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A Few Remarks for Calculations of Prompt Neutron Decay Constant at Delayed Critical of Fast Reactor Systems

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It is clarified that inconsistent selections of fission spectrum at the calculation of the prompt neutron decay constant at delayed critical, α_c , by the eigenvalue calculation leads to the difference in the value of α_c by the amount of more than ten percent. The attension should be paid to the difference in the fission spectrum between the static calculation and the dynamic calculations.

The difference of the prompt neutron decay constant between the eigenvalue calculation and the perturbation calculation is also investigated. At the delayed critical, it is rather small less than a few percent since the effect by the distortion is compensated by the effect by the difference in fission spectrum between the eigenvalue calculation and the perturbation calculation. However, the difference becomes large as the effective multiplication factor decreases.

高速炉系の遅発臨界時における即発中性子減衰定数の計算に対する二，三の所見

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高速炉系の即発中性子減衰定数αを固有値として計算する場合には計算に用いる分裂スペクトルに注意を払わなければならない。即ち，実効増倍係数を計算する場合のような静的な計算においては分裂スペクトルとしては遅発中性子の成分も入れたものを用い，即発中性子減衰定数を計算する動的な計算の場合には即発中性子のみの分裂スペクトルを用いるべきである。これを混同すると遅発臨界時のα，即ちα_Cの値に１０数％の誤差を生ずることがわかった。

また，固有値計算によるα_Cと摂動計算によるα_Cとの差はたかだか数％であるが，これは所謂Kinetic distortionの効果と分裂スペクトルの差による効果が打消し合って見かけ上の一致を良くしていることがわかった。しかし未臨界度が大きくなると両者の差は広がる。

これらの問題は分裂スペクトルが炉内のスペクトルの一部を形成しているという高速炉系の特徴からもたらされたものである。

目次なし

1. Introduction

On fast reactor systems, the prompt neutron decay constant, α, reflects strongly the characteristics of the neutron spectrum especially on the lower energy side. Therefore, the agreement between the calculation and the experiment gives one of the certifications to the cross section set. As reported before, there has been a large discrepancy up to 30 percent between the calculated and the experimental values of α_c, the decay constant at delayed critical.$^{(1)}$ This discrepancy was reduced down to the level of 10 percent by the improvement of the cross section set.$^{(2)}$ At the present time, the refinement of the calculation method should be required.

There are two ways to calculate the decay constant of prompt neutrons. One of them is the perturbation method and the other is the method to seek directly the constant as an eigenvalue of the time dependent Boltzmann equation.

In the following, it will be pointed out that kinetic parameters and the fission spectrum should be carefully used at the performance of time eigenvalue calculations because the fission spectrum is a part of the neutron energy spectrum in cores in the cases of fast reactor systems.

Since the perturbation method is an approximation to the eigenvalue calculation, the accuracy of the prompt neutron decay constant, α, calculated by the perturbation method will also be discussed in comparison with those by the eigenvalue calculation.

2. Fission spectra at static and dynamic calculations

The fission source, F, is composed of the prompt neutron source, F_p, and the delayed one, F_d, as

$$F = F_p + F_d \qquad (1),$$

and

$$F_p = \sum_i \chi_{p,i}(1-\beta_i)\int \nu_i \Sigma_{f,i}\, \phi(\boldsymbol{r}, E)dE \qquad (2),$$

$$F_d = \sum_i \chi_{d,i}\, \beta_i \int \nu_i \Sigma_{f,i}\, \phi(\boldsymbol{r}, E)dE \qquad (3).$$

Before the calculation of time eigenvalue, α, it is necessary to know the effective neutron multiplication factor, K, using the static fission source expressed by Eq. (1). Then, the time eigenvalue corresponding to the effective multiplication factor of K is sought with the prompt fission source as Eq. (2).

