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Applicability of One-Group Diffusion Theory for Burnup Analysis of a Fast Reactor

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Applicability of the One-Group Diffusion Theory for Burnup Analysis of a Fast Reactor

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Applicability of the one-group diffusion theory for burnup analysis of a fast reactor, spherical and core-volume 3,700ℓ, has been studied comparatively with the multi-group diffusion theory in the variation of effective multiplication factor, power distribution and atom densities. The following were revealed.

(1) The variations of power distribution and atom densities can be estimated with good accuracy with one-group reactor constants of the initial state.

(2) The variation of effective multiplication factor can be inferred accurately with one-group reactor constants based on the perturbation theory, even when the spectrum change of reactor core is not taken into consideration.

Burnup characteristics of the fast reactor can thus be estimated fairly accurately by the burnup analysis based on one-group reactor constants of the initial state.

An approximate method developed for representing the burnup-dependent one-group reactor constants is also described and the results of calculation are presented.

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一群拡散理論に基づく高速炉の燃焼解析の検討

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一群拡散理論を用いて，高速炉の燃焼特性である実効増倍率，出力分布，原子数密度を炉心容積3,700ℓの球形，高速炉に対して解析し，その妥当性を検討した。その結果次の結論をえた。(1)高速炉の燃焼特性のうち出力密度と原子数密度とは初期状態の一群炉定数を用いた解析により推定可能である。(2)実効増倍率の時間変化は摂動論に基づく一群炉定数を使って推定することが可能である。従って初期状態で定義される一群炉定数を用いた解析により，かなりの精度で燃焼特性の推定が可能であることが分った。

また，一群炉定数の時間変化が近似的に取扱われ，その結果に検討が加えられた。

Contents

1. Introduction

In order to study the economic and safety aspects of a commercial fast reactor, it is very important to evaluate the neutronic and thermal characteristics of the reactor through the burnup process. These characteristics are evaluated on the basis of the neutron flux and fission distributions obtained from the nuclear burnup analysis. Therefore the nuclear burnup analysis plays an important role in evaluating the behaviors of the whole reactor system including the fuel management.

A fast reactor usually consists of the core, axial and radial blankets. Then the burnup characteristics of the fast reactor should be evaluated using the burnup codes based on two or three dimensional multi-group diffusion equations. These calculations usually need a large amount of computer time for the long-term burnup analysis.

A main purpose of the paper is to get burnup behaviors of the fast reactor with a reasonable computation time using the one-group diffusion theory instead of the multi-group one in the burnup equation without significant sacrifice in its accuracy.

Chapter 2 deals with the expression for the diffusion equation and the depletion equation by one-group constants, which are related to multi-group reactor constants and neutron fluxes. The variations of the one-group constants are evaluated in Chapter 3 through the burnup steps using the burnup code based on one dimensional multi-group diffusion theory. The effects of the variations of these constants are evaluated in Chapter 4 on the burnup characteristics, such as the variations in atom densities, effective multiplication factors and power distributions. The effects of the difference in the methods of producing the one-group transport constants, which are used for obtaining the diffusion coefficient, are studied for the burnup characteristics. The validity of the first order perturbation theory is examined calculating the variations of the effective multiplication factor. With the consideration of the results obtained above, an approximate method is developed in Chapter 5 for estimating the burnup-dependent one-group constants. The method is compared with the burnup analysis based on the multi-group diffusion theory.

2. One-group constants in the burnup equation

The burnup analysis will be performed with the linearized burnup equation which consists of the diffusion equation representing the neutron field and the depletion equation of the burnup nuclides. The multi-group diffusion equation and its boundary conditions are usually given as follows,

$$\nabla D_i(\underline{x},t)\nabla\varphi_i(\underline{x},t)-\Sigma_{T,i}(\underline{x},t)\varphi_i(\underline{x},t)$$

$$+\sum_{j=1}^{i-1}\Sigma^{s}_{j\to i}(\underline{x},t)\varphi_j(\underline{x},t)+\frac{\chi_i}{k(t)}\sum_{j=1}^{n}\nu\Sigma_{f,j}(\underline{x},t)\varphi_j(\underline{x},t)=0 \qquad (1)$$

and

$$\lim_{\delta\to 0} D_i(\underline{x}-\underline{\delta},t)\nabla\varphi_i(\underline{x}-\underline{\delta},t)=\lim_{\delta\to 0} D_i(\underline{x}+\underline{\delta},t)\ \varphi_i(\underline{x}+\underline{\delta},t),$$

$$\lim_{\underline{\delta}\to 0}\varphi_i(\underline{x}-\underline{\delta},t)=\lim_{\underline{\delta}\to 0}\varphi_i(\underline{x}+\underline{\delta},t). \qquad (2)$$

