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ANALYSIS AND CONSTRUCTION OF COMPILER  
OF A PROGRAMMING LANGUAGE  
BY PRECEDENCE GRAMMAR  
WITH PRECEDENCE FUNCTIONS

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Analysis and Construction of Compiler of a  
Programming Language by Precedence Grammar  
with Precedence Functions

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In appearance of the third generation computers with versatile software concepts and facilities, it is felt keenly that construction of the softwares for such computers require tremendous manpower and time. This situation is called "software crisis". To cope with the problem, languages suitable for software construction must be found. GPL is one of these system description languages; it is a modified version of the PL360. It is a precedence language with precedence functions. The implementation techniques of a GPL compiler and usefulness of the language are described.

A N A L Y S I S   A N D   C O N S T R U C T I O N   O F   C O M P I L E R  
O F   A   P R O G R A M M I N G   L A N G U A G E   B Y   P R E C E D E N C E  
G R A M M A R   W I T H   P R E C E D E N C E   F U N C T I O N S

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計算機利用分野の多様化に対応するためのソフトウェアが開発されつつあるが、これらのソフトウェアの開発にはぼう大なマンパワーと時間を要することがわかつってきた。理由のひとつにはソフトウェアの作成に従事する人間の生産性の低さが挙げられる。従来はこれらのソフトウェアを計算機の機械語によって記述していたので生産性を上げることができなかった。そのうえ機械語で書かれたソフトウェアは他の計算機で使うことはできない。そこでソフトウェアを記述するための高水準の言語を開発し、これらの言語によってソフトウェアの作成をおこなり試みが始まられ、現在はこの方法が一般的な傾向になりつつある。GPLはこのようなソフトウェア記述言語のひとつである。本報告ではGPLの言語解析の理論的基礎、コンパイラ実現のための手法、および言語の有用性についての評価を行なった。

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## Symbol Table

#(A)	Number of elements in a finite set A
A*	Set of strings of finite length over a finite set A
(a,b)	A binary(pair) relation between elements a and b
$\delta$	Set of binary relations
$\bar{\delta}$	Complement of the set $\delta$
$\delta^+$	Non null transitive closure of the set $\delta$
G = (V <sub>N</sub> , V <sub>T</sub> , S, P)	Grammar G with set of variables V <sub>N</sub> , set of terminal symbols V <sub>T</sub> , starting symbol S and set of rewriting rules P.
<A>	A variable in V <sub>N</sub>
$\rightarrow$	A binary relation, or a rewriting rule
$=>$	A sequence of rewriting rules
$\stackrel{i}{=}>$	A sequence of rewriting rules whose length is i
$*=>$	Sequences of =>
$=>^+$	Non null transitive closure of =>
$\alpha, \lambda, \rho$	Binary(noncommutative) relations over the set $(V_N \cup V_T)$ . In the Wirth-Weber type precedence grammar, relations $\alpha = \pm$ , $\alpha\lambda^+ = <\cdot$ and $\rho^+\alpha\cup\rho^+\alpha\lambda^+ = \rightarrow$ hold.
$\pm, <\cdot, \rightarrow$	Precedence relations defined by the combinations of $\alpha, \lambda$ and $\rho$ .

## Chapter 1

### Introduction

Since the appearance of the third generation computers with versatile software concepts and facilities, we are forced to realize that constructions of software for these computers require tremendous manpower and time.

This situation is sometimes expressed by the words "software crisis". To get rid of the crisis, we must promote the productivity of programmers, or more generally, of people who want to communicate with computers.

The one of solutions for the promotion of productivity is the introduction of communication languages with computers suitable for software constructions.

The languages for this purpose are called system description languages, or software writing languages and can be classified into two categories of machine independent high level languages and machine oriented high level languages. The machine oriented high level language, which is of our concern in this thesis, allows programmers to specify or to use directly some features of computer.

It is the current trends that we use concepts of formal language theory, especially the concept of context-free grammars presented by N. Chomsky[Cho 60], even for specifications of machine oriented languages. For practical applications, however, it is better to restrict the notion of grammars to some subsets of context-free grammars.

The class of precedence grammars are one of the subsets.

The precedence grammar is originally presented by R. Floyd [Flo 63] and M. Nagao[Naga 63] independently of each other.

The notion of precedence grammars has been extended by Wirth and Weber[Wir 66], Colmerauer[Col 70], Ichbiah and Morse[Ich 70], Inoue[Ino 70], Aho et al.[Aho 72] and others.

Thus we have now a large classes of precedence grammars. Precedence languages derived from the above mentioned grammars are analyzed by aids of precedence matrices. However the sizes of precedence matrices are too big for practical compilers.

Instead of the matrices we may use precedence functions. The precedence functions are two vectors of small sizes and were originally proposed by R. Floyd[Flo 63]. D. Martin [Mar 72] and K. Asai[Asa 72b] have shown that we can use the precedence functions for any precedence grammar.

The adoption of the notion of precedence grammar, i.e., the assumption of explicit existence of phrases, in the analysis of programming language will reduce the compilation speed of compiler.

If the amount of overhead induced by the existence of phrases is not small, it will become another cause of the "software crisis" because we must use the system description language extensively for software constructions.

On the otherhand, the usefulness of a system description language is rather independent of the above mentioned overhead and will be considered from the view points of the easiness to read or write, the memory utilization of compiled program and execution efficiency of compiled program.

The purpose of this thesis is to investigate these problems, to give techniques to solve the problems and to evaluate the results.

In the chapter 2, basic notions on precedence grammars which are relevant to succeeding chapters are given.

In the chapter 3, an approximation theorem for precedence grammars with precedence functions is given.

In the chapter 4, the existence theorem of precedence functions is given.

In the chapter 5, using a software writing language GPL [Asa 72a], i.e., a modified PL360 [Wir 68] as an example, the problems of overhead and usefulness are discussed.

In the chapter 6, a conclusion on the above methods is given.

In the appendices, the language specification, precedence functions, precedence relations and example programs of GPL are given.

## Chapter 2

### Basic Concepts and Definitions

#### 2.1 Vocabulary and String

A vocabulary is any finite set of symbols. A string over a vocabulary  $V$  is a finite concatenation of symbols of  $V$ .

An empty string  $\epsilon$  is a string of null length, i.e., it is a string consisting of no symbol. We also use the term finite sequence of symbols as a synonym for string.

The symbol  $V^*$  denotes the set of all strings over  $V$ , including the empty string.

#### 2.2 Pair Relations

We denote a subset of pair relations over a set  $E$  by  $\rho$ .  $a \rho b$  is an abbreviation of a pair relation  $(a,b) \in \rho$ , where  $a,b \in E$ . The set of pair relations is denoted by  $E \times E$  and complement of  $\rho$  is denoted by  $\bar{\rho} = E \times E - \rho$ . The product  $\rho\sigma$  of relations  $\rho, \sigma$  is defined as

$$a\rho\sigma b \equiv [\text{there exists } c \in E, a\rho c \cap c\sigma b].$$

The closure  $\rho^+$  of  $\rho$  is defined as  $\rho^+ = \bigcup_{i=1}^{\infty} \rho^i$ , where

$\rho^i = \rho \circ \rho^{i-1}$ ,  $a\rho^0 b \equiv [a=b]$ . If the number of elements of the

set  $E$  is  $n$ , then  $\rho^+ = \bigcup_{i=1}^{\infty} \rho^i = \bigcup_{i=1}^n \rho^i$ .

### 2.3 Context-free Grammars

A quadruple  $G = (V_N, V_T, S, P)$  of sets  $V_N, V_T, S$  and  $P$  is called a context-free grammar if it satisfies following conditions;

- (1)  $V_N, V_T$  and  $S$  are finite sets such that  $V_N \cap V_T = \emptyset$ ,  
 $S \in V_N$ ,
- (2)  $P$  is a set of finite pair relations  $\rightarrow$  over the set  $(V_N \cup V_T)^*$  of finite sequences of elements of  $(V_N \cup V_T)$ , including the empty sequence,
- (3) if  $x \rightarrow y$  for  $x, y \in (V_N \cup V_T)^*$  and the length of  $x$  is unity, then  $x$  is an element of  $V_N$ ,
- (4) if  $x \rightarrow y$  for  $x, y \in (V_N \cup V_T)$ ,  $z \in V_N$ ,  $x = u z v$ ,  $y = u w v$ , then there exist  $u, v, w \in (V_N \cup V_T)^*$  such that  $z \rightarrow w$ .

The set  $V_N, V_T$  and  $P$  are called the sets of variables, terminal symbols and rewriting rules, respectively.  $x \Rightarrow y_n$  is an abbreviation of relation  $x \rightarrow y_1 \rightarrow \dots \rightarrow y_n$ , where  $x \in V_N$  and  $y_1, \dots, y_n \in (V_N \cup V_T)^*$ . The language generated by a context-free grammar  $G$  is denoted as  
 $L(G) = \{t \in V_T^* | S \Rightarrow^+ t\}$ .

For the rewriting rule  $x \rightarrow y$  of the above (3),  $x$  and  $y$  are called the left part and right part, respectively.

A context-free grammar  $G = (V_N, V_T, S, P)$  is said to be  $\epsilon$ -free if  $P$  has no rule of the form  $A \rightarrow \epsilon$ , where  $A \in V_N$ .

A string  $w \in (V_N \cup V_T)^*$  is said to be a sentential form if  $S \Rightarrow w$ .

Two grammars  $G_1$  and  $G_2$  are equivalent if  $L(G_1) = L(G_2)$ .

A left part  $A$  of a rewriting rule is sometimes denoted by  $\langle A \rangle$  to distinguish it from terminal symbols. The notation

is called Backus form notation.

The left part or right part of a rewriting rule is sometimes called a phrase.

The term detection of phrases means an operation which recognizes right parts in a finite sequence of symbols.

The term reduction of phrase means an operation which replaces a right part by a left part of a rewriting rule.

The term syntax analysis of a string  $x$  or analysis of a sentence  $x$  means an operation which determines whether  $x \in L(G)$ , i.e.,  $S \Rightarrow^* x$  for the starting symbol  $S$ .

In practical applications in the field of programming languages, the starting symbol  $S$  is <program>. Hence let us assume that the starting symbol appears once for all in only one left part and it does not appear in any right part when we are referring to grammars of programming languages.

A context-free grammar  $G$  is said to be loop-free if it has no sequences of rewriting rules of forms  $A \Rightarrow A$  for any variable  $A$  of  $G$ .

Hereafter let us assume that all grammars are loop-free.

Since we can get an equivalent context-free grammar for any context-free grammar with  $\epsilon$ -rules [Hop 69], let us hereafter consider solely  $\epsilon$ -free grammars.

## 2.4 Simple Precedence Relations

The pair relations  $\alpha, \lambda$  and  $\rho$  between  $A, B \in (V_N \cup V_T)$  of a context-free grammar  $G = (V_N, V_T, S, P)$  are defined as follows;

$A\alpha B \equiv [\text{there exists } U \in V_N, x, y \in (V_N \cup V_T)^*, U \rightarrow xABy]$ ,  
 $A\lambda B \equiv [\text{there exists } y \in (V_N \cup V_T)^*, A \rightarrow By]$ ,  
 $A\rho B \equiv [\text{there exists } x \in (V_N \cup V_T)^*, B \rightarrow xA]$ .

We must take care that the assumption of  $\epsilon$ -free grammar does not mean that any  $x, y$  in the set  $(V_N \cup V_T)^*$  are not empty strings. In fact, for the relations  $\alpha, \lambda$  and  $\rho$ , strings  $x$  and  $y$  are sometimes empty strings.

### Example 2.1

For intuitive understanding of the relations  $\alpha, \lambda$  and  $\rho$ , let us consider the relations over a context-free grammar  $G = (V_N, V_T, S, P)$ , where  $V_N = \{S\}$ ,  $V_T = \{a, b\}$ ,  $P = \{S \rightarrow aSb, S \rightarrow ab\}$ .

For the rewriting rules  $S \rightarrow aSb$  and  $S \rightarrow ab$ , the relations  $\alpha, \lambda$  and  $\rho$  are shown as following:

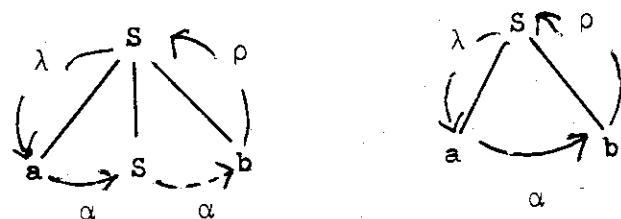


fig. 2.1 Binary relations over  $V_N \cup V_T$ .

In the above figure the arrow from symbol A to B shows a relation  $(A, B)$ .

## 2.5 Precedence Matrix, Precedence Functions

Let us consider noncommutative pair(binary) relations  $\doteq$ ,  $\rightarrow$ ,  $\leftarrow$  and  $\phi$  (empty relation) over the vocabulary  $V_N \cup V_T$ . Since every relation is not commutative, relations over  $V_N \cup V_T$  are represented by a matrix.

A matrix  $M$  is called a precedence matrix if its  $(i,j)$  element is the pair relation  $(S_i, S_j)$  of a context-free grammar  $G = (V_N, V_T, S, P)$ ,  $S_i, S_j \in (V_N \cup V_T)$ .

For a context-free grammar  $G = (V_N, V_T, A, P)$ ,  $V_N = \{A, B, C\}$ ,  $V_T = \{[, ], \lambda\}$ ,  $P = \{\phi_1, \dots, \phi_6\}$ ,  $\phi_1: A \rightarrow CB$ ,  $\phi_2: A \rightarrow [ ]$ ,  $\phi_3: B \rightarrow \lambda$ ,  $\phi_4: B \rightarrow \lambda A$ ,  $\phi_5: B \rightarrow A$ ,  $\phi_6: C \rightarrow [$ , assuming  $\rightarrow \supset \doteq \lambda^+$ , we can get a precedence matrix as Fig.2.2.

	A	B	C	]	[	$\lambda$
A				$\rightarrow$		
B				$\doteq$		
C	$\leftarrow$	$\doteq$	$\leftarrow$	$\leftarrow$	$\leftarrow$	
]				$\rightarrow$		
[	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\doteq$	$\rightarrow$	$\rightarrow$
$\lambda$	$\doteq$		$\leftarrow$	$\rightarrow$	$\leftarrow$	

Fig.2.2 Precedence matrix

As is shown in the above example, the matrix  $M$  is composed of  $N \times N$  elements when the number of elements of set  $(V_N \cup V_T)$  is  $N$ . The values of  $N$ 's are not small for practical programming languages. For example, the value of  $N$  is approximately 500 for Fortran IV language, but we can compress this matrix to two vectors  $f$  and  $g$  with  $N$  elements, respectively. These two vectors are called precedence functions.

Two functions  $f$  and  $g$  with  $N$  values respectively are called the precedence functions of a context-free grammar  $G = (V_N, V_T, S, P)$  if they satisfy following relations for any  $S_i, S_j \in (V_N \cup V_T)$ ;

if  $S_i < S_j$  then  $f(S_i) < g(S_j)$ ,

if  $S_i = S_j$  then  $f(S_i) = g(S_j)$ ,

if  $S_i \rightarrow S_j$  then  $f(S_i) > g(S_j)$ ,

where  $N$  is the number of elements of  $(V_N \cup V_T)$ .

With the precedence matrix, we can find out easily the empty relation between two symbols in an input string. When we use the precedence functions for the analysis of the input string, the detection of erroneous relation is delayed until we cannot find a proper left part for the corresponding right part. This is the demerit of precedence functions.

Thus when we use precedence functions for analysis of strings, we must provide additional procedures for proper error detections and recoveries. There may exist two methods to solve the problem. The one is to do a syntax check of strings before the use of precedence functions. The other method is

to cut off the erroneous part from the string by assuming that the part is correctly processed. The author uses the latter method for the syntactical analysis of the GPL[Asa 72a].

A context-free grammar generally has no precedence functions but we can find equivalent context-free grammars with precedence functions for any given grammar. We will prove the fact in the chapter 4.

## 2.6 Precedence Grammars

In this section we will discuss notions and characteristics of several types of precedence grammars.

### 2.6.1 Notion of Simple Precedence Grammar

A context-free grammar  $G$  is called a simple precedence grammar if for any  $A, B \in (V_N \cup V_T)$ , there exists one and only one of pair relations  $\cdot >$ ,  $\cdot <$ ,  $\cdot =$  and  $\emptyset$  (empty relation).

From hereafter we mean only simple precedence grammars by the term precedence grammars.

A precedence grammar  $G$  is said to be unambiguous simple precedence grammar if it satisfies following two conditions;

- (1) if  $X \rightarrow u$ ,  $Y \rightarrow u$ , then  $X = Y$ ,
- (2)  $(\cdot < \cap \cdot >) \cup (\cdot < \cap \cdot =) \cup (\cdot = \cap \cdot >) = \emptyset$ .

A grammar  $G$  which satisfies the condition (1) is sometimes called invertible.

For practical application, a grammar should be invertible to accomplish a highly efficient analysis of input strings.

Practical programming languages are invertible or they are forced to be invertible in the assumption that they are described by bounded context grammars. A context-free grammar G is bounded context if we can determine a symbol A in an input string is to be in the set  $(V_N \cup V_T)$  by looking finite number of symbols to the left or to the right of the symbol A [Flo 64].

Hereafter let us assume, unless explicitly mentioned, that every grammar is invertible.

#### 2.6.2 Wirth-Weber Type Simple Precedence Grammar

New pair relations over  $V_N \cup V_T$   
 $\doteq = \alpha$ ,  $<= = \alpha\lambda^+$  and  $\rightarrow = \rho^+\alpha \cup \rho^+\alpha\lambda^+$   
 are called the Wirth-Weber type simple precedence relations [Wir 66, Col 70].

An invertible context-free grammar G is said to be an Wirth-Weber type simple precedence grammar if it satisfies the above relations. The language generated by the grammar is called a precedence language. The merit of this grammar is that we can find a unique right part of a rewriting rule in every step of the language analysis. It has however two demerits. Firstly it requires a  $N \times N$  matrix of big size to analyze the language, where  $N = \#(V_N \cup V_T)$ . But this demerit vanishes if we use precedence functions. Secondly we must increase numbers of elements of the sets P and  $V_N$  to remove confictions of precedence relations in the original grammar. The author's experience on Fortran[Asa 70] and McAfee and

Presser's experience on Algol 60 [McA 72] show that we must add a quarter of new rewriting rules and variables to get a precedence grammar. The GPL is described by this type of grammar.

### 2.6.3 Total Precedence Grammar

A. Colmerauer [Col 70] has shown that (a) the condition (2) of 2.6.1 is equivalent to the following (2'), and that (b) G is a precedence grammar if its precedence relations  $\cdot >$ ,  $\cdot <$  and  $\cdot =$  satisfy the following condition (3);

$$(2') (\alpha \lambda^+ \cap \alpha) \cup (\rho^+ \alpha \cap \alpha) \cup (\rho^+ \alpha \lambda^+ \cap \alpha) \cup (\rho^+ \alpha \cap \alpha \lambda^+) = \emptyset,$$

$$(3) \alpha \subset \{ \cdot =, \alpha \lambda^+ \subset \cdot <, \rho^+ \alpha \subset \cdot >, \rho^+ \alpha \lambda^+ \subset \cdot \vee \cdot > \}.$$

The relations which satisfy the above conditions (2') and (3) are called the total precedence relations and the grammar analyzed by the total relations is called a total precedence grammar.

Because of the condition  $\rho^+ \alpha \lambda^+ \subset \cdot \vee \cdot >$ , the total precedence grammar gives a language designer the freedom to set precedence relations more loosely than the Wirth-Weber type precedence grammar. The author's experience with Fortran and GPL however shows that the Wirth-Weber type precedence grammar is sufficient for programming language description. The total precedence language is a language generated by the total precedence grammar. The total precedence language is also analyzed by a  $N \times N$  precedence matrix.

If we use precedence functions, we must provide additional procedures to find out invalid pairs of symbols.

#### 2.6.4 Operator Precedence Grammar

The notion of precedence grammar is originally proposed by R. W. Floyd in the form of operator precedence grammar [Flo 63]. Wirth and Weber have obtained the simple precedence grammar by extending the concept of operator precedence.

A context-free grammar  $G$  is called an operator precedence grammar if it satisfies following conditions;

- (1) for any  $a, b \in V_T$ , there exists one and only one of pair relations  $\leftarrow$ ,  $\rightarrow$ ,  $\doteq$  or  $\phi$ ,
- (2) there does not exist such rewriting rule as  $A \rightarrow xBCy$ , where  $B, C \in V_N$  and  $x, y \in (V_N \cup V_T)^*$ .

The condition (2) means that every rewriting rule of an operator precedence grammar is of the form  $A \rightarrow xbcy$ , or  $A \rightarrow xbBcy$ , where  $b, c \in V_T$ ,  $B \in V_N$  and  $x, y \in (V_N \cup V_T)^*$ ,

The relations  $\doteq$ ,  $\rightarrow$  and  $\leftarrow$  are defined as following;

- (i)  $b \doteq c$  if  $A \rightarrow xbcy$ , or  $A \rightarrow xbBcy$ ,
- (ii)  $b \rightarrow c$  if  $A \rightarrow xBcy$  and  $B \Rightarrow wb$ , or  $B \Rightarrow wbD$ ,
- (iii)  $b \leftarrow c$  if  $A \rightarrow xbCy$  and  $C \Rightarrow cw$  or  $C \Rightarrow Dcw$ ,  
where  $D \in V_N$ ,  $w \in (V_N \cup V_T)^*$ .

This type of grammar has two demerits. The one demerit is the fact that we may execute ambiguous reductions of right parts of rewriting rules. Suppose that an operator grammar has a rewriting rule of the form  $A \rightarrow t_1 B t_2$ , where  $A, B \in V_N$ ,  $t_1, t_2 \in V_T$ . Then we may also detect a right part of the form  $t_1 B' t_2$  in the reduction step since the pair relations

$(t_1, B')$  and  $(B', t_2)$  are not checked by the operator precedence grammar. In practical applications the decision whether we must take  $B$  or  $B'$  is done by semantic routines. The another demerit is the lack of rewriting rules of the form  $A \rightarrow xABy$ ,  $A, B \in V_N$ ,  $x, y \in (V_N \cup V_T)^*$ . In a compiler construction, we often need to assign meanings(semantic) to rewriting rules of the above form. The prohibition of such rewriting rules is sometimes too restrictive to compiler designers.

J. Gray and M. Harrison have obtained interesting results to remove the difficulty by introducing a new type of precedence grammar[Gray 69]. We will sketch the method in the chapter 5 in connection with a problem on the overhead of syntactical analysis procedures.

The merit of operator precedence grammar is the small size of precedence matrix. The size of the matrix is  $N \times N$ , where  $N = \#(V_T)$ .

We can use precedence functions for the analysis of operator precedence grammars. The precedence functions were originally proposed by R. W. Floyd as a reduction method of precedence matrices of operator precedence grammars[Flo 63].

#### 2.6.5 Right Precedence Grammar

For symbols  $B$  and  $C$  in rewriting rules of a form  $A \rightarrow xBCy$  of a context-free grammar, relations  $\prec = \alpha^+ \cup \alpha\lambda^+$  and  $\succ = \rho^+ \cup \rho^+\alpha\lambda^+$  are said to be right precedence relations [Ino 70], where  $B, C \in (V_N \cup V_T)$ ,  $x, y \in (V_N \cup V_T)^*$ .

A context-free grammar  $G = (V_N, V_T, S, P)$  is said to be a right precedence grammar [Ino 70, Ino 72] if it satisfies following conditions;

- (1) the one and only one of relations  $\leq^*$ ,  $\cdot >$  or  $\phi$  holds between any two elements  $A, B$ , where  $A \in (V_N \cup V_T)$ ,  $B \in V_T$ ,
- (2) if there exist rewriting rules such that  $A \rightarrow xy$  and  $B \rightarrow y$ , then  $C \rightarrow uH_i(x)Dv$  and  $D \Rightarrow T_{n-i}(x)Bw$  do not exist, where  $H_i(x)$  is a head(of length i) of the string x,  $T_{n-i}(x)$  is a tail(of length n-i) of the string x and n is the length of the string x.

The demerit of the right precedence grammar is that we need a special procedures to find out the left boundary of a right part. Since the relation  $\leq^*$  means relations  $\leq^*$  or  $\doteq$ , the analyzer must activate a procedure to determine which relation must hold.

