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TWO-DIMENSIONAL SIMULATION OF THE MHD STABILITY(I)

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Two-dimensional Simulation of the MHD Stability (I)

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The two-dimensional computer code has been prepared to study MHD stability of an axisymmetric toroidal plasma with and without the surrounding vacuum region. It also includes the effect of magnetic surfaces with non-circular cross sections.

The linearized equations of motion are solved as an initial value problem.

The results by computer simulation are compared with those by the theory for the cylindrical plasma; they are in good agreement.

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2次元のMHD安定性のシミュレーション（I）

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軸対称トロイダルプラズマのMHD安定性を、外部の真空領域が存在する場合としない場合について調べるために、2次元の計算機コードを開発した。コードは、非円形断面の効果も含んでいる。

線形化された運動方程式を、初期値問題として解いた。

円筒プラズマについて、計算機シミュレーションの計算結果を解析解と比較したところ、よい一致が得られた。

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目 次 な し

§.1 Introduction

Many theoretical works have been devoted to investigate the ideal MHD stability in cylindrical geometry^{1,2)} and also in toroidal one^{3,4)}, since it plays a crucial role for fusion applications. Further, in view of present tokamak experiments, a great deal of attention has been paid to low m-number internal mode, and it is important to investigate these stability theoretically.

Analytical calculation is a very powerful means, but it is restricted to the case of a rather specific class of plasma configurations. So in order to investigate the stability of more realistic plasmas, it is necessary to invoke the computer simulation, which will make possible to solve non-linear three-dimensional problem. Several works have done for the purpose of this goal^{5,6,7,8)}.

In this paper we develop the two-dimensional code to investigate the stability of axisymmetric toroidal plasma against internal and external kink modes including the effect of non-circular cross section with arbitrary plasma current distributions.

The linearized equations of motion are solved in curvilinear coordinate as an initial value problem.

As to the difference scheme, we use simple explicit one to avoid the complexity of program.

In §.2, the equilibrium is obtained for tokamak plasmas. The equilibrium equation is expanded in powers of the inverse aspect ratio. In §.3, the basic equations for the ideal MHD stability are given. In §.4, the boundary conditions for both internal and external kink modes, the former corresponds to fixed boundary and the latter is free boundary, are described. The numerical procedure are given briefly in §.5. In §.6, the growth rates calculated by the present two-dimensional code are compared with the analytical ones for the cylindrical case and a fairly good agreement is obtained between the two.

§.2 Equilibrium

In this section, the equilibrium solutions for tokamak plasma are given.

We consider a toroidal plasma with large aspect ratio, $R/a \gg 1$, where R and a denote the major and the minor radii, respectively.

The MHD equilibrium equations are

$$\vec{\nabla}P_0 = j_0 \times B_0 , \quad (2.1)$$

$$j_0 = \vec{\nabla} \times B_0 ,$$

We use the curvilinear coordinate system (r, θ, ϕ) , where the surfaces of constant r correspond to the magnetic surfaces^{9,10}. (See Fig. 1)

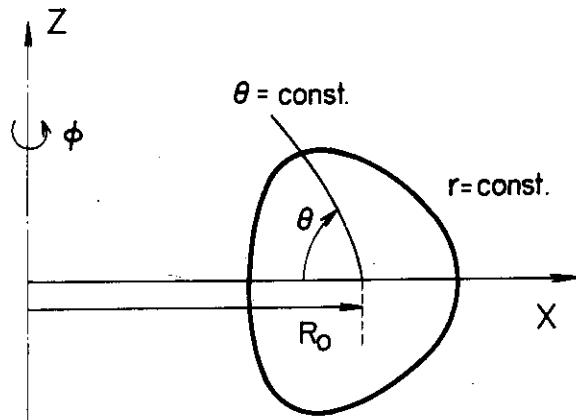


Fig. 1 Curvilinear coordinate (r, θ, ϕ) .

The closed curve, $r=\text{const.}$, is the cross section of a magnetic surface. R_0 is the length of the major radius and the magnetic axis is at $r=0$ ($X=R_0$, $Z=0$)

The transformation between (r, θ, ϕ) coordinate system and cylindrical (X, Φ, Z) system can be written formally as

$$\begin{aligned} X &= X(r, \theta) & , \\ Z &= Z(r, \theta) & , \\ \Phi &= \phi & . \end{aligned} \quad (2.2)$$