The notations in the above equations are usual ones.
The depletion equation can be written as follows,

$$\frac{dN_p^m(t)}{dt}=-Z^m_{a,p}(t_l)N^m_p(t)+Z^{m-1}_{c,p}(t_l)N^{m-1}_p(t),\qquad (m=0,1,2,3,4)$$

$$\frac{dN_p^5(t)}{dt}=2\sum_{m=0}^{4}Z^m_{f,p}(t_l)N^m_p(t),\qquad (t_l\leq t\leq t_{l+1}) \qquad (3)$$

where m= 0, 1, 2, 3, 4 and 5 represent ^{238}U, ^{239}Pu, ^{240}Pu, ^{241}Pu, ^{242}Pu and FP respectively. $N_p^m(t)$ is the atom density of the m-th nuclide averaged over the p-th region at time t. The burnup step time t_l is the discrete time at which the diffusion equation is solved using new atom densities to obtain neutron fluxes in each region. $Z^m_{x,p}(t_l)$ is the reaction rate x(a: absorption, c: capture, f: fission) of the m-th nuclide in the p-th region defined at the burnup step time t_l, and it is given by the following

$$Z^m_{x,p}(t_l)=\frac{1}{V_p}\sum_{i=1}^{n}\int_{V_p}\sigma^{m,p}_{x,i}\ \varphi_i(\underline{x},t_l)\,d\underline{x}\ , \qquad (4)$$

where V_p is the volume of the p-th region, and the neutron fluxes $\varphi_i(\underline{x}, t_l)$ are normalized to give the constant power,

$$\sum_{i=1}^{n} \int_{\text{Reactor}} \Sigma_{f,i}(\underline{x}, t_l)\, \varphi_i(\underline{x}, t_l)\, d\underline{x} = \text{constant}. \tag{5}$$

The burnup characteristics are calculated from the atom densities at each burnup step time. Then it is clear from Eqs. (3), (4) and (5) that one-group constants should be defined so as to conserve the reaction rates in order to calculate the atom densities at each burnup step time accurately. The one-group constants and one-group neutron flux are ordinarily defined as,

$$\langle \sigma_x^m \rangle^p (t_l) = \frac{\sum_{i=1}^{n} \int_{V_p} \sigma_{x,i}^{m,p}(t_l)\, \varphi_i(\underline{x}, t_l)\, d\underline{x}}{\sum_{i=1}^{n} \int_{V_p} \varphi_i(\underline{x}, t_l)\, d\underline{x}} , \tag{6}$$

and

$$\phi(\underline{x}, t_l) = \sum_{i=1}^{n} \varphi_i(\underline{x}, t_l) . \tag{7}$$

The one-group diffusion equation can be related to the multi-group equation using the relations of group constants expressed in Eqs. (6) and (7), except for the diffusion coefficient.

There remains some problems in defining the one-group diffusion coefficients or the one-group transport cross sections which are considered to be equivalent to the diffusion coefficients in the multi-group diffusion equation. The proposed diffinitions are follows,

$$\langle \sigma_{tr}^m \rangle^p (t) = \frac{\sum_{i=1}^{n} \int_{V_p} \sigma_{tr,i}^{m,p}(t)\, \varphi_i(\underline{x}, t)\, d\underline{x}}{\sum_{i=1}^{n} \int_{V_p} \varphi_i(\underline{x}, t)\, d\underline{x}} , \tag{8}$$

$$\langle \sigma_{tr}^m \rangle^p (t) = \left[\frac{\sum_{i=1}^{n} \int_{V_p} \frac{1}{\sigma_{tr,i}^{m,p}(t)}\, \varphi_i(\underline{x}, t)\, d\underline{x}}{\sum_{i=1}^{n} \int_{V_p} \varphi_i(\underline{x}, t)\, d\underline{x}} \right]^{-1} , \tag{9}$$

$$\langle \sigma_{tr}^{m} \rangle^{p}(t) = \frac{\sum_{i=1}^{n} \int_{V_p} \frac{\sigma_{tr,i}^{m,p}(t)}{\Sigma_{T,i}^{p}(t)} \varphi_i(\underline{x},t)\, d\underline{x}}{\sum_{i=1}^{n} \int_{V_p} \frac{1}{\Sigma_{T,i}^{p}(t)} \varphi_i(\underline{x},t)\, d\underline{x}} \quad . \tag{10}$$