The right precedence grammar has two merits. Firstly it reduces the size of precedence matrix to  $\#(V_N \cup V_T) \times \#(V_T)$ . Secondly fusing relations  $\leq^*$  and  $\doteq$  to  $\leq^*$ , it allows the existence of left recursive rewriting rules.

A rewriting rule  $A \rightarrow Bx$  is said to be left recursive if it allows  $A \equiv B$  or  $B \Rightarrow Ay$ , where  $B \in V_N$ ,  $x, y \in (V_N \cup V_T)^*$ .

At the same period J.D. Ichbiah and S.P. Morse have presented the idea of weak precedence grammar[Ich 70] which is almost same as of Inoue's.

## 2.7 Analysis Mechanism of Simple Precedence Grammar

When a context-free language is described by a simple precedence grammar, the language is analyzed by a very simple mechanism. An analysis mechanism of Wirth-Weber type precedence language is given below. In this case the grammar is

assumed to be of the form  $G = (V_N, V_T, \perp S \perp, P)$ . In the figure  $I_K$  is the  $k$  th input symbol and  $S_j$  is the top symbol of the stack. The author uses this mechanism for the analysis of GPL language. In the GPL compiler we use precedence functions. As the result, the check for the invalid pair  $(S_j, I_k)$  is omitted and the check is delayed until we find an empty left part  $t$ (i.e.,  $t = \phi$ ).

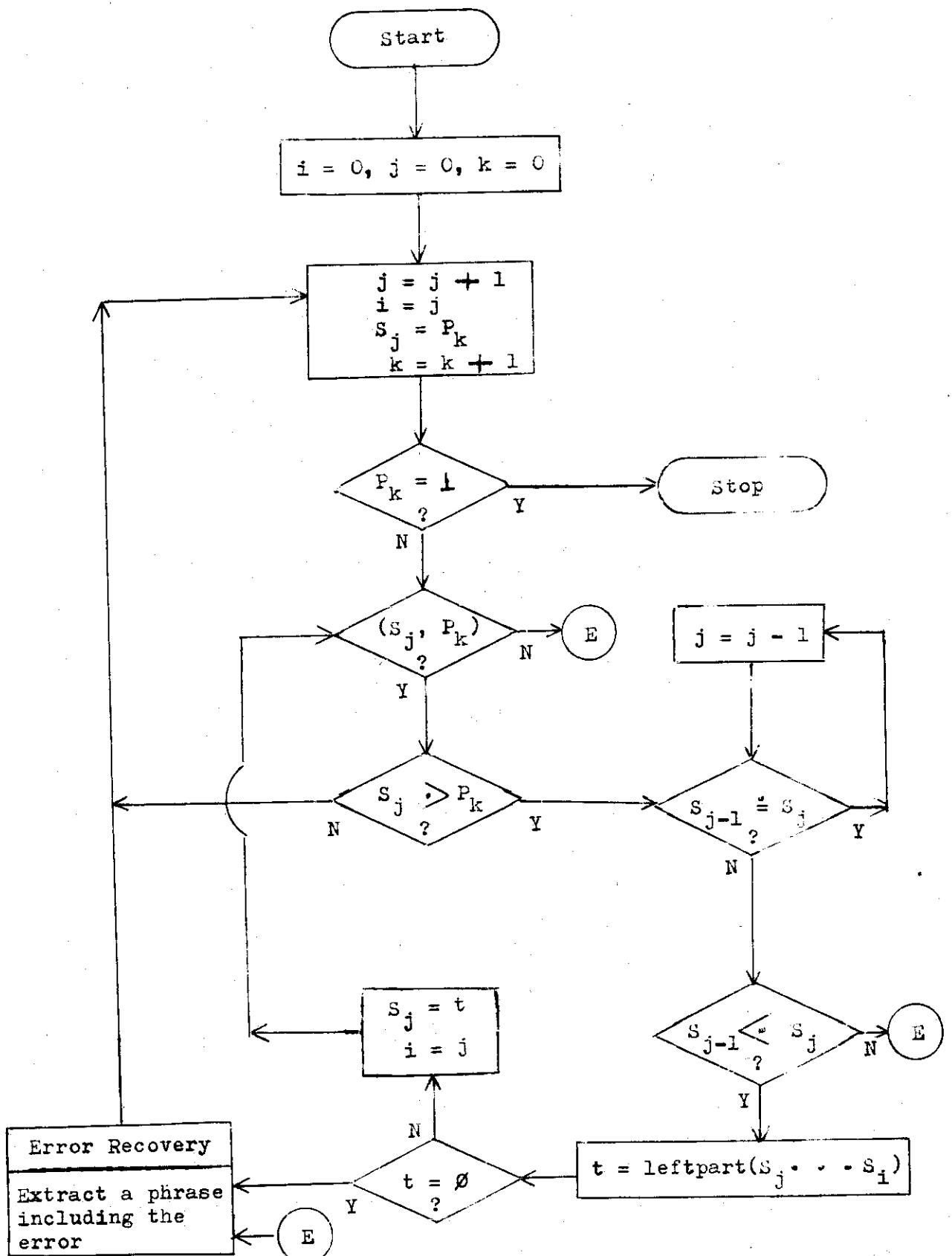


fig. 2.3 Analysis mechanism of simple precedence grammar

## 2.8 Analysis Speed of Simple Precedence Grammar

The analysis speed of the simple precedence language is the most important factor in its compiler construction. The analysis speed of the simple precedence language is proportional to the length of the input string. The fact is proved by A. Colmerauer[Col 67]. The proof of this section is due to K. Inoue[Ino 72] and A. Colmerauer.

Let  $P_1, P_2, \dots, P_q$  be rewriting rules required to analyze an input string  $u$  and let  $h_1, h_2, \dots, h_q$  be lengths of right parts of the rules  $P_1, P_2, \dots, P_q$ , respectively.

For the grammar  $G$  of the above sections, let us assume that  $S \Rightarrow u$ . The string  $\perp u \perp$  analyzed by the mechanism of the above section 2.7 eventually becomes  $\perp S \perp$ . The length of the string  $\perp u \perp$  is equal to  $|u| + 2$  and the length of  $\perp S \perp$  is equal to 3, where  $|u|$  means the length of the string  $u$ .

In every reduction, a right part of length  $h_i$  is replaced by a left part. Hence

$$2 + |u| - \sum_{j=1}^q (h_j - 1) = 3.$$

From the above equation we have

$$\sum_{j=1}^q h_j = |u| + q - 1.$$

In the flowchart fig.2.3, we must execute following operations;

- (i)  $|u| + 2$  replacements of  $S_j = P_k$ ,

- (ii)  $\sum_{i=1}^q h_i$  checks for  $s_{j-1} \leftarrow s_j$ ,  
 (iii) q replacements of  $t = \text{leftpart } (s_j \dots s_i)$ .

Let  $c_1$ ,  $c_2$  and  $c_3$  be weight constants for the above three types of operations, respectively. Then the total operations required are

$$N = c_1 (|u| + 2) + c_2 \sum_{j=1}^q h_j + c_3 q .$$

Replacing  $\sum_{j=1}^q h_j$ , we get

$$N = (c_1 + c_2) |u| + (c_2 + c_3)q + 2c_1 - c_2 .$$

On the otherhand we can show  $q \leq 2P |u| - p$  if

$$S \Rightarrow u, p = \#(P) [\text{Col 67}] .$$

Hence

$$N \leq \{(c_1 + c_2) + 2P(c_2 + c_3)\} |u| + c_4 .$$

The inequality

$$q \leq 2p |u| - P$$

is shown by induction as the following.

- (1) If  $|u| = 1$  then  $|S| = 1$  because G is context-free.
- (2) Let us assume that the inequality is true for u such that  $1 \leq |u| \leq s$ . Since  $|u| > 1$ , there exists a string x such that

$$S \stackrel{q-i}{\Rightarrow} x \stackrel{i}{\Rightarrow} u, |x| > 1, q - i \leq p .$$

Since  $|x| > 1$ , there exist nonempty strings  $x_1, x_2, u_1$  and  $u_2$  such that  $x = x_1 x_2$ ,  $u = u_1 u_2$ ,

$x_1 \xrightarrow{k} u_1, x_2 \xrightarrow{\ell} u_2$ , where  $k + \ell = i$ ,  $|u_1| + |u_2| = s$ .

By the induction hypothesis,

$$|u_1| < s, |u_2| < s, k \leq 2p |u_1| - p,$$

$$\ell \leq 2p |u_2| - p.$$

$$\text{Hence } i = k + \ell \leq 2ps - 2p.$$

$$\text{Since } q - i \leq p, \text{ we have } q \leq 2ps - p.$$

In practical applications, we realize that the number of operations and the time spent for the weight constants sometimes become so big as to make the compiler designer hesitate to adopt these analysis techniques which assume the existence of phrases explicitly.

We will investigate the situation in the chapter 5.

### Chapter 3

#### A Family of Precedence Grammars with Precedence Functions

##### 3.1 Introduction

As we have seen in the section 2 of chapter 2, R.W. Floyd, Wirth and Weber, A. Colmerauer, Inoue and others have shown that we can analyze programming languages by simple procedures if the languages are generated by precedence grammars.

Let  $N$  be the number of vocabulary of the grammar, then the precedence table used in the analysis is represented by  $N \times N$  matrix. The values of  $N$ 's are not small for practical programming languages. For example, the value of  $N$  is approximately 500 for FORTRAN IV language. Since this is not acceptable size for practical compilers, several methods have been proposed to minimize the table size. One of the methods due to K. Inoue reduces the size of the precedence matrix and the other methods due to Floyd, Wirth and Weber use precedence functions  $f, g$  of  $2N$  values instead of the matrix.

For the analysis of programming languages, there may exist following four methods to analyze  $L(G)$  which is generated by a simple precedence grammar  $G$ :

- (1) Every input symbol is scanned semantically and locally, then the analysis of  $L(G)$  is done according to the precedence relations defined for the elements of  $V_T$ .
- (2) Every input symbol is preprocessed as the above mentioned scanning procedure, then the analysis is done according to precedence relations defined for the elements of  $V_N \cup V_T$ .

- (3) Without the preprocess for input symbols, the analysis is done according to the precedence relations defined for the elements of  $V_N \cup V_T$ .
- (4) Without the preprocess for input symbols, the analysis is done according to the precedence functions defined for the elements of  $V_N \cup V_T$ .

Among the above four, the first method is adopted by conventional compilers. This method, however, makes it difficult to attach interpretation rules to rewriting rules by restricting the analysis units to terminal symbols. By the third, it is required to describe every element of  $V = V_N \cup V_T$  strictly and as the results, the number of rewriting rules will increase.

Since the size of  $V$  does not change, number of rewriting rules with same right part will also increase. If we want to eliminate rewriting rules with same right part, we need some scanning procedures for input symbols and thus the third method will tend to approach the second. The second method also involves difficulties in its efficiency and memory requirement.

We may show the point by an example. Let us assume that  $G$  is a simple precedence grammar which generates the FORTRAN IV language. By extracting from  $G$  new sets  $V'_N$ ,  $V'_T$ ,  $\langle \text{expr} \rangle$  and  $P'$ , we can define a new simple precedence grammar  $G'$  which generates FORTRAN IV expressions. In that case numbers of elements of  $V'_N$ ,  $V'_T$ ,  $P'$  and number of relations of A and B, where A and B are elements of  $V_N \cup V_T$ , are about 70, 30, 150 and 1500, respectively. Since most compilers accept only terminal symbols, we may use not  $100 \times 100$ , but  $100 \times 30$  matrix. Even by this reduction, the FORTRAN IV language requires more than 10000

precedence relations. Compression of a relation from one word to three bits reduces amounts of memory requirement, but it induces inefficiency in access procedure to the relations. Since it is a complicated work to give a proper error recognition procedure and to continue the analysis avoiding the erroneous input symbols, conventional compilers have adopted the method (1) with one dimensional precedence relations  $a \triangleleft b \triangleleft \dots \triangleleft c$  for terminal symbols. If there exist precedence functions  $f$  and  $g$  for the grammar  $G$ , we can make use of the method of (4).

Generally a precedence grammar  $G$  has no precedence functions, but there exists a family of simple precedence grammars with precedence functions.

In this chapter the author shows the existence of a non-trivial family of precedence grammars that has precedence functions. The family of the grammars is nontrivial in the sense that the structure is a good approximation of grammars of current programming languages such as FORTRAN IV and Algol-like language PL360. Comparative results for these grammars are also given.

### 3.2 Precedence Grammars with Precedence Functions

In this section let us show the existence of a family of precedence grammars with precedence functions.

#### Definition 3.1

The set  $P$  of rewriting rules of a context-free grammar  $G = (V_N, V_T, S, P)$  is simple if any  $S_i \in (V_N \cup V_T)^*$  appears

once for all in the right parts of rewriting rules.

### Lemma 3.1

A context-free grammar G with simple P is an unambiguous simple precedence grammar.

Proof. The grammar G is invertible and it satisfies the condition (2') of 2.6.3.

### Lemma 3.2

For the grammar G of the above lemma 3.1, integers p and q are unique if they satisfy a relation  $S_i \rho^p \alpha \lambda^q S_j$ , where  $S_i, S_j \in (V_N \cup V_T)$

Proof. Since P is simple, only one relation is permissible for any  $S_i$  and  $S_j$ .

### Definition 3.2

A matrix Q is called a pq-matrix if its (i, j) element is the pair (p, q) of the lemma 3.2 or  $\phi$  (empty).

### Definition 3.3

Notations  $\phi = \phi_n \phi_{n-1} \dots \phi_1$  and there exists  $\phi : x \Rightarrow y_n, \phi \in P^*$  mean that there exists a sequence

$\phi_1 : x \Rightarrow y_1, \phi_2 : y_1 \Rightarrow y_2, \dots, \phi_n : y_{n-1} \Rightarrow y_n$   
for  $\phi_i \in P, x \in V_N, y_i \in (V_N \cup V_T)^*$ ,  $(1 \leq i \leq n)$ .

Let us denote by  $K_i$  the ith row vector, by  $L_j$  the jth

column vector and by  $(p_{i,j}, q_{i,j})$  the  $(i,j)$  element of the pq-matrix.

Lemma 3.3

If the set  $P$  of rewriting rules of a context-free grammar  $G$  is simple, the pq-matrix of  $G$  satisfies following conditions (1) - (4).

(1) If there exists  $j$  such that  $(p_{i,j}, q_{i,j}) = (0,0)$ , then

$$K_i \not\ni (p_{i,j}, q_{i,j}), p_{i,j} > 0.$$

Proof. This is equivalent to a condition that the case of Fig.3.1 does not occur in the precedence matrix (in the following section let us restrict our discussion solely on the Wirth-Weber type precedence relations). If it is not the case, there exist  $\phi_1, \phi_2, \phi_3$  such that

$$\begin{aligned}\phi_1 : U &\rightarrow x_1 S_i S_j y_1, & \phi_1 \in P, \\ \phi_2 : V &\rightarrow x_2 S_k S_j y_2, & \phi_2 \in P, \\ \phi_3 : S_k &\Rightarrow^* x_3 S_i, & \phi_3 \in P^*,\end{aligned}$$

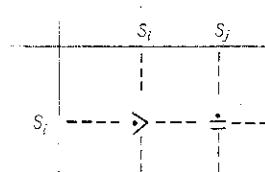


fig.3.1 This is not the case if  $P$  is simple or semi-simple.

or there exist  $\phi_1, \phi_4, \phi_5, \phi_6$  such that

$$\phi_4 : Y \rightarrow x_2 ABy_2, \quad \phi_4 \in P,$$

$$\phi_5 : A \Rightarrow x_3 S_i, \quad \phi_5 \in P^*,$$

$$\phi_6 : B \Rightarrow S_\ell y_3, \quad \phi_6 \in P^*.$$

This contradicts the assumption that  $P$  is simple.

(2) If there exists  $i$  such that  $(p_{i,j}, p_{i,j}) = (0,0)$

then  $L_j \not\ni (0, q_{k,j}), q_{k,j} > 0$ .

Proof. If the case of Fig.3.2 occurs, then there exist

$\phi_1, \phi_2, \phi_3$  such that

$$\phi_1 : U \rightarrow x_1 S_i S_j y_1, \quad \phi_1 \in P,$$

$$\phi_2 : V \rightarrow x_2 S_k C y_2, \quad \phi_2 \in P,$$

$$\phi_3 : C \Rightarrow S_i y_3, \quad \phi_3 \in P^*.$$

This contradicts the assumption that  $P$  is simple.

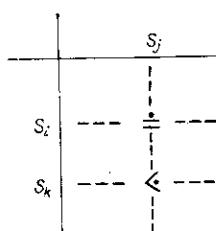


fig.3.2 This is not the case if  $P$  is simple.

(3) If there exists  $j$  such that  $L_j \ni (0, q_{i,j}), q_{i,j} > 0$ ,

then  $K_i \not\ni (p_{i,\ell}, q_{i,\ell}), p_{i,\ell} > 0$ .

Proof. If the case of Fig.3.3 occurs, then since  $S_i \subset S_j$ , there exist  $\phi_1, \phi_2$  such that

$$\begin{array}{ll} \phi_1 : U \rightarrow x_1 S_i V y_1, & \phi_1 \in P, \\ \phi_2 : V \Rightarrow S_j y_2, & \phi_2 \in P^*, \end{array}$$

and since  $S_i \rightarrow S_\ell$ , there exist  $\phi_3, \phi_4$  such that

$$\begin{array}{ll} \phi_3 : W \rightarrow x_3 X S_\ell y_3, & \phi_3 \in P, \\ \phi_4 : X \Rightarrow x_4 S_i, & \phi_4 \in P^*, \end{array}$$

or

$$\begin{array}{ll} \phi_5 : Y \rightarrow x_5 A B y_5, & \phi_5 \in P, \\ \phi_6 : A \Rightarrow x_6 S_i, & \phi_6 \in P^*, \\ \phi_7 : B \Rightarrow S_\ell y_7, & \phi_7 \in P^*. \end{array}$$

$\phi_1$  and  $\phi_4$  or  $\phi_1$  and  $\phi_6$ , however, contradict the assumption.

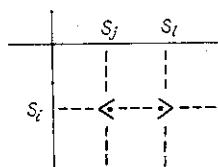


fig.3.3 This is not the case if  $P$  is simple or semi-simple.

(4) If there exist  $i, j, k$  such that  $L_j \ni (p_{k,j}, q_{k,j})$ ,  $p_{k,j} > 0$  and  $L_j \ni (0, q_{i,j})$ ,  $q_{i,j} > 0$ , then  $L_\ell \ni (p_{i,\ell}, q_{i,\ell}) = (0, 0)$  may be the case.

Proof. If the case of Fig.3.4 occurs, then there exist  $\phi_1, \phi_2$  such that

$$\begin{array}{ll} \phi_1 : U \rightarrow x_1 V S_j y_1 & \phi_1 \in P, \\ \phi_2 : V \Rightarrow x_2 S_k, & \phi_2 \in P^*, \end{array}$$

or there exist  $\phi_3, \phi_4, \phi_5$  such that

$$\begin{array}{ll} \phi_3 : U \rightarrow x_3 A B y_3, & \phi_3 \in P, \\ \phi_4 : A \Rightarrow x_4 S_k, & \phi_4 \in P^*, \\ \phi_5 : B \Rightarrow S_j y_5, & \phi_5 \in P^*. \end{array}$$

Moreover there exist  $\phi_6, \phi_7$  such that

$$\begin{array}{ll} \phi_6 : W \rightarrow x_6 S_i C y_6, & \phi_6 \in P, \\ \phi_7 : C \Rightarrow S_j y_7, & \phi_7 \in P^*. \end{array}$$

Since the  $P$  is simple,  $\phi_5 \equiv \phi_7$ ,  $B \equiv C$ ,  $A \equiv S_i$ ,  $\phi_3 \equiv \phi_6$  and  $S_i \neq B (\equiv S_\ell)$  hold.

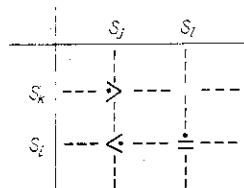


fig.3.4 This may be the case if  $P$  is simple or semi-simple.

#### Lemma 3.4

If a set  $P$  of rewriting rules is simple for a context-free grammar  $G = (V_N, V_T, S, P)$ , then  $f$  and  $g$  of following definition are precedence functions of the grammar  $G$ :

$$f(S_i) = \max_{\lambda} (p_{i,\lambda}), (p_{i,\lambda}, q_{i,\lambda}) \in K_i,$$

$$g(S_j) = \begin{cases} 1/2, & \text{if there exists } k \text{ such that} \\ & L_j \ni (0, q_{k,j}), q_{k,j} > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where  $S_i, S_j, S_k \in (V_N \cup V_T)$ .

Proof.

(1)  $S_i \cdot\cdot S_j$

By (3) of the lemma 3.3,  $f(S_i) = \max_{\ell} (p_{i,\ell}) = 0, \ell=1, \dots, N$ , where  $N$  is the number of elements of  $(V_N \cup V_T)$ . By (2) of the lemma 3.3, there is no  $S_k$  such that  $S_k \neq S_j$ . By the assumption  $S_i \cdot\cdot S_j$ , it follows that  $L_j \ni (0, q_{k,j}), q_{k,j} > 0$  and  $g(S_i) = 1/2$ .

Hence  $f(S_i) < g(S_j)$ .

(2)  $S_i \neq S_j$

Since  $K_i \not\ni (p_{i,\ell}, q_{i,\ell}), p_{i,\ell} > 0$  by (1) of the lemma 3.3,  $f(S_i) = \max_{\ell} (p_{i,\ell}) = 0, \ell=1, \dots, N$ . Since  $L_j \not\ni (0, q_{k,j}), q_{k,j} > 0$  by (2) of the lemma 3.3,  $g(S_j) = 0$ . Thus  $f(S_i) = g(S_j)$ .

(3)  $S_1 \cdot\cdot S_j$

By (3) of the lemma 3.3,  $f(S_k) = \max_{\ell} (p_{k,\ell}) > 1$ . On the otherhand, since  $g(S_j) \leq 1/2$ ,  $f(S_k) > g(S_j)$  holds.

Q.E.D.

Let us show in the following discussion that there exists a wider class of precedence grammars with precedence functions.

Definition 3.5

The set  $P$  of rewriting rules of a precedence grammar  $G = (V_N, V_T, S, P)$  is semi-simple if it satisfies following conditions:

(1)  $p_k > 0$  for all  $k$  if there exists a  $k_0$  such that  $p_{k_0} > 0$ ,  
 where  $p_k$  is an integer of the form  $S_i p_k \alpha \lambda^{q_k} S_j$  for any  $S_i$ ,  
 $S_j \in (V_N \cup V_T)$ .

(2) The pq-matrix whose  $(i,j)$ -element is defined by expres-  
 sions

$$p = \max_k (p_{i,k}), \quad k=1, 2, \dots,$$

$$q = \max_\ell (q_{j,\ell}), \quad \ell=1, 2, \dots$$

satisfies the conditions (1) and (3) of the lemma 3.3.

Where the  $p_{i,k}$  and  $q_{j,\ell}$  are nonnegative integers which  
 satisfy precedence relations  $S_i p_{i,1}^\alpha \lambda^{q_{i,1}} S_j, S_i p_{i,2}^\alpha \lambda^{q_{i,2}} S_j, \dots$ , etc.

When the  $S_i$  or  $S_j$  appears recursively, the value of the  $p_{i,k}$   
 or  $q_{j,\ell}$  is defined as unity. Since the  $P$  is not simple, pair  
 $(p_{i,k}, q_{j,\ell})$  is not unique, so that we have defined the  $(p,q)$   
 element of the matrix in the above fashion.

(3) For the above mentioned pq-matrix,

$K_k \ni (p_{k,\ell}, q_{k,\ell}) = (0, 0)$  or  $\phi$  if there exist  $i, j$   
 such that  $K_i \ni (p_{i,j}, q_{i,j}) = (0, 0)$ ,

$K_k \ni (p_{k,j}, q_{k,j}) = (0, 0)$  and  $K_i \ni (p_{i,\ell}, q_{i,\ell}) = (0, 0)$ .