The fission source at the static calculation is usually expressed as

$$F_s = \sum_i \chi_{s,i} \int \nu_i \Sigma_{f,i} \phi(\boldsymbol{r}, E) dE \quad (4),$$

where $\chi_{s,i}$ is the fission spectrum of nucleus i for static calculations. Therefore, for the nucleus i,

$$\chi_{s,i} = \chi_{p,i}(1-\beta_i) + \chi_{d,i}\beta_i \quad (5).$$

For both static and dynamic calculations, conventional codes usually permit only one kind of the fission spectrum throughout the reactor. Therefore, when there are more than single kind of fissionable nucleus as usually so in fast reactors, it is necessary to define the effective fission spectra of the reactor for both static and prompt neutron sources. Since the difference among the fision spectra of prompt neutrons or among the spectra of delayed neutrons of various kind of nuclie is much less than the difference between spectra of the prompt neutrons and the delayed neutrons of any kind of nucleus, the following relation can be assumed with good approximation:

$$\chi_{s,eff} = (1-\beta_{eff})\chi_{p,eff} + \beta_{eff}\chi_{d,eff} \quad (6),$$

where

$$\chi_{s,eff} = \sum_i \chi_{s,i} S_i / \sum_i S_i \quad (7\text{-a}),$$

$$\chi_{p,eff} = \sum_i \chi_{p,i}(1-\beta_i)S_i / \sum_i (1-\beta_i)S_i \quad (7\text{-b}),$$

$$\chi_{d,eff} = \sum_i \chi_{d,i}\beta_i S_i / \sum_i \beta_i S_i \quad (7\text{-c}),$$

$$\beta_{eff} = \sum_i \beta_i S_i / \sum_i S_i \quad (7\text{-d}),$$

and S_i is the fission neutrons by the nucleus i as

$$S_i = \int \nu_i \Sigma_{f,i} \phi(\boldsymbol{r}, E)\, dE\, d\boldsymbol{r} \quad (8),$$

The eigenvalue, α, calculated with the fission spectrum $\chi_{p,eff}$ is the decay constant of prompt neutrons of the reactor whose effective multiplication factor, K, is obtained by the static calculation with the fission spectrum $\chi_{s,eff}$. Since the energy spectrum of delayed neutrons has a form like a δ-function at the energy region where the importance is relatively low as shown in Fig.1, the effective prompt neutron multiplication factor, K_p, calculated with $\chi_{s,eff}$ is smaller than K_p with $\chi_{p,eff}$. This means that α calculated with $\chi_{s,eff}$ is larger than α with $\chi_{p,eff}$. At delayed critical, this difference between α's is very sensitive to the difference between K_p's since the difference of α is exaggerated by the order of about $1/\beta_{eff}$. Therefore, the fission spectrum at eigenvalue calculations should be consistent to the static fission spectrum at static calculations.

The fission spectrum usually used in various static calculations is, in fact, the fission spectrum of prompt neutrons. To have $\chi_{s,eff}$ by Eq. (6), the value of β_{eff} as well as the fission spectra of $\chi_{p,eff}$ and $\chi_{d,eff}$ should be known. Then, the preliminary calculation with conventional fission spectrum will be required. There remains another problem about the averaging neutron flux to be used in obtaining $\chi_{p,eff}$. Strictly speaking, the spectrum $\chi_{p,eff}$ used at eigenvalue calculations should be obtained by the dynamic flux. However, the effect on $\chi_{p,eff}$ by the difference between the static and the dynamic flux is much less than the difference between $\chi_{s,eff}$ and $\chi_{p,eff}$.

To see the effect of the fission spectrum on α, α_c's were calculated about fast assemblies of FCA I-1(3) and FCA I-5-S(4). FCA I-1 consists of a 20 percent enriched uranium metal core sourrounded by a 30 cm thick blanket of natural uranium metal, while FCA I-5-S is a bare spherical core also composed of only 20 percent enriched uranium metal.

To avoid the confusion, the fission spectra are redefined as

$$\chi_{gv} \equiv \text{the conventional fission spectrum}$$

$$\chi^+ \equiv \chi_{gv}(1-\beta_{eff}) + \chi_{d,eff}\,\beta_{eff}$$

$$\chi^- \equiv (\chi_{gv} - \chi_{d,eff}\,\beta_{eff})/(1-\beta_{eff})$$

If χ_{gv} is regarded as the prompt fission spectrum, $\chi_{p,eff}$, the corresponding static spectrum $\chi_{s,eff}$ is χ^+. On the other hand, when χ_{gv} is considered as $\chi_{s,eff}$, χ^- should be used as the consistent $\chi_{p,eff}$.

