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TRANSIENT HEAT TRANSFER IN TURBULENT FLOW WITH TIME-DEPENDENT HEAT INPUT

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Transient Heat Transfer in Turbulent Flow with Time-Dependent Heat Input

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Transient variation of the heat transfer coefficient is involved in accident analysis of a power reactor and thermal design of a pulse reactor. In these cases, however, the heat transfer coefficient is assumed to be equal to the steady-state one. Validity of this 'quasi-static assumption' is examined.

The transient energy equation is solved numerically for turbulent annular flow. Variations of the heater temperature and the heat transfer coefficient are obtained in stepwise increase and decrease of the heat input.

Variation of the heater temperature is obtained with the quasi-static assumption. The quasi-static assumption is valid if the wall heat capacity is large compared with that of the effective thermal boundary layer in the fluid.

Applied in a pulse reactor, the quasi-static assumption results in a higher fuel surface temperature during pulse heating when the gap is filled with sodium.

発熱が時間的に変化する場合の乱流非定常熱伝達

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非定常状態における熱伝達の時間変化を知ることは，動力炉の安全解析やパルス炉の熱設計にとって重要である。しかし，従来これらの解析や設計においては，熱伝達率はつねに定常値に等しいと仮定されていた。本論文は，この準定常状態の仮定の成立条件を検討するものである。

非定常エネルギ方程式を二重管内乱流について数値的に解き，発熱体温度や熱伝達率の時間変化を求める。発熱は，時間的にステップ状に増加または減少するとする。他方，準定常状態を仮定した計算をも行ない，両者を比較した。その結果，発熱体の熱容量が流体内温度境界層の熱容量に比して十分大きい場合には，準定常状態の仮定が成立することがわかった。

発熱がパルス状に増減する場合の計算をも行ない，パルス炉への応用を議論した。

目 次 な し

1. Introduction

When reactor power changes with time, heat transfer coefficient deviates from its steady state value. The transient heat transfer coefficient is required fro accident analysis of power reactors and thermal design of pulse reactors. The transient value is, however, assumed equal to its steady state value in all the analyses and designs. This is called quasi-static assumption. The present purpose is to examine validity of the quasi-static assumption by analyzing the transient heat transfer for turbulent flow.

Many works have been made for the transient laminar heat transfer, but relatively few works for the transient turbulent heat transfer. The present author made analyses[1,2] and an experiment[3] of the transient turbulent heat transfer for a stepwise power input with a constant flow rate.

He found three non-dimensional parameters relevant to the transient heat transfer. The first is a non-dimensional time Z defined as

$$Z = \frac{\alpha_{st}^{2}\, t}{4(\lambda\ \rho\ c_p)_f} , \qquad (1)$$

The heat transfer coefficient reaches the steady state value roughly when $Z \sim 1$ for a stepwise heat input. The second parameter is

$$\beta = \frac{\alpha_{st}\ H}{(\lambda\ \rho\ c_p)_f} , \qquad (2)$$

where H is the wall heat capacity per a heat transfer area. The variation of the wall temperature is nearly quasi-static, if $\beta < 10$. The third is γ defined as

$$\gamma = \sqrt{\frac{(\lambda\rho\ c_p)_f}{(\lambda\rho\ c_p)_w}} . \qquad (3)$$

This depends on physical properties of the wall and the fluid.

The wall heat capacity H in Eq. (2) is expressed as

$$H = d_w^* (\rho c_p)_w , \tag{4}$$

where d_w^* is the equivalent wall thickness defined as

$$d_w^* = (r_i^2 - r_c^2)/(2r_i). \tag{5}$$

The third parameter γ appears often in solution of the transient thermal conduction equation for a composite solid. The parameters Z and β are rewitten as

$$Z = a_f t/(4 d_f^{*2}) \tag{6}$$

$$\beta = \frac{d_w^* (\rho c_p)_w}{d_f^* (\rho c_p)_f} , \tag{7}$$

where d_f^* is λ_f/α_{st} and the dimension of length.

If the fluid is assumed a semi-infinite solid with a uniform thermal conductivity λ_f, d_f^* is the thickness of the temperature-varying layer which gives the heat transfer coefficient of α_{st} in the steady state (see Fig. 1). Thus, Z is interpreted as the Fourier Number where the characteristic length is the equivalent thermal boundary layer thickness d_f^*. The time for the temperature variation to develop in the equivalent thermal boundary layer is an order of $Z \sim 1$, and it has been found[1] roughly equal to the steady-state time of the heat transfer coefficient for a stepwise heat input.

Equation (7) shows that β is the ratio of the heat capacity of the wall to that of the equivalent thermal boundary layer. When $\beta >> 1$, most of the heat generated is stored in the wall; when $\beta < 1$, more heat is transferred to the fluid than that stored in the wall. So, the transient effect in the heat transfer is prominent when $\beta < 1$.

2. Formulation

2.1 Assumptions

Figure 2 shows the co-ordinate system. Assumptions are (1) a very long tube is assumed, so the flow is fully developed; (2) the inner surface of the heating tube ($r=r_c$) and the outer surface of the annular channel ($r=r_o$) are thermelly insulated; (3) properties are independent of the temperature; (4) turbulent eddy diffusivities, ε_M and ε_H, are constant with time. Validity of the last assumption were discussed in Ref. (1).

2.2 Two-dimensional equations

The flow is assumed constant with time. The momentum equation is

$$\frac{g_c}{\rho_f}\frac{\partial P}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[(\varepsilon_M+\nu)r\frac{\partial u}{\partial r}\right]. \tag{8}$$

The energy equation for the fluid is

$$\frac{\partial T_f}{\partial t} + u\frac{\partial T_f}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[(\varepsilon_H+a_f)r\frac{\partial T_f}{\partial r}\right]. \tag{9}$$

The thermal conduction equation for the heating wall is

$$\frac{\partial T_h}{\partial t} = a_w\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T_h}{\partial r}\right] + \frac{q_G}{H}, \tag{10}$$

where q_G is the heat generation rate per a heat transfer area and is a function of time.

Boundary conditions are

$$\begin{aligned} &\partial T_h/\partial r = 0 && \text{at } r = r_c \\ &T_h = T_f,\ \lambda_w(\partial T_h/\partial r) = \lambda_f(\partial T_f/\partial r),\ u = 0 && \text{at } r = r_i \\ &\partial T_h/\partial r = 0,\ u = 0 && \text{at } r = r_o \\ &T_f = 0 && \text{at } x = 0. \end{aligned} \tag{11}$$

Initial temperature is zero except in Section 4.

The momentum eddy diffusivity used is the Kay's[4] correlation multiplied by the damping factor $[1 - \exp(-y^+/A^+)]$, where y^+ is a non-dimensional distance from a wall and A^+ is a damping constant. The thermal eddy diffusivity ε_H is calculated referring to Mizushina[5]. The eddy diffusivity ratio $\varepsilon_H/\varepsilon_M$ is about 1.0 to 1.2 for the normal fluids.

