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A Numerical Code SELENE To Calculate Axisymmetric Toroidal MHD Equilibria

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A new algorithm to calculate an MHD equilibrium of axisymmetric toroidal plasmas has been devised and applied successfully to the case of a fixed boundary nonlinear eigenvalue problem. The algorithm is based on a combination of a usual finite element method and a new iteration scheme whereby the mesh structure for calculation is corrected for the finite elements always to be along magnetic surfaces. Very high accuracy is thus attained in the vicinity of the magnetic axis and in the regions where magnetic surfaces are closely located.

Keywords: MHD equilibrium, Finite element method, Axisymmetric toroidal plasma, Tokamak plasma, Computer code

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軸対称トーラス・プラズマのＭＨＤ平衡計算コード
（ＳＥＬＥＮＥ）

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（１９７８年１２月１５日受理）

軸対称トーラス・プラズマのＭＨＤ平衡の計算をする為の新しい方法を考案し，固定境界非線形固有値問題として扱われる場合に適用して満足すべき結果を得た。この方法は有限要素法と新しい逐次近似法に基づくものである。逐次近似の過程では，有限要素が常に磁気面に沿って並ぶようにメッシュ構造が補正されつつ計算が進行する。この結果，磁気軸近傍および磁気面の混んでいる付近で高い計算精度が達成される。

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1. Introduction

It is required to develop numerical codes by which one can calculate MHD equilibria of a toroidal plasma accurately at high speed. This requirement comes from the fact that recently numerical codes for MHD equilibrium calculation are incorporated with codes which calculate two-dimensional transports in a tokamak plasma[1-3] and multidimensional MHD stability analyses[4,5]. As for the former, the equilibrium is calculated a hundred times to a thousand times during an analysis. In the latter the overall accuracy of stability analysis is usually limited by accuracy of the equilibrium calculation. It is necessary to calculate equilibrium near a magnetic axis very accurately because conventional numerical codes for equilibrium calculation are formulated on the basis of square meshes in an r-z poloidal plane but stability calculation is carried out on square meshes in ψ-χ plane.

A successful example of equilibrium code in which a system of meshes are constructed with respect to the magnetic surface is that based on the waterbag method by Potter et al.[6] In this method pressure balance is satisfied on a finite number of magnetic surfaces and during iterations residual forces push the magnetic surfaces to equilibrium configuration. Takeda applied similar method[7] to write a numerical code in which a functional of the finite element method made of Grad-Shafranov equation is directly minimized by using a nonlinear programming algorithm. This method is not very successful as for the accuracy and CPU time required for the calculation, because nonlinear programming algorithms are too general and not efficient as compared with other specific iteration schemes.

We developed a new equilibrium code in which the structure of meshes are corrected during iterations so that the ψ value of a certain mesh is always same and the magnetic axis is always on a mesh point. The algorithm is, in principle, the usual solution method of a partial differential equation by the finite element method.

In the next section we describe the method of calculation. We show some examples of results of calculations and also examine the accuracy by calculation of the surface integrals and local stabilty criterion in Section 3. Section 4 summarizes and discusses the algorithm.

2. Method of Calculation

An equation to be solved is the Grad-Shafranov's equilibrium equation[8];

$$\Delta^* \psi = \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = -\mu_0 r j_\phi, \tag{1}$$

$$j_\phi = r\frac{dp}{d\psi} + \frac{1}{\mu_0 r}\frac{1}{T}\frac{dT}{d\psi}, \tag{2}$$

where p and T are plasma pressure and toroidal field functions, respectively, and both are functions of the poloidal flux function ψ.

2.1 Numerical formulation of the problem by the finite element method

As we are mainly interested in the development of a high accuracy and high speed MHD equilibrium code we restrict ourselves to a fixed boundary solution as a nonlinear eigenvalue problem. Therefore, the equation is rewritten as[8]

$$\Delta^* \psi = -\lambda \mu_0 r j_{\phi_0}, \tag{3}$$

$$\lambda j_{\phi_0} = j_\phi, \tag{4}$$

The functional form of the toroidal current density j_{ϕ_0} is given before calculation. The eigenvalue (λ) and eigenvector (ψ) are calculated under the boundary conditions;

$$\psi = 0 \qquad \text{at the plasma boundary,} \tag{5}$$

$$\psi = -1 \qquad \text{at the magnetic axis.} \tag{6}$$

Practical constraints such as magnitude of total toroidal current are imposed by using an appropriate scaling law after a solution is obtained(see 2.3). To obtain the solution of the above problem, iterations are performed as follows,

$$\Delta^* \psi^{n+1} = -\lambda^n r j_{\phi_0}\left(\frac{\psi^n}{\psi_0^n}\right), \tag{7}$$

$$\lambda^{n+1} = -\frac{1}{\psi_0^{n+1}}\lambda^n, \tag{8}$$

where ψ^{n+1} is the (n+1)th step's ψ-value at the magnetic axis. The iteration

is terminated by using a stopping parameter ε as

$$\left|\frac{-1}{\psi_0^{n+1}} - 1\right| \leq \varepsilon \ . \tag{9}$$

Solution of eq.(7) is the usual one based on the finite element method where the object function is derived as a weak form of the equation[9],

$$L(\psi^{n+1}) = (\Delta^*\psi^{n+1}, \psi^{n+1}) - 2(f(\psi^n), \psi^{n+1}), \tag{10}$$

$$f(\psi^n) = -\lambda^n \mu_0 r j_{\phi_0}(-\psi^n/\psi_0^n), \tag{11}$$

where (u,v) denotes a scalar product of functions u and v as

$$(u,v) = \int dS\ u \cdot v. \tag{12}$$

In our problem the functional L is derived in a poloidal plane of the axi-symmetric torus as

$$L(\psi^{n+1}) = -\int r dr dz \left\{ \left(\frac{1}{r}\frac{\partial \psi^{n+1}}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial \psi^{n+1}}{\partial z}\right)^2 - \frac{2\lambda^n}{r} j_{\phi_0}\left(\frac{\psi^n}{\psi_0^n}\right)\psi^{n+1} \right\}. \tag{13}$$

By using an appropriate set of basis functions, the functional L (eq.(13)) is represented by N parameters $(\psi_1^{n+1}, \psi_2^{n+1}, \ldots\ldots \psi_N^{n+1})$ where N is the number of the node points inside the boundary in the upper half plane (Fig.1).

Simultaneous linear equations with respect to ψ_i^{n+1} is immediately derived from eq.(13), that is,

$$\mathbf{A}\ \psi^{n+1} = B^n \ , \tag{14}$$

where

$${}^t\psi^{n+1} = (\psi_1^{n+1}, \psi_2^{n+1}, \ldots\ldots \psi_N^{n+1}), \tag{15}$$

$$B^n \cdot \psi^{n+1} = -2(f(\psi^n), \psi^{n+1}). \tag{16}$$

Thus the solution for the (n+1)-th iteration is obtained. The matrix $\mathbf{A}$ is derived as follows. In the finite element method the integral in eq.(13) is represented by a sum of integrals for all the elements,

$$L = \sum_{elm} L_{elm} \ . \tag{17}$$

An integral for an element (L_{elm}) is represented by an integral of a quadratic form of ψ, $\partial\psi/\partial r$, $\partial\psi/\partial z$ with respect to a matrix F as

$$L_{elm} = \int_{elm} (\psi, \frac{\partial\psi}{\partial r}, \frac{\partial\psi}{\partial z}) F \begin{pmatrix} \psi \\ \frac{\partial\psi}{\partial r} \\ \frac{\partial\psi}{\partial z} \end{pmatrix} dS, \tag{18}$$

where $$F = \begin{pmatrix} 0, & 0, & 0 \\ 0, & 1/r, & 0 \\ 0, & 0, & 1/r \end{pmatrix} . \tag{19}$$

In our case (Fig.1), all elements except for the innermost layer elements are rectangular and the meshes in the innermost layer are triangular. For the rectangular meshes the following isoparametric transformation is effective to obtain a continuous solution[9]. First we transform an arbitrary shaped rectangular mesh in r-z plane into a regular square mesh in ξ-η plane (Fig.2). The transformation is represented by a bilinear forms of ξ and η as

$$r = a_0 + a_1\xi + a_2\eta + a_3\xi\eta, \tag{20}$$

$$z = b_0 + b_1\xi + b_2\eta + b_3\xi\eta, \tag{21}$$

where a_i and b_i are functions of r, z coordinates of the node points of the rectangle. Then we represent the ψ-values in the element by a bilinear form of ξ and η, that is,

$$\psi = (1, \xi, \eta, \xi\eta) S \begin{pmatrix} \psi_i \\ \psi_j \\ \psi_k \\ \psi_\ell \end{pmatrix}, \tag{22}$$

where $$S = \begin{pmatrix} \alpha_i & \alpha_j & \alpha_k & \alpha_\ell \\ \beta_i & \beta_j & \beta_k & \beta_\ell \\ \gamma_i & \gamma_j & \gamma_k & \gamma_\ell \\ \delta_i & \delta_j & \delta_k & \delta_\ell \end{pmatrix} , \tag{23}$$

$$\alpha_i = [\xi_j\eta_j(\xi_k\eta_\ell - \xi_\ell\eta_k) + \xi_k\eta_k(\xi_\ell\eta_j - \xi_j\eta_\ell) + \xi_\ell\eta_\ell(\xi_j\eta_k - \xi_k\eta_j)]/D_{ijk\ell}, \tag{24}$$

$$D_{ijk\ell} = \xi_i\xi_i[\eta_j(\xi_k-\xi_\ell)+\eta_k(\xi_\ell-\xi_j)+\eta_\ell(\xi_j-\xi_k)] -\xi_j\xi_j[\eta_k(\xi_\ell-\xi_i)+\eta_\ell(\xi_i-\xi_k)+\eta_i(\xi_k-\xi_\ell)] +\xi_k\xi_k[\eta_\ell(\xi_i-\xi_j)+\eta_i(\xi_j-\xi_\ell)+\eta_j(\xi_\ell-\xi_i)] -\xi_\ell\xi_\ell[\eta_i(\xi_j-\xi_k)+\eta_j(\xi_k-\xi_i)+\eta_k(\xi_i-\xi_j)] , \quad (25)$$

$$\beta_i = [\eta_j\eta_k(\xi_j-\xi_k)+\eta_k\eta_\ell(\xi_k-\xi_\ell)+\eta_\ell\eta_j(\xi_\ell-\xi_j)]/D_{ijk\ell} , \quad (26)$$

$$\gamma_i = [\xi_j\xi_k(\eta_j-\eta_k)+\xi_k\xi_\ell(\eta_k-\eta_\ell)+\xi_\ell\xi_j(\eta_\ell-\eta_j)]/D_{ijk\ell} , \quad (27)$$

$$\delta_i = [\xi_j(\eta_\ell-\eta_k)+\xi_k(\eta_j-\eta_\ell)+\xi_\ell(\eta_k-\eta_j)]/D_{ijk\ell} . \quad (28)$$

Equations for j, k, and ℓ nodes are obtained by changing the suffixes cyclically in the order of i, j, k, and ℓ. In the triangular elements the following equations are applied,

$$D_{ijk} = \xi_i(\eta_j-\eta_k)+\xi_j(\eta_k-\eta_i)+\xi_k(\eta_i-\eta_j) , \quad (29)$$

$$\alpha_i = (\xi_j\eta_k-\xi_k\eta_j)/D_{ijk} , \quad (30)$$

$$\beta_i = (\eta_j-\eta_k)/D_{ijk} , \quad (31)$$

$$\gamma_i = (\xi_j-\xi_k)/D_{ijk} . \quad (32)$$

Numerical values for the above representation is easily derived in our case and the matrix $\underline{S}$ is obtained as

$$\underline{S} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 \end{pmatrix} , \quad \text{rectangular case,} \quad (33)$$

$$\underline{S} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 2/3 & -1/3 & -1/3 \\ 0 & -1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}, \quad \text{triangular case,} \quad (34)$$

Thus we obtain a set of values for ψ, $\partial\psi/\partial\xi$, $\partial\psi/\partial\eta$ for an arbitrary point in an element as

$$\begin{pmatrix} \psi \\ \partial\psi/\partial\xi \\ \partial\psi/\partial\eta \end{pmatrix} = \begin{pmatrix} 1 & \xi & \eta & \xi\eta \\ 0 & 1 & 0 & \eta \\ 0 & 0 & 1 & \xi \end{pmatrix} \underline{S} \begin{pmatrix} \psi_i \\ \psi_j \\ \psi_k \\ \psi_l \end{pmatrix}, \tag{35}$$

To evaluate the integral for each element (L_{elm}) we should calculate the coefficient matrix $\underline{T}$ for the transformation ($\partial\psi/\partial\xi$, $\partial\psi/\partial\eta \rightarrow \partial\psi/\partial r$, $\partial\psi/\partial z$),

$$\begin{pmatrix} \psi \\ \partial\psi/\partial r \\ \partial\psi/\partial z \end{pmatrix} = \underline{T} \begin{pmatrix} \psi \\ \partial\psi/\partial\xi \\ \partial\psi/\partial\eta \end{pmatrix}, \tag{36}$$

where

$$\underline{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & b_2+b_3\xi & -b_1-b_3\eta \\ 0 & -a_2-a_3\xi & a_1+a_3\eta \end{pmatrix} \frac{1}{J}, \tag{37}$$

where $$J = (a_1+a_3\eta)(b_2+b_3\xi)-(a_2+a_3\xi)(b_1+b_3\eta). \tag{38}$$

By the above procedure we obtain a quadratic form of ψ_i, ψ_j, ψ_k, and ψ_ℓ for the functional L_{elm}. From the quadratic form corresponding to L (eq.(17)) the matrix A is derived.

2.2 Iteration schemes

There are two kinds of iteration process in this algorithm, one is the inner iteration by which the eigenvalue λ is converged, and the other is the outer iteration by which the mesh structure is converged. The most important feature of this algorithm is that the positions of the node points are corrected every outer iteration so that the elements are defined along the magnetic surfaces. The advantages of the algorithm are evident, that is, the accuracy of the calculation near the magnetic axis is extremely good as compared with the usual methods and that of outside of the magnetic axis is also very good even when we treat a high beta toroidal equilibrium.

This algorithm is similar to that of the waterbag method by Potter et al. in that the mesh points are traced every iteration. The waterbag method is, however, principally the Lagrangian calculation and ours is the sequence of the Eulerian calculation for a system of variable meshes. The disadvantage of the former algorithm that the position of the magnetic axis is

difficult to calculate accurately is easily overcome by our method.

The overall iteration procedure is summarized as follows,

(1) Prepare initial meshes.