The metric tensors of (r, θ, ϕ) system are defined as follows;

$$\begin{aligned} g_{rr} &= \left(\frac{\partial X}{\partial r}\right)^2 + \left(\frac{\partial Z}{\partial r}\right)^2 , \\ g_{r\theta} &= \frac{\partial X}{\partial r} \frac{\partial X}{\partial \theta} + \frac{\partial Z}{\partial r} \frac{\partial Z}{\partial \theta} , \\ g_{\theta\theta} &= \left(\frac{\partial X}{\partial \theta}\right)^2 + \left(\frac{\partial Z}{\partial \theta}\right)^2 , \\ g_{\phi\phi} &= X^2 \\ \sqrt{g} &= X \left(\frac{\partial X}{\partial \theta} \frac{\partial Z}{\partial r} - \frac{\partial X}{\partial r} \frac{\partial Z}{\partial \theta} \right) , \\ g^{rr} &= \frac{g_{\theta\theta} g_{\phi\phi}}{g} , \end{aligned} \quad (2.3)$$

$$g^{r\theta} = - \frac{g_{r\theta} g_{\phi\phi}}{g},$$

$$g^{\phi\phi} = \frac{1}{x^2}.$$

We consider equilibrium magnetic field in the following form,

$$\vec{B}_0 = RB_0(f(r)\vec{\nabla}\phi \times \vec{\nabla}r + h(r)\vec{\nabla}\phi), \quad (2.4)$$

where B_0 is constant magnetic field magnitude introduced to make $f(r)$ and $h(r)$ dimensionless. The first term of eq. (2.4) represents the poloidal magnetic field and the second is the toroidal field.

Then, the equilibrium equation (2.1) can be written in (r, θ, ϕ) system as follows,

$$\frac{P'_0(r)}{RB_0^2} + \frac{h(r)h'(r)}{g_{\phi\phi}} + \frac{f(r)}{\sqrt{g}} \left\{ \left(\frac{f(r)}{\sqrt{g}} g_{\theta\theta} \right)' - \frac{\partial}{\partial \theta} \left(\frac{f(r)}{\sqrt{g}} g_{r\theta} \right) \right\} = 0, \quad (2.5)$$

where prime denotes the derivative with respect to r . Once the equilibrium plasma pressure distribution $P_0(r)$ and plasma current distribution $f(r)$ are given, the other equilibrium quantities such as $\Delta(r)$, the displacement of the magnetic surface from the magnetic axis, are determined by solving eq. (2.5) in the inverse aspect ratio expansion.

§.3 Basic Equations

In this section, the basic equations for the ideal MHD stability are described and are rewritten in the form convenient for the numerical calculation.

We adopt the linearized hydromagnetic equations in the following form.

$$\begin{aligned} p_0 \frac{\partial^2 \vec{\xi}}{\partial t^2} &= - \vec{\nabla} P + \vec{j}_0 \times \vec{Q} + \vec{j} \times \vec{B}_0, \\ \vec{j} &= \vec{\nabla} \times \vec{Q}; \\ \vec{Q} &= \vec{\nabla} \times (\vec{\xi} \times \vec{B}_0), \\ P &= -\gamma P_0 \vec{\nabla} \cdot \vec{\xi} - (\vec{\xi} \cdot \vec{\nabla}) P_0, \end{aligned} \quad (3.1)$$

where the quantities with subscript 0 are the equilibrium ones, γ is the

specific heat ratio and \vec{j} , \vec{Q} and P are perturbed current density, magnetic field magnitude and plasma pressure, respectively.

For the sake of convenience of calculation the current densities, both equilibrium and perturbed current densities, are transformed as;

$$\sqrt{g}j \rightarrow j, \quad (3.2)$$

Eqs. (3.1) are rewritten in each vector components are follows;

$$\begin{aligned} \rho_0 \frac{\partial^2 \xi_r}{\partial t^2} &= - \frac{\partial P}{\partial r} + (j^\theta B^\phi - j^\phi B^\theta) + (j_0^\theta Q^\phi - j_0^\phi Q^\theta), \\ \rho_0 \frac{\partial^2 \xi_\theta}{\partial t^2} &= - \frac{\partial P}{\partial \theta} + (j_0^\phi Q^r - j^r B^\phi), \\ \rho_0 \frac{\partial^2 \xi_\phi}{\partial t^2} &= - \frac{\partial P}{\partial \phi} + (j^r B^\theta - j_0^\theta Q^r), \\ Q^r &= B^\theta \frac{\partial \xi^r}{\partial \theta} + B^\phi \frac{\partial \xi^r}{\partial \phi}, \\ Q^\theta &= B^\phi \frac{\partial \xi^\theta}{\partial \phi} - B^\theta \frac{\partial \xi^\phi}{\partial \phi} - \frac{1}{\sqrt{g}} \frac{\partial}{\partial r} (\sqrt{g} B^\theta \xi^r), \\ Q^\phi &= B^\theta \frac{\partial \xi^\phi}{\partial \theta} - \frac{1}{\sqrt{g}} \left\{ \frac{\partial}{\partial r} (\sqrt{g} B^\phi \xi^r) + \frac{\partial}{\partial \theta} (\sqrt{g} B^\phi \xi^\theta) \right\}, \\ j^r &= \frac{\partial Q_\phi}{\partial \theta} - \frac{\partial Q_\theta}{\partial \phi}, \\ j^\theta &= \frac{\partial Q_r}{\partial \phi} - \frac{\partial Q_\phi}{\partial r}, \\ j^\phi &= \frac{\partial Q_\theta}{\partial r} - \frac{\partial Q_r}{\partial \theta}, \\ P &= -\xi^r \frac{\partial P_0}{\partial r} - \frac{\gamma P_0}{\sqrt{g}} \left\{ \frac{\partial}{\partial r} (\sqrt{g} \xi^r) + \frac{\partial}{\partial \theta} (\sqrt{g} \xi^\theta) + \frac{\partial}{\partial \phi} (\sqrt{g} \xi^\phi) \right\}, \end{aligned} \quad (3.3)$$

where the quantities with superscript and subscript mean the contravariant and the covariant components, respectively.

