JAERI-M
6536

AN INTERPOLATION METHOD FOR THE RESPONSE FUNCTION OF RADIATIONS OBTAINED BY PROTON RECOIL TYPE SCINTILLATORS

May 1976

Nobuo SASAMOTO

日本原子力研究所
Japan Atomic Energy Research Institute

この報告書は，日本原子力研究所が JAERI-M レポートとして，不定期に刊行している研究報告書です．入手，複製などのお問合わせは，日本原子力研究所技術情報部（茨城県那珂郡東海村）あて，お申しこしください．

JAERI-M reports, issued irregularly, describe the results of research works carried out in JAERI. Inquiries about the availability of reports and their reproduction should be addressed to Division of Technical Information, Japan Atomic Energy Research Institute, Tokai-mura, Naka-gun, Ibaraki-ken, Japan.

JAERI - M 6536

An Interpolation Method for the Response Function of Radiations obtained by Proton Recoil Type Scintillators

Nobuo SASAMOTO
Division of Reactor Engineering, Tokai, JAERI

(Received April 19, 1976)

A new method is proposed for obtaining the interpolated response function for any radiations measured by the proton recoil type scintillators. This is the method that the interpolation be performed using the reference response functions which are in advance normalized so as their cutoff pulse height agree each other. This method is found to be useful for interpolating the response functions, especially in the vicinity of the cutoff pulse height.

JAERI−M 6536

反跳陽子シンチレータの応答関数の内挿法

日本原子力研究所東海研究所原子炉工学部
笹 本 宣 雄

（1976年4月19日受理）

シンチレータを用いて得られる放射線の応答関数から，精度よいスペクトルを求めるための新しい内挿法について述べた。本方法は関数のカットオフ波高値がすべて一致するように規格化された応答関数を基準として内挿計算を行うものであり，特に関数変化の急激なカットオフ波高値近傍での内挿に有効であることが明らかになった。

目　次　な　し

1. Introduction

To measure the radiation spectrum by a method with which the spectrum cannot be directly obtained, the unfolding procedure is necessary. We encounter the problem in neutron spectroscopy using a proton recoil type scintillator. In the unfolding method, information of the detector response to the monoenergetic radiation is required, which is represented in the form of a matrix. The energy response is usually obtained by Monte Carlo calculation supported by experiment for several specified neutron energies. At present, only few response matrices are usually available, which are given by Verbinski et al.[1,2] for several sizes of NE213 liquid scintillators.*) In principle, the pulse height mesh points of the matrix must be coincident with those of the pulse height spectrum measured. The energy mesh points must be selected according to the energy region of the expected spectrum and the resolution of the scintillator. So, one must prepare his own response matrix corresponding to the experimental setup, by an interpolation procedure using a reference response matrix such as the one by Verbinski et al., or otherwise he must process the measured pulse height spectrum so its pulse height mesh points will be fit to those of the existing response matrix. In both cases, the interpolation procedure is necessary. In general, the energy response for monoenergetic radiation (response function), a set of which constitutes the response matrix, has the maximum pulse height (cutoff pulse height) beyond which the response is uniformly zero.

*) made by Nuclear Enterprises Ltd.

Therefore, it is seen that the interpolation of the response function with respect to radiation energy is meaningless in the region between two cutoff pulse heights of the response functions for lower and higher neutron energies. Actually, the interpolated function in this region is evidently not connected smoothly at the lower cutoff mesh with the response function below this pulse height mesh. Considering the fact that the response function in the vicinity of the cutoff plays a most important role in unfolding the pulse height spectrum, utmost attention must be paid for the interpolation in such a region.

In this report, a new method to produce a interpolated response function is proposed. The response functions utilized for interpolation are all normalized so that their cutoff pulse heights agree to the common one. The interpolated response function is then obtained by an ordinary interpolation procedure using the normalized response functions, and the expected response function is obtained by reversing the interpolated function so that its cutoff may take the original value. To obtain the reasonable results by the above method, it is necessary that the normalized response functions be similar in shape. The Lagrange's formulae for the two and three points interpolations are used in each interpolation process.

2. Interpolation Method

In this section, we will discuss a procedure of obtaining the response function $R_x(p_j)$ of a radiation detector for radiation having energy E_x, using two kinds of response functions $R_1(p_j)$ and $R_2(p_j)$, presented by the dotted lines in

Fig.1, for the radiation energies E_1 and E_2, respectively. The pulse height mesh points p_j's are common to the response functions for any radiation energy. Here, the pulse height mesh point be represented by phmp. As shown in Fig.1, in the region between upper cutoff phmp $p_{1,up}$ and $p_{2,up}$, only $R_2(p_j)$ is useful for interpolation, since $R_1(p_j)$ is zero there. Therefore, the ordinary procedure is not suitable in this region. In order to avoid the problem, following procedure is proposed. As in Fig.2, two response functions used for interpolation are normalized so that their cutoff phmp may agree to a certain common pulse height p_{norm}, which may be set arbitrarily. The p_{norm} is set to be equal to the cutoff phmp of the available response function to the highest radiation energy of interest. This normalization is equivalent to the variable transformation with respect to the pulse height of each response function for any radiation energy. This transformation $R_1(p_j)$ is performed by setting the new phmp $q_{1,j}$ as follows:

$$q_{1,j} = p_j \frac{p_{norm}}{p_{1,up}} \tag{1}$$

The transformed phmp $q_{2,j}$ for $R_2(p_j)$ is similarly given as

$$q_{2,j} = p_j \frac{p_{norm}}{p_{2,up}} \tag{2}$$

Once the cutoff phmp for any radiation energy E_x is determined using both $p_{1,up}$ and $p_{2,up}$,

$$p_{x,up} = p_{1,up} \frac{E_x - E_2}{E_1 - E_2} + p_{2,up} \frac{E_x - E_1}{E_2 - E_1} , \tag{3}$$

the transformed phmp $q_{x,j}$ corresponding to the radiation energy

E_x is obtained using $p_{x,up}$ as

$$q_{x,j} = p_j \frac{p_{norm}}{p_{x,up}} \quad . \tag{4}$$

Let the normalized response functions of $R_1(p_j)$ and $R_2(p_j)$ be denoted by $\tilde{R}_1(q_{1,j})$ and $\tilde{R}_2(q_{2,j})$, respectively. Then, the normalized response function for any radiation energy E_x, $\tilde{R}_x(q_{x,j})$ will be obtained. Using the Lagrange's formula, $\tilde{R}_x(q_{x,j})$ is determined: (cf. Fig.2)

$$\tilde{R}_x(q_{x,j}) = \tilde{R}_1(q_{x,j}) \frac{E_x - E_2}{E_1 - E_2} + \tilde{R}_2(q_{x,j}) \frac{E_x - E_1}{E_2 - E_1} \quad , \tag{5}$$

where $\tilde{R}_1(q_{x,j})$ and $\tilde{R}_2(q_{x,j})$ must be determined prior to the computation of eq.(5).

