_{\text{eff}}$ by the Foldy-Wouthuysen-Tani transformation, we add a potential term $V_i$ ($i=p$, $n$, $\Lambda$, $\Sigma^-$, $\Xi^-$) for each diagonal matrix element of the effective Hamiltonian for baryons. The effective ground state energy density of the system, $\varepsilon_{\text{eff}}$, is obtained after diagonalization of the baryonic part of $H_{\text{eff}}$. We assume a plane-wave type for the classical kaon field as $K^- = f/\sqrt{2} \theta \exp i(k \cdot r - \mu t)$, where $f$ is the meson decay constant, $\theta$ the chiral angle, and $k$ is the kaon momentum.

After expanding the effective energy density $\varepsilon_{\text{eff}}$ with respect to $\theta$ around $\theta = 0$ as $\varepsilon_{\text{eff}} = \varepsilon_{\text{eff}}(\theta = 0) - f^2/2 D_{K^1}^{-1}(\mu, k; \rho_B) \theta^2 + O(\theta^4)$, one obtains the kaon inverse propagator:

$$D_{K^1}^{-1}(\omega, k; \rho_B) = \omega^2 - k^2 - m_K^2 + \frac{1}{f^2} \sum_i \rho_i \Sigma_{K_i} + \frac{1}{f^2} \frac{\rho_p + \frac{1}{2} \rho_n - \frac{1}{2} \rho_{\Sigma^-} - \rho_{\Xi^-}}{2} \omega$$

$$+ \frac{1}{f^2} \left[ \frac{(\rho_p - \rho_{\Lambda})(g_{\Lambda p} k)^2}{2} + \frac{(\rho_n - \rho_{\Sigma^-})(g_{\Sigma^- n} k)^2}{2} + \frac{\rho_{\Lambda} - \rho_{\Xi^-})(g_{\Xi^- \Lambda} k)^2}{2} \right]$$

$$+ \frac{\delta M_{\Lambda N} - \omega + V_{\Lambda} - V_{\Sigma^-}}{2}$$

where $\Sigma_{K_i}$ $(i = p, n, \Lambda, \Sigma^-, \Xi^-)$ are the 'kaon-baryon sigma terms', which give the s-wave scalar attraction, $\delta M_{ij}$ the baryon mass differences, $\rho_i$ the number densities for the baryons $i$, $g_{\Lambda p}$, $g_{\Sigma^- n}$, and $g_{\Xi^- \Lambda}$ are the p-wave axial vector coupling strengths which are expressed in terms of the parameters $D(=0.81)$ and $F(=0.44)$ in the chiral Lagrangian.

B. Potential contribution

For the potential terms $V_i$ in hyperon matter, we refer to the nonrelativistic potential energy density $\varepsilon_{\text{pot}}$ by Balberg and Gal [4]. We define $V_i$ as $V_i = \partial \varepsilon_{\text{pot}}/\partial \rho_i$ by neglecting the momentum dependence of baryon potentials. The coefficients are determined so as to reproduce the saturation properties of the symmetric nuclear matter: the symmetry energy $\sim 30$ MeV, the potential depths of the $\Lambda(=27$ MeV) [4] and the $\Xi^-(=16$ MeV) [5] in nuclear matter. For $V_{\Sigma^-}$, we take the two cases: (I) Following recent analyses of $\Sigma$ hypernuclei [6] and $\Sigma^-$ atom data [4], the potential depth of the $\Sigma^-$ in nuclear matter is taken to be repulsive, i.e., $V_{\Sigma^-}(\rho_B = \rho_0, N = Z) = 23.5$ MeV ($\rho_0 = 0.16$ fm$^{-3}$). (II) As is conventionally used, it is taken to be attractive, i.e., $V_{\Sigma^-}(\rho_B = \rho_0, N = Z) = -27$ MeV.
III. NUMERICAL RESULTS

Here we mainly discuss the results in Case I. The particle fractions as functions of the baryon number density $\rho_B$ are shown in Fig. 1 for the repulsive $V_{\Sigma^-}$.

The $\Lambda$ appears at $\rho_B \sim 0.37$ fm$^{-3}$, and its fraction rapidly increases with density, exceeding the proton fraction at $\rho_B \sim 0.4$ fm$^{-3}$. Soon after the appearance of $\Lambda$, the $\Xi^-$ are mixed at $\rho_B \sim 0.4$ fm$^{-3}$, and its fraction increases with density. On the other hand, the electron fraction rapidly decreases after the appearance of the $\Lambda$ and $\Xi^-$. Our results qualitatively reproduce the results in Ref. [4].

In Fig. 2, we show the dispersion relations of kaonic modes as functions of $|k|$. (a) is for $\Sigma_{Kn}=305$ MeV, $\rho_B=0.50$ fm$^{-3}$ (dashed line) and $\rho_B=0.57$ fm$^{-3}$ (solid line). (b) is for $\Sigma_{Kn}=403$ MeV and $\rho_B=0.48$ fm$^{-3}$.

The branches are characterized by a sign of the residue at their poles of $D_K$, as is the pion case [7]: If $\partial D_K^{-1}/\partial \omega > 0$ ($\partial D_K^{-1}/\partial \omega < 0$) at the pole, the mode has the same quantum number as the $K^-$ ($K^+$). There are three collective particle-hole branches in addition to the $K^-$ and $K^+$ branches which reduce to the free kaons when the kaon-baryon interaction is turned off: the $p\Lambda^{-1}$ which has the same quantum number as the $K^+$, the $\Xi^-\Lambda^{-1}$, and $\Sigma^-n^{-1}$ which have the same quantum number as the $K^-$. (The superscript
The appearance of the $p\Lambda^{-1}$ mode instead of the $\Lambda p^{-1}$ is due to the fact that the $\Lambda$ is more abundant than the proton at this density (Fig. 1). We can see from Fig. 2 that the $\Xi^{-}\Lambda^{-1}$ and $p\Lambda^{-1}$ branches merge at certain momentum $k^2(=984 \text{ MeV})$ and density ($\rho_B=0.57 \text{ fm}^{-3}$). The two excitation modes merge at the $\omega$ axis, where the double-pole condition, $D_K^2=0$ and $\partial D_K^2/\partial \omega=0$, is satisfied. Physically, a pair of the two modes, $[\Xi^{-}\Lambda^{-1}]$ and $[p\Lambda^{-1}]$, are created spontaneously with no cost of energy because the energy of the $[p\Lambda^{-1}]$ mode with the quantum number $K^+$ is to be reversed in sign [7]. Hence the system is unstable with respect to a pair creation of $[\Xi^{-}\Lambda^{-1}]$ and $[p\Lambda^{-1}]$ modes. This instability originates from the $p$-wave kaon-baryon interaction and is similar to that of pion condensation [7].

In Fig. 3, we show the dependence of the excitation energies of kaonic modes on the baryon number density except for the $K^+$. (a) is for $\Sigma_{Kn}=305 \text{ MeV}$ and $|k|=500 \text{ MeV}$, and (b) is for $\Sigma_{Kn}=403 \text{ MeV}$ and $|k|=100 \text{ MeV}$.

For (a), the $K^-$ is repelled far from the remaining particle-hole modes over the relevant densities, and there is no level crossing. The $\Xi^{-}\Lambda^{-1}$ and $p\Lambda^{-1}$ modes merge at $\rho_B \simeq 0.60 \text{ fm}^{-3}$, which corresponds to the instability with respect to $p$-wave condensation. The qualitative feature is also applied to the case at the critical momentum $k^2$ which satisfies $\partial D_K^2/\partial |k|=0$, as well as the double-pole condition. The particle-hole branches such as the $\Xi^{-}\Lambda^{-1}$ and $p\Lambda^{-1}$ do not depend much on the value of $|k|$, nor does the critical point for the $p$-wave condensation.

For $\Sigma_{Kn}=403 \text{ MeV}$, on the other hand, the excitation energy of the $K^-$ is small as compared with the $\Sigma_{Kn}=305 \text{ MeV}$ at a given density due to the larger $s$-wave scalar attraction and the smaller momentum $|k|$, and one can see in Fig. 3 (b) that there are energy gaps between $K^-$ and $\Sigma^{-}n^{-1}$ branches and the $\Sigma^{-}n^{-1}$ and $\Xi^{-}\Lambda^{-1}$ branches owing to the level crossings. As a result of the level crossings, the $\Xi^{-}\Lambda^{-1}$ branch takes over the characteristics of the $K^-$, the excitation energy of which changes appreciably depending on the magnitude of the $s$-wave scalar interaction simulated by $\Sigma_{Kn}$. Thus, when the level crossing occurs, the behavior of the $\Xi^{-}\Lambda^{-1}$ branch is sensitive to the value of $\Sigma_{Kn}$. 

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so that the critical point for the $p$-wave condensation, which is given by the merge point of the $\Xi^{-}\Lambda^{-1}$ and the $p\Lambda^{-1}$ branches, also depends on $\Sigma_{K\Lambda}$.

As seen from the $K^{-}$ branch in Fig. 2, the $K^{-}$ with $k=0$ has a minimum energy $\omega_{\min}(K^{-})$. Due to the substantial decrease of the charge chemical potential $\mu$ with density in the presence of hyperons as compared with the conventional neutron-star matter case, the condition giving a critical density of the $s$-wave $K^{-}$ condensation, $\omega_{\min}(K^{-}) = \mu$, is not met at this density, and the onset of the $p$-wave condensation precedes the $s$-wave $K^{-}$ condensation: $\rho_B^p(p) \sim 0.57$ fm$^{-3}$, $\rho_B^\sigma(s) \sim 0.67$ fm$^{-3}$ for $\Sigma_{K\Lambda}=305$ MeV, and $\rho_B^p(p) \sim 0.48$ fm$^{-3}$, $\rho_B^\sigma(s) \sim 0.50$ fm$^{-3}$ for $\Sigma_{K\Lambda}=403$ MeV.

In Case II (the attractive $V_{\Sigma^{-}}$), the $\Sigma^{-}$ appears first at $\rho_B \sim 0.3$ fm$^{-3}$ as a hyperon-mixing. The onset density of $\Lambda$ becomes a little larger as compared with Case I, while the onset density of $\Xi^{-}$ is pushed up to a high density ($\rho_B \sim 0.68$ fm$^{-3}$). Due to the strong attraction of $V_{\Sigma^{-}}$, the $\Sigma^{-}n^{-1}$ branch is softer than the $\Xi^{-}\Lambda^{-1}$ and $K^{-}$ branches, so that the $\Sigma^{-}n^{-1}$ branch merges first with the $p\Lambda^{-1}$ branch. In Case II, $\rho_B^p(p) \sim 0.64$ fm$^{-3}$ and $\rho_B^\sigma(s) \sim 0.72$ fm$^{-3}$ for $\Sigma_{K\Lambda}=305$ MeV, and $\rho_B^p(p) \sim 0.53$ fm$^{-3}$ and $\rho_B^\sigma(s) \sim 0.54$ fm$^{-3}$ for $\Sigma_{K\Lambda}=403$ MeV. It is to be noted that the $p\Lambda^{-1}$, $\Xi^{-}\Lambda^{-1}$, and $\Sigma^{-}n^{-1}$ branches hardly depend on the magnitude of $\Sigma_{K\Lambda}$, as far as $\Sigma_{K\Lambda}$ is not very large, so that the critical density of the $p$-wave pair condensation of these particle-hole modes changes little even for $\Sigma_{K\Lambda}=0$.

IV. SUMMARY AND CONCLUDING REMARKS

We have shown that a collective $p\Lambda^{-1}$ mode carrying the $K^+$ quantum number appears over the densities where the $\Lambda$ is more abundant than the proton. The system becomes unstable with respect to a creation of $[\Xi^{-}\Lambda^{-1}]$ and $[p\Lambda^{-1}]$ pair or $[\Sigma^{-}n^{-1}]$ and $[p\Lambda^{-1}]$ pair ($p$-wave kaon condensation), which stems from the $p$-wave kaon-baryon interaction. The large mixing of $\Lambda$ as compared with that of the proton is crucial for the appearance of the $p\Lambda^{-1}$ mode. The onset density of this instability is lower than that of the $s$-wave $K^{-}$ condensation for a standard value of the parameter $\Sigma_{K\Lambda}$ simulating the magnitude of the $s$-wave kaon-baryon scalar interaction, and it hardly depends on the value of $\Sigma_{K\Lambda}$ as long as $\Sigma_{K\Lambda}$ is not too large.

Our model used for the $p$-wave kaon-baryon interaction is based on the leading order expansion in the chiral perturbation theory. Higher order terms in chiral expansion are considered to be quantitatively important to kaon dynamics in highly dense matter.

It has to be clarified whether the instability of the system with respect to the $p$-wave condensation leads to a fully condensed phase beyond the critical density. In this context, the EOS of the $p$-wave condensed phase and the characteristic features of the system have to be examined. Mixing of hyperons only already leads to appreciable softening of the EOS. Hence, further development of kaon condensates in hyperonic matter would make the EOS too soft to obtain the observed neutron star masses $\sim 1.4M_\odot$. Relativistic effects may help weaken the softness of the EOS, since it has been shown that the energy gain of kaon condensation coming from the $s$-wave scalar attraction is suppressed by the relativistic effects [2]. In addition, in view of making the EOS consistent with observations, some realistic effects which reduce the $p$-wave kaon-baryon attraction should be taken into account: E.g., (1) vertex renormalization at the $p$-wave kaon-baryon vertices in terms of form factors, and (2) short-range correlations between baryons.
We have not taken into account the effects of the other subthreshold resonances such as \( \Sigma(1385) \) (abbreviated to \( \Sigma^* \)) on the dispersion relations of the kaonic modes. However, the excitation energies of these resonances are of the order \( \sim 450 \text{ MeV} \) so that their branches lie far above the other particle-hole branches, \( p\Lambda^{-1}, \Sigma^- n^{-1}, \Xi^- \Lambda^{-1} \). Furthermore, their coupling strengths to the \( K^- N \) are not so large as compared with \( g_{\Lambda p}, g_{\Sigma^- n}, \text{ and } g_{\Xi^- \Lambda} \). Hence the inclusion of these resonances would not change the results quantitatively.

5. 相対論的平均場理論に基づく中性子星内殻の比熱計算
Relativistic Hartree-Bogoliubov Calculation of
Specific Heat of the Inner Crust of Neutron Stars *

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Abstract

We calculate the specific heat of the inner crust of neutron stars within a local-density approxima-
tion to an improved relativistic Hartree-Bogoliubov theory. Non-uniformness of the system enhances 
the specific heat in particular at low temperatures. The degree of enhancement is similar to that in 
the spherical phase of Elgaroy et al. We examine a schematic interpolation between the results 
of Broglia et al. adopting the Gogny force and ours based on the Lagrangian of the relativistic mean field model.

The inner crust of neutron stars, with densities ranging from the neutron drip density to that of the 
order of the saturation one, is a non-uniform system consisting of a Coulomb lattice of neutron-rich nuclei 
and a sea of free neutrons. This part conveys temperature drop due to neutrino emission in the core 
to the surface. The rate of the conveyance depends on the specific heat and the thickness of the inner 
crust. Since the former is sensitive to superfluidity, we study this non-uniform system as an application 
of the improved description of pairing correlation based on the RMF interaction, that is, a one-boson 
exchange interaction derived from the Lagrangian of the relativistic mean field (RMF) model. As a first 
step of this application, we adopt a local-density approximation and neglect possible non-spherical shapes 
of neutron-rich nuclei, in this talk. In this sense, this is a relativistic counterpart of Ref.[1] and of the 
spherical part of Ref.[2].

The first attempt to describe superfluidity in uniform infinite nuclear matter adopting the RMF 
interaction was done in Ref.[3]; but its result was not very realistic in that the resulting pairing gap 
was too large. The reason can be ascribed to an unphysical behavior of the RMF interaction at high 
momenta: Repulsion is too strong. Note that repulsion also contributes positively to pairing gap at high 
momenta beyond a sign change of the gap function. Tanigawa and one of the present authors proposed a method to determine quantitatively a density-independent parameter to cut the momentum integration 
in the gap equation [4]. Although a more natural way to modulate the high-momentum part smoothly 
was also developed later [5], the simple method in Ref.[4] is enough for the present purpose since only 
the pairing gap at the Fermi surface at each density is necessary under a local-density approximation.

In order to describe the non-uniform system composed of a lattice of neutron-rich nuclei and a sea of 
free neutrons, we adopt the Wigner-Seitz (WS) approximation; each spherical cell contains at its center 
one nucleus permeated by free neutrons which dripped off from it. We parametrize the neutron density

given by a density-dependent Hartree-Fock calculation [6] in terms of a Woods-Saxon shape,

\[ \rho_n(r) = \frac{\rho_{0,n}}{1 + \exp[(r - R_n)/a_n]} + \rho_{\text{ext},n}, \]  

as done in Ref.[1]. We perform calculations at five representative densities conforming to this reference; the parameters entering into Eq.(1) and the radii of the WS cell are summarized in Table 1. Here we introduce a local-density approximation — assume uniform Fermi gas of neutrons at each spatial point. Then the local Fermi momentum is calculated from the local density given by Eq.(1) as

\[ k_F(r) = \left( \frac{3\pi^2 \rho_n(r)}{2} \right)^{1/3}. \]  

Its profile in each WS cell is shown in the left panel of Fig.1. This determines the profile of pairing gap since the gap in the uniform neutron matter can be calculated as a function of \( k_F \) by using the method of Ref. [4]. (Note that the symmetric nuclear matter case was presented there.) The result is shown in the right panel. The local pairing gap shown there gives the local quasiparticle energy,

\[ E(r, p) = ((E(p) - E(k_F(r)))^2 + \Delta(r, p)^2)^{1/2}, \]  

with the single-particle energy \( E(p) = (p^2 + M^*^2)^{1/2} + g_\omega (\omega^0), \) and \( M^* = M - g_\sigma < \sigma > \) being the density-dependent effective neutron mass.

<table>
<thead>
<tr>
<th>( \rho/\rho_0 )</th>
<th>( \rho_{0,n} \text{[fm}^{-3}] )</th>
<th>( \rho_{\text{ext},n} \text{[fm}^{-3}] )</th>
<th>( R_n \text{[fm]} )</th>
<th>( a_n \text{[fm]} )</th>
<th>( R_{\text{out}} \text{[fm]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 0.46</td>
<td>0.114</td>
<td>0.0737</td>
<td>5.0</td>
<td>1.2</td>
<td>15.0</td>
</tr>
<tr>
<td>(b) 0.28</td>
<td>0.101</td>
<td>0.0436</td>
<td>7.0</td>
<td>1.1</td>
<td>20.9</td>
</tr>
<tr>
<td>(c) 0.12</td>
<td>0.098</td>
<td>0.0184</td>
<td>7.4</td>
<td>0.8</td>
<td>29.4</td>
</tr>
<tr>
<td>(d) 0.034</td>
<td>0.104</td>
<td>0.0047</td>
<td>6.8</td>
<td>0.9</td>
<td>35.5</td>
</tr>
<tr>
<td>(e) 0.005</td>
<td>0.108</td>
<td>0.0005</td>
<td>5.9</td>
<td>0.9</td>
<td>45.7</td>
</tr>
</tbody>
</table>

Table 1: Parameters entering into Eq.(1) and the radii of the WS cell.

Figure 1: Profile of the local Fermi momentum (left) and of the local pairing gap (right) in a Wigner-Seitz cell. Five curves in the order from the top to the bottom in the left panel correspond to densities (a) – (e) in Table 1.

Now we are ready to calculate the specific heat of the system. The specific heat is calculated from the entropy,

\[ S = -2 \int d^3r \int \frac{d^3p}{(2\pi)^3} \left\{ f(r, p) \ln f(r, p) + [1 - f(r, p)] \ln [1 - f(r, p)] \right\}. \]  

as

\[ C_V = T \frac{\partial S}{\partial T} \bigg|_V. \]  

\[ - 25 - \]
The phase-space distribution function in Eq.(4) is

$$f(r,p) = \frac{1}{1 + \exp[E(r,p)/T]}$$

where the quasiparticle energy is given by Eq.(3). The final expression is

$$C_{Vn} = \frac{1}{\pi T V_{ws}} \int_{\nu_{ws}} dr r^2 \int dp p^2 \frac{E(r,p)}{\cosh^2[E(r,p)/2T]} \left[ \frac{E(r,p)}{T} - \frac{dE}{dT} \right].$$  \hspace{1cm} (7)

The spatial integration is performed over each WS cell and the momentum integration is done up to $\Lambda = 3.60 \text{ fm}^{-1}$ determined in Ref.[4] to give physical pairing gaps. Note that the temperature dependence of the quasiparticle energy is neglected because only low temperatures are of interest in the present study. In addition to neutrons, degenerate electron gas gives a comparable contribution,

$$C_{Ve} = \frac{1}{3} k_B^2 (m^2 + k_B^2)^{1/2} T,$$

while that from protons is negligible. Then the total specific heat is given by

$$C_{Vtot} = C_{Vn} + C_{Ve}.$$  \hspace{1cm} (9)

The effect of non-uniformness can be seen by comparing the left (showing $C_{Vn}$ for uniform matter) and the center (for non-uniform) panels of Fig.2. One characteristic is that density dependence is weak in the non-uniform system. This is brought about by a density-dependent increase of $C_{Vn}$. The reason why non-uniformness increases it can be inferred from an expression obtained by a weak-coupling approximation for the uniform system [7],

$$\frac{C_{Vn}}{T} \propto \left( \frac{\Delta_0}{T} \right)^{5/2} \exp \left( -\frac{\Delta_0}{T} \right),$$

with $\Delta_0$ being the uniform gap. This is a decreasing function of $\Delta_0/T$ at low temperatures, $\Delta_0/T > 5/2$, and consequently leads to $C_{Vn}$(normal) > $C_{Vn}$(super). Since non-uniformness produces a region where the gap is small, it results in an increase of $C_{Vn}$. This effect is conspicuous in particular at lower temperatures because a small variation in $\Delta_0$ leads to a large one in $\Delta_0/T$. Conversely, difference in $\Delta_0$ is not important at high temperatures.

Figure 2: Temperature dependence of the partial specific heat of uniform neutrons (left), non-uniform neutrons (center) and electrons (right).

Since the contribution from electrons is of the same order (the right panel of Fig.2) as that from the non-uniform neutrons, the enhancement of $C_{Vtot}$,

$$\frac{C_{Vtot}(nu)}{C_{Vtot}(u)} = \frac{C_{Vn}(nu) + C_{Ve}}{C_{Vn}(u) + C_{Ve}},$$

(11)
where "nu" and "u" stand for non-uniform and uniform, respectively, is factor 3 - 4 as shown in Fig.3. Comparing this to Fig.3 in Ref.[1] highlights the difference at low temperatures. The consideration above indicates that the difference originates from the region where pairing gap is small — the interior region of the nucleus at the center of the WS cell. Actually, although the Gogny force adopted in Ref.[1] is known to provide us with pairing properties similar to those given by bare interactions at low densities [8], there is a big difference at high densities — superfluid phase survives up to much above the saturation density (see Fig.5.4 of Ref.[9], for example). In contrast, our cutoff parameter in the RMF description was determined so as to reproduce the pairing properties given by the Bonn-B potential and accordingly pairing gap closes at around the saturation density. This makes the gap in the interior region vanish. In Ref.[2], the non-relativistic version of the Bonn-A potential was adopted, and the results for \(\rho/\rho_0 = 0.058\) and 0.176 in the spherical phase were also presented. Our results resemble theirs. Here one comment is in order on the double-counting problem of short-range correlations which arises when effective interactions are adopted in the gap equation. In principle this problem exists as pointed out in Ref.[2], but an analysis in Ref.[5] indicates that it is not serious practically.

![Figure 3: Temperature dependence of the ratio of the total specific heat. Legend of the curves is the same as in Fig.2.](image)

![Figure 4: Schematic variation of the gap profile (left), resulting temperature dependence of the neutron specific heat (center) and of the ratio of the total specific heat (right), at density (b).](image)

Since the essential difference between Ref.[1] and ours is the gap in the interior region as mentioned above, we examine a schematic interpolation between the two models; we vary the gap in this region by hand as shown in the left panel of Fig.4 for density (b). The resulting specific heats are presented in the center panel. Clearly \(C_{\text{vn}}\) approaches to that in Ref.[1] (see Fig.2 there) as the gap in the interior region is increased, and to that of the uniform matter by a further increase. Finally the total specific heats are
shown in the right panel. This confirms that the vanishing gap in the interior region is the very cause of the enhancement of \( C_{\text{Vtot}} \) at low temperatures.