(4) For the pq-matrix of the above (2),  $i=m$  and  $k=n$  if

there exist  $j, \ell$  such that

$L_j \ni (p_{i,j}, q_{i,j}) = (0, 0)$ ,  $L_j \ni (p_{k,j}, q_{k,j}) = (0, q_{k,j})$ ,

$q_{k,j} > 0$ ,  $L_\ell \ni (p_{m,\ell}, q_{m,\ell}) = (0, 0)$ ,

$L_\ell \ni (p_{n,\ell}, q_{n,\ell}) = (0, q_{n,\ell})$ ,  $q_{n,\ell} > 0$ .

For a precedence grammar  $G$  with semi-simple  $P$  (Fig. 3.5),  
 next theorem holds.

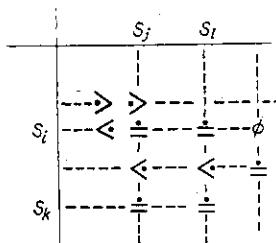


fig.3.5 This may be the case if P is semi-simple.

### Theorem 3.1

A precedence grammar with a semi-simple set of rewriting rules has precedence functions.

Proof. In the sequence of functions f and g defined by following expressions, there exist precedence functions for the grammar G with semi-simple P.

$$f(S_i) = \begin{cases} 1/4, & \text{if there exist } j, k \text{ such that} \\ & K_i \ni (p_{i,j}, q_{i,j}) = (0,0) \text{ and} \\ & L_j \ni (0, q_{k,j}), q_{k,j} > 0, \\ 1/4, & \text{if there exists } j \text{ such that } K_i \ni (p_{i,j}, q_{i,j}) \\ & = (0,0) \text{ and } g(S_j) = 1/4, \\ \max_j(p_{i,j}), & \text{otherwise,} \end{cases}$$
  

$$g(S_j) = \begin{cases} 1/4, & \text{if there exist } i, k \text{ such that } L_j \ni (p_{i,j}, q_{i,j}) \\ & = (0,0) \text{ and } L_j \ni (0, q_{k,j}), q_{k,j} > 0, \\ 1/4, & \text{if there exists } i \text{ such that } L_j \ni (p_{i,j}, q_{i,j}) \\ & = (0,0) \text{ and } f(S_i) = 1/4, \\ 1/2, & \text{if there exists } k \text{ such that } L_j \ni (0,0), \\ & L_j \ni (0, q_{k,j}) \text{ and } q_{k,j} > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where  $s_i, s_j \in (V_N \cup V_T)$ ,  $G = (V_N, V_T, S, P)$ .

For every element of  $V_N \cup V_T$ , let us determine the value of  $f$  and  $g$ . In a case of  $f(s_i) = g(s_j) = 0$  or  $f(s_i) = g(s_j) = 1/4$ , redefinition of the values of  $f$  and  $g$  will occur, but we can redefine them in finite processes.

$$(1) \quad s_i \cdot s_j$$

By  $s_i \cdot s_j$ , it follows that  $(p_{i,j}, q_{i,j}) = (0, q_{i,j})$ ,  $q_{i,j} > 0$ .  $K_i \not\ni (p_{i,t}, q_{i,t})$ ,  $p_{i,t} > 0$  from definition 3.5, (2), we get  $\max_j(p_{i,j}) = 0$ . Hence  $f(s_i) \leq 1/4$ . Since  $q_{i,j} > 0$ ,  $g(s_j) \geq 1/4$ . If we suppose that  $f(s_i) = g(s_j) = 1/4$ , then there are following (a)-(d) cases.

(a) The values of  $f(s_i)$  and  $g(s_j)$  are determined independently of other  $g(s_\ell)$  and  $f(s_k)$ : By the definitions of  $f$  and  $g$ , there exist a row  $K_i$  and a column  $L_j$  (Fig.3.6) such that  $K_i \ni (0,0)$ ,  $L_j \ni (0,0)$  and  $L_j \ni (0, q_{k,j})$ ,  $q_{k,j} > 0$ ,  $i \neq k$ . It is,

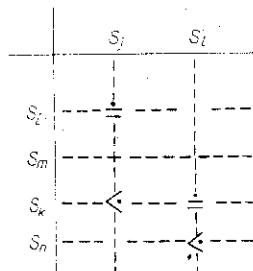


fig.3.6 This is not the case if  $P$  is semi-simple.

however, not the case by (4) of the definition 3.5.

(b) Only the value of  $f(S_i)$  is dependent on  $g(S_\ell)$ ; in this case there are two situations according to the definition of  $g(S_\ell)$ .

(b-1) The value of  $g(S_\ell)$  is independent of other  $f(S_m)$ : there exist  $k, \ell$  such that  $L_\ell \ni (p_{i,\ell}, q_{i,\ell}) = (0,0)$ ,  $L_\ell \ni (0, q_{k,\ell})$ ,  $q_{k,\ell} > 0$ . By the assumption that  $g(S_j)$  does not depend on other  $f(S_k)$ , there also exists  $m$  such that  $L_j \ni (p_{m,j}, q_{m,j}) = (0,0)$ ,  $i \neq m$ . This contradicts with (4) of the definition 3.5.

(b-2) The value of  $g(S_\ell)$  is dependent on  $f(S_m)$ : If the value of  $f(S_m)$  does not depend on other  $g(S_u)$ , this is the same case as the above (a), so that such  $g(S_\ell)$  does not exist.

Suppose that  $g(S_\ell)$  depends on the other  $f(S_m)$ . By the definition of  $f(S_i)$  and  $g(S_\ell)$ , it follows that  $i \neq m$ . Similarly it follows that  $\ell \neq u$ . Continuing this process, we can get  $f$  and  $g$  that are not dependent on other  $g$  and  $f$ , respectively. Since the values of these  $f$  and  $g$  are  $1/4$ , they contradict with (4) of the definition 3.5.

(c) Only the value of  $g(S_j)$  is dependent on other  $f(S_k)$ : The assumption that  $f(S_i) = g(S_j) = 1/4$  leads to a contradiction as in the case of above mentioned (b).

(d) The values of  $f(S_i)$  and  $g(S_j)$  are dependent on other  $g(S_\ell)$  and  $f(S_k)$ : In this case there exist two sequences of functions  $f(S_i) \rightarrow g(S_\ell) \rightarrow \dots$  and  $g(S_j) \rightarrow f(S_k) \rightarrow \dots$

By (3) of the definition 3.5, the two sequences do not become one, i.e., there is no sequence such as  $f(S_i) \rightarrow g(S_\ell)$

becomes one, i.e., there is no sequence such as  $f(S_i) \rightarrow g(S_\ell) \rightarrow f(S_k) \rightarrow \dots$ , (fig.3.7). Hence the two sequences terminate in the case of (a).

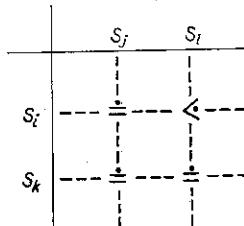


fig.3.7 This is not the case if P is semi-simple.

By (a)-(d), the case  $f(S_i) = g(S_j) = 1/4$  cannot occur if  $S_i \leftrightarrow S_j$ . Hence  $f(S_i) < g(S_j)$ .

(2)  $S_i \neq S_j$

From the condition  $S_i \neq S_j$ , it follows that  $(p_{i,j}, q_{i,j}) = (0,0)$ . Since  $K_i \not\in (p_{i,\ell}, q_{i,\ell})$ ,  $p_{i,\ell} > 0$  by (2) of the definition 3.5,  $\max_j (p_{i,j}) = 0$ . If  $L_j \geq (0, q_{k,j})$ ,  $q_{k,j} > 0$ , then  $f(S_i) = g(S_j) = 1/4$  and  $L_j \geq (0, q_{k,j})$ ,  $q_{k,j} > 0$ , hence  $f(S_i)$  is dependent on other  $g(S_\ell)$ .  $f(S_i) = g(S_\ell)$  holds if  $g(S_j) = 1/4$ .

Suppose  $g(S_j) = 0$  by the assumption  $L_j \not\in (0, q_{k,j})$ ,  $q_{k,j} > 0$ . In this case we can change without contradiction the value of  $g(S_j)$  to  $g(S_j) = f(S_i) = 1/4$ , because for  $S_k$  such that  $S_k \neq S_j$ , by (3) of the definition 3.5 there does not occur  $S_k \leftrightarrow S_\ell$ .

Hence the  $f(S_k)$  does not change the expression  $f(S_i) = g(S_j) = 1/4$ . Similarly we can show that there is no contradiction in changing expression  $f(S_i) = 0$  and  $g(S_j) = 1/4$  to  $f(S_i) = g(S_j) = 1/4$ . In other cases the  $f$  and  $g$  satisfy  $f(S_i) = g(S_j) = 0$ .

Hence  $f(S_i) = g(S_j)$ .

(3)  $S_i \rightarrow S_j$

By (2) of the definition 3.5, it follows that  $K_i \ni (0, q_{i,\ell})$ ,  $q_{i,\ell} \geq 0$  and  $f(S_i) = \max_{\ell} (P_{i,\ell}) > 1$ . On the otherhand,  $g(S_j) \leq 1/2$ . Hence  $f(S_i) > g(S_j)$ . Q.E.D.

As the result of the theorem, we can get two corollaries.

### Corollary 3.1

If the principal minors of a pq-matrix of a precedence grammar are the pq-matrices of the theorem and other elements are empty relations, then the precedence grammar has precedence functions.

The corollary may seem trivial, but it plays an important role in applications. Using this, we can blow up a precedence grammar with precedence functions to a big one.

### Corollary 3.2

In the generalized precedence grammars (grammars of A. Colmerauer), there exists a family of precedence grammars with precedence functions. Since the precedence relations of a precedence grammars are defined as  $\alpha \Leftarrow$ ,  $\alpha \lambda^+ C \Leftarrow$ ,  $P \alpha \lambda^+ C \Leftarrow \cup .$ , the pq-matrix may be specified by assuming  $A \rho^P \alpha B$  instead of  $A \rho^P \alpha \lambda^Q B$  if  $A \rightarrow B$  or  $A \alpha \lambda^Q B$  instead of  $A \rho^P \alpha \lambda^Q B$  if  $A \Leftarrow B$ .

### 3.3 Comparative Results

The language difined by the family of theorem 3.1 may seem too restricted for practical applications. We can however show that the sets of rewriting rules of grammars

which describe currently existing typical programming languages such as Fortran IV and Algol-like language PL360 are almost semi-simple.

Our experience indicates that almost all grammars have no precedence functions in their original forms. The lemmas and the theorem of previous section give us a method to modify a precedence grammar to a precedence grammar with precedence functions.

Since our computer program for precedence relation calculation [Fuji 70] cannot analyze a grammar with more than 300 rewriting rules, we have skeletonized the Fortran IV grammar ignoring types of variables and then have partitioned it into five independent grammars. A grammar for <expr> is shown as fig.3.8. We have computed precedence relations and functions of these grammars by Floyd-Wirth algorithm [Wir 65]. The results are shown in fig.3.9.

For any given context-free grammar, we can find equivalent precedence grammars [Fis 69, Lear 70, McAf 72], but in our modification we used the technique to introduce new terminal symbols to get precedence grammars. Thus the languages obtained are not equivalent to the original ones.

```

1  'EXPRESSION'      ::= 'ARITHMETIC EXPR'
2                                ::= 'LOGICAL EXPR'
3  'ARITHMETIC EXPR'   ::= 'ARITHMETIC EXPR*'
4  'ARITHMETIC EXPR*'  ::= 'TERM'
5                                ::= '+' 'TERM'
6                                ::= '-' 'TERM'
7                                ::= 'ARITHMETIC EXPR*' '+' 'TERM'
8                                ::= 'ARITHMETIC EXPR*' '-' 'TERM'
9  'TERM'                 ::= 'TERM*'
10     ::= 'TERMINAL'
11     ::= 'TERM*' '* 'FACTOR'
12     ::= 'TERM*' '/' 'FACTOR'
13  'FACTOR'              ::= 'FACTOR*'
14  'FACTOR*'             ::= 'PRIMARY'
15                                ::= 'FACTOR*' '** 'PRIMARY'
16  'PRIMARY'              ::= 'PRIMARY.1'
17                                ::= ('ARITHMETIC EXPR')
18  'PRIMARY.1'            ::= CONSTANT
19                                ::= VARIABLE
20                                ::= FUNCTN DESIGNATOR
21                                ::= IDENTIFIER
22                                ::= ARRAY-IDENTIFIER
23                                ::= ARRAY-IDENTIFIER ('SUBSCRIPT LIST')
24  'SUBSCRIPT LIST'       ::= 'SUBSCRIPT LIST*'
25  'SUBSCRIPT LIST*'     ::= 'SUBSCRIPT'
26                                ::= 'SUBSCRIPT LIST*' ',' 'SUBSCRIPT'
27  'SUBSCRIPT'            ::= 'ARITHMETIC EXPR'
28                                ::= 'LOGICAL EXPR'
29  'LOGICAL EXPR*'        ::= 'LOGICAL TERM'
30                                ::= 'LOGICAL EXPR*' .OR. 'LOGICAL TERM'
31  'LOGICAL TERM'         ::= 'LOGICAL TERM*'
32  'LOGICAL TERM*'        ::= 'LOGICAL FACTOR'
33                                ::= 'LOGICAL TERM*' .AND. 'LOGICAL FACTOR'
34  'LOGICAL FACTOR'       ::= 'LOGICAL PRIMARY'
35                                ::= .NOT. 'LOGICAL PRIMARY'
36  'LOGICAL PRIMARY'      ::= 'PRIMARY.1'
37                                ::= 'RELATIONAL EXPR'
38                                ::= ('LOGICAL EXPR')
39  'RELATIONAL EXPR'       ::= 'ARITHMETIC EXPR*' 'REL OP' 'ARITHMETIC EXPR**'
40  'ARITHMETIC EXPR**'    ::= 'ARITHMETIC EXPR*'
41  'REL OP'                ::= .LT.
42                                ::= .LE.
43                                ::= .EQ.
44                                ::= .NE.
45                                ::= .GT.
46                                ::= .GE.
47  'FUNCTN DESIGNATOR'    ::= FUNCTN-IDENTIFIR ('ACTUAL ARG LIST')
48  'ACTUAL ARG LIST'       ::= 'ACTUAL ARG LIST*'
49  'ACTUAL ARG LIST*'     ::= 'ACTUAL ARGUMENT'
50                                ::= 'ACTUAL ARG LIST*' ',' 'ACTUAL ARGUMENT'
51  'ACTUAL ARGUMENT'       ::= 'ARITHMETIC EXPR'
52                                ::= 'LOGICAL EXPR'
53                                ::= EXT-FUN-NAME

```

fig. 3.8 Expression of Fortran IV described by  
a precedence grammar

S	P	$V_N$	$V_T$	$T_p$	$T_f$	K	R	GT	EQ	LT	GT2	LT2
F O R T R A N	Main control	104	49	65	4	1	1	366	143	85	138	128
	Declarations	60	30	24	0	2	7	157	74	43	40	26
	Format st.	49	24	30	1	1	1	433	243	37	153	126
	I/O statements	54	24	25	2	1	4	149	47	37	65	43
	Expression	53	26	22	0	0	6	536	322	31	183	250
	PL360	154	65	64	0	0	13	1531	975	117	439	498
												183

fig. 3.9 Precedence relations of FORTRAN IV and PL360 languages.

Notations of fig. 3.9 .

For a grammar  $G = (V_N, V_T, S, P)$ , $T_p$ : number of newly introduced terminal symbols to make  $G$  a precedence grammar, $T_f$ : number of terminal symbols to make  $G$  a precedence grammar with precedence functions,

K : number of columns which have the form of fig.3.1 or fig.3.3(a few cases of (3) and (4) of the definition 3.5 are not counted here.),

R : number of relations,

GT: number of relations which satisfy  $A \triangleright B$  for  $A, B \in (V_N \cup V_T)$ ,EQ: number of relations which satisfy  $A \equiv B$  for  $A, B \in (V_N \cup V_T)$ ,LT: number of relations which satisfy  $A \triangleleft B$  for  $A, B \in (V_N \cup V_T)$ ,GT2: number of relations which satisfy  $A \triangleright B$  for  $A \in (V_N \cup V_T)$ , and  $B \notin V_T$ ,LT2: number of relations which satisfy  $A \triangleleft B$  for  $A \notin (V_N \cup V_T)$ , and  $B \in V_T$ .

When a precedence grammar  $G = (V_N, V_T, S, P)$  has no precedence functions, we can get a new precedence grammar  $G' = (V_N, V_T \cup \{a, b, c, \dots\}, S, P')$  with semi-simple  $P'$  by introducing new terminal symbols  $a, b, c, \dots$ . The introduction of new terminal symbols becomes possible when the input string is preprocessed and is changed to fit into the  $L(G')$ . This means that the scanning routine for input strings must be rewritten when the grammar  $G'$  is changed to a new grammar  $G''$ . This is the demerit of the technique used in this chapter.

On the otherhand, the lemmas and theorem 3.1 give us an intuitive insight as to whether what type of grammars, or more precisely, what type of set of rewriting rules has precedence functions.

In next chapter let us prove that we can go furthermore, that is, we can get a new precedence grammar with precedence functions without introducing new terminal symbols. But the result of this chapter is still useful when we have to construct a new precedence grammar with precedence functions with minimum modifications because the characteristics represented by the lemmas and the theorem are closely related to any precedence grammar with precedence functions.

### 3.4 Concluding Remarks

From the above discussion we may conclude as follows:

- (1) There exists a family of precedence grammars with precedence functions which is a good approximation of currently existing programming languages such as Fortran

IV and Algol-like language PL360 in its structure.

- (2) The lemmas and the theorem play a role of approximation theory to obtain precedence grammars with precedence functions.
- (3) The introduction of new terminal symbols is inconvenient because we must change the scanning routine of input strings according to the introduction of new terminal symbols.

## Chapter 4

### On Existence of Precedence Functions

As we have seen in the chapter 2, practical applications of precedence grammars meet with a difficulty since sizes of precedence matrices used in the analyses of the grammars become so big. The one of methods to solve the problem is to use precedence functions. Precedence functions, however, have not been considered as tools for analyses of the grammars since precedence grammars generally have no precedence functions. But they are very useful in analysis and construction of a compiler if they exist. Fortunately there exist equivalent precedence grammars with precedence functions for a given precedence grammar and the grammars are natural extensions of the given grammar. The author uses the fact in construction of a compiler of a modified PL360 language named GPL [Asa 72a, Asa 72b].

#### 4.1 Existence of Precedence Functions

In this section let us show the existence of equivalent precedence grammars with precedence functions for any given precedence grammar.

Let  $G = (V_N, V_T, S, P)$  be given precedence grammars.

Two functions  $f$  and  $g$  with  $N$  values respectively are called precedence functions of  $G$  if they satisfy following relations for any  $S_i, S_j \in (V_N \cup V_T)$  ;

if  $S_i < S_j$  then  $f(S_i) < g(S_j)$ ,

if  $S_i \neq S_j$  then  $f(S_i) = g(S_j)$ ,

if  $S_i > S_j$  then  $f(S_i) > g(S_j)$ ,

where  $N$  is the number of elements of  $(V_N \cup V_T)$ .

In this discussion the grammar  $G$  may or may not have precedence functions.

Definition 4.1 Symbols  $f_i$  and  $g_j$

$f_i$  and  $g_j$  are symbols which have one-to-one correspondence with function values  $f(S_i)$  and  $g(S_j)$ , respectively. These symbols have precedence  $f_i <^\circ g_j$  if  $S_i <^\circ S_j$ ,  $f_i =^\circ g_j$  if  $S_i =^\circ S_j$ , or  $f_i >^\circ g_j$  if  $S_i >^\circ S_j$ .

We sometimes denote  $f_i \leq^\circ g_j$  if  $f_i$  and  $g_j$  have precedence  $f_i <^\circ g_j$  or  $f_i =^\circ g_j$ .

Definition 4.2 Sets  $H_1, \dots, H_n$

We denote by  $H_1$  a set  $\{f_1, \dots, f_n, g_1\}$  and by  $H_n$  a set  $\{f_1, \dots, f_n, g_1, \dots, g_n\}$ .

Definition 4.3 Cycle, Monotone Cycle and Set  $B(h)$

If there exists a sequence  $h_1 R_1 h_2 R_2 \dots h_m R_m h_1$  for non-empty relation  $R_i \in \{\cdot>, <^\circ, =^\circ\}$  and  $H_n \ni h_j, j=1, \dots, m$ , we call the sequence as a cycle of  $h_1$ . This cycle is said to be a monotone cycle if a transitive relation  $h_1 <^\circ h_1$  holds for the cycle. In this case we say that the set  $H_n$  contains a monotone cycle. We also denote  $h \in B(h)$  if there exists a monotone cycle of  $h$  and  $h \notin B(h)$  otherwise.

Theorem 4.1

A precedence grammar  $G$  has precedence functions if and only if every element  $h$  in the set  $H$  of the definition 4.2

for the grammar G has no monotone cycle.

Definition 4.4  $f_i$ -line,  $g(S_j, f_i)$ ,  $L(f_i)$ ,  $U(f_i)$

Let us assign integer values  $g(S_j)$  and  $f(S_i)$  to symbols  $g_j$  and  $f_i$  of  $i$  th row  $K_i$  of the precedence matrix M. We adjust the values of  $f(S_i)$  and  $g(S_j)$  to  $f(S_i) \leq g(S_j)$  if  $f_i < g_j$ ,  $f(S_i) = g(S_j)$  if  $f_i = g_j$  and  $f(S_i) > g(S_j)$  if  $f_i > g_j$ , beginning with the first row of M. We call a set  $\{f_i, f(S_i), g_1, g(S_1), \dots, g_N, g(S_N)\}$  and the correspondences  $f_i \leftrightarrow f(S_i)$ ,  $g_j \leftrightarrow g(S_j)$  thus obtained as  $f_i$ -line. The value  $g(S_i)$  on the  $f_i$ -line is sometimes denoted as  $g(S_i, f_i)$ .

Sets  $L(f_i)$  and  $U(f_i)$  are defined by

$$L(f_i) = \{g_j | f_i > g_j, \text{ or } f_i \neq g_j, g_j \in K_i\},$$

$$U(f_i) = \{g_j | f_i \leq g_j, g_j \in K_i\}.$$

Definition 4.5 Slide to the right

We say that a subset A of  $H_N$  is slid to the right by a value  $h(S_i)$  when for all elements  $h$  of A,  $h(S_j)$  of  $h_j$  ( $h_j \in A$ ) is set to  $h(S_i) + 1$  if  $h_i < h_j$  and  $h(S_k)$  of  $h_k$  ( $h_k \in A$ ) is set to  $h(S_j)$  or  $h(S_j) + 1$  if  $h_j \neq h_k$ , or  $h_j < h_k$ , accordingly.

Proof of theorem 1.

Since it is obvious that the condition is necessary, let us show that it is sufficient. The proof procedure is sketched as the following;

- (1) we can make a new  $f_1 \cdot f_2$ -line from  $f_1$  and  $f_2$ -lines if  $h \notin B(h)$ ,  $h \in H_2$ ,

- (2) assuming that we have  $f_1 \dots f_n$ -line under the assumption  $h \notin B(h)$ ,  $h \in H_n$ ,  $n=N-1$ ,
- (3) we show that we can make  $f_1 \dots f_n$ -line from  $f_1 \dots f_{N-1}$ -line and  $f_N$ -line for  $n=N$  if  $h \notin B(h)$ ,  $h \in H_n$ .

Step 1.

(1)  $L(f_1) \cap L(f_2) \ni g_i$

(i) If  $g(S_i, f_1) = g(S_i, f_2)$ , then  $g(S_i)$  on  $f_1$  and  $f_2$ -lines remain unchanged.

(ii) If  $g(S_i, f_1) > g(S_i, f_2)$ , then let  $g(S_i, f_2) = g(S_i, f_1)$  and slide  $U(f_2)$  of  $f_2$ -line to the right. In this case  $U(f_2) \cap L(f_1) = \emptyset$  (empty) or  $L(f_2) \cap U(f_1) = \emptyset$  since assumptions  $U(f_2) \cap L(f_1) \ni g_j$ ,  $L(f_2) \cap U(f_1) \ni g_k$  and redefinition of  $g(S_i, f_2)$  cause a redefinition sequence

$$g(S_i, f_1) \rightarrow g(S_i, f_2) \rightarrow g(S_j, f_2) \rightarrow g(S_j, f_1) \rightarrow g(S_k, f_1) \rightarrow \\ g(S_j, f_2) \rightarrow \dots$$

This contradicts the assumptions that  $g_j \notin B(g_j)$ ,  $g_k \notin B(g_k)$ .

