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SOME PROPERTIES OF THE HIGHER
ORDER PERTURBATION METHOD
(ONE GROUP DIFFUSION APPROXIMATION)

July 1973

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Some Properties of the Higher Order Perturbation Method
(One Group Diffusion Approximation)

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To know properties of the higher order perturbation method, the analytical expressions for the higher order perturbation terms up to the third order are obtained with one-group diffusion approximation in slab geometry. The higher order perturbation term is generally expressed by the product of three independent terms; i.e. the first order reactivity worth, half the level distance, and the geometrical functions depending on the perturbation region. The n 'th order reactivity worth is proportional to the n 'th power of the first order reactivity worth, that is, proportional to the n 'th power of the concentration of the perturbation inserted. The n 'th order reactivity worth is proportional to the $(n-1)$ 'th power of the reciprocal of half the level distance, so that the magnitude of the higher order reactivity worths relative to the first order reactivity worth becomes larger as the infinite multiplication factor in the system approaches to 1 or the radius of the system becomes larger. If the perturbation is uniformly inserted in the whole system of a bare reactor, the neutron flux does not change and the higher order reactivity worths are zero for the perturbation where only absorption cross section changes. The properties of the higher order perturbed fluxes are also examined in detail. Further, convergence criterion is deduced from the analytical expression of the higher order reactivity worths.

高次摂動法の諸性質（一群拡散近似）

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（1973年6月21日受理）

原子炉の計算において、高次摂動法の性質を具体的に調べるために、一群拡散近似で平板系について、直接高次摂動項を3次まで解析的に求めた。高次反応度は一般に、3個の独立な量、即ち一次反応度、固有値のレベル間隔の逆数及び摂動領域の形状のみに関係する幾何学的関数の積で表わされる。 n 次反応度は一次反応度の n 乗に比例するので、加えた摂動の濃度の n 乗に比例する。一次反応度に対する高次反応度の大きさは、 n 次反応度がレベル間隔の逆数の $n-1$ 乗に比例するので、無限媒質における増倍率が1に近くなると、即ち体系の半径が大きくなると、増大する。裸の体系で、吸収断面積のみが変化する摂動が体系全体に一樣に加えられた時には、高次反応度は全て零になる。高次反応度に対する解析的表現から、摂動級数の収斂条件が推論されているが、数学的に厳密な取扱い及び数値例については別に報告する予定である。

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I. Introduction

In a previous paper⁽¹⁾, the general theory of the higher order perturbation method for reactor calculations was given and characteristics of the higher order perturbation method was also numerically examined. However, the higher order perturbation formula given in the previous paper does not include the explicit form of the higher order perturbed fluxes, so that characteristics of the higher order perturbation cannot be directly known. Further, convergence criterion of the perturbation series and error estimate of the first-order perturbation method were not established. The latter is very important since the first-order perturbation technique is widely used in reactor calculations.

If analytical expressions of the arbitrary n 'th term of the higher order perturbation can be obtained, it may be easy to know the convergence criterion and error estimate. However, this is impossible for multi-group energy treatment and also for one-group diffusion approximation.

In the present paper, analytical expressions of the higher order reactivity worth up to the third order are obtained by one-group diffusion approximation in slab geometry, and characteristics of the higher order reactivity worths and perturbed fluxes are examined in detail. Further, convergence criterion is also deduced from these analytical expressions. However, the present treatment is limited up to the third order perturbation, so that, in order to have strict mathematical treatment, the general theory of the perturbation method for a self-adjoint operator given by T. Kato⁽²⁾⁻⁽⁴⁾ must be used. The present treatment is, however, very useful to grasp physical insight into the higher order perturbation method. The strict mathematical treatment of convergence criterion and error estimate of the perturbation method for a self-adjoint operator and numerical examples will be given in a further coming paper.

II. General Theory of the Higher Order Perturbation Method

The brief description of the general theory of the higher order perturbation method is given for its use in next section. Detailed discussions are given in a reference (1).

We will consider a critical reactor system. This reactor is unperturbed. The neutron flux and its adjoint flux in this system can be calculated from the following equations;

$$M\phi = S\phi \quad (1)$$

$$M^*\phi^* = S^*\phi^* \quad , \quad (2)$$

where S is the operator for neutron production due to a fission process and M is the operator for neutron loss due to transport, absorption and scattering, and M^* , S^* are the adjoint operators for M , S respectively. Here, we put

$$Q = M - S \quad , \quad Q^* = M^* - S^* \quad . \quad (3)$$

Then, Eqs. (1) and (2) can be rewritten as

$$Q\phi = 0 \quad , \quad (4)$$

$$Q^*\phi^* = 0 \quad . \quad (5)$$

In a multigroup approximation, Q and Q^* are the matrices, and ϕ and ϕ^* are the vectors.

If the perturbation is introduced in a critical reactor, this perturbation can then be described as changes in the system parameters. If the changes in the system operators M and S are represented by δM and δS , the system operators after the perturbation can be expressed as

$$M' = M + \delta M \quad , \quad (6)$$

$$S' = S + \delta S \quad , \quad (7)$$

and the total change of the system operator is equal to,

$$\delta Q = M - S . \quad (8)$$

When the perturbation δQ has been inserted, the system is no longer stationary. The steady-state reactor equations for this perturbed system can be obtained by introducing an effective multiplication factor for the system as discussed by Henry and Usachev. By this procedure, the neutron flux and its adjoint flux in the unperturbed system are given as

$$M'\phi' = \frac{1}{K'} S'\phi' \quad (9)$$

$$M'^*\phi'^* = \frac{1}{K'} S'^*\phi'^* \quad (10)$$

where K' is the effective multiplication factor for the perturbed system.

The static reactivity is generally defined by the following relation.

$$\rho = 1 - \frac{1}{K'} \quad (11)$$

By this relation, Eqs. (9) and (10) can be rewritten;

$$Q'\phi' = -\rho S'\phi' \quad (12)$$

$$Q'^*\phi'^* = -\rho S'^*\phi'^* \quad (13)$$

where

$$Q' = Q + \delta Q , \quad Q'^* = Q^* + \delta Q^* . \quad (14)$$

To obtain the higher order perturbed flux and reactivity worth, the iterative method well known in quantum mechanics can be used. In place of Eq. (12), we will consider Eq. (15) in which the values δQ and δS are replaced by $\lambda \delta Q$ and $\lambda \delta S$, respectively:

$$(Q + \lambda \delta Q)\phi' = -\rho(S + \lambda \delta S)\phi' \quad (15)$$

For solving this equation, ϕ' and ρ are expressed in the following power series;

$$\phi' = \phi^{(0)} + \lambda \phi^{(1)} + \lambda^2 \phi^{(2)} + \dots \quad (16)$$

$$\rho = \lambda \rho^{(1)} + \lambda^2 \rho^{(2)} + \lambda^3 \rho^{(3)} + \dots \quad (17)$$

where $\phi^{(n)}$ and $\rho^{(n)}$ now correspond to the n'th-order perturbed neutron flux and n'th-order reactivity.

Substituting Eqs. (16) and (17) into Eq. (15) and equating the coefficients of equal powers of λ on both sides of the resultant equation, a series of equations which express the successively higher orders of perturbation can be obtained:

$$Q\phi^{(0)} = 0 \quad (18)$$

$$Q\phi^{(1)} = q^{(1)} - \rho^{(1)} S\phi^{(0)} \quad (19)$$

$$Q\phi^{(2)} = q^{(2)} - \rho^{(2)} S\phi^{(0)} \quad (20)$$

$$\vdots$$

$$Q\phi^{(n)} = q^{(n)} - \rho^{(n)} S\phi^{(0)}, \quad (21)$$

$$\vdots$$

where

$$q^{(n)} = -\delta Q\phi^{(n-1)} - \sum_{i=1}^{n-1} \rho^{(i)} S\phi^{(n-1)} - \sum_{i=1}^{n-1} \rho^{(i)} \delta S\phi^{(n-i-1)}. \quad (22)$$

The higher order perturbation formula for reactivity can be easily obtained by multiplying both sides of Eq. (21) by the adjoint flux $\phi^{(0)*}$ in the unperturbed system and integrating over all regions.

$$\rho^{(n)} = -\langle 0 | \delta Q | n-1 \rangle - \sum_{i=1}^{n-1} \rho^{(i)} \langle 0 | \delta S | n-i-1 \rangle, \quad (23)$$

where

$$\langle 0 | A | m \rangle = \frac{\langle \phi^{(0)*} A \phi^{(m)} \rangle}{\langle \phi^{(0)*} S \phi^{(0)} \rangle} \quad (24)$$

In this derivation, the following relation is used.

$$\langle \phi^{(0)*} S \phi^{(n)} \rangle = 0 \quad . \quad (25)$$

The first, second and third order perturbation formulas not containing $\rho^{(i)}$ explicitly are as follows:

$$\rho^{(1)} = -\langle 0 | \delta Q | 0 \rangle \quad , \quad (26)$$

$$\begin{aligned} \rho^{(2)} = & -\langle 0 | \delta Q | 1 \rangle \\ & + \langle 0 | \delta Q | 0 \rangle \langle 0 | \delta S | 0 \rangle \quad , \end{aligned} \quad (27)$$

$$\begin{aligned} \rho^{(3)} = & -\langle 0 | \delta Q | 2 \rangle \\ & + \langle 0 | \delta Q | 1 \rangle \langle 0 | \delta S | 0 \rangle \\ & - \langle 0 | \delta Q | 0 \rangle [\langle 0 | \delta S | 0 \rangle^2 - \langle 0 | \delta S | 1 \rangle] \quad . \end{aligned} \quad (28)$$

To calculate the n'th-order reactivity $\rho^{(n)}$, it is necessary to know both $\phi^{(i)}$ and $\rho^{(i)}$ up to the (n-1)'th order. To obtain the n'th-order perturbed flux $\phi^{(n)}$ by solving Eq. (24), it is necessary to know $\phi^{(i)}$ up to the (n-1)'th order and $\rho^{(i)}$ up to the n'th order. The zero'th-order neutron flux is known, so that the higher order perturbed fluxes and reactivity worths can be obtained successively in the following order:

$$\begin{aligned} \phi^{(0)} \rightarrow \rho^{(1)} \rightarrow \phi^{(1)} \rightarrow \rho^{(2)} \quad \dots\dots\dots \\ \phi^{(n-1)} \rightarrow \rho^{(n)} \rightarrow \phi^{(n)} \quad \dots\dots\dots \end{aligned} \quad (29).$$

III. Analytical Expression of the Higher Order Perturbation Values

In order to simplify the mathematical treatment, we will consider a bare system in slab geometry and use one-group diffusion approximation. A reactor model is shown in Fig. 1 where the half-thickness of a bare reactor is a and that of a perturbation region is b .