Beyond this first-step calculation, two kinds of refinements are in order; one is to take into account non-spherical shapes of lattice nuclei in the high-density part of the inner crust, the other is the so-called proximity effect beyond the local-density approximation, that is, spread of the pair wave function. The former was examined in Refs. [2, 10] and shown to strengthen non-uniformness, whereas the latter was shown to weaken it [11].

References

6. Dimensional analysis of nuclear shapes in neutron star crusts and in supernova matter

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Neutron rich nuclei in neutron-star crust and in supernova matter have been investigated by various macroscopic model calculations. One of their interesting properties is the existence of exotic nuclear shapes which are expected to realized at about the nuclear density, i.e. the saturation density of symmetric nuclear matter. A simple dimensional analysis can account for the shape change qualitatively in the compressible liquid drop model. In this analysis, the energetically favorable shape is determined as a function of the volume fraction of a nucleus in a unit cell. As this fraction increases, the favorable shape changes from the familiar spherical shape to cylinder, slab, cylindrical hole and spherical hole. It is remarkable that this result is independent of a specific choice of nuclear interactions. However, this analysis cannot treat the transition from nuclei to uniform matter. Hence some of the above shapes may not appear in case that the uniform matter has a smaller energy than the exotic nuclei. As was shown clearly in the compressible liquid drop model by Watanabe et al., the existence of the exotic shapes does depend on nuclear interactions. In this study, it is demonstrated that the key interaction parameters are the asymmetry energy and its density dependence at the nuclear density. Their values may be derived from masses and radii of neutron-rich nuclei. Therefore, it is recommended to perform systematic measurements of masses and radii of neutron-rich nuclei, while theoretical evaluation based on many-body calculations is also needed to compensate for the limitation of neutron-richness achievable in these measurements.
7. 超新星爆発における r プロセス元素合成
r-process nucleosynthesis in supernova explosions

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Abstract
We study the r-process nucleosynthesis in supernovae from different mass ranges of progenitors. We have investigated the two scenarios, the r-process in prompt supernova explosions and in neutrino-driven winds, which might correspond, respectively, to low and high mass ranges of progenitors. We have performed general relativistic hydrodynamical simulations of supernova explosions and neutrino-driven winds with the relativistic EOS table. We have then performed the reaction network calculations of r-process adopting the numerical results of hydro simulations. We have found that the r-process elements are produced in the prompt supernova explosion from a 11 solar-mass star and its abundance pattern agrees very well with the solar abundance pattern up to A=200. The successful r-process in the neutrino-driven winds, on the other hand, can be found in the cases of relatively massive proto-neutron stars.

1、はじめに
最近の天体観測技術の進化により、非常に古い星の元素組成が明らかになってきている。とりわけ非常に鉄の量の少ない星において、鉄よりもずっと重い領域の r プロセス元素が見つかった例は注目に値する [1]。このことは、進化の時間スケールの短い大質量の星を起源とする、超新星爆発のような天体現象を強く示唆するものであり、鉄を種とする 2 次 (secondary) r プロセスではなく、陽子・中性子などの軽い核から作り上げる (primary) r プロセスが起きたことを意味している。さらに興味深いことは、観測された r プロセス元素の組成パターンが太陽系組成 (solar abundance) のパターンとよく一致していることにある。このことは今も昔も同じように r プロセス元素合成過程が起きていることを示唆している。また、質量数 130 付近の比較的軽い核の領域では太陽系組成からの系統的な欠乏も観測されており、 r プロセスの起源は 1 つではなく、2 つ以上である可能性も議論されている。これらのことから考えると、重力崩壊型超新星爆発には 2 種類あって元素合成もそれに従って 2 種類あるのではないかという考え方が
にたどり着く。これらを足がかりに、それぞれ過程から、どれだけの量の$r$プロセス元素が、どのようなパターンで生成されるのかを明らかにして、これらを今後行われる古い星の系統的な観測データと比較し、銀河や宇宙の歴史において重元素がどのようにして作られできたのかを明らかにするのが最終的な研究目的である。ここでは、この考え方方に従って超新星爆発において起きる$r$プロセス元素合成の2つのシナリオについて、数値シミュレーションにより研究を行った結果を簡単に報告したい。詳しくは論文[2,3]を参照されたい。

2. 超新星爆発での$r$プロセス

大質量星の重力崩壊型超新星爆発のメカニズムは、未だ完全には明らかになっていないが、モデル的な、太陽質量の2.0倍程度の親星では遅延型爆発（delayed explosion）により爆発すると考えられている。一方、太陽質量の1.0倍程度の非常に軽い親星では、鉄のコアが十分に小さく、流体力学的に爆発する（prompt explosion）ことも考えられる。この2つの場合に$r$プロセスが起きる可能性としては、前者ではニュートリノ駆動型と呼ばれる原始中性子星からの質量放出を考えられ、後者はコアパウツン時に直接中性子過剩な物質が放出されることが考えられる。これら2つの$r$プロセス合成過程は、親星の質量の範囲（より正確には鉄のコアのサイズの範囲）に対応してきている、古い星から現在に至るまで$r$プロセス元素を作り続けている。というのがここで考えることができる。

3. プロスペクト爆発での$r$プロセス

比較的軽い質量の親星の場合には鉄のコアが小さいために、コアパウツン時に鉄のコアの中ほどで発生する衝撃波は、鉄のコアの表面にまで達すことができ、流体力学的に超新星爆発を起こすことができる可能性がある。この時、大量に物質を放出することになるが、なかでも中性子星の表面に一番近い部分は非常に中性子過剰になる。これは、パウツンおよびシュック通過時に密度温度が高くなるために、鉄が陽子・中性子に溶解放され、同時に電子捕獲反応が急速に進むためである。この部分が放出されるまでに温度が冷える過程で$r$プロセス元索合成が行われる。この考え方はかなり古くから提案されていて[4,5]が、数値シミュレーションで定量的にどれだけの量の$r$プロセス元索が作られるのかは、まだ調べられていなかった。そこで、我々は一般相対論的流体力学の数値シミュレーションを行い、$r$プロセス元索合成の量を調べることを行った。

Figure 1: Evolution of the spatial coordinate for various mass shells in the collapse and prompt explosion of our 11 Msolar model.

図1に流体計算の結果を示した。鉄のコアがつぶれて、バウンスして、そこで発生した衝撃波が外に伝わっていく様子が見えている。中心には原始中性子星が形成されているが、この時に放出される物質の一番内側の部分について着目し、電子捕獲反応の計算、元素合成計算を行った。各流体素片にそって温度、密度、陽子混在度が時間とともに記録されているので、それを用いて核反応ネットワーク計算を行い、それぞれの元素合成の結果をすべて足しあわせることになる。

Figure 2: Final integrated isotopic abundances for ejected material (solid line) compared with the Solar r-process abundances (filled circles).
図2の実験として得られた$\beta$プロセス元の合成の組成である。比較しているのは$\beta$プロセス元素の太陽系組成である。質量数2 00付近までの$\beta$プロセス元素が作られていて、第2、第3ビーグや希土類のサブピークもうまく再現されている。また質量数1 00付近の元素も過不足なく生成されていることがわかる。全体として生成された$\beta$プロセス元素の量も大量すぎることもなく、我々の銀河に存在する$\beta$プロセス元素の量と矛盾することはない。

4、ニュートリノ駆動風での$\beta$プロセス
大質量の親星の場合には、流体力学的に爆発することができず、従って中性子過剩な物質を直接外に放出することはできない。つまり衝撃波は鉄のコアの表面まで達することはできず、ニュートリノによる再加熱などを経て、やや時間をかけて爆発する遅延型超新星爆発が起きることになる。$\beta$プロセスは爆発過程で進行して原始中性子星が形成された後のニュートリノ駆動風により起きると考えられている。これらについても一般相対論的流体数値シミュレーションおよび元素合成ネットワーク計算による研究を行った。これらに関しては昨年度の研究会で報告させていただいたので、詳細は昨年度の報告や論文[2]を参照していただきたさい。最近の研究の中で明らかにてきたことは、$\beta$プロセスがうまく起こるために、膨張の時間スケールが十分短ければよいこと、中性子星の質量が大きい方が有利であるということである。後者は親星の関連を考えると、質量の大きい親星を示唆していると考えられる。

5、まとめ
我々は超新星爆発において$\beta$プロセスの起こる2つのシナリオについて、数値シミュレーションによる研究を進めてきた。まず、軽い星が起源と考えられるブロント爆発において、$\beta$プロセスが非常にうまく起こることがわかった。また、重い星において起こるであろう遅延爆発においてはニュートリノ駆動風を通じて$\beta$プロセスが起こるが、この時形成される中性子星の質量は大きい方がよいということもわかっている。これまでの結果から考えると、$\beta$プロセスの起源は2つあり、軽い星によるブロント爆発が質量数2 00程度までの重い領域の$\beta$プロセス元素を作り、重い星からの遅延爆発でのニュートリノ駆動風が質量数1 30程度の軽い領域の$\beta$プロセス元素を作るのではないかという考え方が浮かび上がってくる。むろん、これらを明らかにするためには両方のシナリオの場合について更なる研究を進めていき、観測データと照らし合わせて検証していく必要がある。本研究会におけるテーマであるハドロン物質の状態方程式における中性子星の構造も重要であり、原研において整備されている核データも強い武器になるであろう。今後も研究会の皆さまと議論をしながら、関連する分野とともに研究を発展させていきたい。
謝辞：上記の研究は寺澤真理子、大槻かおり、鈴木英之、山田章一、Hong Shen、G. J. Mathews、土岐博、梶野敏貴（敬称略）の皆さんとの共同研究として進めているものです。この場を借りてこれまでの研究活動に対し御礼申し上げます。また千葉敏さんをはじめとする原子力研究所のスタッフの方々の篤いサポート活動に感謝いたします。

References


8. 超新星物質の不安定領域におけるフラグメント分布
Fragment Distribution in Unstable Region of Supernova Matter

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概要
We study the possibility of supernova nucleosynthesis in the early stage of explosion in terms of liquid-gas phase transition of supernova matter. We solve the coexistence problem in supernova matter, and find that at low entropies, the adiabatic paths go through the coexistence region. The nuclear chart in supernova matter is also discussed.

1 背景と目的

我々の周りに存在する元素の起源は、原子核・宇宙双方の分野から徐々に明かになってきた。しかし特に重い元素についての理解では、まだ説明が十分な点がある。ここでは既存の理論で説明できなかった重い元素合成の機構として超新星物質の不安定領域における元素合成の可能性を提案したい。

そこでまず、重い元素合成の標準モデルを見ることにする。鉄より重い元素の中で208Biまで安定な元素は、原子核が星の進化の段階で放出される中性子をゆっくりと吸収して作られる(s過程)、さらに重い238Uなどの放射性原子核は早中に中性子捕獲過程であるr過程でのみ合成され得る。このr過程のサイトとして超新星爆発を考えられている。しかしながら以上の様々な標準モデルには、『安定核に陽子が二つ以上束縛した陽子過剰核を合成するような経路が確立されていないこと』や、『現在観測されている元素の背景組成を十分に説明できない』という困難がある。

本研究の目的はこれらの困難を解決するような新しい元素合成の機構を議論し、その可能性を定量的に調べることにある。

2 超新星物質の不安定領域における元素合成の可能性

一方軽い元素の合成に目を向けると、QCD相転移が初期宇宙元素合成に大きな役割を果たし得るなど、核物質の相転移が元素合成に果たす役割は大きい。そこで重い元素合成に対しては核物質の

液相・気相相転移(LG相転移)が上の問題を解決できる重要な要素になり得る。この核物質の相には『体積不安定性から急激な相分離を起こすスピノーダル不安定相』と『原子核と核子気体が共存するLG共存相』の二種類の不安定相がある。核物質の相がこの不安定相の中に入ると、そこで様々な質量数を持つ原子核が作られる。例として重イオン反応で生成される原子核は、LG相転移に伴う不安

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図 1: 太陽系の元素の相対組成

定相での相分離によるものと考えられている。そこで今後は新しい元素合成の機構として、超新星爆発中に LG 相移に伴う不安定領域での原子核破壊により重い元素を合成するという観点のモデルを考えた。

中心に鉄のコアを持つような重い星の超新星爆発では、まず鉄のコアの重力崩壊が始まる。そこで放出されるニュートリノがコアに閉じ込められるとレプトン数が保存され、やがてβ平衡が実現する。

こうした標準的な進行の後、その頃に中心にできた原始中性子星の表面付近で、上で見たような核物質の不安定領域での元素合成がおこり、元素の組成のバックグラウンドが形成され、これを核にして上の過程などによるピークが再現される可能性がある。（図1の太い実線（Sum）を参照）

3 超新星物質での LG 共存相

上記の可能性を定性的に調べる第一歩として、超新星爆発を超新星物質の断熱膨張と仮定し、相互作用に相対論的平均場理論のパラメーターの一つである TM 1 ([3] 参照) を用い液体・気体の相平衡を考慮して、バリオン密度に対する超新星物質の (1) 状態方程式 (EOS)、(2) 相図と断熱経路、(3) 断熱指数を求めた。そこで示す結果は lepton ratio Y_e = 0.3 の場合である。（図2は上から順に (1), (2), (3) に対応している。）核物質の密度の目安となる標準核物質密度は、TM 1 の場合には 0.145 fm^-3 である。縦軸は (1) では圧力、(2) では温度、(3) では断熱指数である。

(1) 超新星物質の密度が一定の場合（実線）と LG 共存相を考慮した場合（点線）の EOS の変化を調べた。

これを見ると LG 共存相を考慮すると、状態方程式が数パーセント変化する事がわかる。

(2) スピノーダル領域と LG 共存相領域に対する断熱経路を求めた。超新星物質の断熱曲線がこれらの不安定領域に入る際のバリオン当たりのエントロピーを見ることで、超新星爆発中にこの
機構が起こり得る経路の情報が得られる。太い実線の内側がLG共存相、破線の内側がスピノダル領域になっている。また細い実線は各エントロピーでの断熱曲線である。

ここで超新星物質が液相・気相相転移をおこす臨界温度は約15MeVである。これは対称核物質の臨界温度や超新星爆発時の温度と同程度に高い。また不安定領域が標準核密度近くまで広がっている。この二つから、非常に大きな領域で『不安定領域での原子核生成』が起こる可能性があり、これは注目すべきである。断熱曲線を見ると超新星爆発で『不安定領域での原子核生成』が起こる条件として『バリオン当たりのエントロピー S/B が 5 以下』が必要である。

(3) 一番下の図は断熱指数である。断熱指数 $\Gamma = d(\log P)/d(\log \rho_B)$ が 4/3 より小さければ、流体力学的不安定性から (2) の不安定領域で作られた元素(原子核)が乱流などの効果で外部に放出される起きる可能性が高くなる。

この指数をみると比較的内側の領域(密度の高い領域) まで $\Gamma = 4/3$ を下回る領域が存在することから、この新しい機構で生成された物質が外部に放出される可能性は十分にある。
4 超新星物質での原子核の存在領域

ここまでの議論では超新星物質の液相を無限、一様かつ正に帯電した相という取り扱いで共存相の条件を求めてきた。しかし上の議論での液相の取り扱いは、実際にはクーロン斥力が無視できないため現実的ではない。そこで以下に示す議論では超新星物質を有限体として取り扱い、LG共存相の内側でのフラグメント分布を調べる。その方法としてはNuclear Statistical Equilibrium(NSE)を考えられるが、これは結合エネルギーに実験値を用いているため、真空中の原子核と異なり負の背景電子分布をもつ超新星物質中の原子核にそのまま適用することはできない。そこで、今回は電子分布によるクーロン力の抑制を取り入れた超新星物質での原子核の質量公式を作り、その原子核の存在領域を求めた。

まず超新星物質の負の背景電子分布を取り込むために、半径Rの原子核の背景に半径$R_0$の球体内全体で電気的中性になるような電荷分布を考えて、これによる結合エネルギーのクーロン項の変化を調べた。簡単のため対相互作用による補正項を落とし、超新星物質の質量公式は次のようになる。

$$B(A,Z) = \alpha_c A - \alpha_s A^2 - \alpha_A \left( \frac{A}{2} - Z \right)^2 A^{-1} - V_c$$

$$V_c = \frac{3(Ze)^2}{5R} \left( 1 - \frac{3}{2} \eta + \frac{1}{2} \eta^3 \right)$$

ただし$\eta = R/R_0$とする。この質量公式での原子核の存在領域は$\partial B/\partial N > 0$かつ$\partial B/\partial Z > 0$である。また核分裂に対する安定性をBohrとWheelerの方法に従って調べた。
ここでクーロンの項は

\[
V_c = V_c^{\text{nuc}} + V_c^{\text{e}} + V_c^{\text{nuc-e}}
\]

\[
V_c^{\text{nuc-e}} = \int d^3r u_c(r) \rho_{\text{nuc}}(r)
\]

(nuc は nucleus, e は electron を意味する) と表せる。したがって、この変形エネルギーは

\[
dE_{\text{def}} = dV_c^{\text{nuc}} + dE_{\text{surf}}^{\text{nuc}} + dV_c^{\text{nuc-e}}
\]

\[
= dE_{\text{def}}^{\text{c}} + V_c^{\text{nuc-e}}
\]

と表せる。今簡単のため、軸対称で \( l = 2 \) の変形を考えて \( |\alpha_0| \ll 1 \) とする。変形するのは原子核 \( R \Rightarrow R + \zeta(\theta, \eta) \) だけであり、変形核の内部では電荷密度は常に \( \rho \) に等しく、外部ではゼロである。以上の様にして求めた原子核の存在領域は図 3 の通りである。図 3 は \( \rho_e = 0.1 \rho_0 \) の場合（レプトンとバリオンの比は約 \( Y = 0.3 \) に対応）を例にとって示したものである。これから負の背景電子分布を取り入れることで超新星物質中での原子核の存在範囲が非常に大きくなり、中性子過剰核と陽子過剰核の両方に広がった重い核まで存在できることが分かった。

参考文献

9. 膨張無限系の分子動力学シミュレーション

Molecular dynamics simulation of expanding infinite matter

Shinpei Chikazumi*, Toshiki Maruyama†, Satoshi Chiba†, Koji Niita‡, Akira Iwamoto*

概要

Multi-fragmentation occurred in an expanding infinite system is studied by using molecular dynamics simulation. To evaluate the secondary decay effect, the time evolution of expanding system is proceeded till all fragments are stabilized completely. The fragment mass distribution from the expansion is compared with a percolation model and the cause of the exponential shape is clarified. The cause of small critical temperature is also discussed.

1 introduction

The molecular dynamics(MD) simulation of expanding infinite system was proposed in order to investigate the mechanism of multi-fragmentation observed in experiments of heavy ion reactions[?]. During the heavy ion reaction, a so-called fireball is created in the participant region. This fireball is expected to have a large radial flow that cannot be investigated within the framework of thermal equilibrium. The point of our model is that this fireball with radial flow is regarded as an expanding nuclear matter. By using expanding infinite system, the fireball can be investigated taking radial flow into account without any difficulty caused by the finite size effects.

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The model of expanding infinite system is described by the molecular dynamics simulation constrained by an advanced periodic boundary condition under which any two points have a relative velocity proportional to its relative distance. This condition lets an infinite system expand at a fixed rate. The system starts to expand at saturation density and becomes unstable as its average density decreases. During expansion, multi-fragmentation occurs because of the instability. In the model of expanding infinite system, this multi-fragmentation process can be investigated systematically by using the two parameters, the initial temperature $T$ and the velocity of expansion $h$.

So far, we obtained the following conclusions.

- The fragment mass distribution depends on only $h$. The initial temperature $T$ does not affect the distribution.

- With large $h$, the fragment mass distribution follows exponential type. While, with small $h$, the distribution follows a power law.

- The power law originated from expansion appears even when the initial temperature is lower than typical critical temperature. Hence, the power law in expansion is different from that of thermal equilibrium system.

The above claims are main points obtained by the previous research[1]. In this report, further details are reported[2].
2 Fragment distribution

2.1 Secondary decay effect

In the previous study, the time evolution of expansion stops when the average density reaches at \(1/20\rho_0\) (\(\rho_0\) is a saturation density). This final density was determined as the density where the interactions between two fragments are weak enough to identify each fragment. However, the fragment originated from expansion might expand by itself. That is, the fragment mass distribution obtained by the previous calculation might include some unstable fragments. We have to obtain the distribution of stable fragments because all fragments observed in the experiment are stable. To obtain stable fragments, we will simply let the system expand more and the final density is set at \(1/1000\rho_0\). The final distribution at the final density are shown in Figure 1 and 2.

The initial temperature is set \(T_{\text{init}} = 30\) MeV and each fragment mass distribution corresponds to different value of \(h\). The feature that the exponential shape with large \(h\) and the power law with small \(h\) are same as the case of \(1/20\rho_0\). When the initial temperature is set at \(T = 5\) MeV, almost the same results are obtained. That is, that initial temperature does not affect the distribution. However, the exponent of the power law is \(\tau = 3.1\) which is quite different from 2.0 for \(1/20\rho_0\). This exponent depends on the initial temperature. In the case of \(T_{\text{init}} = 5\) MeV, \(\tau = 2.7\) is obtained.

In the previous research, we stressed the difference between the exponent of the power law originated from expanding matter and that of the power law of thermal equilibrated system. According to the result from Metropolis sampling method, the matter without expansion at \(1/20\rho_0\) and \(T = 8\) (MeV) has an exponent \(\tau = 2.5\). In the case of expansion, \(\tau = 2.0\) is obtained when the density is at \(1/20\rho_0\). This small exponent seems to be characteristic but it is found that the cause of this small value is unstable fragments. The value of exponent is not available to classify the difference of the two power laws. Nevertheless, it is still characteristic that the power law appears when the initial temperature is small as long as \(h\) is small. In the case of thermal equilibrium system, so-called U-shape like Figure 3(a) is obtained with low temperature. While, this kind of shape never appear in the expanding system.
図 3: The comparison between isothermal fragmentation and expanding fragmentation. The three figures on the left hand side (a), (b) and (c) are isothermal distributions ($T = 5, 8, 18$ MeV) at $\rho = 0.05\rho_0$. The two figures on the right hand side (d) and (e) are the distributions obtained by the expansion ($h = 0.10$).
図 4: Random distribution at $\rho = 0.05$
2.2 Fragment distribution of random configuration

The origin of the distributions should be explained. In order to approach the mechanism of fragmentation, a simulation like percolation model is performed.

A lattice system with periodic boundary condition is prepared. On each lattice point, particles are placed randomly on the condition that the average density is at $1/20\rho_0$. The fragments in the lattice are defined by connecting the two particle located in the neighbor. It is clear that the fragment distribution depends on the distance $r_e$ which is used for evaluating the fragments. The figure 4 shows the fragment mass distributions with different $r_e$. As seen in the figure, the distribution changes from the exponential shape into the power law as the distance $r_e$ increases. This dependence of the distribution to $r_e$ is similar to the dependence to $h$ for the case of expanding system. That $h$ is large corresponds to that $r_e$ is small. In the case of lattice system, $r_e = 1$ is the minimum where particles are most randomly distributed. That is, it can be assessed that the exponential distribution is characteristic of randomness. It is natural that this randomness appears when the system expands rapidly. This is because the randomness originated from the initial state is preserved during expansion when radial velocity overcomes thermal fluctuations. It can be also explained that the randomness does not depend on the initial temperature because the expansion starts at saturation density. While, the power law with large $r_e$ does not directly correspond to that of expanding system. The $r_e$ can be considered as a function represented by both $h$ and interactions. However, the form of this function is not clear so far.

![Graph](image)

图5: The density dependence of pressure in the static nuclear matter. The pressure is calculated on the basis of the virial theorem. Each data point with $(\rho, T)$ corresponds to that of Fig. 6.