(2) Solve the equation for ψ^{n+1},

$$\psi^{n+1} = \Delta^{*-1} \lambda^n f(\psi^n, r). \tag{39}$$

(3) Normalize the ψ values by the value at the magnetic axis and obtain the (n+1)-th eigenvalue λ^{n+1},

$$\lambda^{n+1} = \frac{1}{\psi_0^{n+1}} \lambda^n . \tag{40}$$

(4) If $||\psi_0^{n+1}|-1|$ is not sufficiently small then return to the process (2).

(5) Calculate equi-ψ surfaces and reconstruct meshes.

(6) If $|r_{axis}^{m+1} - r_{axis}^{m}|$ is not sufficiently small then return to the process (2), where m is the number of the outer iteration.

(7) End of iterations.

The flowchart and program structure are shown in Fig.3.

2.3 Scaling of the results

In our calculation the range of the variable ψ is restricted within [-1.0,0.0]. Therefore, it is necessary to transform the variable if the physical quantities such as the total plasma current, maximum pressure and so on are to be adjusted to given values. For this purpose the scaling of equilibrium is often used.[5,10] When an equilibrium is transformed into a new equilkbrium, the new equilibrium quantities $\hat{X}$'s are derived using scaling factor σ and the old quantities X's as

$$\hat{\psi} = \sigma \psi \quad , \tag{41}$$

$$\hat{B}_p = \sigma B_p, \tag{42}$$

$$\hat{j}_\phi = \sigma j_\phi, \tag{43}$$

$$\hat{p} = \sigma^2 p, \tag{44}$$

$$\left(T \frac{dT}{d\psi}\right)\hat{} = \sigma T \frac{dT}{d\psi} , \tag{45}$$

$$\hat{T}^2 = T_a^2 + \sigma^2(T^2 - T_a^2), \tag{46}$$

$$\hat{q}^2 = q^2 \frac{\hat{T}^2}{T^2\sigma^2}, \tag{47}$$

If the prescribed total plasma current is I_p the scaling factor σ is chosen as

$$\sigma = \frac{I_p}{\lambda \iint j_{\phi o} dS} . \tag{48}$$

It is needless to say that, by the scaling, the functional dependences of p and T on Φ are changed from the given ones. In some cases this causes serious problems. But in the cases such as the optimization of the MHD equilibrium with respect to the beta values or two-dimensional transport calculation this is not a very important restriction. Therefore we use the above scaling procedure in our code.

2.4 Surface integrals and local stability criterion

When we apply the results of the equilibrium calculations to the stability and transport analyses, we should calculate magnetic field and many kinds of surface integrals with high accuracy. For convenience of the numerical calculation, these quantities are classified into 5 groups as

(1) Quantities which are derived directly from the equilibrium calculations by the finite element method.
In our numerical program these quantities are evaluated at the center of each element.

$$\psi, \quad \frac{\partial\psi}{\partial r}, \quad \frac{\partial\psi}{\partial z}, \quad r, \quad z, \quad d\ell, \quad T^2, \quad p, \quad j_\phi, \quad s,$$

where $d\ell$ is the line length across an element along the magnetic surface and s is the area of the element.

(2) Surface integrals defined as functions of the magnetic surface.
1. Safety factor: q

$$q = \frac{1}{2\pi} \oint \frac{T}{r^2 B_p} d\ell , \tag{49}$$

2. Other surface quantities which are derived directly from the quantities in the group (1).

$$C_1 = \oint \frac{d\ell}{B_p^3 r^2} , \tag{50}$$

$$C_2 = \oint \frac{d\ell}{B_p^3 r^4} , \tag{51}$$

$$C_3 = \oint \frac{d\ell}{B_p^3} , \tag{52}$$

$$C_4 = \oint \frac{r^2 d\ell}{B_p} , \tag{53}$$

$$C_5 = \oint B_p d\ell , \tag{54}$$

$$C_6 = \oint B_p r^2 d\ell , \tag{55}$$

$$V = \int dV , \tag{56}$$

$$\frac{dV}{d\psi} = 2\pi \oint \frac{d\ell}{B_p} , \tag{57}$$

$$\frac{dU}{d\psi} = 2\pi \oint \frac{d\ell}{B_p r^2} , \tag{58}$$

these quantities are also used for the calculation of the Mercier criterion.

(3) Poloidal coordinate.

$$\chi = \frac{1}{q(\psi)} \int \frac{T d\ell}{B_p r^2} . \tag{59}$$

(4) Volume integrals necessary to evaluate beta values.[5)]

$$I_{pr} = \iint J d\psi d\chi p , \tag{60}$$

$$I_{pr2} = \iint J d\psi d\chi p^2, \tag{61}$$

$$I_{B2} = \iint J d\psi d\chi \left(B_p^2 + \frac{T^2}{r^2}\right), \tag{62}$$

$$I_V = \iint J d\psi d\chi, \tag{63}$$

$$I_I = \iint J d\psi d\chi j_\phi. \tag{64}$$

By using these quantities, the usual beta value(β), the poloidal beta value(β_p) and the reactor plasma beta value(β^*) are given as

$$\beta = 2\frac{I_{pr}}{I_{B2}}, \tag{65}$$

$$\beta_p = \frac{8\pi}{\mu_o}\frac{I_{pr}}{I_I^2}, \tag{66}$$

$$\beta^* = 2\frac{I_{pr}^2 I_V}{I_{B2}}, \tag{67}$$

(5) Quantities obtained from derivatives of the above quantities with respect to ψ.

Magnetic shear S (Eq.68) and ψ derivative of pressure belong to this class.

$$S = -\frac{\frac{d}{d\psi}\left[T \oint (1/r^2 B_p) d\ell\right]}{\left(\frac{dV}{d\psi}\right)^3}. \tag{68}$$

These quantities are transfered to the stability analyses codes and transport analyses codes. The mercier criterion is evaluated in our equilibrium code. The plasma is stable against a local instability if

$$M = M_0 + M_1 + M_2 > 0, \tag{69}$$

for all ψ values, where

$$M_0 = \frac{1}{4} S^2, \tag{70}$$

$$M_1 = \mu_o \frac{dp}{d\psi} \left[C_1 \frac{T\,S}{\left(\frac{dV}{d\psi}\right)^3} + \left(C_2 T^2 + \frac{dU}{d\psi} \right) \frac{\left(\frac{d^2V}{d\psi^2}\right)}{\left(\frac{dV}{d\psi}\right)^6} \right] , \tag{71}$$

$$M_2 = -\mu_o^2 \left(\frac{dp}{d\psi}\right)^2 \left[T^2 \left(C_2 C_3 - C_1^2 \right) + C_3 \frac{dU}{d\psi} \right] \frac{1}{\left(\frac{dV}{d\psi}\right)^6} , \tag{72}$$

3. Results of Calculation

One of the main purpose of development of this code is to calculate accurately wider class of MHD equilibrium, that is, the functional forms of $p(\psi)$ and $T(\psi)$ should be selected arbitrarily as possible. This code can calculate an equilibrium for numerically given distributions of pressure and toroidal field functions, which is very important when we use the code to transport analyses.

In the following, however, we choose the dependences of p and T adopted by ORNL group for the convenience of parameter survey[11]. The pressure of the plasma is expressed by a nonlinear function of ψ as

$$p = \beta_J p_0' \left\{ (\psi-\psi_b) - \frac{\alpha}{2}\left[(\psi-\psi_m)^2 - (\psi_b-\psi_m)^2 \right] \right.$$
$$\left. + \frac{\gamma}{L+1} \left[(\psi-\psi_m)^{L+1} - (\psi_b-\psi_m)^{L+1} \right] \right\} , \quad L \geq 4 \tag{73}$$

$$\gamma = - \frac{1 - \alpha(\psi_m-\psi_b)}{(\psi_b-\psi_m)^L} , \tag{74}$$

where ψ_b and ψ_m are the ψ values at the plasma boundary (ψ_b=0) and the magnetic axis (ψ_m=-1), respectively, and α is a parameter which determines the width of the current column. The toroidal field function T is related to the pressure function as

$$TT' = \left(\frac{1}{\beta_J} - 1 \right) p' \frac{1}{\langle r^{-2} \rangle} \sim \left(\frac{1}{\beta_J} - 1 \right) R_\theta^2 \, p' , \tag{75}$$

where β_J is a parameter which is approximately equal to the poloidal beta value. In the following several examples of the results are presented.

$$M_1 = \mu_o \frac{dp}{d\psi} \left[C_1 \frac{T\,S}{\left(\frac{dV}{d\psi}\right)^3} + \left(C_2 T^2 + \frac{dU}{d\psi} \right) \frac{\left(\frac{d^2V}{d\psi^2}\right)}{\left(\frac{dV}{d\psi}\right)^6} \right] , \tag{71}$$

$$M_2 = -\mu_o^2 \left(\frac{dp}{d\psi}\right)^2 \left[T^2 \left(C_2 C_3 - C_1^2 \right) + C_3 \frac{dU}{d\psi} \right] \frac{1}{\left(\frac{dV}{d\psi}\right)^6} , \tag{72}$$

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(1) Example 1.

Convergence with respect to the number of the finite elements

Poloidal mesh number (MDIV) and radial mesh number (NDIV) are varied from 10 to 30 (Fig.4). The quadratic convergence is shown by the relation between the eigenvalue λ and the number of the meshes (Fig.5).

(2) Example 2.

Accuracy of calculation of magnetic shear

One of most stringent checks of the accuracy of the calculation is the comparison between the numerically obtained magnetic shear and analytically obtained one, as it contains several quantities which are derived through differentiation processes. In Fig.6, the numerically obtained shears for several large aspect ratio plasmas are shown with analytically obtained ones for cylindrical plasmas of corresponding pitch length (L/a). There are slight differences between the analytical and numerical results, especially, near the magnetic axis.

(3) Example 3.

High beta equilibria

To show that the algorithm is especially effective for the analyses of a high beta toroidal equilibria, we calculated the equilibria for the cases of β_J = 1, 2, 3 (Fig.7). In the case of β_J= 3, the magnetic axis is shifted outward by about 40 % of the minor radius of the plasma and the mesh width outside the axis is as small as 1/3 times that inside the axis. Therefore, in such a case, about ten times as many a number of meshes are required if one calculate the equilibrium by the usual rectangular mesh system with the same accuracy.

(4) Example 4.

Scaling of the equilibrium

To see the effect of the scaling, we show the changes of the equilibria for the same p, T functions but different scaling factor in Fig.8.

4. Summary and Discussions

A computer code to calculate an MHD equilibrium of an axisymmetric toroidal plasma is written on the basis of a combination of a new iteration scheme and the finite element method. By this algorithm mesh structure for the calculation is corrected every outer iteration so that the finite elements are always alighned along magnetic surfaces. Consequently, very high

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(2) Example 2.

Accuracy of calculation of magnetic shear

One of most stringent checks of the accuracy of the calculation is the comparison between the numerically obtained magnetic shear and analytically obtained one, as it contains several quantities which are derived through differentiation processes. In Fig.6, the numerically obtained shears for several large aspect ratio plasmas are shown with analytically obtained ones for cylindrical plasmas of corresponding pitch length (L/a). There are slight differences between the analytical and numerical results, especially, near the magnetic axis.

(3) Example 3.

High beta equilibria

To show that the algorithm is especially effective for the analyses of a high beta toroidal equilibria, we calculated the equilibria for the cases of β_J = 1, 2, 3 (Fig.7). In the case of β_J= 3, the magnetic axis is shifted outward by about 40 % of the minor radius of the plasma and the mesh width outside the axis is as small as 1/3 times that inside the axis. Therefore, in such a case, about ten times as many a number of meshes are required if one calculate the equilibrium by the usual rectangular mesh system with the same accuracy.

(4) Example 4.

Scaling of the equilibrium

To see the effect of the scaling, we show the changes of the equilibria for the same p, T functions but different scaling factor in Fig.8.

4. Summary and Discussions

A computer code to calculate an MHD equilibrium of an axisymmetric toroidal plasma is written on the basis of a combination of a new iteration scheme and the finite element method. By this algorithm mesh structure for the calculation is corrected every outer iteration so that the finite elements are always alighned along magnetic surfaces. Consequently, very high

accuracy is realized near the magnetic axis and regions where magnetic surfaces are located closely. For example, in the case of the MHD stability analyses by the ERATO code[5] it is expected that the necessary number of meshes for equilibrium calculations ($N_\psi = N_\chi = N$) is about $N \times N$ (our code) as compared with $N_r \times N_z \sim 6N \times 3N$ (usual codes). High accuracy of the calculation is demonstrated by several examples, especially, by the calculations of surface integrals.

The present numerical code is applicable only for a fixed boundary nonlinear eigenvalue problem as shown in the previous sections. However, practically, numerical codes for a free boundary plasma or for an FCT equilibrium[12] are required. Extension of our code to these problems is not essentially difficult.

Acknowledgments

The authors would like to thank Dr. M. Tanaka for his valuable discussions. They are also very grateful to Drs. S. Mori and Y. Obata for their continuing encouragements.

References

1) Hogan, J.T., "The accessibility of high beta tokamak states", ORNL/TM-6049 May, 1978.

2) Byrne, R.N. and Klein, H.H., J. Computational Phys. 26 352 (1978).

3) Helton, F.J., Miller, R.L., and Rawls, J.M., J. Computational Phys. 24 117 (1977).

4) Grimm, R.C., Greene, J.M., and Johnson, J.L., "Computation of the Magnetohydrodynamic Spectrum in Axisymmetric Toroidal Confinement Systems" in *Methods in Computational Physics* (Academic Press Inc., N.Y., 1976) Vol. 16 p.253.

5) Gruber, R., "ERATO stability code" to be published in Computer Physics Communications.

6) Potter, D., "Waterbag Methods in Magnetohydrodynamics" in *Methods in Computational Physics* (Academic Press Inc., N.Y., 1976) Vol.16 p.43.

7) Takeda, T., "Application of nonlinear optimization to solution of differential and integral equations", IPP 6/147 F3 (1976).

8) Lackner, K., Computer Physics Communications 12 33 (1976).

9) Strang, G. and Fix, G.J., *An Analysis of the Finite Element Method* (Prentice-Hall, Inc., N.J., 1973).

10) Bateman, G. and Peng, Y.-K.M., Phys. Rev. Letters 38 829 (1977).

11) Peng, Y.-K.M., Dory, R.A., Nelson, D.B., and Sayer, R.O., Phys. Fluids 21 467 (1978).

12) Clarke, J.F. and Sigmar, D.J., Phys. Rev. Letters 38 70 (1977).

Figure captions

Fig.1 An example of mesh structure for equilibrium calculation.

Fig.2 Isoparametric transformation.

(a) A triangular mesh

(b) A rectangular mesh

Fig.3 Flow chart (a) and program structure (b) of the SELENE code.

Fig.4 Convergence of the scheme with respect to number of finite elements.

