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TWO-PHASE PRESSURE DROP IN AN EVAPORATING CHANNEL

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On the basis of the drift flux flow model, the conservation of mass, momentum and energy are solved to obtain the profiles of void fraction, pressure drop, vapor velocity and liquid velocity for an evaporating vertical upflow in a channel with uniform heating.

Two situations are considered,i.e., the flow entering the channel is (1) subcooled, and (2) saturated. In the former, the pressure loss vs. flow curve may have a negative slope which could lead to two-phase instabilities. The obtained analytical results explain appearance and disappearance of the negative slope.

The analytical results are found in good agreements with experimental results.

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蒸発チャンネルでの二相流圧損

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朝　日　義　郎

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ドリフトフラックスモデルに基づき，一様加熱垂直チャンネルでのボイド率，圧損，蒸気速度，液体速度のプロファイルを得るために質量，運動量，エネルギの3つの保存則を解いた。

チャンネル入口流がサブクールの時と，飽和であるときとについて考えてある。前者の場合，不安定性を起こす原因となる負勾配が圧損対流動カーブに出現することがある。本報告によって得られた解析結果は，この負勾配の出現消滅を説明しており，実験ともよく一致することもわかった。

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I. Introduction

The prediction of steady state pressure drop of two-phase flow in an evaporating channel is an essential step in the design of a large variety of industrial plants in the power and process industries. The prediction of steady state pressure drop is important also from a dynamical design point of view since thorough understanding of steady states is indispensable for any dynamical analysis of a system.

In the traditional analyses[1] of two-phase flow, use has been made of the "slip" flow model characterized by the notion of slip ratio which usually does not depend on flow rate. For an evaporating vertical upflow, Weiss[4], Thom[5] and Lombardi and Pedrocchi[6], for example, analyzed the pressure drop, regarding the channel as a lumped parameter system. The system is, however, essentially a distributed parameter system described by the conservation equations which are strongly coupled to one another.

It has been experimentally observed[4,7,8] that under certain conditions the pressure loss versus flow curve exhibits a negative slope which may lead to serious two-phase flow instability[9]. The mechanism of the appearance of the negative slope inducing the two-phase flow instability has not yet been quite understood.

Zuber and Staub[10] formulated the two-phase flow problem in terms of the velocity of center of volume and of the drift velocity of the vapor with respect to the former. They derived a void propagation equation and found that their analysis was in good agreements with the experiment[11,12].

In this work, following the formulation by Zuber and Staub[10], the conservation equations for mass, momentum and energy will be solved for an evaporating vertical upflow in a channel with a uniform heat input, and the solutions so obtained will be compared to experimental results.

The symbols used in the various equations are defined as they are introduced into the text. For ready reference, however, they are listed at the end of the text.

II. Drift Flux Flow Model

In this chapter a brief review of the drift flux flow model is given; interested readers can find the details in Ref. (10) and those cited there.

Let u_f and u_g be the local velocities of the liquid and the vapor, respectively. Then the velocity of center of volume is given by,

$$\bar{j} = (1-\alpha)u_f + \alpha u_g \tag{II-1}$$

Let V_{gj} be the local drift velocity of the vapor with respect to the center of volume of the mixture;

$$V_{gj} = u_g - j. \tag{II-2}$$

Then the relative velocity between the phases is given by,

$$u_g - u_f = \frac{1}{1-\alpha} V_{gj}. \tag{II-3}$$

The vapor drift velocity V_{gj} depends on the flow regime; for the churn turbulent bubbly flow[13] ,

$$V_{gj} = 1.41 \left(\frac{\sigma g(\rho_f - \rho_g)}{\rho_f^2} \right)^{1/4} \tag{II-4}$$

and for the slug flow[14],

$$V_{gj} = 0.35 \left(\frac{g(\rho_f - \rho_g)D}{\rho_f} \right)^{\frac{1}{2}} . \tag{II-5}$$

Expressions for some other flow regimes are given in Ref. (10). It can be seen from the above expressions for V_{gj} that the vapor drift velocity does not depend upon the void fraction in case of the churn turbulent bubbly flow and the slug flow. We assume in this work that the flow is in the churn turbulent bubbly flow region or in the slug flow region so that the drift velocity V_{gj} does not depend on the void fraction α.

In this work it will be assumed that the void distribution across the channel is uniform; this may be a good assumption in the case of churn turbulent bubbly flow.

III. Subcooled Region and Boiling Boundary

We assume that there is no subcooled boiling and that the viscosity and density of the subcooled liquid are constant and depend only on the system parameters such as the system pressure.

The continuity equation in the subcooled region is,

$$\frac{\partial G}{\partial z} = \frac{\partial}{\partial z} \rho_f u = 0,$$

from which it follows that the flow rate G is a constant;

$$G = \rho_f u = a \quad \text{constant.} \tag{III-1}$$

Due to incompressibility, the fluid velocity u also remains constant.

The equation of momentum balance is;

$$\frac{\partial}{\partial z} \rho_f u^2 = -\frac{\partial p}{\partial z} - g\rho_f - \frac{fG^2}{2D\rho_f} , \tag{III-2}$$

where the friction factor f may be given by the Blasius relation;

$$f = c\left(\frac{\mu_f}{DG}\right)^n \tag{III-3}$$

where c and n are constant. Integrating Eq. (III-2) from the channel inlet, z = 0, to a point, z, downstream in the subcooled region, we obtain;

$$p(z,\ u) = -(\rho_f g + \frac{fG^2}{2D\rho_f})z + p_{in}, \qquad (\text{III-4})$$

where p_{in} is the channel inlet pressure. The first term of the RHS(right hand side) of Eq. (III-4) gives the pressure drop in the subcooled region as a function of z, u and the system parameters.

The conservation of energy gives,

$$\frac{\partial}{\partial z}\rho_f u\, h = q. \qquad (\text{III-5})$$

Since we assume a uniform heating, Eq. (III-5) can easily be integrated and gives;

$$h(z,\ u) = \frac{q}{\rho_f u}z + h_{in}, \qquad (\text{III-6})$$

where h_{in} is the inlet enthalpy.