3 Instability of thermal equilibrated nuclear matter

It is important to investigate the nuclear matter below saturation density without expansion because it is the basis when discussing the instability of expanding nuclear matter. Figure 5 shows the isothermal curb obtained by Metropolis sampling method. In figure 5, the isothermal line from $T = 3$ to $T = 30$ (MeV) and the ground state of fcc lattice are depicted. Without the fcc lattice, the freedom to make fragments is given in order to compare these results with the case of expanding system. In the figure, $T = 8$ (MeV) is almost a critical temperature. This value is quite smaller than typical value. The cause of this small critical temperature is due to the fact fragmentation is allowed when evaluating the pressure. Generally speaking, the fragmentation lets the system stable so that the unstable region defined as $dP/d\rho < 0$ is narrow. When the homogeneous configuration is assumed, the system becomes unstable easily as the temperature decreases. While, if the fragmentation is allowed, it restores the stability of the system. As a result, the system becomes stable.
図 6: The dependence of energy on the mean nearest neighbor distance for the initial temperature $T_{\text{init}} = 5$ and 30 MeV with $h = 0.10$.

when the temperature is relatively smaller. This fact is described by the sudden change from $T = 3$ to the ground state of the fcc lattice.

4 Expanding infinite system as open system

The model of expanding infinite system does not conserve energy during time evolution. The response of the energy to the density depends on both $h$ and $T_{\text{init}}$. Figure 6 shows the case of $h = 0.1$ and $T_{\text{init}} = 5, 30$ (MeV). In the model of expanding infinite system, the system is constrained as to expand at a fixed rate. Therefore, the energy increases when attractive force dominates and decreases when repulsive force dominates. It is considered that these energy dependence affects the instability of the system. Thus the expanding infinite system should be considered as what is different from the fire ball assumed in the context of heavy ion collision. Though the expanding infinite matter is regarded as a fire ball without surface effects, this point of view is wrong. Nevertheless, from point of the study of nuclear matter, the expanding nuclear matter is still an interesting model. This is because this model is an simple example of open system. The phase transition in the open system has been an attractive topics nowadays[3]. I am plan to study expanding infinite system as an open system from now on.

參考文献


10. Hydrodynamic analysis of anisotropic transverse flow at RHIC

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RHICにおける非等方的な流れの流体モデルによる解析

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abstract: By using a (3+1)-dimensional relativistic hydrodynamic model, we estimate the magnitude of (differential) elliptic flow parameter \( v_2 \) at the BNL-RHIC energy. We compare the centrality and the transverse momentum dependence of \( v_2 \) with the experimental data observed by the STAR Collaboration.

The main goals in the physics of relativistic heavy-ion collisions are not only the discovery of a new state of deconfined nuclear matter, the quark-gluon plasma (QGP), but also the investigation of thermodynamical aspects of its new phase, i.e., the equation of state (EOS), the order of phase transition between the QGP phase and the hadron phase, or the critical temperature [1, 2]. Since the pressure gradient perpendicular to the collision axis causes various transverse collective flow, such as radial flow, directed flow and elliptic flow, these flows observed in the final state are expected to carry the information about the EOS [3]. Elliptic flow especially is believed to be one of the powerful tools to investigate the nuclear equation of state and the degree of thermalization. Elliptic flow is characterized by the second Fourier coefficient of the azimuthal distribution [4]:

\[
v_2 = \frac{\int \cos 2\phi E \frac{dN}{dp^2} dp^3}{\int E \frac{dN}{dp^2} dp^3}.
\]  

(1)

It has been measured in relativistic heavy-ion collisions at various energies. At the BNL Relativistic Heavy-Ion Collider (RHIC), the STAR Collaboration recently observed the transverse momentum and the centrality dependence of \( v_2 \) [5]. In this paper, we quantitatively analyze these data by making use of a relativistic hydrodynamic model and show our preliminary results.

Some groups have already analyzed these data by using a two-dimensional hydrodynamic model [6, 7]. They assume the longitudinal flow can be described by Bjorken’s solution [8] and numerically solve hydrodynamic equations only in the transverse plane. Under this assumption, all physical quantities do not depend on rapidity. On the other hand, the physical quantities actually depend on the rapidity even at the RHIC energy. Therefore fully three-dimensional hydrodynamic simulations are indispensable for a comprehensive understanding of the expansion stage of heavy-ion collisions and the phase transition of nuclear matter. Our 3-D hydrodynamic model shown in Ref. [9, 10] is suited for this purpose.

Relativistic hydrodynamic equations represent (local) energy and momentum conservations

\[
\partial_\mu T^{\mu\nu}(x) = 0,
\]  

(2)

and baryon density conservation

\[
\partial_\mu n_B^\mu(x) = 0, \quad n_B^\mu = n_B u^\mu.
\]  

(3)

For a perfect fluid, the energy-momentum tensor \( T^{\mu\nu} \) is written by

\[
T^{\mu\nu} = (E + P)u^\mu u^\nu - P\eta^{\mu\nu}.
\]  

(4)

Here \( E, P, n_B \) and \( u^\mu \) are, respectively, energy density, pressure, baryon density and four-velocity.

It is needed to specify the EOS of nuclear matter in order to solve the hydrodynamic equations (2) and (3) numerically. We employ a model EOS with first-order phase transition between the QGP phase
and the hadron phase. The QGP phase is assumed to be composed of massless and free u, d, s quarks and gluons. We adopted a resonance gas model with an exclude volume correction for the hadron phase. One can construct a model EOS with first order phase transition on prescription of Gibbs' phase equilibrium condition. $T_{\text{QGP}} = T_{\text{had}}$, $\mu_{\text{QGP}} = \mu_{\text{had}}$ and $P_{\text{QGP}} = P_{\text{had}}$. For further details on the EOS used in this work, see Ref. [11].

Initial conditions in the hydrodynamic model are chosen so that we can reproduce the experimental data of single particle spectra. We use the Cooper-Frye prescription [12] in order to transfer from the hydrodynamic picture to the particle picture. We suppose the hydrodynamic picture is valid above the freeze-out energy density $E_f$. Below $E_f$, the particle picture is rather adequate and the observed spectra reflect the momentum distribution on the freeze-out hypersurface $\Sigma$. The Cooper-Frye formula [12] is written by

$$E \frac{dN}{dp^3} = \frac{g}{(2\pi)^3} \int_\Sigma \exp \left( \frac{p' u_\nu (x) - \mu (x)}{T (x)} \right) \pm 1.$$ (5)

Here $g$ represents the degree of freedom for hadrons under consideration. Numerical results of hydrodynamic simulations are reflected through $T (x), \mu (x), u^\mu (x)$ and $\sigma (x)$. The Cooper-Frye formula is used for particles directly emitted from freeze-out hypersurface. On the other hand, particles feeding from resonances also contribute to the spectra. The contribution from resonance decays plays an important role in understanding the flow phenomena because the resonance decays dilutes the observed elliptic flow owing to decay kinematics [9]. Detailed description to obtain the contribution from resonance decays within our hydrodynamic model are represented in Ref. [10].

We chose initial conditions in this paper so as to reproduce the multiplicity of charged particles at midrapidity. Usually, we use the (pseudo)rapidity distribution to fix the longitudinal profile and the magnitude of energy density at the initial time, and the transverse mass (or momentum) distribution to fix the freeze-out energy density (or temperature). The available experimental data at the RHIC energy are rather limited, so we can say that our tuning of initial conditions is at a rough estimate. It should be noted that tuning of initial conditions is very important in discussing flow phenomena because hydrodynamic models have a little prediction power in comparison with microscopic transport models.

The impact parameter which we chose for central collisions is 2.5 fm. At the initial time $t_i = 1.0$ fm, the energy density and the baryon density at the origin are, respectively, $15 \text{ GeV/fm}^3$ and 0.135 fm$^{-3}$. Longitudinal density profile at the initial time can be represented by Bjorken's solution with a cut near the light cone in order to make the total energy finite; While transverse density profile at the initial time is supposed to be in proportion to the number of wounded nucleons at the transverse area. For nuclear density distribution, we use a standard Woods-Saxon distribution for a gold nucleus. Initial transverse flow is assumed to vanish. The freeze-out energy density $E_f$ can be chosen 132 MeV/fm$^3$, which corresponds to the freeze-out temperature 140 MeV at the zero baryon density. The resultant pseudorapidity density for charged particles becomes 620, where we take account of direct $\pi$, $K$, $p$ and $\bar{p}$, and those feeding from $\rho$, $K^*$, $\omega$ and $\Delta$. On the other hand, observed experimental data show $dN_{ch}/d\eta |_{|\eta|<1} = 555 \pm 35 \text{ (syst.)}$ for 6% central events (PHOBOS) [13] and $dN_{ch}/d\eta |_{|\eta|=0} = 622 \pm 41 \text{ (syst.)}$ (PHENIX) for 5% central events [14]. By changing the impact parameter and unchanging the other parameters, we can easily obtain the initial conditions for non-central collisions. We calculate $v_2$ for various impact parameters and compare these results with experimental data observed by the STAR Collaboration [5]. We choose impact parameters as 2.5, 5.0, 7.5, 10.0 or 12.5 fm. Figure 1 shows the centrality dependence of $v_2$ for charged pions. We average $v_2$ over $|\eta| < 1.3$ and $0.1 < p_t < 2.0$ GeV. Horizontal axis shows the normalized multiplicity of charged particles. The normalization factor is the maximum value of the multiplicity. We should be careful when one compare the hydrodynamic results with the experimental data through the dependence on the normalized multiplicity. Results obtained from hydrodynamics usually show mean values, not including fluctuations; while experimental data always contain fluctuations. This means that $\pi_{\text{max}}$ obtained by the experimental group is always larger than the one from hydrodynamics. Taking account of this fact, our results are slightly shifted to the left direction and could be reasonable agreement with experimental data from central ($b \sim 0$ fm) to semi-central ($b \sim 8$ fm). Other relevant quantity like the impact parameter would be very welcome.

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1 At present, there are a large number of preliminary data at the RHIC energy and these data were presented at the international conference of Quark Matter 2001[1].

2The authors of Ref. [7] took into account this fact and scaled $n_{ch}/n_{\text{max}}$ by a factor 0.95.
when discussing the centrality dependence of elliptic flow. Figure 2 shows $v_2$ as functions of transverse momentum $p_t$ for charged pions. Experimental data, which corresponds to the minimum bias events, are represented by plots with errorbars. Our results for each impact parameters are represented by lines. Results from hydrodynamics are all rising linearly as functions of $p_t$ in this region. Experimental data also show linear dependence with respect to $p_t$ below 1.4 GeV.

In summary, we calculate the magnitude of (differential) elliptic flow $v_2$ for various impact parameters at the BNL RHIC energy by using the fully 3-D relativistic hydrodynamic model. Our results are consistent with the previous analyses in Refs. [6, 7].

In this paper, we rely on the wounded nucleon scaling for initial conditions in non-central collisions. Recently Kolb et al. [15] discuss the initial transverse profile dependence of the multiplicity and the elliptic flow. The energy density profile based on the binary collisions rather than the wounded nucleon parametrization seems to be compatible with the experimental data of charged particle multiplicity per participants. On the other hand, the elliptic flow is not so sensitive to the parametrization scheme. If we change the initial transverse parametrization, our results for the elliptic flow will not be changed significantly.

Since we reproduce the multiplicity only near midrapidity, our tuning of initial parameters at the present paper is not proper to discuss the rapidity dependence of elliptic flow. The freeze-out parameter also should be chosen properly so that we reproduce $n_t$ spectra for hadrons. In addition to the elliptic flow, the directed flow as a function of rapidity will be interesting in understanding the space-time evolution of hot nuclear matter. These analyses are in progress and the results will be shown elsewhere.
References

[1] See, for example, the following website: http://www.rhic.bnl.gov/qm2001/program.html


11. Probing the “Nutcracker” by Two-Pion Correlation

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Abstract

We investigate the “nutcracker” phenomenon by studying two-pion correlation functions of non-central relativistic heavy-ion collisions. We apply the full (3+1)-dimensional hydrodynamic model with first order phase transition. Based on numerical results which shows the “nutcracked” freeze-out hypersurface, we calculate the two-pion correlation functions. In the case of snapshot hypersurfaces which clearly show the nut-shell structure, we find an interesting behavior at high relative momenta. We discuss the possibility of observing the phenomenon in heavy ion experiments.

Many kinds of candidates have been proposed for a signature of creation of the quark-gluon plasma (QGP) at high energy heavy ion experiments [1]. Teaney and Shuryak show that an unusual space-time evolution of hot and dense matter may occur in non-central heavy ion collisions if the phase transition takes place [2]. This phenomenon is named “nutcracker” scenario because the matter distribution looks like a “nut” and two “half-shells” and the characteristic distribution is a consequence of the existence of phase transition between the hadronic phase and the QGP phase. As the scenario exhibits a geometrical feature, pion interferometry is the most suitable tool to confirm the scenario. According to the (anti-)symmetry of identical quanta, particle interferometry gives us a geometrical information such as source sizes and lifetime [3]. In the present paper, we examine the possibility for the two-pion correlation function as a probe of the “nutcracker”. In spite of the detailed analyses of the nutcracker scenario by Kolb et al. [4], the initial condition for the scenario is not clear. Here we select several sets of parameters which give typical structures in space-time evolution and focus our discussion on the behaviors of correlation functions.

Because the scenario happens in non-central collisions, we have to apply the full (3+1)-dimension calculations for the hydrodynamical equations [5]. Here we adopt an equation of state (EoS) with first order phase transition. The QGP phase consists of a free gas of u, d and s-quarks and gluons and the hadronic phase is a free resonance gas with excluded volume correction where heavy resonances are included up to 2 GeV [6]. Assuming the critical temperature \( T_c \) as 160 MeV at zero net baryon density, the bag constant becomes \( B^{1/4} = 233 \) MeV. As for the impact parameter, we put it as 7 fm. Maximum energy density \( E_{\text{max}} \) at the initial state is fixed at 7 GeV/fm³. Though this value is much higher than the value for the 158 GeV/A Pb+Pb collisions at SPS [7], much higher energy density is expected at RHIC.

Whether the nut-shell structure appears or not depends on the shape of the initial transverse profile [4]. We assume that the transverse profile is proportional to the Woods-Saxon function. Here we vary a diffuseness parameter \( \delta_\perp \) of the Woods-Saxon function so that four different scenarios occur, then we compare correlation functions which are obtained from those scenarios. Figure 1 shows the volume fraction distributions of freeze-out hypersurface projected onto the
transverse plane. Here we show only the volume in $|z| \leq 0.5$ fm. Each column corresponds to one scenario. Each row corresponds to chronological snapshots before the freeze-out. The time interval of the snapshots is 0.4 fm, respectively. The leftmost series corresponds to $\delta_r = 0.75$, where two shells and a nut are created at the final stage ($t - t_0 = 6.8$ fm). We call this scenario "nut-shell". The next column corresponds to $\delta_r = 1.0$ ($t - t_0 = 6.0$ fm $\sim 7.6$ fm). Similar structure also appears in this scenario but two shells survive longer than a central nut. So we call the scenario "shell". In the third series, though we can also see the structure in the second figure ($t - t_0 = 6.0$ fm), the shells vanish at the next step and only a central nut has been left. We call the scenario "nut" ($\delta_r = 0.5$). The rightmost series where $\delta_r = 0.25$ shows no unusual space-time evolution. We call the scenario "usual".

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Figure 1: Volume fraction distributions of freeze-out hypersurfaces at each time step. See text for detail.
Assuming that the source is chaotic, the two-pion correlation function can be written as

\[ C(q^\mu, K^\nu) = 1 + R(q^\mu, K^\nu), \]

\[ R(q^\mu, K^\nu) = \frac{1}{N} \int K_\mu d\sigma^\mu(x) \int K_\nu d\sigma^\nu(x') \sqrt{F(k_1, x)F(k_2, x')}F(k_1, x')F(k_2, x)\cos(q_\mu(x-x'))(3) \]

where \( K^\mu = 1/2(k_1^\mu + k_2^\mu) \) is the average momentum of two emitted pions, \( q^\mu = k_1^\mu - k_2^\mu \) is the relative momentum and \( N \) is the normalization given by one-particle distribution. \( F(k, x) \) denotes the Bose-Einstein distribution function which is given as

\[ F(k, x) = \frac{1}{\exp(u_\mu(x)k^\mu/T_T) - 1}. \quad (2) \]

The four velocity of the fluid \( u^\mu(x) \) causes the space-momentum correlation which is observed as collective flow. As is well known, we can obtain the geometrical information on a space-time direction from the correlator as a function of a corresponding component of the relative momentum \[ [8]. \]

As seen in Fig.1, the nut-shell structure appears in the shape of the freeze-out hypersurface. But geometrical information obtained from the correlator corresponds to the source function \( (i.e., \) emission probability of a particle with a momentum from a space-time point), which is deformed by the collective flow through the thermal distribution \( (2). \) First of all, we consider the particles at midrapidity because the nut-shell structure has been created in central region. We fix the transverse momentum as \( K_T = 50 \text{ MeV}. \) Putting the longitudinal relative momentum as \( q_z = 0, \) we have three variables; azimuthal angle \( \phi \) and the transverse relative momentum \( q_x \) and \( q_y. \) (Note that the \( x \) axis is chosen to be parallel to impact parameter.) If we consider the case \( \phi = 0 \ (\phi = \pi/2) \), \( q_x \) is the outward (sideward) component and \( q_y \) is the sideward (outward) component. In the nut-shell structure (see the lowest left figure of Fig.1), shape of the source along \( x \) axis differs from the one along \( y \) axis because the outer shell is cracked on \( y \) axis. Hence, if we probe the source along \( x \) axis and \( y \) axis respectively, the result should show some differences. Therefore, we calculate the sideward correlation functions of both \( \phi = 0 \) and \( \phi = \pi/2 \).

Figure 3 shows the sideward correlation functions. Left figure is the case \( \phi = 0 \) and right one is the case \( \phi = \pi/2. \) In each figure, thick line denotes the correlation function calculated from the "nut-shell" snapshot hypersurface (see the lower left figure of Fig.1). As clearly seen, the correlation function has the second peak at \( q_{\text{side}} \simeq 250 \text{ MeV} \) in the case \( \phi = \pi/2. \) This is the clear signature of the nut-shell structure of the source. Hence, we see the signature of the nutcracker by comparing the sideward correlation function of \( \phi = \pi/2 \) with the one of \( \phi = 0 \) which has no secondary peak. Other four kinds of lines include the whole hypersurface. However, even the "shell" scenario shows no peak at high relative momentum in the case \( \phi = \pi/2. \) In order to discuss more quantitatively, let us use a simple two-dimensional source model with nut-shell structure,

\[ S(x, y) = S_{\text{nut}}(x, y) + S_{\text{shell}}(x, y), \quad (3) \]

\[ S_{\text{nut}}(x, y) = \frac{1 - \alpha}{2 \pi \sigma_x \sigma_y} \exp \left( -\frac{x^2}{2 \sigma_x^2} - \frac{y^2}{2 \sigma_y^2} \right), \quad (4) \]

\[ S_{\text{shell}}(x, y) = \frac{\alpha}{8L \delta_x} \theta(L - |y|) \{ \theta(\delta_x - |x - R_x|) + \theta(\delta_x - |x + R_x|) \}. \quad (5) \]

A nut is modeled into a gaussian source which has the width \( \sigma_x \) and \( \sigma_y \) (eq.(4)). Two half shells are modeled into walls parallel to \( y \) axis with length \( L \) and thickness \( \delta_x. \) The source function
$S(x, y)$ describes the emission probability of particles at transverse coordinate $x$ and $y$. Here we neglect the effect of the collective flow (i.e., space-momentum correlation), for simplicity. A parameter $\alpha$ controls the ratio of the number of particles emitted from the shell to the whole source. We can easily obtain the correlation functions in analytical form,

$$C(q_{\text{side}}, q_{\text{out}} = 0, \phi = \frac{\pi}{2}) - 1 = \left( 1 - \alpha \right) \exp \left\{ -\frac{1}{2} \left( \frac{q_{\text{side}}^2 \sigma_x^2}{q_{\text{side}} \delta_x} \right) \right\} + \alpha \frac{\sin q_{\text{side}} \delta_x}{q_{\text{side}} \delta_x} \cos q_{\text{side}} R_x \right\}^2, \quad (6)$$

$$C(q_{\text{side}}, q_{\text{out}} = 0, \phi = 0) - 1 = \left( 1 - \alpha \right) \exp \left\{ -\frac{1}{2} \left( \frac{q_{\text{side}}^2 \sigma_y^2}{q_{\text{side}} \delta_y} \right) \right\} + \alpha \frac{\sin q_{\text{side}} L}{q_{\text{side}} L} \right\}^2. \quad (7)$$

The nut-shell structure appears in eq.(6) as $\cos(q_{\text{side}} R_x)$. Figure 3 shows the above correlation functions for the cases $\alpha = 0.1$ and $\alpha = 0.3$. Parameters are set to $\delta_x = 0.5, \sigma_x = 2.5, \sigma_y = 4.0, R_x = 2.5$ and $L = 8.0$. In the case $\phi = \pi/2$, we see the peak which characterizes the nut-shell structure for $\alpha = 0.3$ but there is no peak for $\alpha = 0.1$. In our hydrodynamic analysis (case of Fig.2), $\alpha$ is about 0.12. This value is too few for the correlation function to behave as the case of the nut-shell snapshot.

In summary, we analyze the two-pion correlation functions for “nutcracker” scenario. We perform a hydrodynamical calculation to reproduce some unusual structure of freeze-out hypersurface. Based on the results, we calculate the correlation function and consider how the unusual structures affect the correlation function. We find that the second peak of the correlation function at high relative momentum results from the nut-shell structure. However, it is also shown that the number of pions emitted from the shell is not so large that we cannot see any second peak of the correlation function from our freeze-out hypersurface. More detailed discussion will be published later [9].


References

Figure 2: Left: the two-pion sideward correlation function at $\phi = 0$. Right: $\phi = \pi/2$. Thick lines show the function calculated from the nut-shell snapshot. Dotted lines, dashed lines, dashed-dotted lines and thin lines correspond to the "nut-shell" scenario, the "shell" scenario, the "nut" scenario and the "usual" scenario, respectively.

Figure 3: Correlation function calculated from a toy source model.
12. Effect of Phase Transition on QGP fluid in Ultra-relativistic Heavy Ion Collision
超相対論的重イオン衝突におけるQGP流体への相転移の影響

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Abstract
A full (3+1)-dimensional calculation using the Lagrangian hydrodynamics is proposed for relativistic nuclear collisions. The calculation enables us to evaluate anisotropic flow of hot and dense matter which appears in non-central and/or asymmetrical relativistic nuclear collisions. The relativistic hydrodynamical model is related to the equation of the state and the useful for the verification of quark-gluon plasma state. By virtue of the Lagrangian hydrodynamics we can easily trace the trajectory which corresponds to the adiabatic paths in the T-μ plane. We evaluate the directly of the influence of the phase transition to physical phenomena in the ultra-relativistic nuclear collisions. Using our relativistic hydrodynamical model, we discuss the effect of the phase transition on the collective flow.

1 Introduction
The study of the quark-gluon plasma (QGP) is one of the hottest topics in the high energy nuclear physics and many analyses have been done from both experimental and theoretical side to understand the QGP phenomena [1]. Recently anisotropic collective flow phenomena have been measured precisely not only at AGS [2, 3] but also SPS [4, 5] and furthermore the analyses of anisotropic collective flow have already been reported at RHIC by STAR collaboration [6]. Based on the relativistic hydrodynamical model, Rischke reported that the existence of the minimum point in the excitation function of directed flow would suggest the phase transition [7]. Under the first phase transition, energy density has the discontinuity at critical temperature. On the other hand, pressure has no discontinuity as a function of temperature. The sound velocity in the mixed phase vanishes. As a result, the growth of the flow in the case of the existence of the phase transition is suppressed and the behavior of flow with the phase transition is different from one without the phase transition. Hence, the analysis of collective flow is one of the candidates from which we obtain the information of QGP phase. The result by Rischke suggests that the phase transition has already occurred at the AGS energy region. However there are no experimental results which show the production of QGP clearly at the AGS energy region. Here, we discuss how the influence of the phase transition appears on collective flow by using the realistic model.
2 Model Description

The outline of our calculation procedure is as follows: First, we parameterize the initial conditions of energy density, baryon number density and local velocity based on the result of event generator URASiMA [8]. Our event generator URASiMA [9] is characterized by multi-chain model (MCM) by which multi-particle production process can be described successfully.