Irrespective to the above defined one-group transport constants the one-group diffusion equation is written as follows,

$$D^{p}(t) \nabla^2 \phi(\underline{x},t) - \Sigma_a^{p}(t) \phi(\underline{x},t) + \frac{1}{k(t)} \nu\Sigma_f^{p}(t) \phi(x,t) = 0 , \tag{1}'$$

with boundary conditions,

$$\begin{aligned} \lim_{\underline{\delta}\to 0} D^{p}(t) \nabla\phi(\underline{x}_{pp'} - \underline{\delta}, t) &= \lim_{\underline{\delta}\to 0} D^{p'}(t) \nabla\phi(\underline{x}_{pp'} + \underline{\delta}, t) , \\ \lim_{\underline{\delta}\to 0} \phi(\underline{x}_{pp'} - \underline{\delta}, t) &= \lim_{\underline{\delta}\to 0} \phi(\underline{x}_{pp'} + \delta, t) , \end{aligned} \tag{2}'$$

where p' denotes the region neighboring to the region p and $\underline{x}_{pp'}$ denotes the boundary position of the two regions p and p' .

Using the one-group constants, Z in the depletion equation is expressed to be,

$$Z_{x,p}^{m}(t) = \frac{1}{V_p} \langle \sigma_x^{m} \rangle^{p}(t) \int_{V_p} \phi(\underline{x},t)\, d\underline{x} \quad , \tag{4}'$$

and the power normalization to be,

$$\sum_{m=0}^{4} \sum_{p} N_p^{m}(t) \langle \sigma_f^{m} \rangle^{p}(t) \int_{V_p} \phi(\underline{x},t)\, d\underline{x} = \text{constant.} \tag{5}'$$

Burnup analysis may be performed with one-group diffusion equation if the one-group reactor constants including its time dependence can be determined with the knowledge at the initial state.

3. Variation of one-group constants with burnup

One-group constants are calculated from the multi-group constants of ABBN-set[1] by Eqs. (6) and (8) with the weighting fluxes at each burnup step time, which are calculated by the burnup analysis code TORCH[2] with the group constants of ABBN-set.

$$\langle \sigma_{tr}^{m} \rangle^{p}(t) = \frac{\sum_{i=1}^{n} \int_{V_p} \frac{\sigma_{tr,i}^{m,p}(t)}{\Sigma_{T,i}^{p}(t)} \varphi_i(\underline{x},t)\, d\underline{x}}{\sum_{i=1}^{n} \int_{V_p} \frac{1}{\Sigma_{T,i}^{p}(t)} \varphi_i(\underline{x},t)\, d\underline{x}} \quad . \tag{10}$$

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A spherical reactor with core and blanket is analysed. The thermal power of the reactor is assumed to be 1,500 MWt. The core is divided into five regions and the blanket into three in order to investigate the spacial dependence of the burnup characteristics. The system calculated is shown in Fig. 1 and the initial atom densities of each region are shown in Table 1.

One group constants $\langle\sigma_x^m\rangle^p(t)$ vary with burnup through the changes in the neutron spectrum and the self-shielding effect which are brought about by the changes in the atom densities of the burnup nuclides. The variations of one-group constants $\langle\sigma_x^m\rangle^p$ may be expressed by the following equation with the first order approximation for the two effects,

$$\frac{\delta\langle\sigma\rangle}{\langle\sigma\rangle(t=0)} \approx \frac{\delta\langle\sigma\rangle_{sp}}{\langle\sigma\rangle(t=0)} + \frac{\delta\langle\sigma\rangle_{sh}}{\langle\sigma\rangle(t=0)} , \qquad (11)$$

where $\delta\langle\sigma\rangle_{sp}$ and $\delta\langle\sigma\rangle_{sh}$ represent the variations of one-group constants by the change of neutron spectrum and self-shielding effect respectively.