At the boiling boundary z_b, the relation

$$h(z_b,\ u) = h_f(p_0) \qquad (\text{III-7})$$

is satisfied. The symbol p_0 means the system pressure. Eqs. (III-6) and (III-7) give,[15]

$$z_b = u\tau_h, \qquad (\text{III-8})$$

where

$$\tau_h = \frac{\rho_f}{q}(h_f(p_0) - h_{in}) \qquad (\text{III}-9)$$

Eq. (III-8) indicates that the boiling boundary is proportional to the liquid velocity in the subcooled region.

IV. Saturated Region

We assume that the liquid viscosity and the liquid and vapor densities are constant and depend only on the system pressure. It will also be assumed that the enthalpies, h_f and h_g, of the saturated liquid and vapor remain to be the values at the system pressure. We will let symbol ρ_f stand for the densities of both the saturated and subcooled liquid, since these quantities may in the present case take on nearly the same value specified by the system parameters.

The conservation equations in the saturated zone are; for energy,

$$\frac{\partial}{\partial z}[\rho_f(1-\alpha)h_f u_f + \alpha\rho_g h_g u_g] = q \; ; \qquad \text{(IV-1)}$$

for mass,

$$\frac{\partial}{\partial z}(1-\alpha)\rho_f u_f = \Gamma_f \; , \qquad \text{(IV-2)}$$

$$\frac{\partial}{\partial z}\alpha\rho_g u_g = \Gamma_g \; , \qquad \text{(IV-3)}$$

and

$$\Gamma_f + \Gamma_g = 0 \; ;$$

and for momentum,

$$\tau_h = \frac{\rho_f}{q}(h_f(p_0) - h_{in}) \qquad (\text{III-9})$$

Eq. (III-8) indicates that the boiling boundary is proportional to the liquid velocity in the subcooled region.

$$\frac{\partial}{\partial z}[\rho_f(1-\alpha)u_f{}^2+\rho_g\alpha u_g{}^2] = -\frac{\partial p}{\partial z}-[\rho_f(1-\alpha)+\rho_g\alpha]g-\frac{\partial\sigma_f}{\partial z} \quad . \quad (\text{IV-5})$$

The last term of the RHS of Eq. (IV-5) stands for the frictional pressure loss gradient.

First we consider the conservation of energy. Performing the differentiation on the LHS (left hand side) of Eq. (IV-1) and substituting Eqs. (IV-2), (IV-3) and (IV-4), we obtain,

$$\rho_f(1-\alpha)u_f\frac{\partial h_f}{\partial z}+\rho_g\alpha u_g\frac{\partial h_g}{\partial z}+\Gamma_g(h_g-h_f) = q.$$

Since we assume that the saturated enthalpies, h_f and h_g, depend only on the system pressure, the above equation reduces to,

$$\Gamma_g = \frac{q}{h_g-h_f} \quad . \qquad (\text{IV-6})$$

Next, assuming the incompressibility of the vapor and the liquid, we obtain from Eqs. (IV-2) and (IV-3),

$$\frac{\partial}{\partial z}(1-\alpha)u_f = \frac{-\Gamma_g}{\rho_f} \qquad (\text{IV-7})$$

$$\frac{\partial}{\partial z}\alpha u_g = \frac{\Gamma_g}{\rho_g} \quad . \qquad (\text{IV-8})$$

Adding these equations, we have,

$$\frac{\partial j}{\partial z} = \frac{1}{\tau} \qquad (\text{IV}-9)$$

where

$$\tau = \frac{\rho_g \rho_f}{\Gamma_g (\rho_f - \rho_g)} \quad . \qquad (\text{IV}-10)$$

If the flow entering the channel is subcooled, and the channel is sufficiently long, then the flow finally reaches saturation at z_b which is given by Eq. (III-8). First we consider this situation. Later we consider another situation in which the flow entering the channel is already saturated.

Integration of Eq. (IV-9) over $[z_b, z]$ gives,

$$j(z) = j(z_b) + \frac{z - z_b}{\tau} \quad . \qquad (\text{IV}-11)$$

Since $\alpha(z_b)=0$, Eq. (II-1) gives,

$$j(z_b) = u_f(z_b) = u,$$

where we note that u is the velocity of the liquid in the subcooled region. Substituting the above equation into Eq. (IV-11), we obtain,

$$j(z) = u + \frac{z - z_b}{\tau} \quad . \qquad (\text{IV}-12)$$

Substituting Eq. (IV-12) into Eqs. (II-1) and (II-2), we obtain,

$$u_f = \frac{z-z_b}{\tau} + u - \frac{\alpha}{1-\alpha} V_{gj} \qquad (IV\text{-}13)$$

and

$$u_g = \frac{z-z_b}{\tau} + u + V_{gj} \quad . \qquad (IV\text{-}14)$$

Substituting Eq. (IV-14) into Eq. (IV-8) and noting that V_{gj} can be assumed to be a function of the system pressure only, we obtain,

$$\frac{\partial\alpha}{\partial z} = \frac{\frac{\Gamma_g}{\rho_g}\tau - \alpha}{z - z_b + \tau(u+V_{gj})} \quad . \qquad (IV\text{-}15)$$

Integration of Eq. (IV-15) over $[z_b, z]$ gives,

$$\alpha(z,u) = \frac{\rho_f}{\rho_f - \rho_g}\left[1 - \frac{\tau(u+V_{gj})}{z - z_b + \tau(u+V_{gj})}\right] \quad . \qquad (IV\text{-}16)$$

Since we do not consider superheated flow, we require $z_b < H$ and $\alpha(H,u) \le 1$, which gives,

$$z_b < H \le z_b + \left(\frac{\rho_f}{\rho_g} - 1\right)\tau(u+V_{gj}), \qquad (IV\text{-}17)$$

where H stands for the length of the heated part of the channel.