(a) NDIV=MDIV=10

(b) NDIV=MDIV=20

(c) NDIV=MDIV=30

Fig.5 Convergence of eigenvalue λ and the safety factor at the magnetic axis (q_o).

Fig.6 Magnetic shears of large aspect ratio plasmas(numerical) and a cylindrical plasma(analytic).

Fig.7 Examples of results for different values of β_J.

(a) β_J=1

(b) β_J=2

(c) β_J=3

Fig.8 Examples of results for different values of scaling factor σ.

(a) σ=0.028

(b) σ=0.030

(c) σ=0.032

(d) σ=0.034

(e) σ=0.036

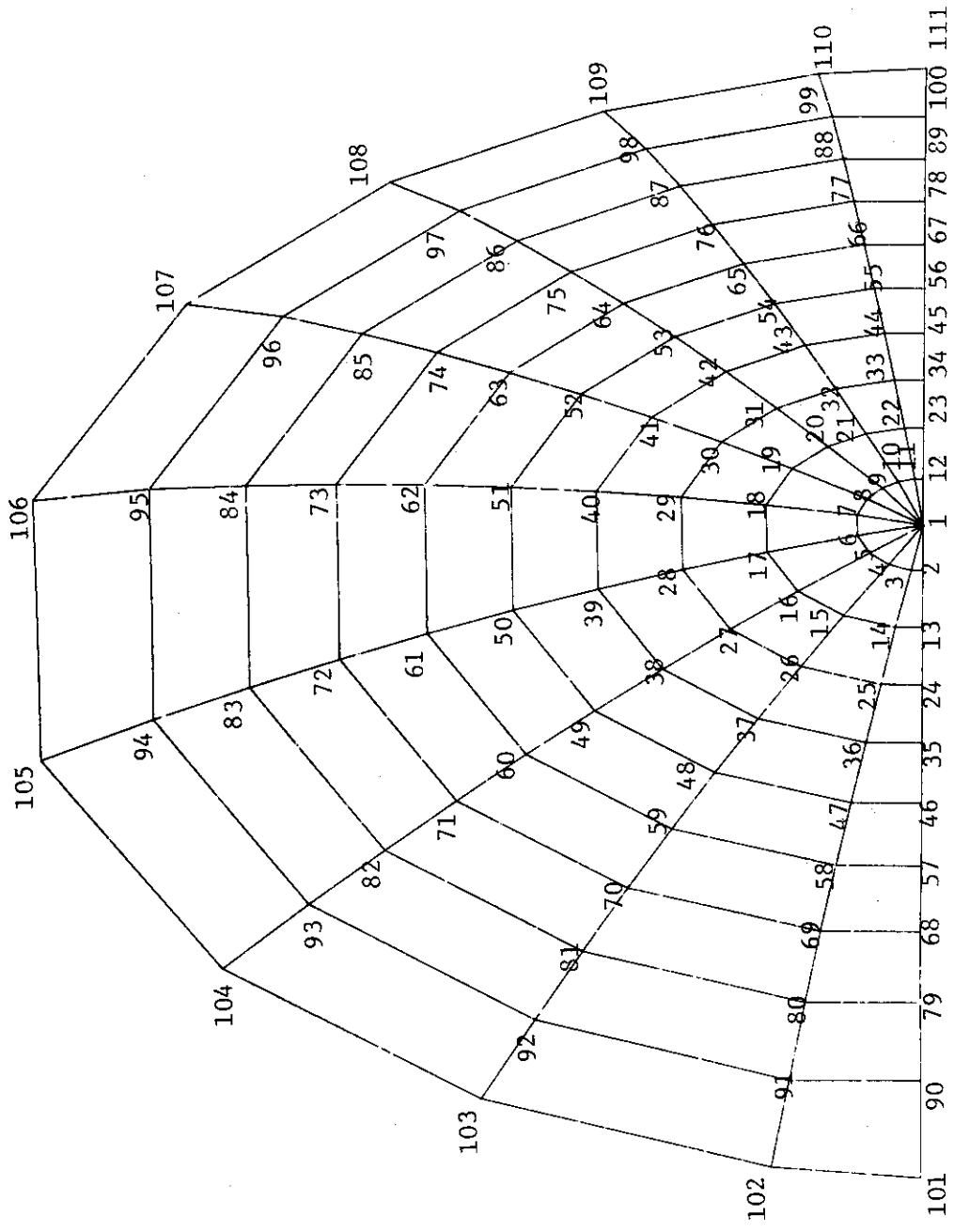
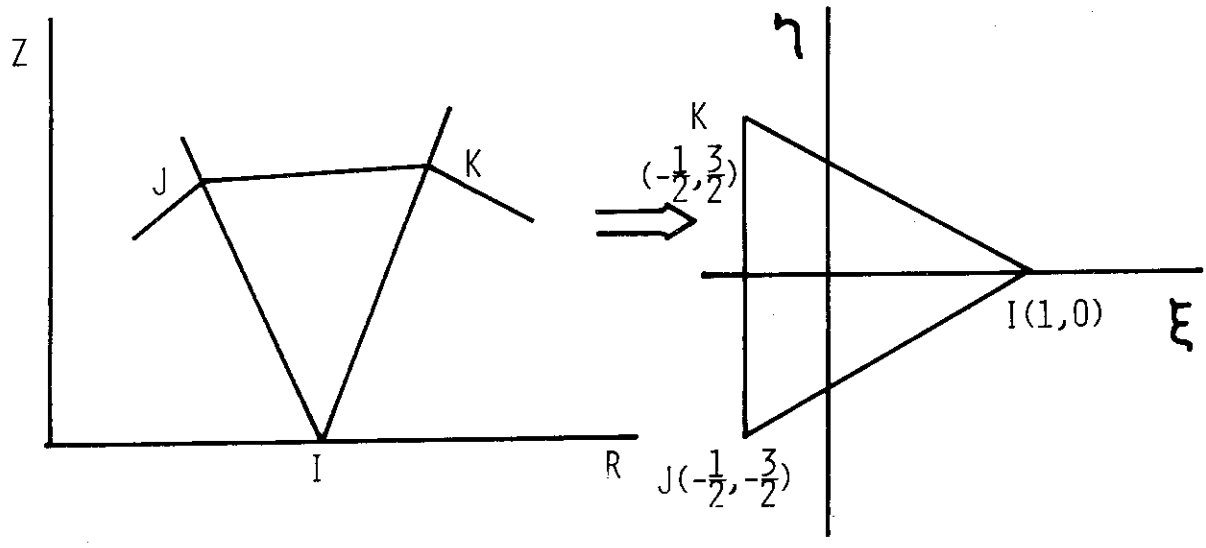
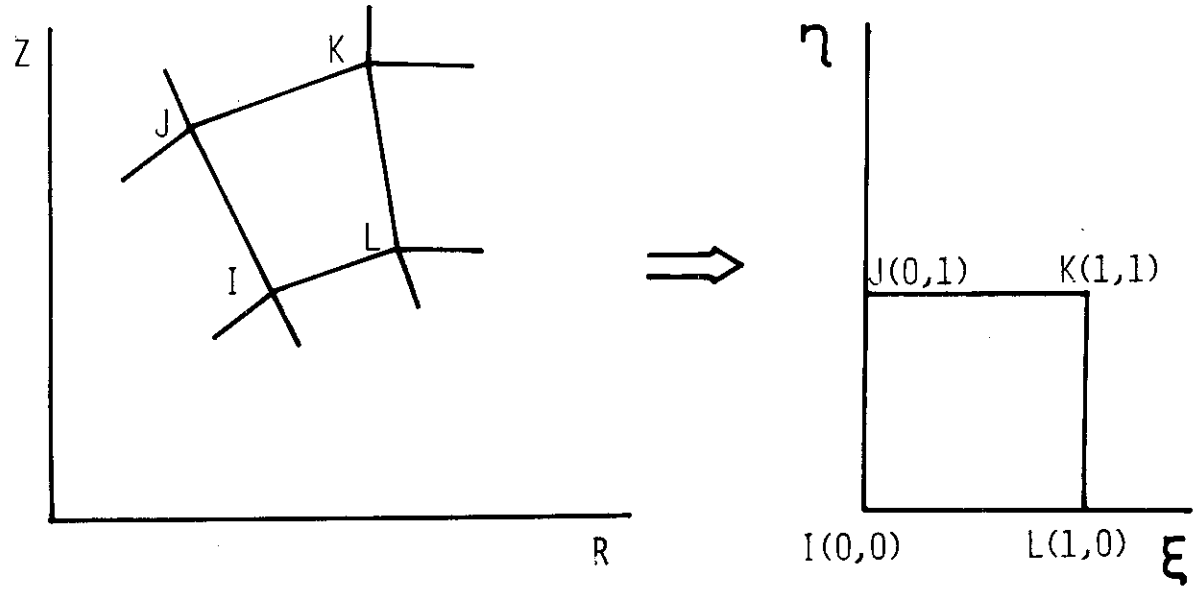


Fig.1 An example of mesh structure for equilibrium calculation.



(a)



(b)

Fig.2 Isoparametric transformation.
(a) A triangular mesh
(b) A rectangular mesh

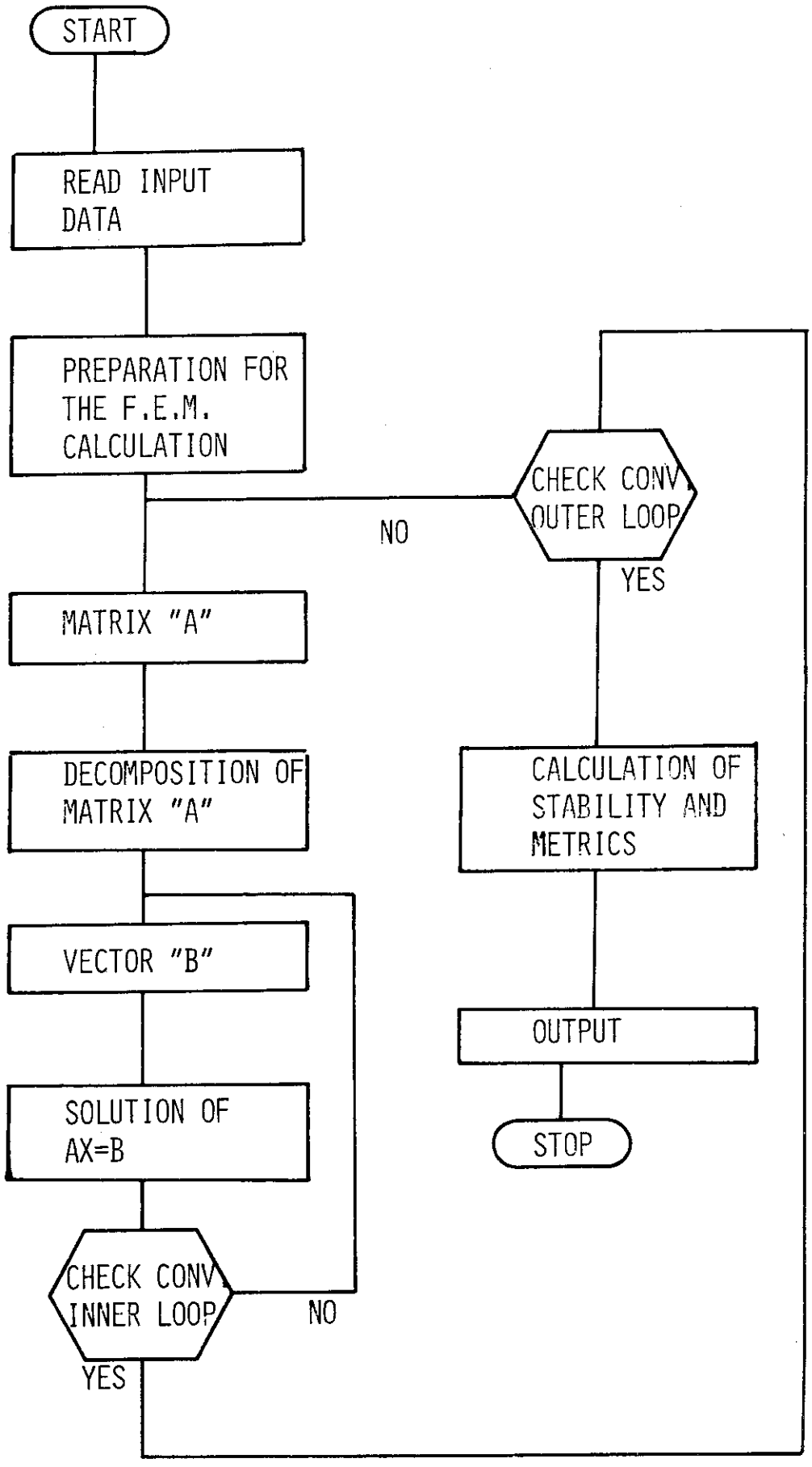


Fig.3 Flow chart (a) and program structure

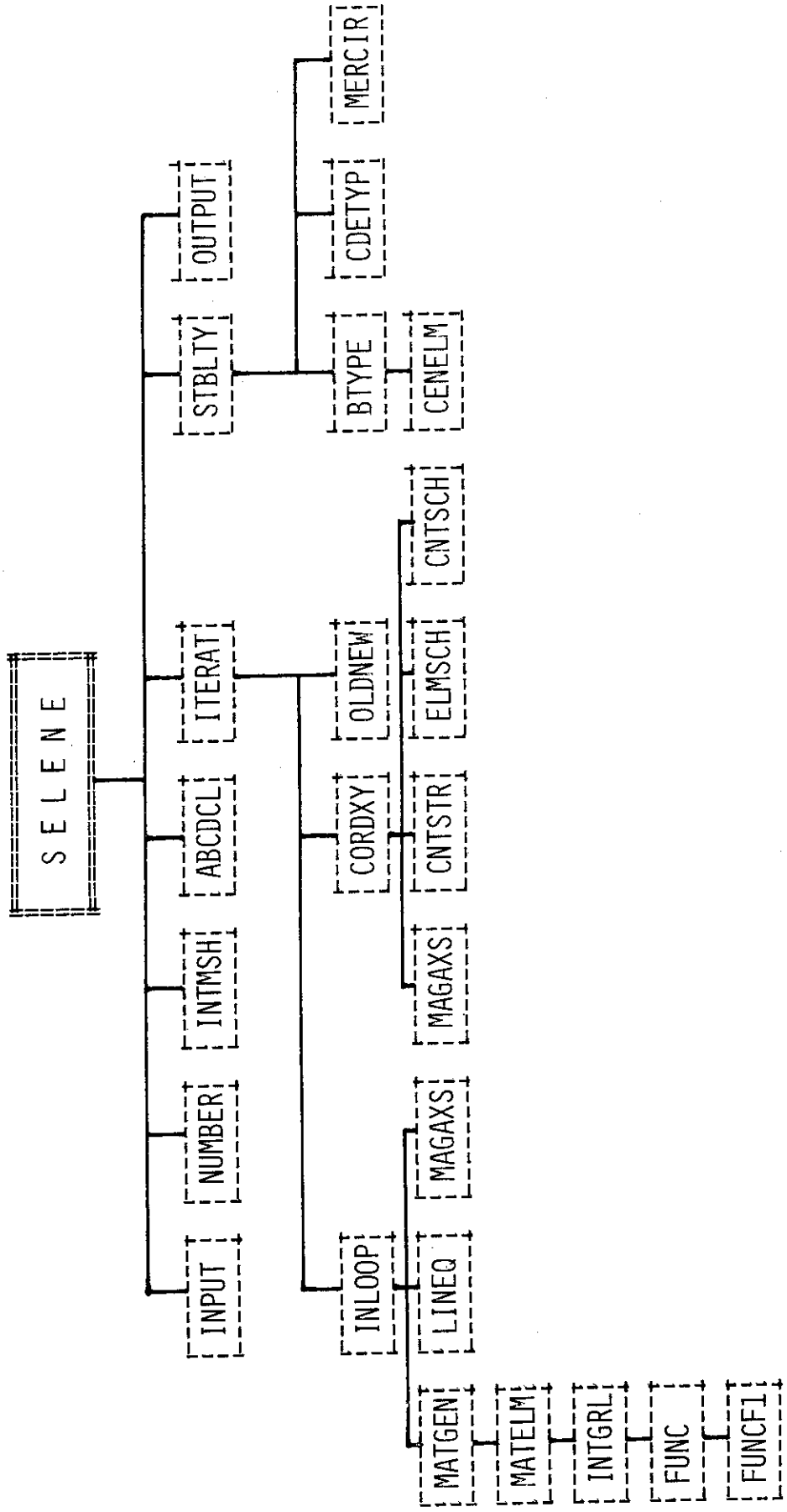


Fig.3 Flow chart (b) of the SELENE code.