Since each perturbed quantity may have different phases in the ϕ direction, they must be divided into even and odd parts with respect to ϕ and are expressed as,

$$\xi^r(r, \theta, \phi) = \xi^{re}(r, \theta) \cos(n\phi) - \xi^{ro}(r, \theta) \sin(n\phi),$$

$$\xi^\theta(r, \theta, \phi) = \xi^{\theta e}(r, \theta) \cos(n\phi) + \xi^{\theta o}(r, \theta) \sin(n\phi),$$

$$\xi^\phi(r, \theta, \phi) = \xi^{\phi e}(r, \theta) \cos(n\phi) + \xi^{\phi o}(r, \theta) \sin(n\phi),$$

$$\begin{aligned}
Q^r(r, \theta, \phi) &= Q^{re}(r, \theta) \cos(n\phi) + Q^{ro}(r, \theta) \sin(n\phi), \\
Q^\theta(r, \theta, \phi) &= Q^{\theta o}(r, \theta) \cos(n\phi) - Q^{\theta e}(r, \theta) \sin(n\phi), \\
Q^\phi(r, \theta, \phi) &= Q^{\phi o}(r, \theta) \cos(n\phi) - Q^{\phi e}(r, \theta) \sin(n\phi), \\
j^r(r, \theta, \phi) &= j^{re}(r, \theta) \cos(n\phi) + j^{ro}(r, \theta) \sin(n\phi), \\
j^\theta(r, \theta, \phi) &= j^{\theta o}(r, \theta) \cos(n\phi) - j^{\theta e}(r, \theta) \sin(n\phi), \\
j^\phi(r, \theta, \phi) &= j^{\phi o}(r, \theta) \cos(n\phi) - j^{\phi e}(r, \theta) \sin(n\phi), \\
P(r, \theta, \phi) &= P^o(r, \theta) \cos(n\phi) - P^e(r, \theta) \sin(n\phi),
\end{aligned} \tag{3.4}$$

where n is the wavenumber in the ϕ direction. The covariant components have the same form as in eqs. (3.4).

Substituting eqs. (3.4) into eqs. (3.3), we obtain the system of equations in the following form,

$$\begin{aligned}
\rho_0 \frac{\partial^2 \xi_r^0}{\partial t^2} &= - \frac{\partial P^o}{\partial r} + (j^{\theta o} B^\phi - j^{\phi o} B^\theta) + (j_0^\theta Q^{\phi o} - j_0^\phi Q^{\theta o}), \\
\rho_0 \frac{\partial^2 \xi_r^e}{\partial t^2} &= - \frac{\partial P^e}{\partial r} + (j^{\theta e} B^\phi - j^{\phi e} B^\theta) + (j_0^\theta Q^{\phi e} - j_0^\phi Q^{\theta e}), \\
\rho_0 \frac{\partial^2 \xi_\theta^e}{\partial t^2} &= - \frac{\partial P^o}{\partial \theta} + j_0^\phi Q^{re} - j^{re} B^\phi, \\
\rho_0 \frac{\partial^2 \xi_\theta^0}{\partial t^2} &= \frac{\partial P^e}{\partial \theta} + j_0^\phi Q^{ro} - j^{ro} B^\phi, \\
\rho_0 \frac{\partial^2 \xi_\phi^e}{\partial t^2} &= n P^e + j^{re} B^\theta - j_0^\theta Q^{re}, \\
\rho_0 \frac{\partial^2 \xi_\phi^0}{\partial t^2} &= n P^o + j^{ro} B^\theta - j_0^\theta Q^{ro}, \\
Q^{re} &= B^\theta \frac{\partial \xi^{ro}}{\partial \theta} - n B^\phi \xi^{re}, \quad Q^{ro} = -B^\theta \frac{\partial \xi^{re}}{\partial \theta} - n B^\phi \xi^{ro}, \\
Q^{\theta o} &= n(B^\phi \xi^{\theta o} - B^\theta \xi^{\phi o}) - \frac{1}{\sqrt{g}} \frac{\partial}{\partial r} (\sqrt{g} B^\theta \xi^{ro}), \\
Q^{\theta e} &= n(B^\phi \xi^{\theta o} - B^\theta \xi^{\phi e}) - \frac{1}{\sqrt{g}} \frac{\partial}{\partial r} (\sqrt{g} B^\theta \xi^{re}), \\
Q^{\phi o} &= B^\theta \frac{\partial \xi^{\phi e}}{\partial \theta} - \frac{1}{\sqrt{g}} \{ \frac{\partial}{\partial r} (\sqrt{g} B^\phi \xi^{ro}) + \frac{\partial}{\partial \theta} (\sqrt{g} B^\phi \xi^{\theta e}) \}, \\
Q^{\phi e} &= -B^\theta \frac{\partial \xi^{\phi o}}{\partial \theta} - \frac{1}{\sqrt{g}} \{ \frac{\partial}{\partial r} (\sqrt{g} B^\phi \xi^{re}) - \frac{\partial}{\partial \theta} (\sqrt{g} B^\phi \xi^{\theta o}) \}, \\
j^{re} &= \frac{\partial Q_\phi^0}{\partial \theta} + n Q_\theta^e, \quad j^{ro} = \frac{\partial Q_\phi^e}{\partial \theta} + n Q_\theta^0,
\end{aligned} \tag{3.5}$$