For the conservation of momentum, we have Eq.(IV-5), the RHS of which can further be manipulated with the help of the continuity equations (IV-7) and (IV-8) and then of Eqs.(IV-13) and (IV-14) as follows;

$$\frac{\partial}{\partial z}[\rho_f(1-\alpha)u_f^{\,2}+\rho_g\ u_g^{\,2}]$$
$$=\rho_f(1-\alpha)u_f\frac{\partial u_f}{\partial z}+\rho_g\alpha u_g\frac{\partial u_g}{\partial z}+\Gamma_g(u_g-u_f)$$
$$=[\rho_f(1-\alpha)+\rho_g\alpha](\frac{u+V_{gj}}{\tau}+\frac{z-z_b}{\tau^2})-\frac{\rho_f V_{gj}}{\tau}$$
$$+\frac{\rho_f V_{gj}}{1-\alpha}(\frac{\Gamma_g}{\rho_f}-u_f\frac{\partial\alpha}{\partial z})\quad , \tag{IV-18}$$

where we have assumed that the drift velocity V_{gj} depends only on the system pressure. Next we note the relationship,

$$\frac{\Gamma_g}{\rho_f}-u_f\frac{\partial\alpha}{\partial z}=-(1-\alpha)(\frac{1}{\tau}-V_{gj}\frac{\partial}{\partial z}\frac{1}{1-\alpha})\quad , \tag{IV-19}$$

which follows from Eqs.(IV-7) and (IV-13). Substituting Eq.(IV-19) into the last term on the RHS of Eq.(IV-18), we finally obtain,

$$\frac{\partial}{\partial z}[\rho_f(1-\alpha)u_f^{\,2}+\rho_g\alpha u_g^{\,2}]$$
$$=\frac{1}{\tau}[\rho_f(1-\alpha)+\rho_g\alpha](u+V_{gj}+\frac{z-z_b}{\tau})-\frac{2}{\tau}\rho_f V_{gj}+\rho_f V_{gj}^{\,2}\frac{\partial}{\partial z}\frac{1}{1-\alpha}\quad . \tag{IV-20}$$

Sustituting Eq. (IV-20) into Eq. (IV-5), we obtain,

$$\frac{\partial}{\partial z}\left(p+\frac{\rho_f V_{gj}^{2}}{1-\alpha}\right) = -[\rho_f(1-\alpha)+\rho_g\alpha]\left(g+\frac{u+V_{gj}}{\tau}+\frac{z-z_b}{\tau^2}\right) + \frac{2\rho_f V_{gj}}{\tau} - \frac{\partial \sigma_f}{\partial z} \quad . \qquad (\text{IV-21})$$

It is conventional to express the frictional two-phase pressure loss as,

$$\frac{\partial \sigma_f}{\partial z} = \frac{f}{2D}\frac{G^2}{\rho_f}\Phi^2 \qquad (\text{IV-22})$$

where the factor Φ^2 is called the two-phase multiplier[5,16,17,18] and G is the total mass flow rate for the two-phase mixture;

$$G = \rho_f u_f(1-\alpha)+\rho_g u_g \alpha \quad . \qquad (\text{IV-23})$$

The factor f is given by Eq. (III-3),

$$f = c\left(\frac{\mu_f}{DG}\right)^n \quad ,$$

where it should be noted that G is given by (IV-23). It should also be noted that the total mass flow rate G does not depend upon z, on account of the continuity equation for the mixture,

$$\frac{\partial G}{\partial z} = 0$$

and hence that the total mass flow rate in the saturated region coincides with that in the subcooled region, i.e.,

$$G(z) = \rho_f u \quad .$$

Substituting Eqs. (IV-16) and (IV-22) and integrating over $[z_b, z]$, we obtain,

$$p(z,u) = p(z_b,u) - \left[\frac{\alpha(z,u)}{1-\alpha(z,u)} \rho_f V_{gj}{}^2 - \frac{\rho_f}{\tau}(V_{gj}-u)(z-z_b)\right] - \rho_f \tau (u+V_{gj}) g \log\frac{z-z_b+\tau(u+V_{gj})}{\tau(u+V_{gj})} - \frac{fG^2}{2D\rho_f}\int_{z_b}^{z} \Phi^2(\alpha,G)dz \quad . \qquad (IV-24)$$

Substituting into Eq. (IV-24), Eq. (III-4) with z replaced by z_b, we obtain,

$$\begin{aligned}\Delta p(z,u) &= p_{in} - p(z,u) \\ &= \left[\rho_f g + \frac{fG^2}{2D\rho_f}\right] z_b \\ &\quad + \left[\frac{\alpha(z,u)}{1-\alpha(z,u)} \rho_f V_{gj}{}^2 - \frac{\rho_f}{\tau}(V_{gj}-u)(z-z_b)\right] \\ &\quad + \rho_f \tau (u+V_{gj}) g \log\frac{z-z_b+\tau(u+V_{gj})}{\tau(u+V_{gj})} \\ &\quad + \frac{fG^2}{2D\rho_f}\int_{z_b}^{z} \Phi^2(\alpha,G)dz \quad . \qquad (IV-25)\end{aligned}$$

Eq. (IV-25) gives under the condition (IV-17) the pressure drop in an evaporating channel with a uniform heating when the flow entering the channel is subcooled. The liquid phase velocity, the vapor phase velocity and the void distribution are given by Eqs. (IV-13), (IV-14) and (IV-16), respectively.

We will next consider another situation where the flow entering the channel is already saturated. The simmilar procedure taken in the preceeding discussions in this chapter can also be followed to solve the problem so that only the final results are presented below.

The void fraction, pressure loss and fluid velocities in this case are given by the following set of equations:

$$\alpha(z,G) = \frac{\frac{\Gamma_g}{\rho_g} z+\alpha_{in}(j_{in}+V_{gj})}{\frac{z}{\tau}+j_{in}+V_{gj}} \qquad \text{(IV-26)}$$

$$\begin{aligned}\Delta p(z,G) &= p_{in}-p(z,G)\\ &= \rho_f V_{gj}{}^2\left[\frac{1}{1-\alpha(z,G)}-\frac{1}{1-\alpha_{in}}\right]\\ &\quad +\rho_{in}\tau g(j_{in}+V_{gj})\log\frac{z+\tau(j_{in}+V_{gj})}{\tau(j_{in}+V_{gj})}\\ &\quad +\frac{\rho_{in}}{\tau}(j_{in}+V_{gj})z-\frac{2\rho_f V_{gj}}{\tau}z\\ &\quad +\frac{fG^2}{2D\rho_f}\int_0^z \Phi^2(\alpha,G)dz \quad .\end{aligned} \qquad \text{(IV-27)}$$

$$u_f(z,G) = -\frac{\alpha(z,G)}{1-\alpha(z,G)}V_{gj}+\frac{z}{\tau}+j_{in} \quad . \qquad \text{(IV-28)}$$

$$u_g(z,G) = V_{gj}+\frac{z}{\tau}+j_{in} \quad . \qquad \text{(IV-29)}$$

$$\rho_{in} = \rho_f(1-\alpha_{in}) + \rho_g\alpha_{in} \ .$$

In the above equations, α_{in} and j_{in} stand for the void fraction and the velocity of the volumetric center of the two-phase flow entering the channel, respectively. The latter can be expressed in terms of the former and the total mass flow rate G as,

$$j_{in} = \frac{G+\alpha_{in}V_{gj}(\rho_f-\rho_g)}{\rho_f(1-\alpha_{in})+\rho_g\alpha_{in}} \quad . \qquad (\text{IV}-30)$$