2.3 Quasi-static Equation

By integrating Eq. (9) from $r = r_i$ to $r = r_o$, one obtains the one-dimensional energy equation:

$$\frac{\partial \bar{T}_f}{\partial t} + \bar{u}\frac{\partial \bar{T}_f}{\partial x} = \frac{1}{(\rho c_p)_f}\frac{2r_i}{(r_o^2 - r_i^2)}\cdot\alpha(T_w - \bar{T}_f), \qquad (12)$$

where $\bar{T}_f$ is the mixed mean temperature of fluid. Boundary condition on the heat transfer surface is

$$-\lambda_w \frac{\partial T_h}{\partial r} = \alpha(T_w - \bar{T}_f) \qquad \text{at } r = r_i \qquad (13)$$

If α's in Eqs. (12) and (13) are assumed equal to the steady state value, these are called "quasi-static equation" and their solution is "quasi-static solution". The solution of the two-dimensional equation is called "transient solution" in the followings.

3. Heating Phase

The two-dimensional energy equation Eq. (9) was solved[1,2] for a stepwise increase in power q_G. Numerical solution of Eq. (9) is compared with the quasi-static solution in Fig. 3. Wall temperature by the quasi-static solution is higher than that by the transient solution.

Variation of heat transfer coefficient is shown in Fig. 4 for various axial positions. The heat transfer coefficient is very high at early times, and decreases down to the steady state value with elapse of time.

The conduction solution in Fig. 4 is a solution of the equation:

$$\frac{\partial T_f}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[(\varepsilon_H + a_f)\, r\frac{\partial T_f}{\partial r}\right], \qquad (14)$$

which is obtained by neglecting the convection term $u\partial T_f/\partial x$ in Eq. (9). Equation (14) is hereafter called the "conduction equation". The heat transfer coefficient in fully developed region (x/De = 25) agrees well with that by the conduction solution. The lines for x/De = 25 and for the conduction solution cannot be drawn separately in the region plotted.

The reason why the convection term is negligible was discussed in Ref. (1). Briefly, it is because the fluid temperature is axially uniform at small times; so, $\partial T_f/\partial x$ is nearly zero and the convection term is neglizible.

Steady state time for wall temperature is shown in Fig. 5. It is defined as the time required for $\Delta T_w = T_w - \overline{T}_f$ to reach 90 - 95 % of the final value.

i) When the quasi-static heat balance of heating wall is dominant.

This case is a heating wall with relatively large heat capacity and small thermal resistance. The steady state time is given by

$$Z_{st} \sim 2.5(\beta/4), \text{or } t_{st,w} \sim 2.5(H/\alpha_{st}), \tag{15}$$

and is shown by the line 2 in Fig. 5.

ii) When the thermal resistance is dominant.

This is the case of a wall with large thermal resistance. The steady state time is dependent on $\beta\gamma$ and given by

$$Z_{st,w} \sim 0.4(\beta\gamma)^2 \text{ or } t_{st,w} \sim 0.16 \frac{d_w^{*2}}{\alpha_w} . \tag{16}$$

This is shown by the lines 3 - 6.

iii) When the transient variation of heat transfer coefficient is dominant.

If the heat capacity and the thermal resistance of wall are small, the steady state time for wall temperature is nearly equal to that for heat transfer coefficient; i.e.,

$$Z_{st,w} \sim Z_{st}, \text{ or } t_{st,w} \sim 4Z_{st,w}(\lambda\rho\ c_p)_f/\alpha_{st}^2 . \tag{17}$$

The line 1 in Fig. 5 shows $Z_{st,w}$ calculated analytically in Ref. (1). Experimental $Z_{st,w}$ obtained in Ref. (3) was nearly half of the analytical one for β less than 1.

The steady state time for wall temperature is maximum of Eqs. (15) - (17). It is illustrated in Fig. 5 by hatched line for $\gamma = 0.1$.

With the quasi-static assumption, the steady state time for wall temperature becomes infinitesimal with decrease of wall heat capacity (see line 2). In fact, however, the steady state time is never less than $Z_{st,w}$. It is because the heat capacity of fluid adjacent to the wall is not zero even if the wall heat capacity is zero.

Validity of the quasi-static assumption is examined in Fig. 6. The ratio of the transient solution to the quasi-static solution for the wall temperature is plotted against the non-dimensional time Z. The ratio is a function of β and γ.

The ratio is dependent on γ at small times. It is known from the transient heat conduction solution for a composite solid that the wall temperature variation at small times is expressed as

$$T_w(t) = \frac{1}{1+\gamma} \frac{q_G}{H} t, \quad t \to 0 \tag{18}$$

that is, the adiabatic temperature rise devided by (1+γ).

The quasi-static equation gives

$$T_w(t) = \frac{q_G}{H} t, \quad t \to 0. \tag{19}$$

The quasi-static assumption thus results in an error of (1+γ) at small times.

So, the quasi-static assumption is valid roughly when $\gamma < 0.1$. This condition holds for gas coolants and fails for liquid ones.

The ratio at large times is mainly dependent on β, and is near unity if β is large. It has been found(1,2) that the quasi-static assumption is valid for wall temperature variation when $\gamma < 0.1$ and $\beta < 10$.

Mean temperature of heater at small times is expressed by Eq. (19), too. Correct mean temperature is obtainable with the quasi-static assumption when $\beta > 10$ or $\beta\gamma > 1$.

4. Cooling Phase

The transient energy equation is solved for a stepwise decrease of power. The heat generation rate q_G is reduced to zero at t = 0. Figure 7 shows variation of wall temperature. The quasi-static solution is lower than the transient solution. The difference is large for small β's and it is nearly negligible for $\beta \gtrsim 10$.

Transient heat transfer coefficient is shown in Fig. 8. It decreases less than the steady state value. For $\beta \gtrsim 10$, however, the transient heat transfer coefficient is roughly equal to the steat state one.

5. Some Applications to Pulse Reactors

Numerical results for pulse heating are shown in Figs. 9 and 10. These are numerical solutions for the same cooling condition with different heating wall thicknesses. The wall is heated by a square pulse of width 0.1 msec. The wall material is U-Zr alloy and the coolant is sodium. All the temperatures are normalized by the inner surface temperature (T_c) at the end of the heating period.

The transient heat transfer coefficient is much higher than the steady state one during the pulse heating. The transient solution for wall temperature is given by Eq. (18); the quasi-static assumption results in a serious error. The inner surface temperature (T_c) is calculated correctly with the quasi-static assumption during the heated period. This is because the pulse width is so short that heater temperature rises adiabatically except near the heat transfer surface.

The heat transfer coefficient in the cooling phase is first higher and soon becomes lower than the steady state one. The ratio α/α_{st} decreases down to 0.3 for the thin heater (Fig. 10) and 0.7 for the thick one (Fig. 9).

