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APPROXIMATE EXPRESSION OF COLLISION
DENSITY AND CORRECTION OF GROUP
ELASTIC REMOVAL CROSS SECTION NEAR
A WIDE SCATTERING RESONANCE

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Approximate Expression of Collision Density and
Correction of Group Elastic Removal Cross
Section near a Wide Scattering Resonance

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An analytical expression of collision density has been developed. The collision density is well represented with this expression for mixtures composed of sodium and lighter nuclides. The present expression has a considerable error for mixtures containing iron, and the conventional asymptotic solution was temporarily adopted for this case.

The elastic removal cross section contained in the JAERI-Fast set was corrected by using the flux calculated with the present method. The errors of the result thus obtained are less than 10 percents at most near the sodium resonance, while they are of a factor of 2 or 3 without correction. The effect of the sodium resonance can be reflected on the elastic removal cross section of other scattering nuclides for which such an effect is ignored in the original JAERI-Fast set. Thus the applicability of the group constants of coarse group structure (with less than 100 groups) is much enlarged by the present correction.

巨大共鳴散乱付近での衝突密度の近似的表現と弾性除去断面積の補正

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巨大共鳴散乱付近の衝突密度を解析的に表現する方法を開発した。この方法によると，Naとそれより軽い核との混合物の衝突密度は良く表わされるが，鉄を含む場合には誤差が多い。そこで鉄を含む場合には，通常の $1/\bar{\sigma}$ 近似を用いた。

JAERI-Fastセット中の弾性除去断面積を，上述の方法で求めた中性子束を用いて補正した。その結果，補正前は200~300%であった誤差が高々10%に減少した。Na共鳴の影響は，他の核種の弾性除去断面積にも反映されるようになった。これにより，小数群の群定数(100群以下)の適用範囲が拡大された。

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1. Introduction

We have developed a simple but enough accurate calculational system which can be used both for analysis and for design of fast reactors. Along this line the JAERI-Fast set^{1,2)}, a set of group constants of 25 or 70 group structure, was produced as the principal library and many computer codes using this set have been developed.

It is well known that group constants depend essentially on the composition of the reactor considered. Such a composition dependence can be classified into two categories: The first one is the self-shielding effect of resonances and the second one is the effect of large scattering resonances on the elastic removal cross section.

The resonance shielding effect is taken into account in the JAERI-Fast set by table look-up of the self-shielding factors of Bandarenko type³⁾. Even the interference with the resonances of ^{238}U is considered for ^{235}U , ^{239}Pu and ^{240}Pu . On the other hand, the treatment of the elastic removal cross section near a wide scattering resonance is not easy, and the JAERI-Fast set ignores this effect and calculates the elastic removal cross section by assuming $1/E$ spectrum. Hence the JAERI-Fast set gives rather poor results near large scattering resonances. This is especially severe near the 2.8 keV resonance of sodium.

Usually this defect in coarse group constants is overcome with supplemental calculations using finer group constants (of 2,000 ~ 200,000 group structure), for example, with ESELEM-4⁴⁾ or SDR^{5,6)}. It is not a simple procedure, however, to connect these supplemental calculations to codes based on the JAERI-Fast set. Hence a simple correction of elastic removal cross sections has been much required. The deviation from $1/E$ spectrum must be known for this correction.

The collision density of a composited medium is not constant in lethargy unit near a scattering resonance. Various attempts have been made so far, in order to obtain a simple

analytical expression of the collision density of the mixture.

The collision density of a mixture is the inverse of $\bar{\xi} = \frac{\sum_i \Sigma_i \xi_i}{\sum_i \Sigma_i}$

(i specifies the nuclide), when each component of mixture has a constant cross section.

Then it might be possible as the simplest approximation to use this expression in the existence of resonance⁷⁾, where Σ_i depends on energy. The collision density with this approximation has a peak or a valley at the resonance energy. The actual peak or valley of the collision density appears, however, at the lower energy. This shift was fairly well expressed by Segev⁸⁾, who expanded the slowing down equation with moments of ξ and truncated it with two terms.

On the other hand, we have tried to treat the mixture approximately as a pseudo element and to obtain the collision density. This method gives a simple expression of the collision density, if the effective alpha of this pseudo nuclide is obtained. It was found that this method gave satisfactory results for mixtures composed of sodium and lighter nuclides such as oxygen or carbon, where the shift of peak or valley is predominant.

A subroutine COLLIS has been developed for the correction of elastic removal cross section with this method. This subroutine is now equipped to EXPANDA-70⁹⁾ and EXPANDA-75¹⁰⁾, one dimensional diffusion codes for homogeneous and for slab lattice heterogeneous system respectively. With this correction, the accuracy of the results with these codes is much improved and thus the applicability of the JAERI-Fast set is much enlarged.

2. Approximate Expression of Collision Density

2.1 General Procedure

The collision density of a non-absorbing mixture is expressed as Eq. (1) in the case of no external source,

$$F(E) = \sum_i \frac{1}{1 - \alpha_i} \int_E^{E/\alpha_i} \frac{h_i(E')}{E'} \cdot F(E') dE' \quad (1)$$

$$\alpha_i = \left(\frac{A_i - 1}{A_i + 1} \right)^2$$

$$h_i(E) = \frac{\sum_s^i \Sigma_s^i(E)}{\sum_j \Sigma_s^j(E)}$$

where i and j specify the nuclides and the other notations are of common usage. h_i is constant if there is no resonance. If a nuclide has a resonance, h_i has a peak for this nuclide and has a valley for the other nuclides near the resonance energy.

We consider a mixture consisting of a resonance scatterer (nuclide 1) and a constant scatterer (nuclide 2). If the two nuclides have the same mass ($A_1 = A_2$), the total collision density is flat in lethargy unit. When A_1 is larger than A_2 , the collision density deviates from the flat distribution and has a low peak near the resonance energy. On the contrary, if A_1 is smaller than A_2 , then the collision density has a shallow valley.

This suggests that any solution for slowing down due to a single scattering element is not necessarily the fundamental solution of Eq. (1) for a composite medium. Mathematically this difficulty is caused by the fact that the kernel in Eq. (1) consists of sum of several kernels, each of which has a different upper bound of integration. If the kernels have the same upper bound, Eq. (1) could be reduced to the problem for a single scattering material.