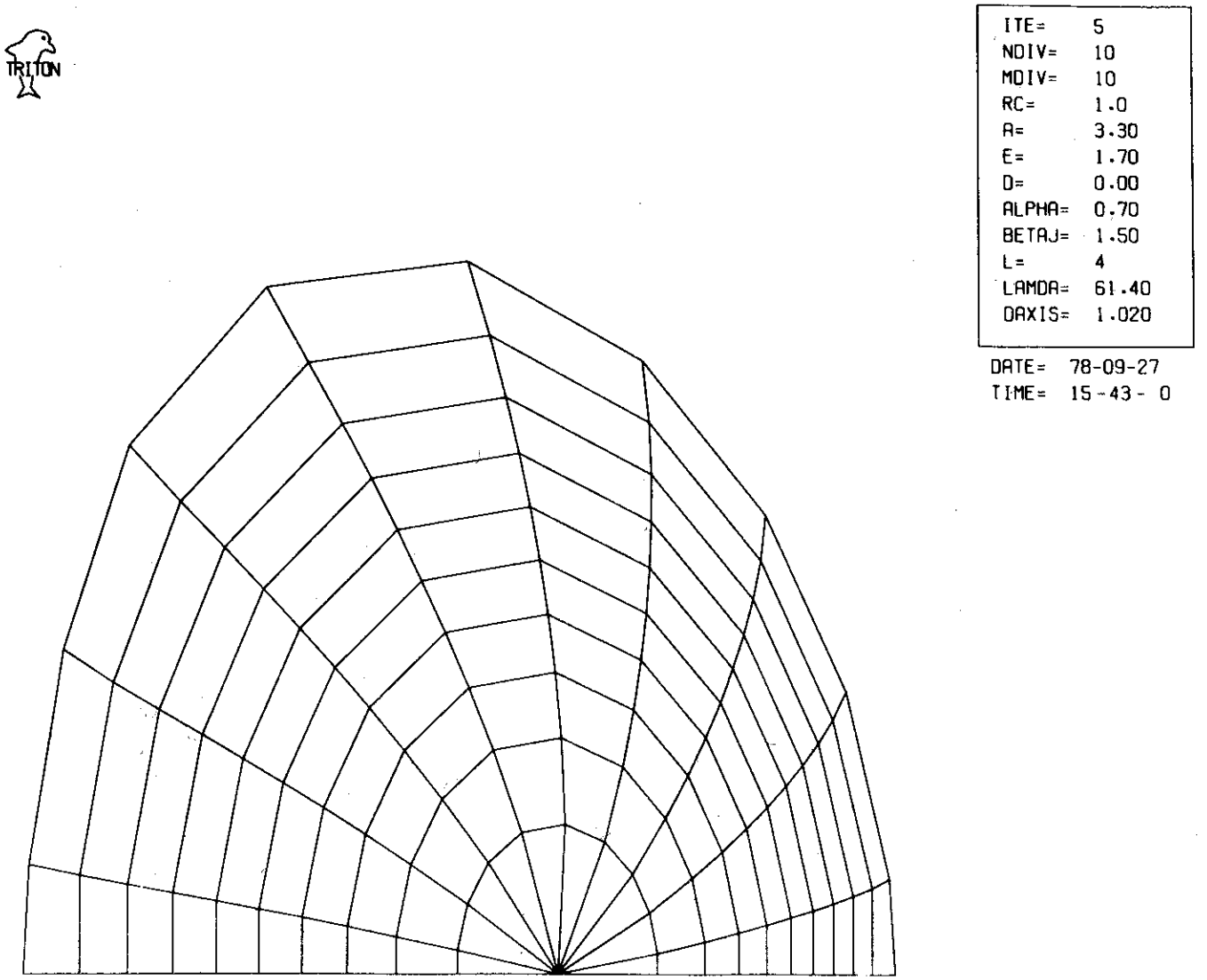


Fig.4 Convergence of the scheme with respect to number of finite elements. (a) NDIV=MDIV=10

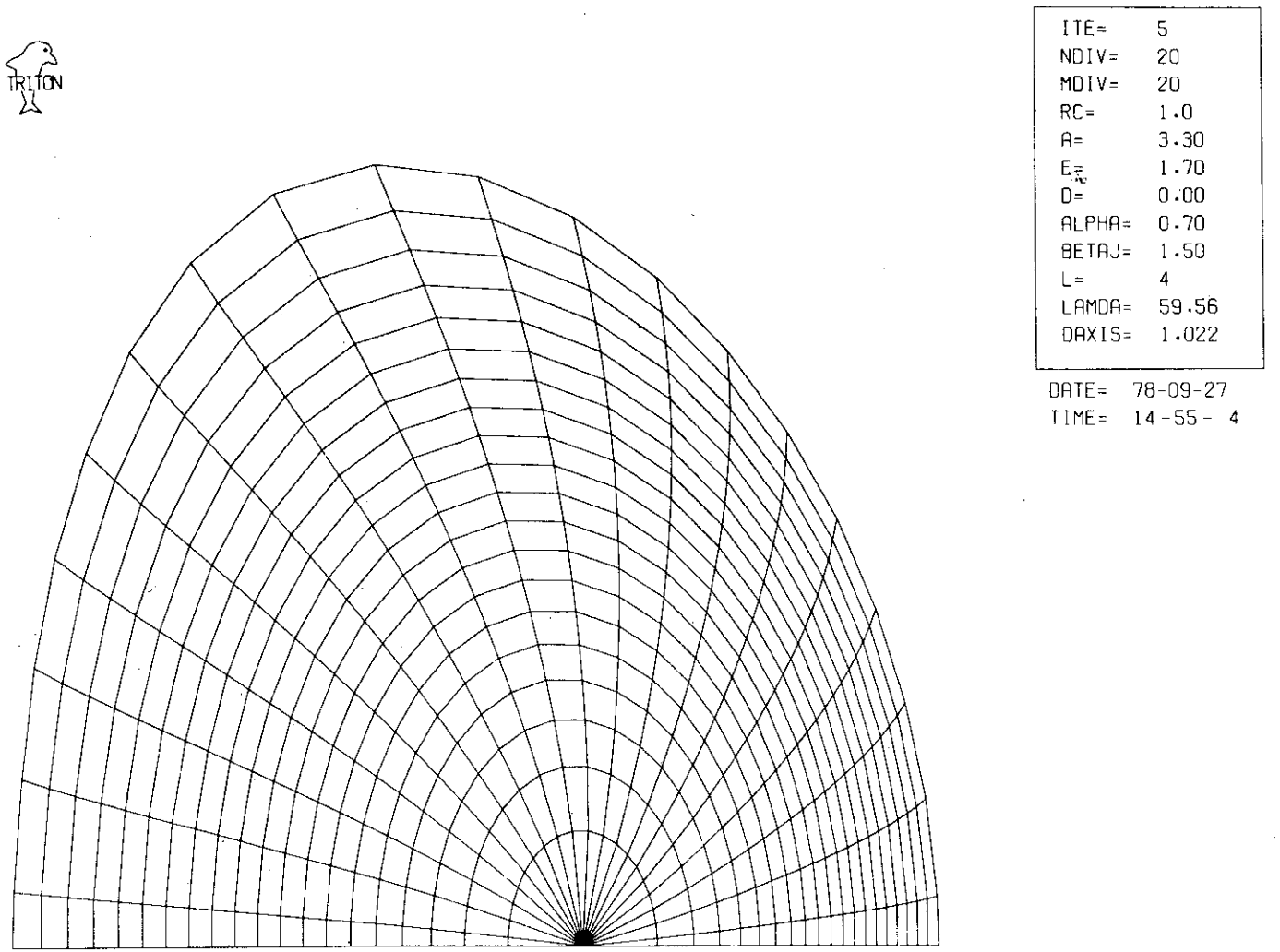


Fig.4 Convergence of the scheme with respect to number of finite elements. (b) NDIV=MDIV=20

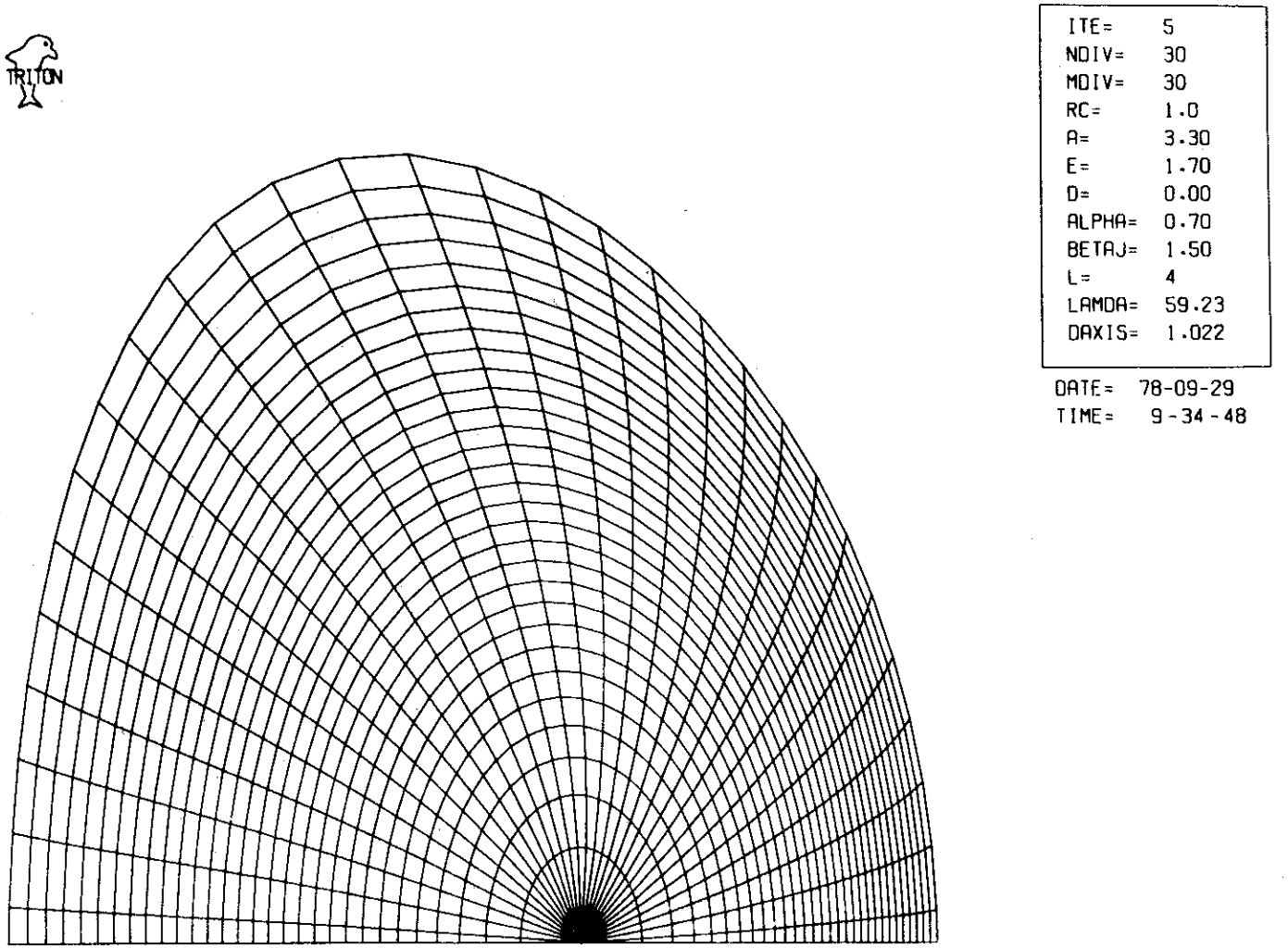


Fig.4 Convergence of the scheme with respect to number of finite elements. (c) NDIV=MDIV=30