The time eigenvalue were calculated by DTF-4 code using JAERI-FAST 25 group set.(5),(6) The fission spectra χ_{gv}, χ^+ and χ^- are summalized in Table 1 and the results of the calculations are shown in Table 2 with the results by the perturbation method. The values of α_c calculated with the prompt fission spectrum identical with the static one become larger than those with the consistent fission spectrum by 14 percent. However, when the consistent set of the fission spectra is employed, the difference of α_c due to the fission spectrum itself, that is, whether $\chi_{p,eff}$ is χ_{gv} or χ^-, is only about 0.5 percent. As seen in the results of FCA I-1, the effect of the blanket on the calculation of α may be also small.

3. Comparison between perturbation and eigenvalue calculations

The reactor equation for the static calculation is written as

$$M\phi_s = \frac{1}{K}\sum_i \chi_{s,i} \int \nu_i \Sigma_{f,i} \phi_s(\boldsymbol{r}, E')\, dE' \qquad (9),$$

and the importance ϕ_s^+ satisfies the equation

$$M^+\phi_s^+ = \frac{1}{K}\sum_i \nu_i \Sigma_{f,i} \int \chi_{s,i} \phi_s^+(\boldsymbol{r}, E')\, dE' \qquad (10),$$

where the operator M includes the process of neutron absorption, scattering and leakage, and M^+ is the adjoint operator of M. On the other hand, the kinetic equation of prompt neutrons may be written as

$$-\frac{\alpha}{v}\phi_p + M\phi_p = \sum_i \chi_{p,i}(1-\beta_i) \int \nu_i \Sigma_{f,i} \phi_p(\boldsymbol{r}, E')\, dE' \qquad (11).$$

Since α is the eigenvalue of Eq. (11), the decay constant of prompt neutrons, α, can be expressed by using Eq. (10) as well as Eq. (11) in the following:

$$\alpha = \frac{1 - K(1 - \beta^*_{eff})}{K \Lambda^*_p} \qquad (12),$$

where

$$\beta^*_{eff} = \frac{1}{N}\int_{\text{reactor}} d\boldsymbol{r} \sum_i \beta_i \left\{\int dE\, \chi_{d,i} \phi_s^+\right\}\left\{\int dE'\, \nu_i \Sigma_{f,i} \phi_p\right\} \qquad (13),$$

and

$$\Lambda^*_p = \frac{1}{N}\int_{\text{reactor}} d\boldsymbol{r} \int dE\, \phi_s^+ \frac{1}{v}\phi_p \qquad (14),$$

and the normalization factor N is given by

$$N = \int_{\text{reactor}} d\boldsymbol{r} \sum_i \left\{\int dE\, [(1-\beta_i)\chi_{p,i} + \beta_i \chi_{d,i}]\phi_s^+\right\}\left\{\int dE'\, \nu_i \Sigma_{f,i} \phi_p\right\} \qquad (15).$$

As mentioned before, when the assumption of the fundamental mode decay is valid, Eq. (12) may give the exact value of the decay constant of prompt neutrons.

By replacing the dynamic flux ϕ_p in Eqs. (13), (14) and (15) to the static flux ϕ_s calculated by Eq. (9), an approximate expression for α can be obtained as

$$\alpha^*_{\mathrm{pert}} = \frac{1 - K\,(1 - \beta^*_{\mathrm{eff,pert}})}{K\Lambda^*_{\mathrm{p,pert}}} \qquad (16),$$

where $\beta^*_{\mathrm{eff,pert}}$ and $\Lambda^*_{\mathrm{p,pert}}$ are defined by the similar equations as Eqs. (13), (14) and (15) using the static flux ϕ_s instead of the dynamic flux ϕ_p. Thus, the perturbation expression, Eq. (16), is an approximation to the eigenvalue calculation.