Bearing the above remarks in mind, we treat the mixture as a pseudo nuclide which

has appropriate α and $h(E)$ defined so as to give the same collision density as Eq. (1):

$$F(E) = \frac{1}{1-\alpha} \int_E^{E/\alpha} \frac{h(E')}{E'} F(E') dE' \quad (2)$$

α must be determined so that $(1-\alpha)E$ is interpreted as the maximum energy loss by a single collision with this pseudo-nuclide. For simplicity, however, α is approximately put as the mean value of α_i so as to satisfy the following relation,

$$\text{Sum}_i \frac{1}{1-\alpha_i} \int_{E/\alpha_i}^{E/\alpha_i} \frac{h_i(E')}{E'} F(E') dE' = 0 \quad (3)$$

It should be noted that α thus defined depends on E and that α is not very different from any α_i , as α_i resembles each other, if we exclude the case of hydrogen.

Then $h(E)$ is obtained as

$$h(E) = (1-\alpha) \text{Sum}_i \frac{h_i(E)}{1-\alpha_i} \quad (4)$$

$h(E)$ is unity at the off-resonance energy and stays near unity even at the resonance energy, as $\text{Sum}_i h_i(E)$ is always normalized to unity and α is not very different from any α_i . $h(E)$ is interpreted as the ratio of elastic scattering to total collision of the pseudo-nuclide.

Near a resonance, the pseudo nuclide is considered to have a virtual absorption and production of neutrons with its collision.

The first order solution of Eq. (2) might be the inverse of lethargy gain $1/\xi E$, where ξ is defined with α as

$$\xi = 1 + \frac{\alpha}{1-\alpha} \log \alpha \quad (5)$$

This solution is inserted in Eq. (2) and we have the second order solution as

$$F(E) = \frac{\bar{h}}{E \xi} \quad (6)$$

where
$$\bar{h} = \int_E^{E/\alpha} \frac{h(E')}{E'^2} dE' \quad (7)$$

Then the flux is
$$\phi(E) = \frac{\bar{h}}{E \xi \cdot \sum_i \Sigma_s^i} \quad (8)$$

Having obtained the flux, the elastic removal cross section of nuclide i is calculated numerically as

$$\sigma_{e.r.} = \int_{E_L}^{E_H} \frac{(E_L - \alpha_i E)}{(1 - \alpha_i) E} \sigma_s^i(E) \phi(E) dE \quad (9)$$

where E_L is the lower boundary of the considered group and E_H the smaller value between E_L/α_i and the upper group boundary.

2.2 α and $h(E)$

We consider a mixture of two scattering nuclides, i.e., a resonance scatterer (nuclide 1), and another nuclide (nuclide 2), whose number densities are N_1 and N_2 respectively. The scattering cross section of the resonance scatterer is put as σ_1^T and those of the nuclides 2 as σ_2 . σ_2 is assumed to be constant through the energy range considered. The absorption by these nuclides is neglected.

σ_1^T is expressed with the single level Breit-Wigner approximation as

$$\sigma_1^T = \sigma_1 + \frac{\sigma_0 \Gamma^2/4}{(E - E_r)^2 + \Gamma^2/4} + \frac{(\sigma_1 \sigma_0 g_J \Gamma_n / \Gamma)^{1/2} \Gamma (E - E_r)}{(E - E_r)^2 + \Gamma^2/4} \quad (10)$$

where E_r is the resonance energy, σ_1 the potential scattering cross section and σ_0 the peak cross section defined as

$$\sigma_0 = \frac{4\pi}{k^2} g_J \frac{\Gamma_n^2}{\Gamma^2}$$

and the other notations are of common usage.

The ratios h_1 and h_2 are expressed according to the procedure given in

Appendix as,

$$\begin{aligned} h_1 &= \frac{N_1 \sigma_1^T}{N_1 \sigma_1^T + N_2 \sigma_2} = a_1 + a_2 \left[\frac{A^+}{E - \gamma^+} + \frac{A^-}{E - \gamma^-} \right] \\ h_2 &= \frac{N_2 \sigma_2}{N_1 \sigma_1^T + N_2 \sigma_2} = a_2 \left[1 - \frac{A^+}{E - \gamma^+} - \frac{A^-}{E - \gamma^-} \right] \quad (11) \end{aligned}$$

where a_1 and a_2 are ratios of constant part of the cross section, and A^\pm and γ^\pm are energy independent complex conjugates.

In solving Eq. (3), the collision density is assumed to be asymptotic $1/E$ form.

Then α can be obtained analytically as given in Appendix.

Having obtained α , $h(E)$ is written as

$$h(E) = A + B \left(\frac{A^+}{E - \gamma^+} + \frac{A^-}{E - \gamma^-} \right), \quad (12)$$

where

$$\begin{aligned} A &= (1 - \alpha) \left(\frac{a_1}{1 - \alpha_1} + \frac{a_2}{1 - \alpha_2} \right) \\ B &= (1 - \alpha) a_2 \left(\frac{1}{1 - \alpha_1} - \frac{1}{1 - \alpha_2} \right). \end{aligned}$$

This procedure can be extended to more general case where more than two scatterers exist. In most cases, however, only one resonance scatterer is important in the energy range considered. Then the other nuclides of constant cross section can be treated as a mixture keeping the slowing down power the same.

2.3 Absorption by Heavy Elements

In actual reactor calculations, absorption by heavy elements cannot be neglected and the asymptotic flux deviates from $1/E$ form. This effect is corrected by using an appropriate spectrum index β , and the collision density is given by

$$F(E) = \frac{\bar{h}}{\Sigma E^\beta} \quad (13)$$

The value of β is obtained by neglecting the resonance scattering. Considering the existence of heavy elements, the collision density is written as

$$F(E) = \left(\Sigma_a^H + \Sigma_S^H + \text{Sum}_i \Sigma_S^i \right) \phi(E)$$

$$= \frac{1}{1-\alpha_H} \int_E^{E/\alpha_H} \Sigma_S^H \phi(E') \frac{dE'}{E'} + \text{Sum}_i \frac{1}{1-\alpha_i} \int_E^{E/\alpha_i} \Sigma_S^i \phi(E') \frac{dE'}{E'} \quad (14)$$

where the suffix H means the heavy element.