The neutron flux in an unperturbed system can be calculated from the following equation⁽⁵⁾.

$$-D \frac{d^2 \phi}{dx^2} + (\Sigma_a - \nu \Sigma_f) \phi = 0, \quad (30)$$

where

ϕ = neutron flux

D = diffusion coefficient

Σ_a = absorption cross section

Σ_f = fission cross section

ν = number of neutrons emitted per fission.

From Eq. (1), the unperturbed flux is

$$\phi = \cos \beta x, \quad (31)$$

where

$$\beta^2 = \frac{\nu \Sigma_f - \Sigma_a}{D} = \left(\frac{\pi}{2a} \right)^2. \quad (32)$$

In the perturbation region, it is assumed that Σ_a and $\nu \Sigma_f$ change and D does not change. Then, the first order reactivity worth is given by⁽¹⁾

$$\rho = - \frac{\delta Q \langle \phi^2 \rangle_b}{\nu \Sigma_f \langle \phi^2 \rangle_a}, \quad (33)$$

where

$$\langle \phi^2 \rangle_b = \int_{-b}^b \phi^2 dx = b + \frac{1}{2\beta} \sin 2\beta b, \quad (34)$$

$$\langle \phi^2 \rangle_a = a, \quad (35)$$

$$\delta Q = \delta \Sigma_a - \delta(\nu \Sigma_f) \quad (36)$$

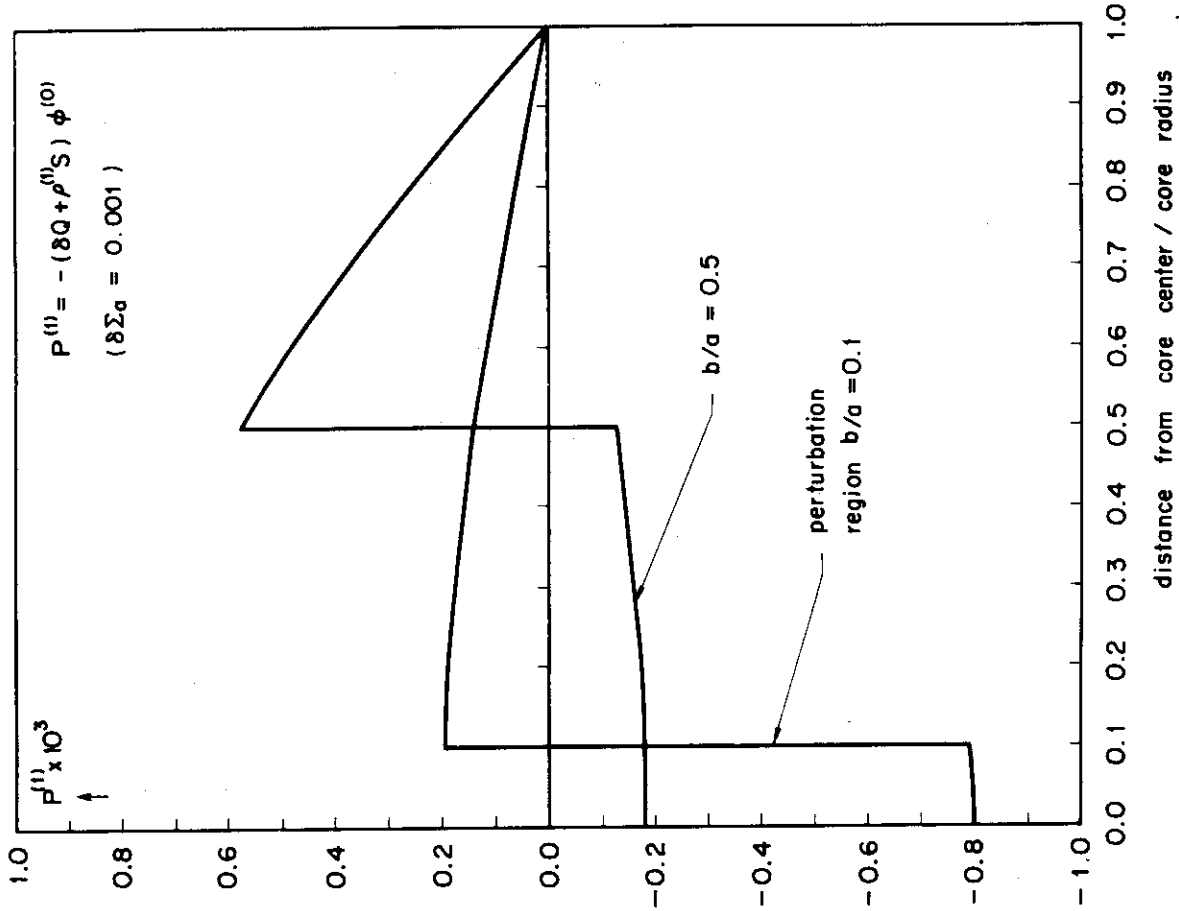


Fig. 2 The spatial distribution of source term of the first order perturbed flux for absorbing material

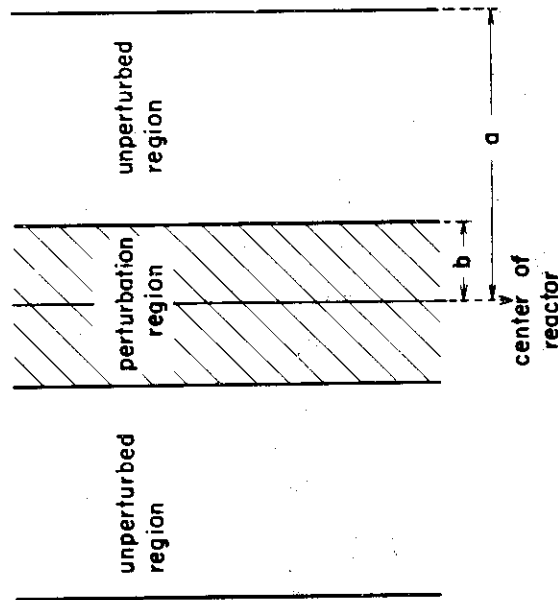


Fig. 1 Reactor model: slab geometry, a half-thickness of reactor is a and that of perturbation region is b .

The first order perturbed flux can be calculated from Eq. (19). This equation becomes for the perturbation region $[-b, 0], [0, b]$

$$-D \frac{d^2 \phi_1^{(1)}}{dx^2} + (\Sigma_a - \nu \Sigma_f) \phi_1^{(1)} = -(\delta Q + \rho^{(1)} \nu \Sigma_f) \phi^{(0)}, \quad (37)$$

and for the unperturbed region $[-a, -b], [b, a]$,

$$-D \frac{d^2 \phi_1^{(1)}}{dx^2} + (\Sigma_a - \nu \Sigma_f) \phi_2^{(1)} = -\rho^{(1)} \nu \Sigma_f \phi^{(0)}. \quad (38)$$

The source terms on the right-hand side of Eqs. (37) and (38) are shown in Fig. 2 in which an absorption cross section is only changed. The source term has a sharply cut-off distribution in the perturbation region and a smoothly varying distribution proportional to the number of neutrons emitted per fission in the unperturbed region.

Using Eqs. (32) and (33), Eqs. (37) and (38) can be rewritten as

$$\frac{d^2 \phi_1^{(1)}}{dx^2} + \beta^2 \phi_1^{(1)} = A_1^{(1)} \cos \beta x \quad (39)$$

and

$$\frac{d^2 \phi_1^{(1)}}{dx^2} + \beta^2 \phi_2^{(1)} = A_2^{(1)} \cos \beta x, \quad (40)$$

where

$$A_1^{(1)} = -\rho^{(1)} \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_b} \frac{\nu \Sigma_f}{D}, \quad (41)$$

$$A_2^{(1)} = \rho^{(1)} \frac{\nu \Sigma_f}{D}. \quad (42)$$

Eqs. (11) and (12) are equal to equations describing the resonance phenomena is forced oscillation⁽⁶⁾. These equations can be solved by the eigen-function expanded or neutron life-cycle method. However, in the present case, it is most simple to solve directly these equations. Solutions of Eqs. (39) and (40) can be easily obtained⁽⁶⁾.

$$\phi_1^{(1)} = C_1 \cos \beta x + C_2 \sin \beta x + \frac{A_1^{(1)}}{2\beta^2} \beta x \sin \beta x, \quad (43)$$

$$\phi_2^{(1)} = C_3 \cos \beta x + C_4 \sin \beta x + \frac{A_2^{(1)}}{2\beta^2} \beta x \sin \beta x . \quad (44)$$

The coefficients C_2 and C_4 are determined from the boundary conditions:

$$\left. \frac{d\phi_1^{(1)}}{dx} \right|_{x=0} = 0 , \quad \phi_2^{(1)}(a) = 0 . \quad (45)$$

Then, $\phi_1^{(1)}$ and $\phi_2^{(1)}$ become

$$\phi_1^{(1)} = C_1 \cos \beta x + \frac{A_1^{(1)}}{2\beta^2} \beta x \sin \beta x , \quad (46)$$

$$\phi_2^{(1)} = C_3 \cos \beta x - \frac{A_2^{(1)}}{2\beta^2} \beta (a-x) \sin \beta x . \quad (47)$$

Two boundary conditions are used to calculate C_1 and C_3 .

$$\phi_1^{(1)}(b) = \phi_2^{(1)}(b) , \quad \left. \frac{d\phi_1^{(1)}}{dx} \right|_{x=b} = \left. \frac{d\phi_2^{(1)}}{dx} \right|_{x=b} . \quad (48)$$

From the boundary conditions, the next relation is only obtained:

$$C_1 = C_3 + \frac{1}{2\beta^2} (A_1 - A_2) \sin^2 \beta b , \quad (49)$$

and each value of C_1 and C_3 cannot be determined. The values C_1 and C_3 are coefficients of $\cos \beta x$. This means that an operation Q in Eq. (19) is equal to that of a critical system, so that the component proportional to $\cos \beta x$ formed by the self-sustaining in chain reaction always remains to be underdetermined. This is undesirable for the present purpose. The higher order perturbed fluxes satisfy the relation (25). Therefore, this relation can be used to obtain coefficients C_1 and C_3 . Then, we have

$$\begin{aligned} C_1 = & -C_3 \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_b} - \frac{A_1^{(1)}}{2\beta^2} \frac{\langle \beta x \sin \beta x \cos \beta x \rangle_b}{\langle \phi^2 \rangle_b} \\ & + \frac{A_2^{(1)}}{2\beta^2} \frac{\langle \beta(a-x) \sin \beta x \cos \beta x \rangle_{a-b}}{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b} \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_b} , \quad (50) \end{aligned}$$

where

$$\langle f(x) \rangle_b = 2 \int_0^b f(x) dx, \quad \langle f(x) \rangle_{a-b} = 2 \int_b^a f(x) dx. \quad (51)$$

From Eqs. (49) and (50), each value of C_1 and C_3 can be determined.