Time dependence of one-group fission cross section $\langle\sigma_f\rangle$ of ^{239}Pu and capture cross section $\langle\sigma_c\rangle$ of ^{238}U are shown by full lines respectively in Fig. 2 and Fig. 3 for the two regions C-1 and B-1. A rather large decrease in the $\langle\sigma_f\rangle$ of ^{239}Pu in region B-1 is caused by the increase in the self-shielding factor which is brought about by the change in the potential cross section of ^{239}Pu from infinite value to finite at the early stage of burnup. Except for the fission cross section of ^{239}Pu in region B-1 the effect of self-shielding change is negligibly small for the variations of one-group constants in the core and blanket. The spectrum term of the above equation is small in the core but considebly large in the blanket, of which the neutron spectrum largely hardens with burnup by the accumulation of ^{239}Pu.

4. Effect of changes of one-group constants on burnup characteristics

The effects of changes in one-group constants are investigated comparing the results of multi-group burnup analysis with those of one-group analysis which uses one-group constants of the initial state. In order to examine the effect of definition of one-group transport cross

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section $\langle\sigma_{tr}\rangle$, three cases are analysed, these are

"EXACT" 25 - group analysis,
"APROX-1" one- group analysis, $\langle\sigma_{tr}\rangle$ = Eq. (8),
"APROX-2" one- group analysis, $\langle\sigma_{tr}\rangle$ = Eq. (9),
"APROX-3" one- group analysis, $\langle\sigma_{tr}\rangle$ = Eq.(10).

Table 2 shows the variations of atom densities through the burnup. The number densities of ^{239}Pu in the blanket are affected considerably by the definition of $\langle\sigma_{tr}\rangle$, and the results of "APROX-2" and "APROX-3" fairly conincide with the results "EXACT", which are obtained by 25-group diffusion calculations.

Fig. 4 displays the variations of the effective multication factor through the burnup. The curves in the figure show convex shape because the initial breeding ratio of the system is greater than unity. Although the effective multiplication factor can be given by one-group calculations of "APROX-2" or "APROX-3" very close to the result of "EXACT" at the initial state, it is seen from Fig. 4 as expected that the burnup dependence of the effective multiplication factor cannot be represented as the changes of the eigenvalues of one-group diffusion equation using one-group constants defined at the initial state.

The material worth W_p^m of the m-th nuclide in the p-th region may be defined by the first order perturbation theory neglecting the changes in self-shielding effect and considering the fact that "APROX-2" reproduce the atom densities of "EXACT" fairly well,

$$W_p^m = \frac{1}{F\cdot V_p}\int_{V_p} d\underline{x} \sum_{i=1}^{n}\Big[\nabla\varphi_i^*(\underline{x})\big[3(D_i^p)^2\sigma_{tr,i}^{m,p}\big]\nabla\varphi_i(\underline{x}) - \varphi_i^*(\underline{x})\sigma_{a,i}^{m,p}\varphi_i(\underline{x}) + \varphi_i(\underline{x})\sum_{j=i+1}^{n}\sigma_{i\to j}^{m,p}(\varphi_j^*(\underline{x})-\varphi_i^*(\underline{x})) + \frac{1}{k(0)}\varphi_i^*(\underline{x})\chi_i\sum_{j=1}^{n}\nu\sigma_{f,j}^{m,p}\varphi_j(\underline{x})\Big], \quad (12)$$

$$F = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{\text{Reactor}}\varphi_i^*(\underline{x})\chi_i\nu\Sigma_{f,j}(\underline{x})\varphi_j(\underline{x})\,d\underline{x},$$

where $\varphi_i^*(\underline{x})$ is the adjoint function of $\varphi_i(\underline{x})$. W_p^m and F are determined by Eq. (12) sorely from the knowledge at the initial state, and may be called a kind of one-group constants. Using W_p^m the variation of the

effective multiplication factor may be represented as follows,

$$\frac{\Delta k(t)}{k(o)} = \sum_{p} \sum_{m=0}^{5} W_m^p \left\{ N_p^m(t) - N_p^m(o) \right\} \cdot V_p \quad . \tag{13}$$

The calculated results are shown in Fig. 4 using the atom densities $N_p^m(t)$ of "EXACT" calculation (P-25) and those of "APROX-2" (P-1). The variations of the effective multiplication factor are estimated accurately defining the one-group constant W_p^m at the initial state.