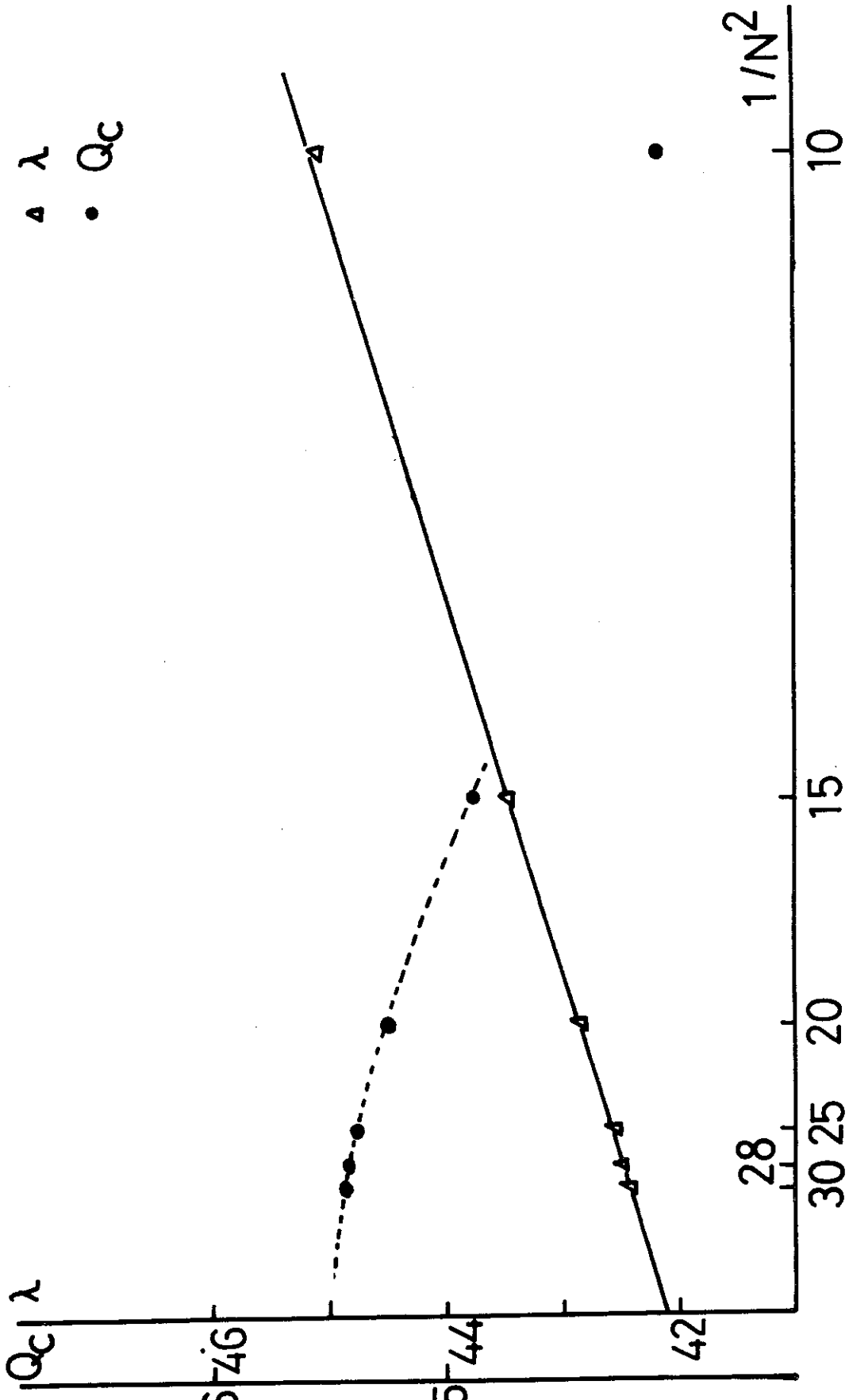


Fig.5 Convergence of eigenvalue λ and the safety factor at the magnetic axis (q_o).

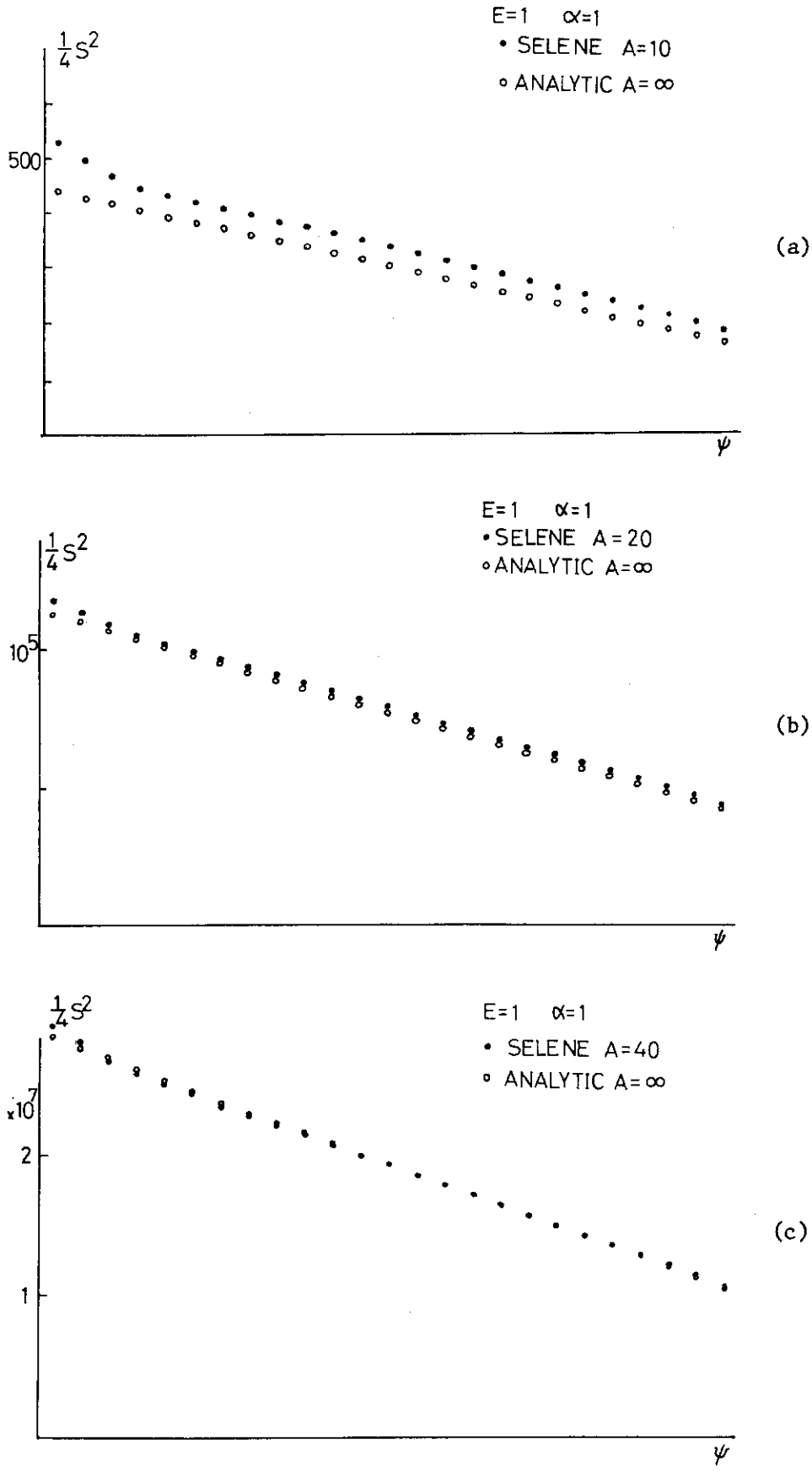


Fig.6 Magnetic shears of large aspect ratio plasmas(numerical) and a cylindrical plasma(analytic).

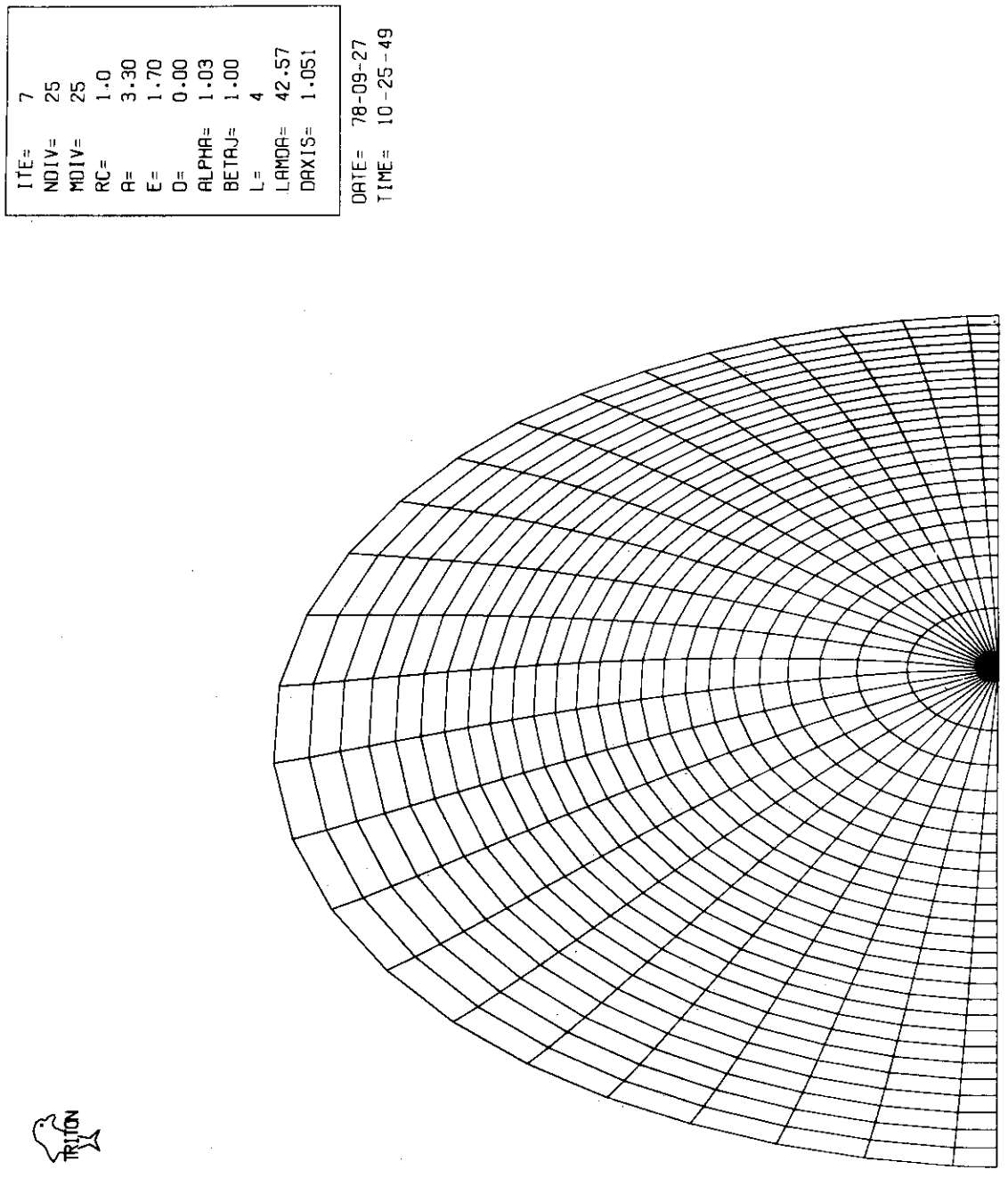


Fig.7 Examples of results for different values of β_J. (a) β_J=1

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ISTOT=	0
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SIGMA=	0.0280
BETA=	0.0071
BETAP=	0.895
BETAST=	0.0108

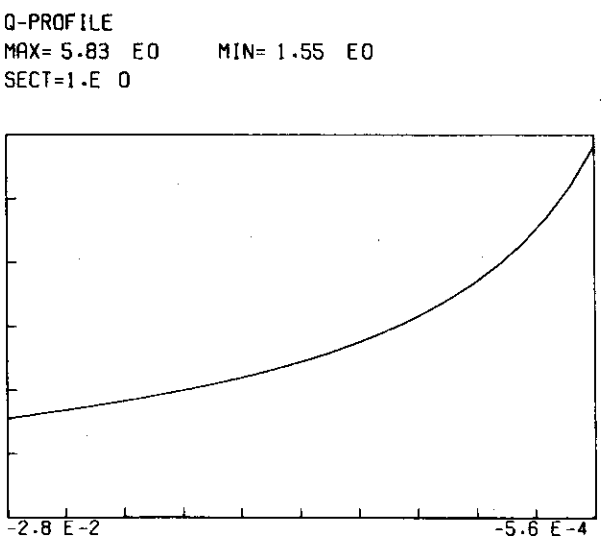
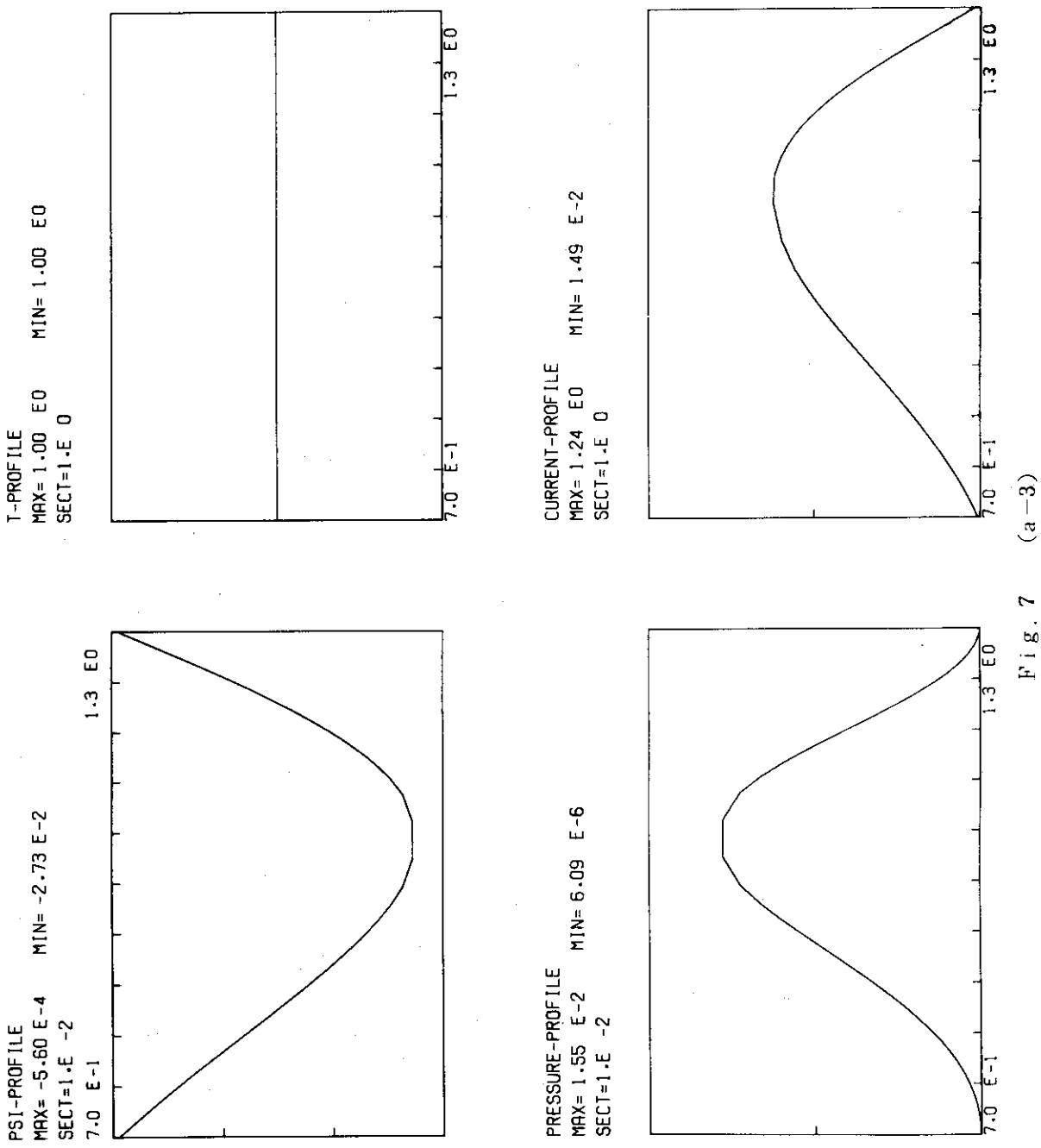


