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The theory by Storrer and Stievenart on the pulsed neutron experiment on fast multiplying system has been applied by Code GS-1 to the pulsed neutron experiments on natural uranium systems and twenty percent enriched uranium bare cores. It predicts well the time behavior of neutrons for the wide range of reactivity from the nearly critical state to the far subcritical state.

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裸の高速増倍系におけるパルス実験のStorrer－Stievenart の理論による解析

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Storrer およびStievenart による高速増倍系におけるパルス中性子実験の理論を適用して，裸の天然ウラン体系および裸のＦＣＡＩ－５炉心におけるパルス中性子実験の解析を行なつた。Storrer 等の理論に基づいて書かれたＧＳ－１コードによる解析の結果，実験体系が裸の体系である場合には，パルス中性子が入射してから或る時間が経過した後の中性子の挙動は，Storrer 等の理論により良く説明されることがわかつた。

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1. Introduction

In 1964, pulsed neutron experiments on natural uranium systemes were performed and the effect of neutron multiplication was clearly observed even at far subcritical state as natural uranium systems.[1] However, at that time there was no theory on the pulsed neutron experiment on fast multiplying systems. It is almost impossible to analyze the experimental results by the conventional model of single exponential decay. Afterwards, Storrer and Stievenart gave a perspective theory on it based on the one-point diffusion model.[2] The applicability of their theory for bare fast systems has been examined for the wide range of reactivity from far subcritical state such as natural uranium systems to nearly critical state.

2. Theory

In the following, the brief description of the theory by Storrer and Stievenart will be given.

In non-multiplying systems, the neutron flux of the i th group of n-group diffusion equation in the case of point model can be described as follows:

$$\Sigma_{t,i}\,\phi_i(t) - \sum_{j=1}^{i-1}\Sigma_{trans,j\to i}\,\phi(t) = -\frac{1}{v_i}\frac{\partial\phi_i(t)}{\partial t} \qquad (1)$$

with

$$\Sigma_{t,i} = D_i B^2 + \Sigma_{a,i} + \Sigma_{rem,i} \qquad (2)$$

where

D : diffusion coefficient,
B^2 : geometrical buckling
Σ_a : absorption cross section
Σ_{rem} : removal cross section by scattering,
Σ_{trans} : transfer cross section from j th group to i th group.

Since the operator forms the triangle matrix in the case of non-multiplying systems, Eq. (1) can be directly solved and the energy spectrum at time t can be expressed as

$$\varphi(E,t) = \sum_i c_i B_i e^{r_i t} \qquad (3),$$

with

$$\gamma_i = -v_i \Sigma_{t,i} \tag{4}.$$

In Eq. (3), $\varphi_i(E,t) = B_i e^{\gamma_i t}$ is an eigen function of Eq. (1) and c_i is a constant determined by the initial condition.

However, in the case of multiplying system, the matrix is usually not a triangle matrix because of "up-scattering" due to fission process and it is toilsome to solve directly the simultaneous equations. Then, Storrer and Stievenart obtained the eigenvalues of multiplying systems from those for non-multiplying systems by making the yield rate of neutrons as the intervene between them.

Denote the spectrum of multiplying system reached from a certain spectrum of pulsed neutron source without suffering of any fission at time t by $\psi(E,t)$, then the neutron yield rate caused by $\psi(E,t)$ at t, $S_1(t)$, can be expressed as

$$S_1(t) = \int_E \nu \Sigma_f(E) \psi(E,t)\, dE \tag{5}.$$

When the pulsed neutron has the spectrum of fission neutrons, the neutron yield rate $K(t)$ for $\psi_\chi(t)$ to which the fission spectrum χ reaches without any fission, can be similarly expressed as

$$K(t) = \int_E \nu \Sigma_f(E) \psi_\chi(E,t)\, dE \tag{6}.$$

Therefore, for two successive generations of fissions, the convolution of

$$S_n(t) = \int_0^t S_{n-1}(t-t') K(t')\, dt' \tag{7},$$

holds. By the Laplace transformation, Eq. (7) becomes as

$$S_n(p) = S_{n-1}(p) K(p) \tag{8}.$$

The total neutron yield rate by all of the generations of fissions is expressed by that

$$S(t) = S_1(t) + S_2(t) + S_3(t) + \cdots\cdots \tag{9}.$$

Equation (9) is Laplace-transformed using Eqs. (7) and (8) as

$$S(p) = S_1(p)\{1+K(p)+K(p)^2+\cdots\cdots\}$$

$$= S_1(p)/\{1-K(p)\} \qquad (10).$$

The eigenvalue of the multiplying systems can be obtained as the solutions, α_i's, of the equation that

$$1 - K(p) = 0 \qquad (11),$$

and the time dependent spectrum can be expressed as

$$\psi(E,t) = \sum_i C_i A_i(E) e^{\alpha_i t}$$

In Eq. (11), $\varphi_i(E,t) = A_i(E)e^{\alpha_i t}$ is one of the eigenfunctions and C_i is one of the constants determined by the initial condition. The $C_i A_i(E)$ can be obtained by the calculation of residues.

After the theory by Storrer and Stievenart, the code named GS-1 was written.(3) The code GS-1 can be applied also for the case of the complex eigenvalue.