In order to solve the relativistic hydrodynamical equation, we need to introduce the equations of state. In the case of equation of state without phase transition, we use the ideal hadron gas including resonances up to 2 GeV. In the case of equation of state with the phase transition model we assume the first order phase transition. Above the phase transition, the thermodynamical quantities are assumed to be determined by QGP gas which is dominated by massless u, d, s quarks and gluons. For the hadron phase we use the excluded volume model [10] which contains all resonances up to 2 GeV [11]. In the mixed phase we introduce the fraction of the volume of the QGP phase, λ(τ, ) (0 ≤ λ ≤ 1).

Finally hadron spectra are obtained by Cooper-Frye formula [12]. We assume that the hadronization process occurs when the temperature and chemical potential of the volume elements cross the boundary which is determined by chemical freeze-out and thermal freeze-out [8].
3 Calculated Results

The directed flow is given by

$$\langle p_x/N \rangle_{\text{dir}} = \frac{1}{N} \int_{-y_{\text{CM}}}^{y_{\text{CM}}} dy \langle p_x/N \rangle(y) \frac{dN}{dy} \text{sgn}(y).$$  \hspace{1cm} (1)

We obtain the directed flow in the central rapidity region, $|y_{\text{CM}}| \leq 0.5$. Figure 3 shows the directed flow with phase transition and without phase transition. The absolute value of the case with phase transition is smaller than that without phase transition. The influence of mixed phase appears in the result of directed flow. From fig.3 we can see that difference of two cases is large from 60 AGeV to 100 AGeV. The effect of the phase transition appears in this incident energy region. At less than 50 AGeV the production of QGP phase is small and the effect of the phase transition does not appear clearly. At more than 100 AGeV the effect of QGP phase becomes large and the effect of the mixed phase is smeared by QGP phase.

Figure 4 shows the trajectory of a fluid element at the central region in phase diagram for each incident energy. In Au+Au 10 AGeV collisions the QGP phase does not exist and in the beginning the volume element moves on the phase boundary (mixed phase) and next enters hadron phase and finally hadronization process occurs. On the other hand in 50, 100, and 150 AGeV collisions there is QGP phase in the beginning of the space-time evolution.

Next we calculate the sound velocity through the trajectory in $T-\mu$ plane (fig.4), which is given by

$$c_s^2 = \left. \frac{\partial p}{\partial \epsilon} \right|_{s/n_B=\text{const}}.$$  \hspace{1cm} (2)

In fig.5 the sound velocity as a function of time of the same volume element in fig.4. We can see that at 10 AGeV the sound velocity in mixed phase is not equal to zero. Therefore
in this energy region the effect of the mixed phase can not be observed clearly, though the volume element remains in the mixed phase. The naive estimation that the sound velocity is equal to zero in mixed phase is not correct. On the contrary, at 50, 100, and 150 AGeV the sound velocity vanishes. At these incident energy region there is possibility that the effect of mixed phase on flow is extracted clearly.

The directed flow which is obtained in experiment is summed for time and all volume elements. In fig.6 the sound velocity for all volume elements of fluid in Au+Au 10, 50, 100, and 150 AGeV collisions. In the case of Au+Au 10 AGeV collisions, the effect of mixed phase is very small. In this incident energy region, we can hardly detect the signal of the production of QGP from the analysis of collective flow. In the case of Au+Au 50, 100 AGeV collisions, the effect of mixed phase appears. However we can not see the strong signal of phase transition such as sound velocity vanishes.

4 Summary

We present (3+1)-dimensional relativistic hydrodynamical model of the Lagrangian hydrodynamics without assuming symmetrical conditions, under the first phase transition. Our algorithm is based on the entropy conservation law and the baryon number conservation law explicitly. In our algorithm we trace the volume elements of fluid along the stream of flux. By using our relativistic hydrodynamical model based on the Lagrangian hydrodynamics, the path of the each volume element in the phase diagram is able to be traced easily. Therefore we can investigate directly how the phase transition takes place and affects physical phenomenon in an ultra-relativistic nuclear collision.

Using this model, we have investigated the effect of the phase transition on anisotropic flow, inputting the various incident energies of collisions. The effect of phase transition on the directed flow is not so clear as Rischke discussed. In actual experiment data we can observe the superposition of the QGP phase, the mixed phase and the hadron phase. We should consider not only the contribution from the mixed phase but also the QGP phase and the hadron phase. Though in the mixed phase the sound velocity is \( c_s^2 \sim 0 \), in the QGP phase it is \( c_s^2 = 1/3 \) and in hadron phase it is \( c_s^2 \sim 0.2 \). Therefore the influence of the mixed phase can be smeared by QGP phase and the hadron phase and it can be hardly detected if the condition of experiment is not suitable for the analysis of the phase transition.

In this calculation the impact parameter is fixed to \( b = 3.0 \) fm. We should investigate the impact parameter dependence of the collective flow. In addition we have to inspect of the validity the initial condition and the freeze-out condition.

References

Figure 5: The sound velocity as a function of time at Au+Au 10 AGeV collisions. We can see that the sound velocity does not vanish in mixed phase.

Figure 6: The sound velocity of all volume elements of fluid as a function of time.

13. 相対論的重イオン衝突における
ハドロン横質量スペクトルの再硬化

Re-Hardening of Hadron Transverse Mass Spectra
in Relativistic Heavy-Ion Collisions

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Abstract

We analyze the spectra of pions and protons in heavy-ion collisions at relativistic energies from 2 A GeV to 65+65 A GeV by using a jet-implemented hadron-string cascade model. In this energy region, hadron transverse mass spectra first show softening until SPS energies, and re-hardening may emerge at RHIC energies. Since hadronic matter is expected to show only softening at higher energy densities, this re-hardening of spectra can be interpreted as a good signature of the quark-gluon plasma formation.

1 Introduction

高エネルギー重イオン反応研究の主たる目的はハドロンの相図において通常核から遠くはなれた高温 and/or 高密度核物質の性質を明らかにすることである。特に、現在はクォーク・グルー・プラズマ (QGP) の生成に興味が持たれている [1, 2]。SPSエネルギーまででの重イオン反応でも QGP生成が期待され、またいくつかの間接的証拠は見出されている。その有力なものは、\( J/\psi \)生成の異常な抑制 [4]、ストレンジエンス・ハドロン生成の増加 (あるいは非常に早い時点でストレンジエンス化学平衡の実現) [5]、低質量レプトン対の増加 [6]、フローナどのにみられるハドロン・スペクトルの軟化 [7, 8]、などがある。これらから「SPSエネルギーの Pb + Pb 衝突では QGPが生成されていると考えるのが自然である (少なくとも QGP生成を否定できない)」といえる。

しかしながら、生成された物質がハドロン物質ではないということ「否定の否定」という形での強い肯定はできていない。これは、ハドロンが複合粒子系から様々な素過程断面積を基本原理から厳密に計算することが必要である。例えば、重イオン反応では \( \pi \) 中間子は \( 2\pi \) から容易に生成され、しかも質量が大きいために \( J/\psi \)の静的吸収分解反応 (低エネルギーでの \( J/\psi + \rho \rightarrow D\bar{D} \)) を起こし得るが、この分解反応の断面積の正確な (あるいは誰もが認める) 評価が難しくいため、ハドロン相での \( J/\psi \)抑制が否定し切れない。
それでは、相転移に伴って一般に見られる、また高エネルギー重イオン反応で実際にハドロン・スペクトルにも現れている物質の軟化現象（softening）はどうであろうか？（エネルギー密度を増加させた場合）物質の軽衰（softening）は、粒子内部自由度の急激な増加のためにエネルギーがその内部自由度に分散するために起こる。一次相転移の場合には、局所にエネルギーが費されるために、粒子当たりの並進運動エネルギーが減少し（あるいは増加せず）、圧力がエネルギー密度に比例して大きくなり、と理解できる。ところがハドロンは複合粒子であるために、QGPへの相転移を起こすと急激な自由度増加があり、具体的には大きな質量をもつ共鳴ハドロンやハドロン・ストリングが大量に生成される場合には、入射エネルギーの多くが質量エネルギーとして費され、softeningが見られるのである。実際、softeningはQGP生成が期待されない粒子当たり数 GeV程度のエネルギーから起こり始めており [7]、ハドロン画像での説明が必要である。我々はこれまでに、上記の「ハドロン自由度の増加」と「高運動量での斥力の減少」という2つの概念を手がかりにして、このsofteningの問題について調べてきた [20, 15]。SIS-AGSのエネルギー領域においては、まず高運動量で核子と中間子の結合が弱くなっているために、1 A GeV 当たりで核力（特に斥力の効果）が小さくなり [15]、「ポテンシャルからsofteningが起こる。その後、共鳴ハドロン、ハドロン・ストリングが大量に生成され始め、「ハドロン自由度からのsoftening」が始まる [15, 20]。後者に関しては注意が必要である。SPS までのエネルギー領域で様々な（ハドロン・ストリングを含む）共鳴ハドロンを取入れた自由度輸送模型が実験データの説明に成功しているが [9, 10, 11, 14, 17]、AGS までのエネルギー領域では少数のオドロン励起状態しか取り込めていない自由度モデルも成功をおさめている [18, 19]。このため、自由度にかかわらずデータを説明可能ではないか、という反論がある。しかし、後者においては必ず有限の粒子生成時間の粒子数粒子生成数が取入れられており、これが実質的には大きな連続自由度の役割を果たすのである [20]。

これらの研究成果を踏まえ、過去の2つの要因により「実験データで見られているsofteningはハドロン画像で説明できる」という結論が得られる。実はこのことは、1960年代にHagedornにより既に提唱されており、複合粒子性からも Exponential的に増大する状態密度のためにハドロン相の温度(α粒子当たりの運動エネルギー)には上限が存在する [16]。このため、圧力の上昇はエネルギー密度の上昇に比べて小さくなる。

それでも、エネルギー密度を十分に大きくして相転移が完了した場合に期待される「物質の再硬化」(re-hardening) は、ハドロン物質で説明できるであろうか？まず、上記の「ハドロン自由度の増加」と「高運動量での斥力の減少」がより大きなエネルギー密度でも続くのであれば、ハドロン画像での説明は非常に困難である。そして、SPSエネルギーでの重イオン衝突で達成されているエネルギー密度以上の自由度で、ハドロン物質の性質が再び変化する (自由度の減少や斥力の増加) ことは期待できない。ハドロン自由度については、既に SPSにおいて連続領域（ストリング）に入っており、より大きな質量密度での急激な変化は無理は無である。核力についても、斥力芯領域に達しているエネルギー以上の斥力の減少を説明しており、斥力芯の内側で大きな斥力芯があるとは考えられない。よって、平衡に達した物質を考える限りにおいては、ハドロン画像での説明是不可能である。

このハドロン・スペクトルの Re-Hardening は、実際に最近の RHIC データに現われている [12, 13]。RHICエネルギー(√s = 130 A GeV)での Au+Au衝突実験で現われる横動径方向の集団運動流速度（radial flow）は、SPSエネルギーでのものと比べて明らかに大きくなっているのである。

実は、この Re-Hardening の Hardening の部分については、現実・理論ともにこれまでの研究で示唆されている。例えば、衝突する系や入射エネルギーの control は難しいが、エムジュンへの宇宙線照射においても、1.5 GeV/fm^3程度のエネルギー密度において平均横運動量の急激な増加が1980年代に既に観測されている [21]。また理論的にも、流体模型計算とカスケード計算を組み合わせた枠組で、SPSエネルギーと比べて RHIC、LHC での平均横運動量の急激な増
加が期待されることが示されている[22]。ただし、我々が主張したいのは、近年の系統的な重イオン衝突実験によって明らかになってきた“Re”-Hardeningである。SIS-AGS-SPSエネルギーで十分にsoft化してのドロン自由度と斥力芯が使い切られると上に見られるRHICエネルギーでのHardeningは、QGP生成の強力な証拠といえるのではなかろうか？

この報告では、mini-Jet生成を通じてparton自由度を部分的に取り込んだhadron-string cascade model(JAM)[17]を用いて、radial flowに見られる最初のhardening領域(SIS-AGS-JHFエネルギーまで)から、softening領域(AGS-JHFからSPS-RHIC(√s = 56 A GeV)エネルギーまで)、そして、re-hardening領域(RHICエネルギー)までの高エネルギー重イオン反応を系統的に分析し、実際にre-hardeningが期待されることを示す。

2 Model

JAM[17]は奈良が中心となって作ったカスケード型であり、様々な共鳴ハドロンやストリング生成が取り入れられている。また、高エネルギーでは、構動的QCDから計算できるparton-parton散乱断面積からアイコナル近似を用いてmini-jet生成断面積を評価し、始状態・終状態でのグレーオン放射を取り入れた模型(Lund model, PYTHIA 6.1)[23]に時空・空間情報を加えることにより、多重mini-jet生成を時空・時間的に追いかけられるようになっている。このような様々な段階の自由度を取り入れているため、JAMは今後の目的に適していると言える。さらに、これまでにAGSエネルギーでの陽子入射反応から重イオン反応[17], SPSエネルギーでの重イオン反応[24]に適用され、実験データをよく再現しているという意味で、十分に自信できる模型でもある。

衝突型加速器(RHIC, LHC)エネルギーでは、JAMはRHIC実験の主たる理論referenceとなっているHIJING[25,26]の空間・時間versionとみなすことができる。この空間・時間versionとの性格から、例えば粒子のエネルギー損失などについてはJAMとHIJINGでは記述の仕方が異なる。HIJINGでは重イオン同士が衝突して通り抜けるまでの間に起こると期待されるmini-jetの数をアイコナル的に評価して発生させるのに対して、JAMではそれぞれのドロン間衝突でのmini-jet生成確率とsoft過程反応確率を評価し、それぞれの段階で素過程反応を起こさせる。この違いのため、RHICエネルギーにおいてHIJINGではπ⁰スペクトルの説明にpartonのエネルギー損失が必要だが、JAMでは多段階の反応でエネルギー損失を起こしたドロンがmini-jet生成を起こす場合があり、partonのエネルギー損失を与えずともデータをほぼ説明する。どちらの理論的取り扱いがより好ましいかは、まだ不明であり、模型の優劣を議論するのは時間早そうであるが、JAMを少なくともRHICエネルギーでの一つの基準として考えても問題ないであろう。

3 Results

JAMによる計算は、

- SIS (2 A GeV, Au+Au)
- AGS (10.6 A GeV, Au+Au)
- JHF (25 A GeV, Au+Au)
- SPS (158 A GeV, Pb+Pb)
- RHIC (√s = 56 and 130 A GeV, Au+Au)

のそれぞれのエネルギーでの中心衝突(衝突径圧0 ≤ b ≤ 3.3 fm)のシミュレーションを行った。これはcentral 350 mbであり、Au+Au, Pb+Pb衝突において実験データが豊富なcentral 5～6%にほぼ対応している。イベント数としては、それぞれの入射エネルギーに対して1000 events以上計算している。実験に比べるとイベント数は少ないが、パイオン、陽子の横質量スペクトルについては(stopping powerが小さくなっているために、mid-rapidityでの分布が小さくなっているため、...
いるRHICエネルギーでのNet陽子(p−p)を除いては統計は十分といえる。ただし、K,Λなどのストレンジネスをもつ粒子については、低エネルギー領域での統計が足りないため、ここでは主としてπ，pについての解析結果を報告する。

計算にあたっては、実験データを見てからのパラメータ調整が無いかように、RHIC実験でAu+Au衝突が観測される以前に公開されたJAM version 1.009.27 (April 21, 2000) を用いた。

Figure 1: Rapidity distribution ($dN/dy$) at the AGS and SPS energies, and pseudorapidity distribution ($dN/dη$) at the RHIC energy. Calculated results are compared with the E802 [27], NA49 [28], PHENIX [29], BRAHMS [30], and PHOBOS [31] data. Collisions with impact parameter $b<3.3$ fm has been taken in the calculations. For experiments, $σ_{trig}=350$ mb for the E802 experiment, 5% for NA49 and PHENIX, and 6% for BRAHMS and PHOBOS.

### 3.1 Rapidity Distribution

まず、Fig. 1 にAGS, SPS, RHICエネルギーでのラピディティ (擬ラピディティ) 分布を示す。パリオンのstopping powerがAGS, SPSエネルギーで若干大きいことを除けば、全体的な傾向は十分よく再現できているといえる。RHICエネルギー ($\sqrt{s} = 130$ A GeV) では、衝突の重
心付近での荷電粒子の発生密度分布が3つの実験グループから報告されている（PHENIX collaboration [29], BRAHMS collaboration [30], and PHOBOS collaboration [31]）。JAMでの計算結果は、これらの実験値をほぼ再現している。また、中心付近での$\rho$と$\bar{\rho}$の粒子数比の計算値は$\rho/\bar{\rho} \approx 0.63$であり、BRAHMS collaboration [30]の実験値（0.61±0.06(stat.)±0.4(syst.)）の誤差の範囲内にある。このことから、AGSからRHICにわたる広いエネルギー領域で、粒子生成やstoppingなどのbulk dynamicsはJAMでよく説明されているといえる。

![Graph](image)

Figure 2: Transverse mass spectra of hadrons at the AGS, SPS and RHIC energies. Calculated results are compared with the E802 [27] and NA49 [28] data. Collisions with impact parameter $b<3.3$ fm has been taken in the calculations. For experiments, $\sigma_{\text{inel}}=350$ mb for the E802 experiment, and 5% for NA49. The exponential lines with the slope parameters $T'$ are shown to guide eyes.

### 3.2 Transverse Mass Spectrum

次に、Fig.2に中心ラピディティでのハドロンの横質量スペクトルを示す。AGS（SPS）エネルギーでは、パイオニン（負電荷ハドロン）のスペクトルがよく再現できていることが分かる。特に注目し
たいのは、高エネルギーでの指数関数的な振る舞いからずれている低エネルギー領域での増加が
よく表れていることである。この部分は音に生産される共鳴ハドロンの崩壊からの寄与が
多くを占めるため、どの程度崩壊ハドロンが存在していたかの指標になる。（崩壊ハドロンを多種
取り入れない小自由度モデルでは、この増加が再現しにくい。）

一方、計算値は陽子の低い横質量において overestimate している。この横運動エネルギーにて
数 100 MeV までの陽子は、後に平均場による後押し効果をもたらす部分である [15, 19]。ところ
がここでは平均場を無視しており、計算上は最後の後押しがない。よって、計算値が overestimate
しているのは (平均場を無視したカスケードモデルでは) 自然なことと解釈できる。

以上より、反応の最終的な段階での粒子崩壊や平均場の効果を受けにくく、「非常に物質の膨張」
という流体力学的な振る舞いが顕著に現われる横質量領域は数 100 MeV 以上ということになる。
また、RHIC エネルギーでの 2 GeV 以上の領域は計算結果では、この領域では指数関数的なス
ペクトルとなっており、それぞれの粒子がほぼ熱平衡な状態から放出されているとの仮定と整合
する。

この指数関数的振る舞いが興味深いのは、入射エネルギー依存性である。AGS エネルギーで
のパイロンと陽子のスペクトルの傾きに有意な差があるのに比べて、SPS エネルギーではほぼ平
行となる。これは AGS-SPS エネルギーにおいてみられる softening である。ところが RHIC エ
ネルギーではなるべくスペクトルの逆スロープ・パラメータパイロンに比べて有意に大きくなっ
ている。この逆スロープ・パラメータの質量依存性は「膨張する火の玉」插絵で考えると分かり
やすい。温度を T、粒子が感じる平均的な radial flow velocity を β とすると、逆スロープ・パラ
メータ T' は、(非対相談的に考えると) 2 次元での平均的運動エネルギーであるから、

\[ T' = T + \frac{1}{2} m \beta^2 \]

とあらわせるであろう。パイロンと陽子の逆スロープ・パラメータの差が大きいことは、大きな
radial flow が存在すること、つまり膨張時に大きな圧力が存在していたことを意味する。これが
RHIC エネルギーにおいて現われる re-hardening である。

Table 1: Minimum and maximum kinetic energies in fitting the transverse mass spectra with a
single exponential. We have tried three sets of parameters, I, II, III and IV. Each value has the
unit in GeV.

<table>
<thead>
<tr>
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<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
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<td></td>
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<td>Δmax</td>
<td>Δmin</td>
<td>Δmax</td>
</tr>
<tr>
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<td>3.0</td>
<td>0.5</td>
<td>2.0</td>
</tr>
<tr>
<td>AGS-RHIC</td>
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<td>3.0</td>
<td>0.5</td>
<td>2.0</td>
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3.3 Re-Hardening

この振る舞いを定量的に議論するため、パイロンと陽子の横質量スペクトルを single exponential
で fit し、その質量依存性から温度 T と radial flow velocity β に分解してみる。ここでは、前述
のように反応の最終段階からの効果が大きい低エネルギーキー領域とともに、(Net 陽子) 計算の統
計が足りないこと、そして初期条件 (mini-jet) に非常に敏感であることの 2 点から大きな横質量
の領域 (m_T > 2 GeV) を除いた領域での fit を行った。また、スペクトルが完全には single
exponential 的で無いため、Table 1 に示した数つかの横質量領域での fit を行った。
Figure 3 にこうした fit の結果を示す。上図にはパイオンと陽子の逆スロープ・パラメータを示している。AGS から SPS のエネルギーでは、入射エネルギーの上昇について、パイオンのスペクトルが非常にゆっくりと硬く（逆スロープ・パラメータが大きく）なっているのに対して、陽子のスペクトルは AGS から SPS エネルギーではほぼ一定であり、fit の領域によってはむしろ軟らかくなっている。そして、RHIC エネルギー（$\sqrt{s} = 130$ A GeV）で急激に硬くなる。

これを $T$ と $\beta$ に分岐したのが中図と下図である。温度はほぼ一定値 ($T \approx 220$ MeV) に近付いていくが、radial flow velocity は RHIC ($\sqrt{s} = 56$ A GeV) までは小さくなり、$\sqrt{s} = 130$ A GeV において大きくなっている。この上昇は非常に急激であり、少なくとも何かの質的な変化があることを期待させる。これまでの議論に従えば、このエネルギーの上昇について、一旦軟らかくなっただ物質が再び硬くなること (Re-Hardening) は、(平衡がある程度達成されているという仮定のまでは) 共存相を越えて QGP が bulk に生成されていることを意味するであろう。

Figure 3: Calculated inverse slope parameters (top) and the extracted temperature (middle) and radial flow parameters (bottom) from SIS to RHIC energies with three fit ranges sets of I (dotted), II (dashed), III (dot-dashed) and IV (solid). See Table 1 for fit ranges I, II, III and IV.
4 Discussions and Conclusion

本報告では、ハドロンの横質量スペクトルの再硬化が RHIC エネルギーで起こることを mini-jet 生成を通じて parton 自由度を取り込んだカスケード模型 JAM により示した。この振る舞いは preliminary であるが最近の RHIC での観測データにも現れている [12, 13]。そしてこの再硬化は、RHIC エネルギーで、部分的に QGP ができてハドロン相と混合しているような共存相を越えて、bulk な QGP が生成されていると考えることにより、もっとも素直に解釈できる。

上の解釈は流体力学的な振る舞いに基づくものであり、当然ながら暗に局所的には平衡がある程度達成されていることを仮定している。この仮定は、RHIC でのデータで支持されているように見える。例えば、(1) 中心衝突ではハドロンの横質量分布が素過程よりもずっとポルツマン的 (single exponential に近い) である [3], (2) 逆スロープ・パラメータの粒子質量依存性が、ほぼ 1 次になっていること [32] は、粒子間の相互作用が十分頻繁に起こっていることを示している。又 (3) より反応初期のダイナミクスに敏感な elliptic flow ($v_2$) が、広い範囲 ($p_t < 2$ GeV) で横運動量に比例するという、流体力学的な振る舞いを示している。など多くの現象が「平衡」がかなりの程度達成されていることを示唆する。計算上、「平衡化」の影響は大きい。例えば、反応初期の baryon-baryon 衝突のみを考えずに計算すると、ハドロンのスペクトルは exponential よりもむしろ power law に近い形となり、中心衝突の実験データを再現しない。

ただし、現時点での JAM においては、異なる mini-jet で生成された parton どうしの相互作用 (parton cascade) を取り入れられていなかったり、ストリングが生成された後に初めて他の jet から作られたストリングやハドロンと相互作用する。しかしながら、シミュレーション計算の上でも RHIC エネルギーにおいて初期に生成された mini-jet がハドロンの多くを強く押し出していることは、parton cascade が取り入れられなければ十分平衡に達するだけの粒子数密度と parton が飛び回る時空体積があることを示唆する。実際に、RHIC での陽子の横質量スペクトルのデータは、ここでの示したものより有意に硬く、re-hardening の様相は実験データの方がより顕著に見えているようである。一般には粒子衝突の頻度が高くなくなる程横質量スペクトルが硬くなるため、parton cascade 過程を取り入れれば、データの再現性がより高まると考えられる。

以上をまとめて、現段階での模型自体は平衡に達した parton gas を生成する仕組みになっておらず、ここで示した re-hardening は素過程としての mini-jet 発生と、そこから作られる大量の粒子が強く相互作用して flow を生み出している結果であるが、実験データは早い段階での流体的振る舞いを支し、かつ計算上も早い段階での「平衡化過程」を取り入れることにより、実験値に近付く結果が得られると期待できる。よって、上述の解釈 (データに現われる re-hardening が QGP 生成を示すこと) とそれが基にしている前提 (平衡がある程度達成されている) は結論として正しいと考える。より堅固な結論を得るためには、実験 (と解析) の側では粒子識別をきちんと行ったスペクトルの確定と、平衡化がどの程度起こっているかを確認する elliptic flow や粒子相関などの他の物理量の測定、理論の側では parton の段階での相互作用を取り入れる、あるいはカスケードと流体模型を組み合わせに、早い段階での平衡化を記述できる枠組の整備、また QCD 相転移の order や (1 次の場合には) 質的の大きさを考慮できる枠組の構築を起こしてさらに分析を起こすこと、などのさまざまな努力が必要であろう。それにしても、粒子スペクトルの入射エネルギー依存性 (radial flow の励起関数）を re-hardening が始まると期待される $\sqrt{s} \simeq 40$ A GeV 近辺で丁寧に測定・計算することが不可欠である。

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References


14. 相対論的重イオン衝突におけるハドロン自由度

Hadronic Degrees of Freedom in Relativistic Heavy Ion Collisions

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The observation of temperature and transverse expansion velocity between BNL-AGS and CERN-SPS suggests the change of property of hadronic matter. In order to study the origin of the fact, it is important to check whether or not pure hadronic scenarios are excluded. We have discussed the temperature and transverse expansion in relativistic heavy-ion collisions using pure hadronic cascade model, HANDEL. We conclude the hadronic matter in AGS energies are understandable in the frame of the hadronic cascade model if we care how much hadronic degrees of freedom are counted.