A large difference exists at small times in the cooling phase between the wall temperatures by the transient and by the quasi-static calculations. This is due to the error in the quasi-static calculation during the heated period.

With lapse of time, the difference in the wall temperatures becomes small for the thick wall (Fig. 9), but stays still significant for the thin wall (Fig. 10).

The inner surface temperature with the quasi-static assumption is roughly exact for the thick wall (Fig. 9) at both small and large times. This is because the inner surface temperature at small times is determined by the transient heat conduction inside the wall; so, it is only slightly affected by the condition on the heat transfer surface.

The value of β for Yayoi-type air-cooled reactors is very large; for example, β of Yayoi is an order of 10^5. It is so large that the quasi-static assumption is valid for transient condition of these reactors.

In case of SORA-type repitive pulse reactors, β is about 10. From the conclusions in Section 3 and 4, one can expect that the quasi-static assumption will not bring a serious error except at small times. As β = 10 is critical, however, further study is preferable. Condition at small times is discussed below.

When a clad exists between a fuel and coolant, Eq. (18) should be applied to the fuel-clad interface not to the clad-coolant interface. If the fuel is assumed in perfect contact with the clad, temperature at the fuel-clad interface rises as

$$T_{f-c}(t) = \frac{1}{1+\gamma_{f-c}} \frac{q_{G,p}}{H} t, \qquad (20)$$

where $q_{G,p}$ is the heat generation rate of the pulse and γ_{f-c} is

$$\gamma_{f-c} = \sqrt{\frac{(\lambda \ \rho \ c_p)_{clad}}{(\lambda \ \rho \ c_p)_{fuel}}} \qquad (21)$$

The fuel does not contact perfectly with the clad, actually. If the gap is assumed an idial annulus, temperature at the fuel surface is given by Eq. (20) with replacement of γ_{f-c} by

$$\gamma_{f-g} = \sqrt{\frac{(\lambda \ \rho \ c_p)_{gap}}{(\lambda \ \rho \ c_p)_{fuel}}} \qquad (22)$$

With the assumption of a constant gap conductance, one gets roughly

$$T_{f-g}(t) \sim \frac{q_{G,p}}{H} t, \tag{23}$$

so an error of $(1+\gamma_{f-g})$ is introduced by the quasi-static assumption. The temperature distribution is shown in Fig. 11 for U-Na interface.

When the gap is filled with gas, γ_{f-g} is an order of 10^{-3}; so the error due to the quasi-static assumption is negligible. When the gap is filled with sodium and the fuel is U-Zr alloy, γ_{f-g} is 1.45. It means that the fuel surface temperature rise is only half of that expected from the quasi-static assumption. This might have a significant effect on the thermal stress inside fuel during pulse heating.

6. Conclusion

The transient heat transfer coefficient becomes higher than the steady state one for step increase of power input; while it becomes lower for step decrease. The quasi-static assumption is roughly valid for calculation of wall temperature if $\beta > 10$ and $\gamma < 0.1$ for step power increase; and if $\beta > 10$ for step decrease.

The fuel surface temperature obtained with the quasi-static assumption is high by a factor of $(1+\gamma)$. If the fuel contacts with gas, γ is an order of 10^{-3}; so the error due to the quasi-static assumption is neglibile. If the fuel contacts with metal or liquid metal, γ is an order of 1, and the error is serious.

Nomenclature

a : thermal diffusivity
c_p : specific heat capacity
D_e : equivalent diameter of an annulus, $= 2(r_o - r_i)$
d_w : thickness of heating wall
g_c : standard acceleration
H : wall heat capacity per unit heat transfer surface
P : pressure
Pr : Prandtl number $= \nu/a_f$

q_G : heat generation rate per unit heat transfer surface
q_n : net heat flux to fluid
Re : Reynolds number = $\bar{u} D_e/\nu$
r : radius
T : temperature
ΔT_w: $T_w - T_f$
t : time
u : velocity
$\bar{u}$: mean velocity
u^* : friction velocity, = $\sqrt{g_c \tau/\rho}$
x : axial distance
y : distance from wall
y^+ : nondimensional distance
Z : nondimensional time
α : heat transfer coefficient, defined by Eq. (13)
β : nondimensional parameter related to wall heat capacity, Eq. (2)
γ : Eq. (3)
ε_H : thermal eddy diffusivity
ε_M : momentum eddy diffusivity
λ : thermal conductivity
ν : kinematic viscosity of fluid
ρ : density
τ : shear stress

Subscripts
c : inner surface of heating wall
f : fluid
g : gap between fuel and clad
h : heater
o : outer wall
quasi : quasi-static solution
st : steady state
tran : transient solution
w : heat transfer wall
0 : initial state
1 : final state

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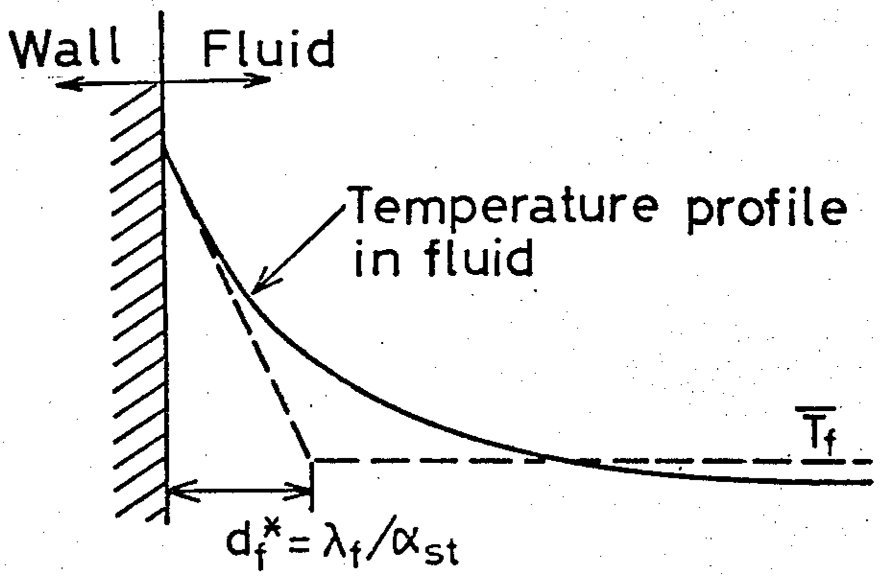

Fig. 1 Equivalant thermal boundary layer thickness.