Finally, the first order perturbed fluxes in the perturbation and the unperturbed regions are as follows.

$$\phi_1^{(1)} = C_{11}^{(1)} \cos \beta x + C_{12}^{(1)} \beta x \sin \beta x \quad (52)$$

$$\phi_2^{(1)} = C_{21}^{(1)} \cos \beta x + C_{22}^{(1)} \beta (a-x) \sin \beta x, \quad (53)$$

where

$$C_{11}^{(1)} = -\frac{1}{4} \frac{k_\infty}{k_\infty - 1} \frac{\delta Q}{\nu \Sigma_f} f_1^{(1)}(b/a) \quad (54)$$

$$C_{12}^{(1)} = \frac{1}{4} \frac{k_\infty}{k_\infty - 1} \frac{\delta Q}{\nu \Sigma_f} 2 \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \quad (55)$$

$$C_{21}^{(1)} = -\frac{1}{4} \frac{k_\infty}{k_\infty - 1} \frac{\delta Q}{\nu \Sigma_f} f_2^{(1)}(b/a) \quad (56)$$

$$C_{22}^{(1)} = \frac{1}{4} \frac{k_\infty}{k_\infty - 1} \frac{\delta Q}{\nu \Sigma_f} 2 \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a}, \quad (57)$$

in which

$$f_1^{(1)}(b/a) = 2 \left[\frac{a-b}{a} \cos^2 \beta b - \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \right] \quad (58, a)$$

$$f_2^{(1)}(b/a) = 2 \left[\frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} - \frac{b}{a} \sec^2 \beta b \right]. \quad (58, b)$$

$$k_\infty = \nu \Sigma_f / \Sigma_a. \quad (59)$$

The values $f_1^{(1)}$, $f_2^{(1)}$, $2(\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b) / \langle \phi^2 \rangle_a$ and $2 \langle \phi^2 \rangle_b / \langle \phi^2 \rangle_a$ dependent only on the value of b/a are shown in Fig. 3. From Eqs. (52)

through (58), the first order perturbed flux has the following properties.

(1) The first order perturbed flux is proportional to the concentration of an inserted perturbation.

(2) The value of $f_1^{(1)}(b/a)$ is always positive for all values of b/a . Therefore, if Σ_a increases in the perturbation region, the first order perturbed flux is, then, negative in the vicinity of the center of the perturbation region. If $\nu\Sigma_f$ increases, the first order perturbed flux is positive in the same region.

(3) The values of $f_1^{(1)}(b/a)$ and $2(\langle\phi^2\rangle_a - \langle\phi^2\rangle_b)/\langle\phi^2\rangle_a$ become zero as the value of b/a approaches 1. Therefore, when a perturbation is uniformly inserted in the whole system, the first order perturbed flux becomes zero. In this case, the exact reactivity worth can be easily obtained.

(4) From the factor $k_\infty/4(k_\infty - 1)$., the first order perturbed flux becomes large as the radius of a reactor becomes large.

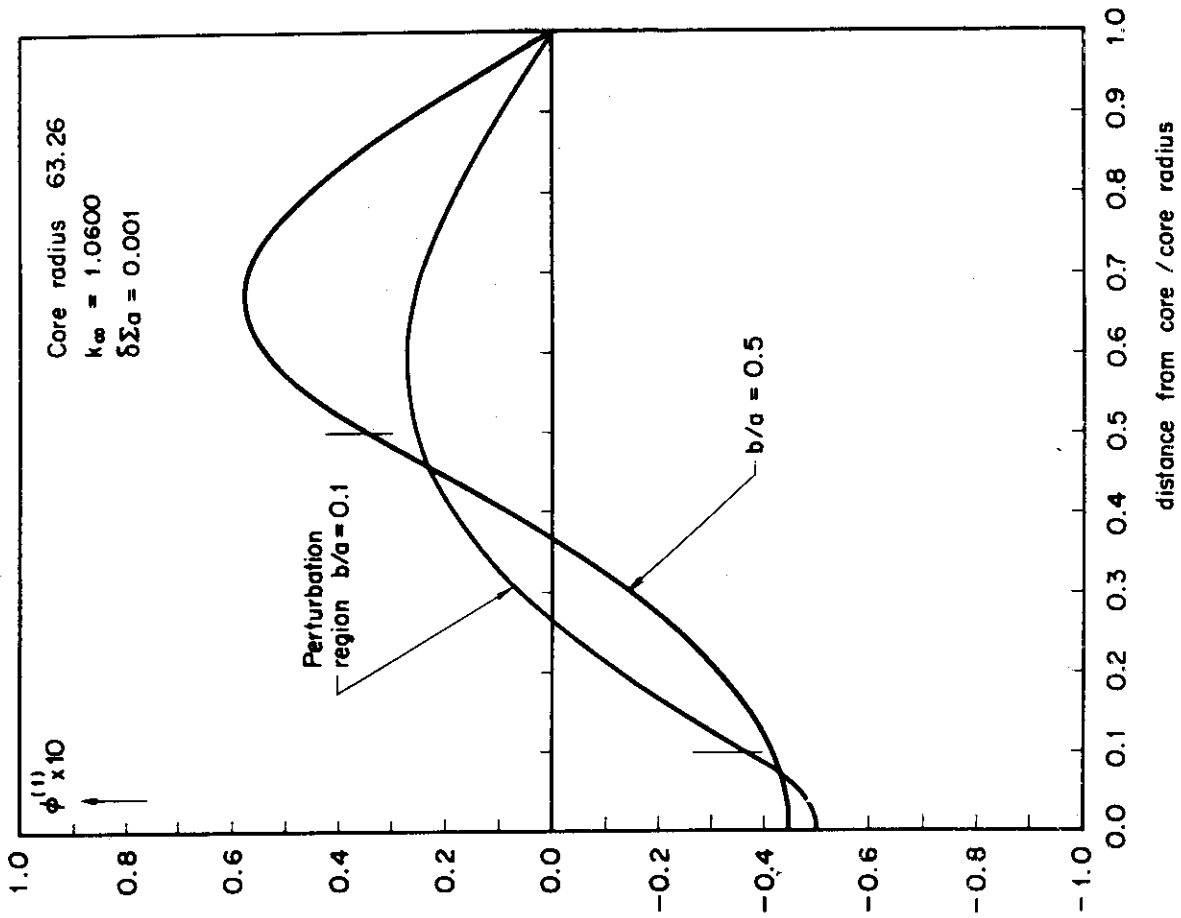


Fig. 4 The spatial distribution of the first order perturbed flux for absorbing material

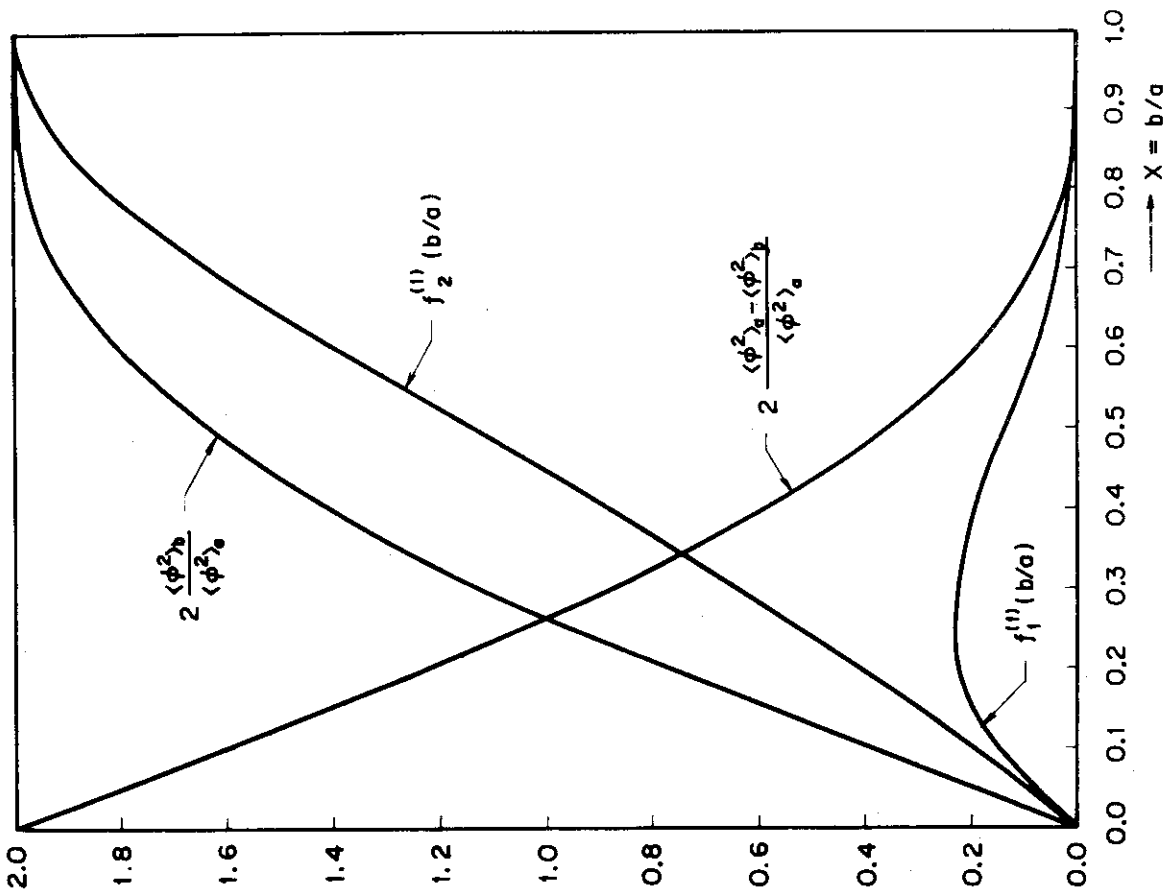


Fig. 3 Geometrical functions of the coefficients of the first order perturbed flux