3. Experiment on natural uranium systems

The pulsed neutron experiment was performed on two natural uranium cubic systems. Their lateral lengths are 20 cm and 30 cm respectively. The pulsed neutrons by D-T reactions and D-D reaction generated by a 2 MeV Van de Graaff accelerator were injected at the center of the system. The decay curves were observed by ^{6}LiI scintillation counters somewhat energy-wisely. The details of the experiment have been described in another issue.(4)

The ^{6}LiI scintillation counter has poor energy resolution due to large Q value and it gives signals of pulse height distribution as schematically shown in Fig. 1 for mono-energy neutrons. Thus, the energy response of ^{6}LiI scintillator is not good enough to discuss about the details of the energy-wise decays and it can be regarded as a kind of integral detector such as a plastic scintillator. The pulse height of the peak for mono-energy neutrons has a linear relation with the neutron energy as shown in Fig. 2 . Therefore, it can be discussed whether there exists the difference or not in the time behavior of decay curves according to the pulse height of detector signals. If all of

decay curves show the same time behavior, it can be said that the neutrons decay asymptotically.

The decay curve of neutrons looks like to be composed of two parts, the initial transient part where the time behavior of each curve differs one another according to the selection of the pulse height and the latter asymptotic part where all decay curves show the same time behavior.(1)

The culculation was performed with 16 group ANL-635 cross section set.(4) The extrapolation distance of the system was obtained by the formula as

$$\overline{\delta} = \int \delta(E) D(E) \phi(E) dE \Big/ \int D(E) \phi(E) dE, \text{ where } \delta(E) = \frac{2}{3} \lambda_{tr}(E)$$

using the energy spectrum calculated by a one-dimensional diffusion code EXPANDA.(5) The results of experiments about 20 cm cubic system are compared in Fig. 3 and about 30 cm cubic system, in Fig. 4. The energy range of the experimental curve was determined from the calibration curve shown in Fig. 2 and they were normarized to the corresponding calculated curves independently. As mentioned before, ^{6}LiI scintillator has poor energy resolution, it is unable to make detail comparison between the experimental curves and the calculated ones. However, it can be said that their time behaviors agree each other as a whole after a certain time of period. The reasons of the difference at the initial part of the decays are (1) that the high energy neutrons make low pulse height pulses as shown in Fig. 1, (2) that the detector is more sensitive to the lower energy neutrons and (3) that the spatial higher modes exist in this stage.

To certify the asymptotic behavior of decay curves observed in natural uranium systems, the similar experiment was carried out on lead assemblies. As shown in Fig. 5, the asymptotic time behavior was not observed in the case of lead assemblies.

As shown in the results of the calculation, the common time behavior is observed for the neutron groups of the higher energy. In order to see that such behavior is due to the neutron multiplication, the calculation have been carried out about the more reactive systems as shown in Figs. 6 and 7. The energy spectrum reaches asymptotic spectrum much faster than the natural uranium systems. To certify this, the calculation was carried out about the fictitious natural uranium system whose ν is zero and the result exhibits no asymptotic behavior as shown in Fig. 8.

From these results, it is clear that the asymptotic behavior observed at natural uranium systems is owing to the neutron multiplication and the calculation by Storrer and Stievenart's theory may predict well the time behavior in natural uranium systems.

4. Experiment on FCA I-5

In the analysis of pulsed neutron experiments, it is essential to know the reactivity of the system. Besides it, it is also required to certify the propriety of the cross section set used in the analysis. If the system can be critical, it is possible to know the reactivity of the subcritical state with more reliability than the case that the system can not be critical. The propriety of the cross section set can be also examined by the critical experiment and the measurement of the prompt neutron decay constant at the delayed critical. By these reasons, a bare fast system named FCA I-5 was assembled. It is a cylindrical core whose components are only 20 percent enriched uranium and stainless steel of the lattice and drawers of the assembly machine. The details of the pulsed neutron experiments carried on FCA I-5 have been explained in another issue.(6) The configurations of each loading of FCA I-5 are shown in Table 1. The experimental results are summarized in Table 2.

At first, the analysis was carried out by DTF-4 code, which is a one-dimentional S_n code and it is able to serch the time eigenvalue by adding $-\alpha/v$ absorption to the balance equation. The analysis was carried out with JAERI-FAST 25 group cross section set.(8),(9) The time eigenvalue search by DTF-4 was carried for cylindrical geometry and it failed to converge for the cases of less reactive than Loading 11-D. Then, the application of GS-1 code was tried. The trancated JAERI-FAST 25 group cross section set as 16 group set was used for the calculations. The comparison between experiments and calculations is shown in Table 3. The results are compared by the value of α/α_c in order to avoid the effect of the difference in the specrum due to inadequacy of the cross section set. The calculated α_c at the delayed critical is larger than the experimental one by 16 percent. The delayed neutron fraction which is required to obtain prompt neutron yield was calculated by SNKPARAM.(10) The lowest eigenvalue for the less reactive Loading than Loading 10-E does not agree with the experimental results as DTF-4 calculations failed.

This discrepancy comes from artificail effect owing to the multi-group structure. Since the experimental decay constants were determined from the decay curves after 0.5 micro second from pulse injection, the calculated energy spectrum after 0.5 micro second was investigated. For all of the cases, the energy spectrum has its peak at sixth group after the initial strong energy higher modes died away. Therefore, the amplitudes CiA_i^6 of Eq. (12) was investigated. They vary as shown in Fig. 9 according to the effective neutron multiplication factor, K_{eff}. At Loading 11-D, the 14 th amplitude (the third amplitude from the bottom) is extraordinally large comparing with others and at Loading 11-E, the 13 th component (the fourth amplitude from the bottom). They are commonly the largest also in other sets of the amplitudes of each group as well as the sixth group. Amplitudes from sixth to eighth are also large but their terms decay rapidly and they are not the commonly largest amplitude for all sets of amplitudes of each group.

The eigenvalues corresponding to these largest amplitude agree well with the experimental results as shown in Table 3. The agreement by GS-1 to the experimental results is observed for the wide renge of neutron multiplication from the nearly critical down to about 0.8 of K_{eff}.