I. はじめに

高エネルギー原子核衝突で生成されるハドロン物質の温度や横膨張の強さを、各種粒子の横質量スペクトルからエネルギーの関数としてプロットされた図が実験的に得られている (Fig. 1, 2)。温度に関しては、低エネルギー領域では、入射エネルギーの増大とともに温度が増加していくが、核子あたりのエネルギーが1GeVを超えるようになったあたりから、温度の増加は鈍くなっていることが示された。

![Graph showing temperature vs. energy](Image)

**Fig. 1.** 温度パラメーターの励起関数。EOS, FOPI, NA49 (一粒子+HBT) に関するそれぞれのデータは、[2-4] からの引用。[1] より許可を得て複製。

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FIG. 3. Au(11.6 GeV/c)+Au 中心衝突 (0 ≤ b ≤ 3.5 fm) における、ラビディティー分布 (下) と横質量分布 (上) を平均場付き RBUU($\sqrt{s_{NN}} = 2.6, 3.5$ GeV)、カスケード RBUU($\sqrt{s_{NN}} = 3.5$ GeV) のそれぞれについて図示した。ここに $\sqrt{s_{NN}}$ は、共鳴粒子生成とストリング生成を分ける不変質量。黒点は E-866 実験の値 [5]。横質量分布に関しては、ラビディティーが $|y_{NN}| < 0.2$ のものを用いた。[6] より許可を得て複製。

一方、低エネルギー領域で入射エネルギーの増大とともに増加している横断截流は、核子あたりのエネ
エネルギーが1GeVを超えるあたりから、その大きさが減少に転じていることが示されている。これらは、ハドロン物質が農間相（クーク・グルー・オン・プラズマ）へ転移していく領域で示される兆候である。\n
即ち、QCD相互作用に近いハドロン物質では、通常の核子とπ中間子に加え、Δ(1232)粒子やρ(760)中間子を始めとする様々な共鳴自由度が発現し、これらの重い質量のハドロンの励起が、温度上昇を妨げ、粒子流を遅くする（＝核物質の軟化）と見なすことができる。

核物質の軟化や温度上昇の鈍化を、ハドロンの自由度を用いた輸送模型であるRBUUを用いて、もう少し定量的に示したのがFig.3である。この図では、1) ストリング自由度を低エネルギー領域から考慮するか否か、2) 平均場を考慮するか否か、を区別した3ケースの結果を、陽子の横質量分布に関して図示している。平均場の導入はハドロン物質を高温で硬いものにする一方、ストリング自由度を低いエネルギーから導入することはハドロン物質を低温で軟かいものにすることができる。このことから、RBUU模型の結果を観察する限り、核物質の軟化や温度の抑制を励起自由度の増大から理解することができるものである。

また、輸送模型に取り入れるハドロン励起自由度と横質量分布の関係を他のハドロン輸送模型に関してみた場合のようなであることも違う。このことに関して著者らは以前に、BNL-AGS領域の重イオン衝突を、低い励起自由度のみを含めた模型HANDELと高い励起自由度まで含めた模型JAMを用いて議論した。その結果は、得られる横質量分布は励起自由度の含め方にはよらない、という予想に反したものであった。\n
この研究では、HANDELに導入されている粒子生成時間の存在が実効自由度として働くことで、高い励起自由度まで含めた模型と同様の横質量分布を与える役割をしていることについて議論する。

II. ハドロン輸送模型JAMとHANDEL

ここでは、2つのハドロン輸送模型JAMとHANDELについて簡単に紹介し、それらがBNL-AGSの入射エネルギーにおいて、どのようなハドロンスペクトルを与えるかについて概観する。

JAM（Jet An Microscopic transport model）[7]は、質量2.0GeVまでのバリオンと同じく1.8GeVまでのメゾノを全種類含む模型である。この模型では、粒子生成は共鳴粒子の崩壊を通じて実現される。また、ある質量を超える共鳴粒子はストリング模型によって崩壊する。この模型は、BNL-AGSからBNL-RHICまでの広いエネルギー領域に適応できることが調べられている。

一方、HANDEL（HAdronic Nucleus-nucleus cascade model）は、軽いハドロン自由度のみを含めた模型である。具体的には、N, N(1440), N(1535), Δ(1232), Λ, Σ, π, η(550), ρ(770), ω(783), K, K*(892)の各アイソスポン自由度を考慮されている。この模型に含まれる低い励起状態の崩壊では、高エネルギー過程での高粒子生成を記述することはできない。そこで、共鳴粒子崩壊による粒子生成に加えて、核子-核子崩壊やπ中間子、核子崩壊で同時に多数の粒子を生成する過程（直接粒子生成）を取り入れている。こうすることによって、HANDELはBNL-AGSからCERN-SPSのエネルギー領域に渡り、素過程におけるπ中間子の生成量に関して、実験値を再現できるようになっている。

これらを用いて計算された、BNL-AGS（核子あたり11.6AGeV/c）で金原子核同士が衝突した時の、ラビディティ分布と横質量分布を、陽子-π中間子においてFig.4に示す。JAM HANDEL両模型がπ*のラビディティ分布の実験値を再現していることから、両模型は粒子生成量を正しく計算しているといえる。また、両模型は両横質量スペクトルの実験値を再現している。詳細に見るとJAMの方が再現性に優れており、おおまかなスペクトルの傾きは両者でほとんど変わらないように見える。横質量分布をボルツマン分布と考えれば、分布の傾きは温度の逆数を示していることになるから、JAMやHANDELは熱的

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に似た性質の物質を与えていることが分かれる。

Au(11.6 GeV/c)+Au → p, π⁺, π⁻ + X (b ≤ 3.3 fm) HANDEL

Au(11.6 GeV/c)+Au → p, π⁺, π⁻ + X (b ≤ 3.3 fm) JAM

FIG. 4. Au(11.6 GeV/c)+Au 中心衝突 (0 ≤ b ≤ 3.3 fm) における、ラピディティー分布 (上) と横質量分布 (下)。ヒストグラムは HANDEL と JAM [7] の計算結果である。一方、黒点は E-866 実験の値 [5]。横質量分布に関しては、ラピディティーが |ηπ| < 0.2 のものを用いた。

自由度の違う 2 つの模型が等しい温度を与え、という上の結果は「与えられたエネルギー密度をもつ、平衡な気体では、自由度の大きいガスほどより低い温度をもつ」という単純な考察では理解できないこと
III. 粒子生成時間と実効のハドロン自由度

粒子生成時間 formation time は、もっともはストリング模型で導入される考え方である。ストリングを崩壊させて粒子生成を起こす際には、ハドロン化するのに有限の時間が必要であり、その時間の大きさは、ストリングの振動の時間と同じ程度だとすれば、弦張力の大ささかから 1 fm/c の程度だと考えられる。ストリング模型に限らず、同時に多粒子を生成するような機構の入った粒子生成模型ではこの粒子生成時間が導入することが、実験を再現する上でも必要となる。前節で議論した 2 つの模型のうち小自由度模型である HANDEL の多粒子発生率にはこの粒子生成時間が導入される。この時間の間には、生成された粒子は他の粒子と相互作用しないようにする。多粒子同時生成は高い不変質量をもった衝突粒子対に対して可能である。このために、粒子生成時間の効果は粒子の運動エネルギーの高い反応の初期のダイナミクスに主に効くはずである。

![図表]

**図 5.** Au(11.6 GeV/c) + Au 中心衝突 (0 ≤ b ≤ 3.3 fm) における、ラビディティー分布 (上) と横質量分布 (下)。ヒストグラムは HANDEL の計算結果である。一方、黒点は E-866 実験の値 [5]。3 通りの粒子生成時間、0.0, 0.8 (通常値), 1.6 fm/c が用いられている。図の詳細は、Fig. 4 と同じである。陽子のラビディティー分布は粒子生成時間が短いほど、重心速度付近に粒子が集中している。横質量分布に関しては陽子・π 中間子とも、粒子生成時間が短いほど、高い横質量成分が増加する。π 中間子のラビディティー分布は、粒子生成時間の選ぶ方には殆ど分からない。

1ラビディティー分布を再現することは、粒子の生成量が再現されていることを意味する。しかしこのことは直ちに、系が同様の温度、または温度変化を与えることを意味しない。反応の発展段階での密度やエネルギー密度の履歴が同じとは限らないからである。
さて、粒子生成時間の役割を調べるために、まず我々は粒子生成時間が導入がハドロンのスペクトルにどのような影響を与えるかを調べた。Fig. 5はHANDELEにおいて、粒子生成時間を0.0, 0.8(標準)，1.6 fm/cの3つの値に変化させた場合の、ラビディティ分布と積質量分布を示す。陽子の重心付近のラビディティ分布を見ると、粒子生成時間を導入しない場合には、阻止能(stopping power)が大きくなることが示されている。これは、反応の初期に粒子生成で生成された粒子が、粒子生成時間の欠落のために直ちに周囲のバリオンと相互作用し、これがバリオンのラビディティが減少を引き起こしたものとみなせる。

一方、積質量分布は、粒子生成時間が果たす熱的な役割についてのヒントを与える。この分布を見ると、粒子生成時間の導入は、積質量の高い成分を減らす働きをしていることが分かる。換言すれば、粒子生成時間の導入は温度や横拡張流を抑制する働きをしていることが分かる。これらの結果は、粒子生成時間の導入は、実効的に新たなハドロン自由度を導入していることになるのではないか、ということを示唆している。

このことを模式的に説明するのがFig. 6である。ここでは、衝突した粒子対から複数の粒子が生成されていく。これらの複数の粒子は、しばらくの間、粒子生成時間に入っており他の粒子と相互作用しない。この生成された粒子全体の系を定まった(不変)質量を持つ塊と見なすことができる。

粒子生成時間の導入が、実効的な新しいハドロン自由度を導入することに対応していることの別の方向からの主張を、Fig. 7に示す。ここでは、周期的境界条件を課された3つのバリオン密度をもつハドロンガスのカロリー曲線が、粒子生成時間が0.0, 0.8 fm/cのそれぞれの場合について、HANDELEの計算結果として示されている。これらの3つの図に共通して見られるのは、粒子生成時間を導入することは、エネルギー密度の増加に対する温度の上昇を抑制することになっている、ということである。

FIG. 6. 同時多粒子生成によって作られた粒子生成時間中の粒子群は定まったエネルギーと運動量を持つ塊とみなすことができる。
FIG. 7. HANDEL によって実験的境界条件を用いて計算した、バリオン密度が$\rho_b = 0.77 \text{ fm}^{-3}$ (左上), 0.39 \text{ fm}^{-3}$ (右上), 0.77 \text{ fm}^{-3}$ (左下) のハドロン物質のカロリー曲線の粒子生成時依存性。それぞれの記号は温度の定義の違いを表す。

ここからも、粒子生成時間の導入は、実効的な新しいハドロン自由度を導入していることになっていると言える。

IV. 重イオン衝突の横質量分布の分析

前節では、横質量分布において、粒子生成時間の導入が実効自由度として働いている可能性を検討した。本節では、このことをもう少し定量的に分析する。横質量 $m_T$ の分布を温度 $T'$ のボルツマン分布で近似する：

$$
\frac{1}{2\pi m_T} \frac{d^2 N}{dm_T dy} \propto \exp \left( \frac{-m_T}{T'} \right)
$$

ここで、左辺は計算から得られるスペクトルであり、これを右辺でフィットすることによって、$T'$ を決めることができる。

さて、温度 $T'$ は横方向の粒子流の運動から決まるが、この粒子流の運動エネルギーの中には純粋に熱的な運動速度$\beta_{th}$ と、横方向への集団運動に起因する運動速度$\beta_T$ の両方が寄与していると考えられる：

$$
T' = \frac{1}{2} m \langle \beta^2 \rangle = \frac{1}{2} m \langle \beta_{th}^2 \rangle + \frac{1}{2} m \langle \beta_T^2 \rangle = T + \frac{1}{2} m \langle \beta_T^2 \rangle
$$

ここに、$T$ は熱的な運動に対応した本当の温度を意味する。これから、それぞれお固有の質量 $m$ をもつ粒子の種類ごとに $T'$ 決定し、これを $m$ の関数としてプロットすれば、$T$ と $\langle \beta_T^2 \rangle$ を決定することができることが分る。
核子とπ中間子についての$T'$を求め、そこから、$T$と$(\beta T)^2$を決定した結果を、Fig. 8とTable 8に示す。この結果から、粒子生成時間の導入は、温度と(集団的)横断流流を抑制する効果があることが示された。

![Graph showing slope parameter vs. mass](image-url)

**Fig. 8.** HANDELによって与えられるAu(11.6 GeV/c)+Au中心衝突(0≤b≤3.3 fm)における、温度$T$と横断流流速度$(\beta T)$のパラメータ$T'$からの決定。3通りの粒子生成時間、0.0, 0.8, 1.6 fm/cが用いられている。ここで用いられた横断流量分布はFig. 5で与えられたものである。

**Table 1.** 輸送模型HANDELで計算されたAu(11.6 GeV/c)+Au中心衝突における、ボルツマン分布の傾きの逆数$T'$からの温度$T$と横断流流速度$(\beta T)$の粒子生成時間依存性

<table>
<thead>
<tr>
<th>$\tau_f$(fm/c)</th>
<th>0.0</th>
<th>0.8</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$(GeV)</td>
<td>0.180</td>
<td>0.167</td>
<td>0.162</td>
</tr>
<tr>
<td>$(\beta T)$</td>
<td>0.486</td>
<td>0.381</td>
<td>0.316</td>
</tr>
</tbody>
</table>

V. まとめと今後の課題

我々は、含める自由度の大きさを変えた2つのハドロン輸送模型を用いた場合に、横断流量分布の傾きが自由度の大きさに依らないように見える原因について分析を行った。その結果、小自由度模型の多粒子同時生成の際に考慮する粒子生成時間の導入は、実用的に追加的なハドロン自由度を導入していることになっていることを、BNL-AGSエネルギー領域の重イオン衝突の横断流量分布を分析するなどによって、結論づけた。

今回の議論の結果は、適切な自由度をあらわす、いわば、何らかの形で取り入れることによって、BNL-AGS領域近傍の重イオン衝突で作られるハドロン物質の励起関数の性質－温度上昇や横断流流の抑制－が、ハドロン相互作用で理解できることを意味している。

この結論でエネルギーをあけるたびに、ハドロン輸送模型で記述されるハドロン物質の中では、ハドロンの自由度が増大し、温度の上昇や横断流流の大きさはますます抑制される。この極限が、Hagedornがかつて議論した限界温度の実現である。流体的な立場から、横断流流の速さを物質の音速と見なすことが許
されるならば、横膨張流の減少は相転移近傍での音速の減少効果と対応させることができる。

ハドロン輸送模型を用いる限り、エネルギーの増加とともにこれらの傾向は如実になるものと思われるが、しかし、実際の自然ではあらゆるところで閉じ込め相の解放があるところで終了し、その先のエネルギー領域の重イオン衝突で生成される物質－クォーク・グルーオン・プラズマにおいては、再びエネルギーの増加とともに温度や横膨張流は増加に転ずるものと考えられる。よって、純粋なハドロン自由度による輸送模型がどのエネルギーまで実験を説明できるか、温度や粒子流などの物理量に注目しながら研究することは、相転移の実現を確固たるものとする上で有用であると考える。

15. Multifragmentation in the 12 GeV proton induced reaction

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\textbf{Abstract}

The intermediate mass fragment (IMF) formation in the 12 GeV proton induced reaction on Au target is analyzed by the quantum molecular dynamics model combined with the JAM hadronic cascade model and the non-equilibrated percolation model. We show that the sideward peaked angular distribution of IMF occur in the multifragmentation at very short time scale around 20 fm/c where non-equilibrated features of the residual nucleus fluctuates the nucleon density and fragments in the repulsive Coulomb force are pushed for the sideward direction.

1 Motivation

Recently, in the KEK 12 GeV proton induced reaction on Au target, interesting multifragmentation phenomena were observed [1]. In the multifragmentation, multiple intermediate mass fragments (IMF) such as O, F, Ne, Na are produced and the angular distribution of IMF has sideward peak around 70 degrees in the laboratory frame. This phenomenon is very mysterious since this can not be understood in a simple statistical picture as discussed above. Therefore, several characteristic mechanisms have been proposed so far such as the shock wave propagation [2], the strange shaped nucleus formation [3] and recoil effects of particle scatterings [4]. However, there is no decisive conclusion and quantitative explanation for the sideward peak of IMF angular distribution, since theoretical analyses of this phenomenon are very difficult. The same experimental conclusion as KEK is obtained at AGS. In the AGS p+Au experiment[4], the laboratory angular distribution of C isotopes come to have sideward peak as the incident energy of proton go beyond 10 GeV. Therefore, it is seemed that some changes occur in the formation mechanism of C isotopes when the incident energy increase. It is expected that these fragment angular distribution anomaly reflects new fragment formation mechanism. And if there is new fragment formation mechanism, that may connected with new property of high excited nuclei produced in the GeV proton induced reaction.

The difficulties of the analysis of IMF formation in p(12 GeV)+Au reaction are summarized as follows. Firstly, appropriate collision cross sections including resonance and string formation are necessary to describe the cascade processes. Here, we adopt the cross sections used in recently developed hadronic cascade model (JAM)[5] which are applicable to the reactions with incident energy of 12 GeV. Next, appropriate mean field dynamics is necessary to describe the formation of residual nucleus. Here, we adopt the QMD mean field interaction which is well tested in the analysis of proton induced reactions.[6] Finally, we have to describe the multifragmentation of residual nucleus to various mass fragments including IMF. This is the
most difficult points. The multifragmentation should reflect the non-equilibrium properties of fragment source since simple statistical equilibrated picture is not applicable to sideward peaking phenomenon as discussed until now. Therefore, we here formulate the new refined non-equilibrated percolation model (NPM) which can describe the multifragmentation by fully taking into account the phase space information of all nucleons inside residual nucleus. By NPM we can take into account the position and momentum fluctuations of nucleons inside fragment source and study the effects of non-equilibrated features of residual nucleus to the IMF formation process.

The purpose of this study is to clarify the mechanism of sideward angular distribution of fragment in the p(12 GeV) + Au reaction by the refined microscopic simulation theory which incorporates high quality collision cross sections given by JAM, standard mean field given by ordinally QMD and multifragmentation description by NPM. We analyze the origin of IMF angular distribution by unified approach to study many fragment observables such as fragment mass and angle and energy distributions quantitatively.

2 Non-equilibrated percolation model without lattice

Recently, the percolation model is proposed as theoretical model which well describe the mass distribution of fragment produced from multifragmentation process in the relativistic energy (> 10 GeV) proton induced reactions [7, 8, 9, 10]. The percolation model describe the projectile and target as nuclear lattices of nucleons, and the essential features of proton induced reactions can be described by using very simple physical assumptions.

The strong assumption of the standard percolation model is that total excitation energy is uniformly distributed over the entire excited system of residual nucleus. That means that residual nucleus reaches equilibrated nuclear matter. In reality, the excitation energy may differ by the local position inside residual nucleus. Therefore, the bond breaking probability should depend on position of the bond. The natural way to decide the position dependence of bond breaking probability is to evaluate the locality of excitation of residual nucleus by using the phase space information of nucleons which belong to residual nucleus. We can get the phase space information of nucleons from the microscopic transport simulation such as QMD. Such locality of excitation may affect the angular distribution of fragment produced from GeV proton induced reaction. And, the assumption of representing the nucleons by sites on a simple cubic lattice in three dimensions is also unnatural. Then, here we formulate following non-equilibrated percolation model without lattice (NPM) combined with QMD. The actual calculation of NPM are performed as follows.

(0) QMD simulates proton induced reaction untilt \( t = t_{sw} \) (fm/c).
(1) Phase space information of nucleons inside resdual nucleus and the physical property of residual nucleus such as charge \( Z_{res} \), mass \( A_{res} \) and excitation energy \( E^* \) are obtained by the method shown at Ref. [11].
(2) All nucleons inside residual nucleus are connected by bonds.
(3) When \( E^* \geq 0.008 \times A_{res} \times c_1 \) (GeV), bonds are broken if the length of bonds are larger than \( R_{cut} \)
When \( E^* < 0.008 \times A_{\text{res}} \times c_1 \) (GeV), bonds are broken if the length of bonds are larger than \( R_{\text{cut}} \times c_2 \) (These procedures are necessary for keeping the stability of residual nucleus when excitation energy of residual nucleus is very low.)

(4) Bond which connects i-th and j-th nucleon is broken by the probability \( p_{b(ij)} \), where \( p_{b(ij)} \) depends on local excitation energy of residual nuclei. \( p_{b(ij)} \) are determined as follows.

\[
p_{b(ij)} = 1 - \exp \left( -\nu E^*(R_{ij}) \right)
\]

Here, \( E^*(R_{ij}) \) is the local excitation energy at local point \( R_{ij} \) which is the center of the bond connecting i-th and j-th nucleon. We calculate \( E^*(R_{ij}) \) as follows,

\[
E^*(R_{ij}) = \sum_k e^*(r_k) \exp \left( -\frac{(R_{ij} - r_k)^2}{a^2} \right),
\]

where \( e^*(r_k) = t_{\text{kin}} - e \) (if \( t_{\text{kin}} - e \leq 0, e^*(r_k) = 0 \)) and \( t_{\text{kin}} \) is kinetic energy of k-th nucleon.