As an example, the spatial distribution of first order perturbed fluxes for an absorbing material is shown in Fig. 4. The second-order reactivity worth is given by Eq. (27). Substituting Eq. (52) into Eq. (27),

$$\begin{aligned} \rho^{(2)} &= \frac{1}{4} \frac{k_{\infty}}{k_{\infty} - 1} (\rho^{(1)})^2 F^{(2)}(b/a) \\ &\quad - \rho^{(1)} \frac{\delta(\nu \Sigma_f) \langle \phi^2 \rangle_b}{\nu \Sigma_f \langle \phi^2 \rangle_a}, \end{aligned} \quad (60)$$

where

$$\begin{aligned} F^{(2)}(b/a) &= 2 \frac{\langle \phi^2 \rangle_a}{\langle \phi^2 \rangle_b} \left[\frac{a-b}{a} \cos^2 \beta b \right. \\ &\quad \left. - \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \left(\frac{3}{2} \frac{b}{\langle \phi^2 \rangle_b} \cos^2 \beta b \right) \right]. \end{aligned} \quad (61)$$

The values of $F^{(2)}(b/a)$ and also the function

$$H^{(2)}(b/a, k) = F^{(2)}(b/a) - 4 \frac{k_{\infty} - 1}{k_{\infty}} \quad (62)$$

are shown in Fig. 5. The latter function is useful for examining the property of $\rho^{(2)}$ when only $\nu \Sigma_f$ changes. Characteristics of the second order reactivity worth depend on those of the first-order perturbed flux.

Properties of the second order reactivity worth are mainly the following:

(1) The second-order reactivity worth is proportional to the second power of the first order reactivity worth, that is, the second power of the concentration of an inserted perturbation. Further, from the factor $k_{\infty}/4(k_{\infty}-1)$, the value of the second order reactivity worth becomes large as the volume of reactor becomes large.

(2) The value of $F^{(2)}(b/a)$ is positive for all values of b/a . Therefore, when Σ_a is only changed, the second order reactivity worth is always positive.

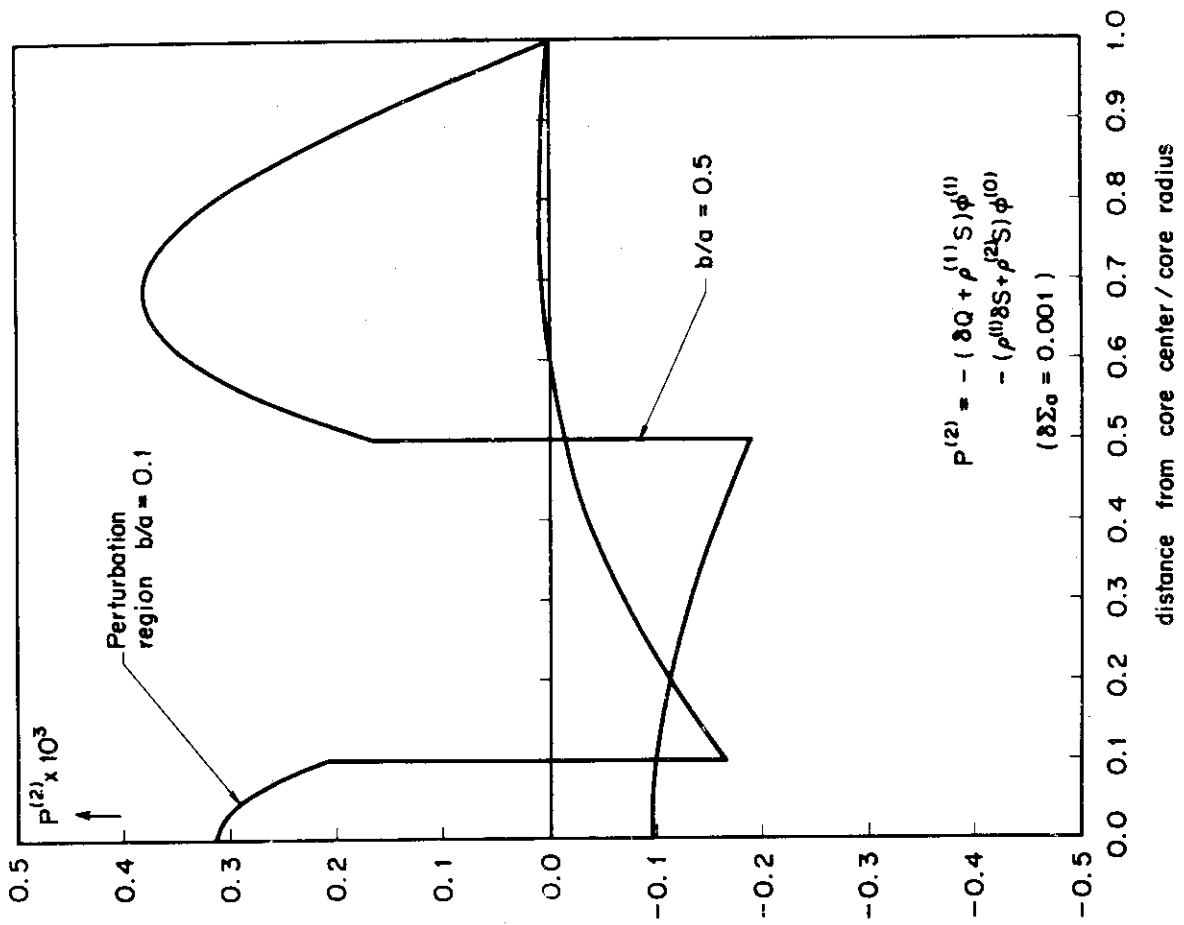


Fig. 6 The spatial distribution of source term of the second order perturbed flux for absorbing material

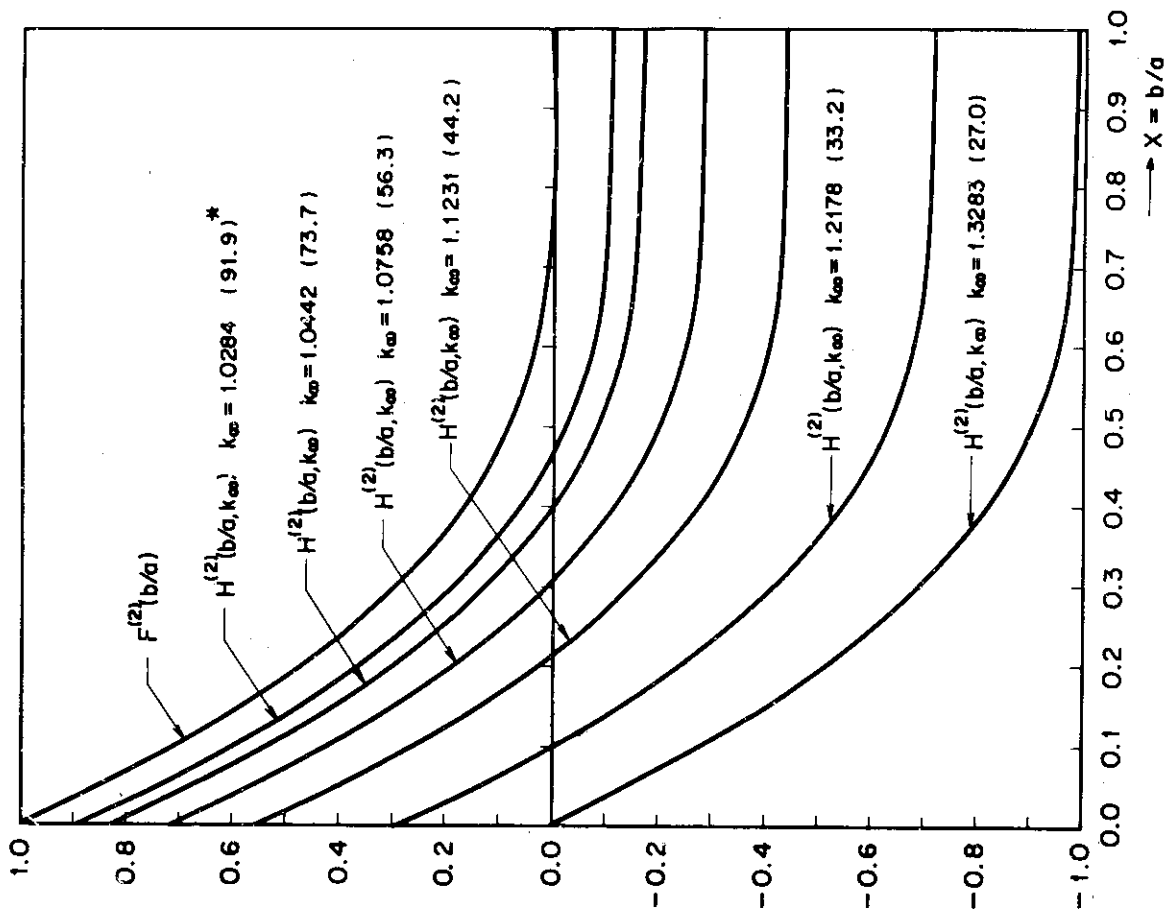


Fig. 5 The geometrical function $F^{(2)}(b/a)$ of the second order perturbation value and $H^{(2)}(b/a, k_{\infty})$ function for various values of k_{∞} . (* the value in a parenthesis is a half-thickness of a bare slab system)

(3) The value of $H^{(2)}(b/a, k_\infty)$ is positive for small values of b/a , and negative for the large values. Therefore, when $\nu \Sigma_f$ is only changed, the second order reactivity worth is positive for small values of b/a and negative for the large values. This property depends on the value of k_∞ , and if the value of k_∞ is larger than $4/3$, the second order reactivity worth is always negative when only $\nu \Sigma_f$ is changed.