Since $q_{x,j}$ does not always agree to any one of the sets of $q_{1,k}$ and $q_{2,k}$ $(k=1,2,\cdots)$, $\tilde{R}_1(q_{x,j})$ and $\tilde{R}_2(q_{x,j})$ must be obtained by the following interpolation formula:

$$\begin{aligned}\tilde{R}_1(q_{x,j}) = {} & \tilde{R}_1(q_{1,k}) \frac{q_{x,j} - q_{1,k+1}}{q_{1,k} - q_{1,k+1}} \cdot \frac{q_{x,j} - q_{1,k+2}}{q_{1,k} - q_{1,k+2}} \\ & + \tilde{R}_1(q_{1,k+1}) \frac{q_{x,j} - q_{1,k}}{q_{1,k+1} - q_{1,k}} \cdot \frac{q_{x,j} - q_{1,k+2}}{q_{1,k+1} - q_{1,k+2}} \\ & + \tilde{R}_1(q_{1,k+2}) \frac{q_{x,j} - q_{1,k}}{q_{1,k+2} - q_{1,k}} \cdot \frac{q_{x,j} - q_{1,k+1}}{q_{1,k+2} - q_{1,k+1}} \quad , \end{aligned} \tag{6}$$

where $q_{1,k}$, $q_{1,k+1}$ and $q_{1,k+2}$ are the nearest three phmp to $q_{x,j}$, on the transformed pulse height axis for radiation energy E_1. $R_2(q_{x,j})$ can also be calculated by the equation similar to

eq.(6) with the replacements of $\tilde{R}_1(q_{1,m})$ by $\tilde{R}_2(q_{2,m})$ (m=k,k+1,k+2) and $q_{1,m}$ by $q_{2,m}$.

Then, the response function $R_x(p_j)$ for any radiation energy is obtained by reversing the obtained $\tilde{R}_x(q_{x,j})$ using the following relations:

$$R_x(p_j) = \tilde{R}_x(q_{x,j}) = \tilde{R}_x(p_j \frac{p_{norm}}{p_{x,up}}) . \quad (7)$$

As an example, an interpolated function $R_x(p_j)$ obtained from $R_1(p_j)$ and $R_2(p_j)$ is shown in Fig.1.

As described previously, the phmp of response matrix must be in accordance with those of measured data. Therefore, it is necessary to calculate the response function $R_x(z)$ at any desired pulse height z. This is obtained using Lagrange's three points formula as follows:

$$R_x(z) = R_x(p_k) \frac{z - p_{k+1}}{p_k - p_{k+1}} \cdot \frac{z - p_{k+2}}{p_k - p_{k+2}} + R_x(p_{k+1}) \frac{z - p_k}{p_{k+1} - p_k} \cdot \frac{z - p_{k+2}}{p_{k+1} - p_{k+2}} + R_x(p_{k+2}) \frac{z - p_k}{p_{k+2} - p_k} \cdot \frac{z - p_{k+1}}{p_{k+2} - p_{k+1}} , \quad (8)$$

where p_k, p_{k+1} and p_{k+2} are the nearest three phmp to any pulse height z.

The flowchart is illustrated in Fig.3 of the interpolation procedure for the response function using Lagrange's formula. The computational code was programmed in order to give shape to

the above method. In the Appendix are included the description of the code : i) description of input data, ii) the list of the input data for sample problem presented in the next section and iii) the program list.

3. Sample Calculations and Discussion

The response functions for 11 specified neutron energies were interpolated by the proposed procedure, referring to the response matrix of a 2"x 2" diam NE213 liquid scintillator given by Verbinski et al. as reference data. The interpolated results are shown in Fig.4 together with the reference data. The neutron energies are so selected that they may take the mean value of each interval of the energy mesh points adopted in the Verbinski's matrix. In Table 1 are listed the neutron energies adopted by Verbinski et al. and those adopted in the present study. The pulse height mesh points are the same as those of the reference matrix.

As evident from Fig.4, the interpolated response functions are all smoothly varying functions up to the cutoff pulse height region. The rigorous examination of the validity for the calculated results is to compare them directly with the experimental results or the same results by Monte Carlo calculation, but the experiment of the Monte Carlo calculation is not always available. On the other hand, the proposed interpolation procedure requires the assumption that the normalized response functions utilized for interpolation be similar in shape. Neglecting this assumption, that is, using two response functions whose energies differ unreasonably,

makes the procedure itself meaningless. The intervals of neutron energy mesh points satisfying the assumption cannot be estimated quantitatively, but it is considered that the mesh intervals, at least, of the response matrix referred to in this study are small enough to satisfy the assumption. So, the interpolated results derived from such reference data are considered to be reasonable. However, this is only true for interpolation, but not for extrapolation. There is no difference in calculational procedure between the two because both use two responses which correspond to the nearest two energies to the neutron energy of interest. However, the extrapolated values are generally not so reliable as the interpolated ones, and besides, their errors cannot be estimated quantitatively. Therefore, it is not desirable to use an extrapolated response matrix for the unfolding method.

The present method may be extensively useful for obtaining the interpolated pulse height distributions for any radiation obtained by proton recoil type scintillators.

Acknowledgement

The author is indebted to S. Ichimori of Tokyo Institute of Technology for his helpful discussions in the early stage of this work. Thanks are also due to Y. Furuta for the valuable comments on the manuscript.

References

1) V. V. Verbinski et al., "The Response of Some Organic Scintillators to Fast Neutrons," Proc. Amer. Nucl. Soc., Special session Fast neutron spectroscopy, ANS-SD-2, 189, (1964).

2) V. V. Verbinski et al., "Calibration of an Organic Scintillator for Neutron Spectroscopy," Nucl. Instr. and Meth. 65 (1968) 8.