(iii) If  $g(S_i, f_1) < g(S_i, f_2)$ , then let  $g(S_i, f_1) = g(S_i, f_2)$  and slide  $U(f_1)$  to the right. As is the above (ii), this does not cause a loop of definitions for  $g_j$ .

(2)  $L(f_1) \cap U(f_2) \ni g_i$

Let  $L(f_1) = L(f_1) \cup \{h | h \leftrightarrow g_i, h \in H_2\}$ , and

$$U(f_2) = U(f_2) \cup \{h | g_i \leftrightarrow h, h \in H_2\}.$$

(i) If  $g(S_i, f_1) = g(S_i, f_2)$ , then values of  $g_i$  on  $f_1$  and  $f_2$ -lines remain unchanged.

(ii) If  $g(S_i, f_1) > g(S_i, f_2)$ , then let  $g(S_i, f_2) = g(S_i, f_1)$  and slide  $g_i$  on  $f_2$ -line to the right. This definition of  $g(S_i, f_2)$  does not change other values of  $g_j$ .

(iii) If  $g(S_i, f_1) < g(S_i, f_2)$ , then let  $g(S_i, f_1) = g(S_i, f_2)$  and slide  $U(f_1)$  of  $f_1$ -line to the right. As is shown in (1) and (2),  $U(f_1) \cap L(f_2) = \emptyset$  and the definition of  $g(S_i, f_1)$  does not cause a loop.

(3)  $U(f_1) \cap L(f_2) \ni g_i$

We can define the value of  $g_i$  as the above (2).

(4)  $U(f_1) \cap U(f_2) \ni g_i$

(i) If  $g(S_i, f_1) = g(S_i, f_2)$ , then values of  $g_i$  on  $f_1$  and  $f_2$ -lines remain unchanged.

(ii) If  $g(S_i, f_1) > g(S_i, f_2)$ , then setting  $g(S_i, f_2) \leq g(S_i, f_1)$ , we slide  $g_i$  on  $f_2$ -line to the right. By the assumption that  $L(f_1) \not\ni g_i$ ,  $L(f_2) \not\ni g_i$ , only value of  $g_i$  on  $f_2$ -line is redefined. Hence this redefinition of the value of  $g_i$  does not cause a loop.

(iii) If  $g(S_i, f_1) < g(S_i, f_2)$ , then setting  $g(S_i, f_1) \leq g(S_i, f_2)$ , we slide  $g_i$  on  $f_1$ -line to the right. As is mentioned in the above (4)-(ii), this redefinition of value does not cause a loop.

(5) Product set is empty

The value of  $g_i$  on  $f_1$ -line remains unchanged.

Definition 8.  $f_1 \cdot f_2$ -line,  $L(f_1/f_1 \cdot f_2)$ ,  $U(f_1/f_1 \cdot f_2)$ ,  $g(S_i, f_1 \cdot f_2)$   
 Iterations of operations of the above (1)-(5) for  $S_i$  ( $i = 1, \dots, n$ ) make values of  $g(S_i, f_1)$  and  $g(S_i, f_2)$  equal on  $f_1$  and  $f_2$ -lines. Hence we can make a line  $f_1 \cdot f_2$  from  $f_1$  and  $f_2$ -lines and denote  $g_i$  on the line with  $g(S_i, f_1 \cdot f_2)$ ,  $L(f_1 \cdot f_2)$  and  $U(f_1 \cdot f_2)$  with  $L(f_1/f_1 \cdot f_2)$ ,  $U(f_1/f_1 \cdot f_2)$ , respectively.

Step 2.

Assume that we have  $f_1 \dots f_{n-1}$ -line. We show that we can make  $f_1 \dots f_n$ -line from  $f_1 \dots f_{n-1}$  and  $f_n$ -lines.

$$(6) L(f_n) \cap L(f_p/f_1 \dots f_{n-1}) \ni g_i, (1 \leq p \leq n-1)$$

Let  $L(f_p/f_1 \dots f_{n-1}) = L(f_p/f_1 \dots f_{n-1}) \cup \{h | h \leq g_i, h \in H_n\}$  and

$$U(g_i/f_1 \dots f_{n-1}) = U(g_i/f_1 \dots f_{n-1}) \cup \{h | g_i \leq h, h \in H_n\}.$$

(i) If  $g(S_i, f_n) = g(S_i, f_1 \dots f_{n-1})$ , then values of  $g(S_i, f_n)$  and  $g(S_i, f_1 \dots f_{n-1})$  remain unchanged.

(ii) If  $g(S_i, f_n) > g(S_i, f_1 \dots f_{n-1})$ , then let  $g(S_i, f_1 \dots f_{n-1}) = g(S_i, f_n)$  and slide  $U(g_i/f_1 \dots f_{n-1})$  of  $f_1 \dots f_{n-1}$ -line to the right by  $g_i$ . By the assumption  $g_i \notin B(g_i)$ , we get  $g_i \not\leq g_i$  on  $f_1 \dots f_{n-1}$ -line.

If  $g_i \in L(f_n)$ ,  $g_k \in U(f_n)$ ,  $g_j \in U(f_q/f_1 \dots f_{n-1})$  and  $g_k \in L(f_q/f_1 \dots f_{n-1})$ , then we have  $g_k \leq f_n \leq g_j (1 \leq q \leq n-1)$  on  $f_1 \dots f_{n-1}$ -line and  $g_j \leq f_n \leq g_k$  on  $f_n$ -line. In this case the definition of  $g_j$  causes loops but it contradicts the assumption that  $g_j \notin B(g_j)$ .

(iii) If  $g(S_i, f_n) < g(S_i, f_1 \dots f_{n-1})$ , then let  $g(S_i, f_n) = g(S_i, f_1 \dots f_{n-1})$  and slide  $f_n \cup U(f_n)$  to the right by  $g(S_i, f_n)$ . If there exist  $g_j$  and  $g_k$  such that  $g_k \in (U(f_n) \cap L(f_q/f_1 \dots f_{n-1}))$  and  $g_j \in (L(f_n) \cap U(f_q/f_1 \dots f_{n-1}))$ , ( $1 \leq q \leq n-1$ ), they satisfy relations  $g_k \leq f_q \leq g_j$  on  $f_1 \dots f_{n-1}$ -line and  $g_j \leq f_n \leq g_k$  on  $f_n$ -line. This contradicts the assumption that  $g_j \notin B(g_j)$  and  $g_k \notin B(g_k)$ .

$$(7) L(f_n) \cap U(f_q/f_1 \dots f_{n-1}) \ni g_j$$

Let  $U(f_p/f_1 \dots f_{n-1}) = U(f_p/f_1 \dots f_{n-1}) \cup \{h | g_i \leq h, h \in H_n\}$  and

$U(g_i/f_1 \dots f_{n-1}) = U(g_i/f_1 \dots f_{n-1}) \cup \{h | g_i < h, h \in H_n\}.$

If there exists  $f_q$  on  $f_1 \dots f_{n-1}$ -line such that  $g_j < f_q$ , ( $1 \leq q \leq n-1$ ), the same discussion as of (6) follows. If no such  $f_q$  exists on  $f_1 \dots f_{n-1}$ -line, there is no trouble to slide  $g_i$  to the right. Similar discussion as of (6)(iii) follows when we slide  $f_n \cup U(f_n)$  on  $f_n$ -line to the right.

$$(8) \quad U(f_n) \cap L(f_p/f_1 \dots f_{n-1}) \ni g_i$$

Let  $U(g_i/f_1 \dots f_{n-1}) = U(g_i/f_1 \dots f_{n-1}) \cup \{h | g_i < h, h \in H_n\}$  and

$$U(f_n) = U(f_n) \cup \{h | g_i < h, h \in H_n\}.$$

We slide  $g_i$  to the right according to  $g(S_i, f_n) \geq g(S_i, f_1 \dots f_{n-1})$ . By the assumption that  $g_j \notin B(g_j)$ , we can define the value of  $g_i$  in finite operations.

$$(9) \quad U(f_n) \cap U(f_p/f_1 \dots f_{n-1}) \ni g_i$$

Let  $U(g_i/f_1 \dots f_{n-1}) = U(g_i/f_1 \dots f_{n-1}) \cup \{h | g_i < h, h \in H_n\}$  and

$$U(f_n) = U(f_n) \cup \{h | g_i < h, h \in H_n\}.$$

We slide  $g_i$  to the right according to  $g(S_i, f_n) \geq g(S_i, f_1 \dots f_{n-1})$ . By the assumption that  $g_j \notin B(g_j)$ , we can define the value of  $g_i$  in finite operations.

(10) Product set is empty.

The value of  $g_i$  on  $f_1 \dots f_{n-1}$  or  $f_n$ -line remains unchanged.

Iterating the above operations we can set all elements of  $H_n$  on a real line. Hence there exist precedence functions if  $h \notin B(h)$  and  $h \in H_N$ .

The precedence grammar  $G$  of following example has no precedence functions since it includes a monotone cycle  $f(\lambda) \prec g([]) \prec f([]) = g([]) \prec f(\lambda)$  if we assume  $\rightarrow \supset p^+ \alpha \lambda^+$ .

#### Example 4.1

For simple precedence grammar  $G = (V_N, V_T, A, P)$ ,  $V_N = \{A, B, C\}$ ,  $V_T = \{[], \lambda\}$ ,  $P = \{\phi_1, \dots, \phi_6\}$ ,  $\phi_1 : A \rightarrow CB$ ,  $\phi_2 : A \rightarrow []$ ,  $\phi_3 : B \rightarrow \lambda$ ,  $\phi_4 : B \rightarrow \lambda A$ ,  $\phi_5 : B \rightarrow A$ ,  $\phi_6 : C \rightarrow []$ , assuming  $\rightarrow \supset p^+ \alpha \lambda^+$ , we can get its precedence matrix as fig.4.1

	A	B	C	)	[	$\lambda$
A				$\supset$		
B				$\doteq$		
C	$\doteq$	$\doteq$	$\doteq$	$\doteq$	$\doteq$	
)				$\supset$		
[	$\supset$	$\supset$	$\supset$	$\doteq$	$\supset$	$\supset$
$\lambda$	$\doteq$		$\supset$	$\supset$	$\doteq$	

fig.4.1 Precedence matrix for  $G$

By the theorem 4.1 a precedence grammar  $G$  has no precedence function if the set  $H = \{f_i, g_j | i, j = 1, \dots, n\}$  of a precedence grammar contains monotone cycles. We can, however, obtain a new set  $H'$  which has no monotone cycle by transforming  $H$  to  $H' = \{f_i, g_j | i, j = 1, \dots, m\}$ ,  $m \geq n$ ,  $H' \supseteq H$  and  $G$  to  $G'$ . Let us note that this transformation preserves a condition  $L(G) = L(G')$ . From this fact we have following theorem.

Theorem 4.2

For a given precedence grammar, there exist equivalent precedence grammars with precedence functions.

Before giving the proof let us sketch the proof procedures briefly.

(1) Suppose that the set  $H = \{f_i, g_j \mid i, j = 1, \dots, n\}$  of a precedence grammar  $G$  contains a monotone cycle  $f_i <^\circ g_k <^\circ f_k <^\circ \dots <^\circ f_i$ .

(2) We can remove the monotone cycle by adding new variables and rewriting rules to  $G$  as following:

(i) When a relation  $f_i <^\circ g_j$  is induced by a relation  $f_i \alpha \lambda^+ g_j$ , we can change both relations to  $f_i \rho^+ \alpha \lambda^+ g_j$  and to  $f_i \rightarrow g_j$  by introducing new variables and new rewriting rules.

(ii) When a relation  $f_i \rightarrow g_j$  is induced by  $f_i \rho^+ \alpha \lambda^+ g_j$ , we can change the relation to  $f_i <^\circ g_j$ .

(iii) When a relation  $g_j <^\circ f_k$  is induced by a relation  $f_k \rho^+ \alpha g_j$ , we can change both to  $f_k \rho^+ \alpha \lambda^+ g_j$  and to  $f_k <^\circ g_j$ , respectively.

(iv) When a relation  $g_j <^\circ f_k$  is induced by a relation  $f_k \rho^+ \alpha \lambda^+ g_j$ , we can change the relation to  $f_k <^\circ g_j$ .

(3) We can remove monotone cycles by the above procedures.

A difficulty, however, arises since removal of one monotone cycle may induce another monotone cycle.

For example, in the following two cycles, a monotone cycle  $C_1$  may be removed by changing  $f_1 <^\circ g_1$  to  $f_1 \rightarrow g_1$ , but the change may induce another monotone cycle  $C_2$ :

$$C_1 : f_1 \leftarrow g_1 \leftarrow f_2 \leftarrow g_2 \leftarrow f_1,$$

$$C_2 : f_1 \leftarrow g_1 \rightarrow f_3 \rightarrow g_3 \rightarrow f_1.$$

Let us, however, change  $C_1$  and  $C_2$  to  $C_1'$  and to  $C_2'$ ;

$$C_1' : f_1 \rightarrow g_1 \leftarrow f_2 \leftarrow g_2 \leftarrow f_1,$$

$$C_2' : f_1 \rightarrow g_1 \rightarrow f_3 \rightarrow g_3 \rightarrow f_1.$$

Then let us change  $g_1 \rightarrow f_3$  to  $g_1 \leftarrow f_3$ , and do the same operations as the above for a newly induced monotone cycle.

Continuing these operations we can obtain a relation

$g_1 \leq f_i$  for any  $f_i$  such that  $f_i R g_1$ , where  $R$  is a precedence relation.

- (4) We have thus removed all monotone cycles of  $g_1$ . By the same operation we can remove all monotone cycles of  $g_2$ ,  
 $\dots, g_n$ .

These operations increase variables and rewriting rules of the grammar, so that the set  $H$  changes to  $H' = \{f_i, g_j\}$   
 $i = 1, \dots, n, n+1, \dots, m\}$ .

Since monotone cycles of  $H'$  can be removed by removing those of  $H$ , the grammar for  $H'$  has precedence functions.

### Proof of theorem 4.2

We can obtain the above theorem by following lemma 1 and lemma 2.

#### Lemma 1.

A grammar  $G^{(n+1)}$  which is derived from a precedence grammar  $G^{(n)} = (V_N^{(n)}, V_T, S, P^{(n)})$  by following operations (P1)-(P3) satisfies conditions

$$(1) L(G^{(n+1)}) = L(G^{(n)}),$$

$$(2) G^{(n+1)} \text{ is a precedence grammar.}$$

(P1) If there exists a rule and a derivation sequence such that

$\phi_1: D \rightarrow x_1 CBy_1$  and  $C \Rightarrow x_2 A$  ( $x_2 \neq \phi$ ),

then replace the rule  $\phi_1$  by new rules  $\phi_1'$ ,  $\phi_2''$

$\phi_1': D \rightarrow x_1 CB'y_1$ ,  $\phi_2'': B' \rightarrow B$

by introducing a new nonterminal  $B'$ ,

where  $\phi_1'$ ,  $\phi_2'' \notin P^{(n)}$  and  $B' \notin (V_N^{(n)} \cup V_T)$ .

(P2) If there exists a rule and a derivation sequence such

that  $\phi_1: D \rightarrow x_1 ACy_1$  and  $C \Rightarrow By_2$  ( $y_2 \neq \phi$ ),

then replace the rule  $\phi_1$  by new rules  $\phi_1'$ ,  $\phi_2''$

$\phi_1': D \rightarrow x_1 A'By_1$ ,  $\phi_2'': A' \rightarrow A$

by introducing a new nonterminal  $A'$ ,

where  $\phi_1'$ ,  $\phi_2'' \notin P^{(n)}$  and  $A' \notin (V_N^{(n)} \cup V_T)$ .

(P3) If there exists a rule such that

$\phi_1: D \rightarrow x_1 ABy_1$  ( $A \neq \phi \neq B$ ),

then replace the rule  $\phi_1$  by new rules  $\phi_1'$ ,  $\phi_2''$

$\phi_1': D \rightarrow x_1 CB'y_1$ ,  $\phi_2'': C \rightarrow A$ ,

or

$\phi_1': D \rightarrow x_1 ACy_1$ ,  $\phi_2'': C \rightarrow B$ ,

where  $\phi_1'$ ,  $\phi_2'' \notin P^{(n)}$  and  $C \notin (V_N^{(n)} \cup V_T)$ .

Proof for (1): It is obvious that  $L(G^{(n+1)}) \supset L(G^{(n)})$  and  $L(G^{(n+1)}) \subset L(G^{(n)})$ . Hence  $L(G^{(n+1)}) = L(G^{(n)})$ .

Proof for (2): The operation (P1) replaces relation  $F_i \rho^+ \alpha B$  by relations  $F_i \rho^+ \alpha B'$  and  $F_i \rho^+ \alpha \lambda^+ B$  for all  $F_i$  such that  $C \Rightarrow xF_i$ . Since  $B'$  is a new symbol, there exists no relation such that  $F_i \alpha B'$ . Hence  $B'$  satisfies the conditions (2') and (3) of 2.6.3 of chapter 2. Similar discussion is applicable to (P2) and (P3).

Lemma 2.

There exist precedence grammars with precedence functions in the set of grammars  $G^{(n)}$  of the above lemma 1.

Proof: Let us assume that  $G^{(1)}$  is a precedence grammar without precedence functions.

Let  $f_1 \leftrightarrow g_2 \leftrightarrow \dots \leftrightarrow f_1$  be one of monotone cycles of  $G^{(1)}$ .

- (1) If all precedence relations of this monotone cycle is of the form  $\rho^+ \alpha \lambda^+$ , we can remove this cycle replacing  $\leftrightarrow$  by  $\rightarrow$ , or  $\rightarrow$  by  $\leftrightarrow$ , respectively.
- (2) If none of the relations is of the form  $\rho^+ \alpha \lambda^+$ , modifying  $G^{(1)}$  by operations of the lemma 1 and inverting precedence relations we can remove the monotone cycle.
- (3) If the inversion of precedence relations causes no new monotone cycles,  $G^{(n)}$  has precedence functions.
- (4) If the inversion of a precedence relation from  $f_1 \leftrightarrow g_2$  to  $f_1 \rightarrow g_2$  causes a new monotone cycle  $f_1 \rightarrow g_2 \rightarrow f_i \rightarrow \dots \rightarrow f_1$ , ( $f_i \neq f_1$ ), then we can modify the grammar so that a relation  $g_2 \leftrightarrow f_i$  holds. Since we can modify the grammar such that a relation  $g_2 \leftrightarrow f_i$  holds for any  $f_i$ , monotone cycles of  $g_2$  are always removable.

Iterating the above procedures (1), (2) and (4), we can obtain  $G^{(n)}$  with precedence functions.

Example 4.2

Introducing a new variable  $A'$  and changing the rewriting rule  $\phi_2 : A \rightarrow [ ]$  to  $\phi'_2 : A \rightarrow A']$ ,  $\phi''_2 : A' \rightarrow [$  of the grammar  $G$  of example 4.1, we can obtain a grammar  $G'$  with precedence functions as following:

S =	A	A'	B	C	]	[	$\lambda$
$f(S) =$	2	1	1	1	2	4	2
$g(S) =$	2	3	1	3	1	3	2

fig.4.2 Precedence functions for  $G'$ 

As many authors have proved[Fis 69, Lear 70, McAf 72], we can get equivalent precedence grammars for any given context-free grammar by using techniques (P1), (P2) and (P3) of theorem 4.2.

From this fact we have following theorem.

#### Theorem 4.3

There exist equivalent precedence grammars with precedence functions for a given context-free grammar.

The author had not yet obtained these theorems when he began to construct the GPL compiler[Asa 72a, Asa 72c].

At that time the author, as we have seen in the chapter 3, adopted following method;

- (i) there exists a family of precedence grammars with precedence functions,
- (ii) by introducing new variables and terminal symbols we can transform any precedence grammar to a grammar of the family.

The above method has, however, some difficulties since the introduction of new terminals enforces the scanning routine of the input symbol to scan symbols context sensitively.

If a grammar has precedence functions depends on frequencies of appearances of terminals in the right parts of rewriting rules, so that it is easy to obtain precedence grammars with precedence functions since numbers of such troublesome terminals are usually a few.

We can use the same procedures to obtain precedence grammars from a context-free grammar and to obtain precedence grammars with precedence functions from a precedence grammar.

This fact assures us that we can obtain an equivalent precedence grammar with precedence functions immediately when we have obtained a precedence grammar from a context-free grammar, and that we may take the term "equivalent grammars" as "equivalent grammars with the same analysis mechanism".

#### 4.3 Concluding Remarks

From the above theorems we have obtained following results for precedence grammars:

- (1) The necessary and sufficient condition that a precedence grammar has precedence functions.
- (2) The existence of equivalent precedence grammars with precedence functions to any given precedence grammar.
- (3) The existence of equivalent precedence grammars with precedence functions to any given context-free grammar.

Recent publication shows that David F.Martin[Mar 72], although the method is different, has independently obtained the same results as the author's.

## Chapter 5

### Experience with the GPL

#### 5.1 Introduction

It is the current trend that not only problem oriented high level programming languages but also simple software writing languages are described and analyzed by context-free grammars. One of the aims of PL360, GPL, or more generally machine oriented languages of these types is to resolve the problem of so called "software crisis" by providing programmers tools to promote their productivity of software.

If a language is used as a tool for software production, it will be used extensively in debugging the software. In that case the compilation speed of the compiler is one of important measures for usefulness of the language. The formalism of analysis, that is, the assumption of explicit existence of phrases will reduce the speed, but to what degree?

In this chapter let us investigate the problem using the GPL as an example.

The usefulness of GPL as a software writing language is also discussed by examples.

The GPL is a software writing language derived from a simple precedence grammar with precedence functions. The language is a modification of the PL360 [Wir 68]. It is different from the PL360 on following three points:

- (1) The GPL compiler produces relocatable binary form object programs.

- (2) Programs written in the GPL may call Fortran subprograms, or they may be called from Fortran programs.
- (3) Except index registers, explicit register specification is not allowed in the language.

The compiler of the language is written in Fortran IV and has been used for two years by the author and others. Using FACOM230-60 computer( $0.92 \mu s$  cycle time), the compiler compiles from 1200 to 1600 source cards in a minute. It produces compact and efficient relocatable form binary object programs which are, in their memory utilization and execution efficiency, comparable to programs written in assembly languages[Asa 73b].

## 5.2 Basic Notions

Let  $G = (V_N, V_T, S, P)$  be a simple precedence grammar with precedence functions.

There exist, however, equivalent precedence grammars with precedence functions for any given precedence grammar. We can get equivalent precedence grammars by modifying any given context-free grammar and we can also get equivalent precedence grammars with precedence functions by modifying a precedence grammar. These are the results of preceding chapter.

The author's experience for the Fortran IV language shows that in the former modification, the values of  $\#(V_N)$ , i.e., number of elements in the set  $V_N$ , and  $\#(P)$  increase about 25 percent. This fact seems coincide with that of J.McAfee and L.Presser for Algol 60[McAf 72]. On the other

hand, in the latter modification, the experience of the author for the Fortran IV and the GPL shows that the values of  $\#(V_N)$  and  $\#(P)$  increase in only a small percentage(in five percent). These two modifications induce stratifications of variables. That is, the modifications are done by introducing a new variable  $A'$ , a new rewriting rule  $\phi: A' \rightarrow A$  and adding the variable and the rule to the original grammar  $G$ . As the results, numbers of such derivations as the form  $A_1 \rightarrow A_2 \rightarrow \dots \rightarrow A_n$  increase, where each  $A_i$  is a variable. Derivation sequences of this form are called chain rules.

By the term semantics of a rule we mean a set of operations for tables and stacks defined by the compiler designer for the rule. We say a chain rule  $A \rightarrow B$  has null semantics if no operation is defined for the rule.

In the analysis process of an input string every phrase, even if it appears in a rule of null semantics, must be detected and reduced to a left part of a rule. Hence the number of chain rules has effects on the efficiency of the compiler.