We note that the total mass flow rate G does not depend upon z. Since we do not consider superheated flow, we require,

$$\alpha(H,G)\leq 1 \quad ,$$

which gives from Eq. (IV-26),

$$(1-\alpha_{in})\frac{\rho_f-\rho_g}{\rho_g}\tau(j_{in}+V_{gj})\geq H \quad . \qquad (\text{IV}-31)$$

Substituting Eq. (IV-30) into Eqs. (IV-26)-(IV-29), we obtain the void fraction, pressure loss and fluid velocities as functions of, among other things, the position z in the channel and the total mass flow rate G.

If the form of the function $\Phi^2(\alpha,G)$ is specified, its integration appearing in Eq. (IV-24) or (IV-27) can be evaluated either analytically or numerically. The two-phase multiplier Φ^2 is a complicated function of the system pressure, the void fraction or steam quality and the flow rate and may even depend on the geometrical configuration of the system. Many experiments have been conducted to obtain correlations for the two-phase multiplier. Thom, for example, reported a correlation for the two-phase multiplier, namely, r_5 in Ref. (5), where Muscettola's experimented results[19] were compared with r_5 and with multipliers given by others[16,20,21]; apparently the two-phase multiplier r_5 showed a better agreement with the experimental results than the other three (see Figs. 14 and 15 in Ref. (5)).

In analogy with the single-phase flow, the frictional two-phase pressure loss may be expressed as,

$$\frac{\partial \sigma_f}{\partial z} = \frac{f_m}{2D} \frac{G^2}{\rho} \qquad (\text{IV}-32)$$

where f_m may be called the two-phase friction factor and

$$\rho = \rho_f(1-\alpha) + \rho_g \alpha \quad .$$

Equating Eqs. (IV-22) and (IV-32), we have,

$$\Phi^2 = \frac{f_m}{f} \frac{\rho_f}{\rho_f(1-\alpha)+\rho_g \alpha} \quad . \qquad (\text{IV}-33)$$

The factor f_m can be evaluated if Φ^2 is given. Numerical values of f_m/f for water-steam mixture are calculated, using the two-phase multiplier r_5 given by Thom[5] and shown in Table 1. Table 1 implies that the two-phase multiplier can be approximated by,

$$\Phi^2 = \frac{\rho_f}{\rho_f(1-\alpha)+\rho_g\alpha} \tag{IV-34}$$

for a wide range of pressure. The two-phase multiplier (IV-34) seems to give good agreements with experimental results for a wide range of mass flow rate as well (see Figs. 14 and 15 in Ref. (5)). It is, however, not considered to give correct values when the void fraction α is close to one.

If we use the two-phase multiplier (IV-34), the last term of the RHS of Eq. (IV-24) becomes,

$$\frac{fG^2}{2D\rho_f}\int_{z_b}^{z}\Phi^2(\alpha,G)dz$$

$$= \frac{fG^2}{2D\rho_f}\,\frac{z-z_b}{\tau(u+V_{gj})}\,[\frac{z-z_b}{2}+\tau(u+V_{gj})] \tag{IV-35}$$

The integration of the last term of the RHS of Eq. (IV-27) can similarly be performed for the two-phase multiplier (IV-34).

Table 1

$$\Phi^2 \frac{\rho}{\rho_f} = \frac{f_m}{f} \qquad \text{(water-steam)}$$

quality by mass	Pressure (psia) 250	600	1250	2100	3000
0.01	1.07	1.06	0.965	—	—
0.02	1.09	1.02	0.953	0.955	—
0.04	1.08	0.980	0.943	0.943	—
0.06	1.06	0.997	0.957	0.940	0.945
0.08	1.04	0.998	0.967	0.938	0.938
0.1	1.03	1.01	0.988	0.943	0.939
0.2	1.00	0.991	0.982	0.944	0.954
0.4	0.993	0.988	0.987	0.951	0.987
0.6	0.992	0.987	0.997	0.968	0.995
0.8	0.986	0.990	1.00	0.992	1.00
1.0	1.00	0.995	1.01	0.996	1.00

V. Comparison of Predicted and Experimental Results

The pressure drop in the heated part of the channel may be given by Eq. (III-4) or (IV-25) or (IV-27). When these equations are referred to in this chapter, the variable z should be interpreted as the length H of the heated part of the channel.

First let the flow entering the channel subcooled. If we plot the pressure drop versus flow rate, we may obtain such a curve[4,8,7,9] as shown in Fig. 1. Let us decrease the flow rate gradually. In zone I all liquid flow occurs and Δp decreases as G decreases, obeying Eq. (III-4). At point A, the flow starts to boil just at the channel exit so that the boiling boundary $z_b = \tau_h G/\rho_f$ coincides with H. As G further decreases, so does the boiling boundary and the exit void fraction increases. The pressure drop Δp may now increase with decreasing G. As the flow rate falls still further, the curve approaches that for all steam flow and again decreases with G. In zone II the pressure drop is given by Eq. (IV-25).

In Figs. 2 and 3, some of the experimental results obtained by Weiss[4] are shown and are compared to the predictions given by Eqs. (III-4) and (IV-25). The data were obtained for tests with an electrically-heated, vertical tube; 0.174 inch i.d., 2 feet heated length and 2.17 feet pressure tap distance. Unfortunately the two-phase multiplier was not reported for this channel so that use was made of the two-phase multiplier (IV-34) for the analysis.