Assuming that

$$\phi(E) = E^{-\beta}$$

$$\alpha^H \doteq 1,$$

the right hand side of Eq. (14) becomes

$$\Sigma_S^H \phi(E) + \text{Sum}_i \frac{\Sigma_S^i}{1-\alpha_i} \cdot \frac{1}{\beta} \phi(E) [1-\alpha_i],$$

and therefore we have

$$\beta = \frac{1}{\Sigma_a^H + \text{Sum}_i \Sigma_S^i} \text{Sum}_i \Sigma_S^i \cdot \frac{1-\alpha_i^\beta}{1-\alpha_i} \quad (15)$$

Eq. (15) can be solved by the iteration of a few times, starting from $\beta = 1$ ($\phi \propto 1/E$).

3. Collision density

The collision density was calculated for various mixtures composed of sodium and some scattering nuclides near the 2.85 keV sodium resonance. The results are compared with the exact solutions obtained by a fine spectrum code RIFH.¹¹⁾

The collision density of mixture composed of sodium and oxygen is shown in Fig. 1 for various number density ratios. The dashed lines are calculated with Eq.(6), the dotted lines are the asymptotic forms of $1/\bar{\xi}$ ($\bar{\xi} = \sum h_i \xi_i$) and the solid lines are the exact solutions calculated with RIFH. The present results agree fairly well with the exact solutions. The same agreement was obtained for mixtures of sodium and carbon.

Mixtures of sodium and iron were examined. The curves of the collision density are given in Fig.2. In these cases, however, the present results do not agree with the exact solutions. The asymptotic $1/\bar{\xi}$ agrees better with the exact solution. The failure of the present method may be explained as follows: The range of integration in Eq.(1) of sodium is more than twice as much as that of iron. Hence the error of α obtained with Eq.(3) may be large. A better and simple method is now searched for.

It was found, on the other hand, that the collision density is approximately constant in lethargy unit, if the effective $\bar{\xi}$ ($= \sum h_i \xi_i$) of the nonresonant nuclides is equal to ξ of sodium. In this case the cross section of nonresonant nuclides is equivalent to the constant part of sodium cross section. We call this state as an equilibrium state in the following discussion.

From the above discussion, the following approximation was concluded to be the most useful at the present time. First the effective $\bar{\xi}$ of the nonresonant nuclides is calculated. If this effective $\bar{\xi}$ is smaller than ξ of sodium (iron rich case), the asymptotic $1/\bar{\xi}$ form is adopted. If the effective $\bar{\xi}$ of nonresonant nuclides is larger than ξ of sodium (a lighter nuclide rich case), the number density of lighter nuclides is divided into

two parts. With one part the equilibrium state is attained. The cross section of the nonresonant nuclides in equilibrium state is added to σ_1^T in Eq.(10). Then the remaining cross section of the lighter nuclide is put as σ_2 .

The usefulness of this approximation is demonstrated in Fig.3, where curves of the collision density are given for three mixtures composed of oxygen, iron and sodium. The number density of case 1 is chosen so as to attain the equilibrium state. The number density of oxygen is 5 times larger in case 2 than in case 1. In case 3, the number density of iron is 5 times larger than case 1 and the collision density is put approximately to $1/\bar{\xi}$. Good agreement is observed in any case between the present result and the exact result.

As a conclusion, the method described in section 2 is very useful for a mixture of sodium and a lighter nuclide. This method is not good for a mixture containing iron, and the conventional $1/\bar{\xi}$ approximation is better for this case. Therefore we adopted the approximation in which the present method or the conventional method is chosen automatically according to the effective $\bar{\xi}$ of the nonresonant scatterers.

4. Correction of Elastic Removal Cross Section

A subroutine named COLLIS has been developed for correcting the elastic removal cross section contained in the JAERI-Fast set. The collision density is calculated approximately with the method described in section 3. The effect of absorption by heavy elements is corrected with the appropriate spectrum index described in section 2, 3.

Each group of the JAERI-Fast set of 70 group structure is divided into 30 subgroups and the flux is calculated for each subgroup. The cross section of sodium is calculated for each subgroup with the resonance parameters. The cross section of other nuclides is assumed to be constant and the values in the JAERI-Fast set are used.

The elastic removal cross sections of a mixture composed of sodium, oxygen and iron (case No. 1 of Fig.3) are shown in Fig.4. The dashed lines are calculated with COLLIS, the solid lines are the exact values calculated with RIFH, and the dotted lines are calculated with 1/E spectrum. The cross sections of oxygen and iron are put as 3.7 and 7 barns respectively for all the groups. The results calculated with COLLIS agree very well with the exact values.

A typical mixture of fast reactor, which contains heavy elements, was examined.

The number densities are as follows:	(unit of $10^{24}/\text{cm}^3$)
U - 238	5.36×10^{-3}
Pu -239	1.39×10^{-3}
Na	8.4×10^{-3}
O	1.38×10^{-2}
Fe	2.1×10^{-2}

The values in the JAERI-Fast set are used for the cross section of oxygen and iron. The elastic scattering and the elastic removal cross sections are given in Table 1 for the light nuclides, and the elastic removal cross sections are shown in Fig.5. The exact values

are calculated with the ultra-fine spectrum code SDR in this case, as the library of heavy elements is not available for RIFH.

The agreement is a little worse in this case than in the previous case. This disagreement is mainly caused by the energy dependence of iron cross sections which are not constant through the energy group. This energy dependence is fairly well represented in SDR, but is ignored in the present calculation. In the previous case, on the other hand, the constant cross section was assumed in calculating the exact value with RIFH.

Any way, the improvement of the elastic removal cross section is excellent with the present correction. The errors are reduced to be several percents at most, while they are of a factor of 2 or 3 without correction near the sodium resonance.

5. Conclusion

An analytical expression of collision density has been developed near a wide scattering resonance. The collision density is well represented with this expression for mixtures composed of sodium and lighter nuclides. A subroutine named COLLIS has been developed for correcting the elastic removal cross section by using the flux calculated with the present method. The errors of the result thus obtained are less than 10 percents at most even near the sodium resonance, while they are of a factor of 2 or 3 without correction.