Fig. 7 (a−2)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CURRENT-PROFILE
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MIN= 1.49 E-2
SECT=1.E 0
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1.3 E0
PSI-PROFILE
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MIN= -2.73 E-2
SECT=1.E -2
7.0 E-1
1.3 E0
PRESSURE-PROFILE
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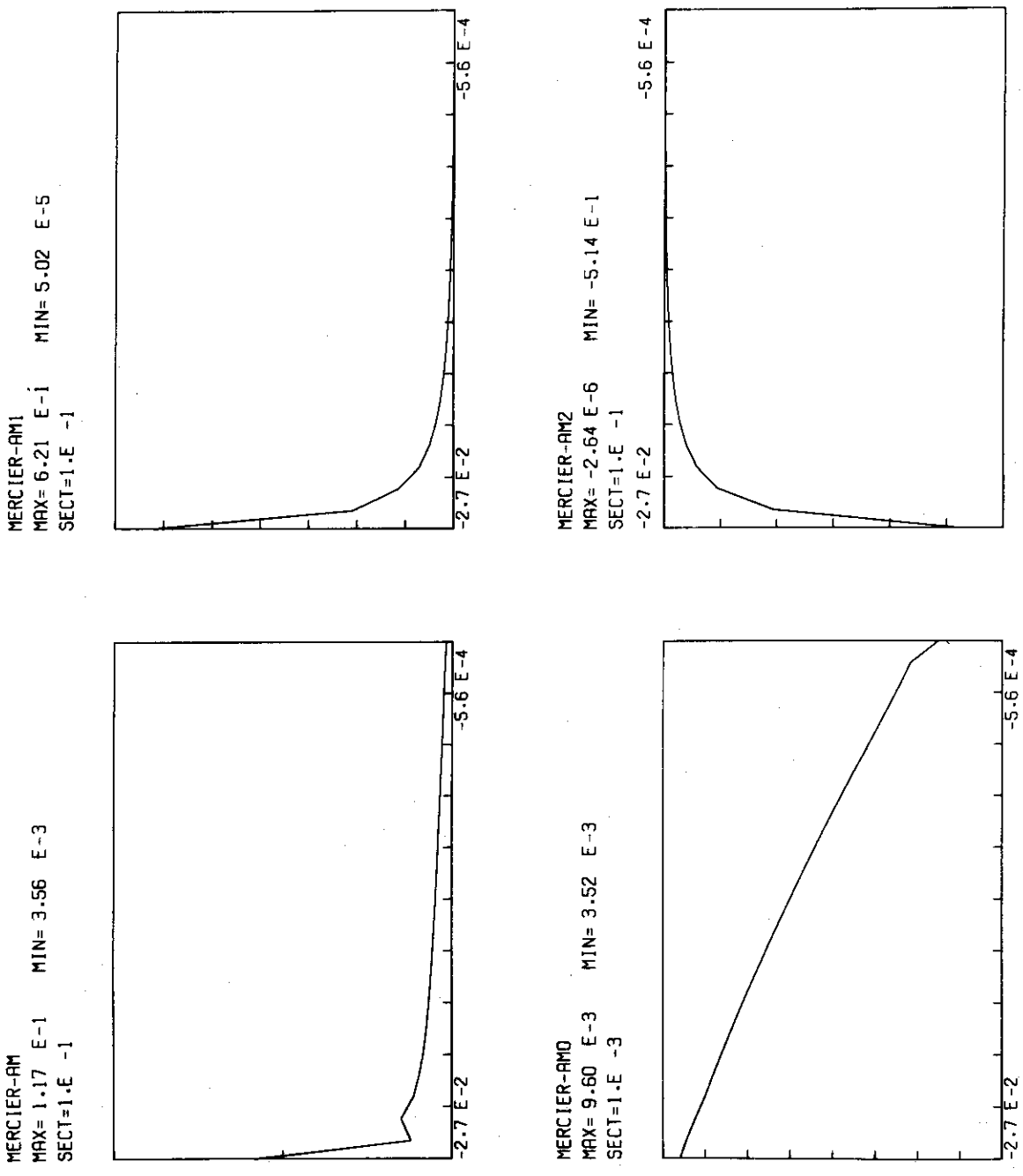
Fig. 7 (a－3)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-2.7 E-2
MERCIER-AM2
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-2.7 E-2
MERCIER-AM
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SECT=1.E -1
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-2.7 E-2
MERCIER-AM0
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SECT=1.E -3
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-2.7 E-2

Fig. 7 (a－4)

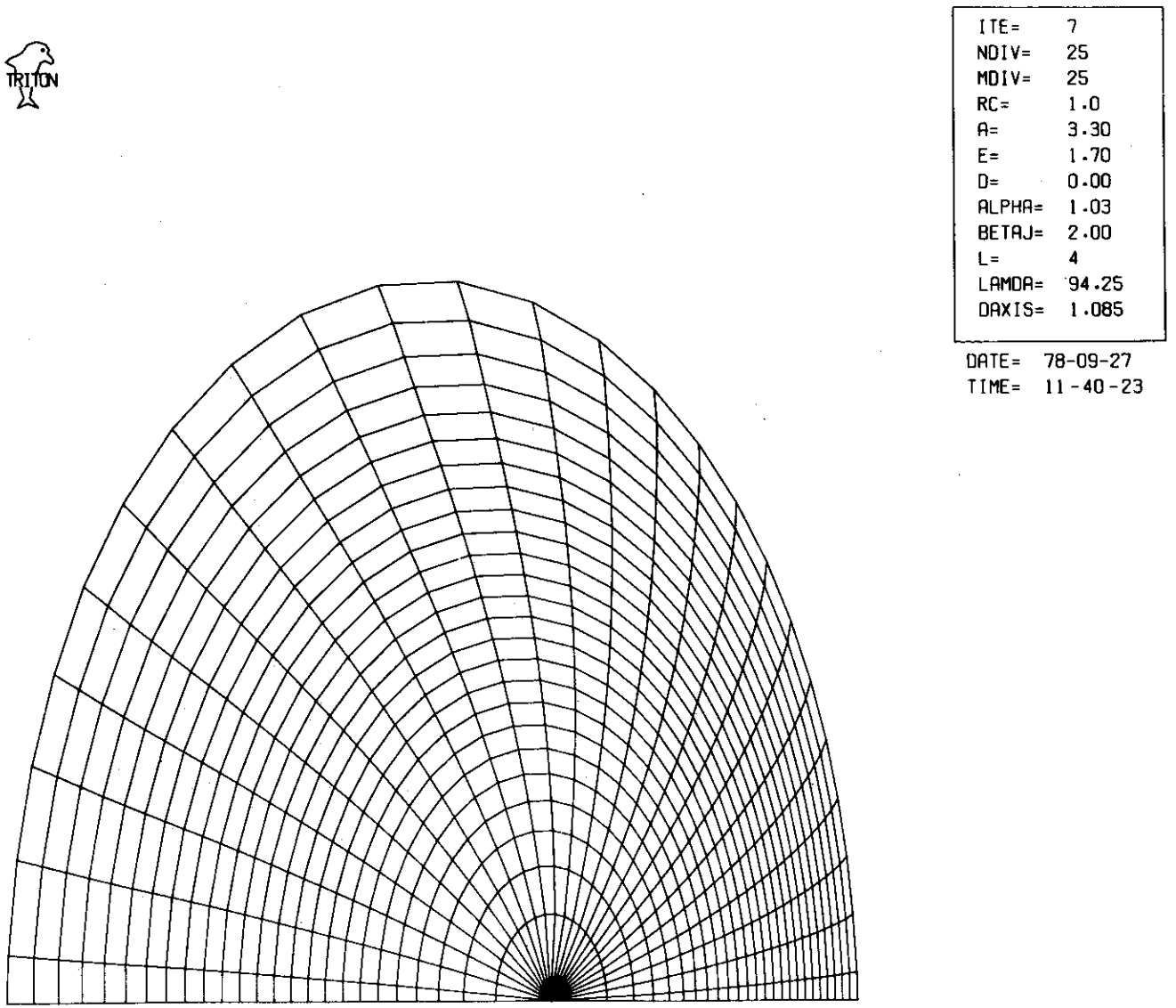

Fig.7 Examples of results for different values of β_J. (b) β_J=2

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ISTOT= 0

QC= 1.29

SIGMA= 0.0280

BETA= 0.0140

BETAP= 2.063

BETAST= 0.0219

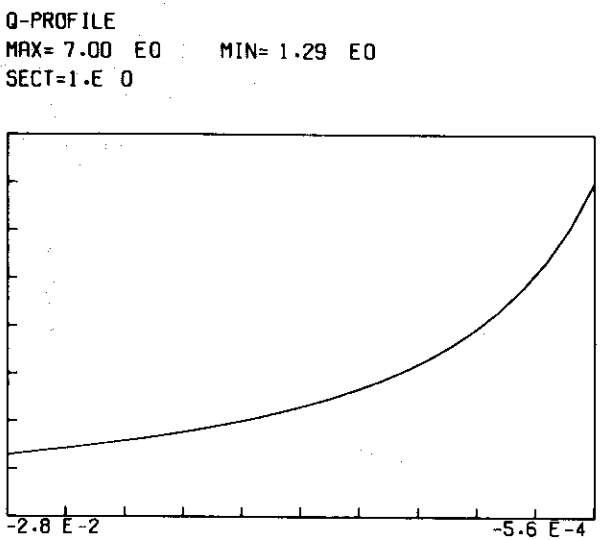
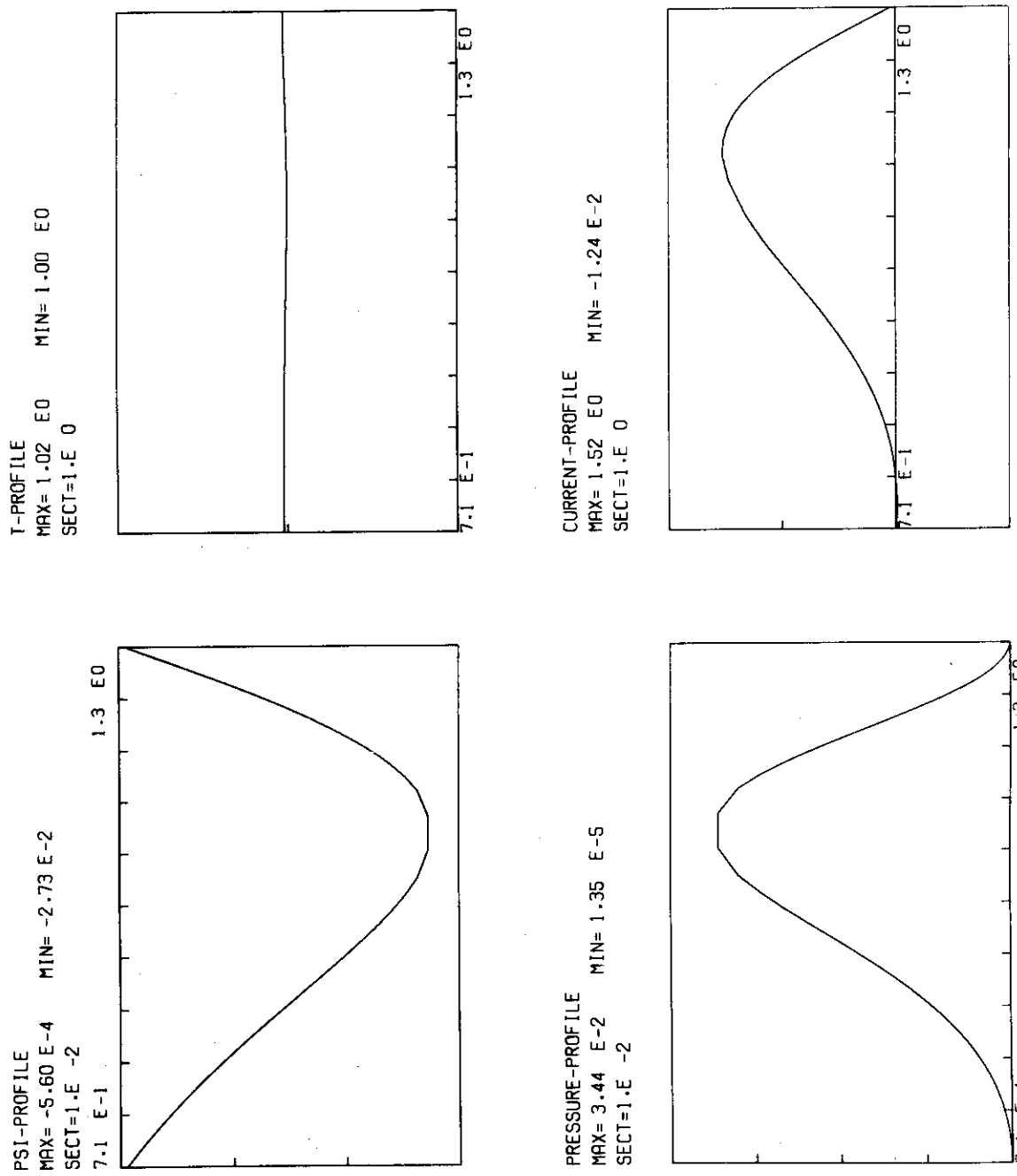


Fig. 7 (b－2)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1.3 E0
PSI-PROFILE
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SECT=1.E -2
7.1 E-1
1.3 E0
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Fig. 7 (b－3)