$$\begin{aligned} j^{\theta o} &= n Q_r^o - \frac{\partial Q_\phi^o}{\partial r} , & j^{\theta e} &= n Q_r^e - \frac{\partial Q_\phi^e}{\partial r} , \\ j^{\phi o} &= \frac{\partial Q_\theta^o}{\partial r} - \frac{\partial Q_r^e}{\partial \theta} , & j^{\phi e} &= \frac{\partial Q_\theta^e}{\partial r} + \frac{\partial Q_r^o}{\partial \theta} , \\ P^o &= -\xi^{ro} \frac{\partial P_0}{\partial r} - \frac{\gamma P_0}{\sqrt{g}} \left\{ \frac{\partial}{\partial r} (\sqrt{g} \xi^{ro}) + \frac{\partial}{\partial \theta} (\sqrt{g} \xi^{\theta o}) \right\} - \gamma P_0 n \xi^{\phi o}, \\ P^e &= -\xi^{re} \frac{\partial P_0}{\partial r} - \frac{\gamma P_0}{\sqrt{g}} \left\{ \frac{\partial}{\partial r} (\sqrt{g} \xi^{re}) - \frac{\partial}{\partial \theta} (\sqrt{g} \xi^{\theta o}) \right\} - \gamma P_0 n \xi^{\phi e}. \end{aligned}$$

Here the covariant components of vectors are related to contravariant components through the metric tensors shown in eqs. (2.3).

§.4 Boundary Conditions

In this section we impose the boundary conditions to eqs. (3.5) for both internal and external kink modes.

In order to avoid the difference in boundary condition at $r=0$ between the $m=1$ (m is a wavenumber in the θ direction) mode and the other modes, we transform ξ^r and Q^r as follows;

$$\begin{aligned} r\xi^r &\longrightarrow \xi^r , \\ rQ^r &\longrightarrow Q^r . \end{aligned} \tag{4.1}$$

Then the boundary conditions at $r=0$ become

$$\begin{aligned} \xi^r &= 0 , \\ Q^r &= 0 , \end{aligned} \tag{4.2}$$

for every case.

In the case of fixed boundary, only boundary conditions at plasma surface ($r=a$) are required and are the same as eq. (4.2).

For free boundary problem, we must obtain the perturbed magnetic field in the vacuum region. In this paper it is obtained analytically in the toroidal coordinate system (μ, η, ϕ) using the toroidal ring functions.

(See Fig. 2)

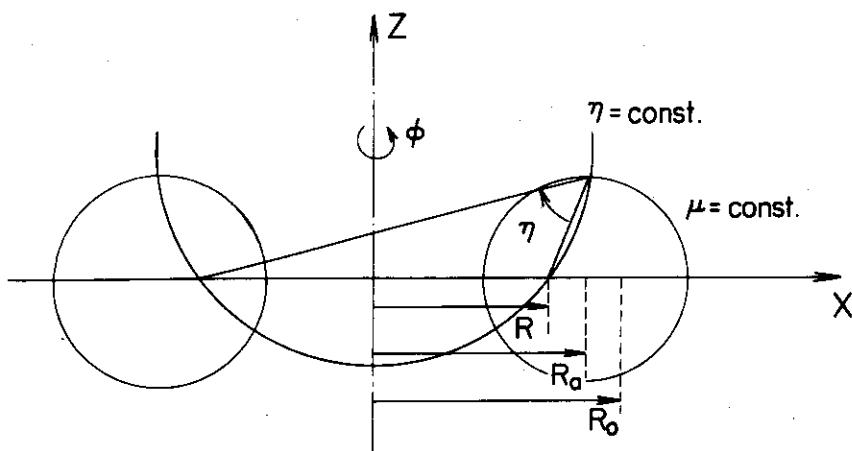


Fig. 2 Toroidal coordinate (μ, η, ϕ) used for the calculation of vacuum magnetic field. R_a is the distance of the center of circular plasma surface and R , the singular point of toroidal coordinate, is expressed by $R=\sqrt{R_a^2-a^2}$

