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# MACROSCOPIC CROSS SECTIONS FOR ANALYZING THE TRANSPORT OF NEUTRAL PARTICLES IN PLASMAS

May 1975

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# Macroscopic Cross Sections for Analyzing the Transport of Neutral Particles in Plasmas

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(Received April 16, 1975)

Algorithms have been developed for calculating the ionization and charge exchange cross sections required for analyzing the neutral transport in plasmas. In our algorithms, the integration of the expression for reaction rate of neutrals with plasmas is performed by expanding the integrand with the use of polynomials. At present, multi-energy-group sets of the cross sections depending on plasma temperature and energy of neutrals can be prepared by means of Maxwellian averages over energy. Calculational results are printed out in the FIDO format.

Some numerical examples are given for several forms of spatial distributions assumed for the plasma ion temperature and source neutral energy.

# プラズマ中の中性粒子輸送解析のための巨視的断面積

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（1975年4月16日受理）

プラズマ中の中性粒子輸送解析に必要なイオン化反応と荷電交換反応断面積を計算するアルゴリズムを開発した。中性粒子とプラズマとの反応率を表す式の積分は被積分関数を多項式で展開して行った。プラズマ温度と中性粒子のエネルギーの空間分布に依存する断面積の多エネルギー群セットは，エネルギーについてマックスウエル分布で平均して求めた。計算結果はFIDO 型式で出力される。プラズマ温度と中性粒子のエネルギーに対していろいろな空間分布を仮定した数値計算例を示す。

Contents

## 1. Introduction

An analysis of the transport of neutrals in plasmas is a fundamental problem concerned directly with the heating of plasmas and the diagnosis of the plasma ion density and temperature.

The present work intends to develop numerical algorithms to calculate the cross sections required for analyzing the neutral transport in plasmas.

We refer the microscopic data for neutral-plasma interactions to M. Gryzinski [1] and A.C. Riviere [2], and average these data with a Maxwellian plasma energy distribution of an assumed plasma temperature in each region.

The integration for obtaing the average cross section is performed by expanding the integrand with the use of polynomials.

## 2. Fundamental Equation

When the neutral particles are injected into a plasma for the purpose of plasma heating or they diffuse into a plasma as impurities, the transport of these neutrals can be analyzed by making application of neutron transport theory, as was first tried by Greenspan [3].

The reaction cross section in the neutral transport equation can be defined in a way similar to those used in neutron transport equation.

In the present work, we assume neutral particles to be of the same type of isotope as for the plasma ions.

The neutrals are ionized by plasma ions and electrons, or exchange their charges with plasma ions. These processes can be written schematically as follows:

$$n(E) + e^- \longrightarrow n^+(E) + 2e^-, \tag{1}$$

$$n(E) + n^+(E') \longrightarrow n^+(E) + n^+(E') + e^-, \tag{2}$$

$$n(E) + n^+(E') \longrightarrow n(E') + n^+(E), \tag{3}$$

where $n(E)$, $n^+(E)$ and $e^-$ denote the neutral particles, plasma ions with energy E, and electrons, respectively.

The ionization reactions (1) and (2) result in the loss of neutral particles. They can, therefore, be treated analogously to the absorption (capture) reaction in neutron transport phenomena. In the charge-exchange reaction (3), there is no net change in the number of neutrals but the

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The reaction rate of neutrals with plasmas can be generally expressed as

$$\Sigma_j(r,\ E,\boldsymbol{\Omega})\cdot\phi(r,\ E,\boldsymbol{\Omega}) = \int_0^{\infty} dE' \int_{4\pi} d\boldsymbol{\Omega}' |v-v'|$$

$$\times\ \sigma_j(E_r)N_j(r,\ E',\boldsymbol{\Omega}')\ \frac{1}{|v|}\ \phi(r,\ E,\boldsymbol{\Omega}), \qquad (4)$$

where j stands for the type of interactions, that is, j = 1, 2 and 3 mean respectively the ionization by electrons, the ionization by plasma ions and the charge exchange with plasma ions. In addition, E is the energy of neutral particles, $\phi(r,\ E,\boldsymbol{\Omega})$ is the neutral flux, $\sigma_j(E_r)$ is the microscopic reaction cross-section of neutrals with energy $E_r$ corresponding to the relative velocity of $|v-v'|$ and $N_j(r,\ E',\boldsymbol{\Omega}')$ is the plasma density distribution.

When the plasma distribution is isotropic, Eq. (4) can be written as

$$\Sigma_j(r,\ E) = \int_0^{\infty} dE'\ \frac{1}{2} \int_{-1}^{1} d\mu' \left[1 + \left(\frac{v'}{v}\right)^2 - 2\left(\frac{v'}{v}\right)\mu'\right]^{1/2}$$

$$\times\ \sigma_j(E_r)N_j(r,\ E')\ , \qquad (5)$$

where $\mu'$ is the cosine of the scattering angle $(\boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')$.
We now assume that the plasma density distribution is a Maxwellian:

$$N_j(r,\ E') = N_j(r)\ \frac{2}{\sqrt{\pi}}\ (kT_j(r))^{-3/2} \sqrt{E'}\ e^{-E'/kT_j(r)}\ , \qquad (6)$$

where $T_j(r)$ is the plasma temperature distribution. The assumption of the Maxwellian distribution may be inappropriate for some problems. But the use of the Maxwellian distribution as weighting function for averaging cross sections may not be so bad even in such cases and we have no evidences what distributions other than the Maxwellian we have to take. The present calculational algorithm can be improved further as the plasma characteristics become clearer through the increase and refinement of experimental data.

## 3. Calculational Algorithm

The integration of Eq. (5) can be performed by using several numerical methods such as Chebyshev quadrature and Gaussian quadrature. In this report, however, we have adopted a different algorithm stated below.

### 3.1 Change of Variables

In the beginning, let $E = \frac{m_n}{2} v^2$ and $E' = \frac{m_j}{2} v'^2$, where $m_n$, $m_j$, v and v' are mass of neutral particles, mass of j-particles (ions or electrons), velocity of neutral particles and velocity of j-particles, respectively. Using these relations, substitution of Eq. (6) into Eq. (5) gives

$$\Sigma_j(r, E) = N_j(r) \frac{1}{\sqrt{\pi}} (kT_j(r))^{-3/2} \int_0^\infty dE' \int_{-1}^1 d\mu' \left[ E' + \frac{m_n}{m_j} \frac{E'^2}{E} - 2E' \sqrt{\frac{m_n}{m_j} \frac{E'}{E}} \mu' \right]^{1/2} \cdot e^{-E'/kT_j(r)} \times \sigma_j\left(E' + \frac{m_j}{m_n} E - 2\sqrt{\frac{m_j}{m_n} E E'} \mu'\right) \quad . \tag{7}$$

by noting that $E_r = E_r(|v-v'|) = \frac{m_j}{2} |v-v'|^2$.

Now, let

$$\eta(\mu') = E' + \frac{m_j}{m_n} E - 2 \sqrt{\frac{m_j}{m_n} E E'} \mu' \quad . \tag{8}$$

Then, we have

$$\eta(-1) = (\sqrt{E'} + \sqrt{\frac{m_j}{m_n} E})^2 \equiv \eta_U(E') , \tag{9}$$

$$\eta(1) = (\sqrt{E'} - \sqrt{\frac{m_j}{m_n} E})^2 \equiv \eta_L(E') , \tag{10}$$

and

$$d\mu' = -\frac{1}{2} \sqrt{\frac{m_n}{m_j}} \frac{d\eta}{\sqrt{E \cdot E'}} \tag{11}$$

Substituting Eqs. (8), (9), (10) and (11) into Eq. (7), we get

$$\Sigma_j(r, E) = N_j(r) \frac{1}{2\sqrt{\pi}} (kT_j(r))^{-3/2} \cdot \frac{m_n}{m_j} \frac{1}{E} \int_0^\infty dE' e^{-E'/kT_j(r)} \times \int_{\eta_L(E')}^{\eta_U(E')} d\eta \sqrt{\eta}\, \sigma_j(\eta) \quad . \tag{12}$$