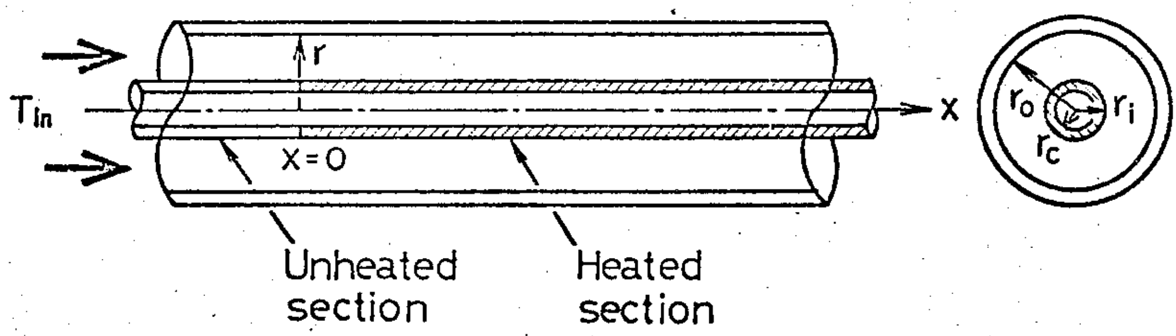

Fig. 2 Co-ordinate system.

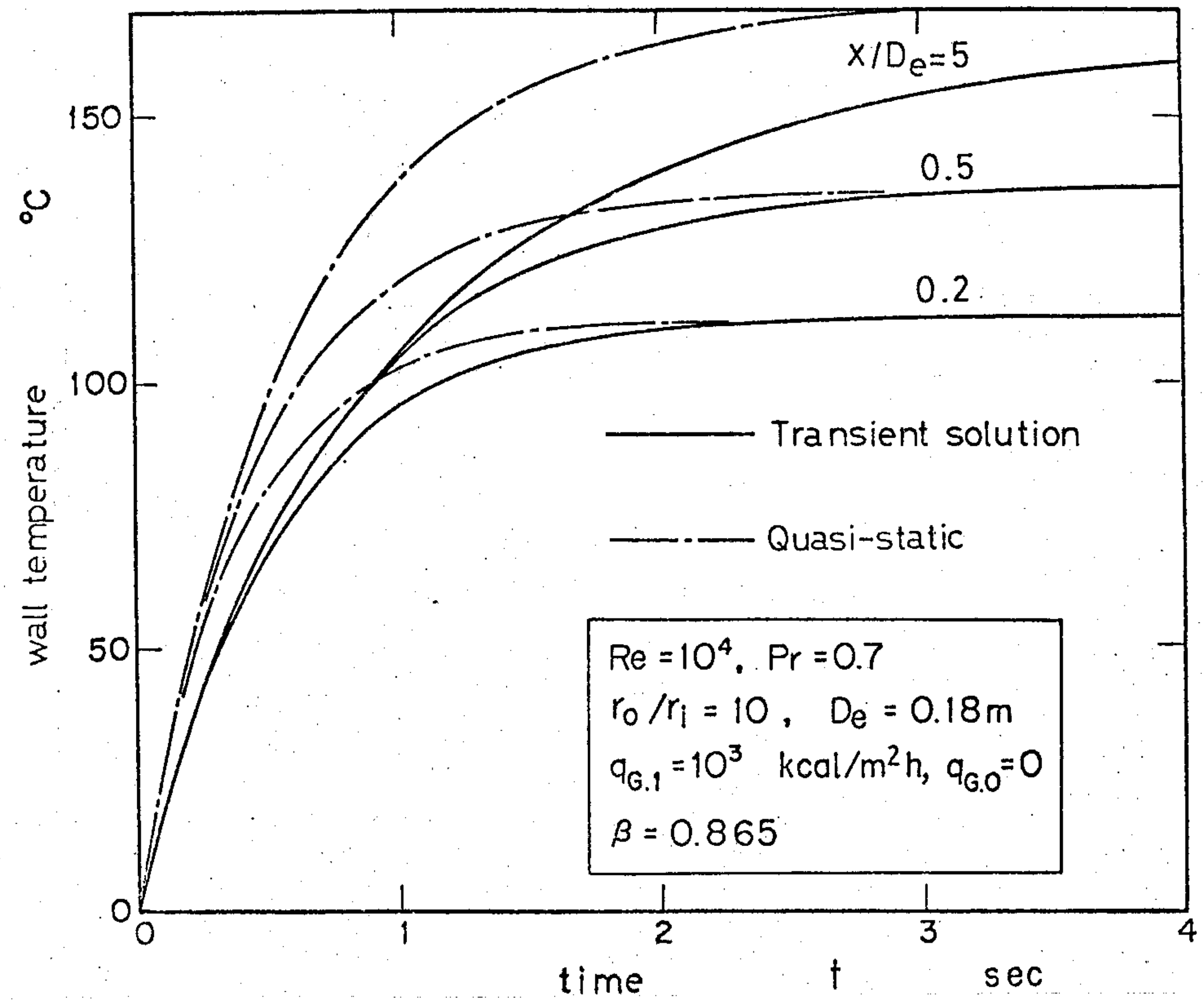

Fig. 3 Variation of wall temperature for stepwise power increase in comparison with quasi-static calculation.