5. Conclusion

The theory by Storrer and Stievenart can predict well the time behavior of neutrons for the wide range of the reactivity from the nearly critical state to the far subcritical state if the system can be regarded as a bare system.

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Table 1 FCA I-5 configurations

material composition (10^{-2}nuclei/cm^3)			dimension (cm)				Volume (ℓ)
	core	lattice *	core		radius	length	
^{235}U	0.7836	——	sphere		25.12	——	66.40
^{238}U	3.1130	——	cylinder	Loading 8	24.52	40.64	76.76
C	0.1653	0.1285	〃	9	22.02	〃	61.87
Fe	0.6631	0.4473	〃	10	19.45	〃	48.30
Ni	0.0898	0.0536	〃	11	16.77	〃	35.91

* The equivalent thickness of the lattice is 85.2 cm

Table 3 Comparison between experimental and calculated values of reactivity and α/α_c

Loading	Reactivity (−% k/k)			α/α_c			
	Calculation (JF)		Experiment	Calculation (JF)			Experiment
	DTF-4 (lattice)	GS-1		DTF-4 (lattice)	GS-1 by the lowest eigenvalue	GS-1 by eigenvalue of peak amplitude	
8 - D	0.0	0.0	0.0	1.00	1.00	1.00	1.00
9 - D	4.83	4.82	4.30±0.26	7.44	7.20	7.20	7.25±0.14 (1)
10 - D	11.0	11.5	11.7±0.7	16.0	1.52	1.52	16.5±0.3
11 - D	20.5	21.5	22.0±1.8	35.0	21.8	25.5	24.9±0.8
11 - E	26.0	26.7	29.3±2.0	——	21.9	31.6	28.8±1.5

(1) α was estimated as 1.16×10^6 sec^{-1} from Table 2

Table 2 Experimental reactivity and decay constant of FCA I-5

core gemetry	Loding and rod condition *	measured reactivity (%,-ΔK/K)	measured α from the decay curve after 0.5 μ sec (10^6 sec^{-1})		
			center		15cm from center
			^{235}U	^{238}U	^{235}U
Sphere	D	0.615	0.319 ±0.002	——	——
	C/R #1 full out	1.09_9	0.438 ±0.002	——	——
	C/R #4 full out	1.27_4	0.471 ±0.003	——	——
	C/R #1&4 full out	1.75_8	0.594 ±0.003	——	——
Cylinder	8-delayed critical	0	0.160±0.002	——	——
	8 - 1	0.620	0.299±0.002	——	——
	8 - 2	1.24_0	0.431±0.002	——	——
	8 - 4	2.48_0	0.737±0.005	0.721 ±0.004	——
	8 - E	4.53_4	1.17 ±0.01	1.16 ±0.01	
	9 - D	4.30_2	——	——	——
	9 - 4	7.21_9	1.70 ±0.01	1.71 ±0.01	1.69 ±0.02 (^{238}U: 1.73±0.02)
	9 - E	9.90_8	2.23 ±0.02	2.20 ±0.02	——
	10 - D	11.6_8	2.65 ±0.03	2.55 ±0.03	2.66 ±0.04
	10 - 4	14.9_1	3.13 ±0.04	3.05 ±0.04	2.96 ±0.05
	10 - E	18.3_7	3.46 ±0.06	3.51 ±0.05	3.34±0.05
	11 - D	22.0_4	3.98 ±0.06	4.00 ±0.06	3.46 ±0.08
	11 - 4	25.5_5	4.13 ±0.09	4.27 ±0.09	2.8 ±0.3
	11 - E	29.2_5	4.60 ±0.09	4.20 ±0.12	——

* Rod conditions D means all rods are inserted.
E " " " " extracted
1.2.4 is the number of rods extracted