### 5.3 Computational Procedures of Precedence Functions

Several methods have been proposed to compute values of precedence functions. We may classify these methods into two categories. The one of them is originally proposed by Floyd [Flo 63] and is used by Wirth [Wir 65]. The author's procedure[in the previous chapter] also falls into the same category. The other one is given by J.R.Bell [Bell 69]. Bell uses a  $2N \times 2N$  Boolean matrix to compute precedence functions, where  $N = \#(V_N \cup V_T)$  in case of Wirth-Weber type precedence grammar

[Wir 66] and A. Colmerauer type[Col 70] precedence grammar, or  $N = \#(V_T)$  in case of operator precedence grammar. In most cases the values of N's of practical compilers exceed 100, for example, as for the GPL(modified PL360), which is described by Wirth-Weber type precedence grammar, the value of  $\#(V_N \cup V_T)$  exceeds 160. In the construction of the GPL compiler, the author and others[Fuji 70] provided two procedures of Wirth and Bell. The procedure of Wirth computes values of precedence functions by checking non-null elements of a  $N \times N$  matrix, where  $N = \#(V_N \cup V_T)$ . The computational procedures become recursive, but the number of procedures pushed down by the recursiveness is usually small. The procedure of Bell computes a  $2N \times 2N$  Boolean matrix  $\sum_{i=1}^N M^{(i)}$ , where  $M^{(i)} = M \times M^{(i-1)}$ ,  $M^{(1)} = M$ .

By the procedure of Wirth, using FACOM230-60 computer (0.92 micro second cycle time), it required 7 seconds to get the precedence functions of the GPL of about 2000 relations.

On the other hand, Bell's procedure seems to require very much time to compute the functions.

Another merit of Wirth's procedure is that using it we can modify a precedence grammar to an equivalent precedence grammar with precedence functions. As is shown in the previous chapter a precedence grammar has no precedence functions if and only if it has a cycle of the form  $f(S_i) < g(S_j) < \dots < f(S_i)$  for an element  $S_i$  of  $(V_N \cup V_T)$ . We may make use of Wirth's procedure to find the cycle, and when we find the cycle, we can remove the cycle by modifying the original grammar.

#### 5.4 Computation of Precedence Relations and Precedence Functions

The GPL is a language derived from a precedence grammar with precedence functions. To get the data for control of syntactical analysis of the input program, following procedures are used:

- (1) The grammar is described by the Backus form,
- (2) the grammar is modified to a precedence grammar,
- (3) the grammar is modified to a precedence grammar with precedence functions,
- (4) the data for the compiler are computed.

The grammar thus obtained is guaranteed to be equivalent to the original one[Fis 69, Lear 70, McAf 72, Mar 72 and Asa 72b].

As is shown in the fig.5.1, these procedures compose a computer program[Fuji 70]. It is, however, better to modify the grammar by clerical operations, so that we can usually avoid appearances of rewriting rules of same right part under the assumption that the grammar is bounded context.

The fig.5.2 shows our method to retrieve left parts of rewriting rules. This method requires a small amount of time and memory for the retrieval of a left part.

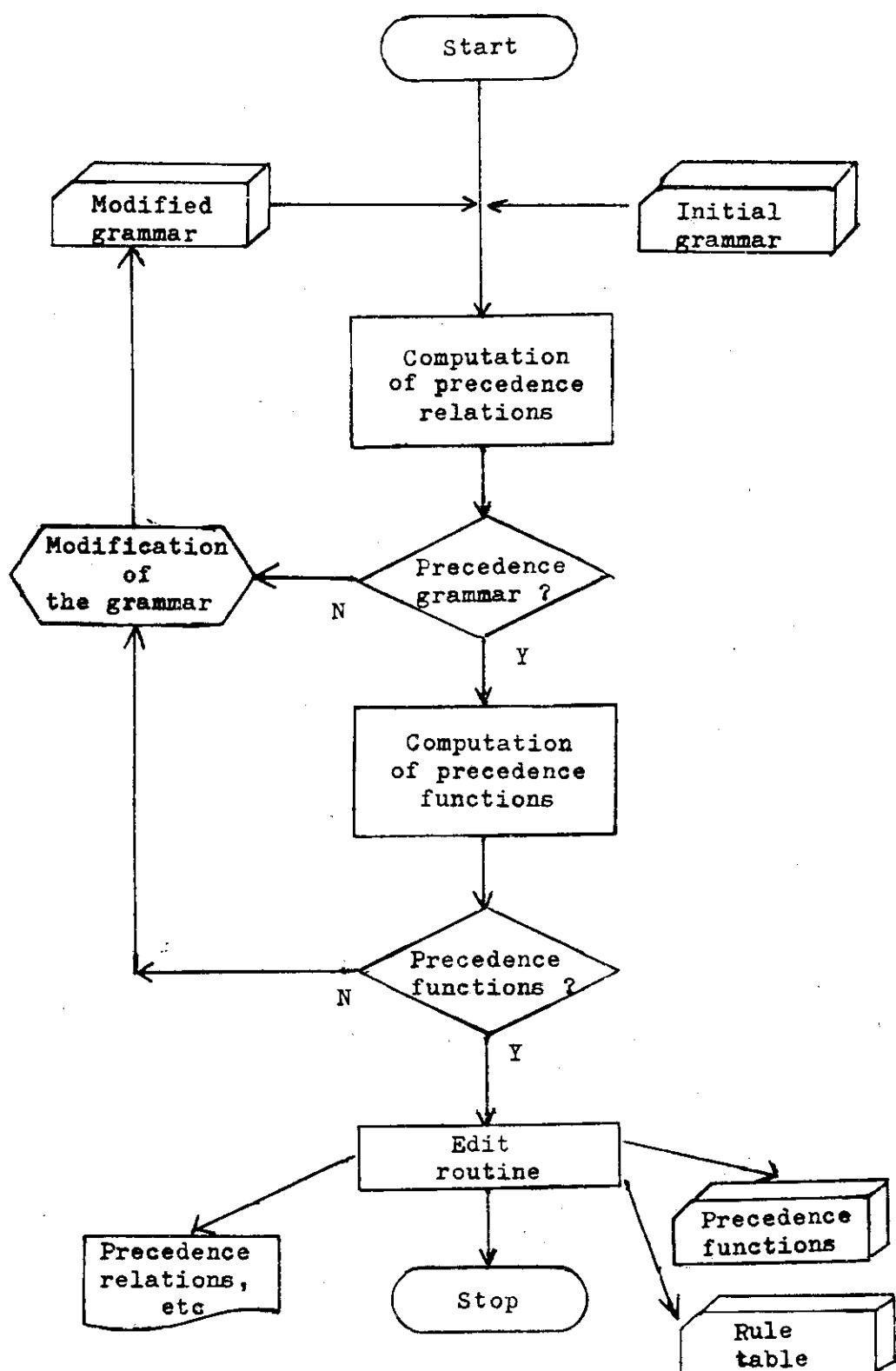
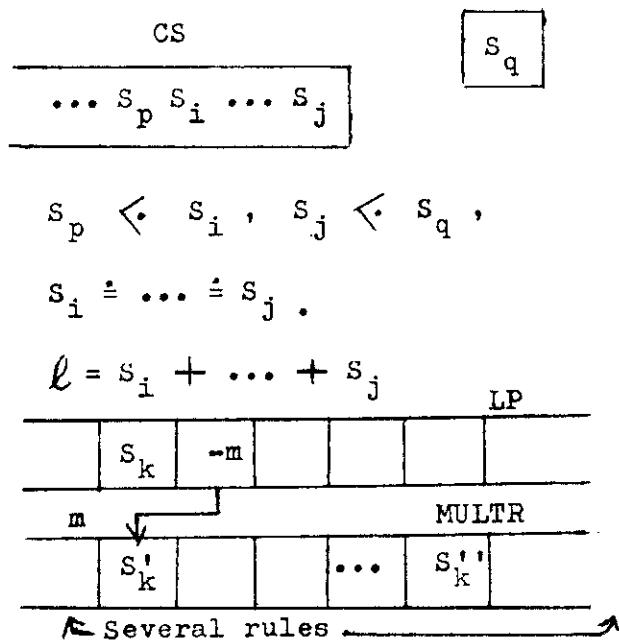


fig. 5.1 Computation of precedence grammars  
and precedence functions



#### Explanation:

- The retrieval of the left part  $s_k$  of a rewriting rule  $s_k \rightarrow s_i \dots s_j$  is done by following procedures;
1. Every symbol  $s_i, \dots, s_j$  in the control stack CS is converted to a numerical value.
  2. The compiler begins the retrieval when relations  $s_p \leftarrow s_i, s_j \rightarrow s_q$  hold for a stack symbol  $s_p$  and for an input symbol  $s_q$ .
  3. The sum  $\ell = s_i + \dots + s_j$  gives a location of the LP table. If  $LP(\ell) > 0$ , then the value  $LP(\ell)$  is the  $s_k$ .
  4. If  $LP(\ell) < 0$ , then there exist several rules of which right parts have the sum  $\ell$ . These rules are stored in a table MULTR. The value  $|LP(\ell)|$  gives first location of corresponding rules. Collating right parts of the stored rules and the symbol string  $s_i \dots s_j$  in the control stack, we find the  $s_k$  in the MULTR.

fig. 5.2 Retrieval of a left part of a rule

### 5.5 Detection and Reduction of Phrases

As is mentioned in the previous section, the modification of a context-free grammar to a precedence grammar and a precedence grammar to a precedence grammar with precedence functions has a tendency to increase numbers of chain rules. In a practical compilation process, we often give null semantics to chain rules. For a proper syntactical analysis, however, a phrase B of a chain rule  $A \rightarrow B$  must be detected and reduced to A. Hence the number of chain rules of null semantics appeared in the analysis process has of great importance on the efficiency of the compiler.

The fig.5.3 shows Floyd's Treesort3 program written in the GPL. The fig.5.4 and fig.5.5 show a statistics obtained by the GPL compiler in the compilation process of the Tree-sort3 program.

```

*GPL      FASP

.BEGIN

PROCEDURE TSORT3(M,N)  $
COMMENT ALGORITHM 245. TREESORT3 BY ROBERT W. FLOYD.
THE COMM. OF THE ACM., DEC., 1964.  $
BEGIN ARRAY 100 REAL M$ INTEGER N$
PROCEDURE EXCHANGE(X,Y)$
BEGIN REAL X,Y,T$
T=X$ X=Y$ Y=T$
END $
PROCEDURE SIFTUP(I,N)$
BEGIN INTEGER I,N,J,K$ REAL COPY$
K=I$ X1=K$ COPY=M(X1)$
LOOP.. J=2*K$ X1=K$ X2=J$
IF J.LE.N THEN
BEGIN IF J.LT.N THEN
BEGIN IF M(X2+1).GT.M(X2) THEN X2=X2+1$ END$
IF M(X2).GT.COPY THEN
BEGIN M(X1)=M(X2)$ K=X2$ GOTO LOOP$ END$
END$
M(X1)=COPY$
END $
INTEGER I$
I=N/2 $
L0.. SIFTUP(I,N)$ I=I-1$ IF I.GE.2 THEN GOTO L0 $
I=N $
L1.. BEGIN SIFTUP(1,I)$ X3=I$ EXCHANGE(M(1),M(X3))$
END$
I=I-1$ IF I.GE.2 THEN GOTO L1 $
END$ .

```

\*

fig.5.3 Treesort3 program described by GPL

Item	Number
used rules	507
null rules	247
chain rules	387
multi-rules	198
retrieval	383
length	760

fig. 5.4 Numbers of procedures used for detection  
and reduction of phrases

Length	1	2	3	4	5	6	7	8	9
Number	120	57	18	15	2	1	3	4	10

fig. 5.5 Chain rules

The fig.5.4 shows

- (1) The total number of rules used is equal to 507.
- (2) The number of rules with null semantics is equal to 247.
- (3) Chain rules of length unity cannot be compressed their lengths furthermore, but for a chain rule of the form

$A_1 \rightarrow A_2 \rightarrow \dots \rightarrow A_n$ , the sequence  $A_2 \rightarrow A_3 \rightarrow \dots \rightarrow A_n$  is either introduced to make the original grammar as a precedence grammar or it is contained in the original grammar.

The fig.5.5 shows the numbers of these sequences. In any case it is desirable to compress the sequences to forms  $A_1 \rightarrow A_n$  at the design stage of the grammar or in the compilation process to accomplish an efficient compilation.

The number of chain rules, excluding rules of length unity and including rules with null semantics, is equal to 387.

- (4) As is shown in the fig.5.2, the left and right parts of a rule are represented by numerical values. The left part of a rule is represented by an additive sum of the right part. Hence for a numerical value of a right part, there sometimes correspond more than one left part. Let us call this one to many correspondence as a collision of variables.

The number of variables which caused collisions in the retrieval processes of left parts is equal to 198.

- (5) The number of collations operated between the control stack and the rule table to get proper left parts in case of collisions is equal to 383.
- (6) The total length of right parts which appeared in the process of compilation is equal to 760.

As is mentioned before, the precedence grammar has a tendency to include many chain rules. As the result, by the adoption of a precedence language the compiler will be forced to detect and reduce many variables with null semantics.

The statistics of fig.5.4 and 5.5 show that the time spent for detections and reductions of rules with null semantics is not small.

Since we have no space here to list up the syntax of the GPL(see Appendix A), let us discuss the problem by another example.

The fig.3.8 shows the syntax of expression of Fortan IV described in the Wirth-Weber type precedence grammar. This precedence grammar has precedence functions. The fig.5.6 shows compilation processes of a sentential form  $\perp (a \times b + c \times d) \perp$  by the grammar.

Stack	Prece-dence	Input symbol	Operation	Used rules
$\perp$	$\leq$	(		
(	$\leq$	a		
( a	$\geq$	$\times$	$a \Rightarrow \eta_1$	21, 19, 16, 14, 13, 10
( $\eta_1 \times$	$\leq$	b		
( $\eta_1 \times b$	$>$	+	$b \Rightarrow \eta_2$	21, 19, 16, 13
		repeat	$\eta_1 \times \eta_2 \Rightarrow \eta_1$	11
( $\eta_1$	$>$	+		9, 4
		repeat		
( $\eta_1 +$	$\leq$	c		
( $\eta_1 + c$	$>$	$\times$	$c \Rightarrow \eta_1$	21, 19, 16, 14, 13, 10
( $\eta_1 + \eta_2 \times$	$\leq$	d		
( $\eta_1 + \eta_2 \times d$	$>$	)	$d \Rightarrow \eta_3$	21, 19, 16, 14, 13
( $\eta_1 + \eta_2 \times \eta_3$	$>$	)	$\eta_2 \times \eta_3 \Rightarrow \eta_2$	10, 9
( $\eta_1 + \eta_2$	$>$	)		
		repeat	$\eta_1 + \eta_2 \Rightarrow \eta_1$	
( $\eta_1$	$>$	)		7, 3
( $\eta_1)$	$>$	$\perp$		
		repeat		
$\perp$	$\doteq$	$\perp$		17, 14, 13, 10, 9, 4, 3, 1

fig. 5.6 Analysis of a sentential form  $\perp (a \times b + c \times d) \perp$

Numbers listed in the "Rules" field of fig.5.6 show numbers of rewriting rules used to analyze the sentential form. The purpose of the compiler is to do actions listed in the "Operation" field. As is shown in this example, the amount of time used for the "Rules" field is much more bigger than that of the "Operation" field. Since the right precedence grammar of Inoue[Ino 70] or grammars of this type [Ich 70, Aho 72] can accept left recursive rules, by the adoption of these grammars we can reduce numbers of rules in the "Rules" field.

The statistics of the examples show that the adoption of a conventional analysis method which does not assume the existence of phrases explicitly in the compilation will reduce the cost minimum.

## 5.6 The Methods of Gray and Harrison

J.Gray and M. Harrison[Gray 69] have presented two methods to solve the problem. The one is a method to skip rewriting rules in the time of analysis. In a sequence of chain rules we may skip detections, reductions and retrievals of several phrases regarding that they are already processed by the analyzer. The other method is to use a precedence grammar with smaller number of phrases. Gray and Harrison have presented the canonical precedence grammar for this purpose.

Let us give a rough sketch of the first method.

Let  $G = (V_N, V_T, \perp S \perp, P)$  be a context-free grammar. If there exist  $x, w \in (V_N \cup V_T)^*$ ,  $y \in V_T^*$ ,  $A \in V_N$  such that

$u = xAy$ ,  $v = xwy$  and the rule  $(A \rightarrow w)$  is in  $P$  for  $u, v \in (V_N \cup V_T)^*$ , then the derivation  $u \Rightarrow v$  is called a canonical derivation and is denoted by  $u \stackrel{R}{\Rightarrow} v$ .

Let  $H$  be a subset of  $P$  and a sequence  $P_1, P_2, \dots, P_k$  ( $\in P^*$ ) be canonical derivations of a sentential form  $x$ , then a mapping  $\Phi(P_1 P_2 \dots P_k)$  defined by a homomorphism

$$\Phi(P_i) = \begin{cases} P_i, & P_i \in H, \\ \phi, & \text{otherwise} \end{cases}$$

is called a  $H$ -sparse generation of  $x$  by the set  $H$ .

Gray and Harrison have shown that introducing the subset  $H$  and the mapping  $\Phi$  for a given context-free grammar  $G$ , we can find a grammar  $G'$  such that  $L(G) = L(G')$ . This assertion is the theoretical background of conventional approach which we have been used from early days of compiler construction.

It is however difficult for a given precedence grammar to find a new precedence grammar with smaller number of chain rules because most of chain rules contained in the original grammar are introduced to make the grammar has precedence relations. Hence, from practical point of view, it is better to apply the homomorphism  $\Phi$  on the same grammar  $G$ .

For better understanding of  $\Phi$ , let us consider the grammar for Fortran IV expression of fig.3.8. Since the grammar  $G$  has precedence functions, we can make use of them to activate the application of  $\Phi$ .

- (i) Let  $f, g$  be precedence functions,  $a$  be an identifier in a sentential form  $\lfloor a \rfloor$ . Since  $f(a) > g(\lfloor)$ , we can start

the operation of table look-up to find  $\Phi(P_1 \dots P_m)$ . This is an operation to find  $A(\in V_N)$  such that  $A \xrightarrow{*} a$ .

- (ii) If  $\Phi$  is not defined for this  $a$ , we go back to normal analysis procedure of precedence grammar.
- (iii) If  $A$  exists and  $f(A) > g(\perp)$  holds, we can replace  $a$  by  $A$ . If  $f(A) < g(\perp)$  holds, we must search other  $\Phi$ . If no such  $\Phi$  exists, we go back to step (ii).

By the above procedures we can analyze a derivation sequence  $A \xrightarrow{*} a$  quickly. When every derivation in the sequence has null semantics, this is one of best analysis mechanisms for compilers. In general many rewriting rules may fall into the domain of  $\Phi$  and as the result, a lot of memory will be required to store the sequences of canonical derivations. Hence it is better to use this mechanism only for rules which appear frequently.

Now let us sketch the second method.

Let  $G = (V_N, V_T, \perp, S, P)$  be a context-free grammar. A set  $T$  is called a token set of  $G$  if it satisfies following two conditions;

- (1)  $V_T \subseteq T$ ,
  - (2) if  $A \in T$  and  $(A \rightarrow x) \in P$ ,
- then  $x \in (V_N \cup V_T)^* T (V_N \cup V_T)^*$ .

A token set  $T$  is said to be a strong operator set (SOP) if it satisfies following condition;

if  $(A \rightarrow xBCy) \in P$  for  $y \in (V_N \cup V_T)^*$ , where  $A, B, C \in (V_N \cup V_T)$ , then  $B \in T$  or  $C \in V_T$ .

Gray and Harrison have proved that we can define prece-

dence relations over  $T$  if a token set  $T$  of a context-free grammar  $G$  is a strong operator set. The precedence relations are Floyd's operator precedence relations if  $T = V_T$  and Colmerauer's total precedence relations if  $T = (V_N \cup V_T)$ .

The operator precedence grammar does not permit a rewriting rule of the form  $A \rightarrow xBCy$ ,  $B, C \in V_N$ . This restriction is inconvenient for assignment of meanings in compiler constructions. In addition to it, the restriction will promote the grammar  $G$  to have many rewriting rules of the form  $A \rightarrow B$ , where  $A, B \in V_N$ . On the otherhand, the size of precedence matrix of total precedence relations is sometimes too big for practical compiler implementation. Thus the methods of Gray and Harrison are good approaches to the solution of overhead in syntactical analysis induced by the existence of phrases.

### 5.7 Error Recovery Procedures

It is easy to detect syntactical errors in a precedence language, but it is not so easy to keep track of the analysis process activating appropriate recovery procedures. Recovery procedures for a syntactical error involve operations of backtracking for tables and stacks. Thus error recovery is not only a problem of syntax but also a problem of semantics.

R.P. Leinius[Lei 70] has presented a method for detection and recovery of syntactical errors of precedence language. Leinius has classified the error detection of precedence language into following four types:

- (1) Type 0: There is no precedence relation between the top symbol  $R_i$  of the stack and incoming input symbol  $a_k$ . This error is denoted by  $(R_i ? a_k)$ .
- (2) Type 1: There does not exist a right part of rewriting rule such that  $R_j \dots R_i$  which is in the stack and is detected to be a phrase.
- (3) Type 2: There exists a variable A such that  $A \rightarrow R_j \dots R_i$ , but it causes relation  $(R_{j-1} ? A)$ .
- (4) Type 3: There exists a variable A such that  $A \rightarrow R_j \dots R_i$ , but it causes the relation  $(A ? a_k)$ .

Leinius attempts to recover these errors by reduction of phrases. The recovery procedure is described as follows;

- (i) it extracts a phrase which contains the error, then
- (ii) it reduces the phrase or the right part which contains the phrase to a variable. If there is no such variable, it repeats from the step (i).

On the otherhand, the GPL compiler uses not precedence relations but precedence functions for syntax analysis, so that any syntactical error becomes the form of type 1.

For the first version of the GPL compiler, the author had adopted following simple method for error recovery.

- (a) An identifier is checked its left and right symbols to see if it is a procedure name. If it is an undefined identifier, it is defined automatically by the compiler.
- (b) When a phrase is detected, the analyzer checks next input symbol to see if its appearance is legitimate.

By this method the compiler must terminate the processing

on encountering a type 1 error. This situation forced the programmer to make rerun his program many times. In the second version, the method is changed to recover from errors of type 1. The analyzer, on encountering the type 1 error, continues to read input symbols until it finds an end of statement mark in the input string. Then the analyzer begins to pop up symbols in the control stack until it finds a symbol <block head> or <blockbody>. The variable <block head> indicates a state that the analyzer is processing declaration statements and the variable <blockbody> indicates a state that the analyzer is processing executable statements. By this method we can now recover from almost all errors.

### 5.8 GPL as a Software Writing Language

The usefulness of a software writing language may be considered from points of view of (1) the easiness to read and write, (2) the memory utilization of programs written by the language, (3) the execution efficiency of the written programs.

The example of fig.5.3 shows that a program written in the GPL is superior to a program written in assembly languages for the point (1). Since it is difficult to give generalized discussion on the points (2) and (3), let us consider the problem by an example.

The Fortran IV compiler of IBM7044 consists of five phases. The role of the first phase(it is called the SCAN) is, (i) to classify an input string and to convert it into internal codes, (ii) to find out almost all syntactical errors in the input

string.

The original SCAN routine is written in the assembly language according to a set of flowcharts. We have written a new routine(let us call it as SCAN2) in the GPL according to the same flowcharts. The SCAN2 routine is implemented on the FACOM230-60 computer. Since both FACOM230-60 and IBM7044 are 36 bits word machines with similar instructions and same numbers of instructions for an operation, we may compare the memory utilization of SCAN and SCAN2 routines. The memory utilizations of SCAN's and SCAN2's instruction parts are 7100 and 7800 words, respectively. Hence the ratio of instruction lengths becomes 1 : 1.1. Considering the ratio, we may conclude that the execution efficiency of program written in the GPL is almost same as a program written in an assembly language.

The difference of 700 words between the SCAN and SCAN2 is caused by following reasons:

- (a) The SCAN2 consists of 40 program units (90 procedures in total) of the GPL. Every procedure written in the GPL requires 3 words for its procedure entry and return operation ( $90 \times 3 = 270$  words).
- (b) Every procedure declaration has two data words to get proper locations of its formal parameters ( $90 \times 2 = 180$  words).
- (c) The most simplest form of "case statement" of the GPL corresponds to the "computed goto statement" of the Fortran language. We used two case statements of 50 branches each in the SCAN2. Each statement requires 50 additive instructions ( $2 \times 50 = 100$  words).

(d) The "for statement", or "while statement" are used in 25 parts in the SCAN2. These statements require additive 2 instructions, respectively ( $25 \times 2 = 50$  words).

The difference of the residual 100 words is caused by the facts that we have often used one word for one character, and that we cannot specify accumulators explicitly in the GPL.