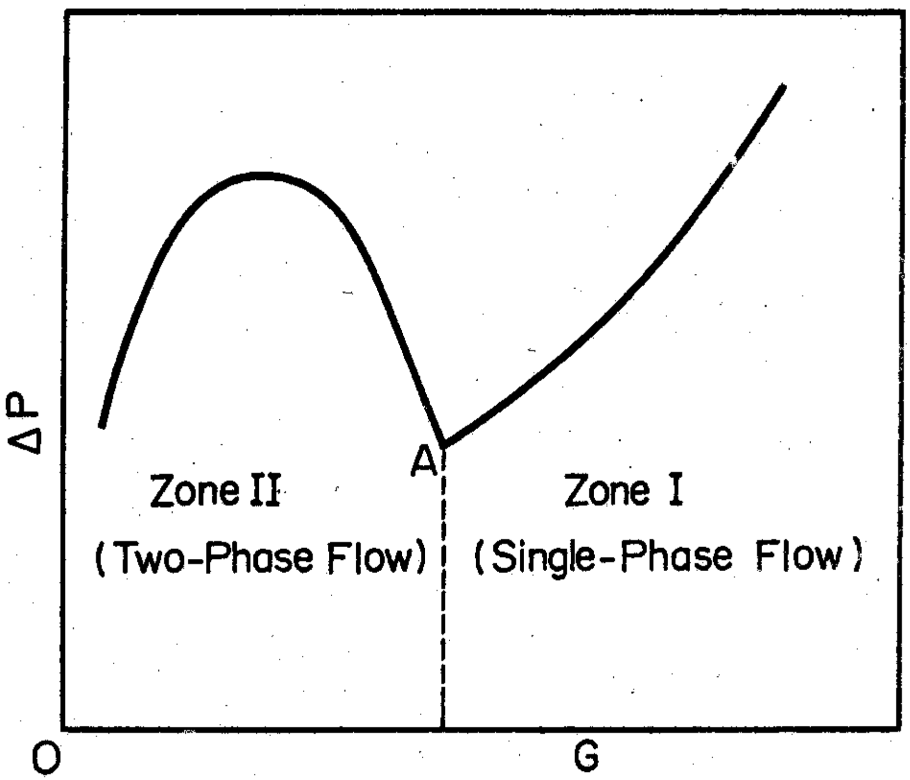

Fig. 1 Pressure drop versus flow curve illustrating a possible peaking in the two-phase zone

If proper multipliers were used, the results would be improved. The drift velocity V_{gj} was calculated from Eq. (II-4). The values c=0.316 and n=0.25 were used for the Blausius relation.

Every experimental curve in Figs. 2 and 3 shows a negative slope in the saturated zone. It is, however, considered important[22] from a stability point of view that the pressure loss versus flow curve does not have a negative slope. Such a case is presented in Fig. 4 which was calculated again from Eqs. (IV-25) and (IV-35) for a typical channel of boiling water nuclear reactors. The exit void fraction in Fig. 4 was calculated from Eq.(IV-16). The drift velocity, V_{gj}, and the friction factor, f, were calculated in the same way as mentioned above.

If the flow entering a channel is already saturated, the pressure drop in the heated part of the channel is given by Eq. (IV-27). In Ref. (23), measurements are reported for an annular tube (Element No. 85) having the following dimensions; the inside diameter of the external tube, the outside diameter of the internal tube, the heated length and the distance between the pressure taps are 0.825 cm, 0.502 cm, 50 cm and 67.4 cm, respectively. Tables 2, 3 and 4 show comparisons between some of the experimental results in Ref. (23) and the predicted values by Eq. (IV-26) and (IV-27). In calculating the pressure drop, use was made of the two-phase multiplier obtained for the same channel by Muscettola[19]. The drift velocity, V_{gj}, and the friction factor, f, were calculated in the same way as mentioned above.