We now deal only with hydrogen atoms as the neutral particles, though our method can be applied directly to other atoms. An analytic expression for each $\sigma_j(\eta)$ is given by Gryzinski [1] and Riviere [2] based on the classical theory:

$$\sigma_1(\eta) = \frac{6.56 \times 10^{-14}}{(13.605)^2} g(x) \ \mathrm{cm}^2 , \tag{13}$$

where

$$g(x) = \frac{1}{x} \left(\frac{x-1}{x+1}\right)^{3/2} \left[1 + \frac{2}{3}\left(1 - \frac{1}{2x}\right) \log_e(2.7 + \sqrt{x-1})\right] ,$$

$$x = \frac{\eta}{13.605} .$$

$$\sigma_2(\eta) = 3.6 \times 10^{-12} \frac{1}{\eta} \log_{10}(0.1666\eta) \ \mathrm{cm}^2 ,$$

$$\text{for } \eta > 150 \text{ keV} , \tag{14}$$

$$\sigma_2(\eta) = 10^{-34.833} \cdot \eta^{8.156} \cdot 10^{-0.8712(\log_{10}\eta)^2} \ \mathrm{cm}^2 ,$$

$$\text{for } \eta < 150 \text{ keV} , \tag{15}$$

$$\sigma_3(\eta) = \frac{0.6937 \times 10^{-14} \times (1-0.155 \log_{10}\eta)^2}{1 + 0.1112 \times 10^{-14} \times \eta^{3.3}} \ \mathrm{cm}^2 , \tag{15}$$

## 3.2 Domain of Integrations

In order to integrate Eq. (12), we must examine the range of integrations for the ionization by electrons (j = 1) because of the restriction $\eta \geq 13.605$ coming from Eq. (13).

We have from Eq. (9) the following relation:

$$\eta_U(E') = \left(\sqrt{E'} + \sqrt{\frac{m_j}{m_n} E}\right)^2 \geq 13.605,$$

which gives

$$E' \geq \left(\sqrt{13.605} - \sqrt{\frac{m_j}{m_n} E}\right)^2 = E'_\alpha . \tag{16}$$

On the other hand, from Eq. (10),

$$\eta_L(E') = (\sqrt{E'} - \sqrt{\frac{m_j}{m_n}E})^2 \geq 13.605$$

which gives

$$E' \geq (\sqrt{13.605} + \sqrt{\frac{m_j}{m_n}E})^2 = E'_\beta \ . \qquad (17)$$

The domain of integration for j = 1 is thus obtained as the shaded region shown in Fig. 1.

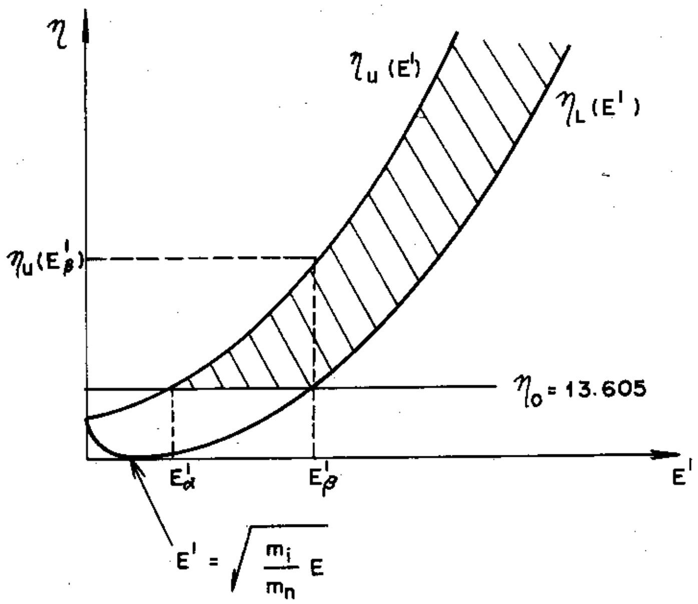


Fig. 1 The domain of integration for j = 1

Noting that $x(E') = \eta(E')/\eta_o$, $x_U(E') = \eta_U(E')/\eta_o$, $x_L(E') = \eta_L(E')/\eta_o$ and $d\eta = \eta_o dx$, Eq. (12) can be rewritten as follows:

$$\Sigma_1(r, E) = A_1(r, E) \cdot [\int_{E'_\alpha}^{E'_\beta} dE' e^{-E'/kT_1(r)} \int_1^{x_U(E')} dx \eta_o^{3/2} \sqrt{x}\ \sigma_1(x)$$

$$+ \int_{E'_\beta}^{\infty} dE' e^{-E'/kT_1(r)} \int_{x_L(E')}^{x_U(E')} dx \eta_o^{3/2} \sqrt{x}\ \sigma_1(x)], \qquad (18)$$

where

$$A_1(r, E) = N_1(r) \frac{1}{2\sqrt{\pi}} (kT_1(r))^{-3/2} \frac{m_n}{m_1} \cdot \frac{1}{E} \ . \qquad (19)$$

It may be worthwhile to mention that the first term of Eq. (18) is almost negligible for hydrogen atoms with relatively low energy. In fact, for hydrogen atoms, $m_e/m_n = 1/1836.14$ and hence $E'_\alpha = (\sqrt{\eta_o} - \sqrt{E/1836.14})^2$, $E'_\beta = (\sqrt{\eta_o} + \sqrt{E/1836.14})^2$, and $x_U(E'_\beta) = (\sqrt{\eta_o} + 2\sqrt{E/1836.14})^2/\eta_o$. Therefore for E << 1836 eV, $E'_\alpha \doteqdot E'_\beta$, $x_U \doteqdot 1$ and $\sigma_1(x)|_{x=1} = 0$ from Eq. (13).

### 3.3 Approximate Expressions

Since the integrand $\sigma_j(\eta)$ has a complicated expression, we expand $\sqrt{\eta}\,\sigma_j(\eta)$ by using polynomials of degree n $P^j_n(\eta)$ so as to minimize $L_2$ norm, $\| \sqrt{\eta}\,\sigma_j(\eta) - P^j_n(\eta) \|_2$. This approximation can be achieved by the least-square method or function minimization algorithm.

Then, Eq. (12) becomes

$$\Sigma_j(r, E) = A_j(r, E) \int_0^\infty dE' e^{-E'/kT_j(r)} \int_{\eta_L}^{\eta_U} d\eta P^j_n(\eta)$$

$$= A_j(r, E) \int_0^\infty dE' e^{-E'/kT_j(r)} \int_{\eta_L}^{\eta_U} d\eta \sum_{\ell=1}^{n+1} a_\ell \eta^{\ell-1} , \qquad (20)$$

where $A_j(r, E)$ is defined as Eq. (19), and the coefficients $a_\ell$, $\ell = 1, \ldots, n+1$ are known quantities.

Now, Eq. (20) can be integrated termwise over $\eta$ as follows:

$$\Sigma_j(r, E) = A_j(r, E) \sum_{\ell=1}^{n+1} a_\ell \int_0^\infty dE' e^{-E'/kT_j(r)} \int_{\eta_L}^{\eta_U} d\eta \cdot \eta^{\ell-1}$$

$$= A_j(r, E) \sum_{\ell=1}^{n+1} \frac{a_\ell}{\ell} \int_0^\infty dE' e^{-E'/kT_j(r)} [\, \eta^\ell_U(E') - \eta^\ell_L(E') \,] , \qquad (21)$$

Since

$$\eta^\ell_U(E') = \left(\sqrt{E'} + \sqrt{\frac{m_j}{m_n} E}\right)^{2\ell}$$

$$= \sum_{r=0}^{2\ell} {}_{2\ell}C_r \sqrt{E'}^{\,2\ell-r} \cdot \sqrt{\frac{m_j}{m_n} E}^{\,r} , \qquad (22)$$

$$\eta_L^{\ell}(E') = (\sqrt{E'} - \sqrt{\frac{m_j}{m_n}E})^{2\ell}$$

$$= \sum_{r=0}^{2\ell} {}_{2\ell}C_r \sqrt{E'}^{\,2\ell-r} (-\sqrt{\frac{m_j}{m_n}E})^r , \qquad (23)$$

in which ${}_nC_r$ is the binomial coefficients $\frac{n!}{r!(n-r)!}$,

$$\eta_U^{\ell}(E') - \eta_L^{\ell}(E')$$

$$= \sum_{r=0}^{2\ell} {}_{2\ell}C_r \sqrt{E'}^{\,2\ell-r} [\sqrt{\frac{m_j}{m_n}E}^{\,r} - (-\sqrt{\frac{m_j}{m_n}E})^r]$$

$$= 2\sum_{\substack{r=1 \\ (\text{odd})}}^{2\ell-1} {}_{2\ell}C_r \sqrt{E'}^{\,2\ell-r} \sqrt{\frac{m_j}{m_n}E}^{\,r} . \qquad (24)$$