This memory ratio is attained under the restriction that all procedures of SCAN2 are, as the SCAN routine, loaded in a same segment. The SCAN or SCAN2 has about 370 external references. When the references are not resolved in a loading unit, i.e., in a segment, they will be connected with each other via address constants. In our case we need three words for an address constant. Whether the restriction of a same segment is removed or not is a control card option of the GPL compiler. If the restriction is not specified, we need another 1000 words for the address constants.

Let us consider another example. The fig.5.7 shows a routine used to make storages free in a dynamic memory allocation scheme (see Appendix D). The sheme is called as Buddy system[Knu 68]. The routine of fig.5.7 written in GPL uses 186 machine words of FACOM230-60 computer.

The fig5.8 shows a rewritten routine of fig.5.7 by using the "function" feature of GPL. This routine has almost same form as programs written in the assembly language and uses 168 machine words. Thus we can reduce the size of original routine in 10 percent.

The GPL compiler is written rather redundantly in the Fortran langugage. It compiles the SCAN2 of 40 program units,

```

.BEGIN
COMMENT*****THIS ROUTINE RETURNS A BLOCK OF 2**N LOCATIONS STARTING AT ****
*   THIS ROUTINE RETURNS A BLOCK OF 2**N LOCATIONS STARTING AT   *
*   ADDRESS I.                                                 *
*****PROCEDURE FREE(I,N) *
BEGIN
SEGMENT BASE /ALLOC/ *
INTEGER GETVALUE,FREEVALUE *
ARRAY 39 INTEGER AREA *
ARRAY 38 INTEGER A SYN AREA(2) *
INTEGER MASKF=3777770000000, MASKB=0000007777770,
TAGON=1777777777770,TAGOFF=4000000000000000 *
INTEGER XR1,XR2,XR3,XR4,XR5 *
INTEGER M=5,SIZE=32 *
SEGMENT BASE LOCAL *
INTEGER I,N,T,U,K,L,P,Q,J *
LABEL TW *
FUNCTION STX(3,0452150000000) *

XR1=X1 * XR2=X2 * XR3=X3 * XR4=X4 * XR5=X5 *
IF N.LT.0 OR N.GT.M THEN GO TO ERRORExit *
IF N.EQ.M THEN BEGIN INIT * GO TO ENDFREE * END *
X1=I *
IF A(X1).LT.0 THEN GO TO ERRORExit *
L=I * J=N *
K=N+SIZE *
CHECK.. X1=K * X2=J * X4=1 *
STX(3,5,TW) *
TW.. T=1 SHLA 0 - 1 *
U=L AND T *
IF U.NE.0 THEN GO TO ERRORExit *
P=T+1 XOR L *
X2=P *
IF A(X2).GT.0 THEN GO TO PUTON *
COMBINE.. X3=A(X1) AND MASKF SHRA 18 *
IF X3.NE.P THEN
BEGIN
IF X3.EQ.K THEN GO TO PUTON *
X1=A(X3) AND MASKF SHRA 18 *
IF X1.NE.P THEN GO TO COMBINE *
IF X1.EQ.K THEN GO TO PUTON *
END *
X4=A(X2) *
T=A(X2) AND MASKF *
Q=A(X4) AND TAGOFF *
A(X4)=A(X4) AND MASKB OR T OR Q *
X5=T SHRA 18 *
A(X5)=X4 *
J=J+1 * K=K+1 *
IF X2.LT.L THEN L=X2 *
GO TO CHECK *
PUTON.. X2=L *
T=A(X1) AND MASKF *
A(X2)=A(X2) AND MASKB OR T *
X3=T SHRA 18 *
A(X3)=X2 * A(X2)=X1 *
T=L SHLA 18 *
A(X1)=A(X1) AND MASKB OR T *
A(X2)=A(X2) OR TAGOFF *
GO TO ENDFREE *
ERRORExit.. FREEVALUE=-1 *
ENDFREE.. X1=XR1 * X2=XR2 * X3=XR3 * X4=XR4 * X5=XR5 *
END *
END.

```

Fig. 5.7 The Free routine written in GPL

```

.BEGIN
COMMENT*****THIS ROUTINE RETURNS A BLOCK OF 2**N LOCATIONS STARTING AT *
* ADDRESS I.
*****
PROCEDURE FREE(I,N) *
BEGIN
SEGMENT BASE /ALLOC/ *
INTEGER GETVALUE,FREEVALUE *
ARRAY 39 INTEGER AREA *
ARRAY 38 INTEGER A SYN AREA(2) *
INTEGER MASKF=3777770000000, MASKB=0000007777770,
TAGCN=1777777777770,TAGCFF=4000000000000 *
INTEGER XR1,XR2,XR3,XR4,XR5 *
INTEGER M=5,SIZE=32 *
SEGMENT BASE LOCAL *
INTEGER I,N,T,U,K,L,P,Q,J,TEMP1 *
LABEL TW,COMBINE,LABEL1,PUTON,ERROREXIT *
FUNCTION STX(3,0452150000000), LA(2,0103100000000),
LXA(3,4440100000000), JUMP(1,4000100000000),
TL(1,1701100000000), TE(1,0702100000000),
TH(1,0700100000000), TNE(1,1702100000000),
ANDCP(1,0106100000000), RAL(6,003200000220) *
FUNCTION STA(1,0231100000000) *

XR1=X1 * XR2=X2 * XR3=X3 * XR4=X4 * XR5=X5 *
LA(0,N) *
TL(0) *
JUMP(ERROREXIT) *
TH(M) *
JUMP(ERROREXIT) *
IF N.EQ.M THEN BEGIN INIT * GO TO ENDFREE * END *
X1=I *
LA(2,AREA) *
TL(0) *
JUMP(ERROREXIT) *
L=I * J=N *
K=N+SIZE *
CHECK.. X1=K * X2=J * X4=1 *
STX(3,5,TW) *
TW.. T=1 SHLA 0 - 1 *
ANDOP(L) *
TNE(0) *
JUMP(ERROREXIT) *
P=T+1 XOR L *
X2=P *
LA(3,AREA) *
TH(0) *
JUMP(PUTON) *
COMBINE.. LA(2,AREA) *

```

Fig.5.8 The refined Free Routine (continued)

```

ANDOP(MASKF) *
RAL *
LXA(4,0,0) *
TE(P) *
JUMP(LABEL1) *
TE(K) *
JUMP(PUTON) *
LA(4,AREA) *
ANDOP(MASKF) *
RAL *
LXA(2,0,0) *
TNE(P) *
JUMP(COMBINE) *
TE(K) *
JUMP(PUTON) *
LABEL1.. X4=A(X2) *
T=A(X2) AND MASKF *
Q=A(X4) AND TAGOFF *
A(X4)=A(X4) AND MASKB OR T OR Q *
X5=T SHRA 18 *
A(X5)=X4 *
J=J+1 * K=K+1 *
IF X2.LT.L THEN L=X2 *
GO TO CHECK *
PUTON.. X2=L *
T=A(X1) AND MASKF *
A(X2)=A(X2) AND MASKB OR T *
X3=T SHRA 18 *
A(X3)=X2 * A(X2)=X1 *
T=L SHLA 18 *
A(X1)=A(X1) AND MASKB OR T *
A(X2)=A(X2) OR TAGOFF *
GO TO ENDFREE *
ERROREXIT.. FREEVALUE=-1 *
ENDFREE.. X1=XR1 * X2=XR2 * X3=XR3 * X4=XR4 * X5=XR5 *
END *

```

Fig.5.8 The refined Free routine

4300 source cards in 200 seconds (by the cpu time of FACOM230-60 of 0.92  $\mu$ s machine cycle) and produces the binary object program for the linkage editor. If the listing of the source program is not specified, it compiles the SCAN2 in 140 seconds.

The compilation speed is due to the in-core, one pass compiling method of the compiler. The adoption of the precedence grammar, or more generally, the phrase structure grammar explicitly for the analysis of the input string reduces the compilation speed to a certain extent.

### 5.9 Concluding Remarks

From the above discussions, we have following conclusions:

- (1) Using the precedence grammar in compiler construction, the compiler designer can get insight into the working process of the compiler at the stage of the language design.
- (2) Adoption of a precedence grammar, or more generally, a phrase structure grammar in the analysis process of a compiler reduces the efficiency of the compiler.
- (3) When the language is suitably designed, the object programs produced by the compiler are comparable to programs written in assembly languages in their memory utilization and execution efficiency.

## Chapter 6

### Conclusion

The recognition that we are in the situation of so called "software crisis" has made us to feel the needs for methods to promote productivity of programmers, and for methods to develop techniques of design automation of software.

To promote the productivity of programmers, considerable effort has been devoted to develop suitable software writing languages for these several years[Fel 68], and almost all of these languages are described by context-free grammars.

To develop techniques for design automation of software, we must assume some structures for the software. But we should notice a fact that the introduced structures always accompany unexpected, undesirable overhead.

In the design of compiler, the structure often means the phrase structure grammar. In our case we have selected the precedence grammar because of reasons that (i) it allows the implementation of very simple analysis mechanism, (ii) it can analyze a sentence efficiently, (iii) it is able to recover from syntactical errors by simple procedures, (iv) it gives unique analysis for the input string. Moreover if we had once constructed a computer program to compute precedence relations, we can use the program for the analysis of many precedence grammars.

The precedence grammar which uses the matrix for language analysis requires much space and time. That is, the analysis method to use precedence matrix is an obstacle to practical

applications of precedence grammars. But we can, as we have seen, avoid the difficulty any time using precedence functions.

Thus we have selected the precedence grammar as the desired structure of language description, but the introduced structure of course induces the overhead. We have seen that the amount of the overhead is rather big. If we want to reduce the overhead, we must eliminate many rules with null semantics.

A conventional method[Sam 60] which does not assume the existence of phrases explicitly attains efficient compilation, but it lacks theoretical validity. An intermediate method of Gries[Gri 68] will give efficient analysis speed but it seems to require a large amount of space for subroutines.

The method of Gray and Harrison[Gray 69] is another one and it also seems to require an amount of space for tables to look up.

Whether there is a method with theoretical validity that guarantees to use less space and time is, to the author's knowledge, an unsolved problem.

On the otherhand, the usefulness of a software writing language is rather independent of the overhead, as we have seen in the application of GPL, if the overhead is in tolerable degree. The statistics about memory utilization and execution efficiency of programs written in GPL show that these types of languages such as PL360, GPL, etc. are useful tools for software construction.

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## Appendix A

## Syntax of GPL

----- EACH OPTION OF OUTPUT -----

0 0 0 0 1

NUMBER OF CHARACTER IN A WORD = 4

\*\*\* WRITE OUT OF INPUT RULES (FORM 2) \*\*\*

LEFT PART BEGINS WITH 11 COLUMN  
 RIGHT PART BEGINS WITH 28 COLUMN

1		'ID'	...=	IDI
2		'T NUMBER'	...=	IDT
3		'STRING'	...=	IDS
4		'X REG'	...=	IDX
5		'FUNC ID'	...=	IDF
6		'DPARAM ID'	...=	IDD
7		'PROC ID'	...=	IDP
8		'B REG'	...=	IDB
9		'LABEL ID'	...=	IDL
10		'EXTERN ID'	...=	EXTERNAL-BASE-ID
11	2	'T CELL ID'	...=	'ID'
12	3	'T CELL'	...=	'T CELL ID'
13	6			'T CELL1' )
14	7			'T CELL2' )
15	8	'T CELL1'	...=	'T CELL2' 'ARITH OP' 'T NUMBER'
16	9			'T CELL3' 'T NUMBER'
17	10	'T CELL2'	...=	'T CELL3' 'X REG'
18	11	'T CELL3'	...=	'T CELL ID' (
19	15	'ARITH OP'	...=	+
20	16			-
21	17			*
22	18			/
23	19			++
24	20			--
25	61	'REL OP'	...=	.LT.
26	62			.EQ.
27	63			.GT.
28	64			.LE.
29	65			.GE.
30	66			.NE.
31	21	'LOG OP'	...=	AND

```

32      22          OR
33      23          XOR
34      24  'SHIFT OP'  ..-= SHLA
35      25          SHRA
36      26          SHLL
37      27          SHRL
38      12  'UNARY OP'  ..-= ABS
39      13          NEG
40      14          NEGABS
41          'EQU'
42          'X REG EXP1'  ..-= 'X REG'
43          'X REG EXPR'  ..-= 'X REG EXP1'
44          'X REG EXP1'  'ARITH OP'  'T NUMBER'
45          'X REG ASS'   ..-= 'X REG'  'X REG EXPR'
46          'X REG'  'T CELL EXP*'
47          'T CELL EXP1' ..-= 'T CELL'
48          'T NUMBER'
49          'STRING'
50          'T CELL EXP2' ..-= 'UNARY OP'  'T CELL'
51          'UNARY OP'  'T NUMBER'
52          'UNARY OP'  'STRING'
53          'A T CELL'
54          'T CELL EXP1'
55          'T CELL EXP2'
56          'T CELL EXP1'  'ARITH OP'  'T CELL'
57          'T CELL EXP1'  'ARITH OP'  'T NUMBER'
58          'T CELL EXP1'  'LOG OP'    'T CELL'
59          'T CELL EXP1'  'LOG OP'    'T NUMBER'
60          'T CELL EXP1'  'SHIFT OP'  'T NUMBER'
61          'T CELL EXP*' ..-= 'T CELL EXP1'
62          'T CELL ASS'   ..-= 'T CELL'  'T CELL EXP*'
63      44  'FUNC1'     ..-= 'FUNC2'  'T NUMBER'
64          'FUNC2'  'X REG'
65      46          'FUNC2'  'T CELL'
66      47          'FUNC2'  'STRING'
67          'FUNC2'  'LABEL ID'
68          'FUNC2'  'PROC ID'
69      48  'FUNC2'     ..-= 'FUNC ID'  (
70      49          'FUNC1'  ,
71          'PROC1'     ..-= 'PROC2'  'T CELL'
72          'PROC2'  'T NUMBER'
73          'PROC2'  'STRING'
74          'PROC2'     ..-= 'PROC ID'  (
75          'PROC1'  ,
76          'MULT ASS1'   ..-= (( 'T CELL'
77          (( 'T NUMBER'
78          (( 'STRING'
79          'MULT ASS2'   ..-= 'MULT ASS3' ,
80          'MULT ASS3'   ..-= 'MULT ASS1'
81          'MULT ASS1'   'MULT ASS2'  'T CELL'
82          'MULT ASS2'   'MULT ASS2'  'T NUMBER'
83          'MULT ASS2'   'MULT ASS2'  'STRING'
84          'MULT ASS4'   ..-= 'MULT ASS3' ))

```

```

85      'MULT ASS'    . . = 'MULT ASS4' 'T CELL EXP1'
86      'VECTOR EXPR' . . = 'MULT ASS4'
87      50      'CASE SEQ'    . . = CASE 'X REG' OF BEGIN
88      51      'CASE SEQ'    . . = 'STATEMENT' *
89      'SIMPLE ST'   . . = 'T CELL' 'X REG EXPR'
90          'T CELL' 'EQU' 'B REG'
91          'B REG' = 'T CELL'
92          'T CELL ASS'
93          'X REG ASS'
94          'MULT ASS'
95          'T CELL' 'VECTOR EXPR'
96          NULL
97          54      GOTO 'ID'
98          55      'PROC ID'
99          56      'PROC1' )
100         57      'FUNC ID'
101         58      'FUNC1' )
102         59      'CASE SEQ' END
103         60      'BLOCKBODY' END
104        67      'NOT'     . . = 'NOT'
105        'CONDITION' . . = 'X REG' 'REL OP' 'T CELL'
106        'X REG' 'REL OP' 'T NUMBER'
107        'T NUMBER' 'REL OP' 'X REG'
108        'T CELL' 'REL OP' 'X REG'
109        'T CELL' 'REL OP' 'T CELL'
110        'T CELL' 'REL OP' 'STRING'
111        'T CELL' 'REL OP' 'T NUMBER'
112        'T NUMBER' 'REL OP' 'T CELL'
113        72      OVERFLOW
114        74      'T CELL'
115        75      'NOT' 'T CELL'
116        76      'COMP COND' . . = 'CONDITION'
117        77      'COMP AOR'   . . = 'COMP COND' AND
118        78      'COMP AOR'   . . = 'COMP COND' OR
119        79      'COND THEN' . . = 'COMP COND' THEN
120        80      'TRUE PART' . . = 'SIMPLE ST' ELSE
121        81      'WHILE'     . . = WHILE
122        82      'COND DO'   . . = 'COMP COND' DO
123        83      'ASS STEP'  . . = 'X REG ASS' STEP 'T NUMBER'
124      'LIMIT'     . . = UNTIL 'X REG'
125        86      UNTIL 'T CELL'
126        87      UNTIL 'T NUMBER'
127        88      'DO'       . . = DO
128        89      'STATEMENT*' . . = 'SIMPLE ST'
129        90      IF 'COND THEN' 'STATEMENT*'
130        91      IF 'COND THEN' 'TRUE PART' 'STATEMENT*'
131        92      'WHILE' 'COND DO' 'STATEMENT*'
132        93      FOR 'ASS STEP' 'LIMIT' 'DO' 'STATEMENT*'
133        94      'STATEMENT*' . . = 'STATEMENT*'
134        95      'SI T TYPE' . . = SHORT INTEGER
135        96      INTEGER
136        97      LOGICAL

```

138	98	REAL
139	99	LONG REAL
140	100	BYTE
141	101	CHARACTER
142		LABEL
143	102	'T TYPE'
144	103	..= 'SI T TYPE'
145	104	ARRAY 'T NUMBER' 'SI T TYPE'
146	105	'T DECL1'
147	106	..= 'T TYPE' 'ID'
148	107	'T DECL2'
149	108	..= 'T DECL3'
150	109	'T DECL4'
151	110	..= 'T DECL5'
152	111	'T DECL6'
153	113	..= 'T DECL7'
154		..= 'T DECL1'
155		'T DECL1' = 'T NUMBER'
156	116	'T DECL1' = 'STRING'
157		'T DECL5' )
158	117	'LP'
159	118	..= 'FUNC DC1'
160	119	..= 'FUNC DC7'
161	120	'FUNC DC2'
162	121	..= 'FUNC DC3'
163	122	'FUNC DC4'
164	123	..= 'FUNC DC5'
165	124	'FUNC DC6'
166	125	..= 'FUNC DC7'
167	126	'SYN DC1'
168		..= 'T TYPE' 'ID' SYN
169		'SI T TYPE' REGISTER 'ID' SYN
170	127	'SI T TYPE' ADCON 'ID' SYN
171	128	'SI T TYPE' SLCON 'ID' SYN
172	129	'SYN DC3' 'ID' SYN
173		'SYN DC1' 'T CELL'
174	131	'SYN DC2'
175	132	'SYN DC1' 'T NUMBER'
176	133	'SYN DC1' 'X REG'
177	134	'SEG HEAD'
178	135	..= 'PROC HD1'
179	136	'PROC HD2'
180		..= 'PROC HD3'
181		'PROC HD4'
182	138	..= 'PROC HD5'
183	139	'PROC HD6'
184		..= 'PROC HD7'
185		'PROC HD2D'
186	140	..= 'DECL'
187	141	'T DECL7'
188	142	'FUNC DC7'
189	143	'SYN DC2'
190		'PROC HD6' 'STATEMENT*' BASE 'B REG'

```

191           'SEG HEAD'   BASE   'EXTERN ID'
192           'SEG HEAD'   BASE   LOCAL
193      145   'LABEL DEF'   .. = 'ID'    ..
194      146   'BLOCKHEAD'   .. = BEGIN
195      147           'BLOCKHEAD'   'DECL'   *
196      148   'BLOCKBODY'   .. = 'BLOCKHEAD'
197      149           'BLOCKBODY'   'STATEMENT'   *
198      150           'BLOCKBODY'   'LABEL DEF'
199      151   'PROGRAM'    .. = . 'STATEMENT'   .

```

199 RULES

## Appendix B

## Precedence Functions for GPL

\*\*\* PRINT OF PRECEDENCE FUNCTIONS \*\*\*

I	F(I)	G(I)	
1	13	ID	6
2	9	T NUMBER	5
3	6	STRING	5
4	9	X REG	5
5	12	FUNC ID	3
6	5	DPARAM ID	1
7	12	PROC ID	5
8	11	B REG	3
9	5	LABEL ID	5
10	2	EXTERN ID	3
11	12	T CELL ID	6
12	9	T CELL	5
13	4	T CELL1	6
14	4	T CELL2	6
15	5	T CELL3	4
16	5	ARITH OP	9
17	5	REL OP	4
18	5	LOG OP	4
19	5	SHIFT OP	5
20	5	UNARY OP	9
21	3	EQU	10
22	4	X REG EXP1	9
23	4	X REG EXPR	3
24	3	X REG ASS	10
25	6	T CELL EXP1	10
26	6	T CELL EXP2	10
27	4	T CELL EXPR	10
28	4	T CELL EXP*	9
29	3	T CELL ASS	3
30	4	FUNC1	3
31	5	FUNC2	3
32	4	PROC1	3
33	5	PROC2	6
34	5	MULT ASS1	6
35	5	MULT ASS2	6
36	4	MULT ASS3	5
37	10	MULT ASS4	3
38	3	MULT ASS	9
39	3	VECTOR EXPR	3
40	1	CASE SEQ	3
41	2	SIMPLE ST	3
42	5	NOT	3
43	6	CONDITION	2
44	5	COMP COND	2
45	2	COMP AOR	2
46	2	COND THEN	1
47	2	TRUE PART	2
48	1	WHILE	3
49	2	COND DO	1
50	1	ASS STEP	1

51	1	LIMIT	1
52	2	DO	1
53	2	STATEMENT*	2
54	1	STATEMENT	1
55	8	SI T-TYPE	9
56	6	T-TYPE	9
57	11	T DECL1	9
58	6	T DECL2	9
59	7	T DECL3	9
60	5	T DECL4	9
61	4	T DECL5	9
62	4	T DECL7	9
63	7	LP	5
64	6	FUNC DC1	9
65	12	FUNC DC2	9
66	5	FUNC DC3	9
67	4	FUNC DC4	9
68	5	FUNC DC5	9
69	4	FUNC DC6	9
70	4	FUNC DC7	9
71	5	SYN DC1	9
72	4	SYN DC2	9
73	6	SYN DC3	9
74	9	SEG HEAD	9
75	6	PROC HD1	9
76	12	PROC HD2	9
77	1	PROC HD3	9
78	4	PROC HD4	9
79	1	PROC HD5	9
80	2	PROC HD6	9
81	1	PROC HD2D	9
82	1	DECL	8
83	8	LABEL DEF	1
84	8	BLOCKHEAD	3
85	1	BLOCKBODY	3
86	1	PROGRAM	1
87	14	IDI	7
88	11	IDT	6
89	6	IDS	6
90	12	IDX	6
91	13	IDF	3
92	5	IDD	2
93	13	IDP	6
94	12	IDB	4
95	5	IDL	6
96	2	EXTERNAL-BASE-ID	4
97	12	)	4
98	8	(	12
99	8	+	5
100	8	-	5
101	8	*	5
102	8	/	5
103	8	++	5
104	8	--	5

105	8	.LT.	10
106	8	.EQ.	10
107	8	.GT.	10
108	8	.LE.	10
109	8	.GE.	10
110	8	.NE.	10
111	8	AND	5
112	8	OR	5
113	8	XOR	5
114	7	SHLA	5
115	7	SHRA	5
116	7	SHLL	5
117	7	SHRL	5
118	8	ABS	6
119	8	NEG	6
120	8	NEGABS	6
121	5	=	11
122	5	A	5
123	8	,	4
124	5	((	6
125	12	))	4
126	5	CASE	3
127	3	OF	9
128	11	BEGIN	3
129	11	*	1
130	3	NULL	3
131	6	GOTO	3
132	3	END	1
133	8	.NOT.	3
134	6	OVERFLOW	3
135	8	THEN	5
136	8	ELSE	2
137	8	WHILE	3
138	8	DO	5
139	5	STEP	3
140	5	UNTIL	2
141	1	IF	3
142	1	FOR	3
143	10	SHORT	10
144	9	INTEGER	10
145	9	LOGICAL	10
146	9	REAL	10
147	10	LONG	10
148	9	BYTE	10
149	9	CHARACTER	10
150	9	LABEL	10
151	5	ARRAY	9
152	8	FUNCTION	9
153	8	SYN	13
154	6	REGISTER	8
155	6	ADCON	8
156	6	SLCON	8
157	10	SEGMENT	9
158	8	PROCEDURE	9