(5) Nucleons still connected to each other via the remaining bonds are identified as fragments.

(6) Mass and charge of fragments are determined and momentum \( P_f \) and position \( R_f \) of fragments are determined as \( P_f = \sum_i A_i p_i \) and \( R_f = \sum_i A_i m_i r_i \) in which \( p_i, r_i \) and \( m_i \) are momentum, position and mass of nucleons inside fragments and \( A_i \) are mass number of fragments.

By using NPM, we can simulate the multifragmentation from non-equilibrium residual nucleus. The parameters of this model such as \( R_{\text{cut}}, \nu, a, c, c_1, c_2 \) are determined to fit the mass distribution of fragments produced from p(11.5 GeV)+Au reaction. The value of parameters of NPM are shown in Table 1.

<table>
<thead>
<tr>
<th>( R_{\text{cut}} ) (fm)</th>
<th>( \nu ) (1/GeV)</th>
<th>( a ) (fm)</th>
<th>( e ) (GeV)</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>600</td>
<td>2.0</td>
<td>0.045</td>
<td>0.2</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Figure 1: Calculated fragment mass distributions in p(12 GeV)+Au reaction. Dashed histograms and solid histograms in left, middle and right panels display calculated results with QMD and QMD followed by NPM at time steps 15 fm/c, 20 fm/c and 30 fm/c. Solid points show the experimental data of p(11.5 GeV)+Au reaction [12]. In QMD simulation, events are restricted to value of the impact parameter \( b \leq 7.5 \) fm.
Calculated fragment mass distributions in \( p(12 \text{ GeV}) + \text{Au} \) reaction are shown in Fig. 1. Dashed histograms and solid histograms in left, middle and right panels display calculated results with QMD and QMD followed by NPM at time steps 15 fm/c, 20 fm/c and 30 fm/c. Solid points show the experimental data of \( p(11.5 \text{ GeV}) + \text{Au} \) reaction [12]. Dashed histograms represent the mass distribution of residual nucleus. The mass of residual nucleus \( A_{\text{res}} \) distributes among \( 160 \sim 197 \). This means that, in QMD simulation, \( 0 \sim 30 \) high energy nucleons are emitted by the high energy collisions at early time steps. Then, the decay of residual nucleus to various mass fragments \( (A_f=1 \sim 197) \) are described by NPM as shown by solid histograms. One can see that the characteristic U-shape curve of the experimental mass distribution is well reproduced by our calculation at each time step. This is a consequence of the contributions of different impact parameters with different local excitation energies.

3 Results

3.1 IMF formation mechanism

![IMF production position distribution](image)

Figure 2: Calculated IMF(\( 6 \leq Z \leq 20 \)) position distributions in xy-plane. Solid points display the IMF positions in 5000 events of QMD at time steps 15 fm/c, 20 fm/c and 30 fm/c followed by NPM. In upper and lower panels, events are restricted to those satisfying \( b \leq 3 \text{ fm} \) and \( 3 \text{ fm} \leq b \leq 7.5 \text{ fm} \).

We now apply QMD combined with NPM to the IMF production process from the \( p(12 \text{ GeV}) + \text{Au} \) reaction. In Fig. 2, IMF(\( 6 \leq Z \leq 20 \)) position distributions in xy-plane calculated by QMD followed by NPM are shown. Solid points display the IMF positions in 5000 events of QMD at time steps 15 fm/c, 20 fm/c and 30 fm/c followed by NPM. In upper panels, IMF positions in the central collision events satisfying \( b \leq 3 \text{ fm} \) are shown. We can see that IMFs are produced from doughnut shaped region. This is because the nucleon density of residual nucleus at the center of the residual nucleus is decreased by the energetic collisions of hadrons and IMFs are rarely produced in this region. In addition, the nucleons at the center of the residual nucleus have relatively high local excitation energy reflecting the collision effects and the bonds between such nucleons are easily broken and IMFs are rarely produced at the center of the residual nucleus. Therefore, IMF is mainly produced from doughnut shaped region in which the number of collision is very small and excitation energy is low. In the lower panels, IMF positions in the peripheral collision events satisfying \( 3 \text{ fm} \leq b \leq 7.5 \text{ fm} \) are shown. We can see that IMFs are produced from a new moon shaped region. In the peripheral collision events, highly excited region is shifted to the peripheral region. As time passes, the nucleons in the peripheral excited region move and collide with other particles and the energy is transported.
to other particles. This transport process is very rapid since the total amount of excitation energies of residual nuclei in the peripheral collision events are smaller than that in the central collision events. Then peripheral excited region shrinks to narrow region at time step 30 fm/c.

In Fig.3, calculated IMF(6 ≤ Z ≤ 20) position distributions in xz-plane, restricted by |y| ≤ 2.0 fm, are shown. Solid points display the IMF positions in 5000 events of QMD at time steps 15 fm/c, 20 fm/c and 30 fm/c followed by NPM. In upper panels, IMF positions in the central collision events satisfying b ≤ 3 fm are shown. We can see from xz-plane that IMF produced point distributions clearly have holes along the collision occuring zone. In lower panels, IMF positions in the peripheral collision events satisfying 3 fm ≤ b ≤ 7.5 fm are shown. We can also see that the IMF positions are concentrated on the peripheral region at early time steps at t=15 fm/c and as time passes IMF come to be produced from whole region of residual nucleus except for the center of the residual nucleus with lower nucleon density due to collision effects. This is caused by rapid energy transport by collisions.

3.2 Analysis of IMF energy distribution

After the fragments being simultaneously produced through the multifragmentation of the residual nucleus, fragments are dispersed by the Coulomb repulsion. In the uniform Coulomb expansion of fragments, the total fragment kinetic energy is calculated as the sum of the thermal and the Coulomb energies. In order to describe the Coulomb expansion of fragments, here we consider the following classical Hamiltonian $H$,

$$ H = \sum_{i=1}^n \sqrt{P_{fi}^2 + m_{fi}^2} + \sum_{i<j} \frac{Z_{fi}Z_{fj}e^2}{R_{fi} - R_{fj}} $$

where $m_{fi}$, $R_{fi}$, $Z_{fi}$ and $P_{fi}$ are respectively, the mass, position, charge, and momentum of the i-th fragment. The Newtonian equation of motion obtained from Eq. 3 are then integrated using standard numerical methods. The initial values of the $R_{fi}$ and $P_{fi}$ are obtained by NPM as discussed in Sec. 2. After the time evolution of the Coulomb expansion process until the total energy of each fragment ceases to show appreciable changes, we obtain kinetic energies of fragments at final states. Here, we adopt time step $\Delta t=0.5$ fm/c and integration until 500 time steps are carried out.

In Fig. 4, comparison of calculated energy distributions of O, F and Ne produced from p(12 GeV)+Au reaction with the experimental data are shown. Left, middle and right panels
Figure 4: Comparison of calculated energy distributions of O, F and Ne produced from p(12 GeV)+Au reaction with the experimental data. Left, middle and right panels show the energy distributions of O, F and Ne. Upper, middle and lower panels show the energy distributions emitted at 30, 70 and 110 degrees from incident proton direction. Solid histograms and dashed histograms show the calculated results with QMD at time steps 20fm/c combined with NPM followed by the Coulomb expansion process and the calculated results with QMD at time steps 20 fm/c combined with NPM. Solid points are experimental data of Ref. [13] show the energy distributions of O, F and Ne. Upper, middle and lower panels show the energy distributions emitted at 30, 70 and 110 degrees from incident proton direction. Solid histograms and dashed histograms show the calculated results with QMD at time steps 20fm/c combined with NPM followed by the Coulomb expansion process and the calculated results with QMD at time steps 20 fm/c combined with NPM. Solid points are experimental data of Ref. [13] From this results we can see that there is large effects in the energy distribution of IMF. If we take into account only kinetic energy of fragment calculated by QMD combined with NPM, the low energy peak around 50 Mev is never reproduced. And the results with QMD combined with NPM following the Coulomb expansion process well reproduce the experimental data. This fact show us that the Coulomb expansion process has very important role in the fragmentation phenomenon.

3.3 Analysis of IMF angular distribution

Now we can obtain the IMF angular distribution. In Fig. 5, calculated time dependence of the angular distribution of IMF(6 ≤ Z ≤ 20) is shown. Dotted, dash-dotted, dashed and solid histograms in left and right panels display the results with QMD at time steps 15fm/c, 20fm/c, 22.5fm/c and 30 fm/c combined with NPM followed by the Coulomb expansion and that is not followed by the Coulomb expansion. Each cross section is multiplied by 0.4, 0.6, 0.75 and 1.0. By the comparison of left and right panels we can see the large effects of the Coulomb expansion process such that forward peaked angular distributions are shifted toward sideward peaked angular distributions by the Coulomb expansion. This mechanism is connected with the IMF production point in the residual nucleus. As shown at Fig. 2 and Fig. 3, IMF is produced from the doughnut shaped region in the violent central collision. Then the Coulomb
force among fragments push the IMF for sideways of the doughnut region even if IMF has the forward angular distribution before the Coulomb expansion. In the peripheral collisions, the rapid transport of local excitation energy occur as time passes and IMF production position become uniform in the residual nucleus except for the center of residual nucleus with low nucleon density by the effects of collisions. Then total impact parameter integrated angular distribution of IMF shifted to sideward peaked one to forward peaked one until t = 30 fm/c even after the Coulomb expansion process as show at left panel of Fig. 5. Therefore, our study suggests the possibility that IMF angular distribution may have sideward peak at around 90 degrees if IMF formation occur at very short time scale around 20 fm/c.

Figure 5: Calculated time dependence of the angular distribution of IMF(6 ≤ Z ≤ 20). Dotted, dash-dotted, dashed and solid histograms in left and right panels display the results with QMD at time steps 15fm/c, 20fm/c, 22.5fm/c and 30 fm/c combined with NPM followed by the Coulomb expansion and that is not followed by the Coulomb expansion. Each cross section is multiplied by 0.4, 0.6, 0.75 and 1.0.

4 Summary

To summarize, we have analyzed the formation mechanism of IMF produced from the 12 GeV proton induced reaction on Au target with QMD combined with the hadronic cascade model JAM and the non-equilibrated percolation model(NPM). NPM can take into account the local excitation in the residual nucleus. Our model has been quantitatively justified by showing good reproduction of fragment mass distributions and fragment energy distributions. By using this model we have shown that IMFs are produced from a doughnut shaped region and a new moon shaped one inside residual nucleus in the central and peripheral collision events. We suggests that combined effects of such geometry of IMF production point distribution and Coulomb expansion of IMF may lead to the sideward peaked angular distribution. Analyzing the angular distribution of IMF, our study shows the possibility that fragmentation to IMF occur at very short time scale around 20 fm/c. The remaining interesting problem is to study the mechanism of the change of peak of the IMF angular distribution appearing as increasing incident proton energy. The study on our approach will be very useful in analyzing such problems.

5 Acknowledgements

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References


16. Monte Carlo Study of SU(2) QCD with Finite Chemical Potential
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Abstract
We study SU(2) lattice QCD with finite chemical potential at $\beta = 1.6$ by using Wilson fermions. The ratio of fermion determinants is evaluated at each step of the Metropolis link update by Woodbury formula. We calculate the baryon number density, the Polyakov lines, and the pseudo-scalar and vector-meson mass on $4^4$ and $4^3 \times 8$ lattices. Preliminary data show the pseudo-scalar meson becomes massive at around $\mu = 0.4$, which indicates the chiral symmetry restoration. We also analyze the behavior of the fermion determinant and eigen value distributions of the determinant, which show a peculiar “Shell-and-Bean” pattern near the transition.

1 INTRODUCTION
Because of the well known problem that action becomes complex, the progress in lattice QCD study of the finite density has been rather slow in contrast to the finite temperature calculation. Indeed, after the first QCD dynamical quark simulation with the chemical potential was done for SU(2)[1], to our knowledge, no full SU(3) QCD calculations had been tried. Though many trials based on novel ideas have been done [2, 3, 4, 5, 6], any definite result has not yet been obtained. Furthermore, Stephanov shows that the quench approximation does not correspond to the correct $N_f = 0$ limit of full QCD [7].

Due to the progress in analytical investigations, in order to obtain some informations on real QCD, it has been proposed recently to study finite density region of the QCD-like theories, instead of real SU(3)-QCD [8, 9]. The QCD-like theories, such as SU(2) QCD, models with quarks in the adjoint representation and QCD at finite isospin density, are expected to have less difficulties in numerical analyses. In these years, there are indeed high activities in Monte Carlo calculations with dynamical quark of these models [10, 11, 12, 13, 14].

In this paper, we report our recent work on SU(2) QCD with Wilson fermions to study finite density states.

2 ALGORITHM
The chemical potential, $\mu$, is introduced in the fermion action, $\bar{\psi} W \psi$, as [16, 15],

\[ W(x, x') = \delta_{x, x'} - 2 \kappa \sum_{i=1}^{3} \left\{ (1 - \gamma_i) U_i(x) \delta_{x', x+i} + (1 + \gamma_i) U_i^\dagger(x') \delta_{x', x-i} \right\} \]

\[ -\kappa \left\{ e^{i \alpha} (1 - \gamma_4) U_4(x) \delta_{x', x+4} + e^{-i \alpha} (1 + \gamma_4) U_4^\dagger(x') \delta_{x', x-4} \right\}. \] (1)

- 87 -
Figure 1: Baryon number density divided by $T^3$ as a function of $\mu$.

Figure 2: Polyakov loop expectation value as a function of $\mu$ for $\kappa=0.158$, 0.180 and 0.185.

Little is known about the behavior of dynamical fermion simulations when the chemical potential is introduced. We therefore adopt an algorithm where the ratio of the determinant,

$$\frac{\det W(U + \Delta U)}{\det W(U)} = \det(I + W(U)^{-1} \Delta W)$$

(2)

is evaluated explicitly at each Metropolis update process, $U \rightarrow U + \Delta U$, where $\Delta W \equiv W(U + \Delta U) - W(U)$ [17, 18, 19]. An essential ingredient of the algorithm is Woodbury formula,

$$(W + \Delta W)^{-1} = W^{-1} - W^{-1} \Delta W (I + W^{-1} \Delta W)^{-1} W^{-1}.$$  

(3)

Suppose we update link variables $U_\mu(x)$'s only on a subset $H$ of whole lattice. Though $\Delta W \neq 0$ only on $H$, Woodbury formula (3) still holds on $H$, and in this case, we can get the ratio of the fermion determinant as far as $U_\mu(x)$'s are locally updated only inside $H$. We take a $2^4$ hypercube as $H$. When we move to the next hypercube, $(W^{-1})_H'$s are initialized by CG method.

3 RESULTS

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\kappa = 0.158$</th>
<th>$\kappa = 0.180$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.775(31)</td>
<td>1.327(51)</td>
</tr>
<tr>
<td>0.2</td>
<td>1.722(09)</td>
<td>1.305(37)</td>
</tr>
<tr>
<td>0.4</td>
<td>1.738(11)</td>
<td>1.428(51)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.649(18)</td>
<td>1.346(65)</td>
</tr>
</tbody>
</table>

$\rho$

| 0.0   | 1.882(45)        | 1.509(41)        |
| 0.2   | 1.783(13)        | 1.490(41)        |
| 0.4   | 1.788(36)        | 1.660(49)        |
| 0.5   | 1.798(46)        | 1.482(100)       |

Table1: Pseudo-scalar and vector masses at $\mu=0.0, 0.2, 0.4$ and 0.5 for $\kappa=0.158$ and 0.180.

Figure 3: $\pi$ and $\rho$ masses as a function of the chemical potential $\mu$ after extrapolation to the chiral limit.

First we calculate the expectation value of the Baryon number density,

$$< n > = \frac{1}{V_s} \frac{\partial}{\partial \mu} \log Z$$

(4)

where $V_s$ is the spatial volume $N_x \times N_y \times N_z$. In Fig.1, we plot $< n > / T^3$, as a function of $\mu$. The dotted line corresponds to the free quark case obtained by setting $U_\mu(x) = 1$ and $\kappa = 1/8$. With larger $\kappa$, the density approach to the free case.
The Polyakov line $< L >$ also increases as a function of $\mu$ as shown in Fig.2. When the hopping parameter, $\kappa$, becomes large from 0.158 to 0.185, values of Polyakov line increase. Since the lattice size is small, no sharp increase of $< L >$ is seen, but growing up of the $< L >$ indicate that quarks become free from the confinement force at $\mu > 0.4$. The behavior of $< L >$ suggests that we are near the phase transition.

We evaluate pseudo scalar and vector meson masses for $\mu = 0.0, 0.2, 0.4$ and 0.5. Because the lattice is small, we fit propagators at $N_t = 2, 3, 4, 5$ and 6 by one-pole fit. The result is shown in Table 1. Although the data are still very preliminary, we extrapolate them to the chiral limit (Fig.3). Error bars are very large, but the pion mass becomes massive at around $\mu \sim 0.4$, which means that the chiral symmetry is restored in these regions.

In Fig.4, we plot eigen value distributions of $W$ for $\mu = 0.0, 0.4, 0.6$ and 0.8 at $\kappa = 0.156$ on $4^4$ lattice. As $\mu$ increases, the distribution of eigen values, $\lambda_i$, becomes wide in the Real axis, and $\text{Min Re}(\lambda) < 0$; on the other hand, $\lambda_i$'s scatter sparsely around origin due to dynamical fermion simulation which includes det$W$ in the measure. These behaviors are expected one. But the behavior of the eigenvalues around the real axis is new to us. As $\mu$ deviates from zero, the region near the real axis becomes dilute, but it seems that a new group appears when $\mu$ becomes further large, and near the phase transition there are outer and inter groups like the shell and bean. This shell-and-bean structure occurs when the calculation breaks, and might be related with the phase transition.

![Figure 4: Eigen value distribution of W for $\mu=0.0, 0.4, 0.6$ and 0.8 at $\kappa = 0.156$ on $4^4$ lattice.](image)

4 CONCLUDING REMARKS

We present numerical study of SU(2) QCD with the chemical potential with Wilson fermions at $\beta = 1.6$. Although the lattice is very small, most data suggests we are reaching the confinement/deconfinement phase transition.

We employ an algorithm which takes into account the ratio of fermion determinant exactly, and has large Markov step, but we suffer from numerical instability and can not go over the phase transition. Near the phase transition, the distribution of eigen values of $W$ shows a peculiar "Shell-and-Bean"
structure. Since the calculation is done at strong coupling region, the strange behavior of $\det W$ might be related with rough configurations far from the continuum. We plan to continue the analysis by using improved gauge actions to clarify the point.

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References

17. Abelian Projection at the Multi-Instanton

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ABSTRACT
We study full non-Abelian, Abelian projected lattice field configurations built up from random instanton gas configurations in the continuum. We study the instanton contribution to the $\bar{Q}Q$ force with respect to whether various versions of Abelian dominance hold. We show that the lattice used to discretize the instanton gas configurations has to be sufficiently coarse ($a \approx 2\bar{\rho}$ compared with the instanton size $\bar{\rho}$) such that maximal Abelian gauge projection as well as the monopole gas contribution to the $\bar{Q}Q$ force reproduce the non-Abelian instanton-mediated force in the intermediate range of linear quasi-confinement.

I. INTRODUCTION
A few years ago, we has started to systematically study the properties of instanton systems with respect to confinement by using multi-instanton simulations [1]. The instanton is a quasipointlike topological object in Euclidean space-time and, being selfdual, satisfies the Yang-Mills field equations [2]. Mixed instanton-antiinstanton configurations [3] (gas or liquid) only locally are approximate solutions, but they offer the attractive possibility to solve, in a semiclassical-like fashion, the $U_A(1)$ problem (explain the large $\eta'$ mass) [4] and to explain chiral symmetry breaking [5,6]. (See Ref. [7] for a recent review.) One main focus of our previous work on instantons, neglected by others, was to explore under which circumstances a dense, however random instanton system could provide confinement. In the beginning [1] it has been pointed out that the monopoles induced in the instanton system and detected by Abelian projection have the clustering property held necessary for confinement. As for the $\bar{Q}Q$ force itself, in Ref. [1] quantitatively the conclusion has been reached, after some refinements, that with the widely accepted density of $1$ fm$^{-4}$ of instantons plus antiinstantons and an average instanton size of $\bar{\rho} = 0.4$ fm, only $40\%$ of the static $\bar{Q}Q$ force could be reproduced at distances around $R \approx 1$ fm.

This result was achieved using an instanton size distribution based on the idea of freezing of the strong coupling constant at large distance. This assumption leads to a dimensionally dictated behavior of the size distribution like $dn(\rho)/d\rho \propto \rho^{-5}$ in the infrared. One should remember that the main parameter of any such model, the instanton density and the instanton size distribution are phenomenological input, beyond justification coming from a truly semiclassical approximation. The latter is available only for a single instanton [8], however afflicted with the famous infrared divergence.

Phenomenologically, in the instanton liquid model instantons are occupying Euclidean space with a density of $n = N/V = 1$ fm$^{-4}$, and with an average radius $\bar{\rho} = 1/3$ fm one gets a packing fraction $f = n\bar{\rho}^4 \approx (1/3)^4$. For our purposes we fix only an average instanton radius $\bar{\rho} = 0.4$ fm. We study the influence of instanton density, considering 1fm$^{-4}$ as the "physical" value. This leaves some room for the choice of the size distribution.

This paper is dedicated to a critical reappraisal of the instanton model to describe, besides other features of the Yang-Mills vacuum, its confinement property, and we shall clearly point out the deficits of this model. Although seemingly insuf-
ficient in quantitative respect, it might be interesting to see to what extent the contribution of instantons to the confining force depends on the $\rho$ distribution. Also the question whether the instanton generated $\bar{Q}Q$ force can be reproduced in accordance to Abelian, monopole or vortex dominance needs some clarification. One has to answer the question at which scale the model can be replaced by an effectively Abelian model with condensing Abelian defects. Monopole and $Z(N)$ vortex mechanisms are presently the leading candidates for an effective infrared description comprising confinement. We shall see that these descriptions are applicable to the instanton mediated force as well. This is expected, corroborating earlier work [10] and the widely-studied interrelation between instantons and monopoles [11,12] and the newer studies concerning the interplay of instantons and vortices [13,14], respectively.

Said in another way, what we want to clarify is the complementarity between the explicit semiclassical-like description in terms of continuous instanton fields on one hand and the monopole aspects on the other. The latter degrees of freedom seem to become physically dominant in the infrared. This is the place where, for our purposes, the lattice discretization appears: it is an infrared matching scale between the instanton picture and the gauge singularities which become manifest in the result of gauge fixing and Abelian projection. If the discretization scale is chosen too small, monopole and vortex degrees of freedom can be identified as well. However, a complementary description of the instanton mediated force can be achieved only if the matching scale is somewhere between the size of and the distance between instantons.

For the sake of clarity we stress that this paper is not a lattice study. The lattice is employed here only to enable the necessary coarse-graining of a continuum model. In a lattice gauge theory investigation the role of the discretization scale $a$ would be completely different. There $a$ is an ultraviolet cut-off which permits, at the cost of a running bare lattice coupling $\beta \propto 1/g^2$, not to deal explicitly with fluctuations of smaller and smaller wave lengths. In this case, the requirement of scale invariance in the limit $aA \propto \exp\left(-\frac{6}{11} N_c \beta\right) \to 0$ is indeed crucial and must be confirmed for any dimensionful quantity to be physical. Instantons can be found which are stochastically generated within the sample of fields. Being lumps of gauge invariant topological density they are of immediate physical importance. As mentioned above, their average size is relatively well-defined, independently of the lattice scale $a$ (for suitable methods of suppressing the shortest fluctuations living on the lattice). For the confining Abelian defects the situation is somewhat different. They are identified by gauge fixing (which is practically performed on the lattice of scale $a$), and the scale invariance of the corresponding density, of the distribution of length or area etc. are controversial. Moreover, it is generally agreed that in order make these defects condensing they have to be defined with some extension (blocked monopoles or thick vortices), a scale which becomes decoupled, in the continuum limit, from the lattice scale $a$. As far as instantons are discussed as a possible microscopic mechanism to induce Abelian defects, finding the correct matching scale is tantamount to define this extension.