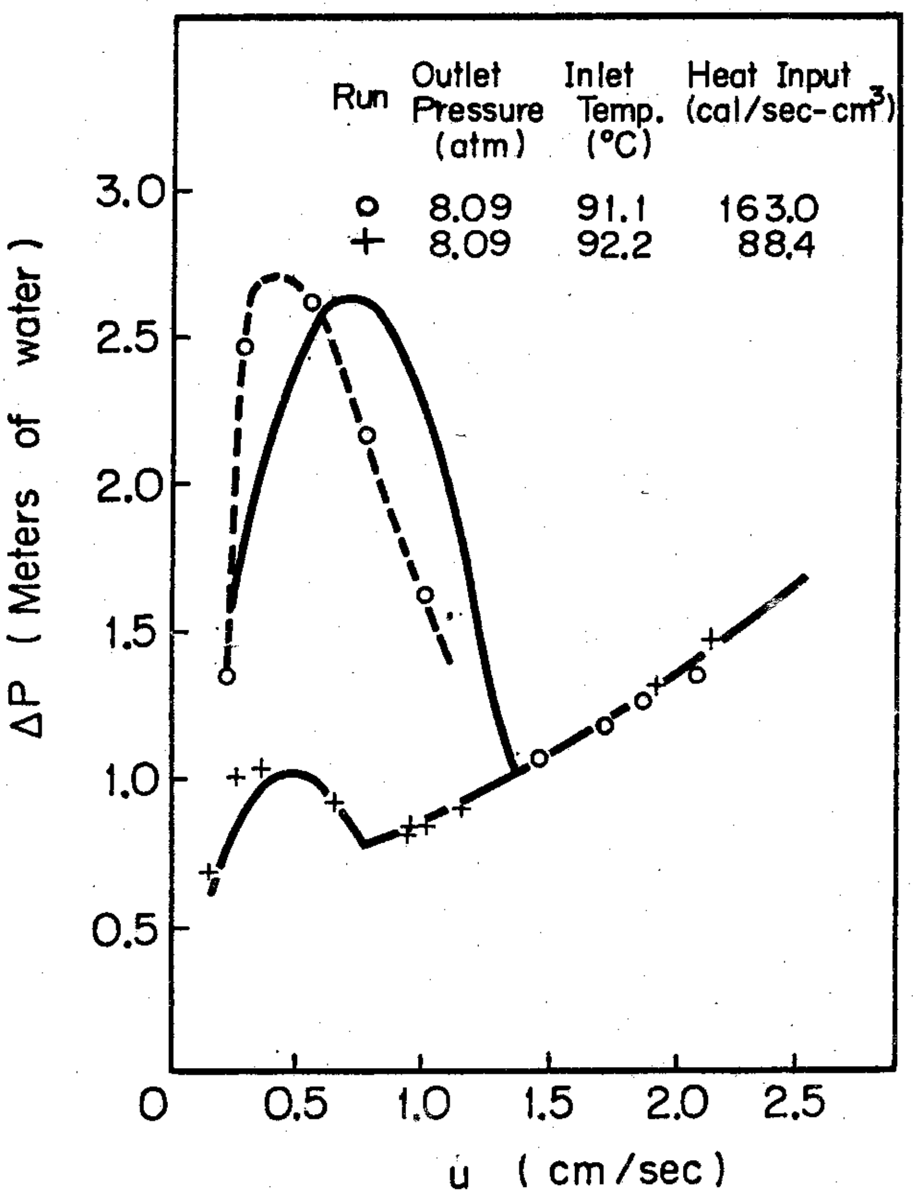

Fig. 2 Comparison of the predicted and experimental pressure loss versus flow curves. The experimental data (indicated by circles and plus signs) are taken from Ref.(4).

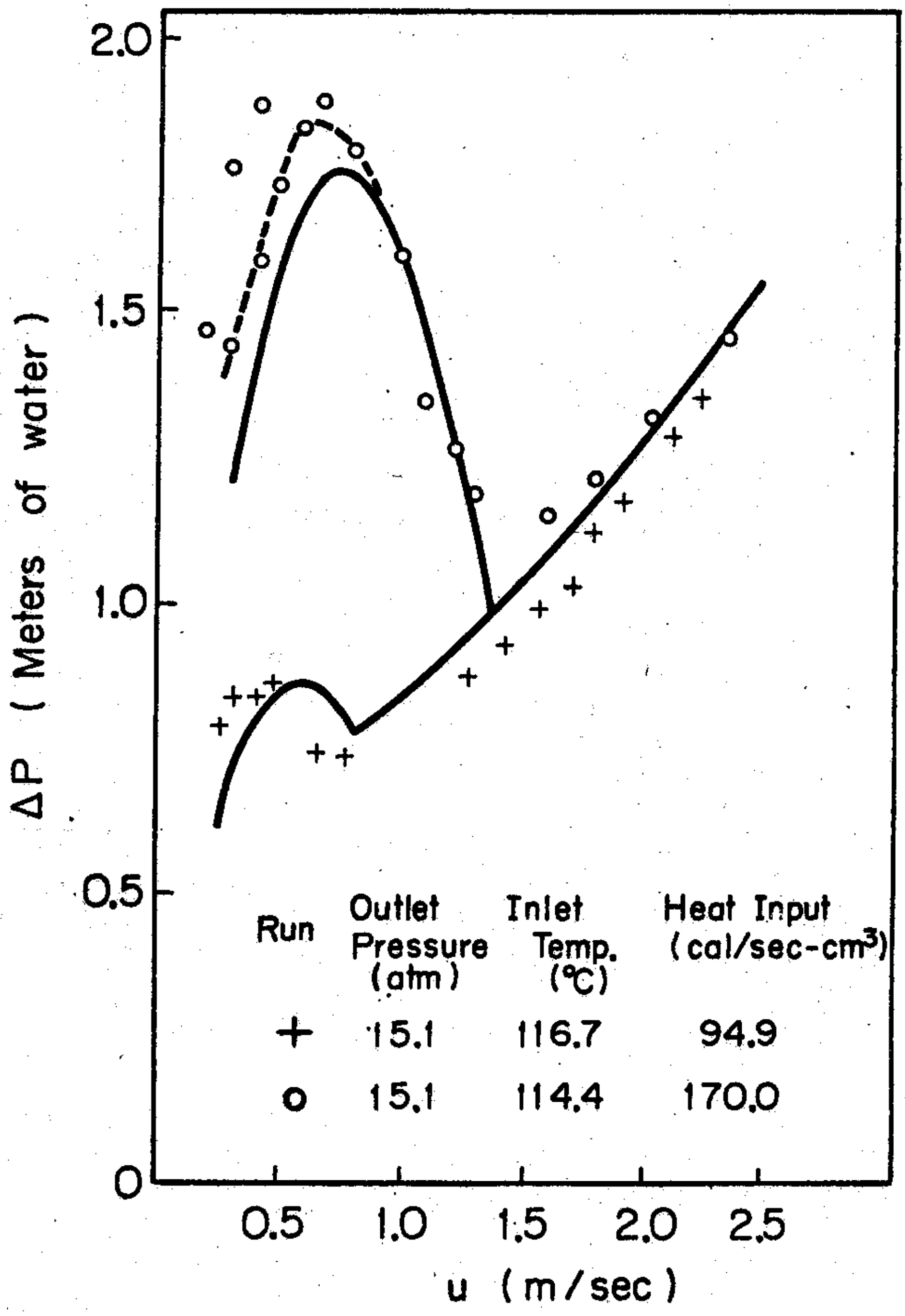

Fig. 3 Comparison of the predicted and experimental pressure loss versus flow curves. The experimental data 9indicated by circles and plus signs) are taken from Ref.(4).