159	3	BASE	9
160	2	LOCAL	3
161	8	"	13
162	1	"	1

INDEX NO.	SYMBOL	REL. NO.	SYMBOL	* INDEX NO.	SYMBOL	REL. NO.	SYMBOL
*** RELATIONS ***							
1	1 ID	.GT.	98 (	2	1 ID	.GT.	16 ARITH OP
3	1 ID	.GT.	99 *	4	1 ID	.GT.	100 -
5	1 ID	.GT.	101 *	6	1 ID	.GT.	102 /
7	1 ID	.GT.	103 +	8	1 ID	.GT.	104 --
9	1 ID	.GT.	108 LOG OP	10	1 ID	.GT.	111 AND
11	1 ID	.GT.	112 OR	12	1 ID	.GT.	113 XOR
13	1 ID	.GT.	119 SHIFT OP	14	1 ID	.GT.	114 SHLA
15	1 ID	.GT.	115 SHRA	16	1 ID	.GT.	116 SHLL
17	1 ID	.GT.	117 SHRL	18	1 ID	.GT.	28 T CELL EXP*
19	1 ID	.GT.	27 T CELL FXP1	20	1 ID	.GT.	25 T CELL EXP1
21	1 ID	.GT.	121 =	22	1 ID	.GT.	26 T CELL EXP2
23	1 ID	.GT.	123 *	24	1 ID	.GT.	125 ))
25	1 ID	.GT.	129 *	26	1 ID	.GT.	23 X REG EXPR
27	1 ID	.GT.	22 X REG EXP1	28	1 ID	.GT.	21 EQU
29	1 ID	.GT.	39 VECTOR FXP1	30	1 ID	.GT.	97 )
31	1 ID	.GT.	17 REL OP	32	1 ID	.GT.	105 .LT.
33	1 ID	.GT.	106 .EQ.	34	1 ID	.GT.	107 .GT.
35	1 ID	.GT.	108 .LE.	36	1 ID	.GT.	109 .GE.
37	1 ID	.GT.	110 .NE.	38	1 ID	.GT.	135 THEN
39	1 ID	.GT.	136 ELSE	40	1 ID	.GT.	138 DO
41	1 ID	.GT.	139 STEP	42	1 ID	.GT.	52 DO
43	1 ID	.EQ.	153 SYN	44	1 ID	.EQ.	161 ::
45	1 ID	.GT.	162 ARITH OP	46	2 T NUMBER	.GT.	97 )
47	2 T NUMBER	.GT.	16 ARITH OP	48	2 T NUMBER	.GT.	99 +
49	2 T NUMBER	.GT.	100 -	50	2 T NUMBER	.GT.	101 *
51	2 T NUMBER	.GT.	102 /	52	2 T NUMBER	.GT.	103 ++
53	2 T NUMBER	.GT.	104 --	54	2 T NUMBER	.GT.	18 LOG OP
55	2 T NUMBER	.GT.	111 AND	56	2 T NUMBER	.GT.	112 OR
57	2 T NUMBER	.GT.	113 XOR	58	2 T NUMBER	.GT.	119 SHIFT OP
59	2 T NUMBER	.GT.	114 SHLA	60	2 T NUMBER	.GT.	115 SHRA
61	2 T NUMBER	.GT.	116 SHLL	62	2 T NUMBER	.GT.	117 SHRL
63	2 T NUMBER	.GT.	123 *	64	2 T NUMBER	.GT.	125 ))
65	2 T NUMBER	.GT.	129 *	66	2 T NUMBER	.EQ.	117 REL OP
67	2 T NUMBER	.LT.	105 .LT.	68	2 T NUMBER	.LT.	106 .EQ.
69	2 T NUMBER	.LT.	107 .GT.	70	2 T NUMBER	.LT.	108 .LE.
71	2 T NUMBER	.LT.	109 .GE.	72	2 T NUMBER	.LT.	110 .NE.
73	2 T NUMBER	.GT.	135 THEN	74	2 T NUMBER	.GT.	136 ELSE
75	2 T NUMBER	.GT.	138 DO	76	2 T NUMBER	.GT.	139 STEP
77	2 T NUMBER	.GT.	51 LIMIT	78	2 T NUMBER	.GT.	140 UNTIL
79	2 T NUMBER	.GT.	52 DO	80	2 T NUMBER	.EQ.	55 SI T TYPE
81	2 T NUMBER	.LT.	143 SHORT	82	2 T NUMBER	.LT.	144 INTEGER
83	2 T NUMBER	.LT.	142 LOGICAL	84	2 T NUMBER	.LT.	146 REAL
85	2 T NUMBER	.LT.	147 LONG	86	2 T NUMBER	.LT.	148 BYTE
87	2 T NUMBER	.LT.	149 CHARACTER	88	2 T NUMBER	.LT.	150 LABEL
89	2 T NUMBER	.GT.	162 *	90	3 STRING	.GT.	16 ARITH OP
91	3 STRING	.GT.	99 *	92	3 STRING	.GT.	100 -
93	3 STRING	.GT.	101 *	94	3 STRING	.GT.	102 /
95	3 STRING	.GT.	103 ++	96	3 STRING	.GT.	104 --
97	3 STRING	.GT.	18 LOG OP			.GT.	111 AND

```

99   3 STRING          .GT. 112 OR
     101  3 STRING          .GT. 119 SHIFT OP
     103  3 STRING          .GT. 115 SHRA
     105  3 STRING          .GT. 117 SHRL
     107  3 STRING          .GT. 125 )
     109  3 STRING          .GT. 129 *
     111  3 STRING          .GT. 135 THEN
     113  3 STRING          .GT. 136 ELSE
     115  4 X REG          .GT. 139 STEP
     117  4 X REG          .GT. 97 )
     119  4 X REG          .GT. 99 +
     121  4 X REG          .GT. 101 *
     123  4 X REG          .GT. 103 ++
     125  4 X REG          .EQ. 123 X REG EXP1
     127  4 X REG          .LT. 121 =
     129  4 X REG          .LT. 127 T CELL EXP1
     131  4 X REG          .EQ. 129 OF
     133  4 X REG          .EQ. 117 REL OP
     135  4 X REG          .LT. 106 EQ,
     137  4 X REG          .LT. 108 LE,
     139  4 X REG          .LT. 110 NE,
     141  4 X REG          .GT. 112 OR
     143  4 X REG          .GT. 136 ELSE
     145  4 X REG          .GT. 139 STEP
     147  4 X REG          .GT. 162 *
     149  5 FUNC ID         .GT. 129 *
     151  5 FUNC ID         .GT. 162 -
     153  6 DPARAM ID        .GT. 97 )
     155  7 PROC ID          .EQ. 98 (
     157  7 PROC ID          .GT. 97 )
     159  7 PROC ID          .GT. 162 -
     161  8 B REG           .EQ. 121 =
     163  8 B REG           .GT. 162 -
     165  9 LABEL ID         .GT. 97 )
     167  11 T CELL ID        .EQ. 98 (
     169  11 T CELL ID        .GT. 99 +
     171  11 T CELL ID        .GT. 101 *
     173  11 T CELL ID        .GT. 103 ++
     175  11 T CELL ID        .GT. 18 LOG OP
     177  11 T CELL ID        .GT. 112 OR
     179  11 T CELL ID        .GT. 119 SHIFT OP
     181  11 T CELL ID        .GT. 115 SHRA
     183  11 T CELL ID        .GT. 117 SHRL
     185  11 T CELL ID        .GT. 127 T CELL EXP1
     187  11 T CELL ID        .GT. 121 =
     189  11 T CELL ID        .GT. 123 +
     191  11 T CELL ID        .GT. 129 *
     193  11 T CELL ID        .GT. 22 X REG EXP1
     195  11 T CELL ID        .GT. 39 VECTOR EXP1
     197  11 T CELL ID        .GT. 17 REL OP
     199  11 T CELL ID        .GT. 106 EQ,
     201  11 T CELL ID        .GT. 108 LE,
     203  11 T CELL ID        .GT. 110 NE,
     100  3 STRING          .GT. 113 XOR
     102  3 STRING          .GT. 114 SHLA
     104  3 STRING          .GT. 116 SHLL
     106  3 STRING          .GT. 123 *
     108  3 STRING          .GT. 129 *
     110  3 STRING          .GT. 135 THEN
     112  3 STRING          .GT. 138 DO
     114  3 STRING          .GT. 162 ARITH OP
     116  4 X REG          .GT. 100 -
     118  4 X REG          .GT. 102 /
     120  4 X REG          .GT. 104 -
     122  4 X REG          .GT. 129 *
     124  4 X REG          .LT. 105 LT,
     126  4 X REG          .LT. 107 GT,
     128  4 X REG          .LT. 109 GE,
     130  4 X REG          .GT. 111 AND
     132  4 X REG          .GT. 135 THEN
     134  4 X REG          .GT. 138 DO
     136  4 X REG          .GT. 52 DO
     138  4 X REG          .EQ. 98 (
     140  4 X REG          .GT. 136 ELSE
     142  4 X REG          .GT. 123 ,
     144  4 X REG          .GT. 123 *
     146  4 X REG          .GT. 129 *
     148  5 FUNC ID         .GT. 136 ELSE
     150  5 FUNC ID         .GT. 136 ELSE
     152  6 DPARAM ID        .GT. 123 ,
     154  7 PROC ID          .GT. 123 *
     156  7 PROC ID          .GT. 129 *
     158  7 PROC ID          .GT. 136 ELSE
     160  8 B REG           .GT. 129 *
     162  8 B REG           .GT. 136 ELSE
     164  9 LABEL ID         .GT. 123 ,
     166  10 EXTERN ID        .GT. 123 *
     168  11 T CELL ID        .GT. 111 AND
     170  11 T CELL ID        .GT. 113 XOR
     172  11 T CELL ID        .GT. 114 SHLA
     174  11 T CELL ID        .GT. 104 --
     176  11 T CELL ID        .GT. 111 AND
     178  11 T CELL ID        .GT. 113 XOR
     180  11 T CELL ID        .GT. 114 SHLA
     182  11 T CELL ID        .GT. 116 SHLL
     184  11 T CELL ID        .GT. 28 T CELL EXP1
     186  11 T CELL ID        .GT. 25 T CELL EXP1
     188  11 T CELL ID        .GT. 26 T CELL EXP2
     190  11 T CELL ID        .GT. 125 )
     192  11 T CELL ID        .GT. 23 X REG EXP1
     194  11 T CELL ID        .GT. 21 EOU
     196  11 T CELL ID        .GT. 97 )
     198  11 T CELL ID        .GT. 105 LT,
     200  11 T CELL ID        .GT. 107 GT,
     202  11 T CELL ID        .GT. 109 GE,
     204  11 T CELL ID        .GT. 135 THEN

```

```

205   11 T CELL ID
      207  11 T CELL ID
      209  11 T CELL ID
      211  12 T CELL
      213  12 T CELL
      215  12 T CELL
      217  12 T CELL
      219  12 T CELL
      221  12 T CELL
      223  12 T CELL
      225  12 T CELL
      227  12 T CELL
      229  12 T CELL
      231  12 T CELL
      233  12 T CELL
      235  12 T CELL
      237  12 T CELL
      239  12 T CELL
      241  12 T CELL
      243  12 T CELL
      245  12 T CELL
      247  12 T CELL
      249  12 T CELL
      251  12 T CELL
      253  14 T CELL2
      255  14 T CELL2
      257  14 T CELL2
      259  14 T CELL2
      261  15 T CELL3
      263  15 T CELL3
      265  16 ARITH OP
      267  16 ARITH OP
      269  16 ARITH OP
      271  16 ARITH OP
      273  16 ARITH OP
      275  17 REL OP
      277  17 REL OP
      279  17 REL OP
      281  17 REL OP
      283  17 REL OP
      285  17 REL OP
      287  18 LOG OP
      289  18 LOG OP
      291  18 LOG OP
      293  18 LOG OP
      295  18 LOG OP
      297  19 SHIFT OP
      299  20 UNARY OP
      301  20 UNARY OP
      303  20 UNARY OP
      305  20 UNARY OP
      307  20 UNARY OP
      309  21 EQU

      .GT. 136 ELSE
      .GT. 139 STEP
      .GT. 162 +
      .GT. 99 +
      .GT. 101 *
      .GT. 103 ++
      .GT. 18 LOG OP
      .GT. 112 OR
      .GT. 19 SHIFT OP
      .GT. 115 SHRA
      .GT. 117 SHRL
      .LT. 27 T CELL EXPR
      .LT. 121 =
      .GT. 123 +
      .GT. 129 *
      .LT. 22 X REG EXP1
      .EQ. 39 VECTOR EXPR
      .EQ. 17 REL OP
      .LT. 106 EQ,
      .LT. 108 LE,
      .LT. 110 NE,
      .GT. 136 ELSE
      .GT. 139 STEP
      .GT. 162 +
      .EQ. 97 )
      .LT. 99 +
      .LT. 101 *
      .LT. 103 ++
      .EQ. 2 T NUMBER
      .EQ. 4 X REG
      .EQ. 2 T NUMBER
      .EQ. 12 T CELL
      .LT. 1 D
      .LT. 13 T CELL1
      .LT. 15 T CELL3
      .LT. 11 T CELL ID
      .LT. 87 IDI
      .LT. 14 T CELL2
      .EQ. 2 T NUMBER
      .EQ. 4 X REG
      .EQ. 3 STRING
      .EQ. 12 T CELL
      .LT. 1 D
      .LT. 13 T CELL1
      .LT. 15 T CELL3
      .LT. 88 ID
      .LT. 17 REL OP
      .LT. 276 17 REL OP
      .LT. 278 17 REL OP
      .LT. 280 17 REL OP
      .LT. 282 17 REL OP
      .LT. 284 17 REL OP
      .LT. 286 17 REL OP
      .LT. 288 18 LOG OP
      .LT. 290 18 LOG OP
      .LT. 292 18 LOG OP
      .LT. 294 18 LOG OP
      .LT. 296 19 SHIFT OP
      .LT. 66 IDT
      .LT. 11 T CELL ID
      .LT. 87 IDI
      .LT. 14 T CELL2
      .LT. 87 IDI
      .LT. 13 T CELL1
      .LT. 15 T CELL3
      .LT. 88 ID
      .LT. 89 IDS
      .LT. 11 T CELL ID
      .LT. 87 IDI
      .LT. 14 T CELL2
      .LT. 87 IDI
      .LT. 13 T CELL1
      .LT. 15 T CELL3
      .LT. 88 ID
      .LT. 89 IDS
      .LT. 94 IDB

206   11 T CELL ID
      208  11 T CELL ID
      210  12 T CELL
      212  12 T CELL
      214  12 T CELL
      216  12 T CELL
      218  12 T CELL
      220  12 T CELL
      222  12 T CELL
      224  12 T CELL
      226  12 T CELL
      228  12 T CELL
      230  12 T CELL
      232  12 T CELL
      234  12 T CELL
      236  12 T CELL
      238  12 T CELL
      240  12 T CELL
      242  12 T CELL
      244  12 T CELL
      246  12 T CELL
      248  12 T CELL
      250  12 T CELL
      252  13 T CELL1
      254  14 T CELL2
      256  14 T CELL2
      258  14 T CELL2
      260  14 T CELL2
      262  15 T CELL3
      264  15 T CELL3
      266  16 ARITH OP
      268  16 ARITH OP
      270  16 ARITH OP
      272  16 ARITH OP
      274  17 REL OP
      276  17 REL OP
      278  17 REL OP
      280  17 REL OP
      282  17 REL OP
      284  17 REL OP
      286  17 REL OP
      288  18 LOG OP
      290  18 LOG OP
      292  18 LOG OP
      294  18 LOG OP
      296  19 SHIFT OP
      .EQ. 12 T CELL
      .LT. 1 ID
      .LT. 13 T CELL1
      .LT. 14 T CELL2
      .LT. 2 T NUMBER
      .EQ. 2 T NUMBER
      .LT. 88 ID
      .LT. 89 IDS
      .LT. 11 T CELL ID
      .LT. 87 IDI
      .LT. 14 T CELL2
      .LT. 87 IDI
      .LT. 13 T CELL1
      .LT. 15 T CELL3
      .LT. 88 ID
      .LT. 89 IDS
      .LT. 94 IDB

```

```

311 22 X REG EXP1          16 ARITH OP
312 22 X REG EXP1          99 +
313 22 X REG EXP1          .LT. 100 -
314 22 X REG EXP1          .LT. 101 *
315 22 X REG EXP1          .LT. 102 /
316 22 X REG EXP1          .LT. 103 ++
317 22 X REG EXP1          .LT. 104 --
318 22 X REG EXP1          .LT. 129 *
319 22 X REG EXP1          .GT. 136 ELSE
320 22 X REG EXP1          .GT. 139 STEP
321 22 X REG EXP1          .GT. 162 .
322 23 X REG EXP1          .GT. 162 .
323 23 X REG EXP1          .GT. 136 ELSE
324 23 X REG ASS          .GT. 139 STEP
325 23 X REG EXP1          .GT. 162 .
326 24 X REG ASS          .GT. 139 STEP
327 24 X REG ASS          .GT. 136 ELSE
328 24 X REG ASS          .GT. 162 .
329 24 X REG ASS          .GT. 162 .
330 25 T CELL EXP1          .GT. 16 ARITH OP
331 25 T CELL EXP1          .GT. 99 +
332 25 T CELL EXP1          .GT. 100 -
333 25 T CELL EXP1          .GT. 101 *
334 25 T CELL EXP1          .GT. 102 /
335 25 T CELL EXP1          .GT. 103 ++
336 25 T CELL EXP1          .GT. 104 --
337 25 T CELL EXP1          .GT. 111 AND
338 25 T CELL EXP1          .GT. 111 XOR
339 25 T CELL EXP1          .GT. 112 OR
340 25 T CELL EXP1          .GT. 113 SHLL
341 25 T CELL EXP1          .GT. 114 SHLA
342 25 T CELL EXP1          .GT. 115 SHRA
343 25 T CELL EXP1          .GT. 116 SHRL
344 25 T CELL EXP1          .GT. 117 SHRL
345 25 T CELL EXP1          .GT. 118 LOG OP
346 25 T CELL EXP1          .GT. 119 SHIFT OP
347 25 T CELL EXP1          .GT. 120 LOG OP
348 25 T CELL EXP1          .GT. 121 LOG OP
349 25 T CELL EXP1          .GT. 122 LOG OP
350 25 T CELL EXP1          .GT. 123 LOG OP
351 26 T CELL EXP2          .GT. 99 +
352 26 T CELL EXP2          .GT. 100 -
353 26 T CELL EXP2          .GT. 101 *
354 26 T CELL EXP2          .GT. 102 /
355 26 T CELL EXP2          .GT. 103 ++
356 26 T CELL EXP2          .GT. 104 --
357 26 T CELL EXP2          .GT. 111 AND
358 26 T CELL EXP2          .GT. 112 OR
359 26 T CELL EXP2          .GT. 113 XOR
360 26 T CELL EXP2          .GT. 114 SHLL
361 26 T CELL EXP2          .GT. 115 SHLA
362 26 T CELL EXP2          .GT. 116 SHRL
363 26 T CELL EXP2          .GT. 117 SHRA
364 26 T CELL EXP2          .GT. 118 LOG OP
365 26 T CELL EXP2          .GT. 119 SHIFT OP
366 26 T CELL EXP2          .GT. 120 LOG OP
367 26 T CELL EXP2          .GT. 121 LOG OP
368 26 T CELL EXP2          .GT. 122 LOG OP
369 26 T CELL EXP2          .GT. 123 LOG OP
370 27 T CELL EXP2          .GT. 124 LOG OP
371 27 T CELL EXP2          .GT. 125 LOG OP
372 27 T CELL EXP2          .GT. 126 LOG OP
373 27 T CELL EXP2          .GT. 127 LOG OP
374 27 T CELL EXP2          .GT. 128 LOG OP
375 27 T CELL EXP2          .GT. 129 LOG OP
376 27 T CELL EXP2          .GT. 130 LOG OP
377 27 T CELL EXP2          .GT. 131 LOG OP
378 27 T CELL EXP2          .GT. 132 LOG OP
379 27 T CELL EXP2          .GT. 133 LOG OP
380 27 T CELL EXP2          .GT. 134 LOG OP
381 27 T CELL EXP2          .GT. 135 LOG OP
382 27 T CELL EXP2          .GT. 136 LOG OP
383 27 T CELL EXP2          .GT. 137 LOG OP
384 27 T CELL EXP2          .GT. 138 LOG OP
385 27 T CELL EXP2          .GT. 139 LOG OP
386 27 T CELL EXP2          .GT. 140 LOG OP
387 27 T CELL EXP2          .GT. 141 LOG OP
388 27 T CELL EXP2          .GT. 142 LOG OP
389 27 T CELL EXP2          .GT. 143 LOG OP
390 28 T CELL EXP2          .GT. 144 LOG OP
391 28 T CELL EXP2          .GT. 145 LOG OP
392 28 T CELL EXP2          .GT. 146 LOG OP
393 28 T CELL EXP2          .GT. 147 LOG OP
394 29 T CELL ASS          .GT. 148 LOG OP
395 29 T CELL ASS          .GT. 149 LOG OP
396 29 T CELL ASS          .GT. 150 LOG OP
397 30 FUNC1              .GT. 151 LOG OP
398 30 FUNC1              .GT. 152 LOG OP
399 31 FUNC2              .GT. 153 LOG OP
400 31 FUNC2              .GT. 154 LOG OP
401 31 FUNC2              .GT. 155 LOG OP
402 31 FUNC2              .GT. 156 LOG OP
403 31 FUNC2              .GT. 157 LOG OP
404 31 FUNC2              .GT. 158 LOG OP
405 31 FUNC2              .GT. 159 LOG OP
406 31 FUNC2              .GT. 160 LOG OP
407 31 FUNC2              .GT. 161 LOG OP
408 31 FUNC2              .GT. 162 LOG OP
409 31 FUNC2              .GT. 163 LOG OP
410 31 FUNC2              .GT. 164 LOG OP
411 31 FUNC2              .GT. 165 LOG OP
412 31 FUNC2              .GT. 166 LOG OP
413 31 FUNC2              .GT. 167 LOG OP
414 31 FUNC2              .GT. 168 LOG OP
415 31 FUNC2              .GT. 169 LOG OP
416 32 PROC1              .GT. 170 LOG OP