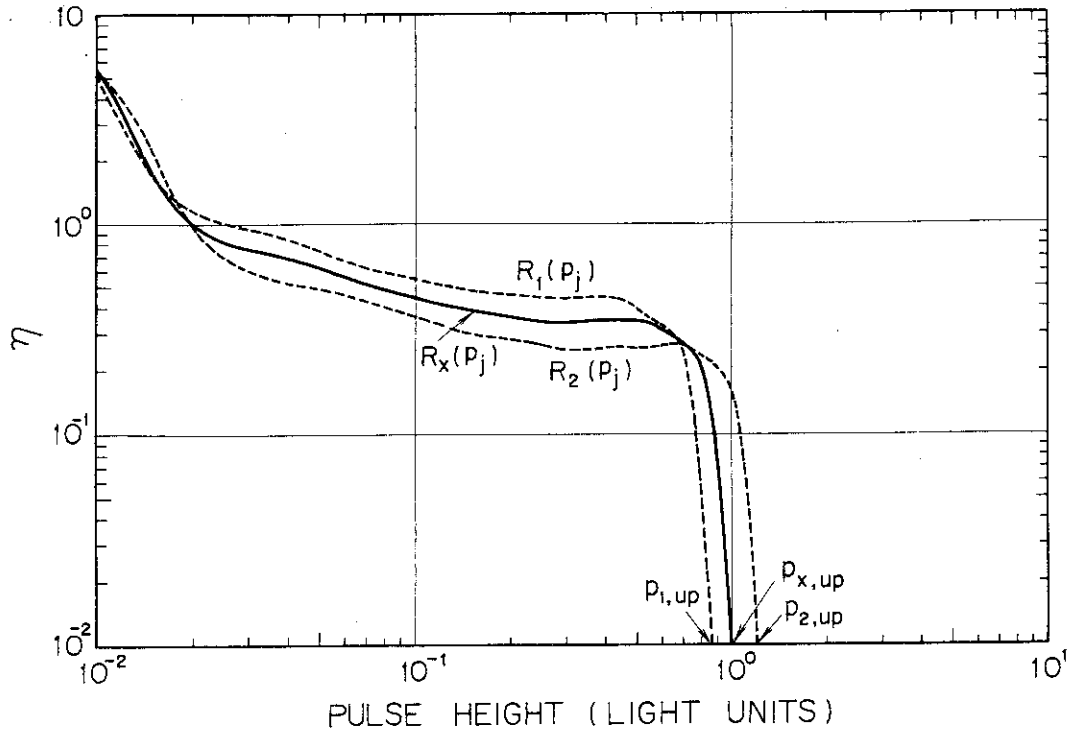

Fig. 1 The response functions on the original pulse height axis. $R_1(p_j)$ and $R_2(p_j)$ are the reference functions, and $R_x(p_j)$ is the interpolated one.

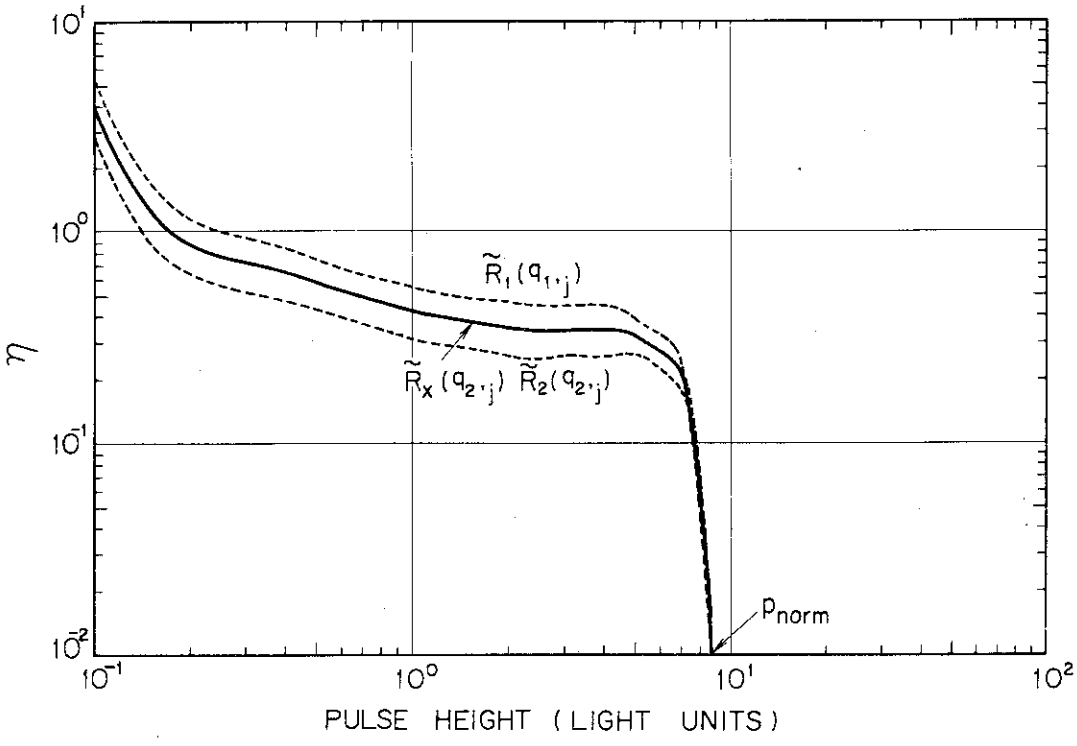

Fig. 2 The response functions on the transformed pulse height axis. (Normalized functions).