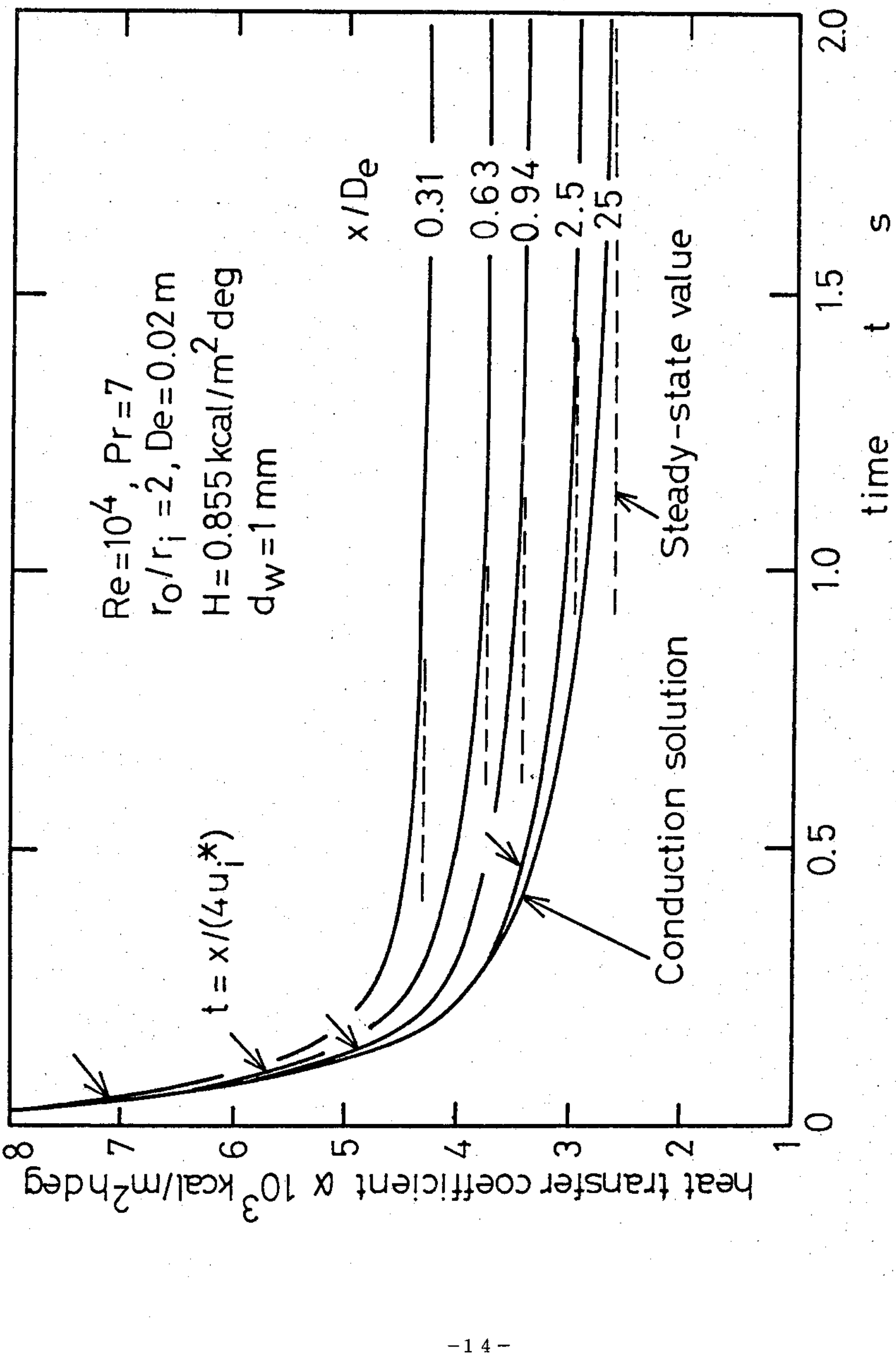

Fig. 4 Transient heat transfer coefficient for stepwise power increase.

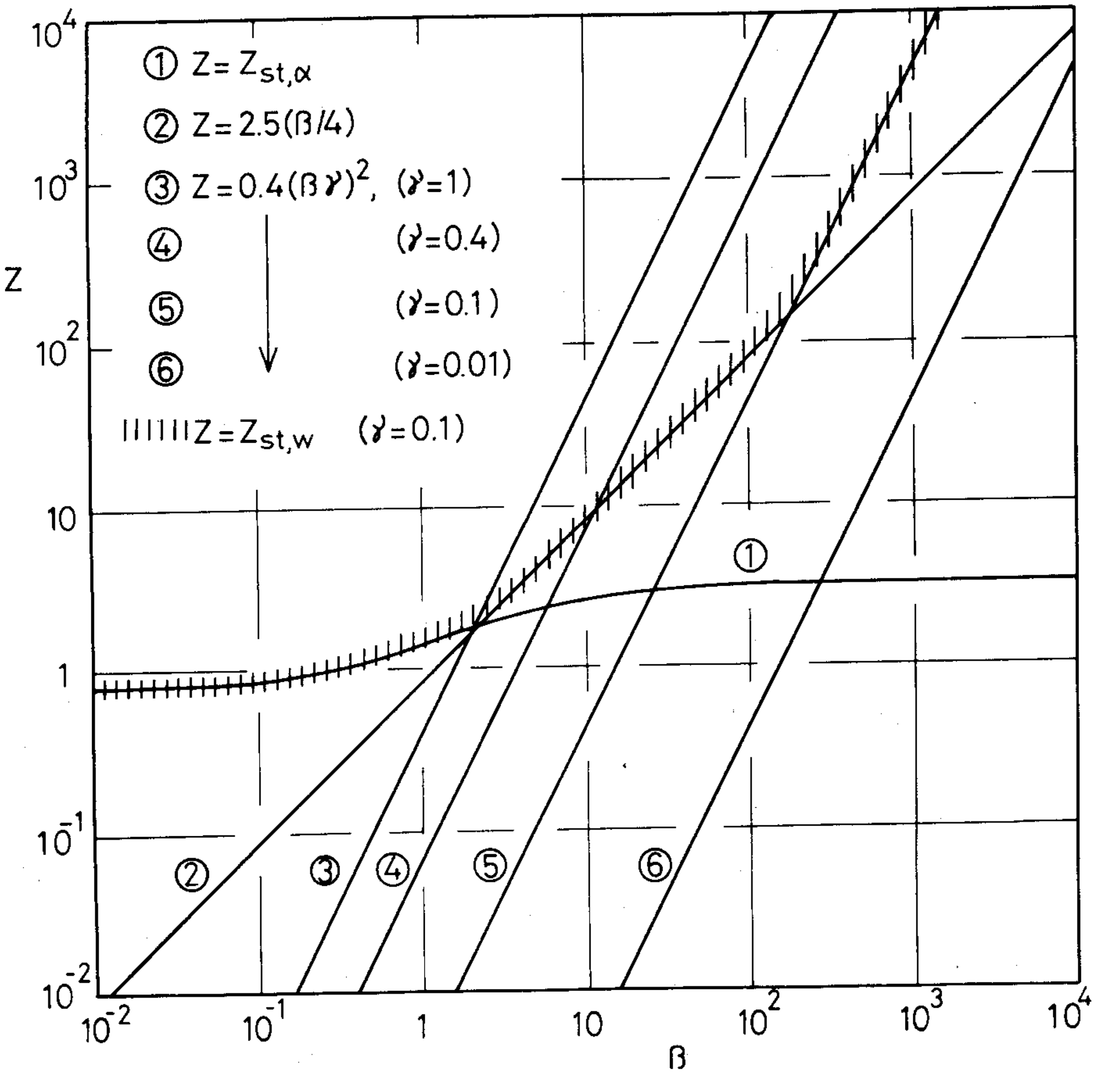

Fig. 5 Steady state time for wall temperature.