The difference of α values calculated by the pertrubation method from those by the eigenvalue calculation comes from the difference between the static flux ϕ_s and the dynamic flux of prompt neutrons, ϕ_p. The relation between the energy spectra of ϕ_s and ϕ_p is illustrated schematically in Fig. 2 with the static and the prompt fission spectra. In the higher energy region where the difference is mainly caused by the difference of the fission spectra, ϕ_s becomes smaller than ϕ_p. Then, $\Lambda^*_{\mathrm{p,pert}}$ may be larger than the exact value Λ^*_p. When there are included fissionable nuclei with the threshold and their β's are larger than those of fissionable nuclei without the threshold, $\beta^*_{\mathrm{eff,pert}}$ becomes smaller than β^*_{eff} because of their fission reactions only in the higher energy region. When there is ^{238}U, this tendency is emphasized by its large fraction of delayed neutrons.

In the lower energy region, on the other hand, ϕ_s becomes smaller than ϕ_p by the effect of $-\alpha/v$ term in Eq. (16) which may be regarded as a kind of negative absorption of the macroscopic cross section of α/v. In the lower energy region, there is no significant source to affect β^*_{eff} such as in the higher energy region.

From the above discussion, it may be clarified that the difference of α^*_{pert} from the eigenvalue α is determined by the competition between the negative effect in the higher energy region and the positive one in the lower energy region. Since the value of α is, roughly speaking, proportional to $(1-K_p)$, the positive effect on α^*_{pert} may strongly depend on the reactivity. Therefore, with decreasing the reactivity, α values obtained by the perturbation method will be more positive than those by the eigenvalue calculations.

To see the difference in α value between the perturbation method and the direct eigenvalue calculation, calculations were performed about FCA I-5-S at several reactivities. The perturbation calculations were carried out by a code SNKPARAM(7), which is a modified code of DTF-4 to calculate the kinetic parameters by the perturbation theory. The eigenvalue calculations were performed by DTF-4 code. For both types of calculation, JAERI-FAST 25 group set was used.

Since α values may strongly depend on the effective multiplication factor, calculations of eigenvalues in the static and dynamic calculations were performed as finely as possible. The results are summarized in Table 3. The fission spectra χ_{gv} and χ^- were used at the static and dynamic calculations, respectively. At delayed critical, the perturbation calculation gives the smaller value of α by about 2 percent, while it overestimates α value by about 2 percent at subcritical state of $K = 0.94$. The over-stimation becomes about 6 percent at highly subcritical state of $K = 0.90$.

The difference between the perturbation method and the eigenvalue calculation was also estimated for a soft spectrum system of FCA III-2 which has a core of 20 percent enriched uranium metal diluted with graphite(8). As seen in Table 3, the difference at delayed critical in only 0.6 percent because of the stronger positive effect in the lower energy region comparing with the cases of the cores of the harder spectrum.

4. "Real" and "Importance" formulations

There is a certain confusion in the treatment of the kinetic parameters, such as the effective delayed neutron fraction or the prompt neutron life time, in connection with the reactivity as pointed out by Spinrad(9). At the calculation of α values, the attention should be paid on the systematic use of the "real" or "importance" formulation of the reactor equations.

By the importance formulation, that is, by the perturbation method, the decay constant α is given by

$$\alpha^* = \frac{1 - K\,(1 - \beta^*_{\mathrm{eff,pert}})}{\ell^*_{\mathrm{p,pert}}} \qquad (17),$$

where $\ell^*_{\mathrm{p,pert}}$ is the effective prompt neutron life time defined as

$$\ell^*_{\mathrm{p,pert}} = K \Lambda^*_{\mathrm{p,pert}} \qquad (18).$$

On the other hand, α is also written in the "real" formulation as follows;

$$\alpha = \frac{1 - K_p}{\ell_p} \qquad (19),$$

where K_p is the neutron multiplication factor as

$$K_p = \frac{\text{prompt neutron production rate}}{\text{prompt neutron disappearance rate}}$$

and ℓ_p is the prompt neutron life time defined as

$$\ell_p = \frac{\text{prompt neutron population}}{\text{prompt neutron disappearance rate}}$$

From the physical point of view, the right hand side of Eq. (19) represents the ratio of the prompt neutron decreasing rate to the prompt neutron population and that is the original definition of the decay constant of prompt neutrons.