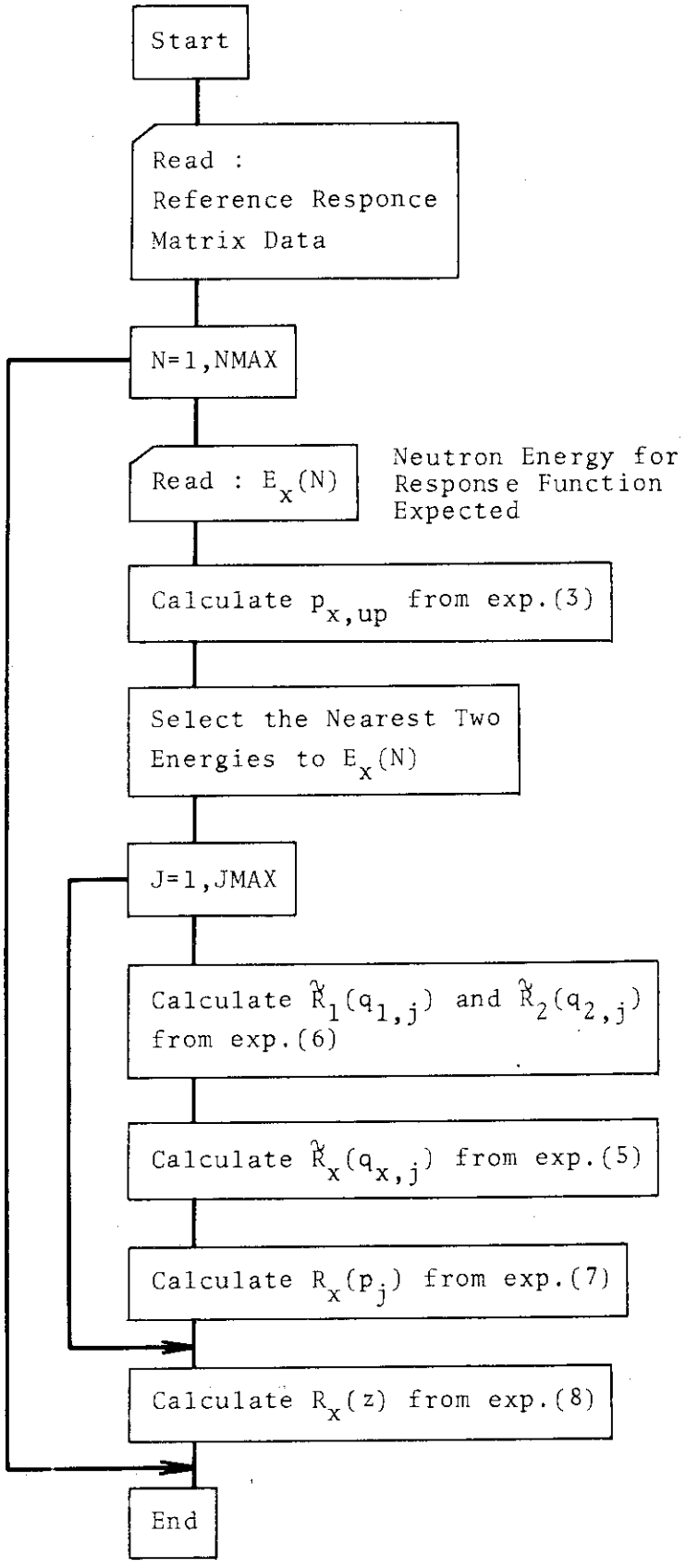

Fig. 3 Flowchart of the interpolation procedure for the response function.

Table 1.

The neutron energies of the response functions in Fig.4.

No.	E_n (MeV)	No.	E_n (MeV)	No.	E_n (MeV)	No.	E_n (MeV)
1	0.335 *)	2	0.47	3	0.60 *)	4	0.77
5	0.946 *)	6	1.10	7	1.25 *)	8	1.40
9	1.56 *)	10	1.74	11	1.91 *)	12	2.45
13	2.98 *)	14	3.47	15	3.96 *)	16	5.00
17	5.97 *)	18	7.00	19	8.12 *)	20	9.56
21	11.00 *)	22	12.70	23	14.40 *)		

*) reference energies

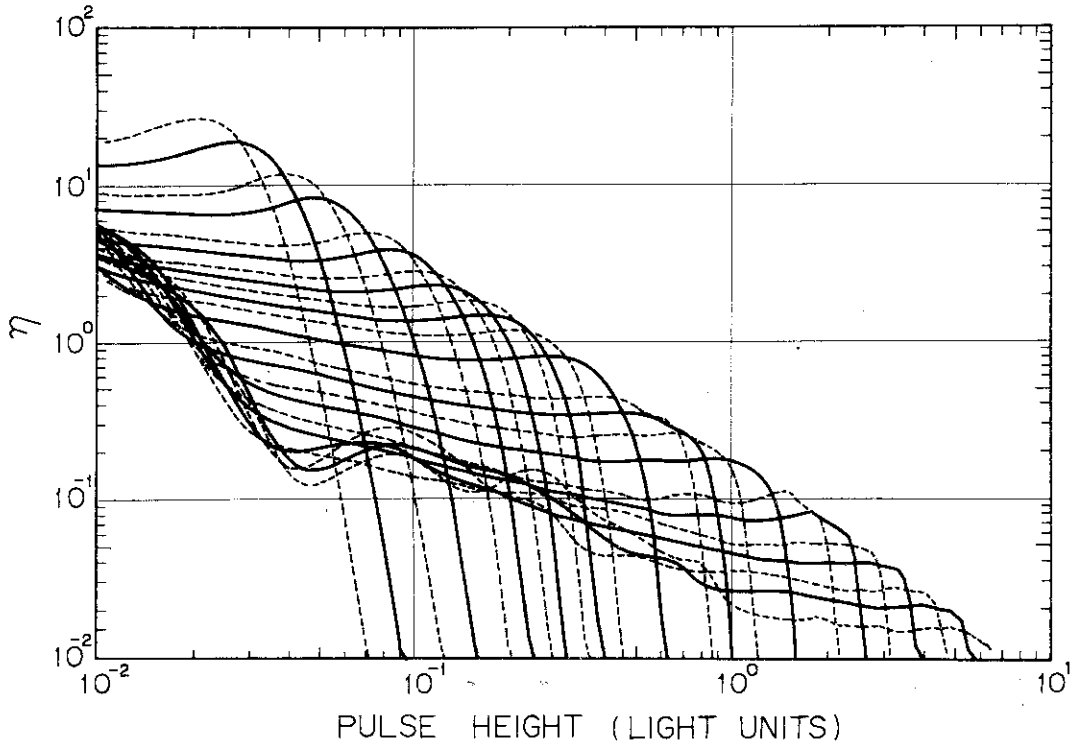

Fig. 4 The interpolated response functions (solid lines) and the reference functions (dotted lines). The corresponding neutron energies are listed in Table 1.

Appendix The description of the program

(1) Input data format

1. JMAX, KMAX, NMAX (10I6)
2. (EN(J), J = 1, JMAX) (6F12.5)
3. (PH(K), K = 1, KMAX) (6F12.5)
4. (DD(J,K), K = 1, KMAX) (6F12.5)

   repeat (J = 1, JMAX)
5. (X(N), N = 1, NMAX) (6F12.5)

(2) Detail description for input

1. JMAX Number of neutron energy meshes of the response matrix referred to.

   KMAX Number of pulse height meshes of the response matrix referred to.