We will then calculate the second order perturbed flux. This flux can be calculated from Eq. (20). This equation becomes for the perturbed region

$$-D \frac{d^2 \phi_1^{(2)}}{dx^2} + (\Sigma_a - \nu \Sigma_f) \phi_1^{(2)} = -(\delta Q + \rho^{(1)} \nu \Sigma_f) \phi_1^{(1)} - (\rho^{(1)} \delta(\nu \Sigma_f) + \rho^{(2)} \nu \Sigma_f) \phi_1^{(0)}, \quad (63)$$

and for the unperturbed region

$$-D \frac{d^2 \phi_2^{(2)}}{dx^2} + (\Sigma_a - \nu \Sigma_f) \phi_2^{(2)} = -\rho^{(1)} \nu \Sigma_f \phi_2^{(1)} - \rho^{(2)} \nu \Sigma_f \phi_2^{(0)}. \quad (64)$$

The spatial distribution of the source term on the right-hand side of Eqs. (63) and (64) is shown in Fig. 6 where the absorption cross section is only changed. Equations (63) and (64) can be rewritten as

$$\frac{d^2 \phi_1^{(2)}}{dx^2} + \beta^2 \phi_1^{(2)} = A_1^{(2)} \cos \beta x + B_1^{(2)} \beta x \sin \beta x, \quad (65)$$

$$\frac{d^2 \phi_2^{(2)}}{dx^2} + \beta^2 \phi_2^{(2)} = A_2^{(2)} \cos \beta x + B_2^{(2)} \beta (a-x) \sin \beta x, \quad (66)$$

where

$$A_1^{(2)} = \left[\frac{1}{4} \frac{k_\infty}{k_\infty - 1} (\rho^{(1)})^2 [F^{(2)}(b/a) - \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_b} f_1^{(1)}(b/a)] + \rho^{(1)} \frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \right] \frac{\nu \Sigma_f}{D}, \quad (67)$$

$$B_1^{(2)} = \frac{1}{4} \frac{k_\infty}{k_\infty - 1} (\rho^{(1)})^2 \cdot \left(\frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_b} \right)^2 \frac{\nu \Sigma_f}{D} , \quad (68)$$

$$A_2^{(2)} = \frac{1}{4} \frac{k_\infty}{k_\infty - 1} (\rho^{(1)})^2 (F^{(2)}(b/a) + f_2^{(1)}(b/a)) \\ - \rho^{(1)} \frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \frac{\nu \Sigma_f}{D} , \quad (69)$$

$$B_2^{(2)} = -\frac{1}{4} \frac{k_\infty}{k_\infty - 1} (\rho^{(1)})^2 \frac{\nu \Sigma_f}{D} . \quad (70)$$

Equations (65) and (66) can be easily solved by the same procedure as for the first order perturbed flux. The solutions satisfying the boundary conditions Eq. (45) are as follows.

$$\phi_1^{(2)} = C_1 \cos \beta x \\ - \frac{B_1^{(2)}}{4\beta^2} \beta^2 x^2 \cos \beta x + \frac{D_1^{(2)}}{4\beta^2} \beta x \sin \beta x , \quad (71)$$

$$\phi_2^{(2)} = C_3 \cos \beta x \\ - \frac{B_2^{(2)}}{4\beta^2} \beta^2 (a-x)^2 \cos \beta x - \frac{D_2^{(2)}}{4\beta^2} \beta (a-x) \sin \beta x , \quad (72)$$

where

$$D_1^{(2)} = 2A_1^{(2)} + B_1^{(2)} , \quad D_2^{(2)} = 2A_2^{(2)} - B_2^{(2)} . \quad (73)$$

From the boundary conditions Eq. (48), the following relation is obtained:

$$C_1 = C_3 + \frac{B_1^{(2)}}{4\beta^2} \beta^2 b^2 - \beta b \sin 2\beta b \\ + \frac{B_2^{(2)}}{4\beta^2} \beta^2 (a-b)^2 + \beta (a-b) \sin 2\beta b \\ + \frac{D_1^{(2)}}{4\beta^2} \sin^2 \beta b - \frac{D_2^{(2)}}{4\beta^2} \sin^2 \beta b . \quad (74)$$

Further, from Eq. (25),

$$C_1 = \frac{B_1^{(2)}}{4\beta^2} K_1 - \frac{D_1^{(2)}}{4\beta^2} K_2 - \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_b} \left[C_3 + \frac{B_2^{(2)}}{4\beta^2} K_3 - \frac{D_2^{(2)}}{4\beta^2} K_4 \right], \quad (75)$$

where

$$K_1 = \frac{\langle \beta^2 x^2 \cos^2 \beta x \rangle_b}{\langle \phi^2 \rangle_b}, \quad K_2 = \frac{\langle \beta x \sin \beta x \cos \beta x \rangle_b}{\langle \phi^2 \rangle_b}, \quad (76)$$

and

$$K_3 = \frac{\langle \beta^2 (a-x)^2 \cos^2 \beta x \rangle_{a-b}}{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}, \quad K_4 = \frac{\langle \beta (a-x) \sin \beta x \cos \beta x \rangle_{a-b}}{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}. \quad (77)$$

Using Eqs. (74) and (75), C_1 and C_3 can be determined.

Finally, the second order perturbed fluxes in both the region are

$$\begin{aligned} \phi_1^{(2)} = & C_{11}^{(2)} \cos \beta x + C_{12}^{(2)} \beta^2 x^2 \cos \beta x \\ & + C_{13}^{(2)} \beta x \sin \beta x, \end{aligned} \quad (78)$$

$$\begin{aligned} \phi_2^{(2)} = & C_{21}^{(1)} \cos \beta x + C_{22}^{(2)} \beta^2 x^2 \cos \beta x \\ & + C_{23}^{(2)} \beta (a-x) \sin \beta x, \end{aligned} \quad (79)$$

where

$$\begin{aligned} C_{11}^{(2)} = & \frac{1}{16} \left(\frac{k_\infty}{k_\infty - 1} \right)^2 \left(\frac{\delta Q}{\nu \Sigma_f} \right) f_1^{(2)}(b/a) \\ & + \frac{1}{4} \frac{k_\infty}{k_\infty - 1} \frac{\delta Q}{\nu \Sigma_f} \frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} f_2^{(2)}(b/a), \end{aligned} \quad (80)$$

$$C_{12}^{(2)} = -\frac{1}{16} \left(\frac{k_\infty}{k_\infty - 1} \right)^2 \left(\frac{\delta Q}{\nu \Sigma_f} \right)^2 2 \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a}, \quad (81)$$

$$c_{13}^{(2)} = \frac{1}{16} \left(\frac{k_{\infty}}{k_{\infty} - 1} \right)^2 \left(\frac{\delta Q}{\nu \Sigma_f} \right)^2 f^{(2)}(b/a) \\ - \frac{1}{4} \frac{k_{\infty}}{k_{\infty} - 1} \frac{\delta Q}{\nu \Sigma_f} \frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} \cdot 2 \cdot \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a}, \quad (82)$$

and

$$c_{21}^{(2)} = \frac{1}{16} \left(\frac{k_{\infty}}{k_{\infty} - 1} \right)^2 \left(\frac{\delta Q}{\nu \Sigma_f} \right)^2 f_4^{(2)}(b/a) \\ + \frac{1}{4} \frac{k_{\infty}}{k_{\infty} - 1} \frac{\delta Q}{\nu \Sigma_f} \frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} f_5^{(2)}(b/a), \quad (83)$$

$$c_{22}^{(2)} = - \frac{1}{16} \left(\frac{k_{\infty}}{k_{\infty} - 1} \right)^2 \left(\frac{\delta Q}{\nu \Sigma_f} \right)^2 2 \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a}, \quad (84)$$

$$c_{23}^{(2)} = - \frac{1}{16} \left(\frac{k_{\infty}}{k_{\infty} - 1} \right)^2 \left(\frac{\delta Q}{\nu \Sigma_f} \right)^2 f_6^{(2)}(b/a) \\ - \frac{1}{4} \frac{k_{\infty}}{k_{\infty} - 1} \frac{\delta Q}{\nu \Sigma_f} \frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} 2 \frac{\langle \phi^2 \rangle_b^2}{\langle \phi^2 \rangle_a^2}. \quad (85)$$

The functions $f_i^{(2)}(b/a)$ ($i=1, \dots, 6$) in Eqs. (80) through (85) are dependent only on the value of b/a , but very complicated.

$$f_1^{(2)}(b/a) = \left[2 \left(\frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \right)^2 g_1 - \frac{\langle \phi^2 \rangle_b^2}{\langle \phi^2 \rangle_a^2} g_2 \right. \\ \left. + \frac{1}{2} f_3^{(2)}(b/a) g_3 + \frac{1}{2} f_5^{(2)}(b/a) g_4 \right], \quad (86)$$

$$f_2^{(2)}(b/a) = 2 \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \left[\frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} g_4 - \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} g_3 \right], \quad (87)$$

$$f_3^{(2)}(b/a) = 2 \frac{\langle \phi^2 \rangle_b^2}{\langle \phi^2 \rangle_a^2} F^{(2)}(b/a) - \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} f_1^{(1)}(b/a) \\ + \left(\frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_b} \right)^2, \quad (88)$$

and

$$f_4^{(2)}(b/a) = 2 \left[\left(\frac{\phi_a^2 - \phi_b^2}{\phi_a^2} \right)^2 (g_1 - h_1) - \frac{\phi_b^2}{\phi_a^2} (g_2 - h_2) \right. \\ \left. + \frac{1}{2} f_3^{(2)}(b/a) (g_3 - h_3) + \frac{1}{2} f_6^{(2)}(b/a) (g_4 + h_4) \right], \quad (89)$$

$$f_5^{(2)}(b/a) = 2 \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \left[\frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} (g_4 + h_4) - \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} (g_3 - h_3) \right], \quad (90)$$

$$f_6^{(2)}(b/a) = 2 \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} F^{(2)}(b/a) + f_2^{(1)}(b/a) + 1, \quad (91)$$

where

$$g_1 = \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} K_1 + \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} h_1 \quad (92)$$

$$g_2 = \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} (h_3 - K_3) \quad (93)$$

$$g_3 = \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} h_3 - \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} K_2 \quad (94)$$

$$g_4 = \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} (K_4 - h_4), \quad (95)$$

in which

$$h_1 = \beta^2 b^2 - \beta b \sin 2\beta b, \quad h_2 = \beta^2 (a-b)^2 + \beta (a-b) \sin 2\beta b \\ h_3 = h_4 = \sin^2 \beta b \quad (96)$$

The coefficients of the second order perturbed flux are very complex. Properties of the second order perturbed flux in the perturbation region can be known only by examining the property of the coefficient $c_{11}^{(2)}$. The functions $f_1^{(2)}(b/a)$ and

$$f_F^{(2)}(b/a, k_\infty) = f_1^{(2)}(b/a) - 4 \frac{k_\infty - 1}{k_\infty} f_2^{(2)}(b/a) \quad (97)$$

are shown in Fig. 7.