We believe that this study contains some lessons for other attempts of an explicit modeling of the QCD vacuum. The observed scale sensitivity of the monopole or center vortex description seems to be a more general feature of the complementarity between semiclassical continuum models for non-perturbative vacuum structure and condensing defects. It should be remembered that the concept of Abelian and monopole dominance had been introduced as a property of gauge fields in the infrared [15–20], not
necessarily on the lattice. Practically, however, all evidence comes from doing the gauge fixing and Abelian projection on the lattice for gauge fields generated by lattice simulation. In the present work the instanton model continuum configurations will be discretized with the purpose to perform the same steps following [18,19] for the monopole part and [21–23] for the vortex part of the heavy quark force. Explicitly calculating these contributions, it turns out that the discretization scale \( \alpha \) is an influential infrared coarse-graining parameter and must be chosen in correspondence to the typical size and density of the disordering continuum non-perturbative configurations forming the vacuum. This is what our instanton model clearly illustrates.

II. INSTANTON CONFIG. AND THEIR DISCRETIZATION

We base our studies of the \( \bar{Q}Q \) force and possible complementary descriptions in terms of monopoles on a model which comprises the Yang-Mills vacuum as an ensemble of random collections of instantons and antiinstantons. The interaction is partly taken into account in the size distribution given below.

In order to fix the scale of our continuum model, we choose an average instanton size \( \bar{\rho} = 0.4 \text{ fm} \). The average size is most solidly defined by the profile of the topological density after a few cooling steps. Our choice realistically applies to \( SU(2) \) Yang-Mills theory. Unless stated otherwise (when we study the density dependence of the instanton mediated force in section 3) the density is chosen \( N/V = (N_I + N_{\bar{I}})/V = 1 \text{ fm}^{-4} \) with the packing fraction \( f = \bar{\rho}^4 N/V = 0.0256 \).

We adopt the sum ansatz [24] in terms of instanton and anti-instanton solutions in the singular gauge where an instanton is written as

\[
A_{\mu}(x) = \frac{2i}{(x-z)^2} \frac{\rho^2}{[(x-z)^2 + \rho^2]} \tau^a \cdot \bar{\eta}^{\mu\nu}(x-z)_{\nu} \eta^{\mu\nu}(x-z)_{\nu},
\]

Here \( \rho \) and \( z \) denote the instanton size and the space-time position of the instanton center, respectively. The instanton solution can be rotated in color space by the color orientation matrix \( O \). The 't Hooft symbol \( \eta^{\mu\nu} \) is defined as \( \bar{\eta}^{\mu\nu} \equiv \epsilon^{\mu\nu}(1 - \delta^{\mu\nu} - \delta^{\mu\nu} + \delta^{\mu\nu}) \). The anti-selfdual solution \( A^{\mu}_I \) is obtained replacing \( \bar{\eta}^{\mu\nu} \) by \( \eta^{\mu\nu} \equiv (-1)^{\delta^{\mu\nu} + \delta^{\mu\nu}} \bar{\eta}^{\mu\nu} \). The instanton solutions have several gluonic collective modes related to variations of parameters like size and position (five collective degrees of freedom). For pure \( SU(2) \) gauge theory, the color orientation matrix is characterized by 3 parameters (the Euler angles). For \( N_c \), colors the number of collective parameters (and gluonic zero modes) generalizes to \( 4N_c \).

Actually, we generate the ensemble of instantons and anti-instantons by randomly placing \( z_k \) in a 4-dimensional Euclidean continuum box. The (adjoint) color orientations \( O_k \) are taken randomly with the Haar measure, and the instanton sizes \( \rho_k \) are sampled according to the following size distribution:

\[
dn(\rho)/d\rho = \alpha \rho^{b-5} \exp(-\beta \rho^2/\bar{\rho}^2)
\]

with \( b = 11N_c/3 \). Here \( \alpha \) and \( \beta \) are fixed by normalizing to the space-time density as \( \int_{0}^{\infty} dn(\rho) = N/V \) and the average size \( \int_{0}^{\infty} \rho \ dn(\rho) = \bar{\rho} N/V \). In the explicit configurations, the instanton number \( N_I \) is taken equal to that of the antiinstantons \( N_{\bar{I}} \).

In our actual calculation, we cover the random multi-instanton configuration by a lattice similar to Ref. [1]. For the calculation of the fully non-Abelian force the discretization actually would not be necessary. In fact, we choose a sufficiently fine lattice spacing of \( a = 0.05 \text{ fm} \) (this should be compared with the average instanton size of \( \bar{\rho} = 0.4 \text{ fm} \)). For this case (of the finest lattice) the link variables are given simply as \( U_{\mu}(x) = \exp[iaA_{\mu}(x + \frac{1}{2} \bar{\mu}a)] \) using the gluon field of the multi-instanton system on the mid-point of the link \( l = \{ x, \mu \} \). In connection with Abelian gauges and Abelian projec-
tions, we will apply a coarser discretization scale. The $SU(2)$ link variables are constructed by integrating the vector potential defined on the continuum space as

$$U_l = P \exp \left[ \int_x^{x+\hat{\mu}a} dx' \mu_A(x') \right]$$

$$= P \prod_{j=1}^{l} \exp \left[ \hat{a} A_{\mu}(x + (j - \frac{1}{2}) \hat{\mu}a) \right].$$

Here, the path from $x$ to $x + \hat{\mu}a$ has been subdivided into smaller segments with step size of $\hat{a} = a/l$, and the above path ordered exponential has been calculated as a product over finer links defined on these segments. Actually, we take $\hat{a} = 0.05$ fm as the segment size. This construction becomes increasingly important when the discretization scale of the lattice gets comparable with or larger than the average instanton size.

Periodicity of the lattice gauge field configurations has been enforced by placing the 4-dimensional Euclidean box of size $V = (4.8 \text{ fm})^4$ (to be covered by the lattice) into a bigger box. The 3-dimensional boundary in each of the eight directions is extended to a 4-dimensional slab of thickness 1.2 fm continuing the basic box. In each of these slabs copies (phantom instantons) are placed of instantons which are near to the opposite boundary. These are also included into the sum ansatz [24] representing the continuum vector potential. Then, along the links restricted to the basic box the above construction is performed.

III. ABELIAN PROJECTIONS

In this section, we want to clarify how concepts like Abelian dominance, monopole dominance can be applied to semiclassical-like multi-instanton configurations. We shall compare the fully non-Abelian force discussed in section 2 with the results obtained after standard techniques of gauge fixing and Abelian projection have been applied to the multi-instanton fields and the respective contributions to the static $Q\bar{Q}$ force have been evaluated. Technically, as a coarse-graining device, the continuum configurations are latticized, and the results will depend critically on the lattice spacing compared with the average instanton size.

For this study the periodic Euclidean box of size $V = (4.8 \text{ fm})^4$ is discretized with four different lattice spacings, $a = 0.2$ fm, $a = 0.4$ fm, $a = 0.6$ fm, and $a = 0.8$ fm, corresponding to lattices $24^4$, $12^4$, $8^4$ and $6^4$, respectively. In all cases, we have chosen $\hat{a} = 0.05$ fm as the segment size (integration step) to construct the links (path ordered exponential) of the lattice configuration. How periodicity is enforced has been described in section 2.

First, we consider the maximally Abelian gauge [31] (MAG), which is defined by maximizing the functional $R_{MA} = \sum_{x,\mu} \text{tr} [U_{\mu} \tau^3 U^\dagger_{\mu} \tau^3]$ with $U_{\mu} = U_{\mu}^0 + i\tau^3 U_{\mu}^1$. In the MA gauge, the $SU(2)$ link variable $U_{\mu}(x)$ is decomposed as

$$U_{\mu} = M_{\mu} u_{\mu}$$

$$= \begin{pmatrix} \sqrt{1 - |c_{\mu}|^2} & -c_{\mu}^* \\ c_{\mu} & \sqrt{1 - |c_{\mu}|^2} \end{pmatrix} \begin{pmatrix} e^{i\theta_{\mu}} & 0 \\ 0 & e^{-i\theta_{\mu}} \end{pmatrix},$$

where the Abelian angle variable $\theta_{\mu}$ and the non-Abelian variable $c_{\mu}$ are defined in terms of $U_{\mu}(x)$ as $\tan \theta_{\mu} = U_{\mu}^1 / U_{\mu}^0$, $c_{\mu} e^{i\theta_{\mu}} = [-U_{\mu}^2 + iU_{\mu}^1]$. To clarify the contribution of Abelian components to the static force, we consider the Abelian projection of full non-Abelian link variables $U_{\mu}$ to the Abelian ones $u_{\mu}$. This is tantamount to replacing, in a Yang-Mills vacuum configuration put into MA gauge, of the $SU(2)$ link variables by $U(1)$ link variables, $U_{\mu} \rightarrow u_{\mu}$. Before this is done the off-diagonal parts $U_{\mu}^1$ and $U_{\mu}^2$ of gluon fields have been minimized by the gauge transformation which has to be found iteratively.

One can further decompose the diagonal gluon component $\theta_{\mu}$ into the monopole part $\theta_{\mu}^{\text{mo}}$ and the photon part $\theta_{\mu}^{ph}$. Using the forward derivative
\( \partial_\mu f(x) \equiv f(x + \mu a) - f(x) \), the 2-form \( \theta_{\mu\nu}(x) \equiv \partial_\mu \theta_\nu(x) - \partial_\nu \theta_\mu(x) \) defines the field strength which is separated as follows

\[
\theta_{\mu\nu}(x) = \bar{\theta}_{\mu\nu}(x) + 2\pi n_{\mu\nu}(x) \tag{5}
\]

into a gauge invariant regular field strength \( \bar{\theta}_{\mu\nu}(x) \equiv \text{mod}_{2\pi} \theta_{\mu\nu}(x) \in (-\pi, \pi] \) and a Dirac string part \( n_{\mu\nu}(x) \in \mathbb{Z} \).

From each part of the field strength the photon part \( \theta_{\mu\nu}^{ph}(x) \) and the monopole part \( \theta_{\mu\nu}^{mo}(x) \) of the \( U(1) \) vector potential can be reconstructed, for instance \( \theta_{\mu\nu}^{mo}(x) = 2\pi \sum_{x'} \square^{-1}(x-x') \partial_\nu n_{\mu\nu}(x') \), using the lattice Coulomb propagator \( \square = \partial_\mu \partial_\mu' \), where \( \partial_\mu' \) denotes the backward derivative. Then, we can construct the Abelian projected Wilson loop as

\[
\langle W_{\text{abel}}(C) \rangle = \langle \exp \left[ i \oint_C \theta_{\mu}(x) \, dx_\mu \right] \rangle \tag{6}
\]

which contains, besides a "photonic" Wilson loop the monopole projected Wilson loop

\[
\langle W_{\text{mon}}(C) \rangle = \langle \exp \left[ i \oint_C \theta_{\mu}^{mo}(x) \, dx_\mu \right] \rangle \tag{7}
\]

as a uniquely defined factor.

In Fig. 1, we show the instanton mediated non-perturbative force for the standard density \( 1 \text{ fm}^{-4} \) and fixed average instanton size \( \bar{\rho} = 0.4 \text{ fm} \). Instanton sizes were sampled according to the exponentially damped size distribution (2). The simulation data are based on a statistics of 1000 multiinstanton configurations for each discretization scale.

As shown in Fig. 1(a), for a lattice spacing much smaller than the typical size of instantons, the Abelian force calculated after Abelian projection from (6) does not reproduce the non-Abelian force. Moreover, the Abelian force is practically reproduced after the monopole component of the Abelian field has been removed (by the "photonic" Wilson loops alone). The monopole contribution to the heavy charge force turns out to be smaller by a factor of two.

For comparison we show in Fig. 1(b) the same for a coarser lattice of lattice spacing \( a = 0.4 \text{ fm} = \bar{\rho} \). In this case, the Abelian force has increased but it is still far from reproducing the non-Abelian force.

This trend continues when we consider a even coarser lattice in Fig. 1(c) with \( a = 0.6 \text{ fm} = 1.5 \bar{\rho} \). In Fig. 1(d) we show the coarsest lattice with \( a = 0.8 \text{ fm} = 2\bar{\rho} \). Now the slopes of the potentials calculated in the various projections are in agreement with Abelian, monopole with an ordering of the quasi-string tensions \( \sigma_{\text{SU}(2)} > \sigma_{\text{Abelian}} > \sigma_{\text{mono}} \). Here, of course, the potential can be looked at only at one or two lattice spacings. Therefore the values of the string tension should be considered only as rough estimates.

IV. SUMMARY

We have studied the static quark potential induced by a random instanton liquid. We have made the present study in the sequel of the previous work [1] where an infrared suppression of the instanton size distribution \( dn(\rho) \propto \rho^{-x} \, d\rho \) has been assumed. In this paper, we adopt a size distribution as \( dn(\rho) / d\rho = \alpha \rho^{b-5} \exp(-\beta \rho^2 / \rho^3) \) (suggested by lattice instanton searches) adapted to a spacetime density of \( N/V = 1 \text{ fm}^{-4} \) and an average instanton size \( \bar{\rho} = 0.4 \text{ fm} \). We find that the gross effects do not depend on the detailed form of the size distribution.

We have investigated the Abelian of the gauge field configurations built up by random instanton configuration. The gauge fixing and subsequent projection has been performed introducing lattices of different lattice spacing, and we have studied the effect of changing the discretization scale, in particular checking lattice spacings \( a = 0.2, 0.4, 0.6 \) and \( 0.8 \text{ fm} \). In the finer lattice case, the monopole contribution to the heavy charge force turns out to be much smaller than the Abelian projected force. With larger and larger discretization scale, Abelian, monopole becomes restored, which is almost perfectly illus-
trated for $a = 2 \bar{\rho}$. Then the quasilinear non-Abelian force can be almost reproduced by the corresponding projected Wilson loops.

We stress this result, although the random instanton liquid model as such does not appear to be a realistic model for the confining aspect of the Yang-Mills vacuum, because this seems to reflect a more general feature to be kept in mind for more sophisticated semiclassical models.

We see that in the multi-instanton liquid, at $a = 2 \bar{\rho}$, Abelian dominance of the heavy charge force amounts to about 90%, which is mainly due to the Abelian monopole component. The singular part of the Abelian gauge potential accounts for 80% of the Abelian force.

V. ACKNOWLEDGMENT

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FIG. 1.

The non-Abelian static potential of fundamental charges compared with the Abelian projection, its monopole component and with the potential in Z(2) center projection, for different lattice spacings (a) $a = 0.20$ fm, (b) $a = 0.40$ fm, (c) $a = 0.60$ fm and (d) $a = 0.80$ fm. The instanton size and density is fixed to $\tilde{\rho} = 0.4$ fm and $N/V = 1$ fm.
18. The JAERI-KEK Joint Project on High Intensity Proton Accelerator and Overview of Nuclear Transmutation Experimental Facilities

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Abstract:
A status of the JAERI/KEK joint project on High Intensity Proton Accelerator is overviewed. It is highlighted that Experimental facilities for development of the accelerator driven system (ADS) for nuclear transmutation technology is proposed under the project.

1. ADS for nuclear transmutation
Nuclear energy system can not be completed without establishing its nuclear waste management system. Although it has been addressed that the principle scenario of the waste stream in Japan is the deep land burial of high level waste (HLW), the HLW still remains as potential hazards. Accordingly it is encouraged to pursue a way to reduce HLW to mitigate a potential load of the waste management. The accelerator driven system (ADS) has been recognized world wide as an attractive option for the nuclear transmutation of HLW. Japan Atomic Energy Research Institute has proposed an ADS concept for the nuclear transmutation of HLW. By the ADS, a hazard level of HLW is estimated to be reduced 1/200 comparing to a case without ADS introduction in 500 years.

2. Key technology needs for ADS
To realize ADS, fundamental researches and technical developments are required in various areas involving spallation target technology, sub-critical reactor physics, hybrid system operation and controls, the nuclear transmutation process, thermal-hydraulics, and material developments, etc. Among them, the development of the material for a proton beam window of the spallation target and the sub-critical reactor physics driven by the high energy proton beam are identified most important to evaluate the technical feasibility for ADS.
3. Overview of the joint project on high intensity proton accelerator

In the meantime, a project on high intensity proton accelerator has been jointly proposed by JAERI and KEK, and has approved to start in 2001 by the government. The project, namely JKJ project, aims at explore forefront sciences in various research fields. The proposal has two phases. The “Phase 1” accelerator complex consists of

- 400-MeV normal-conducting Linac,
- 600-MeV Linac (superconducting) to increase the energy from 400 to 600 MeV,
- 3-GeV synchrotron ring, which provides proton beams at 330 μA (1 MW), and
- 50-GeV synchrotron ring, which provides proton beams at 15 μA (0.75 MW).

In addition, an upgrade towards 5-MW proton beam power at the few GeV energy region is proposed as a “Phase 2” project of the present proposal.

At the initial stage, the normal conducting 400 MeV Linac will be used as an injector to the 3-GeV ring. At the stage when the superconducting 600 MeV Linac becomes stable, however, this 600-MeV Linac will be switched as the injector to the 3 GeV ring.

At the 50-GeV Proton Synchrotron (PS) nuclear/particle physics experiments using kaon beams, antiproton beams, hyperon beams and primary proton beams are planned. Using kaon beams, production of strangeness in nuclear matter become possible, and the study of the influence of nuclear matter on this impurity probe of a strange particle will be performed. Experiments on kaon rare decays, such as $K^0 \rightarrow \pi^0 \nu \bar{\nu}$ to measure CP matrix elements, an experiment on neutrino oscillation from $\nu_\mu$ to $\nu_\tau$ using the Super-Kamiokande as a detector, etc. will also be carried out.

The 3-GeV ring will be used to provide beam power of 1 MW. Extensive physics programs which cover nuclear/particle physics, condensed matter physics, materials sciences and structural biology will be carried out there. Among them the major highlights are materials sciences and structural biology using neutrons produced in proton+nucleus spallation reactions. Since a neutron has a magnetic moment but no electric charge, neutrons can be used for the study of magnetic properties of matter. Also, since the neutron has a mass which is similar to that of the hydrogen atom, neutrons can probe sensitively the location and dynamic behavior of hydrogen atoms in materials.

The role of hydrogen atoms in biological cells is of particular interest in life science and, there, the neutron beams play a crucial role for these studies. In addition to neutrons, muon beams are also important in which $\mu$SR (muon spin rotation/relaxation), muon catalyzed fusion, and other materials sciences can be conducted. Also, particle-physics experiments such as a $\mu N \rightarrow eN$ conversion experiment can be performed. Radioactive beams produced from the 3-GeV PS are also useful to nuclear/astro physics research.
Finally, the high-current 600-MeV Linac will be used for R&D for the accelerator-driven nuclear transmutation.

The accelerator and experimental facilities are to be constructed in the southern area of Tokai site. JAERI team identified that the high intensity proton beam to be available in the project is extremely useful to initiate the ADS development. As the first step of the research program, two experimental facilities for ADS are proposed to be built.

4. Experimental facilities for ADS research under the joint project

They are (1) Accelerator Material Irradiation Facility, and (2) ADS Physics Experimental Facility. Proton beams of 600 MeV with 0.33 mA (200 kW at maximum) to these facilities through the superconducting Linac. The facility is designed to be located at the area between NUCEF and Linac line. The Accelerator Material Irradiation Facility is to test materials of the beam window and the spallation target system with lead-bismuth (Pb-Bi) as the first candidate of target/coolant material. On a preliminary evaluation, dose rates of more than 10 dpa (Displacement per Atom) per year could be achieved. The ADS Physics Experimental Facility uses low power proton beam up to 10 W. The low power of 10 W beam is extracted from main H 200 kW beam via a laser charge exchange scheme the idea of which is developed by the JAERI team. Basic sub-critical reactor physics, e.g., sub-criticality, reactivity, power profile, etc. and reactor power control with the beam power are to be studied by using the low power proton beam. For this purpose, a critical assembly with maximum reactor power of 500 W is to be constructed. It will be the first demonstration anywhere in the world of the sustained stable integral operation of a spallation target and a fast neutron sub-critical core driven by a proton beam. This talk deals with the conceptual study of two experimental facilities, giving a baseline design and a preliminary safety study.

As the research and development for Accelerator Material Irradiation Facility, a liquid Pb-Bi loop for the material test was installed in JAERI/Tokai at the end of January. The loop was successfully operated at 450 degree centigrade with 50 degree centigrade of temperature difference for more than 1200 hours.

As the safety analysis for ADS physics experimental facility, the influence of the hypothetical accident was preliminary evaluated. It was shown that the dose rate around the facility can be controlled at low level by the multiple mechanisms of an emergency reactor shutdown.

The group for the experimental facility design was unified with the research
group for the transmutation system in this April. This new group, named as "Nuclear Transmutation Group", will make broad research and development for the partitioning and transmutation technology to reduce the environmental burden of long-lived high level waste, as well as the development of the ADS Experimental Facility.

5. Status of ADS facilities and summary

Unfortunately, construction of the ADS facilities are postponed to the second phase due mainly to the financial constraint. Even though the ADS facility is still defined in the second phase, we have to work on hard to show more attractive scenario of ADS to have strong support from the society and to prepare more firm design of facility itself. In particular, the delay should be utilized as a leading time to establish the reactor safety analysis with respect to the proton beam injection to the sub-critical core.
19. Alpha cluster condensation in $^{12}\text{C}$ and $^{16}\text{O}$

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Abstract

Based on a microscopic framework, a new $\alpha$–cluster wave function is proposed which is of the $\alpha$–particle condensate type. Applications to $^{12}\text{C}$ and $^{16}\text{O}$ show that the states of low density close to the 3 respective 4 $\alpha$–particle threshold in both nuclei possibly are of this type. It is conjectured that all self-conjugated 4n nuclei may show similar features.
20. Structure of $N \sim 20$ unstable nuclei studied by the Monte Carlo shell model
モンテカルロ含有模型による$N=20$領域不安定核の構造

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Abstract

The structure of neutron-rich nuclei around $N = 20$ is studied by the Monte Carlo shell model. The disappearance of the magic number of $N = 20$ is a main subject of this study. The two-neutron separation energies, energy levels, and $B(E2)$ values of yrast states of even-$A$ Ne, Mg, and Si isotopes are calculated from $N = 16$ to 24 systematically, in agreement with the experiment. The mechanism of the disappearance is discussed in terms of the effective single-particle energy which depends strongly on the proton number.

1 Introduction

The structure of nuclei around $N = 20$ shows quite different features between stable and unstable ones: the magic structure of $N = 20$ persists in the former region, whereas it disappears in the latter around $Z = 12$. The disappearance of the $N = 20$ magic number is induced by the dominance of the 2-particle 2-hole ($2p2h$) excitations from the $sd$ shell to the $pf$ shell in the ground state. The particle-hole excited configurations can gain energy associated with the deformation, while they lose energy in terms of the spherical single-particle energy. The region where the energy of the $2p2h$ state is lower than that of the $0p0h$ is referred to as the “island of inversion” [1]. The island is considered to consist of several Ne, Na, and Mg isotopes of $N \geq 20$. A large $B(E2; 0^+ \rightarrow 2^+)$ value of $^{32}$Mg [2], suggesting a large deformation, is a typical evidence for the disappearance of the magic number.

In the present study, we have performed a shell-model calculation in the $N = 20$ region systematically, based on the Monte Carlo shell model (MCSM) [3]. In the MCSM calculation, the mixing, which is missing in the “island of inversion” picture, between different number of particle-hole excitations is included naturally. Yrast properties including the energy levels and the $B(E2)$ values are calculated [4], and are compared with experimental quantities. The mechanism of the disappearance of the magic number is discussed by introducing the effective single-particle energy.
2 Shell model calculation

In order to describe the particle-hole excitations from the sd to pf shells within the framework of the shell model, the $0f_{7/2}$ and $1p_{3/2}$ orbits as well as the full sd shell are included as a valence shell. With this large shell-model space, one cannot diagonalize the Hamiltonian directly for almost all nuclei: for $^{32}\text{Mg}$ the $M=0$ basis dimension reaches $\sim 10^9$, going beyond current shell-model frontier. In the present study, we adopt the Monte Carlo shell model calculation [3]. The Monte Carlo shell model is characterized by the importance truncation to the exact solution [5]. The Hamiltonian is diagonalized with several tens of bases which are generated stochastically and selected carefully. The symmetries such as the angular momentum are restored with the projection technique. As a result, a very precise solution is obtained, compared with the exact solution. The feasibility of the MCSM is confirmed in pf-shell nuclei whose basis dimensions become more than $10^6$.