   NMAX Number of neutron energies for which the interpolated response functions are to be obtained.
2. EN Neutron energy of the reference matrix.
3. PH Pulse height of the reference matrix.
4 DD Element of the reference matrix.
5. X Neutron energy for which the response function is to be obtained.

(3) Input data list

```
....*....1....*....2....*....3....*....4....*....5....*....6....*....7....*....8

  12    150     11
0,3350        0,6000       0,9450       1.2500       1,5600       1,9100
2,9800        3,9600       5,9700       8.1200      11,0000      14,4000
0,0100        0.0105       0,0110       0.0115       0,0120       0.0126
0,0132        0.0138       0.0145       0.0151       0.0158       0.0166
0,0174        0,0182       0,0191       0.0200       0.0209       0.0219
0,0229        0.0240       0,0251       0.0263       0,0275       0,0288
0,0302        0,0316       0,0331       0,0347       0.0363       0,0380
0,0398        0.0417       0,0437       0,0457       0,0479       0,0501
0,0525        0,0550       0,0575       0,0603       0,0631       0,0661
0,0692        0,0724       0,0759       0,0794       0,0832       0,0871
0.0912        0.0955       0.1000       0,1047       0.1096       0.1148
0,1202        0,1259       0.1318       0.1380       0,1445       0.1514
0,1585        0.1660       0.1738       0.1820       0,1905       0.1995
0,2089        0.2188       0,2291       0.2399       0,2512       0.2630
0,2754        0,2884       0.3020       0.3162       0.3311       0,3467
0,3631        0,3802       0,3981       0.4169       0.4365       0.4571
0,4786        0.5012       0.5248       0,5495       0,5754       0.6026
0.6310        0,6607       0,6918       0,7244       0.7586       0.7943
0,8318        0.8710       0.9120       0.9550       1,0000       1.0471
1,0965        1.1482       1.2023       1.2589       1,3183       1.3804
1,4454        1,5136       1.5849       1.6596       1,7378       1.8197
1,9055        1.9953       2,0893       2.1878       2,2909       2,3988
2,5119        2.6303       2.7542       2,8840       3,0200       3,1623
3,3113        3.4674       3,6308       3.8019       3,9811       4,1687
4,3652        4,5709       4,7863       5.0119       5.2481       5.4954
5,7544        6,0256       6,3096       6,6069       6,9183       7,2444
7,5858        7.9433       8.3176       8.7096       9,1201       9,5499
18,8324      19.0890      19,3719      19.7163      20,1452      20,6636
21,2583      21,9029      22,5671      23.2248      23,8596      24,4637
25,0316      25,5504      25,9906      26,3007      26,4102      26,2396
25,7152      24.7859      23.4362      21.6932      19,6252      17,3319
14,9305      12.5399      10.2663       8,1925       6,3732       4,8343
 3,5766       2.5820       1,8194       1.2519       0,8416       0,5530
 0,3553       0,2233       0.1373       0,0827       0,0488       0,0282
 0,0153       0,0089       0,0048       0,0026       0.0014       0,0007
 0,0004       0,0002       0,0001       0,0000       0.0000       0.0000
   .             .            .            .            .            .
   .             .            .            .            .            .
   .             .            .            .            .            .
   .             .            .            .            .            .
 8,9265       8,8842       8,8255       8.7670       8,7222       8,6988
 8,6968       8.7104       8,7304       8,7478       8,7570       8,7572
```

....*....1....*....2....*....3....*....4....*....5....*....6....*....7....*....8

8.7519	8.7480	8.7537	8.7768	8.8240	8.9012
9.0131	9.1640	9.3567	9.5923	9.8687	10.1805
10.5183	10.8688	11.2142	11.5307	11.7886	11.9529
11.9859	11.8527	11.5273	10.9983	10.2736	9.3801
8.3613	7.2707	6.1647	5.0955	4.1057	3.2250
2.4698	1.8442	1.3429	0.9536	0.6604	0.4460
0.2937	0.1886	0.1181	0.0721	0.0429	0.0249
0.0141	0.0078	0.0042	0.0022	0.0012	0.0007
0.0003	0.0001	0.0000			
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
5.3772	5.3200	5.2499	5.1770	5.1094	5.0524
5.0070	4.9709	4.9391	4.9067	4.8701	4.8279
4.7807	4.7309	4.6807	4.6317	4.5847	4.5394
4.4949	4.4508	4.4071	4.3644	4.3235	4.2854
4.2508	4.2206	4.1957	4.1771	4.1656	4.1629
4.1704	4.1596	4.2222	4.2697	4.3335	4.4140
4.5105	4.6204	4.7385	4.8569	4.9651	5.0504
5.0997	5.1002	5.0417	4.9170	4.7234	4.4527
4.1409	3.7682	3.3581	2.9261	2.4891	2.0538
1.6633	1.3059	0.9938	0.7331	0.5374	0.3985
0.2573	0.1571	0.0902	0.0485	0.0242	0.0112
0.0047	0.0018	0.0006	0.0003	0.0001	0.0000
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
3.9674	3.8694	3.7728	3.6822	3.6021	3.5349
3.4804	3.4364	3.3991	3.3648	3.3306	3.2952
3.2589	3.2227	3.1879	3.1550	3.1241	3.0946
3.0656	3.0363	3.0063	2.9757	2.9445	2.9128
2.8806	2.8478	2.8145	2.7807	2.7468	2.7132
2.6807	2.6499	2.6217	2.5970	2.5765	2.5610
2.5512	2.5476	2.5506	2.5604	2.5768	2.5999