Although both formulations essentially give the same time behavior as

$$\alpha = \alpha^* \qquad (20),$$

there is a difference between $\beta^*_{\mathrm{eff,pert}}$ and $1-K_p$ or $\ell^*_{\mathrm{p,pert}}$ and ℓ_p. As an example, these kinetic parameters at delayed critical were calculated about FCA I-5-S by use of SNKPARAM with JAERI-FAST 25 group set. The results for both of two formulations are shown in Table 4. Although the difference between α_c and α_c^* is only about 2 percent, the difference between ℓ_p and $\ell^*_{\mathrm{p,pert}}$ or $1-K_p$ and $\beta^*_{\mathrm{eff,pert}}$ is up to 10 percent. Thus, we should be careful to use kinetic parameters systematically.

5. Conclusion

The consistent use of the fission spectrum should be kept at the calculation of the time eigenvalue. When the static perturbation calculation is performed, the static fission spectrum which includes the delayed neutron source should be used, while, when the direct time eigenvalue calculation is carried out, the dynamic fission spectrum of only prompt neutron source should be used, respectively. The confusion of the use of fission spectrum leads to the difference in α_c value by the amount of more than ten percent for some cases of fast reactor systems.

The correct introduction of the kinetic distortion into the calculation of α makes the value of α smaller than the value by the static perturbation calculation. This effect is large for the small size core where the neutron energy spectrum has the sharp decrease at the lower part of the spectrum than its peak.

At the perturbation calculations, the effect of the kinetic distortion is cancelled out by the effect of the use of the static fission spectrum which makes α value large. Thus, the perturbation calculation gives an apparently close value to that by the direct time eigenvalue calculation with the consistent use of the fission spectrum within a few percent. However, the difference between them becomes large as the effective multiplication factor decreases.

We should be careful to the consistent use of the kinetic parameters following to "real" or "adjoint" formulation. The difference in the values of the kinetic parameters between the two kinds of formulations is up to ten percent in some cases of fast reactor systems.

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Table 1 Comparison of effective fission spectra

energy group	χ_{gv}	I-5-S		I-1	
		χ^+	χ^-	χ^-_{core}	χ^-_{sys}
1	0.0161	0.01597	0.01623	0.01623	0.01624
2	0.0886	0.08787	0.08934	0.08933	0.08938
3	0.1837	0.18218	0.18523	0.18520	0.18532
4	0.2701	0.26786	0.27236	0.27231	0.27248
5	0.2024	0.20072	0.20409	0.20406	0.20418
6	0.1407	0.14312	0.13826	0.13830	0.13812
7	0.0610	0.06514	0.05682	0.05691	0.05660
8	0.0238	0.02365	0.02395	0.02395	0.02396
9	0.0093	0.00922	0.00938	0.00938	0.00938
10	0.0030	0.00298	0.00303	0.00302	0.00303
11	0.0013	0.00129	0.00131	0.00131	0.00131
12 ~ 25	0	0	0	0	0
total	1.0000	1.00000	1.00000	1.00000	1.00000

Table 2 Calculated α_c with various combination of fission spectra

Assembly	χ_s	χ_p	α_c ($\times 10^5$ sec^{-1})	
			eigenvalue	perturbation
I - 5	χ_{gv}	χ^-	1.981	1.938
	χ_{gv}	χ_{gv}	2.264	
	χ^+	χ_{gv}	1.971	1.928
	χ^+	χ^+	2.250	
I - 1	χ_{gv}	χ^-_{sys}	1.515	1.492
	χ_{gv}	χ^-_{core}	1.532	

Table 3 Relation between prompt neutron decay constants by perturbation and by eigenvalue calculation

FCA Assembly	I-5			III-2
K	1.0000	0.9439	0.9008	1.0000
α_{pert} (10^5 sec^{-1})	1.928	18.10	32.42	28.11
α_{eig} (10^5 sec^{-1})	1.971	17.81	30.66	28.29
$\alpha_{pert}/\alpha_{eig}$	0.978	1.016	1.057	0.994

Table 4 Comparison of kinetic paremeters at delayed critical of FCA I-5

Real formulation	Importance formulation
$1-k_p = 8.081 \times 10^{-3}$	$\beta^*_{eff} = 7.195 \times 10^{-3}$
$\ell_p = 4.100 \times 10^{-8}$ sec	$\ell^*_p = 3.731 \times 10^{-8}$ sec
$\alpha_c = 1.971 \times 10^5$ sec^{-1}	$\alpha^*_c = 1.928 \times 10^5$ sec^{-1}