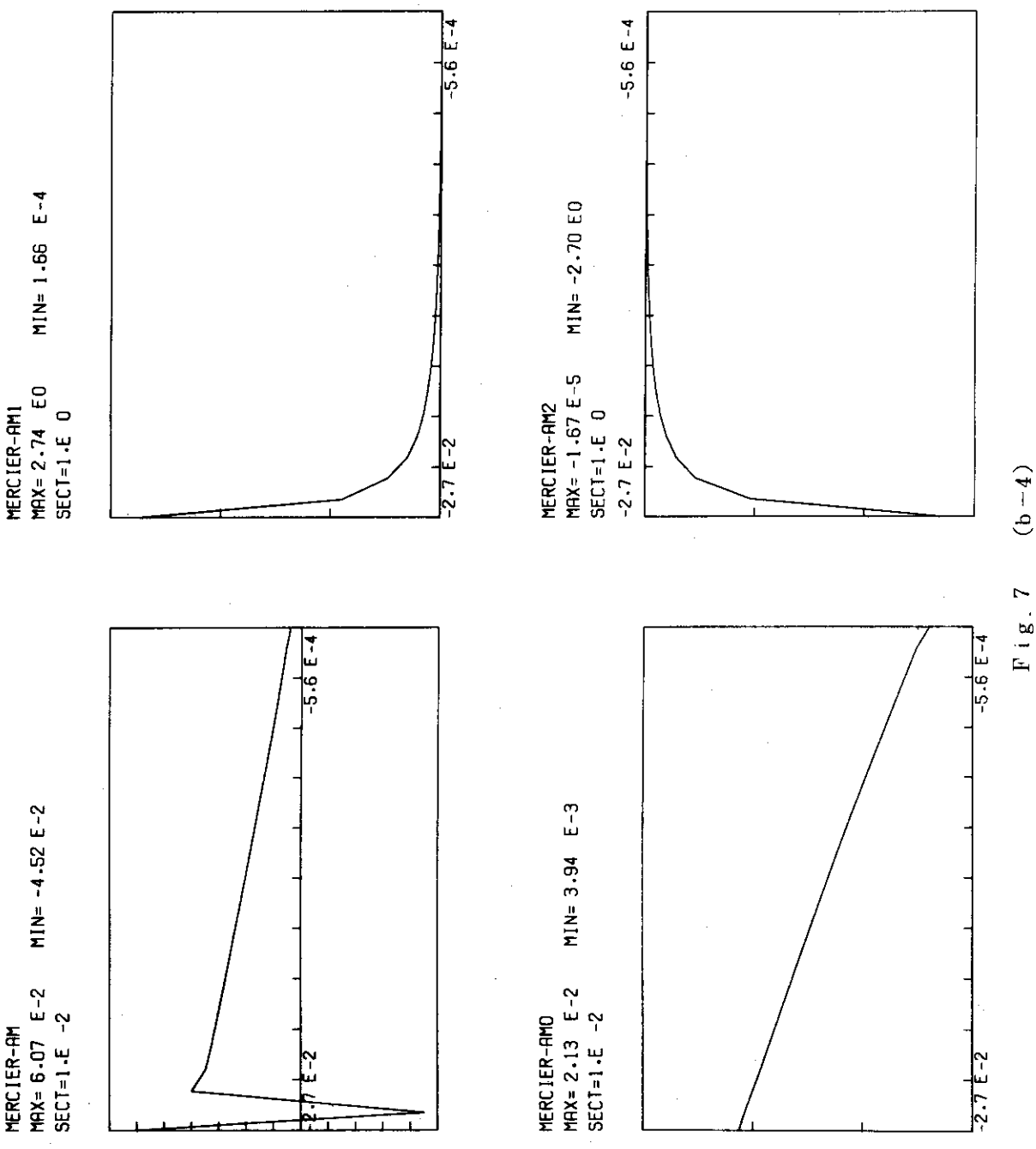
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Fig. 7 (b－4)

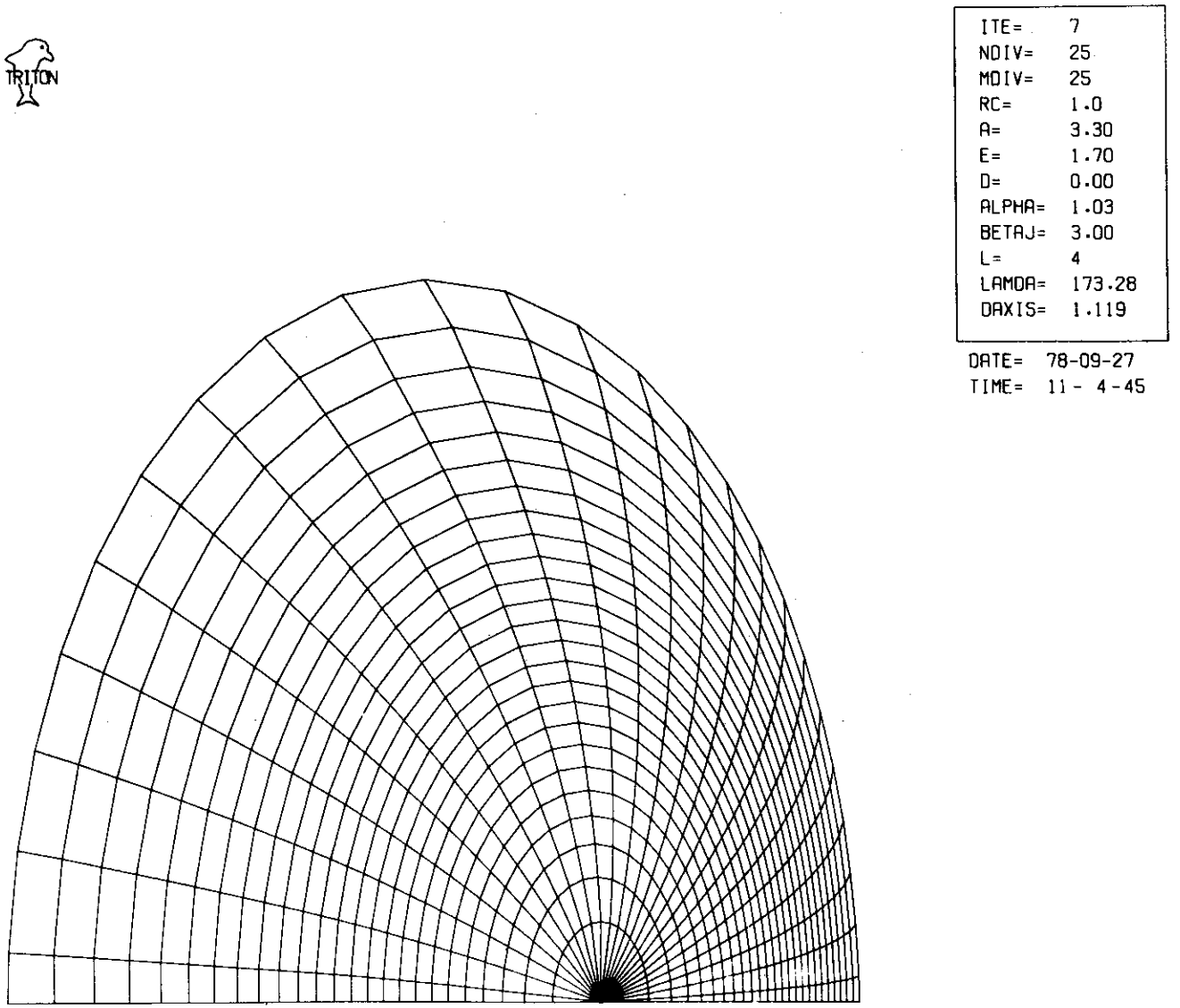

Fig.7 Examples of results for different values of β_J. (c) β_J=3

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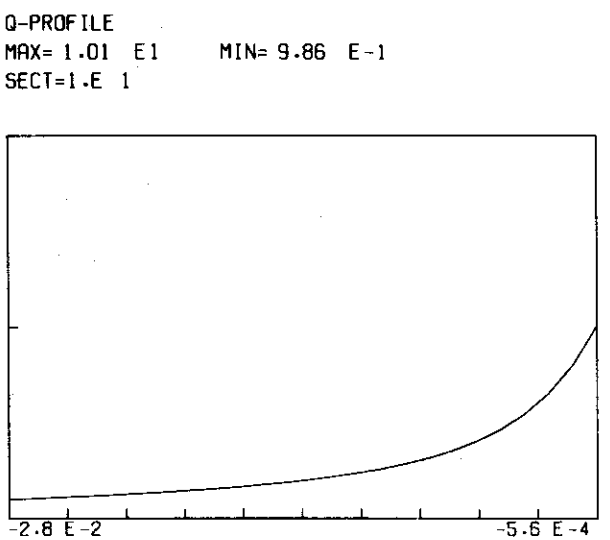
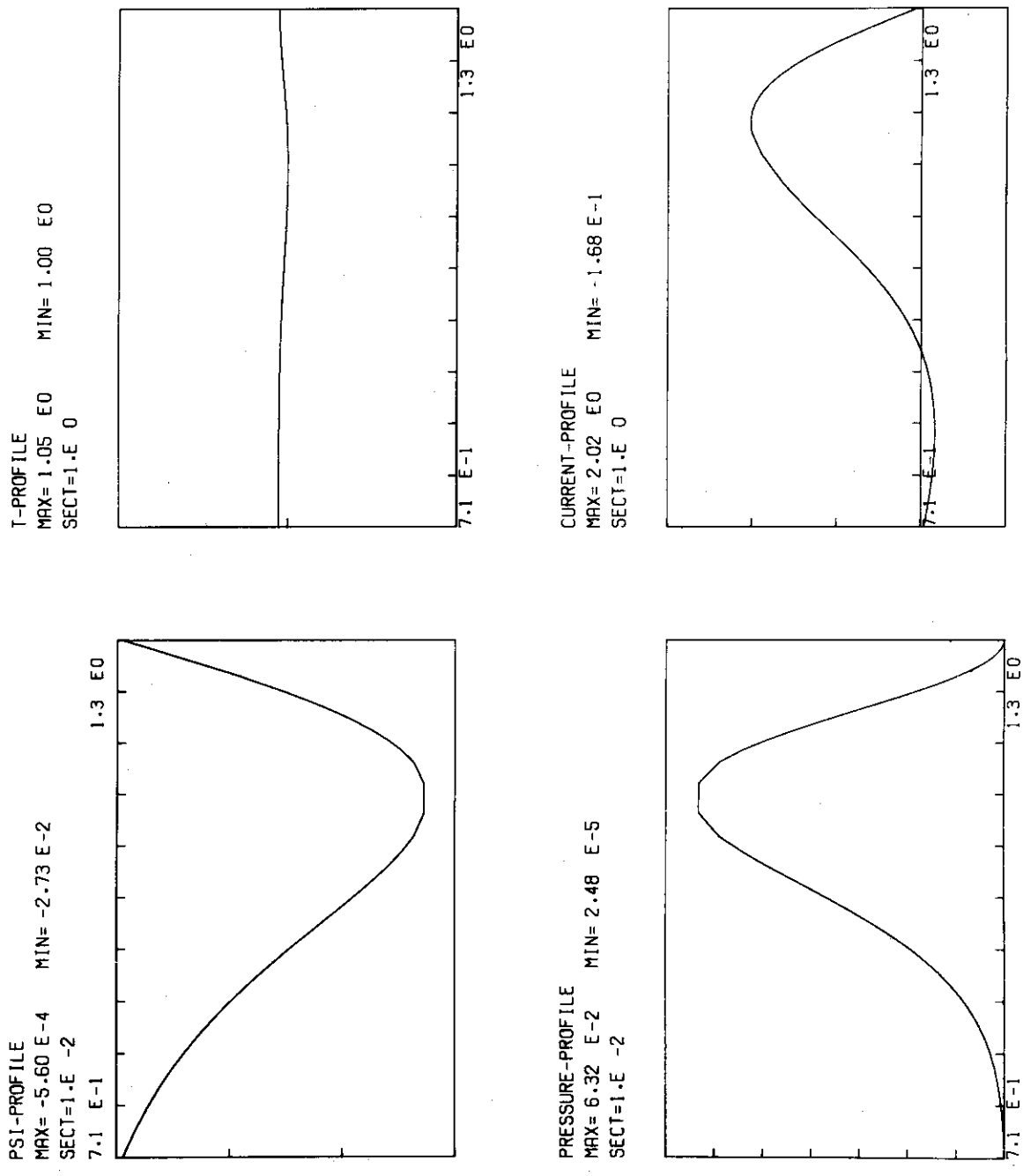


Fig. 7 (c-2)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1.3 E0
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SECT=1.E -2
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1.3 E0
PRESSURE-PROFILE
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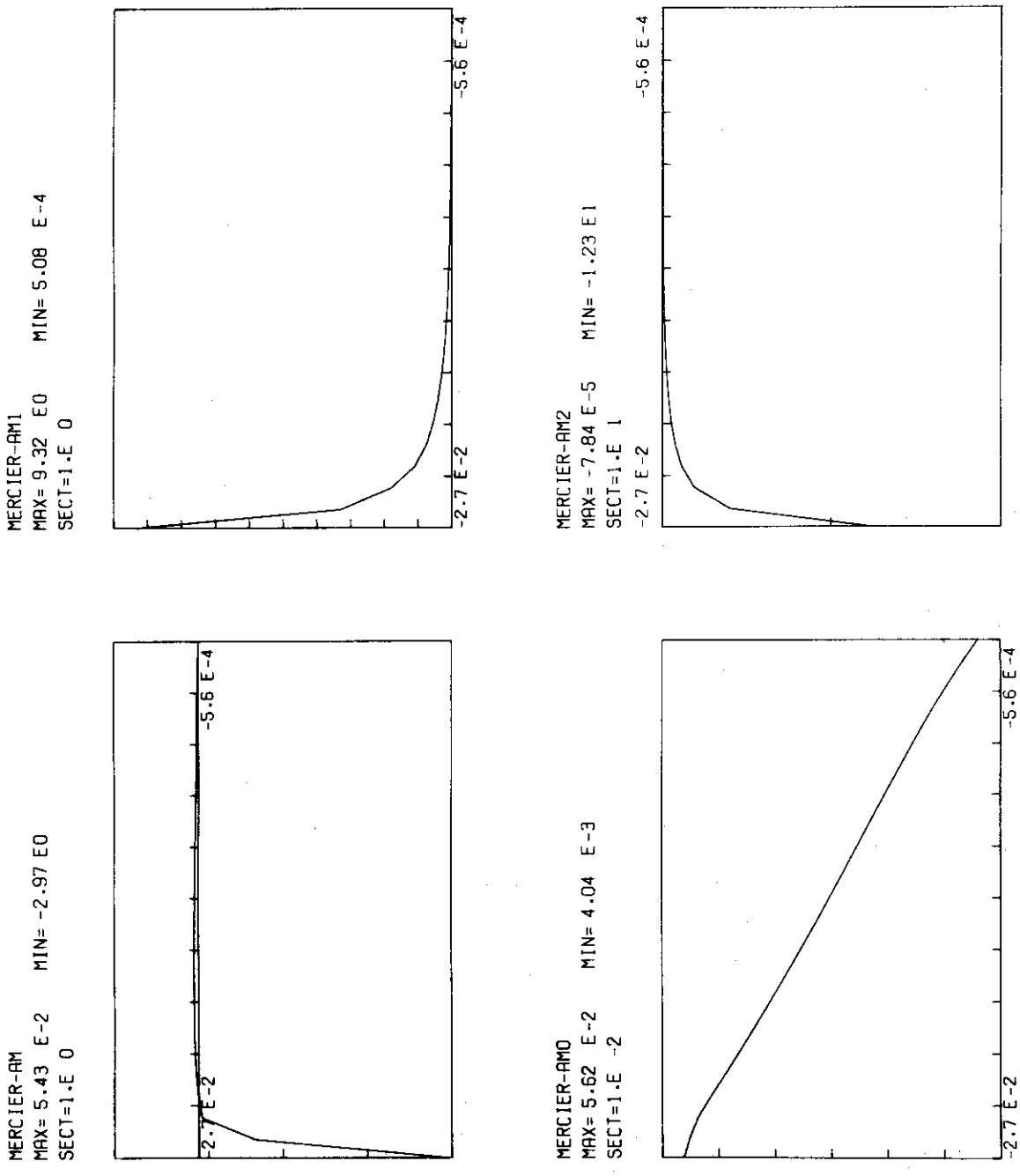
Fig. 7 (c－3)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-2.7 E-2
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MIN= -2.97 E0
SECT=1.E 0
-5.6 E-4
-2.7 E-2
MERCIER-AM0
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Fig. 7 (c－4)