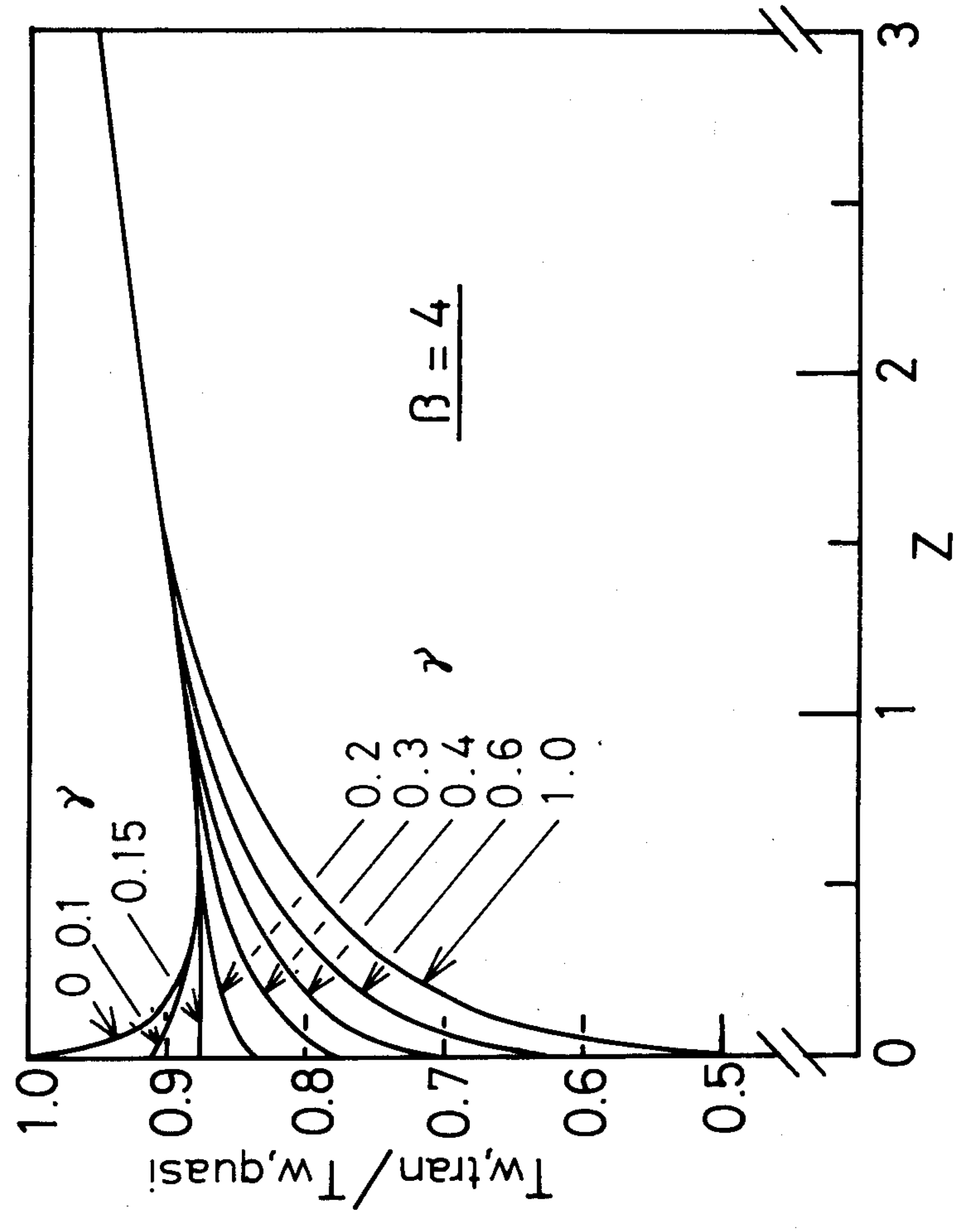

Fig. 6 Comparison of wall temperatures with and without quasi-static assumption for stepwise power increase.

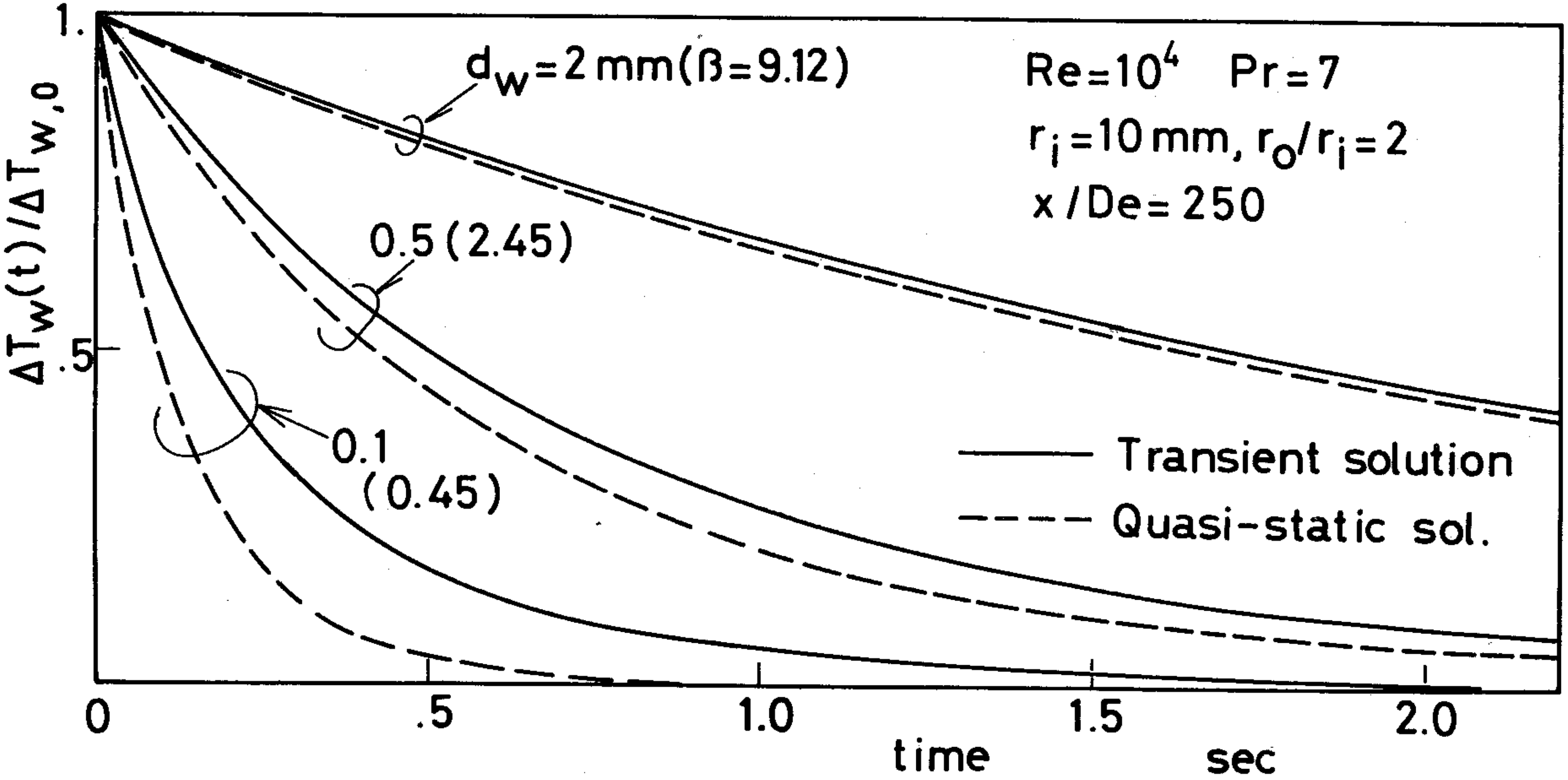

Fig. 7 Variation of wall temperature for stepwise power decrease in comparison with quasi-static calculation.