Burnup dependent power distributions are shown in Table 3. The power distribution in the core is well approximated by "APROX-1". "APROX-2" and "APROX-3" estimate the power lower than the "EXACT" results but the differences are small. In the blanket the power densities of "APROX-2" and "APROX-3" coincide well with the results of "EXACT", and "APROX-1" gives lower estimations of the blanket power. These tendencies do not change through the burnup.

It is known from the above discussions that the burnup characteristics of atom densities and power densities can be properly determined with the one-group constants of the initial state and that the effective multiplication factor cannot be obtained accurately by the one-group diffusion theory with initial one-group constants, and it is necessary for the purpose to define the material worth by the perturbation theory.

5. Approximate method for representing variation of one-group constants

Here the approximate method is examined treating the time dependence of one-group reactor constants.

The coupling between burnup regions is governed by the boundary condition of Eq. (2) and it may be reasonable to assume that the coupling constant or the buckling,

$$B_{i,p}^2 = \frac{-\int_{V_p} \nabla^2 \varphi_i(\underline{x}, t)\, d\underline{x}}{\int_{V_p} \varphi_i(\underline{x}, t)\, d\underline{x}} \quad , \tag{14}$$

of each region changes only a little with burnup because the spectrum dependence of the diffusion coefficient or the transport cross section is

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of each region changes only a little with burnup because the spectrum dependence of the diffusion coefficient or the transport cross section is

small. With this assumption each burnup region may be treated independently for obtaining weighting functions for one-group constants, solving diffusion equations for bare systems,

$$\left[D_i^p(t)B_{i,p}^2+\Sigma_i^p(t)\right]\phi_i^p(t)-\sum_{j=1}^{i-1}\Sigma_{j\to i}^p(t)\phi_j^p(t)=\chi_i \quad . \tag{15}$$

The above equations can be easily solved from the top group. With these weighting functions one-group constants can be calculated at each burnup step time. $\langle\sigma_{tr}\rangle$ is calculated by Eq. (9). This approximation is named "APROX-4" and calculational results are shown in Fig. 2 ~ 5 and Table 2, 3. Fig. 2 and Fig. 3 show that one-group fission cross section $\langle\sigma_f\rangle$ of ^{239}Pu and capture cross section $\langle\sigma_c\rangle$ of ^{238}U coincide with the calculation "EXACT" within the error of about 1 %. It may be said that the results by "APROX-4" reproduce very well the burnup characteristics of the atom densities, the effective multiplication factor and the power distribution of the calculation "EXACT".

6. Conclusions

The applicability of one-group diffusion theory is examined in this paper for the burnup analysis of a fast reactor. Variations of one-group constants with burnup are examined firstly using weighting fluxes by the multi-group burnup analysis. Then fixing the one-group constants at initial state, one-group burnup analysis is performed in order to estimate the effects of changes in one-group constants for burnup characteristics of a fast reactor. It is shown that the agreements are very good between multi-group calculations and one-group calculations, which use one-group constants of the initial state, on the power distribution, atom densities, breeding gain and burnup but not on the effective multiplication factor. Using the material worth, which is determined by the perturbation theory, the burnup variations of the effective multiplication factor $(\delta k/k)$ are estimated within the error of 0.3 % in the present analysis, and also for other reactor systems they are expected to be fairly well estimated.

The approximate method has been developed in order to determine the time dependent one-group constants using the knowledge at the initial state and the burnup variations of the atom densities. Weighting functions

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The approximate method has been developed in order to determine the time dependent one-group constants using the knowledge at the initial state and the burnup variations of the atom densities. Weighting functions

for one-group constants are calculated independently for each region at each burnup step time assuming that the group dependent buckling for each region changes only a little with burnup and then solving the multi-group diffusion equations for the bare system. The method can accurately estimate the time dependent one-group constants.