The present expression has a considerable error for mixtures containing iron, and the conventional asymptotic solution was temporarily adopted for this case. This drawback is now under investigation. It can be concluded, however, that the applicability of the group constants of coarse group structure (with less than 100 groups) is much enlarged by the present correction.

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Appendix. Calculation of α

As the cross section of resonance nuclide is expressed as Eq. (10), the total scattering cross section of the mixture is

$$\Sigma_S = N_2 \sigma_2 + N_1 \left\{ \sigma_1 + \frac{\sigma_0 + (\sigma_1 \sigma_0 g_J \Gamma_n / P)^{\frac{1}{2}} 4(E-E_r) / P}{(E-E_r)^2 + \Gamma^2 / 4} \times \frac{\Gamma^2}{4} \right\} \quad (A-1)$$

and the ratio $h_2 \equiv N_2 \sigma_2 / \Sigma_S$ is written as

$$h_2 = \frac{R \sigma_2}{\sigma_1 + R \sigma_2} \cdot Q(E) \quad (A-2)$$

$$Q(E) = \frac{(E-E_r)^2 + \Gamma^2 / 4}{(E-E_r)^2 + \frac{\Gamma^2}{4} + \frac{\sigma_0 \Gamma^2 / 4 + (\sigma_1 \sigma_0 g_J \Gamma_n / P)^{\frac{1}{2}} \Gamma (E-E_r)}{\sigma_1 + R \sigma_2}}$$

where R is N_2 / N_1 .

Define

$$a_2 \equiv \frac{R \sigma_2}{\sigma_1 + R \sigma_2}$$

$$f \equiv 1 + \frac{\sigma_0}{\sigma_1 + R \sigma_2}$$

$$f_{int} \equiv \frac{(\sigma_1 \sigma_0 g_J \Gamma_n / P)^{\frac{1}{2}}}{\sigma_1 + R \sigma_2}$$

then

$$h_2 = a_2 \cdot \frac{(E-E_r)^2 + \Gamma^2 / 4}{(E-E_r)^2 + f_{int} \Gamma (E-E_r) + f \cdot \Gamma^2 / 4}$$

$$\begin{aligned}
&= a_2 \left\{ 1 - \frac{f_{int} (E - E_r) \Gamma + (f-1) \Gamma^2 / 4}{(E - E_r)^2 + f_{int} (E - E_r) \Gamma + f \Gamma^2 / 4} \right\} \\
&= a_2 \left\{ 1 - \frac{2 E_{int} (E - E_r) + (f-1) \Gamma^2 / 4}{(E - E_r + E_{int} - \frac{\Gamma}{2} i) (E - E_r + E_{int} + \frac{\Gamma}{2} i)} \right\} \\
&= a_2 \left\{ 1 - \frac{A^+}{E - r^+} - \frac{A^-}{E - r^-} \right\} \tag{A-3}
\end{aligned}$$

where

$$\begin{aligned}
E_{int} &\equiv \frac{1}{2} f_{int} \cdot \Gamma \\
r &\equiv (f - f_{int})^{\frac{1}{2}} \Gamma \\
A^{\pm} &\equiv E_{int} \mp i \frac{\Gamma^2}{4r} [(f-1) - 2f_{int}^2] \\
r^{\pm} &\equiv (E_r - E_{int}) \pm \frac{i}{2} r
\end{aligned}$$

Then h_1 can be expressed as

$$\begin{aligned}
h_1 &= 1 - h_2 \\
&= a_1 + a_2 \left\{ \frac{A^+}{E - r^+} - \frac{A^-}{E - r^-} \right\} \tag{A-4}
\end{aligned}$$

where

$$a_1 = \frac{\sigma_1}{\sigma_1 + R \sigma_2}$$

in solving Eq. (3) we assume that the energy dependence of h_i is small and that $F(E)$ can be approximately proportional to $1/E$. Thus Eq. (3) becomes

$$\sum_i \frac{1}{1-\alpha_i} \int_{E/\alpha_i}^{E/\alpha_i} \frac{1}{(E')^2} h_i(E') dE' = 0 \quad (A-5)$$

Expanding the integrands into partial fractions, we have

$$\begin{aligned} & \frac{1}{1-\alpha_2} \int_{E/\alpha_2}^{E/\alpha_2} \frac{h_2(E')}{(E')^2} dE' \\ &= \frac{a_2}{1-\alpha_2} \left[\left(1 + \frac{A^+ \bar{r} + A^- \bar{r}^+}{r^+ r^-} \right) (\alpha - \alpha_2) \frac{1}{E} \right. \\ & \quad \left. - \frac{A^+}{(r^+)^2} \log \frac{E - \alpha_2 \bar{r}^+}{E - \alpha \bar{r}^+} - \frac{A^-}{(r^-)^2} \log \frac{E - \alpha_2 \bar{r}^-}{E - \alpha \bar{r}^-} \right] \end{aligned} \quad (A-6)$$

Similar results are obtained for h_1 .

As α is not very different from any α_i , the following approximation is possible:

$$\begin{aligned} \log \frac{E - \bar{r}^\pm \alpha_i}{E - \bar{r}^\pm \alpha} &= - \log \frac{E - \bar{r}^\pm \alpha_i + (\alpha_i - \alpha) \bar{r}^\pm}{E - \bar{r}^\pm \alpha} \\ &\approx - \frac{(\alpha_i - \alpha) \bar{r}^\pm}{E - \bar{r}^\pm \alpha} \end{aligned} \quad (A-7)$$

Then Eq. (A-5) becomes

$$\begin{aligned} & \frac{\alpha - \alpha_1}{1 - \alpha_1} \left[\left\{ a_1 - a_2 \frac{A^+ \bar{r}^- + A^- \bar{r}^+}{r^+ r^-} \right\} \frac{1}{E} + a_2 \left\{ \frac{A^+}{r^+(E - \bar{r}^+ \alpha_1)} + \frac{A^-}{r^-(E - \bar{r}^- \alpha_1)} \right\} \right] \\ & + \frac{a_2(\alpha - \alpha_2)}{1 - \alpha_2} \left[\left(1 + \frac{A^+ \bar{r}^- + A^- \bar{r}^+}{r^+ r^-} \right) \frac{1}{E} - \left\{ \frac{A^+}{r^+(E - \bar{r}^+ \alpha_2)} + \frac{A^-}{r^-(E - \bar{r}^- \alpha_2)} \right\} \right] \\ & = 0 \end{aligned} \quad (A-8)$$