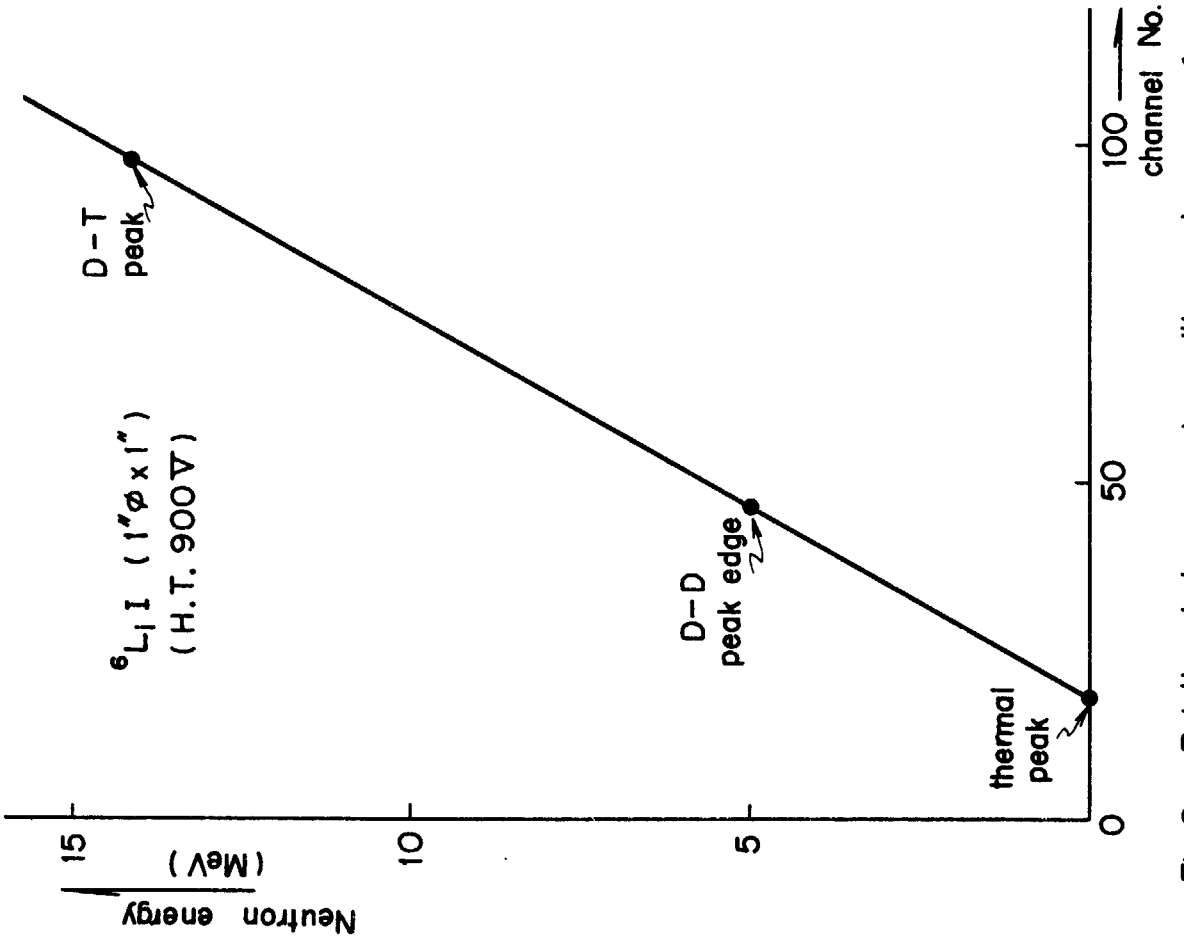

Fig. 2 Relation between peak position and energy of incident neutrons

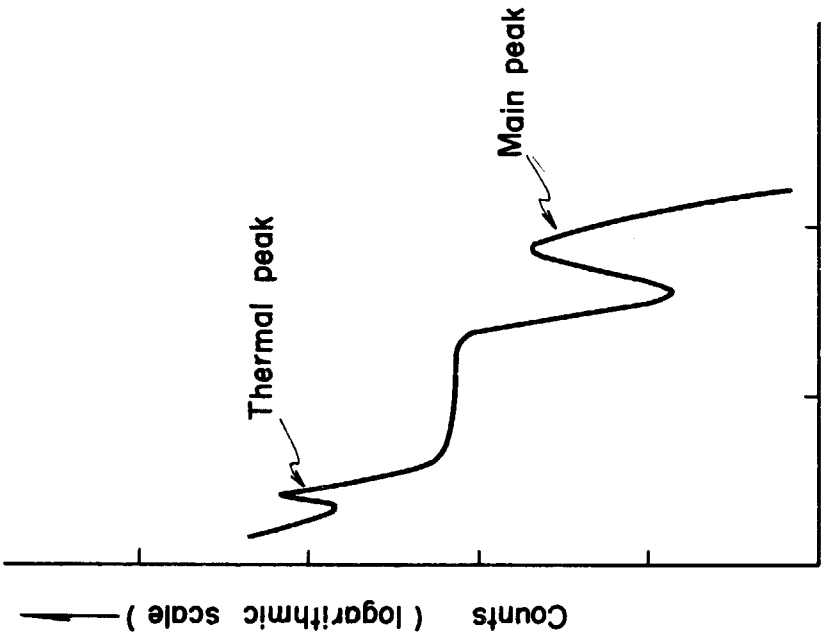

Fig. 1 Schematic pulse height distribution by ^{6}Li I scintillator for mono-energetic neutrons