Substituting Eq. (24) into Eq. (21) and integrating it termwise over E', we get

$$\Sigma_j(r, E) = A_j(r, E) \sum_{\ell=1}^{n+1} \frac{a_\ell}{\ell} \cdot 2 \sum_{\substack{r=1 \\ (\text{odd})}}^{2\ell-1} {}_{2\ell}C_r \sqrt{\frac{m_j}{m_n}E}^{\,r}$$

$$\times \int_0^\infty dE' e^{-E'/kT_j(r)} (E')^{\frac{2\ell-r}{2}}$$

$$= 2A_j(r, E) \sum_{\ell=1}^{n+1} \frac{a_\ell}{\ell} \sum_{r=1}^{\ell} {}_{2\ell}C_{2r-1} \sqrt{\frac{m_j}{m_n}E}^{\,2r-1}$$

$$\times \int_0^\infty dE' e^{-E'/kT_j(r)} (E')^{\ell-r+1/2} . \qquad (25)$$

Denoting the integral of Eq. (25) as $I_{\ell,r}$, it is written as follows:

$$I_{\ell,r} = [kT_j(r)]^{\ell-r+3/2} \int_0^\infty dt e^{-t} t^{\ell-r+1/2}$$

$$= [kT_j(r)]^{\ell-r+3/2} \cdot \Gamma(\ell-r+3/2) , \qquad (26)$$

where $\Gamma(x)$ is the gamma function having a property of

$$\Gamma(1/2) = \sqrt{\pi} , \quad \Gamma(x+1) = x\Gamma(x) .$$

For the ionization by electrons, we can derive an equation similar to Eq. (25) from Eq. (18) as follows:

$$\Sigma_1(r, E) = A_1(r, E)\left[\sum_{\ell=1}^{m+1}\frac{a_\ell}{\ell}\int_{E'_\alpha}^{E'_\beta} dE' e^{-E'/kT_1(r)}\cdot(x_U^\ell(E') - 1)\right.$$

$$\left.+\sum_{\ell=1}^{n+1}\frac{b_\ell}{\ell}\int_{E'_\beta}^{\infty} dE' e^{-E'/kT_1(r)}(x_U^\ell(E') - x_L^\ell(E'))\right]. \quad (27)$$

where coefficients $a_\ell$, $\ell = 1, \ldots, m+1$, and $b_\ell$, $\ell = 1, \ldots n+1$ are known quantities. Noting that $x_U(E') = \eta_U(E')/\eta_o$, $x_L(E') = \eta_L(E')/\eta_o$, we can rewrite Eq. (27) as

$$\Sigma_1(r, E) = A_1(r, E)\left\{\sum_{\ell=1}^{m+1}\frac{a_\ell}{\ell}\left[\left(\frac{1}{\eta_o}\right)^\ell\sum_{r=0}^{2\ell}{}_{2\ell}C_r\sqrt{\frac{m_j}{m_n}E}^{\,r}\right.\right.$$

$$\left.\times\int_{E'_\alpha}^{E'_\beta} dE' e^{-E'/kT_1(r)}\sqrt{E'}^{\,2\ell-r} - \int_{E'_\alpha}^{E'_\beta} dE' e^{-E'/kT_1(r)}\right]$$

$$+2\sum_{\ell=1}^{n+1}\frac{b_\ell}{\ell}\left(\frac{1}{\eta_o}\right)\sum_{r=1}^{\ell}{}_{2\ell}C_{2r-1}\sqrt{\frac{m_j}{m_n}E}^{\,2r-1}$$

$$\left.\times\int_{E'_\beta}^{\infty} dE' e^{-E'/kT_1(r)}\cdot(E')^{\ell-r+1/2}\right\}. \quad (28)$$

Denoting the integral of the first term of Eq. (28) as $I_1(\ell, r)$ and the third term as $I_2(\ell, r)$, they are expressed by the use of incomplete gamma function of the first kind $\gamma(x, p)$ as follows:

$$I_1(\ell, r) = \int_0^{E'_\beta} dE' e^{-E'/kT_1(r)}(E')^{\ell-r/2} - \int_0^{E'_\alpha} dE' e^{-E'/kT_1(r)}(E')^{\ell-r/2}$$

$$= [kT_1(r)]^{\ell-r/2+1}\cdot[\gamma(\ell-r/2+1, E'_\beta/kT_1(r))$$

$$-\gamma(\ell-r/2+1, E'_\alpha/kT_1(r))], \quad (29)$$

$$I_2(\ell, r) = \int_0^{\infty} dE' e^{-E'/kT_1(r)}(E')^{\ell-r+1/2} - \int_0^{E'_\beta} dE' e^{-E'/kT_1(r)}(E')^{\ell-r+1/2}$$

$$= [kT_1(r)]^{\ell-r+3/2}[\Gamma(\ell-r+3/2) - \gamma(\ell-r+3/2,$$

$$E'_\beta/kT_1(r))]. \quad (30)$$

The $\gamma(x, p)$ can be expanded in a series by Legendre's formula:

$$\gamma(x,p) = e^{-p} \sum_{n=0}^{\infty} \frac{p^{x+n}}{x(x+1)(x+2)\ldots(x+n)} \quad . \tag{31}$$

The second term of Eq. (28) can be integrated analytically. And hence from Eq. (25) or Eq. (28), we can calculate the macroscopic cross section $\Sigma_j(r, E)$ for each j depending on plasma temperature $T_j(r)$ and energy of neutral particles E.

We divide the plasma region into NR-subregions with the average temperature $T_j(r_k)$:

$$T_j(r_k) = \frac{\int_{r_{k-1}}^{r_k} dr\ 2\pi r\ T(r)}{\pi(r_k^2 - r_{k-1}^2)} \quad , k=1,\ldots, NR,$$

and the energy into NE-energy groups with the average value $E_i$ to calculate the discrete value $\Sigma_j(r_k, E_i)$.

## 4. Numerical Results

We consider here the following three problems and their results are shown in Figs. 2 ~ 14.

[A] The interaction of hydrogen atoms with hydrogen plasmas in the Princeton ST Tokamak. The physics data are as follows [3]:

(1) Plasma density profile is given by

$$N(r) = N(o) [0.8(1 - r^2/a^2) + 0.2] ,$$

where N(o) is the peak density $3 \times 10^{13}$ cm$^{-3}$ and a = 14 cm is the minor radius of the Tokamak.

(2) Plasma ion temperature is constant, 20 eV.

(3) Plasma electron temperature changes radially as

$$T_e(r) = T_e(o) [0.9(1 - r^2/a^2) + 0.1] ,$$

The $\gamma(x, p)$ can be expanded in a series by Legendre's formula:

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〔A〕 The interaction of hydrogen atoms with hydrogen plasmas in the Princeton ST Tokamak. The physics data are as follows 〔3〕:

(1) Plasma density profile is given by

$$N(r) = N(o)\,[\,0.8(1 - r^2/a^2) + 0.2\,] ,$$

where N(o) is the peak density $3 \times 10^{13}$ $cm^{-3}$ and a = 14 cm is the minor radius of the Tokamak.

(2) Plasma ion temperature is constant, 20 eV.

(3) Plasma electron temperature changes radially as

$$T_e(r) = T_e(o)\,[\,0.9(1 - r^2/a^2) + 0.1\,] ,$$

where $T_e(o)$ = 200 eV is the peak electron temperature.

(4) Energy of the neutral particles (source neutrals) are 3 eV whereas the secondary particles originated through the change exchange reaction have energies of 20 eV.

The results are shown in Fig. 2 representing the temperature dependence of the cross section for ionization by electrons $\Sigma_1(r, E)$. The plasma region is divided into 10 subregions with an average temperature $T(\bar{r}_k)$. The cross section for the ionization by plasma ions $\Sigma_2(E)$, and for the charge exchange with them $\Sigma_3(E)$ are given also in Fig. 2 for E = 3 eV and 20 eV.