....*....1....*....2....*....3....*....4....*....5....*....6....*....7....*....8

2.6290	2.6637	2.7029	2.7451	2.7882	2.8291
2.8539	2.8878	2.8962	2.8843	2.8484	2.7854
2.6937	2.5722	2.4211	2.2418	2.0500	1.8697
1.6398	1.3975	1.1520	0.9139	0.6940	0.5016
0.3431	0.2207	0.1327	0.0741	0.0382	0.0180
0.0077	0.0030	0.0010	0.0003	0.0001	0.0000
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
3.4259	3.3120	3.2043	3.1060	3.0197	2.9467
2.8862	2.8358	2.7917	2.7503	2.7088	2.6657
2.6208	2.5749	2.5290	2.4539	2.4399	2.3970
2.3546	2.3126	2.2710	2.2302	2.1909	2.1539
2.1198	2.0892	2.0623	2.0390	2.0189	2.0009
1.9841	1.9670	1.9487	1.9284	1.9057	1.8811
1.8553	1.8292	1.8038	1.7800	1.7583	1.7391
1.7224	1.7083	1.6968	1.6881	1.6828	1.6812
1.6839	1.6913	1.7035	1.7204	1.7414	1.7655
1.7911	1.8158	1.8369	1.8512	1.8524	1.8400
1.8261	1.7968	1.7504	1.6858	1.6021	1.4987
1.3745	1.2293	1.0641	0.8836	0.6965	0.5154
0.3541	0.2232	0.1276	0.0655	0.0298	0.0119
0.0041	0.0012	0.0003	0.0001	0.0000	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
2.8613	2.6860	2.5352	2.4080	2.3032	2.2190
2.1527	2.1004	2.0582	2.0222	1.9895	1.9584
1.9282	1.8989	1.8709	1.8442	1.8186	1.7934
1.7680	1.7418	1.7147	1.6867	1.6583	1.6300
1.6022	1.5753	1.5496	1.5252	1.5021	1.4803
1.4594	1.4391	1.4190	1.3990	1.3789	1.3587
1.3387	1.3190	1.2998	1.2814	1.2637	1.2466
1.2302	1.2142	1.1988	1.1839	1.1699	1.1570
1.1455	1.1356	1.1276	1.1215	1.1173	1.1149
1.1143	1.1157	1.1192	1.1251	1.1309	1.1332
1.1451	1.1590	1.1737	1.1875	1.1982	1.2034
1.2004	1.1875	1.1637	1.1287	1.0833	1.0280

```
....*....1....*....2....*....3....*....4....*....5....*....6....*....7....*....8

0.9623      0.8845      0.7917      0.6824      0.5582      0.4265
0.2993      0.1897      0.1068      0.0526      0.0223      0.0080
0.0024      0.0006      0.0001      0.0000
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
5.0525      4.3794      3.8086      3.3252      2.9169      2.5731
2.2849      2.0441      1.8436      1.6771      1.5392      1.4253
1.3319      1.2557      1.1943      1.1450      1.1056      1.0737
1.0476      1.0253      1.0060      0.9885      0.9723      0.9569
0.9417      0.9265      0.9109      0.8946      0.8776      0.8599
0.8415      0.8224      0.8029      0.7832      0.7634      0.7440
0.7250      0.7068      0.6896      0.6735      0.6586      0.6447
0.6319      0.6200      0.6089      0.5985      0.5887      0.5793
0.5702      0.5611      0.5520      0.5428      0.5334      0.5242
0.5154      0.5073      0.5003      0.4944      0.4891      0.4835
0.4800      0.4770      0.4741      0.4709      0.4671      0.4627
0.4579      0.4532      0.4488      0.4453      0.4430      0.4418
0.4417      0.4423      0.4433      0.4443      0.4453      0.4463
0.4475      0.4485      0.4484      0.4455      0.4382      0.4256
0.4081      0.3873      0.3664      0.3484      0.3353      0.3255
0.3134      0.2902      0.2480      0.1872      0.1192      0.0614
0.0247      0.0075      0.0017      0.0003      0.0000
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
 .           .           .           .           .           .
5.2401      4.9632      4.6368      4.2718      3.8822      3.4838
3.0914      2.7177      2.3722      2.0614      1.7884      1.5542
1.3574      1.1952      1.0639      0.9591      0.8763      0.8112
0.7597      0.7185      0.6849      0.6570      0.6332      0.6127
0.5948      0.5791      0.5653      0.5532      0.5426      0.5333
0.5250      0.5175      0.5104      0.5035      0.4965      0.4892
0.4816      0.4736      0.4651      0.4564      0.4474      0.4384
0.4295      0.4208      0.4123      0.4040      0.3959      0.3879
0.3799      0.3719      0.3638      0.3556      0.3475      0.3397
0.3321      0.3251      0.3186      0.3127      0.3073      0.3021
0.2978      0.2940      0.2907      0.2876      0.2847      0.2817
0.2786      0.2752      0.2716      0.2678      0.2640      0.2603
0.2568      0.2536      0.2510      0.2494      0.2491      0.2501
0.2525      0.2555      0.2584      0.2602      0.2604      0.2590
0.2567      0.2548      0.2542      0.2554      0.2583      0.2622
0.2658      0.2670      0.2643      0.2569      0.2458      0.2327
0.2201      0.2090      0.1983      0.1831      0.1567      0.1160
```