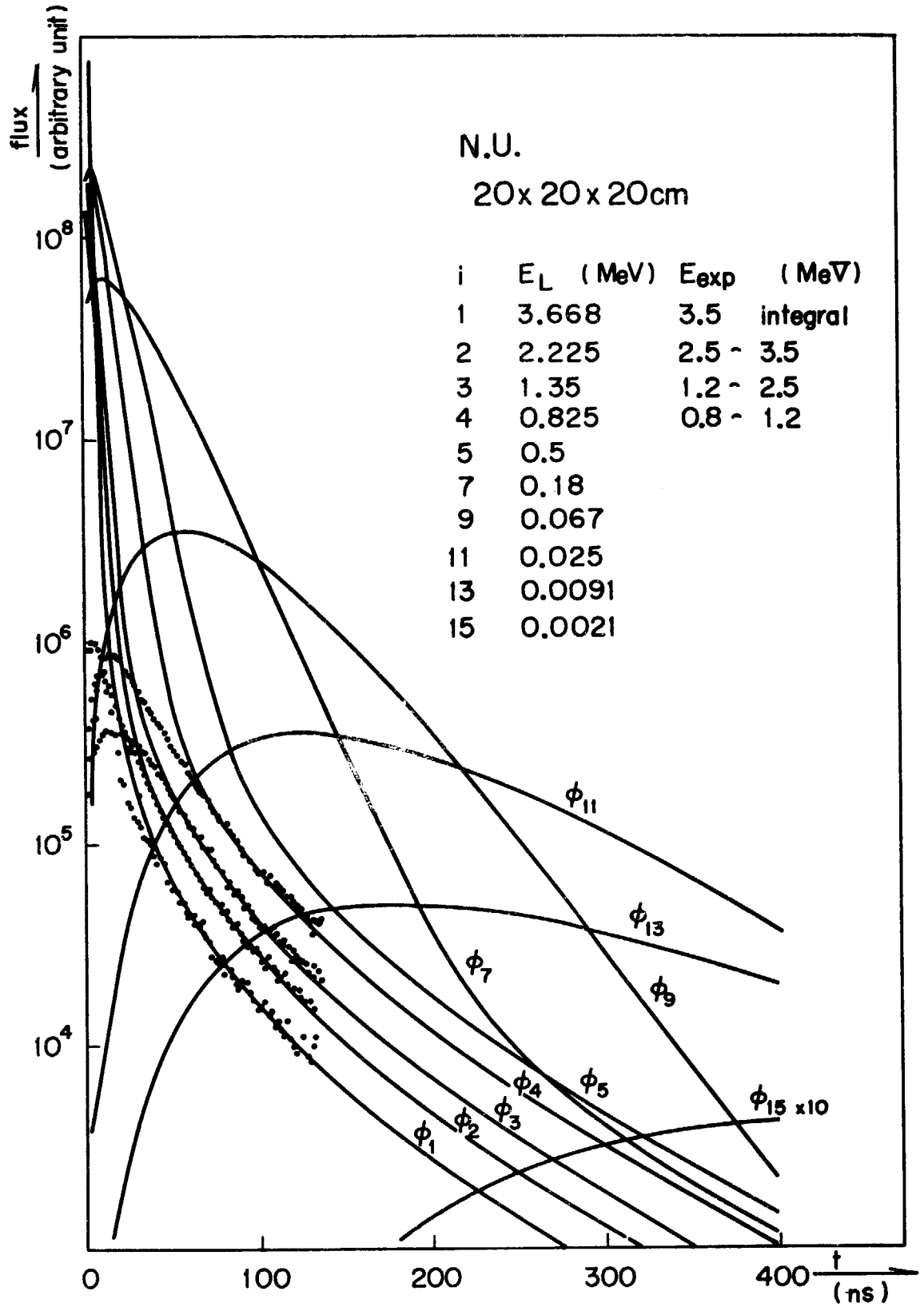

Fig. 3 Energy Dependent Fast Neutron Flux Decay Curves in a System of Natural Uranium of 20 cm Cube

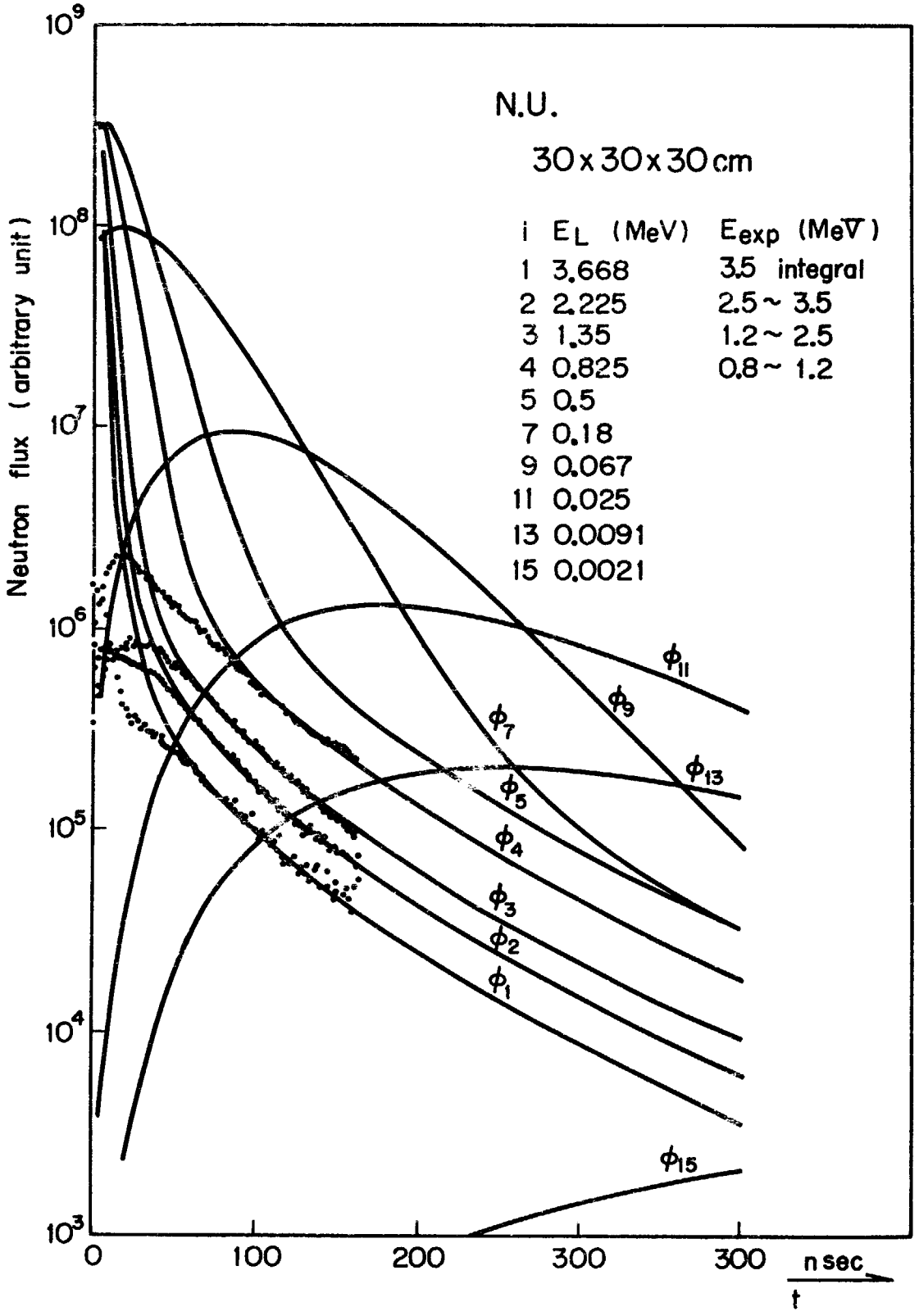

Fig. 4 Energy dependent fast neutron flux decay curves in a system of natural uranium of 30cm cube.