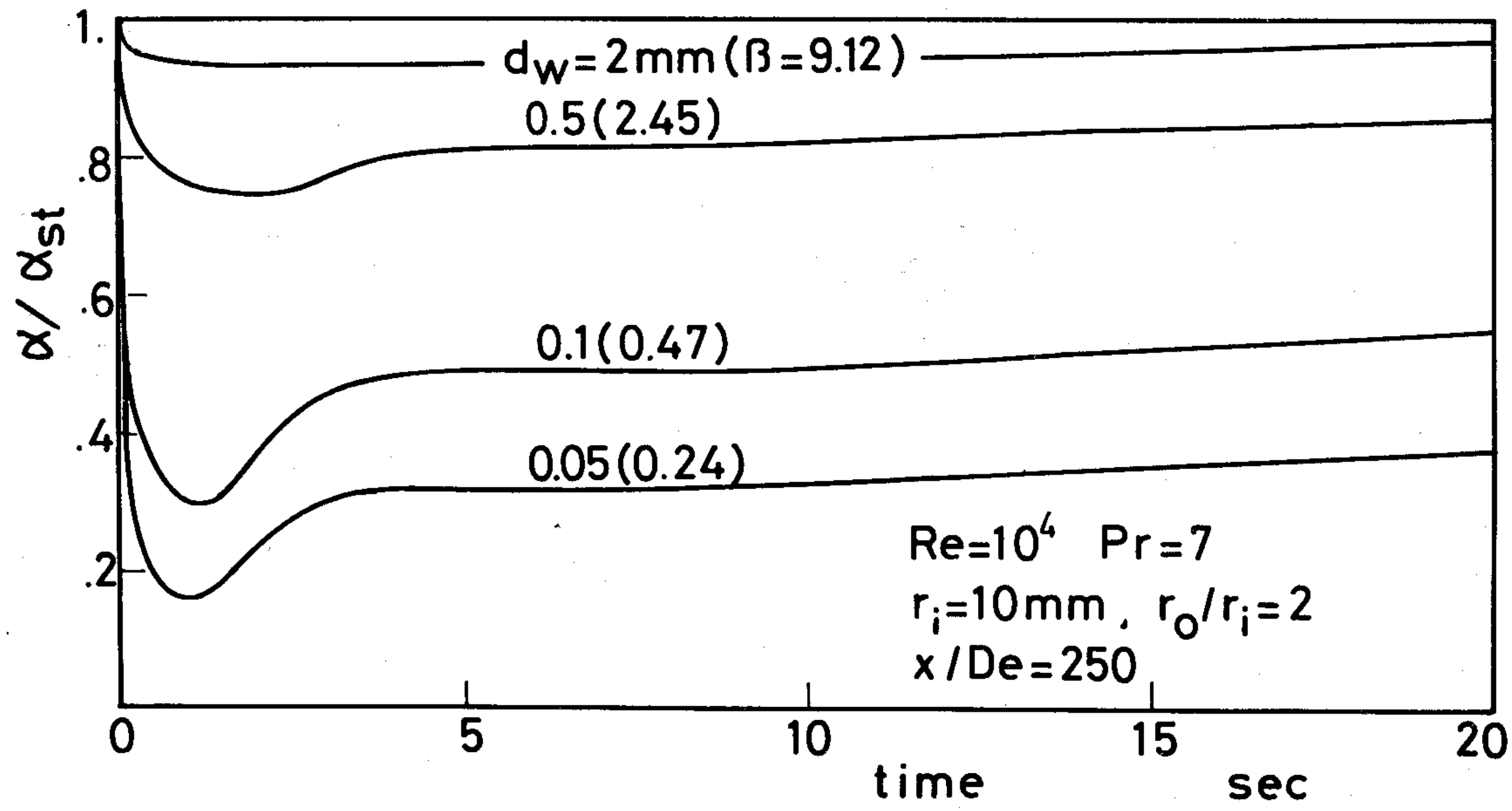

Fig. 8 Transient heat transfer coefficient for stepwise power decrease.

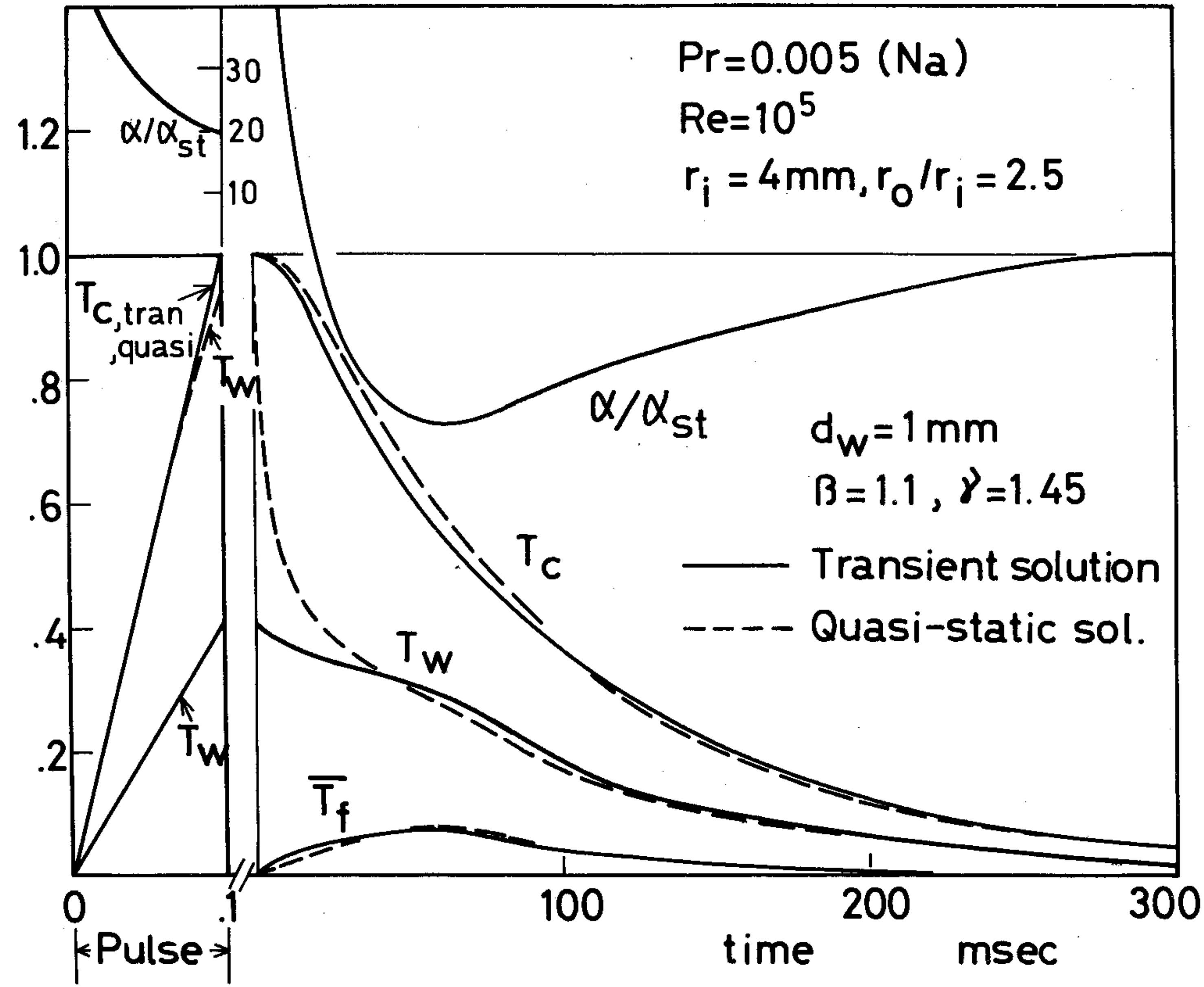

Fig. 9 Variation of temperature and heat transfer coefficient for pulse heating.