[B] The same problem as [A] except for the following assumptions.

The plasma ion temperature and the source neutral energy also varies radially as

$$T_i(r) = T_i(o)\,[\,0.9(1 - r^2/a^2) + 0.1\,] \;,$$

$$E\,(r) = E(o)\,[\,0.9(1 - r^2/a^2) + 0.1\,] \;,$$

where $T_i(o)$ = 100 eV is the peak ion temperature and E(o) = 100 eV is the peak neutral energy.

The results are shown in Figs. 3 - 7 for each reaction depending on plasma temperature T(r) and neutral energy E. Fig. 8 shows the total cross section for this problem which can be expressed by

$$\Sigma_t(\bar{r}_k,\ j\text{-group}) = \Sigma_a(\bar{r}_k,\ j) + \Sigma(\bar{r}_k\!:\ E_j \rightarrow E_k),$$

where $\Sigma_a(\bar{r}_k, j)$ is the absorption cross section written as

$$\Sigma a(\bar{r}_k,\ j\text{-group}) = \Sigma_1(\bar{r}_k,\ E_j,\ T_e = T_e(r_k)) + \Sigma_2(\bar{r}_k,\ E_j,\ T_i = T_i(\bar{r}_k)),$$

and $\Sigma(\bar{r}_k\!: E_j \rightarrow E_k)$ is the scattering cross section from the j-th group to k-th group.

〔C〕 The same problem as〔B〕except that the distribution function of the plasma temperature and source neutral energy is $1 - (r/a)^4$.
The results are shown in Figs. 9 ~ 14 corresponding respectively to Figs. 3 ~ 8 of the problem〔B〕.

Fig. 15 shows the distribution functions of the plasma temperature and neutral energy for the problems〔A〕,〔B〕and〔C〕.

A parametric servey has been performed by using these cross sections to estimate the characteristics of neutral-plasma interactions. The results will be shown in a separate report.

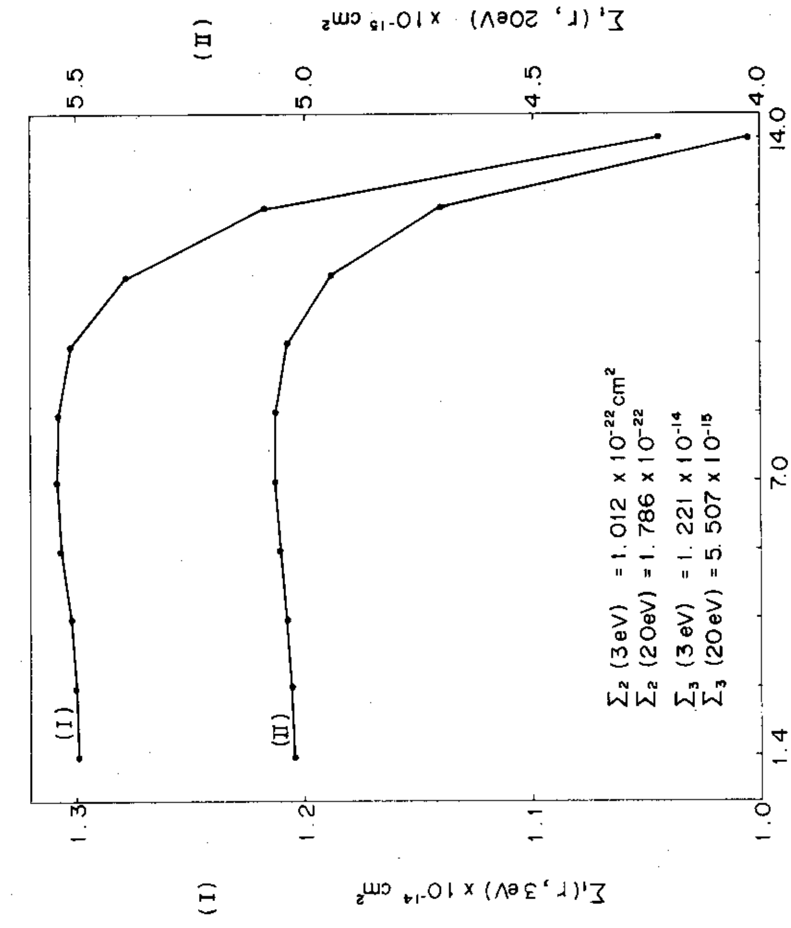


Fig. 2 Cross sections for the ionization of hydrogens by electrons for the problem [A]

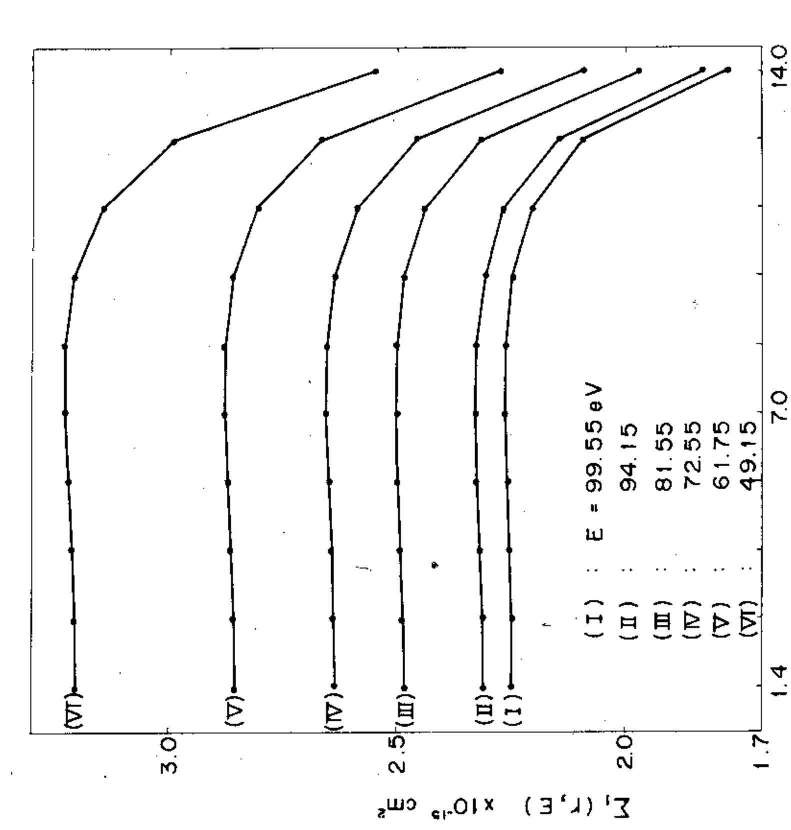


Fig. 3 Cross sections for the ionization of hydrogens by electrons for the problem [B]

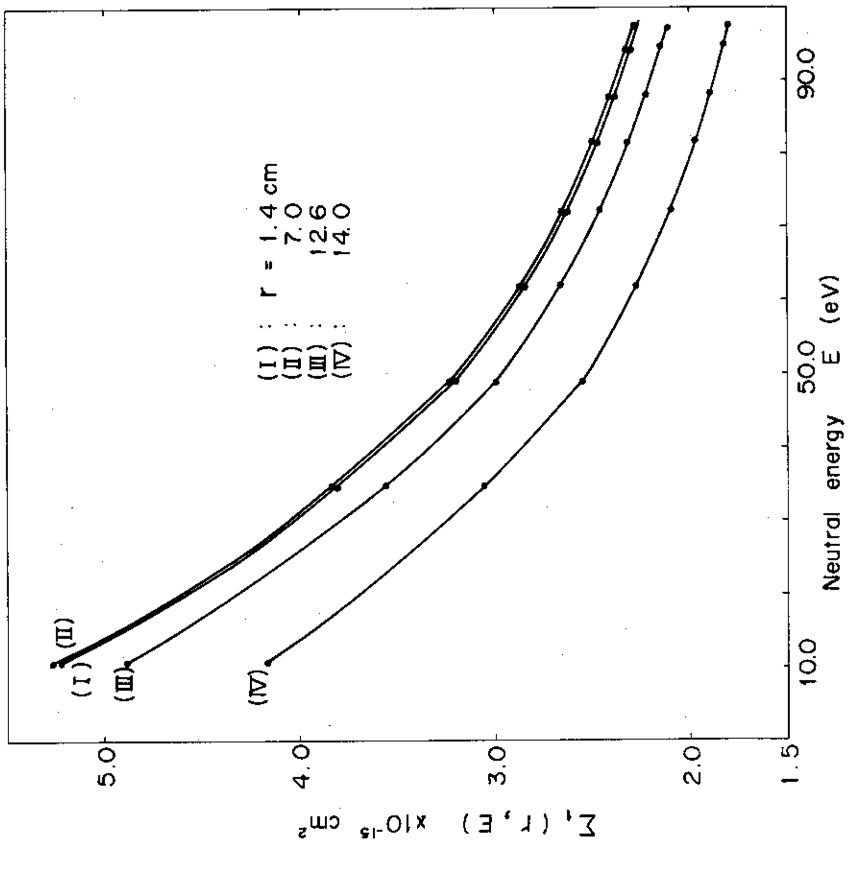


Fig.4 Cross sections for the ionization of hydrogens by electrons as a function of neutral energy