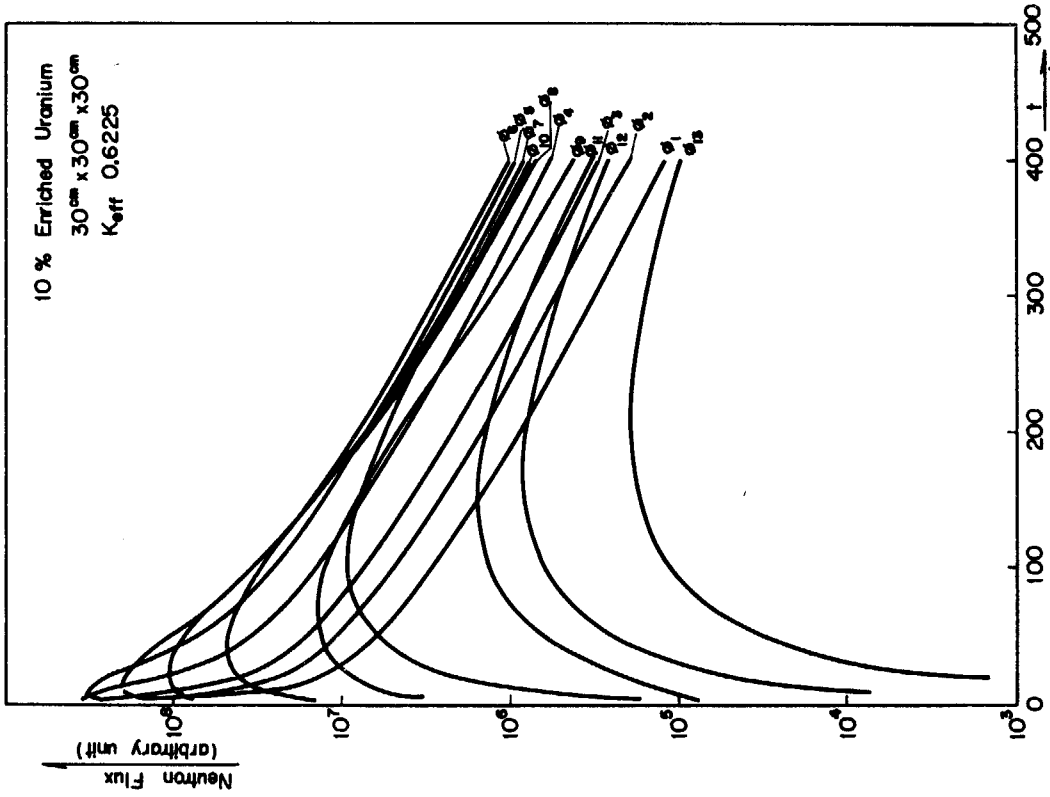

Fig. 6 Fast Neutron Decay in a Cubic 10 % Enriched Uranium of 30 cm Edge Length

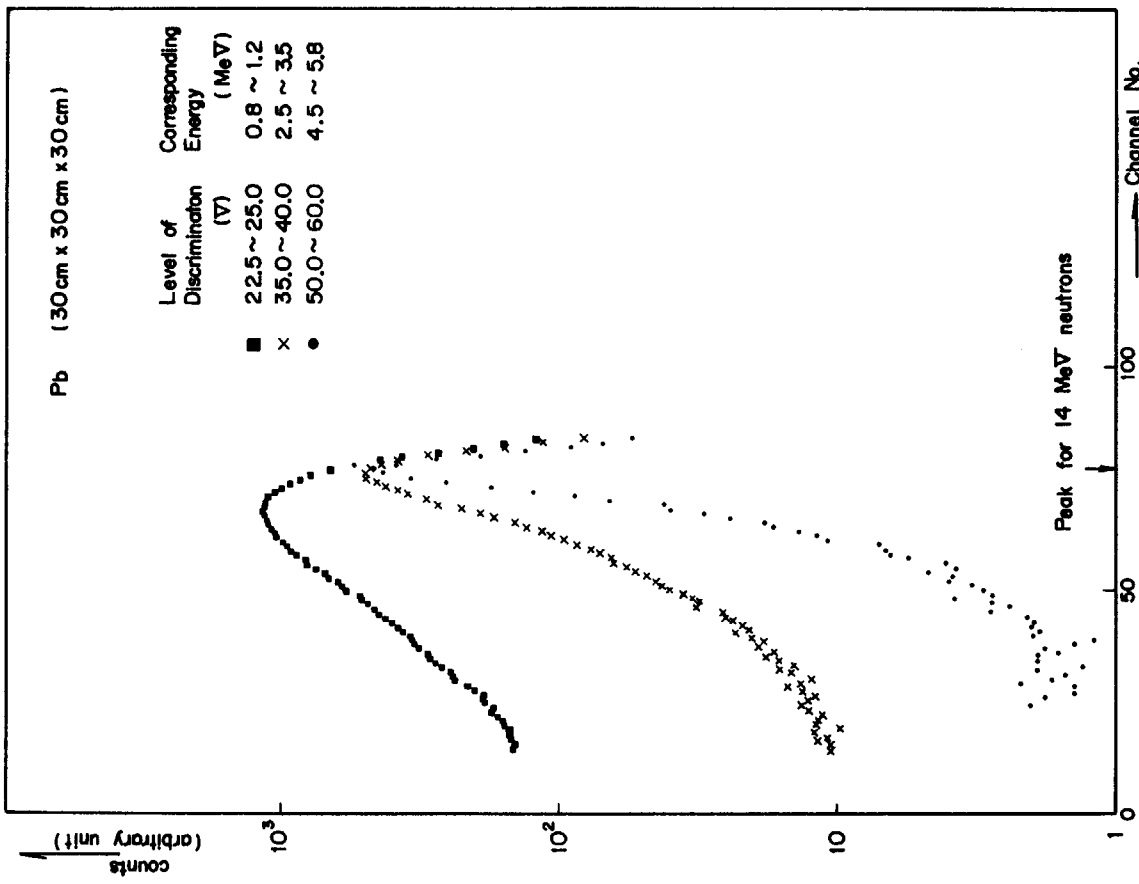