Finally α is expressed as

$$\alpha = \frac{\text{Numerator}}{\text{Denominator}} \quad (A-9)$$

where

$$\begin{aligned} \text{Numerator} \\ = \frac{1}{E} & \left[\frac{a_1 d_1}{1-d_1} + \frac{a_2 d_2}{1-d_2} + a_2 \left\{ \frac{d_2}{1-d_2} - \frac{d_1}{1-d_1} \right\} \frac{A^+ \bar{r}^- + A^- \bar{r}^+}{\bar{r}^+ \bar{r}^-} \right] \\ & + \frac{a_2 d_1}{(1-d_1)} - \frac{(A^+ \bar{r}^- + A^- \bar{r}^+) E - \{A^+ (\bar{r}^-)^2 + A^- (\bar{r}^+)^2\} d_1}{\bar{r}^+ \bar{r}^- (E - \bar{r}^+ d_1) (E - \bar{r}^- d_1)} \\ & - \frac{a_2 d_2}{(1-d_2)} - \frac{(A^+ \bar{r}^- + A^- \bar{r}^+) E - \{A^+ (\bar{r}^-)^2 + A^- (\bar{r}^+)^2\} d_2}{\bar{r}^+ \bar{r}^- (E - \bar{r}^+ d_2) (E - \bar{r}^- d_2)} \end{aligned}$$

Denominator

$$\begin{aligned} = \frac{1}{E} & \left[\frac{a_1}{1-d_1} - \frac{a_2}{1-d_2} - a_2 \left\{ \frac{1}{1-d_2} - \frac{1}{1-d_1} \right\} \frac{A^+ \bar{r}^- + A^- \bar{r}^+}{\bar{r}^+ \bar{r}^-} \right] \\ & + \frac{a_2}{(1-d_1)} - \frac{(A^+ \bar{r}^- + A^- \bar{r}^+) E - \{A^+ (\bar{r}^-)^2 + A^- (\bar{r}^+)^2\} d_1}{\bar{r}^+ \bar{r}^- (E - \bar{r}^+ d_1) (E - \bar{r}^- d_1)} \\ & - \frac{a_2}{(1-d_2)} - \frac{(A^+ \bar{r}^- + A^- \bar{r}^+) E - \{A^+ (\bar{r}^-)^2 + A^- (\bar{r}^+)^2\} d_2}{\bar{r}^+ \bar{r}^- (E - \bar{r}^+ d_2) (E - \bar{r}^- d_2)} \end{aligned}$$

If more accurate value is required, the approximation of Eq. (A-7) can be avoided

with the iteration of a few times.

Table 1. The elastic scattering and the elastic removal cross sections of Na, O and Fe (barns)

Group	En (keV)	*	Na		O		Fe	
			σ_{el}	σ_{er}	σ_{el}	σ_{er}	σ_{el}	σ_{er}
32	4.65-3.6	A	29.2	10.6	3.7	1.49	5.50	0.50
		B	27.4	12.4	3.7	1.74	5.47	0.86
		C	30.7	11.0	3.7	1.45	5.32	0.54
33	3.6-2.78	A	162.4	55.7	3.7	1.02	5.26	0.23
		B	157.0	87.2	3.7	1.72	5.26	0.73
		C	162.6	55.6	3.7	1.04	5.34	0.23
34	2.78-2.15	A	90.5	26.9	3.7	2.36	5.94	1.55
		B	95.3	11.6	3.7	1.73	5.94	0.87
		C	98.5	28.5	3.7	2.33	6.10	1.59
35	2.15-1.66	A	18.2	5.09	3.7	1.90	6.85	1.14
		B	18.9	4.46	3.7	1.72	6.85	0.99
		C	20.9	5.52	3.7	1.84	6.88	1.10
36	1.66-1.29	A	8.21	2.46	3.7	1.81	7.64	1.13
		B	8.61	2.46	3.7	1.76	7.64	1.11
		C	9.87	2.82	3.7	1.74	7.64	1.08
37	1.29-1.0	A	5.36	1.64	3.7	1.74	8.43	1.16
		B	5.47	1.65	3.7	1.75	8.43	1.23
		C	5.95	1.54	3.7	1.63	8.40	1.07

*

- A : The present results
 B : The uncorrected values of JAERI-Fast set
 C : The results obtained by SDR