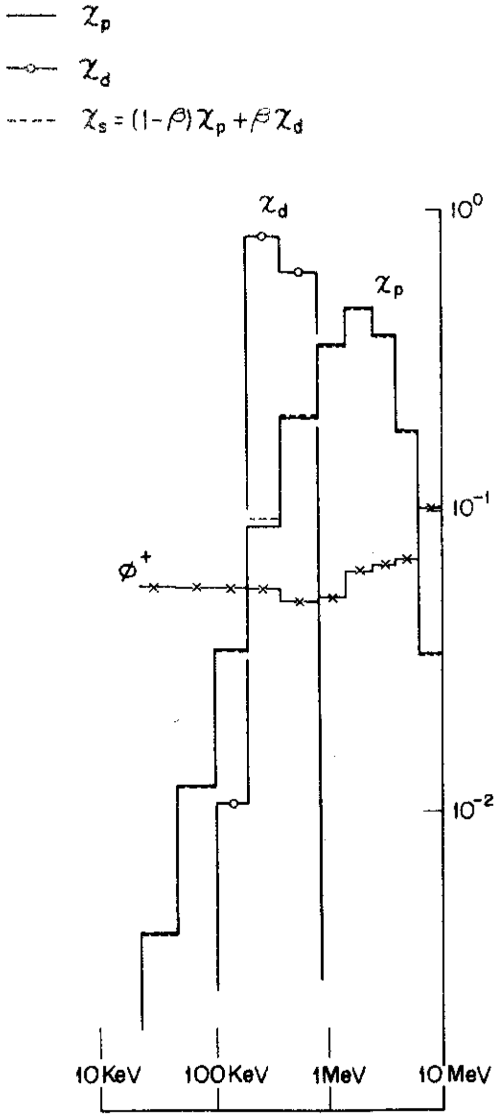

Fig.1 Fission Spectra and adjoint flux of FCA I-5

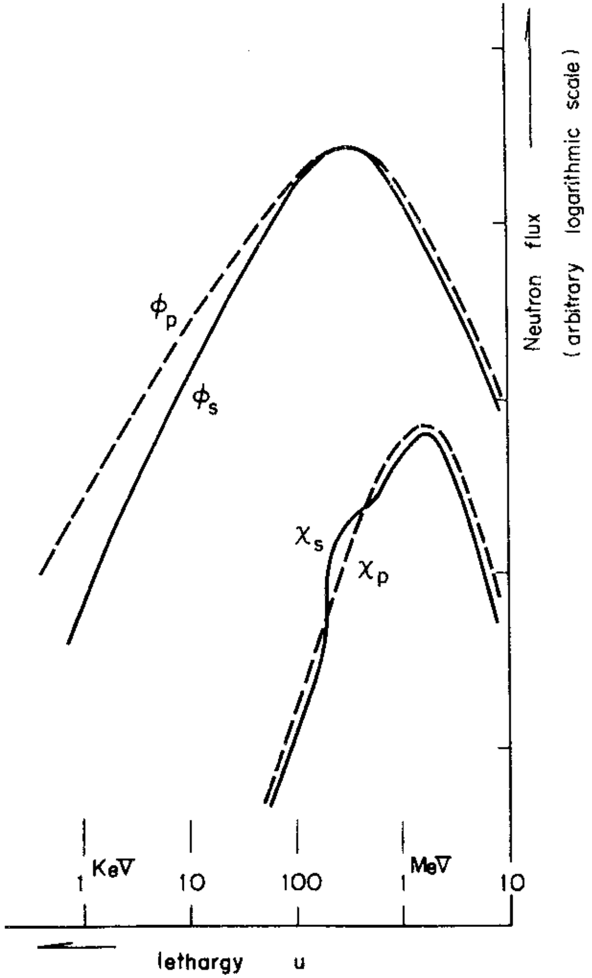

Fig. 2 Schematic comparison of neutron energy spectra and fissio spectra between static and dynamic calculations