....*....1....*....2....*....3....*....4....*....5....*....6....*....7....*....8

0.0687	0.0303	0.0094	0.0019	0.0003	0.0000
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
6.1040	5.7848	5.4035	4.9734	4.5115	4.0367
3.5571	3.1178	2.7004	2.3226	1.9883	1.6983
1.4510	1.2437	1.0716	0.9308	0.8166	0.7245
0.6506	0.5914	0.5439	0.5058	0.4749	0.4498
0.4291	0.4117	0.3968	0.3837	0.3719	0.3612
0.3513	0.3420	0.3333	0.3250	0.3172	0.3097
0.3026	0.2959	0.2893	0.2830	0.2768	0.2707
0.2648	0.2589	0.2531	0.2473	0.2415	0.2356
0.2297	0.2237	0.2176	0.2114	0.2052	0.1991
0.1933	0.1878	0.1825	0.1776	0.1732	0.1688
0.1645	0.1603	0.1559	0.1515	0.1471	0.1428
0.1388	0.1353	0.1326	0.1306	0.1291	0.1281
0.1271	0.1259	0.1241	0.1219	0.1195	0.1171
0.1152	0.1137	0.1125	0.1111	0.1092	0.1067
0.1038	0.1011	0.0994	0.0990	0.1003	0.1024
0.1044	0.1058	0.1066	0.1069	0.1064	0.1049
0.1024	0.0996	0.0971	0.0953	0.0946	0.0951
0.0965	0.0984	0.1003	0.1024	0.1057	0.1097
0.1121	0.1100	0.1027	0.0929	0.0850	0.0804
0.0732	0.0543	0.0274	0.0082	0.0013	0.0001
0.0000	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
6.0191	5.7125	5.3460	4.9322	4.4877	4.0306
3.5778	3.1432	2.7373	2.3666	2.0343	1.7406
1.4842	1.2625	1.0724	0.9107	0.7741	0.6598
0.5649	0.4869	0.4235	0.3726	0.3322	0.3005
0.2760	0.2572	0.2429	0.2321	0.2238	0.2176
0.2126	0.2085	0.2050	0.2017	0.1985	0.1952
0.1919	0.1885	0.1850	0.1813	0.1776	0.1737
0.1698	0.1657	0.1617	0.1576	0.1538	0.1501
0.1465	0.1439	0.1414	0.1392	0.1374	0.1356
0.1339	0.1320	0.1299	0.1276	0.1250	0.1223
0.1197	0.1171	0.1148	0.1129	0.1117	0.1112
0.1112	0.1112	0.1112	0.1103	0.1066	0.1059
0.1025	0.0989	0.0956	0.0929	0.0910	0.0897
0.0888	0.0880	0.0872	0.0861	0.0847	0.0830
0.0811	0.0788	0.0764	0.0740	0.0718	0.0698
0.0681	0.0664	0.0647	0.0628	0.0606	0.0588
0.0575	0.0565	0.0552	0.0535	0.0520	0.0512
0.0511	0.0514	0.0516	0.0516	0.0512	0.0506
0.0508	0.0520	0.0529	0.0529	0.0524	0.0519
0.0520	0.0523	0.0521	0.0514	0.0507	0.0500
0.0490	0.0476	0.0456	0.0392	0.0246	0.0085
0.0012	0.0001	0.0000	.	.	.

....*....1....*....2....*....3....*....4....*....5....*....6....*....7....*....8

4.4761	4.3357	4.1763	4.0051	3.8291	3.6539
3.4819	3.3126	3.1429	2.9683	2.7845	2.5886
2.3801	2.1608	1.9350	1.7082	1.4866	1.2761
1.0817	0.9068	0.7533	0.6218	0.5116	0.4212
0.3484	0.2912	0.2470	0.2139	0.1899	0.1736
0.1636	0.1589	0.1587	0.1624	0.1693	0.1788
0.1903	0.2033	0.2170	0.2310	0.2446	0.2572
0.2684	0.2776	0.2845	0.2886	0.2898	0.2881
0.2835	0.2763	0.2669	0.2557	0.2432	0.2297
0.2157	0.2014	0.1872	0.1735	0.1612	0.1508
0.1395	0.1296	0.1213	0.1147	0.1097	0.1060
0.1033	0.1010	0.0985	0.0953	0.0910	0.0856
0.0789	0.0716	0.0643	0.0577	0.0523	0.0483
0.0458	0.0444	0.0439	0.0440	0.0442	0.0442
0.0439	0.0433	0.0423	0.0412	0.0400	0.0388
0.0378	0.0369	0.0362	0.0355	0.0349	0.0344
0.0343	0.0343	0.0344	0.0346	0.0348	0.0349
0.0349	0.0345	0.0340	0.0335	0.0330	0.0327
0.0323	0.0315	0.0307	0.0301	0.0296	0.0291
0.0281	0.0274	0.0269	0.0265	0.0260	0.0260
0.0265	0.0263	0.0258	0.0260	0.0270	0.0277
0.0271	0.0264	0.0260	0.0258	0.0259	0.0242
0.0211	0.0145	0.0049	0.0005	0.0000	
2.6675	2.5281	2.3982	2.2786	2.1701	2.0727
1.9853	1.9051	1.8288	1.7526	1.6733	1.5887
1.4980	1.4018	1.3014	1.1986	1.0954	0.9934
0.8939	0.7980	0.7062	0.6194	0.5381	0.4631
0.3950	0.3344	0.2819	0.2375	0.2013	0.1729
0.1518	0.1373	0.1286	0.1248	0.1250	0.1283
0.1338	0.1407	0.1484	0.1565	0.1645	0.1723
0.1797	0.1863	0.1919	0.1961	0.1983	0.1983
0.1956	0.1904	0.1827	0.1732	0.1624	0.1512
0.1403	0.1305	0.1224	0.1165	0.1129	0.1110
0.1111	0.1135	0.1182	0.1246	0.1322	0.1402
0.1476	0.1535	0.1570	0.1575	0.1546	0.1485
0.1400	0.1300	0.1194	0.1091	0.0997	0.0916
0.0850	0.0796	0.0755	0.0720	0.0689	0.0656
0.0621	0.0585	0.0552	0.0521	0.0494	0.0469
0.0447	0.0432	0.0422	0.0417	0.0410	0.0393
0.0359	0.0313	0.0267	0.0234	0.0215	0.0205
0.0197	0.0191	0.0186	0.0182	0.0178	0.0175
0.0174	0.0174	0.0173	0.0174	0.0177	0.0180
0.0179	0.0171	0.0161	0.0156	0.0157	0.0157
0.0154	0.0153	0.0156	0.0157	0.0152	0.0145
0.0142	0.0141	0.0143	0.0146	0.0150	0.0152
0.0149	0.0147	0.0154	0.0150	0.0144	0.0140
0.0132	0.0124	0.0119	0.0079	0.0020	0.0001
0.0000					
0.47	0.77	1.10	1.40	1.74	2.45
3.47	5.00	7.00	9.56	12.70	