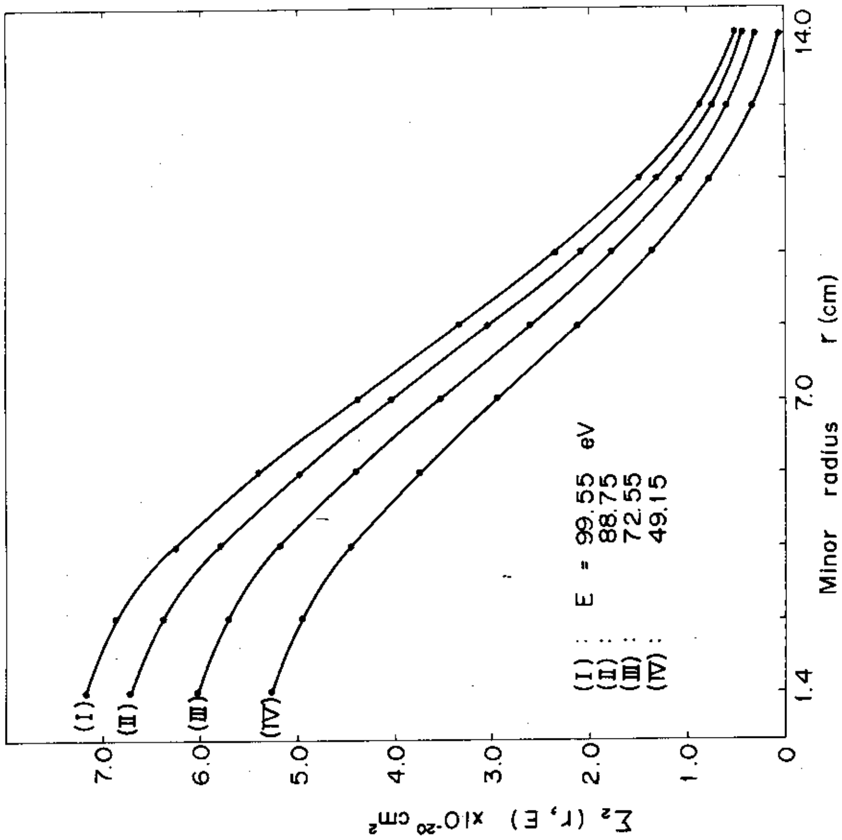


Fig. 5 Cross sections for the ionization of hydrogens by plasma ions

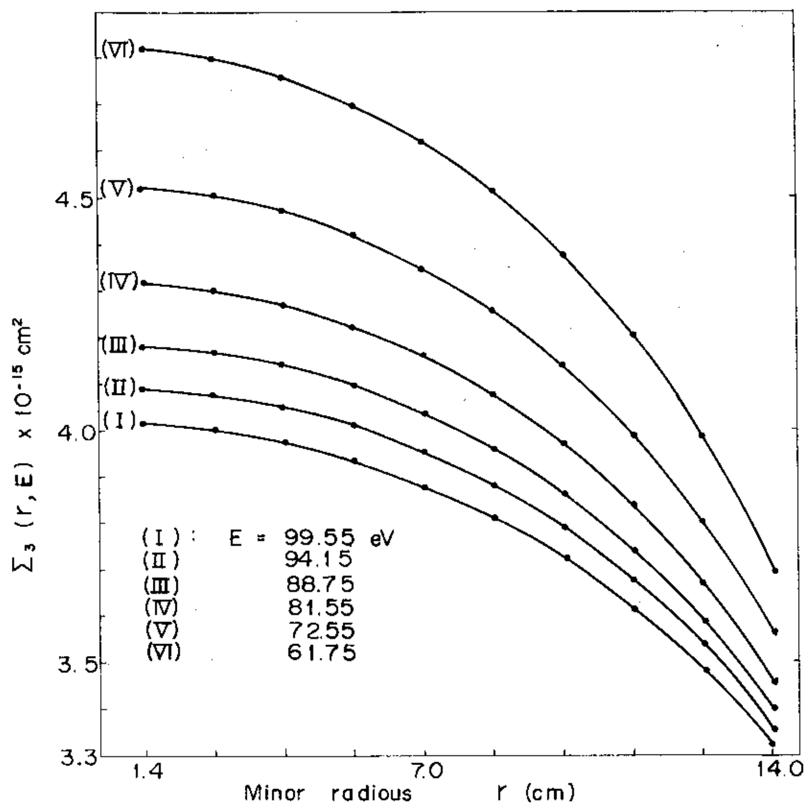


Fig. 6 Cross section for the charge exchange with plasma ions.

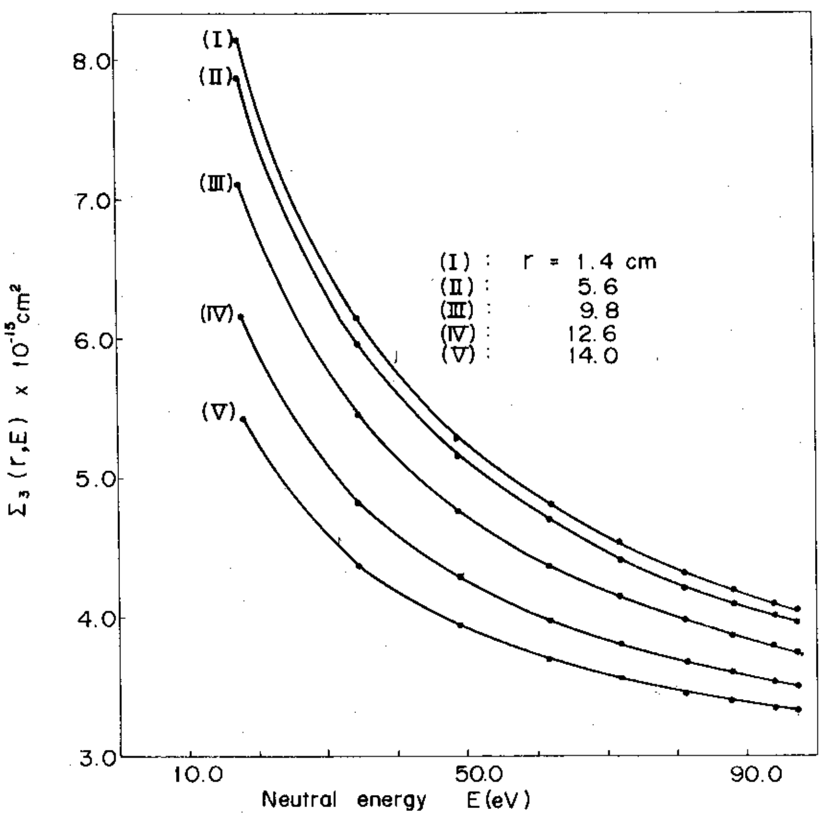


Fig. 7 Cross section for the charge exchange with plasma ions as a function of neutral energy