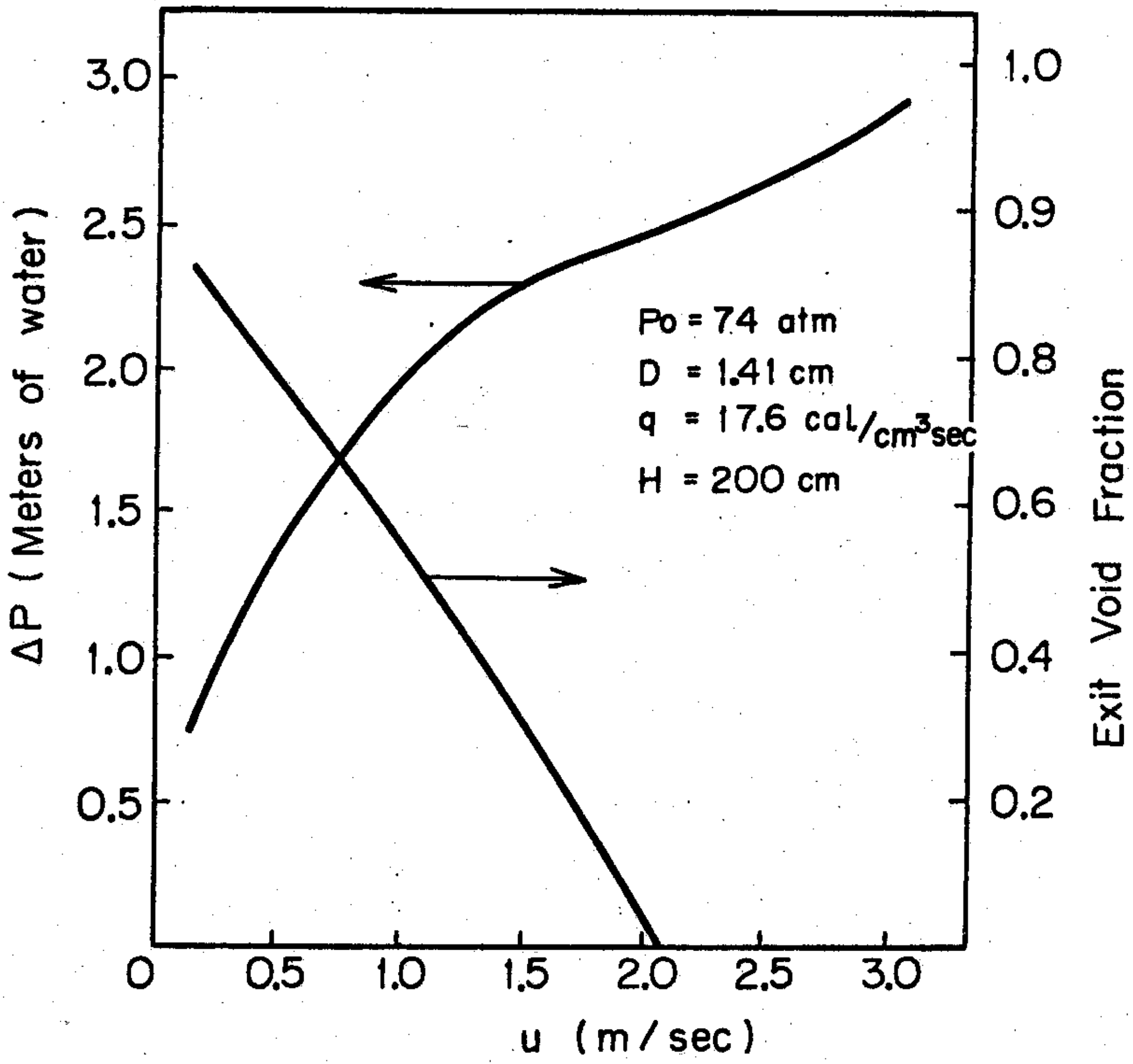

Fig.4 Pressure drop and exit void fraction versus flow curves calculated for a typical channel of boiling water nuclear reactors.

Table 2. Comparison of predicted pressure drop and exit quality with experimental results by Adorni et al.[23]
$p_0 \cong 72$ atm and $G \cong 228$ g/cm^2-sec.

Experiment No.	Inlet Quality by Weight	Heat Flux (watt/cm^2)		Exit Quality by Weight		Pressure Drop (KgW/cm^2)	
		external	internal	Experimental	Predicted	Experimental	Predicted
394	0.195	65.5	38.3	0.298	0.289	1.099	1.15
400	0.123	63.9	38.2	0.224	0.215	1.000	1.01
405	0.066	80.8	38.2	0.186	0.176	0.844	0.900
411	0.0	110.7	38.2	0.145	0.142	0.708	0.739
416	0.192	63.6	63.9	0.310	0.300	1.110	1.18
422	0.118	73.0	63.9	0.247	0.236	1.067	1.06
427	0.057	117.0	64.0	0.235	0.222	1.005	1.02
432	0.0	125.0	61.9	0.168	0.172	0.849	0.835

Table 3 Comparison of predicted pressure drop and exit quality with experimental results by Adorni et al[23].
$p_0 \simeq 71$ atm and $G \simeq 149$ g/cm^2-sec.

Experiment No.	Inlet Quality by Weight	Heat Flux (watt/cm^2)		Exit Quality by Weight		Pressure Drop (KgW/cm^2)	
		external	internal	experimental	predicted	experimental	predicted
450	0.392	27.6	38.2	0.481	0.472	0.718	0.888
455	0.315	42.0	37.2	0.431	0.420	0.756	0.820
460	0.245	63.3	37.2	0.398	0.381	0.735	0.758
466	0.194	73.8	40.3	0.365	0.349	0.669	0.697
472	0.128	72.3	36.9	0.291	0.277	0.531	0.557
477	0.079	63.7	36.9	0.229	0.215	0.423	0.433
482	0.028	61.9	38.2	0.175	0.162	0.374	0.334
491	0.126	67.6	69.8	0.318	0.300	0.576	0.591
496	0.083	74.8	70.6	0.288	0.269	0.525	0.522

Table 4 Comparison of predicted pressure drop and exit quality with experimental results by Adorni et al[23].
$p_0 \simeq 71$ atm and $G \simeq 99$ g/cm^2-sec.

Experiment No.	Inlet Quality by Weight	Heat Flux (watt/cm^2)		Exit Quality by Weight		Pressure Drop (KgW/cm^2)	
		external	internal	experimental	predicted	experimental	predicted
523	0.293	54.2	35.8	0.492	0.466	0.449	0.453
528	0.212	45.6	37.2	0.390	0.368	0.358	0.356
533	0.142	73.2	38.3	0.394	0.362	0.344	0.334
548	0.146	70.9	87.2	0.470	0.429	0.416	0.386
553	0.209	61.2	63.7	0.469	0.437	0.416	0.408
530	0.212	151.6	38.2	0.666	0.611	0.560	0.540
537	0.063	170.9	37.2	0.552	0.505	0.466	0.423

VI. Conclusion

Solving the conservation equations on the basis of the drift flux flow model, the void fraction, pressure drop, vapor velocity and liquid velocity were obtained for an evaporating, vertical channel with uniform heating.