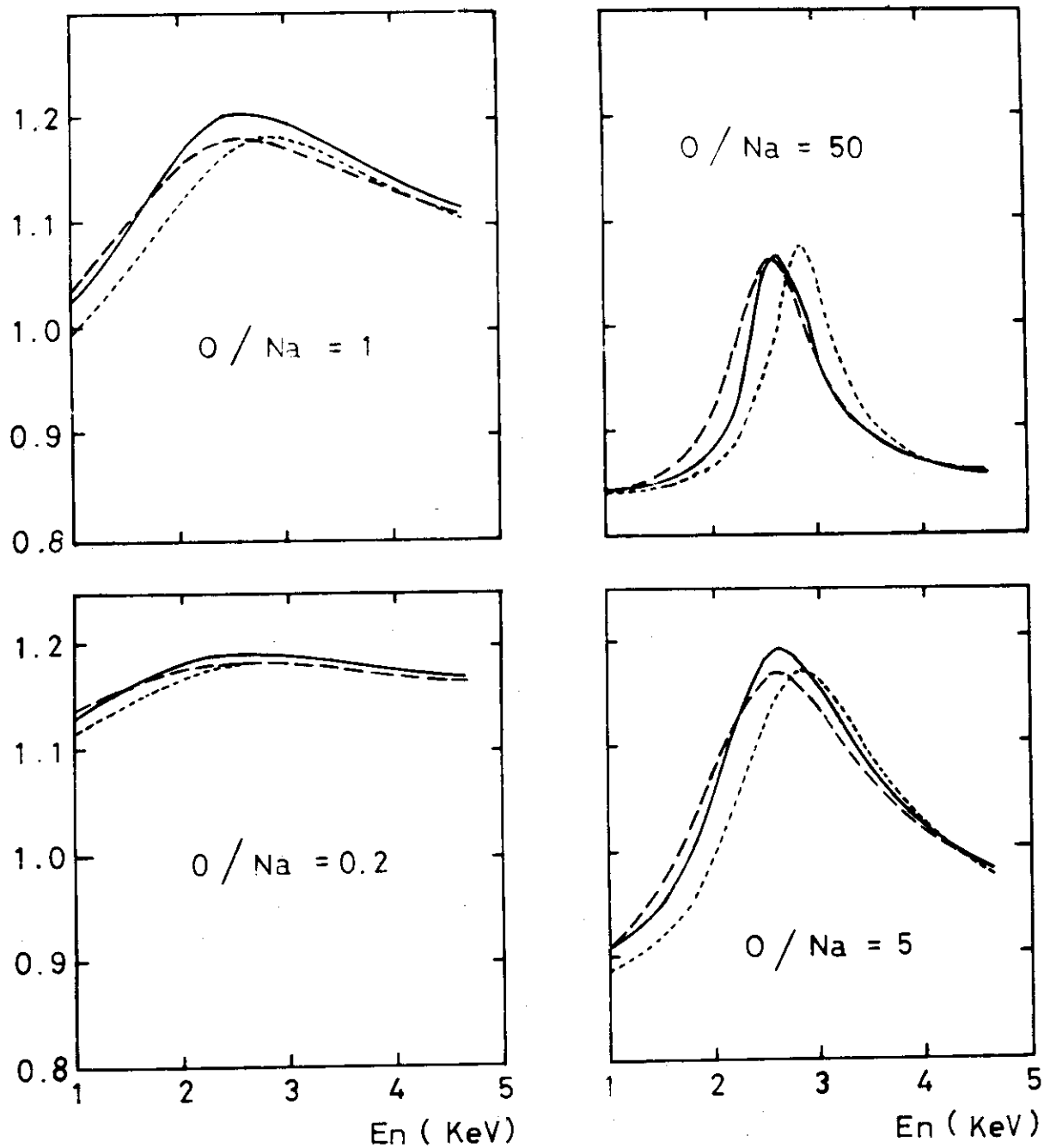


Fig. 1. Collision density of mixtures composed of sodium and oxygen for various number density ratios.

- : the present results.
- : the exact solutions.
- : the asymptotic solutions.

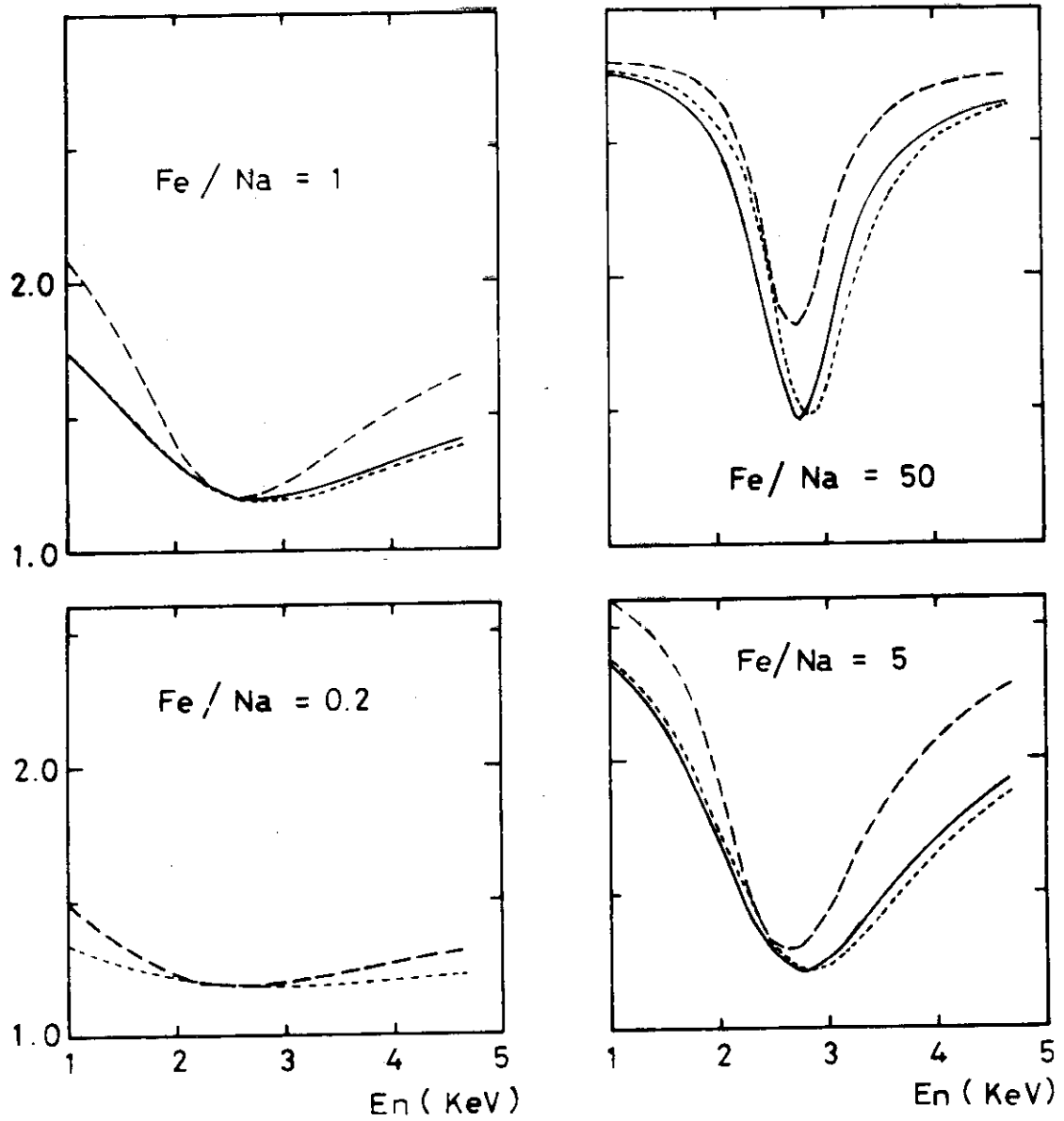


Fig. 2. Collision density of mixtures composed of sodium and iron for various number density ratios.