The primary objective of the paper is to examine the applicability of the one-group diffusion theory for the burnup analysis. One-group constants vary considerably with burnup but the variations do not affect the burnup characteristics so drastically. And approximate method may be proposed to take the variations of one-group constants into account. Therefore, if one apply this method carefully with the considerations described above, the neutronic calculations in the burnup analysis may be simplified.

Acknowledgement

The authors are very grateful to Dr. M. Nozawa and Dr. M. Hirata for the valuable discussions to the results obtained, and also to Miss C. Aoki for preparing the manuscript.

References

1. T. Suzuki; Fast Reactor One-dimentional Burnup Analysis Code "TORCH", JAERI-memo 2312 (1966).
2. I. I. Bondarenko et al.; Group Constants for Nuclear Reactor Calculations, Consultants Bureau, New York (1964).

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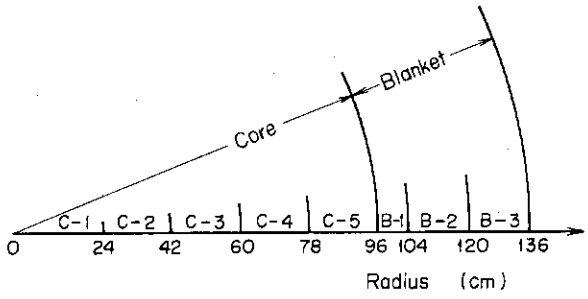

Fig. 1 Schematic diagram of the spherical reactor system calculated.

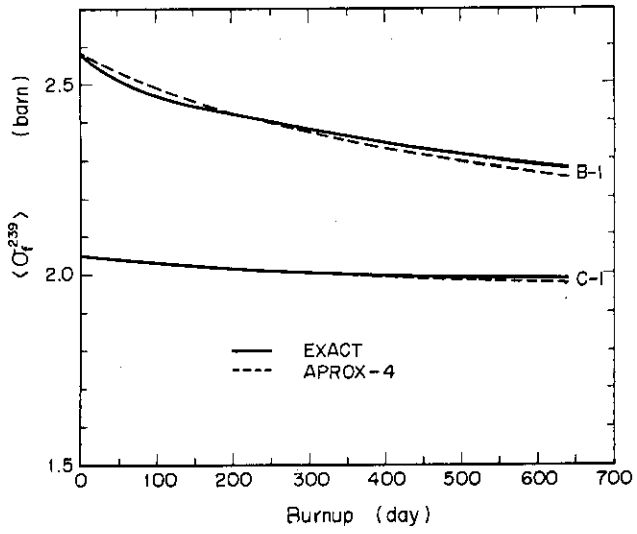

Fig. 2 Burnup variations of one-group fission cross sections of ^{239}Pu.

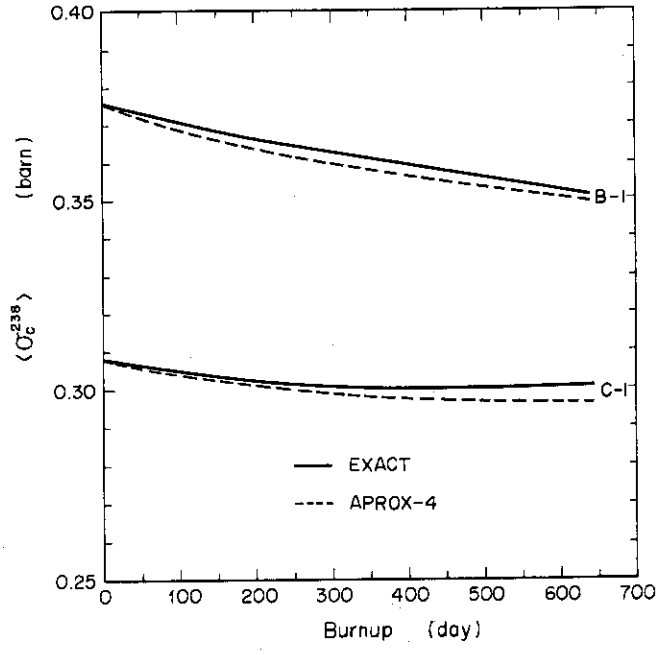

Fig. 3 Burnup variations of one-group capture cross sections of ^{238}U.