```





```

629 49 COND DO
631 49 COND DO
633 49 COND DO
635 49 COND DO
637 49 COND DO
639 49 COND DO
641 49 COND DO
643 49 COND DO
645 49 COND DO
647 49 COND DO
649 49 COND DO
651 49 COND DO
653 49 COND DO
655 49 COND DO
657 49 COND DO
659 49 COND DO
661 49 COND DO
663 49 COND DO
665 49 COND DO
667 49 COND DO
669 50 ASS STEP
671 51 LIMIT
673 52 DO
675 52 DO
677 52 DO
679 52 DO
681 52 DO
683 52 DO
685 52 DO
687 52 DO
689 52 DO
691 52 DO
693 52 DO
695 52 DO
697 52 DO
699 52 DO
701 52 DO
703 52 DO
705 52 DO
707 52 DO
709 52 DO
711 52 DO
713 53 STATEMENT*
715 54 STATEMENT
717 55 SI T TYPE
719 55 SI T TYPE
721 55 SI T TYPE
723 56 T TYPE
725 57 T DECL1
727 58 T DECL2
729 59 T DECL3
731 59 T DECL3
733 60 T DECL4

.EQ. 53 STATEMENT*
.LT. 12 T CELL
.LT. 1 ID
.LT. 13 T CELLI
.LT. 15 T CELL3
.LT. 94 IDB
.LT. 24 X REG ASS
.LT. 90 IDX
.LT. 37 MULT ASS4
.LT. 34 MULT ASS1
.LT. 35 MULT ASS2
.LT. 131 GOTO
.LT. 93 IDP
.LT. 33 PROC2
.LT. 91 IDF
.LT. 31 FUNC2
.LT. 126 CASE
.LT. 84 BLOCKHEAD
.LT. 141 IF
.LT. 137 WHILE
.LT. 51 LIMIT
.LT. 52 DO
.EQ. 53 STATEMENT*
.LT. 12 T CELL
.LT. 1 ID
.LT. 13 T CELLI
.LT. 15 T CELL3
.LT. 94 IDB
.LT. 24 X REG ASS
.LT. 90 IDX
.LT. 37 MULT ASS4
.LT. 34 MULT ASS1
.LT. 35 MULT ASS2
.LT. 131 GOTO
.LT. 93 IDP
.LT. 33 PROC2
.LT. 91 IDF
.LT. 31 FUNC2
.LT. 126 CASE
.LT. 84 BLOCKHEAD
.LT. 141 IF
.LT. 137 WHILE
.GT. 129 *
.EQ. 129 *
.GT. 1 ID
.EQ. 154 REGISTER
.EQ. 156 SLCON
.LT. 87 IDI
.LT. 14 T CELL2
.LT. 8 B REG
.LT. 29 T CELL ASS
.LT. 4 X REG
.LT. 14 T CELL
.LT. 87 IDI
.LT. 30 FUNC1
.LT. 40 CASE SEQ
.LT. 85 BLOCKBODY
.LT. 128 BEGIN
.LT. 48 WHILE
.LT. 142 FOR
.LT. 140 UNTIL
.LT. 138 DO
.LT. 41 SIMPLE ST
.LT. 11 T CELL
.LT. 87 IDI
.LT. 30 FUNC1
.LT. 38 MULT ASS
.LT. 36 MULT ASS3
.LT. 14 T CELL2
.LT. 8 B REG
.LT. 29 T CELL ASS
.LT. 4 X REG
.LT. 7 PROC ID
.LT. 32 PROC1
.LT. 5 FUNC ID
.LT. 30 FUNC1
.LT. 130 NULL
.LT. 40 CASE SEQ
.LT. 85 BLOCKBODY
.LT. 128 BEGIN
.LT. 48 WHILE
.LT. 142 FOR
.GT. 162 *
.EQ. 162 *
.GT. 87 IDI
.EQ. 155 ADCON
.EQ. 1 ID
.GT. 129 *
.TITLE. 53 STATEMENT*
716 54 STATEMENT
718 55 SI T TYPE
720 55 SI T TYPE
722 56 T TYPE
724 57 T DECL1
726 57 T DECL1
728 58 T DECL2
730 59 T DECL3
732 59 T DECL3
734 60 T DECL4

```













1371 112 OR .GT. 14 T CELL2  
 1373 112 OR .GT. 2 T NUMBER  
 1375 112 OR .GT. 43 CONDITION  
 1377 112 OR .GT. 90 IDX  
 1379 112 OR .GT. 42 NOT  
 1381 113 XOR .GT. 12 T CELL  
 1383 113 XOR .GT. 1 ID  
 1385 113 XOR .GT. 13 T CELL1  
 1387 113 XOR .GT. 15 T CELL3  
 1389 113 XOR .GT. 88 IDT  
 1391 114 SHLA .GT. 88 IDT  
 1393 115 SHRA .GT. 88 IDT  
 1395 116 SHLL .GT. 88 IDT  
 1397 117 SHRL .GT. 88 IDT  
 1399 118 ABS .GT. 11 T CELL ID  
 1401 118 ABS .GT. 87 IDI  
 1403 118 ABS .GT. 14 T CELL2  
 1405 118 ABS .GT. 2 T NUMBER  
 1407 118 ABS .GT. 3 STRING  
 1409 119 NEG .GT. 12 T CELL  
 1411 119 NEG .GT. 1 ID  
 1413 119 NEG .GT. 13 T CELL1  
 1415 119 NEG .GT. 15 T CELL3  
 1417 119 NEG .GT. 88 IDT  
 1419 119 NEG .GT. 89 IDS  
 1421 120 NEGABS .GT. 11 T CELL ID  
 1423 120 NEGABS .GT. 87 IDI  
 1425 120 NEGABS .GT. 14 T CELL2  
 1427 120 NEGABS .GT. 2 T NUMBER  
 1429 120 NEGABS .GT. 3 STRING  
 1431 121 .EQ. 4 X REG  
 1433 121 .EQ. 12 T CELL  
 1435 121 .LT. 1 ID  
 1437 121 .LT. 13 T CELL1  
 1439 121 .LT. 15 T CELL3  
 1441 121 .LT. 88 IDT  
 1443 121 .LT. 89 IDS  
 1445 121 .LT. 118 ABS  
 1447 121 .LT. 120 NEGABS  
 1449 121 .EQ. 37 MULT ASS4  
 1451 121 .LT. 34 MULT ASS1  
 1453 121 .LT. 35 MULT ASS2  
 1455 121 .GT. 94 IDB  
 1457 121 .LT. 98 {  
 1459 122 A .LT. 11 T CELL ID  
 1461 122 A .LT. 87 IDI  
 1463 122 A .LT. 14 T CELL2  
 1465 123 .GT. 2 T NUMBER  
 1467 123 .GT. 4 X REG  
 1469 123 .GT. 12 T CELL  
 1471 123 .GT. 1 ID  
 1473 123 .GT. 13 T CELL1  
 1475 123 .GT. 15 T CELL3  
 1372 112 OR .GT. 15 T CELL3  
 1374 112 OR .GT. 88 IDT  
 1376 112 OR .GT. 4 X REG  
 1378 112 OR .GT. 134 OVERFLOW  
 1380 112 OR .GT. 133 NOT.  
 1382 113 XOR .GT. 11 T CELL ID  
 1384 113 XOR .GT. 87 IDI  
 1386 113 XOR .GT. 14 T CELL2  
 1388 113 XOR .GT. 2 T NUMBER  
 1389 114 SHLA .GT. 2 T NUMBER  
 1392 115 SHRA .GT. 2 T NUMBER  
 1394 116 SHLL .GT. 2 T NUMBER  
 1396 117 SHRL .GT. 2 T NUMBER  
 1398 118 ABS .GT. 12 T CELL  
 1399 118 ABS .GT. 1 ID  
 1400 118 ABS .GT. 13 T CELL1  
 1402 118 ABS .GT. 15 T CELL3  
 1404 118 ABS .GT. 88 IDT  
 1406 118 ABS .GT. 89 IDS  
 1408 118 ABS .GT. 3 STRING  
 1410 119 NEG .GT. 11 T CELL ID  
 1412 119 NEG .GT. 87 IDI  
 1414 119 NEG .GT. 14 T CELL2  
 1416 119 NEG .GT. 2 T NUMBER  
 1418 119 NEG .GT. 3 STRING  
 1420 120 NEGABS .GT. 12 T CELL  
 1422 120 NEGABS .GT. 1 ID  
 1424 120 NEGABS .GT. 13 T CELL1  
 1426 120 NEGABS .GT. 15 T CELL3  
 1428 120 NEGABS .GT. 88 IDT  
 1430 120 NEGABS .GT. 89 IDS  
 1432 121 .LT. 90 IDX  
 1434 121 .LT. 11 T CELL ID  
 1436 121 .LT. 87 IDI  
 1438 121 .LT. 14 T CELL2  
 1440 121 .EQ. 2 T NUMBER  
 1442 121 .EQ. 3 STRING  
 1444 121 .EQ. 20 UNARY OP  
 1446 121 .LT. 119 NEG  
 1448 121 .EQ. 122 A  
 1450 121 .LT. 36 MULT ASS3  
 1452 121 .LT. 124 CC  
 1454 121 .GT. 8 B REG  
 1456 121 .EQ. 63 LP  
 1458 122 A .EQ. 12 T CELL  
 1460 122 A .LT. 1 ID  
 1462 122 A .LT. 13 T CELL1  
 1464 122 A .LT. 15 T CELL3  
 1466 123 .GT. 86 IDT  
 1468 123 .GT. 90 IDX  
 1470 123 .GT. 11 T CELL ID  
 1472 123 .GT. 87 IDI  
 1474 123 .GT. 14 T CELL2  
 1476 123 .GT. 3 STRING



```

•GT. 81 PROC HD2D
•GT. 54 STATEMENT
•GT. 41 SIMPLE ST
•GT. 11 T CELL
•GT. 10
•GT. 87 ID1
•GT. 14 T CELL2
•GT. 8 B REG
•GT. 29 T CELL ASS
•GT. 4 X REG
•GT. 38 MULT ASS
•GT. 36 MULT ASS3
•GT. 124 CC
•GT. 130 NULL
•GT. 7 PROC ID
•GT. 32 PROC1
•GT. 5 FUNC ID
•GT. 30 FUNC1
•GT. 40 CASE SEQ
•GT. 65 BLOCKBODY
•GT. 128 BEGIN
•GT. 128 WHILE
•GT. 48 WHILE
•GT. 142 FOR
•GT. 82 DECL
•GT. 57 T DECL1
•GT. 55 SI T TYPE
•GT. 144 INTEGER
•GT. 146 REAL
•GT. 148 BYTE
•GT. 150 LABEL
•GT. 58 T DECL2
•GT. 60 T DECL4
•GT. 70 FUNC DC7
•GT. 68 FUNC DC5
•GT. 66 FUNC DC3
•GT. 64 FUNC DC1
•GT. 72 SYN DC2
•GT. 73 SYN DC3
•GT. 79 PROC HD5
•GT. 77 PROC HD3
•GT. 75 PROC HD1
•GT. 74 SEG HEAD
•GT. 81 PROC HD2D
•GT. 129 *
•GT. 1621 129 *
•GT. 1613 129 *
•GT. 1615 129 *
•GT. 1617 129 *
•GT. 1619 129 *
•GT. 1621 129 *
•GT. 1623 129 *
•GT. 1625 129 *
•GT. 1627 129 *
•GT. 1629 129 *
•GT. 1631 129 *
•GT. 1633 129 *
•GT. 1635 129 *
•GT. 1637 129 *
•GT. 1639 129 *
•GT. 1641 129 *
•GT. 1643 129 *
•GT. 1645 129 *
•GT. 1647 129 *
•GT. 1649 129 *
•GT. 1651 129 *
•GT. 1653 129 *
•GT. 1655 129 *
•GT. 1657 129 *
•GT. 1659 129 *
•GT. 1661 129 *
•GT. 1663 129 *
•GT. 1665 129 *
•GT. 1667 130 NULL
•GT. 1669 130 NULL
•GT. 1671 131 GOTO
•GT. 1673 132 END
•GT. 136 ELSE
•GT. 112 T CELL
•GT. 1 ID
•GT. 13 T CELL1
•GT. 15 T CELL3
•GT. 94 IDB
•GT. 24 X REG ASS
•GT. 90 IDX
•GT. 37 MULT ASS4
•GT. 34 MULT ASS1
•GT. 35 MULT ASS2
•GT. 131 GOTO
•GT. 93 IDP
•GT. 33 PROC2
•GT. 91 IDF
•GT. 31 FUNC2
•GT. 126 CASE
•GT. 84 BLOCKHEAD
•GT. 141 IF
•GT. 137 WHILE
•GT. 132 END
•GT. 62 T DECL7
•GT. 145 LOGICAL
•GT. 147 LONG
•GT. 149 CHARACTER
•GT. 151 ARRAY
•GT. 61 T DECL5
•GT. 59 T DECL3
•GT. 69 FUNC DC6
•GT. 67 FUNC DC4
•GT. 65 FUNC DC2
•GT. 152 FUNCTION
•GT. 71 SYN DC1
•GT. 80 PROC HD6
•GT. 78 PROC HD4
•GT. 76 PROC HD2
•GT. 158 PROCEDURE
•GT. 157 SEGMENT
•GT. 83 LABEL DEF
•GT. 136 ELSE
•GT. 1 ID
•GT. 129 *
•GT. 162 END
•GT. 1674 132 END
•GT. 1676 133 *NOT.
•GT. 1678 133 *NOT.
•GT. 1680 133 *NOT.
•GT. 1682 134 OVERFLOW
•GT. 1684 134 OVERFLOW
•GT. 1686 135 THEN
•GT. 1688 135 THEN
•GT. 111 T CELL1
•GT. 111 T CELL
•GT. 87 ID1
•GT. 14 T CELL2
•GT. 111 AND
•GT. 135 THEN
•GT. 93 STATEMENT*
•GT. 12 T CELL

```

```

1689 135 THEN          11 T CELL 10      1 10
1691 135 THEN          87 1D           13 T CELL1
1693 135 THEN          87 1D           15 T CELL3
1695 135 THEN          14 T CELL2
1697 135 THEN          8 B REG
1699 135 THEN          29 T CELL ASS
1701 135 THEN          4 X REG
1703 135 THEN          38 MULT ASS
1705 135 THEN          36 MULT ASS3
1707 135 THEN          124 CC
1709 135 THEN          130 NULL
1711 135 THEN          32 PROC1
1713 135 THEN          1D
1715 135 THEN          5 FUNC 1D
1717 135 THEN          30 FUNC1
1719 135 THEN          40 CASE SE@*
1721 135 THEN          85 BLOCKBODY
1723 135 THEN          BEGIN
1725 135 THEN          128 BEGIN
1727 136 ELSE          48 WHILE
1729 136 ELSE          142 FOR
1731 136 ELSE          53 STATEMENT*
1733 136 ELSE          1725 135 THEN
1735 136 ELSE          1716 135 THEN
1737 136 ELSE          1728 135 THEN
1739 136 ELSE          1720 135 THEN
1741 136 ELSE          1722 135 THEN
1743 136 ELSE          1724 135 THEN
1745 136 ELSE          1734 136 ELSE
1747 136 ELSE          1736 136 ELSE
1749 136 ELSE          1738 136 ELSE
1751 136 ELSE          1740 136 ELSE
1753 136 ELSE          1742 136 ELSE
1755 136 ELSE          1744 136 ELSE
1757 136 ELSE          1746 136 ELSE
1759 136 ELSE          1748 136 ELSE
1761 136 ELSE          1750 136 ELSE
1763 136 ELSE          1752 136 ELSE
1765 136 ELSE          1754 136 ELSE
1767 137 WHILE          1756 136 ELSE
1769 137 WHILE          1758 136 ELSE
1771 137 WHILE          1760 136 ELSE
1773 137 WHILE          1762 136 ELSE
1775 137 WHILE          1764 136 ELSE
1777 137 WHILE          1766 137 WHILE
1779 137 WHILE          1768 137 WHILE
1781 137 WHILE          1770 137 WHILE
1783 137 WHILE          1772 137 WHILE
1785 138 DO             1774 137 WHILE
1787 138 DO             1776 137 WHILE
1789 138 DO             1778 137 WHILE
1791 138 DO             1780 137 WHILE
1793 138 DO             1782 137 WHILE
1690 135 THEN          133 NOT
1692 135 THEN          133 STATEMENT*
1694 135 THEN          133 COND DO
1696 135 THEN          137 ID
1698 135 THEN          137 ID
1700 135 THEN          137 ID
1702 135 THEN          137 ID
1704 135 THEN          137 ID
1706 135 THEN          137 ID
1708 135 THEN          137 ID
1710 135 THEN          137 ID
1712 135 THEN          137 ID
1714 135 THEN          137 ID
1716 135 THEN          137 ID
1718 135 THEN          137 ID
1720 135 THEN          137 ID
1722 135 THEN          137 ID
1724 135 THEN          137 ID
1726 135 THEN          137 ID
1728 136 ELSE          137 ID
1730 136 ELSE          137 ID
1732 136 ELSE          137 ID
1734 136 ELSE          137 ID
1736 136 ELSE          137 ID
1738 136 ELSE          137 ID
1740 136 ELSE          137 ID
1742 136 ELSE          137 ID
1744 136 ELSE          137 ID
1746 136 ELSE          137 ID
1748 136 ELSE          137 ID
1750 136 ELSE          137 ID
1752 136 ELSE          137 ID
1754 136 ELSE          137 ID
1756 136 ELSE          137 ID
1758 136 ELSE          137 ID
1760 136 ELSE          137 ID
1762 136 ELSE          137 ID
1764 136 ELSE          137 ID
1766 136 ELSE          137 ID
1768 137 WHILE          137 ID
1770 137 WHILE          137 ID
1772 137 WHILE          137 ID
1774 137 WHILE          137 ID
1776 137 WHILE          137 ID
1778 137 WHILE          137 ID
1780 137 WHILE          137 ID
1782 137 WHILE          137 ID
1784 137 WHILE          137 ID
1786 136 DO             137 ID
1788 138 DO             137 ID
1790 138 DO             137 ID
1792 138 DO             137 ID
1794 138 DO             137 ID
1691 1D               94 IDB
1693 1D               24 X REG ASS
1695 1D               90 IDX
1697 1D               37 MULT ASS4
1699 1D               34 MULT ASS1
1701 1D               35 MULT ASS2
1703 1D               91 IDF
1705 1D               131 GOTO
1707 1D               93 IDP
1709 1D               33 PROC2
1711 1D               91 IDF
1713 1D               31 FUNC2
1715 1D               126 CASE
1717 1D               84 BLOCKHEAD
1719 1D               141 IF
1721 1D               137 WHILE
1723 1D               47 TRUE PART
1725 1D               41 SIMPLE ST
1727 1D               11 T CELL 1D
1729 1D               87 IDI
1731 1D               14 T CELL2
1733 1D               8 B REG
1735 1D               29 T CELL ASS
1737 1D               4 X REG
1739 1D               38 MULT ASS
1741 1D               36 MULT ASS3
1743 1D               124 CC
1745 1D               130 NULL
1747 1D               7 PROC 1D
1749 1D               32 PROC1
1751 1D               5 FUNC 1D
1753 1D               30 FUNC1
1755 1D               40 CASE SE@*
1757 1D               85 BLOCKBODY
1759 1D               128 BEGIN
1761 1D               48 WHILE
1763 1D               142 FOR
1765 1D               44 COMP COND
1767 1D               4 X REG
1769 1D               2 T NUMBER
1771 1D               12 T CELL
1773 1D               14 T CELL
1775 1D               13 T CELL 1D
1777 1D               13 T CELL
1779 1D               13 T CELL3
1781 1D               42 NOT
1783 1D               45 COMP AOR
1785 1D               41 SIMPLE ST
1787 1D               11 T CELL 1D
1789 1D               87 IDI
1791 1D               14 T CELL2
1793 1D               8 B REG

```





## Appendix D

## Example Programs Written in GPL

```

1      COMMON /ALLOC/ IGET,IFREE,AREA(39)
2      DIMENSION INDEX(20)
3      IGET=0
4      IFREE=0
5      DO 10 I=1,39
6      10 AREA(I)=0
7      CALL INIT
8      CALL WRITEI
9      DO 1  I=1,5
10     J=I-1
11     CALL GET(J)
12     IF(IGET.LT.0) GO TO 9
13     INDEX(I)=IGET
14     1 CALL WRITEG(J)
15     DO 2  I=1,5
16     J=I-1
17     CALL FREE(INDEX(I),J)
18     IF(IFREE.LT.0) GO TO 9
19     2 CALL WRITEF(INDEX(I),J)
20     DO 3  I=1,8
21     CALL GET(2)
22     IF(IGET.LT.0) GO TO 9
23     INDEX(I)=IGET
24     3 CALL WRITEG(2)
25     DO 4  I=1,8,2
26     CALL FREE(INDEX(I),2)
27     IF(IFREE.LT.0) GO TO 9
28     4 CALL WRITEF(INDEX(I),2)
29     DO 5  I=2,8,2
30     CALL FREE(INDEX(I),2)
31     IF(IFREE.LT.0) GO TO 9
32     5 CALL WRITEF(INDEX(I),2)
33     CALL FREE(0,5)
34     9 CALL WRITEI
35     1000 STOP
36     END

```

```

1      SUBROUTINE WRITEI
2      COMMON /ALLOC/ IGET,IFREE,AREA(39)
3      WRITE(6,1)
4      1 FORMAT(1X,'INIT')
5      GO TO 100
6      ENTRY WRITEG(N)
7      WRITE(6,2) N
8      2 FORMAT(1X,'GET(1,I2,1)')
9      GO TO 100
10     ENTRY WRITEF(L,M)
11     WRITE(6,3) L,M
12     3 FORMAT(1X,'FREE(1,I3,1,I3,1)')
13     100 WRITE(6,101) AREA
14     101 FORMAT(1X,10013,/,1X,10013))
15     RETURN
16     END

```

```

1      SUBROUTINE PRINT(I,J,K,L)
2      WRITE(6,10) I,J,K,L
3      10 FORMAT(1H0, '*** ', 5015)
4      RETURN
5      END

```

```

1      SUBROUTINE LIST(I)
2      WRITE(6,1) I
3      1 FORMAT(1X,'A-REG =',012)
4      RETURN
5      END

```

```

• BEGIN
COMMENT***** THE BUDDY SYSTEM CODED BY M. TOMIYAMA(JAERI) USING GPL. ****
* THIS ROUTINE INITIALIZES VALUES OF THE BUDDY SYSTEM. *
***** ****

PROCEDURE INIT *
BEGIN
SEGMENT BASE /ALLOC/ *
INTEGER GETVALUE, FREEVALUE *
ARRAY 39 INTEGER AREA *
ARRAY 38 INTEGER A SYN AREA(2) *
INTEGER MASKF=3777770000000, MASKB=0000007777770,
TAGON=1777777777770, TAGOFF=4000000000000 *
INTEGER XR1,XR2,XR3,XR4,XR5 *
INTEGER M=5,SIZE=32 *
SEGMENT BASE LOCAL *
INTEGER T1 *

XR1=X1 * XR2=X2 *
T1=0 *
X1=SIZE+M * T1=X1 * A(X1)=0 *
A(0)=#1 SHLA 18 OR T1 OR TAGOFF *
X2=SIZE *
X1=X1-1 *
FOR X2=X2 STEP 1 UNTIL X1 DO
BEGIN
T1=0 *
T1=X2 *
A(X2)=#1 SHLA 18 OR T1 *
END *
X1=XR1 * X2=XR2 *
END *
END .

```

```

.BEGIN
COMMENT***** THIS ROUTINE FINDS AND RESERVES A BLOCK OF 2**N LOCATIONS, OR *
* REPORTS FAILURE(GETVALUE=-1).
***** PROCEDURE GET(N) *
.BEGIN
SEGMENT BASE /ALLOC/ *
INTEGER GETVALUE,FREEVALUE *
ARRAY 39 INTEGER AREA *
ARRAY 38 INTEGER A SYN AREA(2) *
INTEGER MASKF=3777770000000, MASKB=0000007777770,
TAGON=1777777777770,TAGOFF=4000000000000 *
INTEGER XR1,XR2,XR3,XR4,XR5 *
INTEGER M=5,SIZE=32,X=38 *
SEGMENT BASE LOCAL *
INTEGER U,V,W,P,N,K *
LABEL TW *
FUNCTION STX(3,045215000000) *

XR1=X1 * XR2=X2 * XR3=X3 * XR4=X4 * XR5=X5 *
((V,W))=0 *
IF N.LT.0 OR N.GT.M THEN GO TO ERROREXIT *
K=N *
X1=N+SIZE * V=X1 * W=X1 *
FINDBLOCK.. X2=A(X1) SHRA 18 *
IF X2.EQ.V THEN
BEGIN
X1=X1+1 * V=X1 *
K=K+1 *
IF V.GT.X THEN GO TO ERROREXIT *
GO TO FINDBLOCK *
END *
REMOVE.. GETVALUE=X2 *
V=A(X2) AND MASKF *
X3=V SHRA 18 *
A(X1)=A(X1) AND MASKB OR V *
A(X3)=X1 *
A(X2)=A(X2) AND TAGON *
CHECK.. IF X1.EQ.W THEN GO TO FOUND *
SPLIT.. ((P,U))=0 *
X1=X1-1 * U=X1 * X4=1 * X2=K-1 * K=X2 *
STX(3,5,TW) *
TW.. P=1 SHLA U + GETVALUE *
X2=P * X5=U *
U=U SHLA 18 *
A(X2)=A(X2) AND MASKB OR U *
A(X2)=X5 *
A(X2)=A(X2) OR TAGOFF *
A(X1)=P SHLA 18 + P *
GO TO CHECK *
ERROREXIT.. GETVALUE=-1 *
FOUND.. X1=XR1 * X2=XR2 * X3=XR3 * X4=XR4 * X5=XR5 *
END *

```

Corrigenda

Page	Line	Error	Correction
1	L 1	are	is
2	U 6	classes	class
6	U 8	analysis	analysis
7	U 7	sometimes	sometimes
32	U 2	value	values
34	L 9	>	> 0
41	L 7	(	)
51	U 1	grammars	grammar
	U 8	exists	exist
	L 10	;	:
80	U 9	speed	efficiency
Abstract	L 8	require	requires