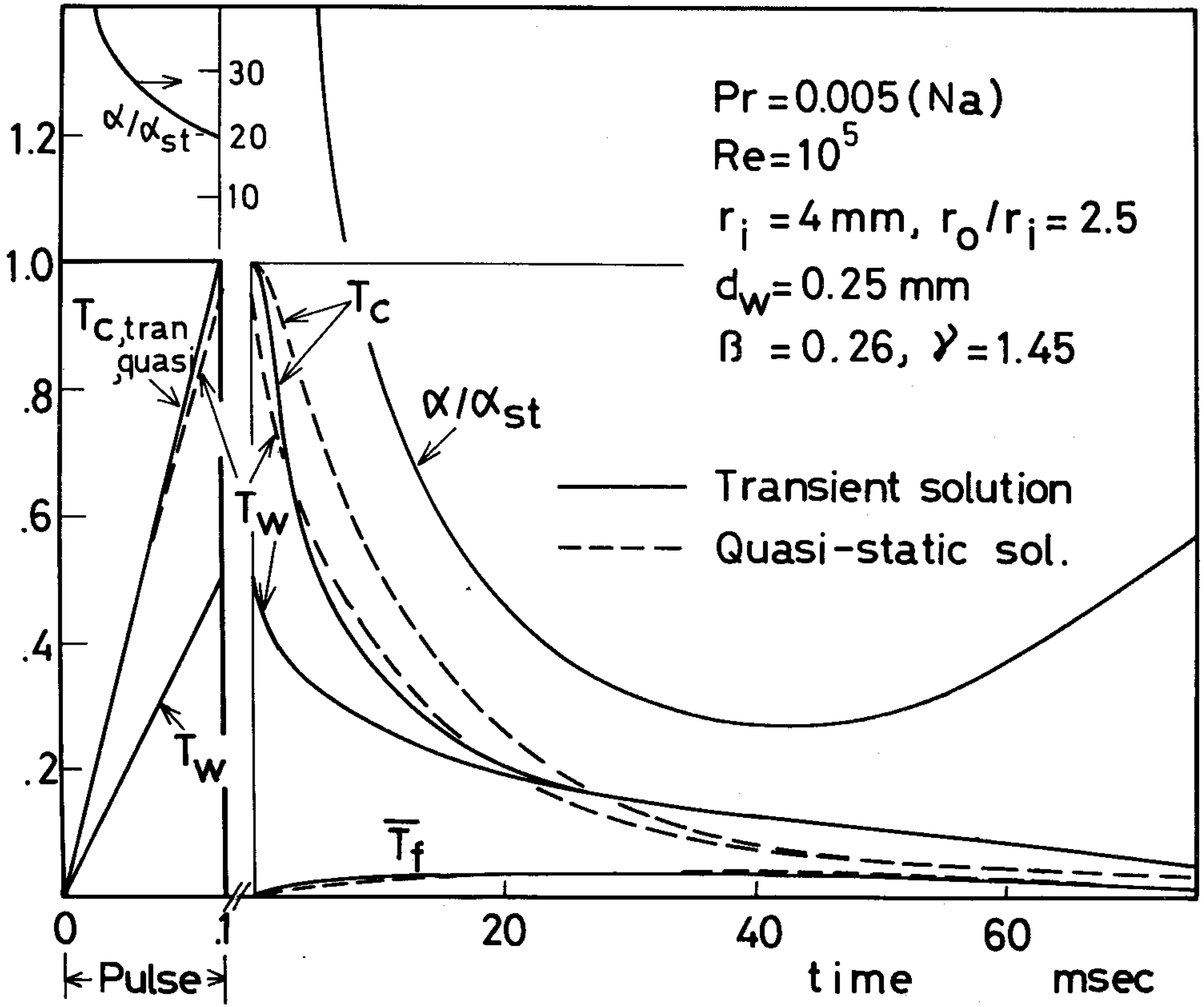

Fig. 10 Variation of temperature and heat transfer coefficient for pulse heating.

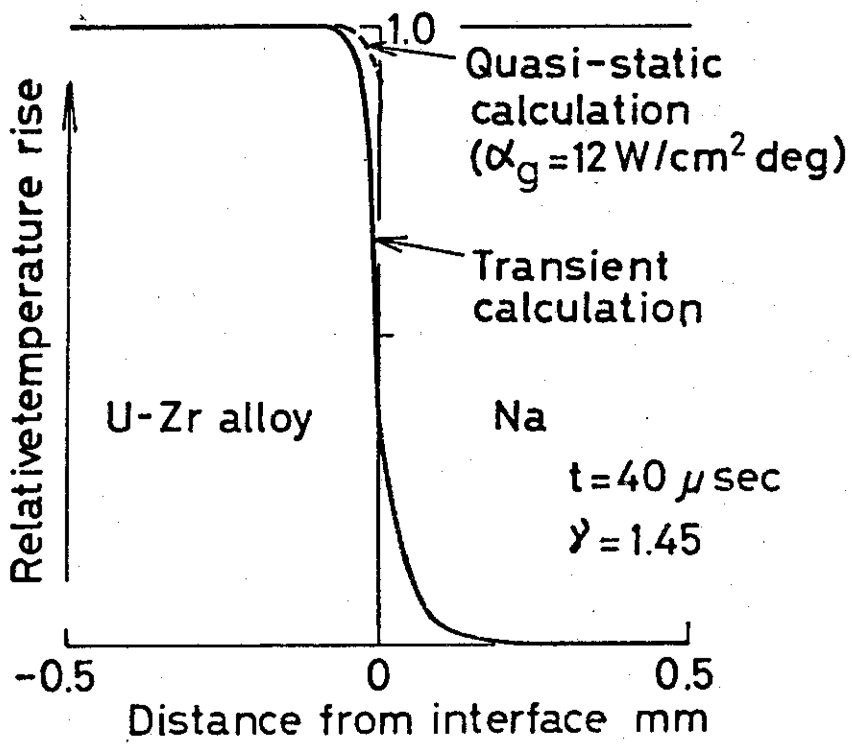

Fig. 11 Transient temperature distribution near U-Na interface.