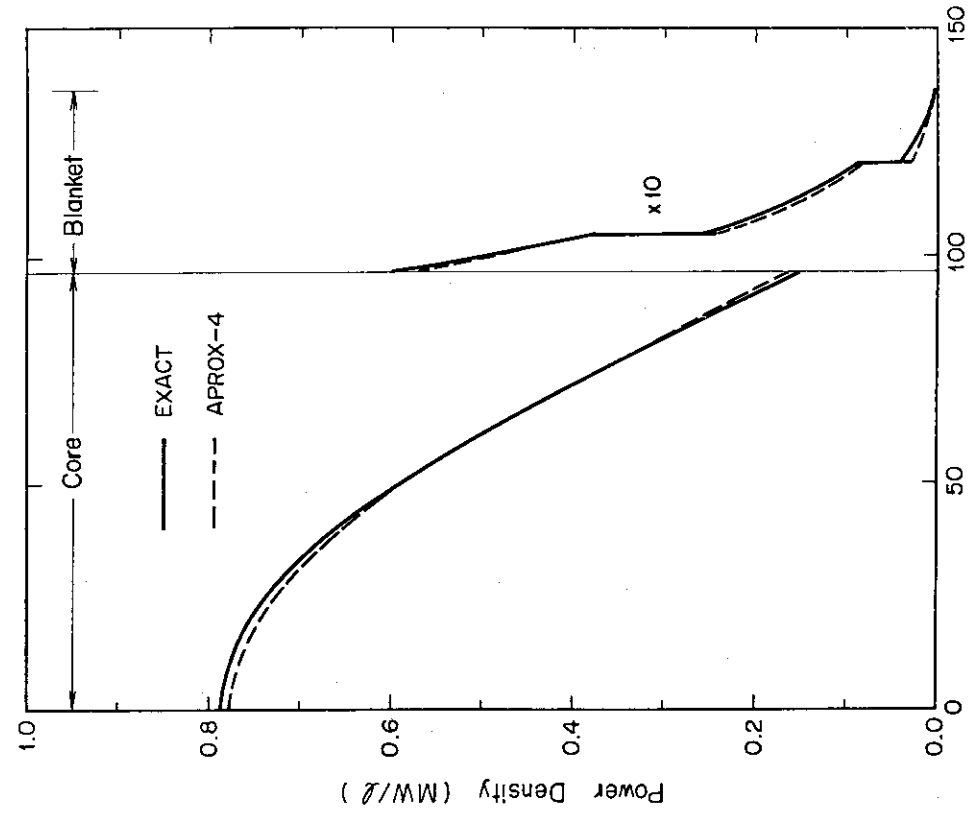

Fig. 5 Comparison of power distributions at 640 days of burnup between 25-group calculation "EXACT" and one-group calculation "APROX-4".