— — — : the present results
 — — — : the exact solutions
 ····· : the asymptotic solutions

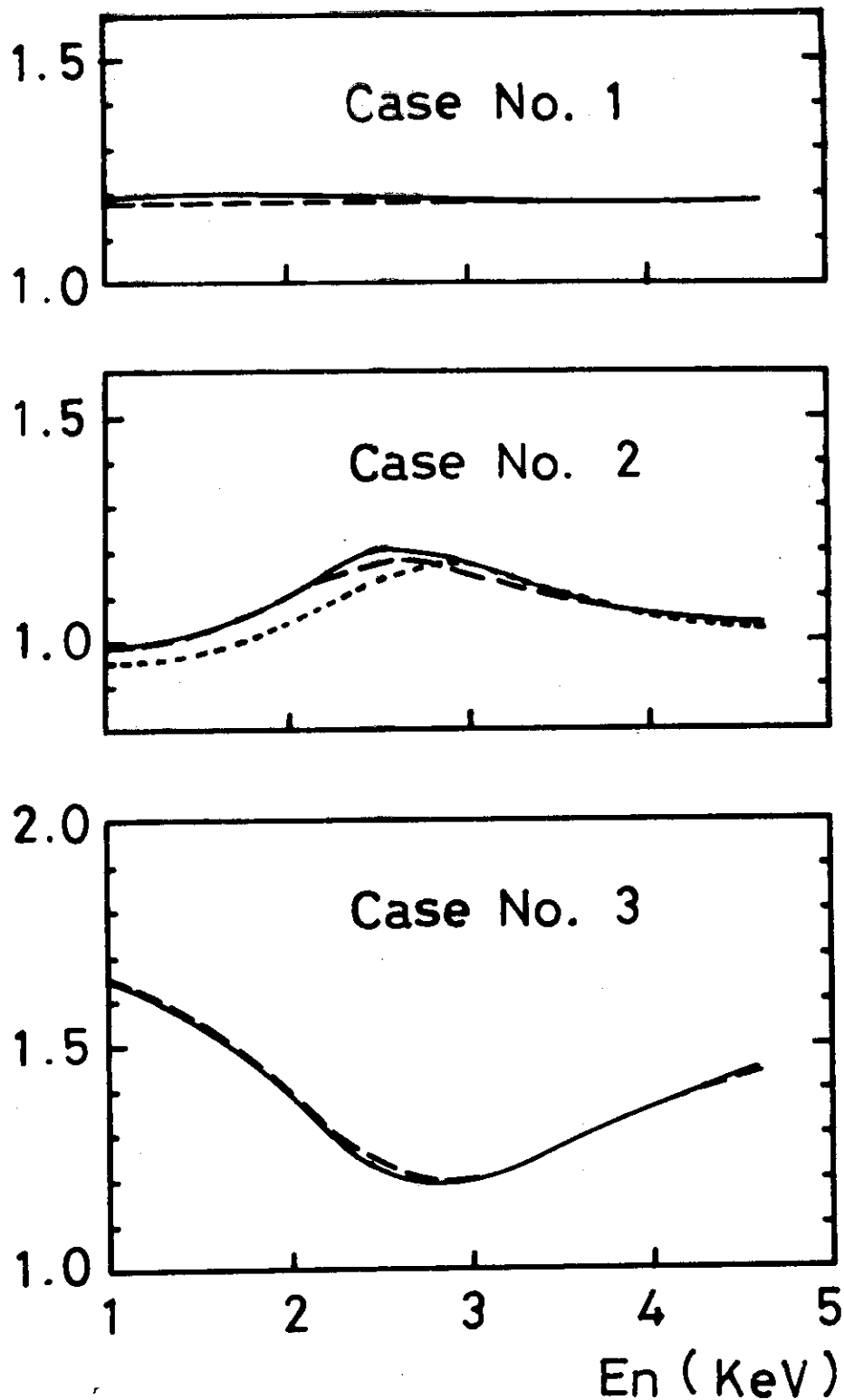


Fig. 3. Collision density of mixtures composed of sodium, oxygen and iron. Case 1 is the equilibrium case, Case 2 the oxygen rich case and Case 3 the iron rich case.
 - - - - : the present results.
 ——— : the exact solutions.
 : the asymptotic solutions.