The toroidal coordinate (μ, η, ϕ) are related to the cylindrical coordinate (X, ϕ, Z) through the following relations,

$$\begin{aligned}\mu &= \frac{1}{2} \ln \frac{(X+R)^2 + Z^2}{(X-R)^2 + Z^2}, \\ \eta &= \tan^{-1} \frac{2RZ}{X^2 + Z^2 - R^2},\end{aligned}\quad (4.3)$$

and the metric coefficients are written as follows;

$$\begin{aligned}g_\mu &= g_\eta = \frac{R}{\cosh \mu - \cos \eta}, \\ g_\phi &= \frac{R \sinh \mu}{\cosh \mu - \cos \eta},\end{aligned}\quad (4.4)$$

The general solution of the Laplace's equation in the toroidal coordinate Ψ_e can be written as follows;

$$\begin{aligned}\Psi_e &= \sqrt{x-\cos \eta} \sum_{k,n} \left[\{A_k P_{k-1/2}^n(x) + B_k Q_{k-1/2}^n(x)\} \cos(k\eta) \cos(n\phi) \right. \\ &\quad \left. + \{C_k P_{k-1/2}^n(x) + D_k Q_{k-1/2}^n(x)\} \sin(k\eta) \sin(n\phi) \right],\end{aligned}\quad (4.5)$$

where $x=\cosh \mu$, $P_{k-1/2}^n(x)$ and $Q_{k-1/2}^n(x)$ are the toroidal ring functions.

In eq. (4.5), the terms $\cos(k\eta) \sin(n\phi)$ and $\sin(k\eta) \cos(n\phi)$ are omitted by considering the form of the internal solutions (3.4).

Since $\vec{Q}_{ex} = \vec{\nabla} \Psi_e$, the perturbed magnetic field in the vacuum region can be written as follows;

$$\begin{aligned}
 Q_\mu &= \sum_{k,n} [(X_A \cdot A_k + X_B \cdot B_k) \cos(n\phi) + (X_C \cdot C_k + X_D \cdot D_k) \sin(n\phi)] , \\
 Q_\eta &= \sum_{k,n} [(Y_A \cdot A_k + Y_B \cdot B_k) \cos(n\phi) + (Y_C \cdot C_k + Y_D \cdot D_k) \sin(n\phi)] , \\
 Q_\phi &= \sum_{k,n} [(Z_C \cdot C_k + Z_D \cdot D_k) \cos(n\phi) + (Z_A \cdot A_k + Z_B \cdot B_k) \sin(n\phi)] .
 \end{aligned} \tag{4.6}$$

Here A_k , B_k , C_k , and D_k are constants to be determined from the boundary condition and X_A , X_B , X_C , X_D , etc. are defined as follows;

$$\begin{aligned}
 X_A &= \frac{\sinh \mu}{g_\mu \sqrt{y}} \left\{ \frac{P_{k-1/2}^n(x)}{2} + y P_{k-1/2}^{n'}(x) \right\} \cos(k\eta) , \\
 X_B &= \frac{\sinh \mu}{g_\mu \sqrt{y}} \left\{ \frac{Q_{k-1/2}^n(x)}{2} + y Q_{k-1/2}^{n'}(x) \right\} \cos(k\eta) , \\
 X_C &= \frac{\sinh \mu}{g_\mu \sqrt{y}} \left\{ \frac{P_{k-1/2}^n(x)}{2} + y P_{k-1/2}^{n'}(x) \right\} \sin(k\eta) , \\
 X_D &= \frac{\sinh \mu}{g_\mu \sqrt{y}} \left\{ \frac{Q_{k-1/2}^n(x)}{2} + y Q_{k-1/2}^{n'}(x) \right\} \sin(k\eta) , \\
 Y_A &= \frac{P_{k-1/2}^n(x)}{g_\eta \sqrt{y}} \frac{\sin \eta \cos(k\eta)}{2} - k_y \sin(k\eta) , \\
 Y_B &= \frac{Q_{k-1/2}^n(x)}{g_\eta \sqrt{y}} \frac{\sin \eta \cos(k\eta)}{2} - k_y \sin(k\eta) , \\
 Y_C &= \frac{P_{k-1/2}^n(x)}{g_\eta \sqrt{y}} \frac{\sin \eta \sin(k\eta)}{2} + k_y \cos(k\eta) , \\
 Y_D &= \frac{Q_{k-1/2}^n(x)}{g_\eta \sqrt{y}} \frac{\sin \eta \sin(k\eta)}{2} + k_y \cos(k\eta) , \\
 Z_A &= -\frac{n\sqrt{y}}{g_\phi} P_{k-1/2}^n(x) \cos(k\eta) , \quad Z_B = -\frac{n\sqrt{y}}{g_\phi} Q_{k-1/2}^n(x) \cos(k\eta) , \\
 Z_C &= \frac{n\sqrt{y}}{g_\phi} P_{k-1/2}^n(x) \sin(k\eta) , \quad Z_D = \frac{n\sqrt{y}}{g_\phi} Q_{k-1/2}^n(x) \sin(k\eta) ,
 \end{aligned} \tag{4.7}$$

where $x = \cos \mu$, $y = x - \cos \eta$ and the prime denotes the derivative with respect to x .