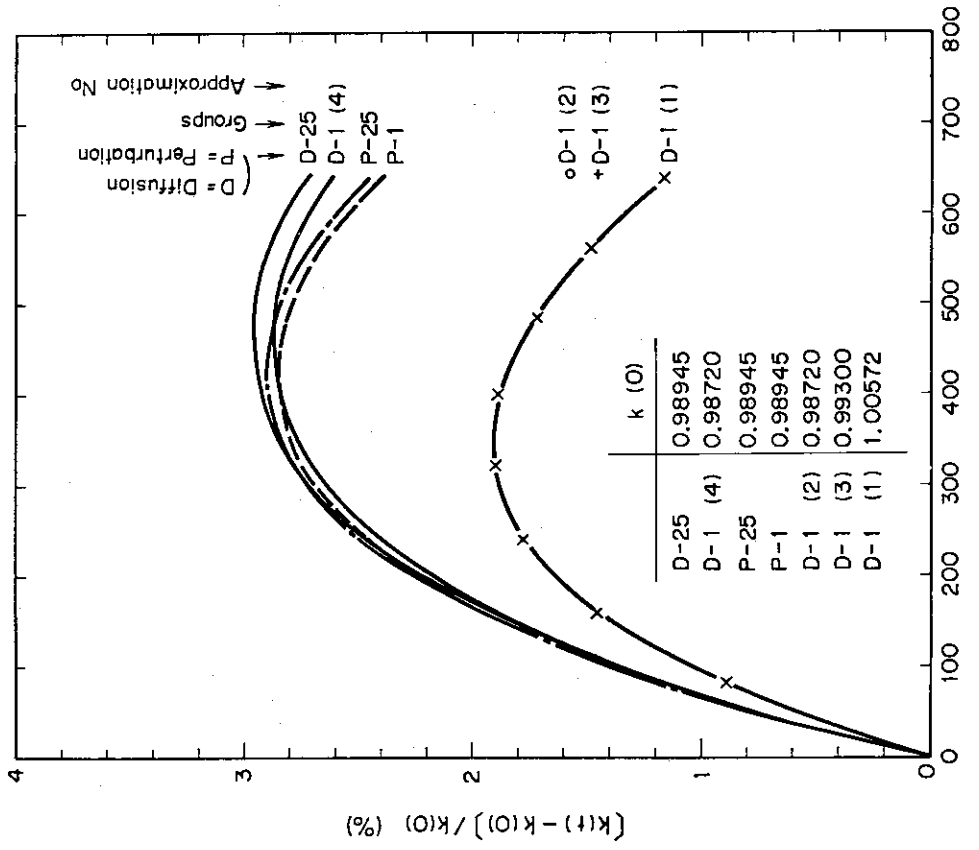

Fig. 4 Comparison of burnup dependences of the effective multiplication factor between different approximations.

Table 1 Atom densities of initial state.

(10^{21}/cm^3)

Nuclide \ Region	Core	Blanket
^{239}Pu	0.9356	0.0
^{240}Pu	0.2088	0.0
^{238}U	8.859	9.972
O	20.01	20.01
Na	8.740	8.740
Cr	2.486	2.486
Fe	9.003	9.003
Ni	1.346	1.346

Table 2 Burnup changes of atom densities in region C-1 (Core) and B-1 (Blanket).

Region ; C-1 (10^{21}/cm^3)

Burn-up	Nuclide	EXACT	APROX-1	APROX-2	APROX-3	APROX-4
320 days	^{239}Pu	0.9760	0.9740	0.9742	0.9741	0.9754
	^{240}Pu	0.2699	0.2724	0.2712	0.2714	0.2685
	^{241}Pu	0.0342	0.0350	0.0345	0.0346	0.0336
	F P	0.7462	0.7439	0.7274	0.7308	0.7265
	^{238}U	7.974	7.975	7.994	7.990	7.997
640 days	^{239}Pu	0.9507	0.9432	0.9453	0.9449	0.9496
	^{240}Pu	0.3078	0.3142	0.3127	0.3130	0.3056
	^{241}Pu	0.0512	0.0537	0.0531	0.0532	0.0506
	F P	1.440	1.440	1.409	1.415	1.409
	^{238}U	7.245	7.243	7.274	7.268	7.281

Region ; B-1

320days	^{239}Pu	0.1467	0.1409	0.1482	0.1466	0.1474
	^{238}U	9.802	9.809	9.799	9.801	9.801
640days	^{239}Pu	0.2696	0.2599	0.2716	0.2691	0.2703
	^{238}U	9.638	9.650	9.633	9.637	9.637

Table 3 Comparison of burnup changes of power densities between 25-group calculation "EXACT" and one-group calculations "APROX-1" ~ "APROX-4".

(unit ; MW/ℓ)

Burnup Days	Region	25 Group	1 Group Diffusion			
		EXACT	APROX-1	APROX-2	APROX-3	APROX-4
0	C-3	0.619	0.618	0.610	0.612	0.610
	C-5	0.248	0.248	0.252	0.251	0.252
	B-1	0.0146	0.0132	0.0139	0.0137	0.0139
	B-2	0.00455	0.00342	0.00371	0.00364	0.00371
320	C-3	0.604	0.607	0.597	0.599	0.597
	C-5	0.242	0.242	0.245	0.245	0.246
	B-1	0.0327	0.0304	0.0330	0.0324	0.0328
	B-2	0.0111	0.00906	0.0104	0.0100	0.0101
640	C-3	0.573	0.577	0.568	0.569	0.569
	C-5	0.246	0.247	0.249	0.249	0.250
	B-1	0.0490	0.0468	0.0504	0.0497	0.0487
	B-2	0.0175	0.0150	0.0172	0.0167	0.0161