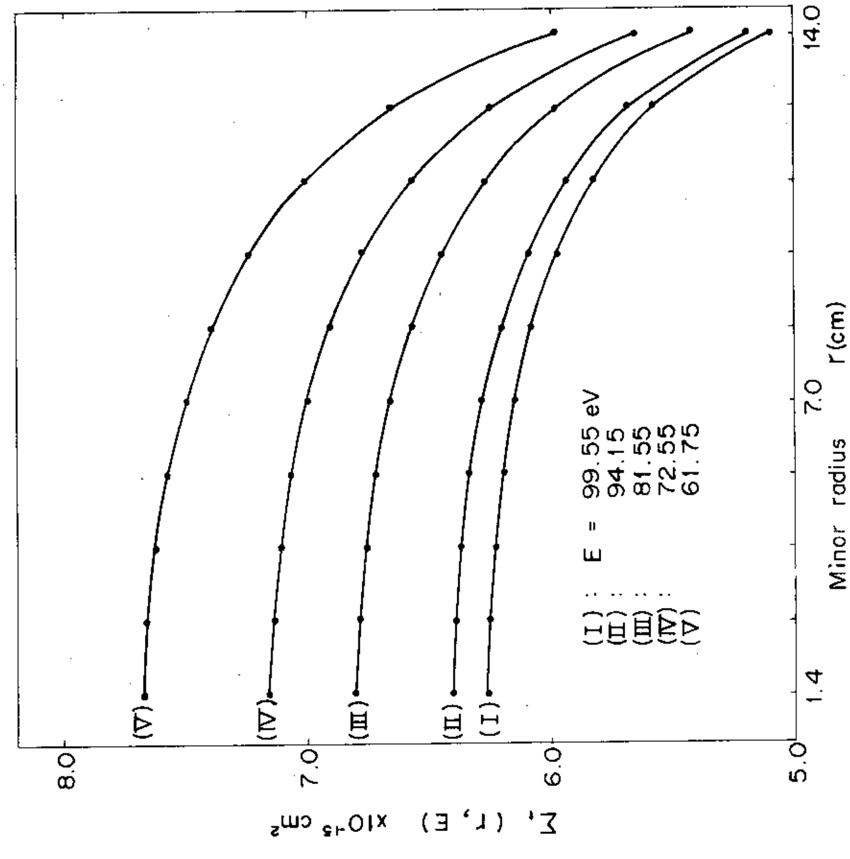


Fig. 8 Total cross sections of neutrals with energy E for the problem [B]

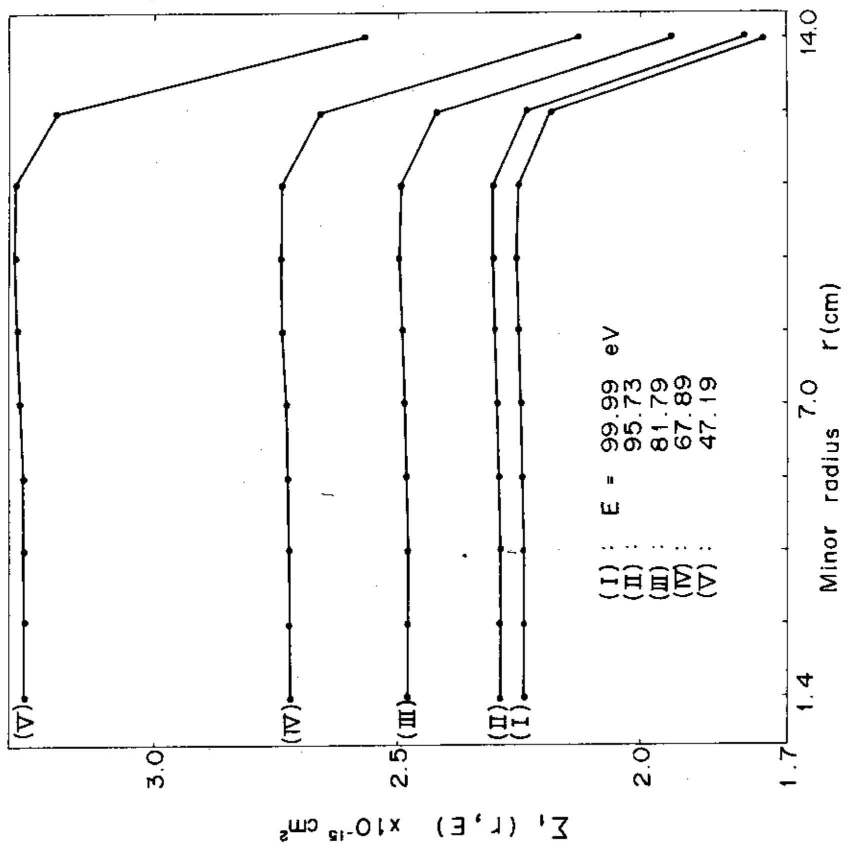


Fig. 9 Cross sections for the ionization of hydrogens by electrons for the problem [C]

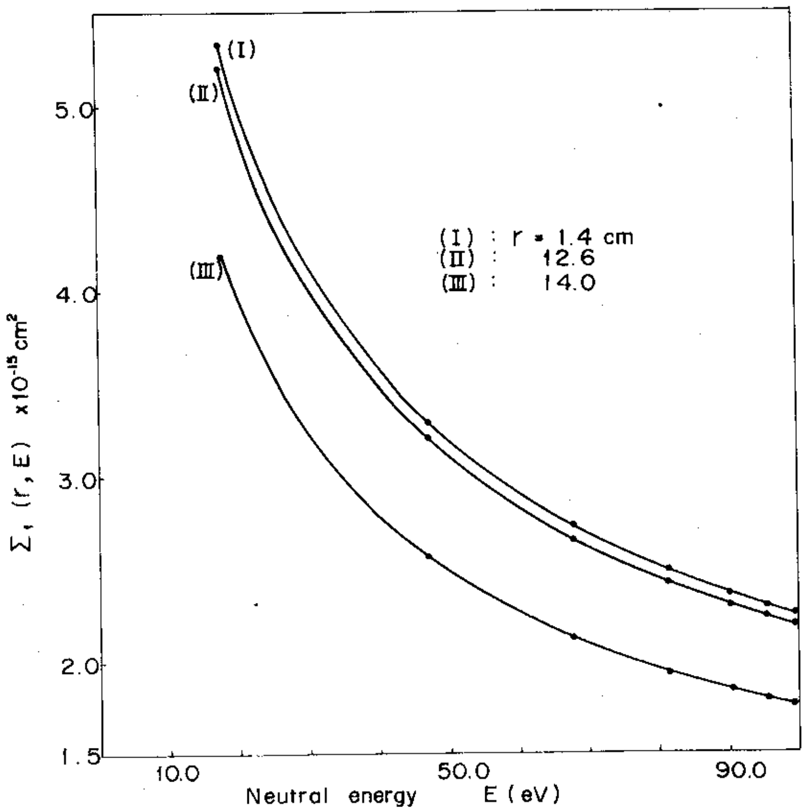


Fig. 10 Cross sections for the ionizations of hydrogens by electrons as a function of neutral energy