First the situation was considered where the flow entering the channel is subcooled. The boiling boundary was found proportional to the inlet flow velocity. The void fraction, liquid and vapor velocities and pressure drop were given at each point of the channel by Eqs. (IV-16), (IV-13), (IV-14) and (IV-25), respectively. The pressure drops calculated from Eq. (IV-25) were found in good agreements with the experimental results. The most important of all is the fact that Eq. (IV-25) is capable of demonstrating both appearance and disappearance of a negative slope in the pressure drop versus flow curve. The appearance of a negative slope is considered very important[22] from a stability point of view. In the case of a typical channel of boiling water nuclear reactors Eq. (IV-25) indeed gives the pressure drop versus flow curve without a negative slope.

Secondly another situation was considered where the flow entering the channel is already saturated. The problem was solved similarly and the solutions were given by Eqs. (IV-26)-(IV-29). The pressure drop and the exit quality by weight obtained from Eqs. (IV-27) and (IV-26), respectively were again found in good agreements with experimental results.

Knowledge of flow regime is required for evaluation of the drift velocity. Since nothing was reported about the flow regime in the experimental works to which our analysis was compared in this work, the flows were assumed throughout to be churn turbulent bubbly flow. In spite of this rather crude approximation the analytical results were found in quite good agreements with the experimental results.

It is important from a dynamical point of view that the steady states can be described either by Eqs. (Ⅳ-13), (Ⅳ-14), (Ⅳ-16) and (Ⅳ-25) or by Eqs. (Ⅳ-26)-(Ⅳ-29), which are exactly the equilibria of the time dependent conservation equations, since dynamical analysis, e.g., stability analysis, of any system is impossible without exact specification of steady states or equilibria.

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References

1 The references are too many to list here. See for example; G. Katuri, J. Stepanek and F.A. Holland, Brit. Chem. Eng., 16, 333 (1971) and 16, 511 (1971)

2 A.B. Jones, KAPL-2170 (1961)

3 Some other references about slip ratio can be found in; L.S. Tong, "Boiling Heat Transfer and Two-Phase Flow", John Wiley & Sons, Inc., New York (1965)

4 D.H. Weiss, ANL-4916 (1952)

5 J.R.S. Thom, Int. J. Heat Transfer, 7, 709 (1964)

6 C. Lombardi and E. Pedrocchi, Energia Nucleare, 19: No. 2, 91 (1972)

7 A.H. Stenning and T.N. Veziroglu, Proceedings of the 1965 Heat Transfer and Fluid Mechanics Institute, Stanford University, 1965

8 G. Yadigaroglu and A.E. Bergles, Report DSR-74629-3, M.I.T., Cambridge, Mass., Dec. 1969

9 M. Ledinegg, Die Wärme, 61, 891 (1938)

10 N. Zuber and F.W. Staub, Nucl. Sci. Eng., 30, 268 (1967)

11 F.W. Staub, N. Zuber and G. Bijwaard, Nucl. Sci. Eng., 30, 279 (1967)

12 F.W. Staub and N. Zuber, Nucl. Sci. Eng., 30, 296 (1967)

13 T. Harmathy, AICHE Journal, 6, 281 (1960) , F.N. Peebles and H.J. Garber, Chem. Engr. Sci., 49, 88 (1953)

14 E.T. White and R.H. Beardmore, Chem. Engr. Sci., 17, 351 (1962)

15 N.Zuber, Sym. on Two-Phase Flow Dynamics, Eindhoven, Vol.1, p.107,1967

16 R.C. Martinelli and D.B. Nelson, Trans. ASME, 70, 695 (1948)

17 C.J. Baroczy, Chem. Eng. Progr. Symp. Ser., No. 64, 62, 232

18 D. Chisholm, Int. J. Heat Mass Transfer, 16, 347 (1973)

19 M. Muscettola, U.K.A.E.A. Reactor Group report AEEW-R284, H.M. Stationery Office (1963)

20 S. Levy, J. Heat Transfer, Trans. ASME, 82C, 113 (1960)

21 J.F. Marchaterre, J. Heat Transfer, Trans. ASME, 83C, 503 (1961)

22 H.S. Isbin, R.H. Moen and D.R. Mosher, AECU-2994 (1954)

23 N. Adorni, et al., CISE-R-31, CISE, Milano, 1961

Nomenclature: [MLT] System of Units with Q Defined by $Q = ML^2/T^2$

D	=	Hydraulic diameter (L)
f	=	Friction factor (-)
g	=	Acceleration due to gravity (L/T^2)
G	=	Mass flow rate (M/TL^2)
h	=	Enthalpy of subcooled liquid (Q/M)
h_{in}	=	h at the channel inlet (Q/M)
h_f	=	Enthalpy of saturated liquid (Q/M)
h_g	=	Enthalpy of saturated vapor (Q/M)
j	=	Velocity of center of volume (L/T)
p	=	Pressure (M/T^2L)
p_o	=	System pressure (M/T^2L)
p_{in}	=	p at the channel inlet (M/T^2L)
q	=	Heat input (Q/TL^3)
u	=	Velocity of subcooled liquid (L/T)
u_g	=	Velocity of saturated vapor (L/T)
u_f	=	Velocity of saturated liquid (L/T)
V_{gj}	=	Vapor drift velocity with respect to j (L/T)
α	=	Void fraction (-)
Γ	=	Rate of mass formation per unit volume (M/TL^3)
σ	=	Surface tension (M/T^2)
ρ	=	Density of two-phase mixture (M/L^3)
ρ_f	=	Liquid density (saturated and subcooled) (M/L^3)
ρ_g	=	Vapor density (M/L^3)
τ_h	=	Time constant defined by Eq. (III-9) (T)

τ = Time constant defined by Eq. (Ⅳ-10) (T)

μ_f = Viscosity of liquid (M/LT)

$\frac{\partial \sigma_f}{\partial z}$ = Frictional pressure gradient (M/T^2L^2)

Δp = Pressure drop (M/T^2L)

subscript

in = channel inlet

m = two-phase mixture

f = liquid

g = vapor

o = system