Properties of the second order perturbed flux are mainly as follows:

(1) The second order perturbed flux is proportional to the second power of the concentration of an inserted perturbation. From the factor $(k_\infty/(k_\infty - 1))^2$, the magnitude of the second order perturbed flux relative to the unperturbed flux becomes large as the radius of the system becomes large.

(2) When Σ_a is only changed, the second order perturbed flux in the vicinity of the center of the perturbation region is positive if $X = b/a$ is smaller than 2.43, and negative if X is larger. When $\nu\Sigma_f$ is only changed, this property further depends on the value of k , and as the value of the becomes larger, the range of X , in which the second order perturbed flux is positive in the vicinity of the center of the perturbation region becomes smaller.

As an example, the spatial distribution of second-order perturbed fluxes for an absorbing material is shown in Fig. 8.

The third order reactivity worth can be calculated from Eq. (28). Inserting Eqs. (52) and (78) into Eq. (28),

$$\begin{aligned} \rho^{(3)} = & \frac{1}{16} \left(\frac{k_\infty}{k_\infty - 1} \right)^2 (\rho^{(1)})^3 F^{(3)}(b/a) \\ & - \frac{1}{4} \frac{k_\infty}{k_\infty - 1} (\rho^{(1)})^2 \frac{\delta(\nu\Sigma_f)}{\nu\Sigma_f} \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} H^{(3)}(b/a) \\ & + \rho^{(1)} \left(\frac{\delta(\nu\Sigma_f)}{\nu\Sigma_f} \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \right), \end{aligned} \quad (98)$$

where

$$\begin{aligned} F^{(3)}(b/a) = & \frac{\langle \phi^2 \rangle_a}{\langle \phi^2 \rangle_b} f_1^{(2)}(b/a) - 2 \left(\frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_b} \right)^2 K_1 \\ & + \left(\frac{\langle \phi^2 \rangle_a}{\langle \phi^2 \rangle_b} \right)^2 K_3 f_3^{(2)}(b/a), \end{aligned} \quad (99)$$

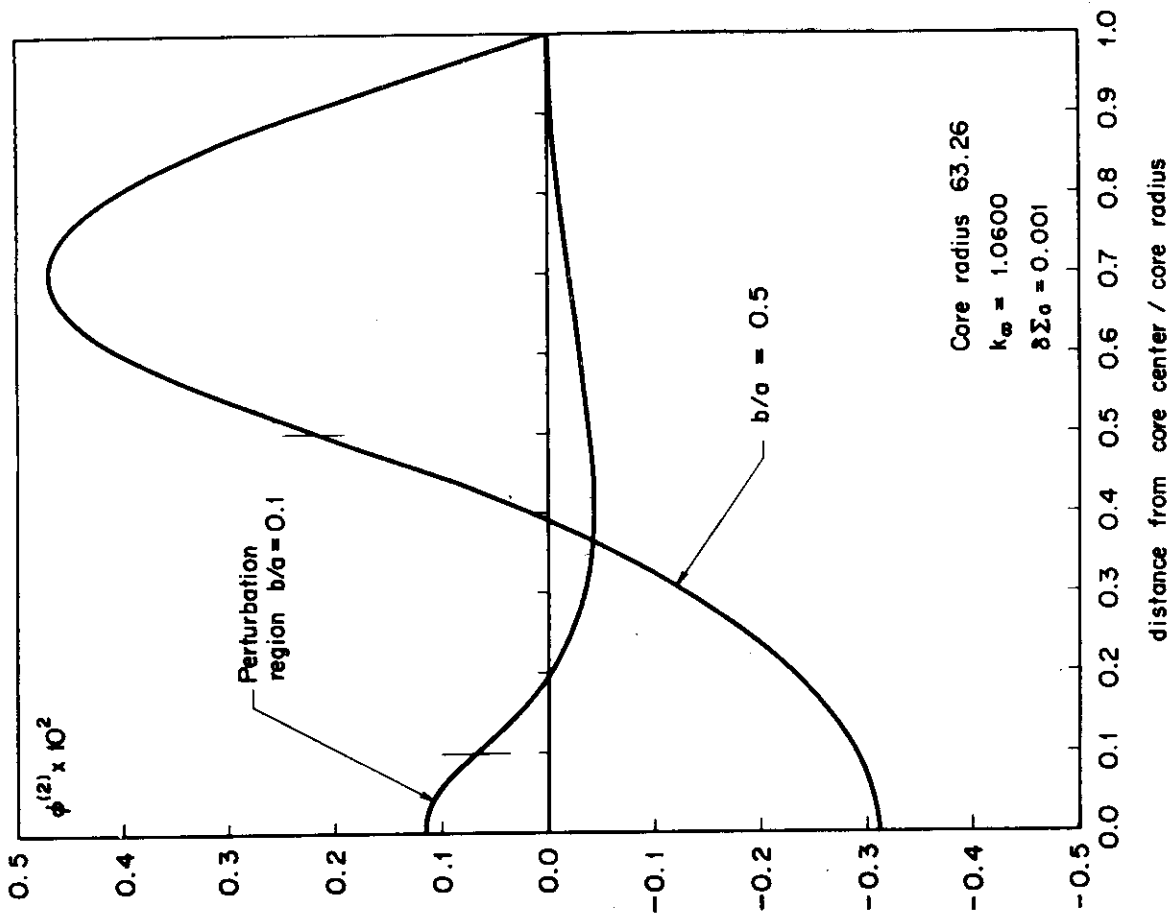


Fig. 8 The spatial distribution of the second order perturbed flux for absorbing material

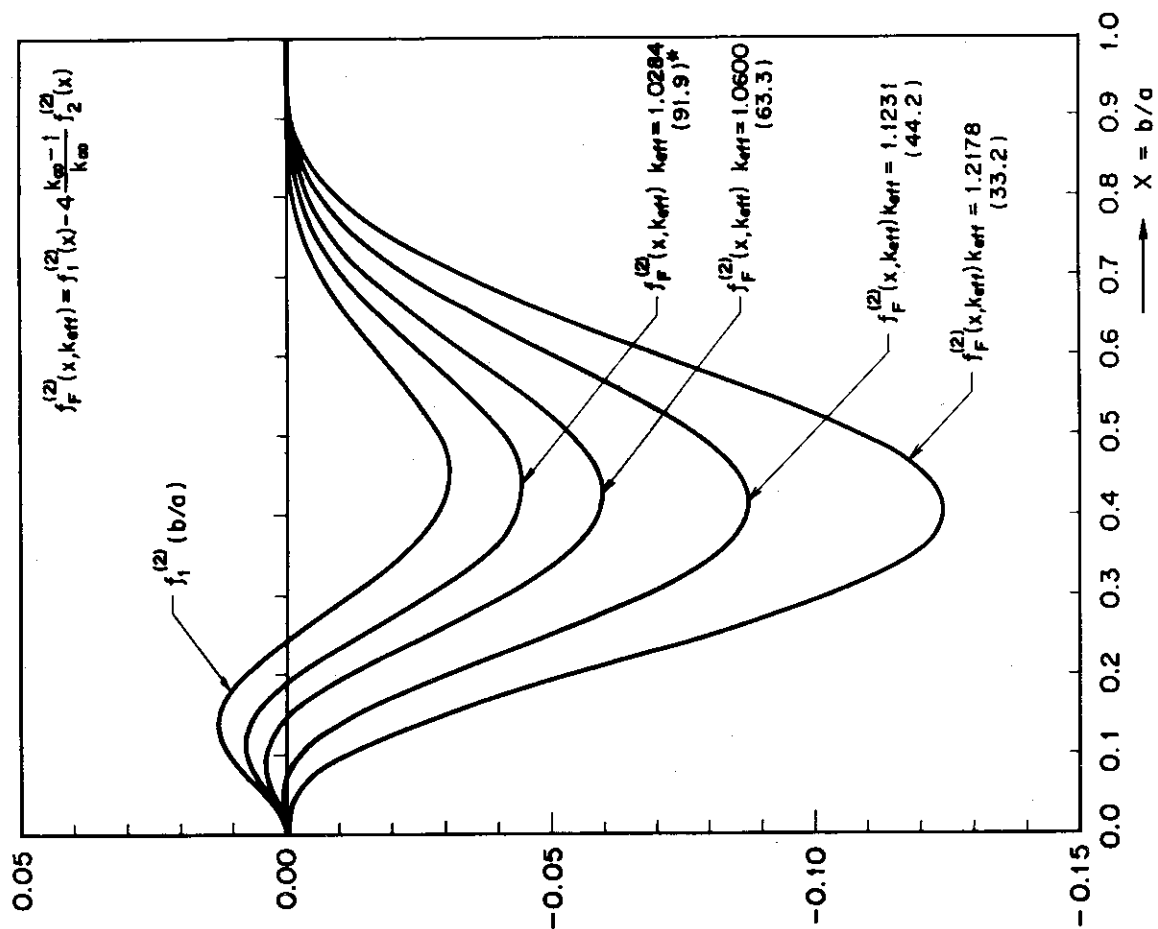


Fig. 7 The geometrical function $f_1^{(2)}(b/a)$ for the second order perturbed flux and function $f_F^{(2)}(x, k_{eff})$ for various values of k_{eff} . * the value in a parenthesis is a half-thickness of a bare slab system

and

$$H^{(3)}(b/a) = 2 \frac{\langle \phi^2 \rangle_a - \langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_b} K_2 - \left(\frac{\langle \phi^2 \rangle_a}{\langle \phi^2 \rangle_b} \right)^2 f_2^{(2)}(b/a) + 2F^{(2)}(b/a) = 3F^{(2)}(b/a) \quad (100)$$

The two functions $F^{(3)}(b/a)$ and

$$F_f^{(3)}(b/a, k_\infty) = F^{(3)}(b/a) - 4 \frac{k_\infty - 1}{k_\infty} H^{(3)}(b/a) + \left(4 \frac{k_\infty - 1}{k_\infty} \right)^2 \quad (101)$$

are shown in Fig. 9. From this figure and Eq. (98), the third order reactivity worth has the following properties.