Using the perturbed magnetic field obtained above, we adopt the following boundary conditions,

- A) The normal component of the magnetic field vanishes at the perfectly conducting shell ($r=b$), that is

$$\vec{Q}_{ex} \cdot \vec{e}_r = 0 \quad . \quad (4.8)$$

B) The continuity of the normal component of the perturbed magnetic field at plasma surface ($r=a$),

$$\vec{Q}_{ex} \cdot \vec{e}_r = Q_{in}^r \quad . \quad (4.9)$$

C) The pressure balance equation at the plasma surface ($r=a$ ¹¹),

$$-\gamma P_0 \nabla \cdot \vec{\xi} + \vec{B}_{in} \cdot \vec{Q}_{in} = \vec{B}_{ex} \cdot \vec{Q}_{ex} + \frac{\xi_r}{2} \left(\frac{dB_{ex}^2}{dr} - \frac{dB_{in}^2}{dr} \right) \quad , \quad (4.10)$$

where

$$\vec{Q}_{ex} = Q_\mu \vec{u}_\mu + Q_\eta \vec{u}_\eta + Q_\phi \vec{u}_\phi \quad . \quad (4.11)$$

Here u_μ , u_η and u_ϕ are the unit vectors in the toroidal coordinate (μ, η, ϕ), respectively.

§.5 Numerical Procedure

In this section, the numerical procedure used in the present two-dimensional code are described briefly.

All variables are normalized by the following characteristic quantities;

- 1) minor radius of a plasma ring: a ,
- 2) toroidal magnetic field at $r=a$: B_0 ,
- 3) plasma density averaged over minor cross section: $\langle \rho \rangle$,
- 4) Alfvén velocity measured by B_0 : $v_f = B_0 / \langle \rho \rangle$,

and other associated normalization factors are time: $\tau = a/v_f$, pressure: B_0^2 and the growth rate: τ^{-1} .

As to the difference scheme, we adopt a simple explicit scheme to avoid the complexity of program.

We use the growing parameter α^6), which is very useful to obtain marginal stability condition.

In the r direction, we use half integral meshes for ξ^θ , ξ^ϕ , Q^ϕ , j^r and P and integral meshes are used for the other variables. On the other hand, only integral meshes are used in the θ direction. Considering the symmetry of upper and lower plane of minor cross section, we calculate the solution

only in upper half plane. (See Fig. 3).

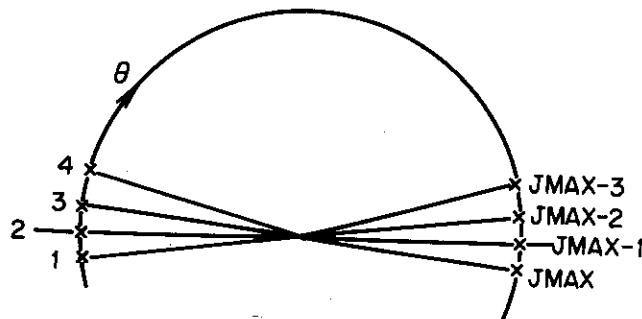


Fig. 3 Grid points in the θ direction. The grid points of $J=2$ and $J=JMAX-1$ represent $\theta=0$ and $\theta=\pi$, respectively.

§.6 Numerical Results for Cylindrical Plasmas

As the first step to check the validity and accuracy of the code, we have calculated the growth rates of MHD instability in cylindrical plasma by use of our two-dimensional code and compared them with the analytical results of Shafranov.

The growth rate of kink mode for cylindrical plasma given by Shafranov is¹⁾

$$\gamma^2 = \frac{B_a^2}{a^2} [2(m-nq_a) - \frac{2(m-nq_a)^2}{1-(a/b)^2}] , \quad (6.1)$$

where B_a and q_a are poloidal magnetic field and so-called safety factor at plasma surface ($r=a$), m and n are poloidal and toroidal wavenumber, and b is the radius of the conductive shell, respectively.

The cylindrical plasma configuration used for the numerical calculation are as follows;

$$\begin{aligned} f(r) &= r f_a & , \\ P_0(r) &= \beta_p f_a^2 (1-r^2) & , \\ h(r) &= \sqrt{1 - 2f_a^2(1-\beta_p)(1-r^2)} & , \end{aligned} \quad (6.2)$$

where $f_a = f(a)$ and β_p is poloidal beta.

The boundary condition for cylindrical case is the same as toroidal one, except that the perturbed magnetic field in the vacuum region is expressed by the modified Bessel functions.

The growth rates calculated by the two-dimensional code are shown in Fig. 5. It is seen that numerical growth rates almost agree with analytical results. To obtain this agreement, the number of grid points used in the θ direction in the upper half plane are 19 for the case of $m=2$ mode, 35 grid points are used for $m=3$ and $m=4$ modes.

For higher m modes, more and more mesh points in θ are needed to obtain good agreement with the analytical results.