Fig. 5 Neutron decays in lead assembly (30cm x 30cm x 30cm)

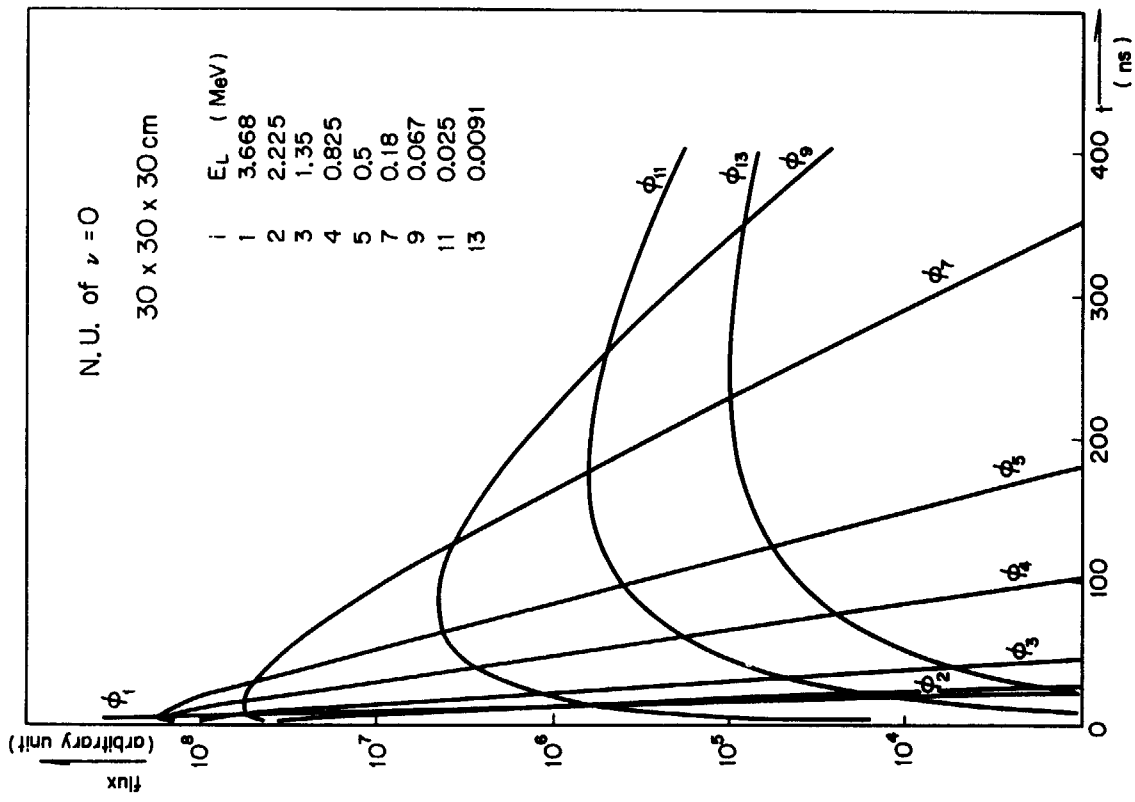

Fig. 8 Energy Dependent Fast Neutron Flux Decay Curves in a Fictitious Natural Uranium of $\nu = 0$