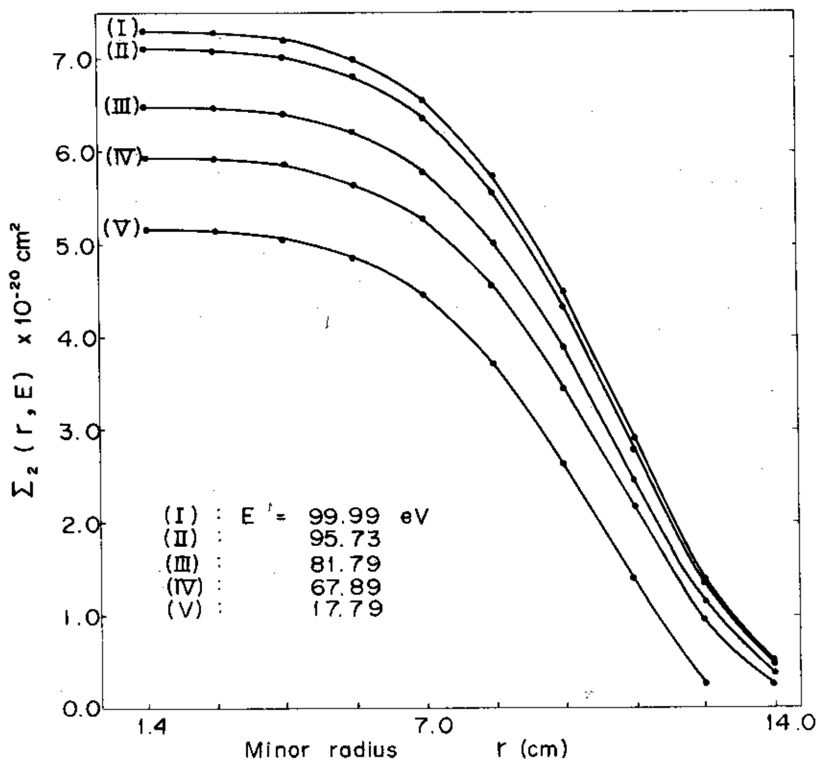


Fig. 11 Cross sections for the ionization of hydrogens by plasma ions

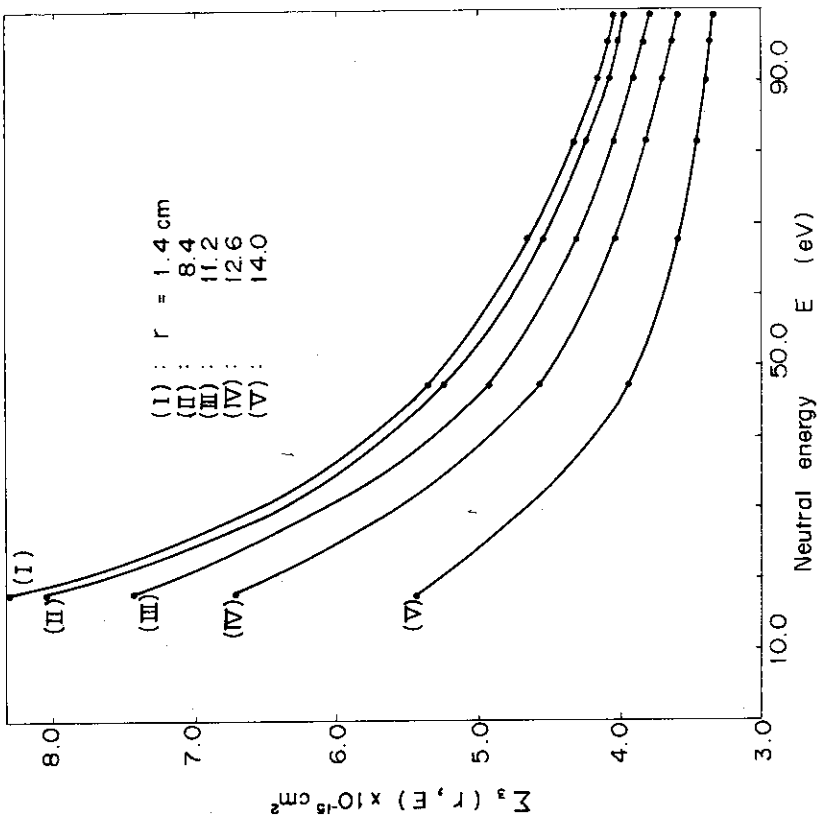


Fig. 13 Cross sections for the charge exchange with plasma ions as a function of neutral energy

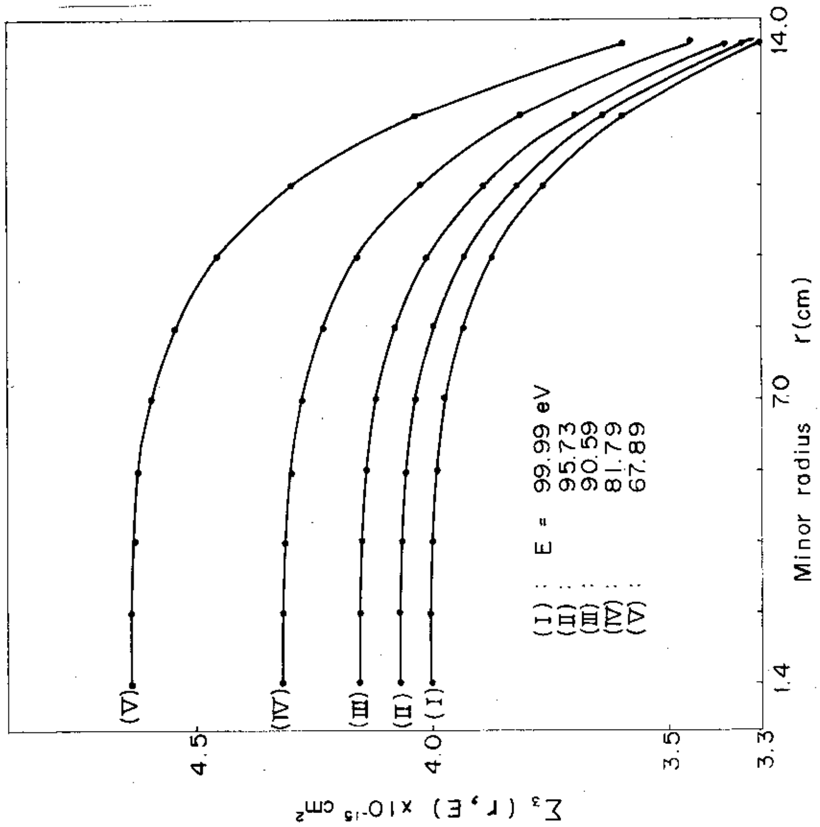


Fig. 12 Cross sections for the charge exchange with plasma ions

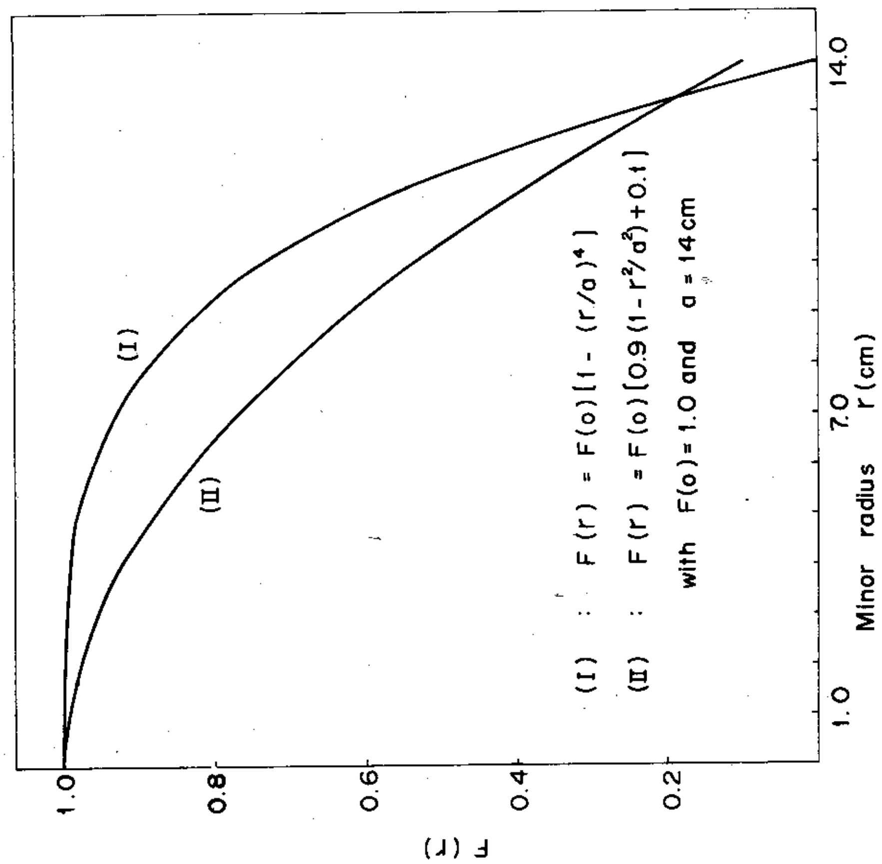


Fig. 15 Distribution functions for plasma temperature and neutral energy.