In the r direction, 11 grid points are used.

The eigen functions almost agree with the analytical ones.

Fig. 6 shows the perturbed pressure distributions for the $m=2$ and $m=3$ modes.

The code is now being used to calculate the case of toroidal plasma configuration with circular cross section.

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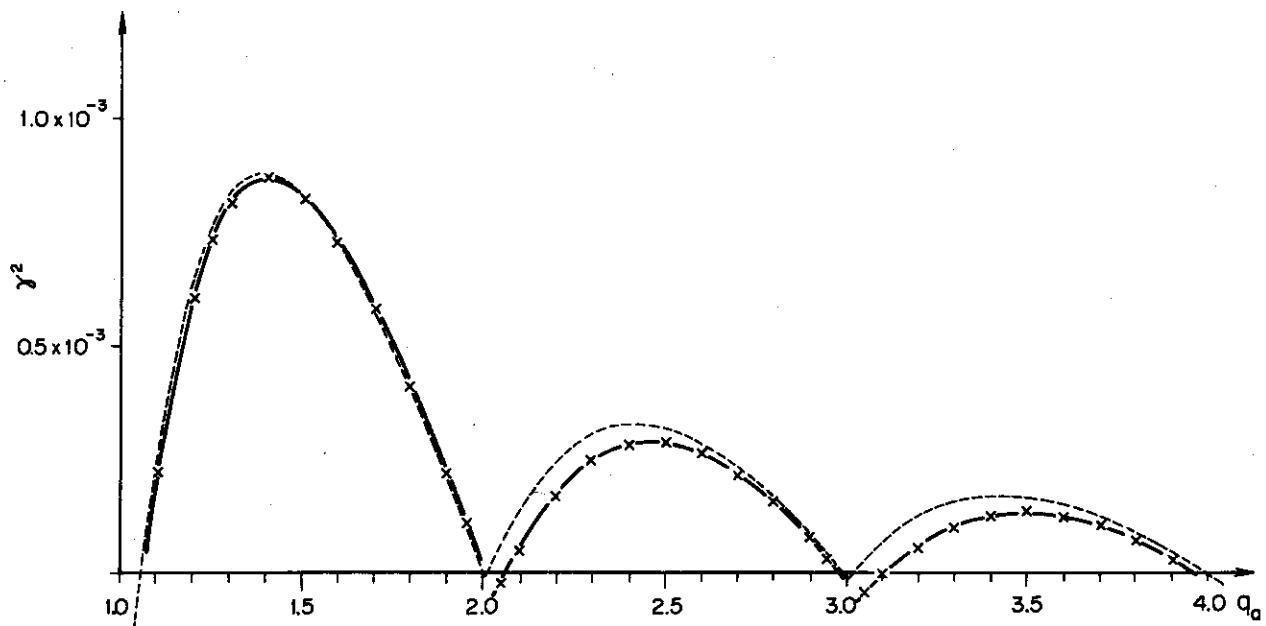


Fig. 4 Graph of normalized growth rate calculated by one-dimensional code versus safety factor at plasma surface $r=a$. Dotted curves are analytical growth rate obtained from eq. (6.1).