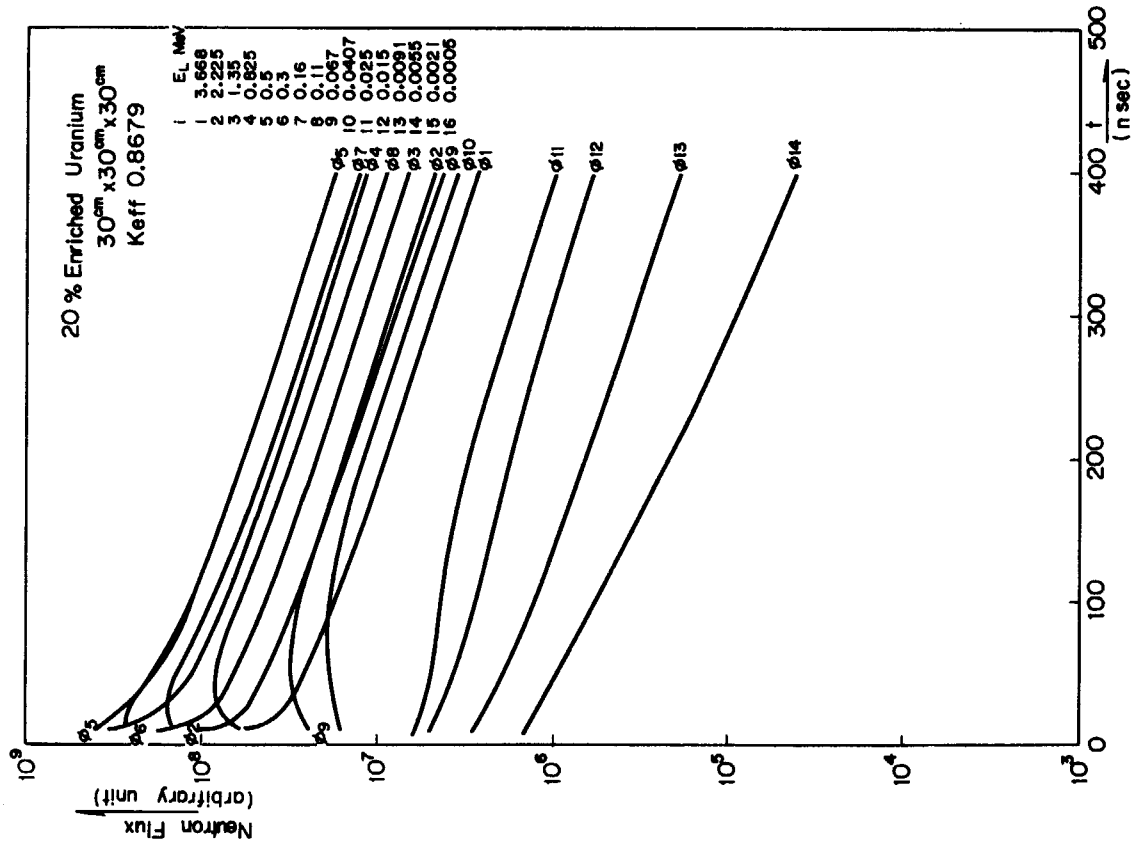

Fig. 7 Fast Neutron Decay in a Cubic 20 % Enriched Uranium of 30cm of Edge Length

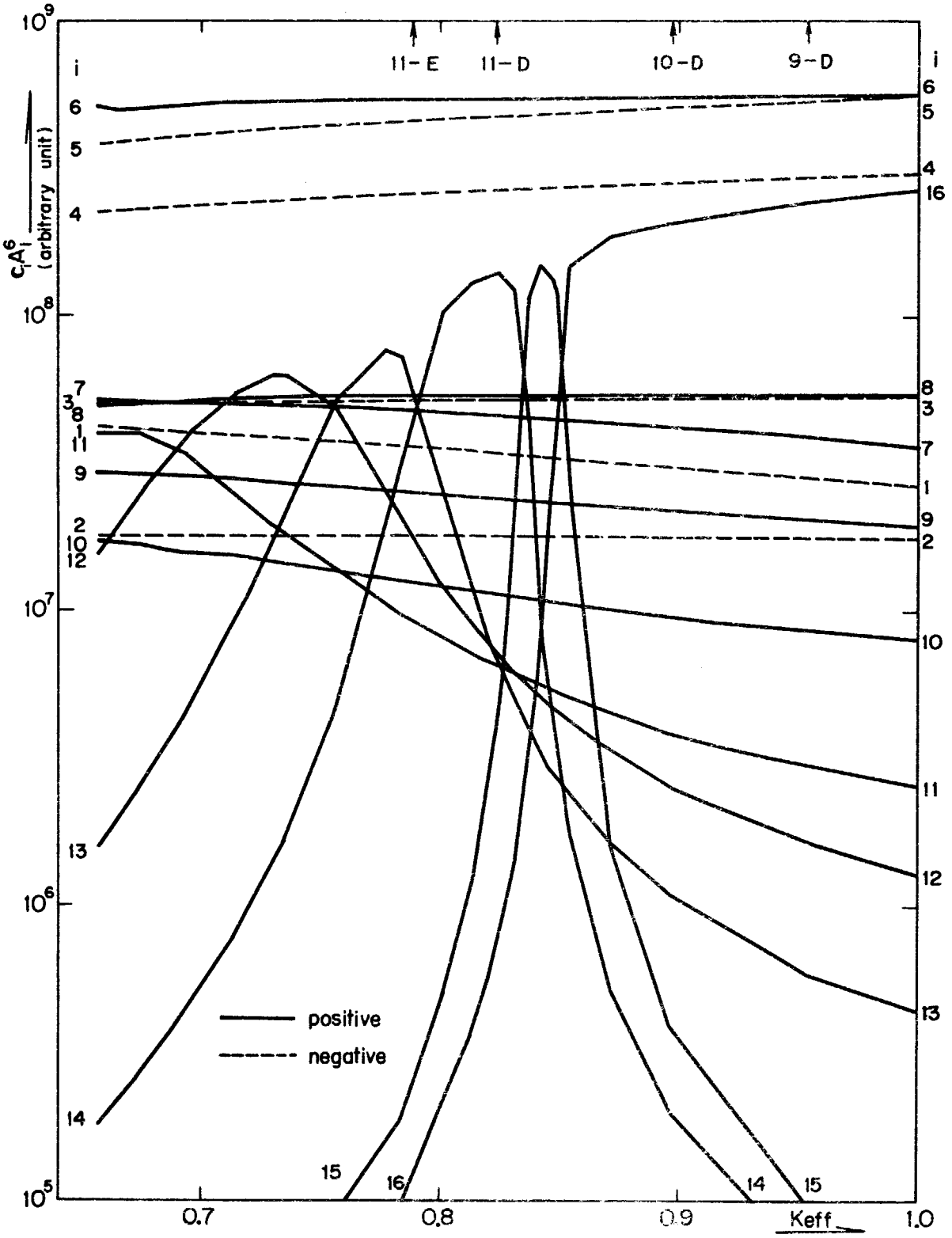

Fig. 9 Amplitudes of decay curve of peak group (6 th group)