- (1) The third order reactivity worth is proportional to the third power of the first order reactivity worth, that is, proportional to the third power of the concentration of the inserted perturbation. From the factor $(k_\infty/4 (k_\infty - 1))^2$, the magnitude of the third order reactivity worth relative to the first order reactivity worth increases with radius of the system.
- (2) From Fig. 9, the third order reactivity worth becomes positive or negative, depending on the value of $X = b/a$ and k_∞ . When only Σ_a increases, the third order reactivity worth is negative if X is smaller than 2.25, and positive if X is larger.

Finally, the reactivity worth up to the third order is as follows:

$$\rho = \rho^{(1)} \left[1 + \frac{1}{4} \frac{k_\infty}{k_\infty - 1} \rho^{(1)} F^{(2)}(b/a) - \frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} + \frac{1}{16} \left(\frac{k_\infty}{k_\infty - 1} \right)^2 (\rho^{(1)})^2 F^{(3)}(b/a) \right]$$

cont'd

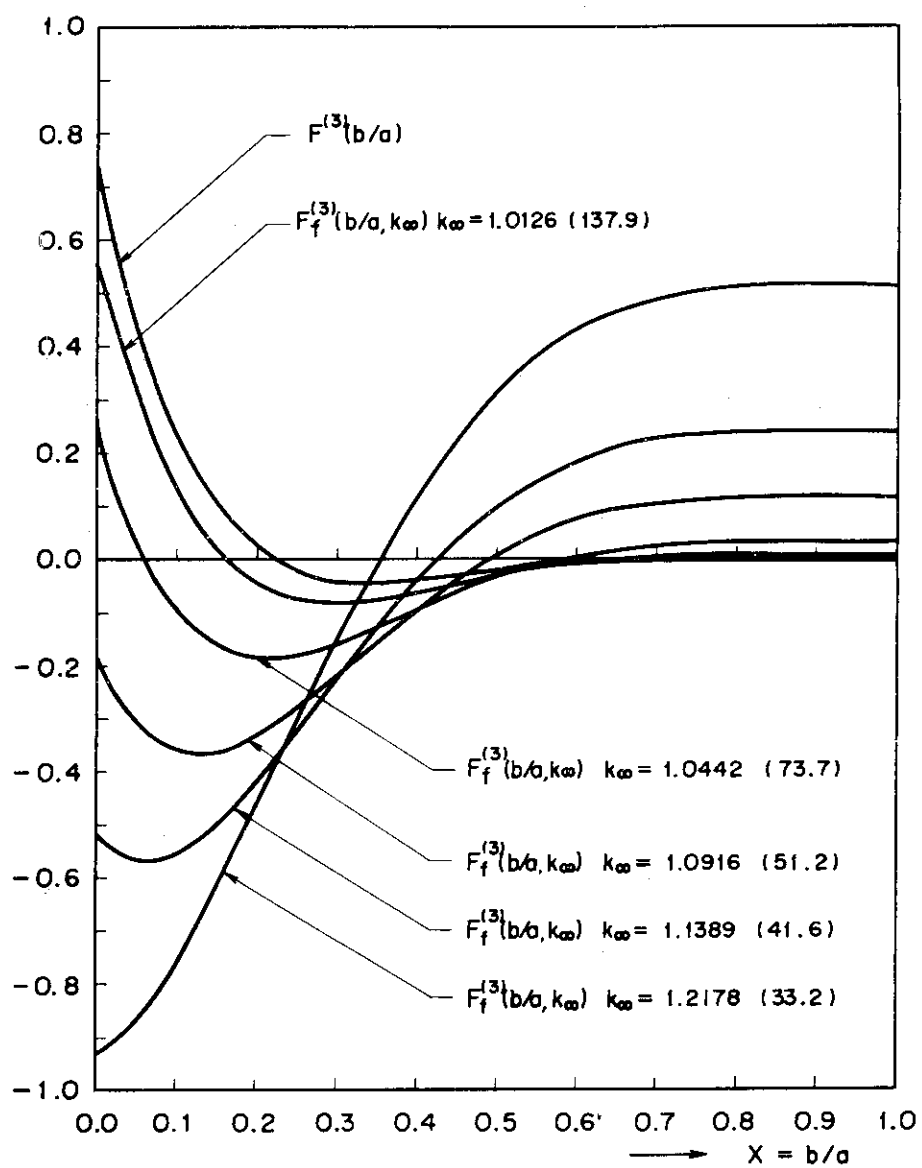


Fig. 9 The geometrical functions $F^{(3)}(b/a)$ and $F_f^{(3)}(b/a, k_\infty)$ of the third order reactivity worth.

$$\begin{aligned}
& - \frac{1}{4} \frac{k_{\infty}}{k_{\infty} - 1} \rho^{(1)} \frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} H^{(3)}(b/a) \\
& + \left(\frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} \frac{\langle \phi^2 \rangle_b}{\langle \phi^2 \rangle_a} \right)^2 \\
& + \dots \dots \dots] \quad (102)
\end{aligned}$$

The first, second and third order reactivity worths are shown in Figs. 10 and 11 as a function of $X = b/a$. When the perturbation is inserted in the whole system, the second and third order reactivity worths are zero for a pure absorbing material, and $\delta(\nu \Sigma_f)/\nu \Sigma_f$ and $(\delta(\nu \Sigma_f)/\nu \Sigma_f)^2$ respectively for a pure fission material. In this case, the perturbed flux is equal to the unperturbed flux, and the exact reactivity worth can be easily obtained.

$$\begin{aligned}
\rho &= - \frac{\delta \Sigma_a - \delta(\nu \Sigma_f)}{(\nu \Sigma_f)'} \\
&= - \frac{\delta \Sigma_a - \delta(\nu \Sigma_f)}{\nu \Sigma_f} \left[1 - \frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} + \left(\frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} \right)^2 - \dots \right] \quad (103)
\end{aligned}$$

For the present problem, the higher order perturbation series converges if

$$1 > \left| \frac{\delta(\nu \Sigma_f)}{\nu \Sigma_f} \right|, \quad \infty > \left| \frac{\delta \Sigma_a}{\nu \Sigma_f} \right|. \quad (104)$$

However, convergent criterion of the perturbation series is not so simple for the generalized problem.

From Eq. (102), the higher order perturbation series for a pure absorbing material is

$$\begin{aligned}
\rho &= \rho^{(1)} \left[1 + \frac{1}{4} \frac{k_{\infty}}{k_{\infty} - 1} \rho^{(1)} F^{(2)}(b/a) \right. \\
&\quad + \frac{1}{16} \left(\frac{k_{\infty}}{k_{\infty} - 1} \right)^2 (\rho^{(1)})^2 F^{(3)}(b/a) \\
&\quad + \dots \dots \dots] \quad , \quad (105)
\end{aligned}$$