(4) The program list

```
....*....1....*....2....*....3....*....4....*....5....*....6....*....7....*....8

C      THE CODE FOR OBTAINING A INTERPOLATED RESPONSE FUNCTION
       DIMENSION X(100),EN(100),DD(25,160),Y(100,200),PH(200),NC(3)
      *          ,NA(3),NB(3),PK(200),PJ(200),PI(25,200),R(30),LW(30)
      *          ,LOW(100)
       COMMON/COM1/ JMAX,KMAX
C      JMAX/NUMBER OF NEUTRON ENERGY MESHES OF THE REFERENCE MATRIX
C      KMAX/NUMBER OF PULSE HEIGHT MESHES OF THE REFERENCE MATRIX
C      NMAX/NUMBER OF NEUTRON ENERGIES FOR WHICH THE INTERPOLATED
C           RESPONSE FUNCTIONS ARE TO BE OBTAINED
C      EN   /NEUTRON ENERGY OF THE REFERENCE MATRIX
C      PH   /PULSE HEIGHT OF THE REFERENCE MATRIX
C      DD   /ELEMENT OF THE REFERENCE MATRIX
C      X    /NEUTRON ENERGY FOR WHICH THE RESPONSE FUNCTION IS TO BE OBTAINED
   11 FORMAT(10I6)
   12 FORMAT(6F12.5)
   25 FORMAT(1H0,   12X,'PLS HEIGHT',3X,6F10.5   )
   26 FORMAT(1H ,6X,I3,F10.4,6X,6F10.5)
   27 FORMAT(1H0, 9X,'    TOTAL ',6X,6F10.5)
   28 FORMAT(1H )
   29 FORMAT(1H1//,29X,6('(',I2,')',6X))
      READ(5,11) JMAX,KMAX,NMAX,NPLOT,NPUNCH,NDISK
      READ(5,12) (EN(J),J=1,JMAX)
      READ(5,12) (PH(K),K=1,KMAX)
      DO 117 J=1,JMAX
      READ(5,12) (DD(J,K),K=1,KMAX)
  117 CONTINUE
      READ(5,12) (X(N),N=1,NMAX)
      DO 141 J=1,JMAX
      DO 142 K=1,KMAX
      IF(DD(J,K).LT.1.0E-06) GO TO 1141
  142 CONTINUE
      LW(J)=K
      GO TO 141
 1141 LW(J)=K-1
  141 CONTINUE
      DO 143 N=1,NMAX
      DO 144 J=1,JMAX
      IF(ABS(X(N)-EN(J)).LT.1.0E-04) GO TO 145
  144 CONTINUE
      CALL SERIAL(EN,X(N),JMAX,NC)
      CC1=FLOAT(LW(NC(1)))
      CC2=FLOAT(LW(NC(2)))
      LOW(N)=FK(CC1,CC2,EN(NC(1)),EN(NC(2)),X(N))
      GO TO 143
  145 CONTINUE
      LOW(N)=LW(J)
  143 CONTINUE
      DO 100 N=1,NMAX
      DO 109 JJ=1,JMAX
      IF(ABS(EN(JJ)-X(N)).LT.1.0E-04) GO TO 108
  109 CONTINUE
      CALL SERIAL(EN,X(N),JMAX,NA)
      DO 102 K=1,KMAX
      PK(K)=PH(K)*PH(KMAX)/PH(LOW(N))
      DO 103 I=1,3
```

```
....*....1....*....2....*....3....*....4....*....5....*....6....*....7....*....8

      PI(NA(I),K)=PH(K)*PH(KMAX)/PH(L*(NA(I)) )
  103 CONTINUE
  102 CONTINUE
      DO 105 K=1,KMAX
      DO 104 I=1,3
      DO 106 K0=1,KMAX
      PJ(K0)=PI(NA(I),K0)
  106 CONTINUE
      DO 101 K0=1,KMAX
      IF(ABS(PJ(K0)-PK(K)).LT.1.0E-04) GO TO 1101
  101 CONTINUE
      CALL SERIAL(PJ,PK(K),KMAX,NB)
      N1=NA(I)
      R(NA(I))=FN(DD(N1,NB(1)),DD(N1,NB(2)),DD(N1,NB(3))
     *           ,PJ(NB(1))    ,PJ(NB(2))    ,PJ(NB(3))    ,PK(K) )
      IF(R(NA(I)).GE.0.0) GO TO 104
      R(NA(I))=FK(DD(N1,NB(1)),DD(N1,NB(2)),PJ(NB(1)),PJ(NB(2)),PK(K))
      GO TO 104
 1101 CONTINUE
      R(NA(I))=DD(NA(I),K0)
  104 CONTINUE
      Y(N,K)=FK(R(NA(1)),R(NA(2)),EN(NA(1)),EN(NA(2)),X(N))
  105 CONTINUE
      GO TO 100
  108 CONTINUE
      DO 107 K=1,KMAX
      Y(N,K)=DD(JJ,K)
  107 CONTINUE
  100 CONTINUE
      DO 1116 N=1,NMAX
      Y(N,KMAX+1)=0.0
      DO 1116 K=1,KMAX
      Y(N,KMAX+1)=Y(N,KMAX+1)+Y(N,K)
 1116 CONTINUE
      NROW = 6
      NN=NMAX/NROW
      DO 115 N1=1,NN+1
      NS=(N1-1)*NROW+1
      IF(NS.GT.NMAX) GO TO 118
      NF=NS+NROW-1
      IF(NF.GT.NMAX) NF=NMAX
      WRITE(6,29) (N,N=NS,NF)
      WRITE(6,25) (X(N),N=NS,NF)
      DO 116 K=1,KMAX
      IF(MOD(K,10).EQ.1) WRITE(6,28)
      WRITE(6,26) K,PH(K),(Y(N,K),N=NS,NF)
  116 CONTINUE
      WRITE(6,27) (Y(N,KMAX+1),N=NS,NF)
  115 CONTINUE
  118 CONTINUE
      STOP
      END
      SUBROUTINE SERIAL(Z,S,MAX,I1)
      DIMENSION Z(1),X(200),I1(3)
      DO 102 J=1,MAX
```

```
....*....1....*....2....*....3....*....4....*....5....*....6....*....7....*....8

      X(J)=ABS(Z(J)-S)
  102 CONTINUE
      DO 103 I=1,3
      XMIN=1.0E+20
      DO 101 J=1,MAX
      IF(X(J).GE.XMIN) GO TO 101
      XMIN=X(J)
      JMIN=J
  101 CONTINUE
      I1(I)=JMIN
      X(JMIN)=1.0E+30
  103 CONTINUE
      RETURN
      END
      FUNCTION FN(C1,C2,C3,E1,E2,E3,Z)
C  INTERPORATION BY POLYNOMIAL OF SECOND ORDER
      FN=C1*(Z-E2)*(Z-E3)/((E1-E2)*(E1-E3))
     1   +C2*(Z-E1)*(Z-E3)/((E2-E1)*(E2-E3))
     2   +C3*(Z-E1)*(Z-E2)/((E3-E1)*(E3-E2))
      RETURN
      END
      FUNCTION FK(C1,C2,E1,E2,Z)
C  INTERPOLATION BY LINEAR EXPRESSION
      FK=C1*(Z-E2)/(E1-E2)
     *   +C2*(Z-E1)/(E2-E1)
      RETURN
      END
```