3 Results

The properties of the yrast state of even-$A$ Ne, Mg, and Si are calculated, including the two-neutron separation energies, energy levels, and $B(E2)$ values. A part of the results is shown now. Figure 1 illustrates the energy levels of Ne and Mg isotopes.

![Energy levels of Ne and Mg isotopes](image)

Figure 1: Energy levels of Ne and Mg isotopes. The symbols, solid lines, dashed lines are the experiment, MCSM calculation, and sd-shell model calculation with the USD interaction [7]. The closed symbols denote experimental results which are published following the MCSM calculation.

For a $N=20$ nucleus of $^{32}\text{Mg}$, the $2^+_1$ energy level is lower than that of the sd-shell model where all of the states are composed of the closed-shell $\nu(sd)^{12}$ configurations, being in agreement.
with the experimental value. The ground state of $^{32}\text{Mg}$ is dominated by the $2p2h$ configurations, while the mixing between the $0p0h$ and $4p4h$ configurations occurs to some extent. We predict that the ground state is deformed for $^{30}\text{Ne}$. For $N = 16$, the $0p0h$ ($=sd$) configurations become dominant, as suggested by the agreement between experiment and the $sd$-shell model. In the ground state of $N = 18$ isotopes, the mixing between the $0p0h$ and $2p2h$ configurations develops, particularly in Ne: the $2_{1}^{+}$ energy level is located considerably lower than that of the $sd$-shell model prediction. This lowering has been measured recently [6], giving nice agreement with the prediction of MCSM.

4 Effective single-particle energy

Now we discuss why the disappearance of the magic number occurs in a restricted region around $Z = 12$. As mentioned in Sect. 1, $(sd)^{n}$ and $(sd)^{n-2}(pf)^{2}$ configurations compete in energy: the former has the advantage of the spherical single-particle energy, while the latter gains the deformation energy. Thus, the spherical single-particle energy plays a significant role in the competition. The spherical single-particle energy varies as the mass number changes, since the valence nucleons affect it similarly to the single-particle energy of $^{16}\text{O}$ core. In terms of the shell model, this varying single-particle energy can be defined by introducing the effective single-particle energy. Namely, a contribution of the valence nucleons to the single-particle energy is included via an “average interaction”. The average interaction corresponds to the monopole interaction [8] whose angular dependence is averaged out. For a detailed discussion about the effective single-particle energy, see Ref. [4].

The effective single-particle energies for $N = 20$ nuclei are shown in Fig. 2. Around stable nuclei with $Z \sim 20$, the energy gap between the $0d_{3/2}$ and $0f_{1/2}$ is as large as $\sim 6$ MeV, which is large enough to be a stable spherical shell structure. For smaller $Z$, the gap becomes narrow: it is $\sim 4$ MeV for Mg and $\sim 3$ MeV for Ne. The deformed configurations are superior with the
narrowing shell gap. In fact, the MCSM calculation shows that the disappearance never occurs for $^{32}$Mg by shifting the gap to $\sim 6$ MeV artificially.

The transition of the shell structure from unstable to stable nuclei is summarized as follows. For $Z = 8$, the $0d_{3/2}$ orbit is pushed toward the $pf$ shell by the strong spin-orbit splitting. This is related to a new $N = 16$ magic number mentioned recently [9]. As the $Z$ increases, the $0d_{3/2}$ lowers very steeply due to the strong $T = 0$ $0d_{3/2}-0d_{5/2}$ monopole interaction. It shifts the magic number from 16 to 20. For Ne and Mg, the gap is still small, overridden easily by the deformed configurations. The proton numbers $Z = 10$ and 12 favor large prolate deformation, which helps these nuclei be deformed.

5 summary

A systematic shell-model calculation for unstable nuclei around $N = 20$ has been performed based on the Monte Carlo shell model. The disappearance of the magic number is produced, and we gave predictions of energy levels and $B(E2)$ for more neutron-rich nuclei. The deformation of $^{34}$Mg is predicted to be larger than that of $^{32}$Mg, which is in excellent agreement with recent experimental data [10]. In terms of the shell model interaction, the monopole interaction characterizes the shell gap. Owing to strong $T = 0$ $0d_{3/2}-0d_{5/2}$ monopole interaction, the $sd-pf$ gap becomes narrower as $Z$ decreases, constituting a main factor of the disappearance together with the preference for prolate deformation in $Z = 10$, 11, and 12 isotopes.

References

21. Modification of AMD wave functions and application to the breaking of the N=20 magic number
Masaaki Kimura and Hisashi Horiuchi. Dept. of Phys Kyoto University

Abstract
By using the deformed Gaussian instead of the spherical one, we have modified the AMD (Antisymmetrized Molecular Dynamics) wave functions. The calculation results with this modified AMD shows the drastic improvement of the deformation properties of Mg isotopes. This improvement means that this new version of AMD can treat the deformation of mean field properly than before and the deformation of mean field is important in Mg isotopes. With this new version of AMD, we have also calculated 32Mg in which the breaking of magic number N=20 is experimentally known. In this nucleus, β-energy surface is also drastically changed by the modification of AMD wave function. Our results show that this nucleus is indeed deformed and neutron's 2p2h state is dominant in its ground state. This ground state reproduces the experimental data and shows the breaking of the magic number N=20 clearly. Additionally, near the ground state, there is also very interesting state which has neutron's 4p4h structure and shows parity violating density distribution and cluster-like nature.

Framework of AMD and its modification
In the AMD, the Slater Determinant of single particle wave functions represents wave function of the system. In this study, we have used the parity projected wave function instead of single Slater Determinant.

\[ |\Phi^\pm\rangle = |\Phi\rangle \pm P_x |\Phi\rangle, \quad |\Phi\rangle = \text{det}[\psi(r_j)] \]  \hspace{1cm} (Eq.1)

The single particle wave function \( \psi \) consists from spatial part (\( \phi \)), spin (\( \sigma \)) and isospin part (\( \tau \)). In the old version of AMD, spatial part of each single particle wave function is represented in spherical Gaussian form.

\[ \psi(r_j) = (2\nu/n\mu)^{3/4} \exp(-\nu(r_j - Z_j)^2) \]  \hspace{1cm} (Eq.2)

In this study, we employ the deformed Gaussian as the single particle wave function instead of spherical one. This deformed single particle wave function has independent widths to each direction \( x, y \) and \( z \).

\[ \psi(r_j) = (8\nu_x \nu_y \nu_z / \pi)^{1/4} \exp(-\sum_{\sigma=x,y,z} \nu_\sigma (r_j - Z_j)_{\sigma}^2) \]  \hspace{1cm} (Eq.3)

Using this deformed spherical wave function, we expect the following effects.
- When the mean-field is deformed, single particle wave function should be also deformed. So, using this deformed wave function, we can treat the deformation of mean field better.
- This deformed wave function will activate Is force, because Is force is sensitive to the deformation of single particle wave function.

Here, in Eq2 and 3, complex variables \( Z_i \) are variational parameters and independent to each nucleon. The width of the Gaussian \( \nu \) (Eq2), \( \nu \sigma \) (Eq3, \( \sigma = x, y \) and \( z \)) are common to all nucleons and they are also variational parameters. In Eq3, \( \nu x, \nu y \) and \( \nu z \) are determined independently by the variational calculation.

Spin and isospin parts of single particle wave functions are follows.

\[ X_u = \alpha \chi_1 + \beta \chi_2, \quad X_u = P \text{ or } N \]  \hspace{1cm} (Eq.4)

Here, in Eq.4, spin up and down component parameters \( \alpha \) and \( \beta \) are independent to each nucleon and determined by the variational calculation. As the effective nuclear force, we employ the Gogny force. Coulomb force is approximated by the sum of seven Gaussians.
Results and Discussions

First, we will compare the deformation properties of Ne and Mg isotopes. Proton quadrupole moments of Mg isotopes are shown in Fig.1. We can see that the quadrupole moment of Mg isotopes becomes drastically large by using the deformed single particle wave functions instead of spherical one. Because there are no experimental data of neutron rich Mg isotopes, we plotted the calculation results of HF with Skyrme-II force made by Tajima et.al. Except 32Mg, deformations of Mg isotopes become large by using the deformed Gaussian and the results of modified AMD and HF-SIII are close to each other. This means that the use of deformed Gaussian indeed can treat the deformation of mean field properly and in Mg isotopes, mean-field deformation is important. On the other hand, old version of AMD fails to reproduce the deformations of Mg isotopes and thus it fails to represent the deformation of mean field.

Next, we will show the more detailed result about 32Mg. In Fig2, \( \beta \)-deformation energy surface (energy surface as the function of the deformation parameter \( \beta \)) is shown. In this figure, the energy surface of 32Mg calculated with old version of AMD and modified version of AMD are shown.

Here we again see the drastic effect of the deformed single particle wave functions. The old version of AMD gives very hard energy surface against the deformation. On the other hand, modified version of AMD gives rather flat surface. So, also in this nucleus, the deformation of mean field is important. When the angular momentum is projected to \( 0^+ \), deformed state becomes the ground state as shown in Fig2 (dashed line) and this ground state reproduces the features of the experimental data as shown in Table1. So, we can say that by using the deformed single particle wave function, the deformation of 32Mg is reproduced and thus the deformation of mean field plays an important role in the breaking of magic number \( N=20 \).
Table 1: Comparison with experimental data of 32Mg. The values shown with bold font are the results of present calculation.

Next, we will study the ph properties of 32Mg appearing on the energy surface. In Fig3, the single particle energies of neutrons that are obtained by reconstructing the HF single particle Hamiltonians from the AMD wave functions are shown.

In this figure, neutron single particle energies are shown as the function of the deformation parameter $\beta$. There are two neutrons (spin up and down) are in each line. We can see clearly that the closed shell nature of 32Mg at the spherical ($\beta=0$) state. But, as the deformation becomes large, energies of 0d3/2 orbit go up and around $\beta \sim 0.3$, its energy goes above the intruder pf-orbit. So, in the deformed regions ($\beta > 0.3$), the neutron 2p2h states are dominant and the ground state of 32Mg is also dominated by 2p2h state. This result shows clearly the breaking of the magic number $N=20$, namely two neutrons breaks $N=20$ shell closure and go up to the pf-shell orbit. When we proceed to more deformed region, we see that another pf-shell orbit comes down and four neutrons go up to the pf-shell orbit (4p4h state). This 4p4h state is also deeply bounded and very close to the ground state, only a few MeV above the ground state.

When we see the density distributions, we know that each ph state has different characters. In Fig4, proton and neutrons density distributions of each ph state (0p0h, 2p2h and 4p4h states) are shown. The density distributions of 0p0h states are almost spherical and reflects the N=20 shell closure nature of this state. On the other hand, density distributions of 2p2h and 4p4h states are deformed. This reflects the fact that the two and four neutrons are in the pf-shell orbit. Though both 2p2h and 4p4h states are deformed, there is a difference between these states. The 2p2h state has parity symmetric density distributions and show smooth density distributions. On the other hand, the 4p4h state has parity violating density distributions and there is narrow part in the density distributions. This is the
appearance of the clustering feature that is not seen in the 2p2h state. Though we have mentioned that the deformation of the mean field is important in 32Mg, clustering feature is also important in the neutron 4p4h state of 32Mg. This feature indicates that there is competition between mean field structure and cluster structure and this competition may take place in neutron 2p2h state, though we cannot see it clearly because only the mean field state appears in the present calculations.

As the conclusion, projecting the angular momentum to 0+ state, the ground state of 32Mg is deformed. This ground state reproduces the features of the experimental data such as 2+ excitation energy and E2 transition probability. There is also neutron 4p4h state near the ground state which has the parity violating density distributions and shows the clustering feature. This indicates that there is the competition between the mean field structure and the cluster structure.

Summary

In this study, we have introduced the new version of the AMD that uses the deformed Gaussian as the single particle wave function. This new version of AMD is expected to represent the deformation of mean field better and to activate the ls force. We have applied this new version of AMD to Mg isotopes to see the effect of the modification. The quadrupole moment of Mg isotopes are drastically becomes large and become close to the HF-SIII results. This means that the deformation of mean field is important in Mg isotopes and the new version of AMD can treat these deformations of mean field properly. We have also applied this new model to the 32Mg that is the magic number N=20 breaking nucleus. In this nucleus, we again see the drastic effect of the deformed single particle wave function. Using the deformed single particle wave function, the β-energy surface of 32Mg becomes almost flat. And after angular momentum projection to 0+ state, the deformed state becomes the ground state. This deformed state reproduces the features of the experimental data and has the neutron 2p2h structure showing the breaking of the magic number N=20 clearly. There is also neutron 4p4h state near the ground state that has the parity violating density distributions and shows the clustering feature. This indicates that there is the competition between the mean field structure and the cluster structure.

References

22. CLUSTERING EFFECT OF $^{19}$B IN ITS FRAGMENTATION

H. TAKEMOTO, H. HORIUCHI$^1$ and A. ONO$^2$

Abstract

We investigate how clustering structure of the neutron dripline nucleus, $^{19}$B, is reflected in the heavy-ion reaction. We compare $^{19}$B fragmentation with $^{13}$B fragmentation in $^{14}$N-target reactions calculated by antisymmetrized molecular dynamics (AMD) [1] where $^{13}$B which is the neutron closed shell nucleus has no clustering feature in its structure, while $^{19}$B which is the neutron drip-line nucleus has a well developed cluster structure in its ground state. The clustering structure of the $^{19}$B nucleus is predicted by the molecular orbital model [2] and the AMD model [3], but has not yet been confirmed experimentally.

The formalism of AMD is described in detail in Ref.[1], and we give only an outline of the AMD method below. In AMD, the wave function of the A-nucleon system $|\Phi\rangle$ is described by a Slater determinant,

$$|\Phi\rangle = \frac{1}{\sqrt{A!}} \det [\varphi_i (j)], \quad \varphi_i = \phi_{z_i} \chi_{\alpha_i}, \quad (\alpha_i = p \uparrow, p \downarrow, n \uparrow, n \downarrow) \quad (1)$$

where

$$\phi_{z_i} = \left( \frac{2\nu}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\nu \left( \frac{r - Z_i}{\sqrt{\nu}} \right)^2 + \frac{1}{2} Z_i^2 \right], \quad (2)$$

Here, $\chi_{\alpha_i}$ and $\phi_{z_i}$ represent the spin-isospin wave function and the spatial wave function of the $i$th single particle state, respectively. $Z = \{Z_i\}$ represent the positions of the centers of Gaussians. The time development of $Z$ is determined by the time-dependent variational principle. When we apply AMD to heavy ion reactions, nucleon-nucleon collision processes should be incorporated. We have done it by transforming coordinates $Z$ to the physical coordinates $W = \{W_i\}$, whose real and imaginary parts can be interpreted as the positions and momenta of nucleons, respectively. We stop AMD calculations at a certain time $t = t_{ew}$ when produced fragments are thermally equilibrated, and calculate the evaporation process of these fragments by a multistep statistical decay code.

Before AMD simulations we have to prepare ground states of colliding nuclei. The wave function for the ground state, which minimize the expectation value of the Hamiltonian, is determined by the frictional cooling method [3], concerning the properties of ground states of B isotopes. Figure 1 display density distributions of B isotopes based on the AMD wave functions. Here we use the Gogny force as an effective force. AMD calculations show the development of clustering structure of B isotopes with an increase of neutron number; namely, the $^{13}$B nucleus, which is a neutron closed shell nucleus, has the shell-model-like spherical structure, and gradually the cluster structure develops as the neutron number increases, and the $^{19}$B nucleus, which is a neutron dripline nucleus, has a well developed cluster structure. This development of clustering structure of B isotopes with an increase of neutron number is considered to be a unique structure in unstable neutron-rich nuclei to bind

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neutrons as many as possible by a few protons. Such clustering structure of neutron-rich nuclei has not been confirmed yet experimentally.

![Density distributions of ground states of B isotopes based on the AMD wave functions obtained by the frictional cooling method. Open and filled circles represent centers of Gaussians of protons and neutrons, respectively.](image)

Clustering structure of the $^{18}$B nucleus is reflected as the "dynamical" cluster breakup of the $^{18}$B nucleus into He and Li isotopes in its fragmentation. As is shown in FIG. 2(a), there is the abundance of He and Li isotopes in $^{19}$B fragmentation compared with in $^{18}$B fragmentation at the end of dynamical processes in the $^{14}$N-target reaction at 35 MeV/nucleon. From FIG. 2(c), we can see that most of these He and Li isotopes are produced simultaneously in $^{19}$B fragmentation by reflecting its cluster structure, while most of Li isotopes are produced with the emission of H isotopes in $^{18}$B fragmentation [4]. However, if we consider statistical decay of excited fragments produced during dynamical processes, most of He and Li isotopes are produced simultaneously in both $^{18}$B and $^{19}$B fragmentation, as is shown in FIG. 2(d).

![Charge distributions before statistical decay in $^{14}$N-target reactions at 35 MeV/nucleon and Charge distributions after statistical decay in $^{14}$N-target reactions at 35 MeV/nucleon](image)

![Charge distributions in coincidence with the Li isotope before statistical decay in $^{14}$N-target reactions at 35 MeV/nucleon and Charge distributions in coincidence with the Li isotope after statistical decay in $^{14}$N-target reactions at 35 MeV/nucleon](image)

FIG. 2 (a) and (b) show charge distributions before and after statistical decay, respectively, and (c) and (d) show charge distributions in coincidence with the Li isotope before and after statistical decay, respectively. Solid and broken lines indicate those from $^{18}$B and $^{19}$B fragmentation in $^{14}$N-target reactions at 35 MeV/A, respectively.
FIG. 3 (a) Incident-energy dependence of coincident cross sections between He and Li isotopes after statistical decay. (b) Ratios of coincident cross sections between He and Li isotopes at the end of dynamical processes to that after statistical decay. Solid and broken lines indicate $^{19}$B and $^{13}$B fragmentation, respectively, and the dotted line in (a) indicates the gain of the coincident cross section in $^{19}$B + $^{14}$N reactions from before to after statistical decay.

The coincident cross section between He and Li isotopes after statistical decay in $^{19}$B fragmentation decreases with the incident energy increasing, while that in $^{13}$B fragmentation is independent on the incident energy, as is shown in FIG. 3(a). The coincident cross section in $^{19}$B fragmentation is about twice as large as that in $^{13}$B fragmentation at 25 MeV/nucleon. As the incident energy increases, its difference becomes small, and they are almost the same at 100 MeV/nucleon. By comparing the gain of the coincident cross section in $^{19}$B fragmentation from before to after statistical decay, which is indicated by the dotted line in FIG. 3(a), with the coincident cross section in $^{13}$B fragmentation, we see that they are almost the same. Here the simultaneous production of He and Li isotopes in $^{13}$B fragmentation is mainly due to statistical decay processes, as is shown in FIG. 3(b) which displays ratios of coincident cross sections between He and Li isotopes at the end of dynamical processes to those after statistical decay. Therefore we find that the larger value of the coincident cross section between He and Li isotopes in $^{19}$B fragmentation than that in $^{13}$B fragmentation at low incident energies is due to the “dynamical” cluster breakup of the $^{19}$B nucleus by reflecting the cluster structure of the $^{19}$B nucleus. As the incident energy increases, the dynamical component of simultaneous production of He and Li isotopes in $^{19}$B fragmentation decreases, which is shown in FIG. 3(a), and the coincident cross sections become almost the same in $^{13}$B and $^{19}$B fragmentation at 100 MeV/nucleon, that is to say, the cluster structure of the $^{19}$B nucleus is not reflected in its fragmentation at high incident energies.

From these results, we suggest that it is possible to examine the cluster structure of the $^{19}$B nucleus in its ground state experimentally by using fragmentation mechanism, in particular, by using a coincident experiment between He and Li isotopes. AMD calculations indicate that if we observe the larger value of the coincident cross section between He and Li isotopes in $^{19}$B fragmentation than that in $^{13}$B fragmentation, this is due to the “dynamical” cluster breakup of the $^{19}$B nucleus by reflecting the cluster structure of the $^{19}$B nucleus. We stress here that it is important to verify such a clustering structure as the $^{19}$B nucleus experimentally since such a structure is expected to be a new aspect in neutron-rich nuclei.

References
Appendix

第3回「極限条件におけるハドロン科学」研究会 プログラム

平成13年1月29日(月)〜31日(水) 日本原子力研究所 先端基礎交流棟大会議室

10:00 ～ 安岡 弘志 原研 挨拶

セクション1 中性子星1 10:15 ～ 11:45 座長：千葉 敏
10:15 ～ 10:45 工 勝隆 京大理 A duality in quark ferromagnetism
10:45 ～ 11:15 丸山 智幸 日大生物資源 Spontaneous Spin-Polarization and Phase Transition in Highly Dense Matter
11:15 ～ 11:45 コンドラチェフ 原研 Magnetodynamics of Crusty Nuclear Matter

昼 食

セクション2 中性子星2 13:15 ～ 14:45 座長：工 勝隆
13:15 ～ 13:45 武藤 巧 千葉工大資源 ハイブロン物質中のK中間子凝縮
13:45 ～ 14:15 松崎 昌之 福教大学物理 相対論的平均場理論に基づく中性子星内殻の比熱計算
14:15 ～ 14:45 親松 和浩 愛知淑徳大 Dimensional analysis of nuclear shapes in neutron-star crusts and in supernova matter

コーヒー・ブレイク

セクション3 超新星・元素合成 15:15 ～ 16:45 座長：親松 和浩
15:15 ～ 15:45 住吉 光介 沼津高専 Scenarios of r-process in collapse-driven supernovae
15:45 ～ 16:15 石塚 知香子 北大理 超新星物質の不安定領域におけるフラグメント分布
16:15 ～ 16:45 近角 真平 原研 膨張無限系の分子動力学シュミレーション

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セクション4 高エネルギー反応 10:00～11:30 座長：大西 明
10:00～10:30 平野 哲文 早大理工 桥本 恒平、津田 佳一
Hydrodynamical analysis of elliptic flow at SPS and RHIC
10:30～11:00 森田 健司 早大理工 2π相関を用いた、高エネルギー重イオン衝突における“Nutcracker”シナリオの解析
11:00～11:30 野中 千穂 広大理 超相対論的重イオン衝突におけるQGP流体への相転移の影響

昼食

セクション5 高エネルギー反応・QCD 13:15～15:40 座長：平野 哲文
13:15～13:40 大塚 直彦 北大理 粒子生成時間とハドロン自由度
13:40～14:10 大西 明 北大理 AGS–SPS–RHICにおける粒子横質量スペクトルの変化
14:10～14:40 平田 雄一 北大理 12GeV陽子入射反応におけるマルチフラグメンテーション
14:40～15:10 室谷 心 徳山女短 SU(2)格子ゲージ理論による有限密度ハドロン状態の研究
15:10～15:40 福島 昌宏 阪大RCNP QCDにおけるトポロジカルオブジェクトの性質

コーヒーブレイク

特別セクション 16:10～17:00 座長：井頭 政之
16:10～17:00 池田 裕二郎 原研 大強度陽子加速器計画と核変換実験施設の概要

送迎バス 先端基礎交流棟玄関前 17:45発 → 阿漕ヶ浦クラブ

懇親会 18:00～20:30
セクション6 有限多体核子系 9:30 ～ 11:30 座長：松崎 昌之

9:30 ～ 10:00 東崎 昭弘 信州大繊維 α凝縮状態の微視的研究
（鈴木）

10:00 ～ 10:30 宇都野 稔 原研 モンテカルロ殻模型計算によるN=20領域不安定核の構造

10:30 ～ 11:00 木村 眞明 京大理 32Mg原子核でのN=20魔法数破れに関する研究

11:00 ～ 11:30 竹本 宏輝 原研 19Bのクラスター構造を反映したフラグメント生成機構

送迎バス 先端基礎交流棟玄関前 11:45発 → 東海駅
### 国際単位系 (SI) と換算表

#### 表1 SI基本単位および補助単位

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<tr>
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#### 表2 SIと補用単位

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#### 表4 SIと統一的に維持される単位

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