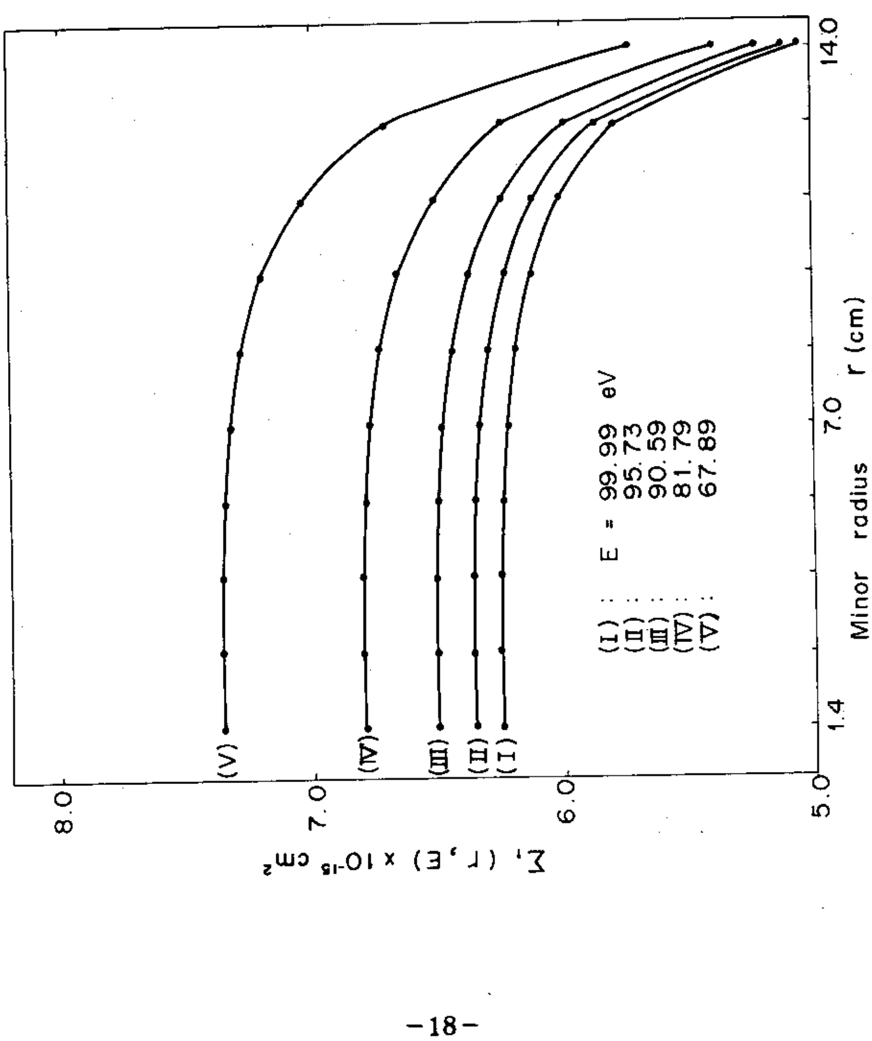


Fig. 14 Total cross section of neutrals with energy E for the ploblem (C)

## 5. Arrangement of the Cross Section Data

The absorption, energy transfer and total cross sections calculated in each subregion with average temperature $T(\bar{r}_k)$ are printed out in the FIDO format as shown in Fig. 16. In this arrangement, we must take care of the treatment of the so-called self-scattering reaction.

In neutral transport phenomena, the self-scattering means the charge exchange reaction:

$$H(E) + H^+(E) \longrightarrow H^+(E) + H(E) \; , \tag{32}$$

and Fig. 16 is shown under the assumption that the neutral particles originated by this reaction fly out to other regions with a different temperature without suffering this reaction again. The scheme shown in Fig. 16 is therefore called as the one-collision approximation.

In the case where the region is infinite, the reaction (32) and its reverse are repeated infinite times before the neutral particles are ionized by absorption reactions, and hence the reaction (32) has no participation in the net change of the number of neutrals. We can therefore ignore the self-scattering reaction (32) and set the contribution to the total cross section to be zero. This is called as the infinite collision approximation.

Strictly speaking, the reaction is in between one-collision and infinite collision approximation, and we can treat it in a way of changing the energy group index whenever the reaction occures.

In practice, however, we may use the one-collision approximation by making the region with constant temperature as small as necessary.

## Acknowledgements

The authors wish to express their gratitude to Dr. T. Asaoka for his advice and help about the present work. We are also much indebted to Dr. M. Tanaka for arousing our interest in the work done by Greenspan. Acknowledgement is also due to Mr. S. Inoue for his valuable discussions on the polynomial fitting. The numerical computations were performed on FACOM 230-60 computer of the JAERI.

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| | First Region $T(r_1)$ | | | Second Region $T(r_2)$ | | | | Tenth Region $T(r_{10})$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Group 1 | Group 2 | Group 10 | Group 1 | Group 2 | Group 10 | | Group 1 | Group 2 | Group 10 |
| $\Sigma_a(r_k,E_j)$ | $\Sigma_a(r_1,E_1)$ | $\Sigma_a(r_1,E_2)$ | $\Sigma_a(r_1,E_{10})$ | $\Sigma_a(r_2,E_1)$ | $\Sigma_a(r_2,E_2)$ | $\Sigma_a(r_2,E_{10})$ | | $\Sigma_a(r_{10},E_1)$ | $\Sigma_a(r_{10},E_2)$ | $\Sigma_a(r_{10},E_{10})$ |
| $\nu\Sigma_f(r_k,E_j)$ | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| $\Sigma_t(r_k,E_j)$ | $\Sigma_a(r_1,E_1)+$ $\Sigma(r_1:E_1\to E_1)$ | $\Sigma_a(r_1,E_2)+$ $\Sigma(r_1:E_2\to E_1)$ | $\Sigma_a(r_1,E_{10})+$ $\Sigma(r_1:E_{10}\to E_1)$ | $\Sigma_a(r_2,E_1)+$ $\Sigma(r_2:E_1\to E_2)$ | $\Sigma_a(r_2,E_2)+$ $\Sigma(r_2:E_2\to E_2)$ | $\Sigma_a(r_2,E_{10})+$ $\Sigma(r_2:E_{10}\to E_2)$ | | $\Sigma_a(r_{10},E_1)+$ $\Sigma(r_{10}:E_1\to E_{10})$ | $\Sigma_a(r_{10},E_2)+$ $\Sigma(r_{10}:E_2\to E_{10})$ | $\Sigma_a(r_{10},E_{10})+$ $\Sigma(r_{10}:E_{10}\to E_{10})$ |
| UP- scattering | $\Sigma(r_1:E_{10}\to E_1)$ | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| | $\Sigma(r_1:E_9\to E_1)$ | 0 | 0 | 0 | $\Sigma(r_2:E_{10}\to E_2)$ | 0 | | 0 | 0 | 0 |
| | $\Sigma(r_1:E_8\to E_1)$ | 0 | 0 | 0 | $\Sigma(r_2:E_9\to E_2)$ | 0 | | 0 | 0 | 0 |
| | . | . | . | . | . | . | ... | . | . | . |
| | . | . | . | . | . | . | ... | . | . | . |
| | . | . | . | . | . | . | ... | . | . | . |
| | | | | | $\Sigma(r_2:E_4\to E_2)$ | | | | | |
| | $\Sigma(r_1:E_2\to E_1)$ | 0 | 0 | 0 | $\Sigma(r_2:E_3\to E_2)$ | 0 | | 0 | 0 | 0 |
| Self- scattering | $\Sigma(r_1:E_1\to E_1)$ | 0 | 0 | 0 | $\Sigma(r_2:E_2\to E_2)$ | 0 | | 0 | 0 | $\Sigma(r_{10}:E_{10}\to E_{10})$ |
| Down- scattering | 0 | 0 | 0 | 0 | $\Sigma(r_2:E_1\to E_2)$ | 0 | | 0 | 0 | $\Sigma(r_{10}:E_9\to E_{10})$ |
| | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | $\Sigma(r_{10}:E_8\to E_{10})$ |
| | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | . |
| | . | . | . | . | . | . | | . | . | . |
| | . | . | . | . | . | . | | . | . | $\Sigma(r_{10}:E_3\to E_{10})$ |
| | . | . | . | . | . | . | | . | . | $\Sigma(r_{10}:E_2\to E_{10})$ |
| | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | $\Sigma(r_{10}:E_1\to E_{10})$ |

Fig. 16 Cross section arrangement for the case of 10 groups and 10 regions