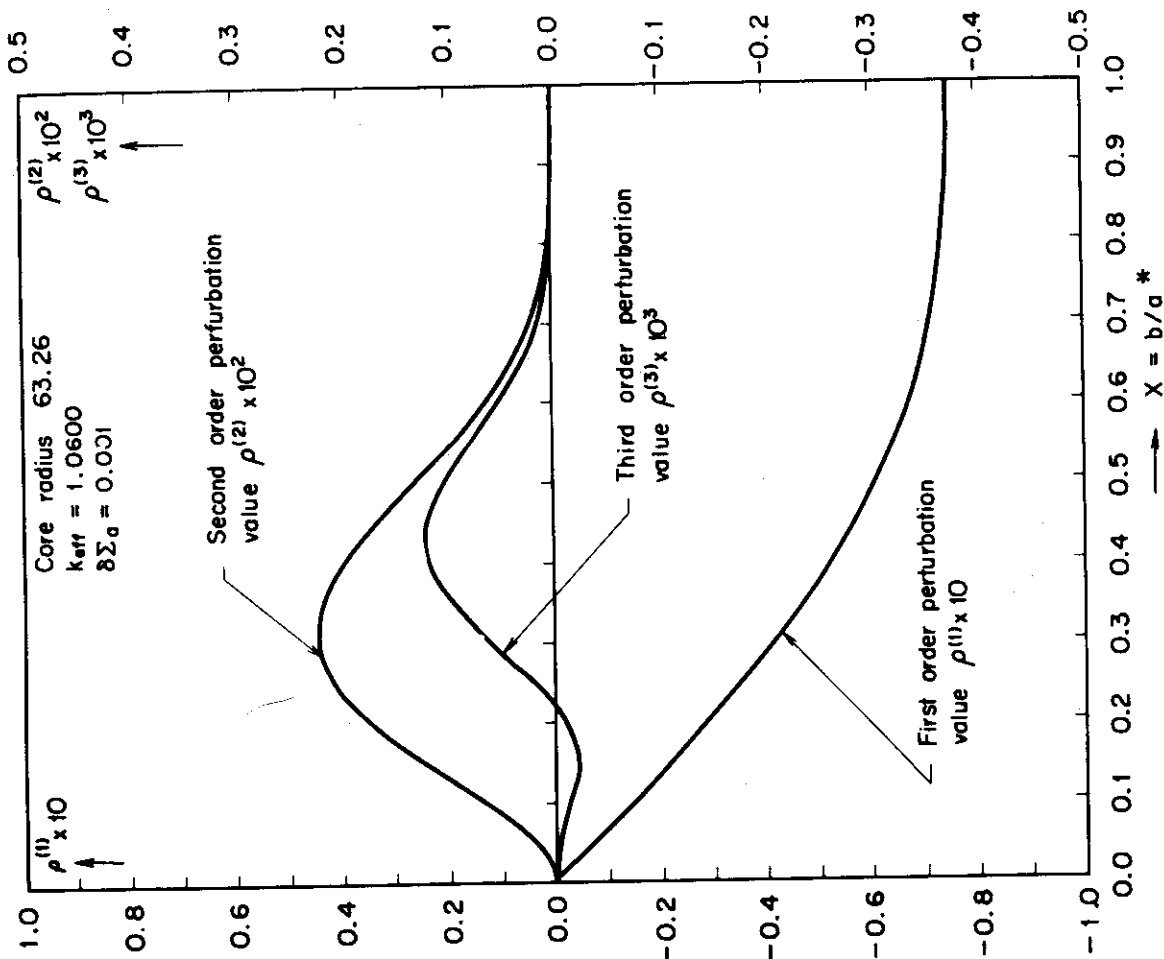


Fig. 10 The region dependence of the first, second and third order perturbation values of absorbing material. * b : half-thickness of the perturbation region, a : half-thickness of bare system.

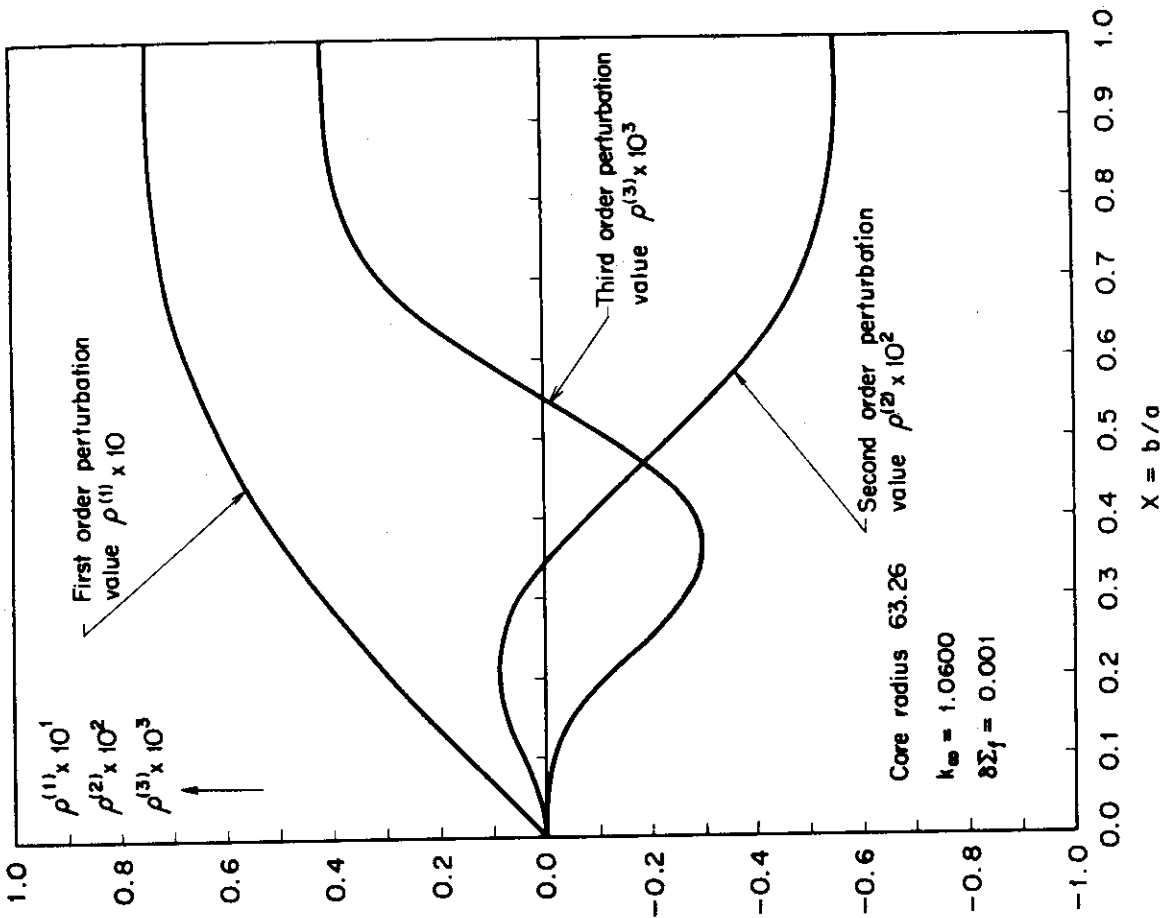


Fig. 11 The region dependence of the first, second and third order perturbation values of a pure fissionable material. b : half-thickness of the perturbation region, a : half-thickness of a bare system

in which, from Figs. 5 and 9, the function $F^{(2)}(b/a)$ and $F^{(3)}(b/a)$ satisfy:

$$\begin{aligned} 1 &\geq F^{(2)}(b/a) \geq 0, \\ 1 &> |F^{(3)}(b/a)|. \end{aligned} \quad (106)$$

Therefore, convergence criterion of the perturbation series for a pure absorbing material may be deduced from Eq. (105).

$$4 \frac{k_{\infty} - 1}{k_{\infty}} > |\rho^{(1)}|. \quad (107)$$

The factor $4(k_{\infty} - 1)/k_{\infty}$ has the following physical significance. The reactor equation in one-group diffusion approximation can be expressed in eigen-value equation.

$$-D \frac{d^2 \phi_n}{dx^2} + \Sigma_a \phi_n = \frac{1}{K_{\text{eff},n}} \nu \Sigma_f \phi_n, \quad (108)$$

where $K_{\text{eff},n}$ is the eigen value. It is 1 for a just critical system and generally

$$1 = K_{\text{eff},0} > K_{\text{eff},1} > K_{\text{eff},2} > \dots > K_{\text{eff},n} > \dots \quad (109)$$

The reactivity worth is defined as

$$\rho = 1 - \frac{1}{K_{\text{eff},n}} \quad (110)$$

Therefore, Eq. (108) can be rewritten to the one with eigen value .

$$D \frac{d^2 \phi_n}{dx^2} + (\nu \Sigma_f - \Sigma_a) \phi_n = \rho_n \nu \Sigma_f \phi_n. \quad (111)$$

The solution of this equation is

$$\phi_n = \cos B_n x \quad (112)$$

where

$$B_n^2 = \frac{(1 - \rho_n) \nu \Sigma_f - \Sigma_a}{D} \quad (113)$$

From the boundary condition

$$\phi_n(a) = 0, \quad (114)$$

$$\frac{(1 - \rho_n) \nu \Sigma_f - \Sigma_a}{D} = (2n + 1)^2 \left(\frac{\pi}{2a} \right)^2. \quad (115)$$

For a just critical system,

$$\frac{\nu \Sigma_f - \Sigma_a}{D} = \left(\frac{\pi}{2a} \right)^2. \quad (116)$$

Therefore, the eigen value ρ_n is given by

$$\rho_n = - \left[(2n + 1)^2 - 1 \right] \frac{k_\infty - 1}{k_\infty}. \quad (117)$$

Then,

$$\rho_0 = 0, \quad (118)$$

$$\rho_1 = -8 \frac{k_\infty - 1}{k_\infty}, \quad (119)$$

$$\rho_2 = -24 \frac{k_\infty - 1}{k_\infty}, \quad (120)$$

⋮

The level distance between ρ_0 and ρ_1 is

$$d = 8 \frac{k_\infty - 1}{k_\infty}. \quad (121)$$

Thus, the factor $4(k_\infty - 1)/k_\infty$ in the left hand side of Eq. (107) is equal to half the level distance between just a critical state and the first excited state.

For the general case, we must consider Eq. (102). The function $H^{(3)}(b/a)$ becomes larger than 1, and satisfies

$$3 \geq H^{(3)}(b/a) \geq 0 \quad . \quad (122)$$

Therefore, if the following conditions are satisfied, the perturbation series will be convergent.

$$4 \frac{k_{\infty} - 1}{k_{\infty}} > |\rho^{(1)}| \quad , \quad (123)$$

$$\frac{1}{3} > \left| \frac{\delta(\nu \Sigma_f) \langle \phi^2 \rangle_b}{\nu \Sigma_f \langle \phi^2 \rangle_a} \right| \quad . \quad (124)$$

However, the present criterion is derived from Eq. (102), and the more higher order terms than the third order cannot be easily obtained. In order to have strict mathematical treatment, the general theory of self-adjoint operator by T. Kato must be used.

IV. Conclusion

Applying the general theory of the perturbation method to the practical reactor calculations with one-group diffusion approximation, properties of the higher order perturbation method have been studied and the convergence criterion of the perturbation series have also deduced from the analytical expressions of the higher order reactivity worths. However, the present analytical derivation is limited up to the third order reactivity worth, so that, in order to have strict mathematical treatment, the general theory of self-adjoint operator given by T. Kato must be used. This calculation and numerical examples will be given in a further coming paper.

The convergence criterion for a pure absorbing material is given by the condition that the first-order reactivity worth is smaller than half the level distance between just a critical state and the first excited state. This is reasonable from physical considerations and may be true in a strict mathematical treatment.

The present treatment is limited to one-group diffusion approximation, so that further extension due to multi-group energy treatment seems to be necessary. However, the multi-group energy treatment is very difficult. Even if the treatment is limited to two-group diffusion approximation, analytical expression of the higher order perturbation terms cannot be easily obtained.

V. Acknowledgment

The author expresses his sincere thanks to Dr. J. Hirota for his encouragement and support of the present work. He is also indebted to Dr. H. Kuroi for his discussions.

IV. Conclusion

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