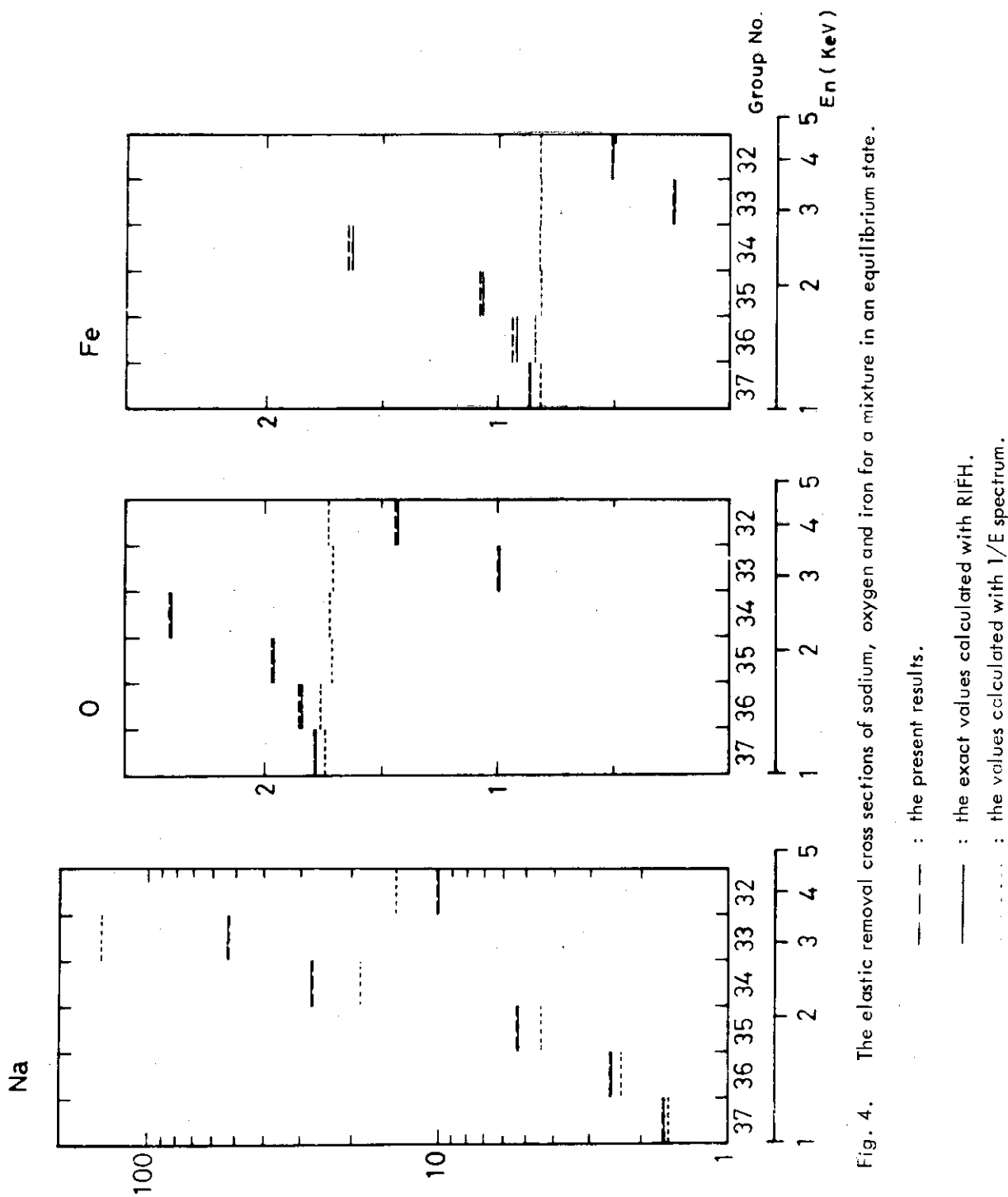


Fig. 4. The elastic removal cross sections of sodium, oxygen and iron for a mixture in an equilibrium state.

- : the present results.
- : the exact values calculated with RIFH.
- : the values calculated with 1/E spectrum.

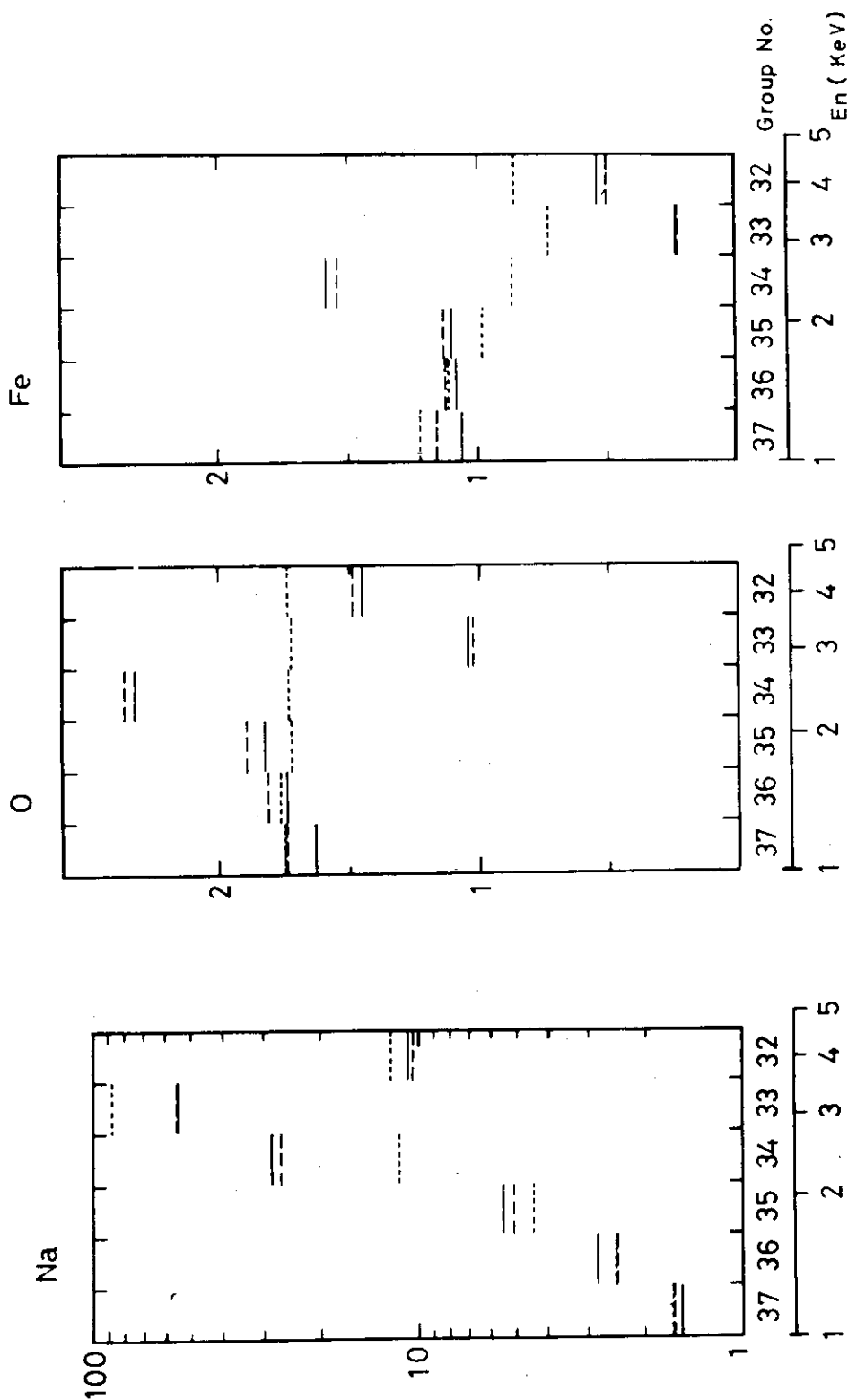


Fig. 5. The elastic removal cross sections of sodium, oxygen and iron for a mixture composed of U-235, Pu-239, Na, O and Fe.

- : the present results.
- - - : the exact values calculated with SDP.
- · · : the uncorrected values in the JAERI-Fast set