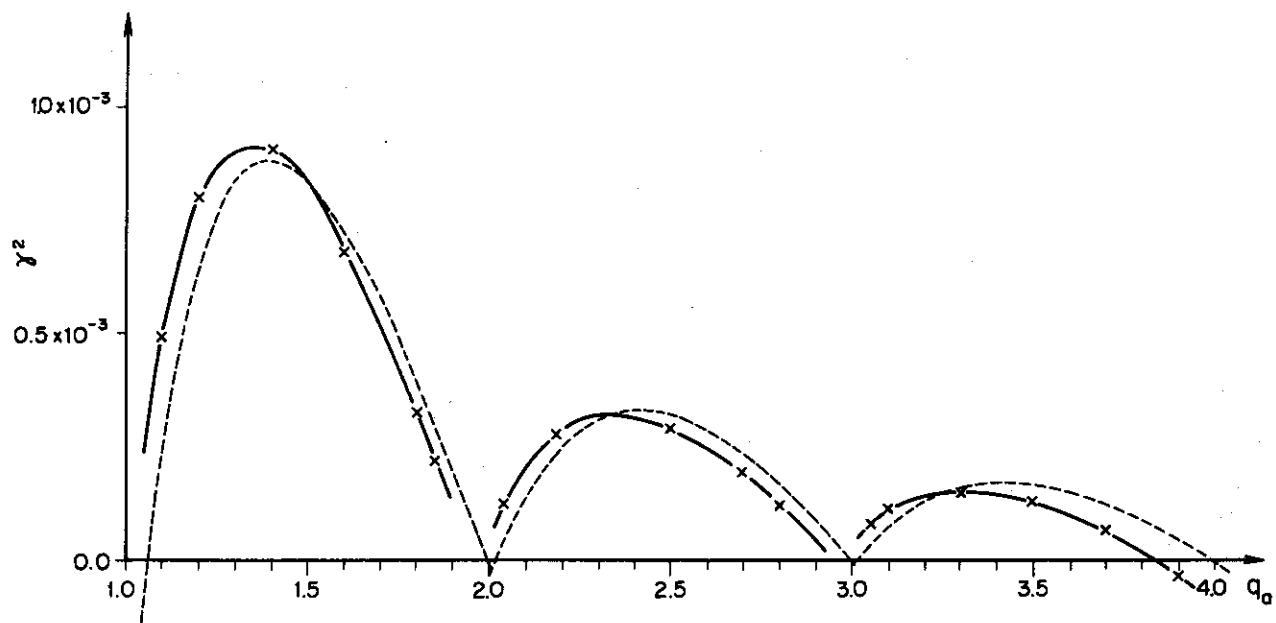


Fig. 5 Graph of normalized growth rate calculated by two-dimensional code versus safety factor at plasma surface $r=a$. Dotted curves are analytical growth rate obtained from eq. (6.1).

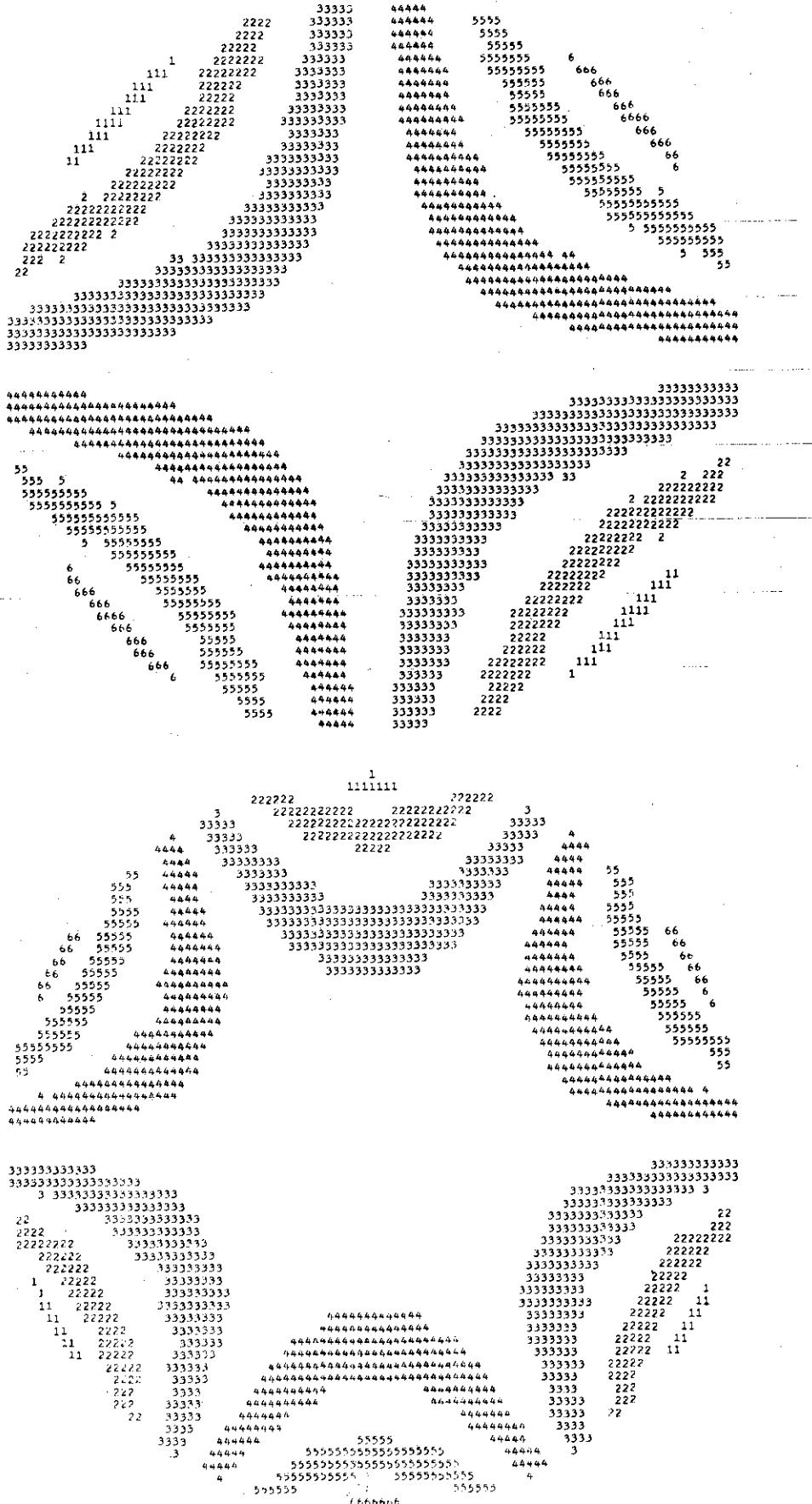


Fig. 6 Perturbed plasma pressure distribution for
 (1) $m=2$ mode and (2) $m=3$ mode.

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