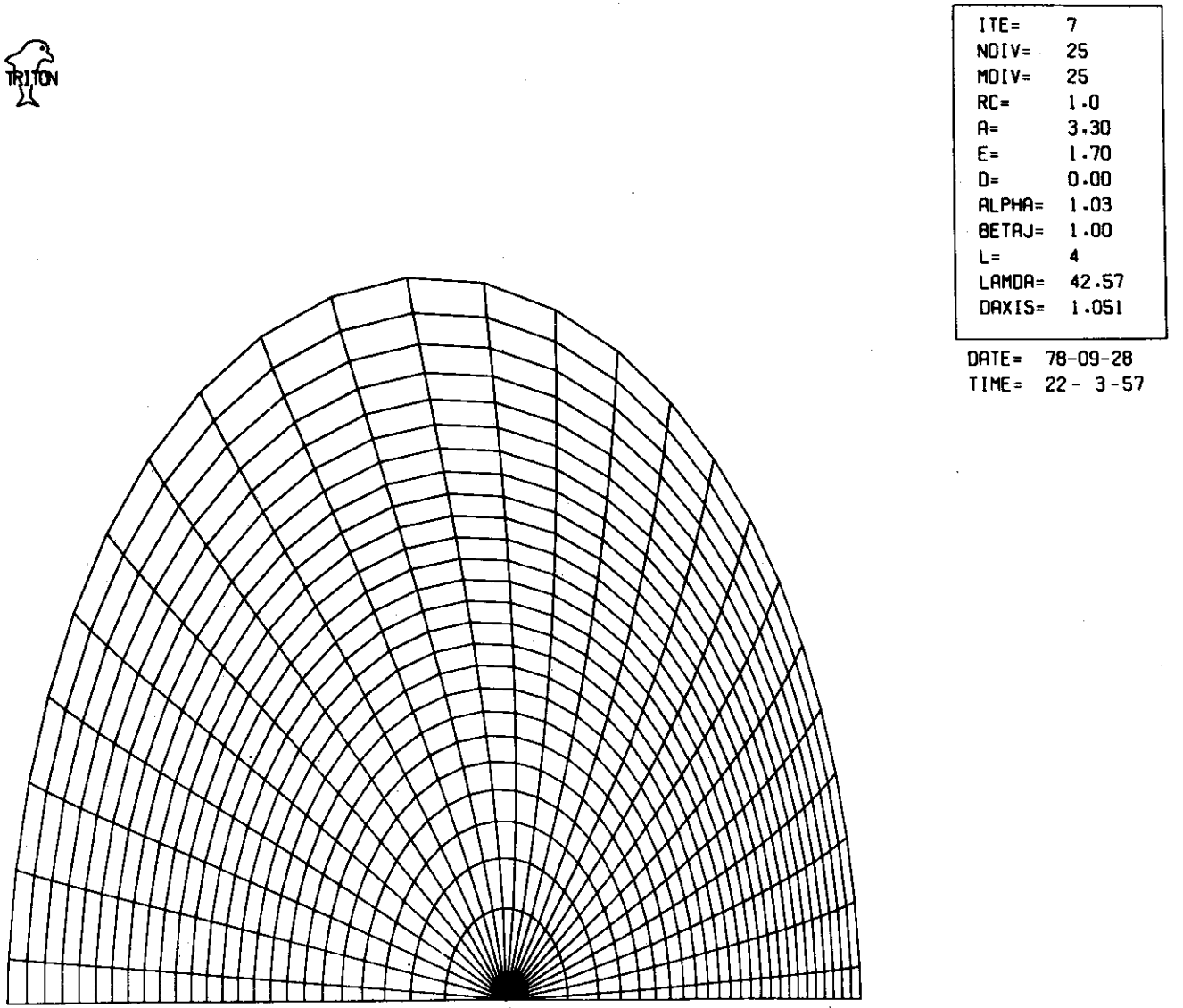

Fig.8 Examples of results for different values of scaling factor σ.

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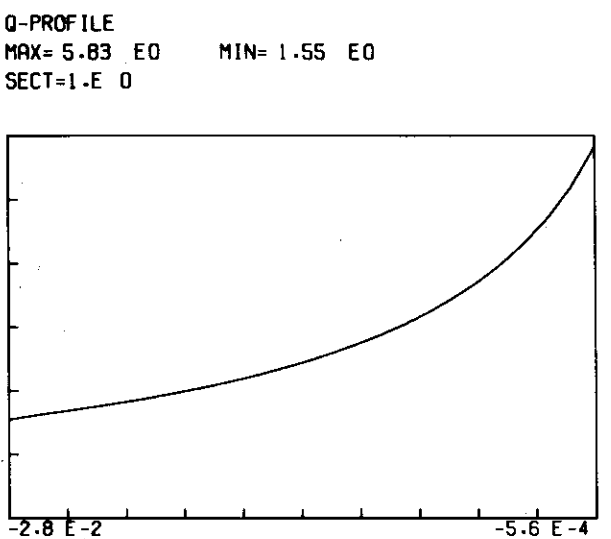
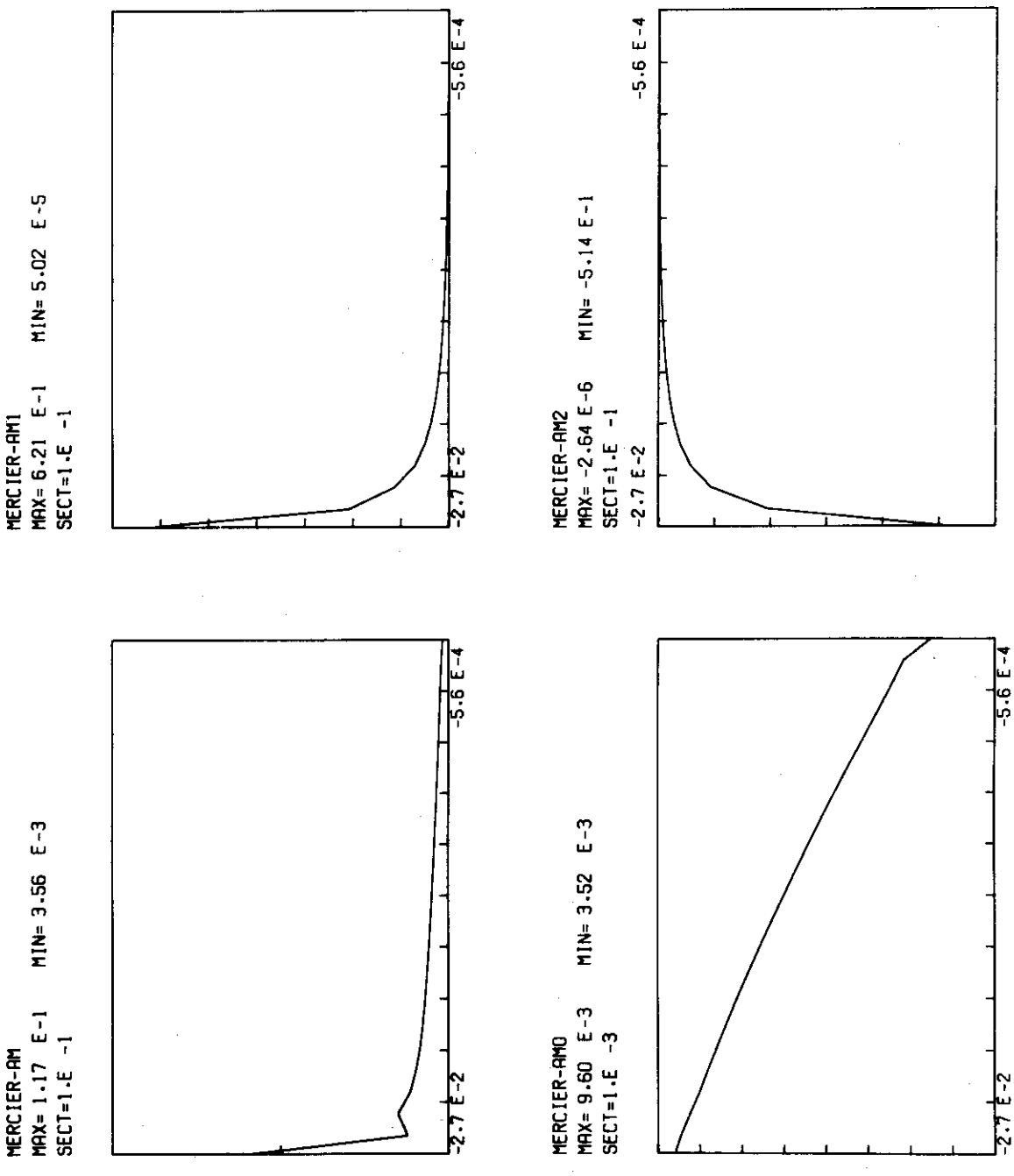


Fig.8 Examples of results for different values of scaling factor σ.
(a) σ=0.028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MERCIER-AM
MAX= 1.17 E-1 MIN= 3.56 E-3
SECT=1.E -1
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-2.7 E-2
MERCIER-AM0
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SECT=1.E -3
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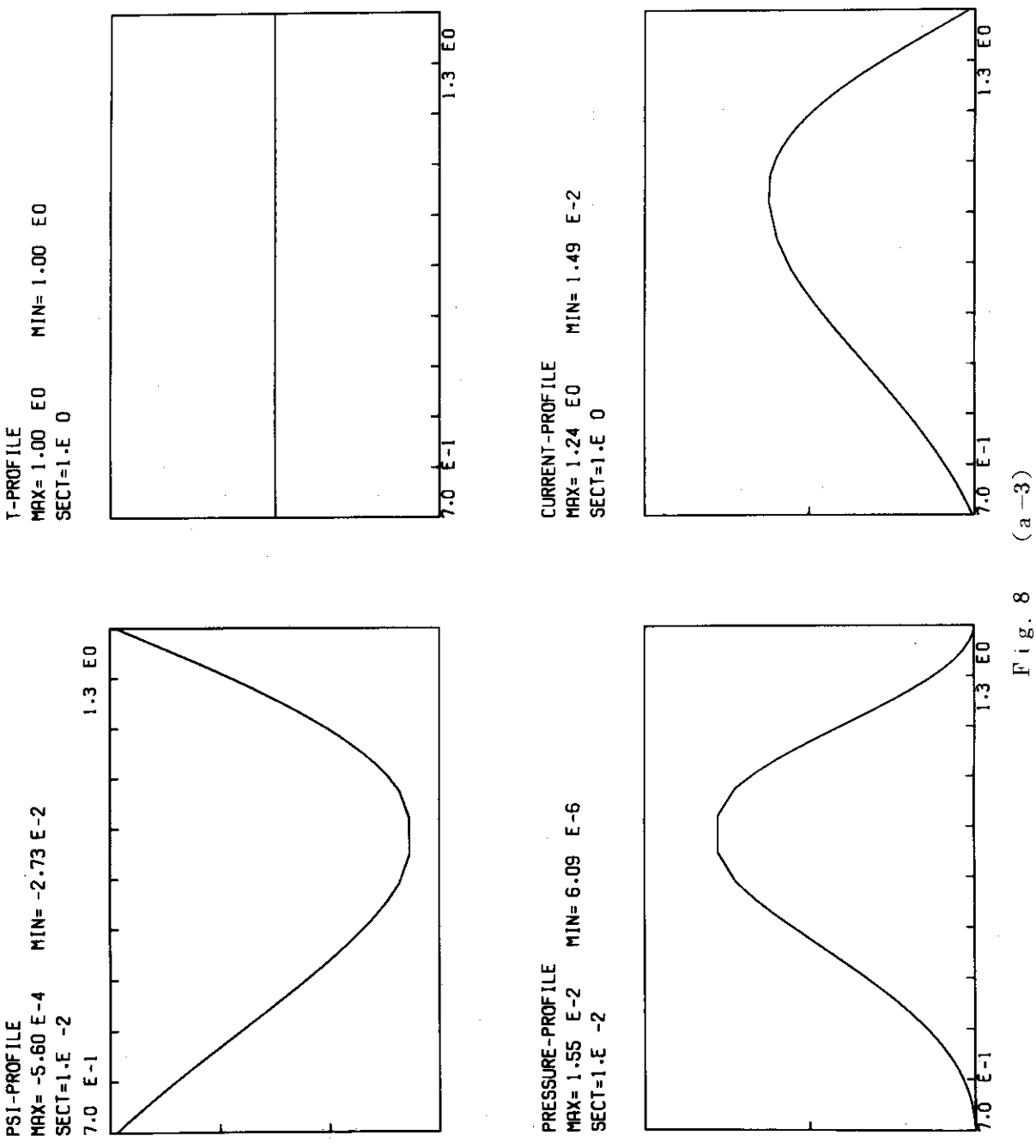
-2.7 E-2

Fig. 8 (a－2)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CURRENT-PROFILE
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MIN= 1.49 E-2
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7.0 E-1
1.3 E0
PRESSURE-PROFILE
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MIN= 6.09 E-6
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1.3 E0

Fig. 8 (a－3)

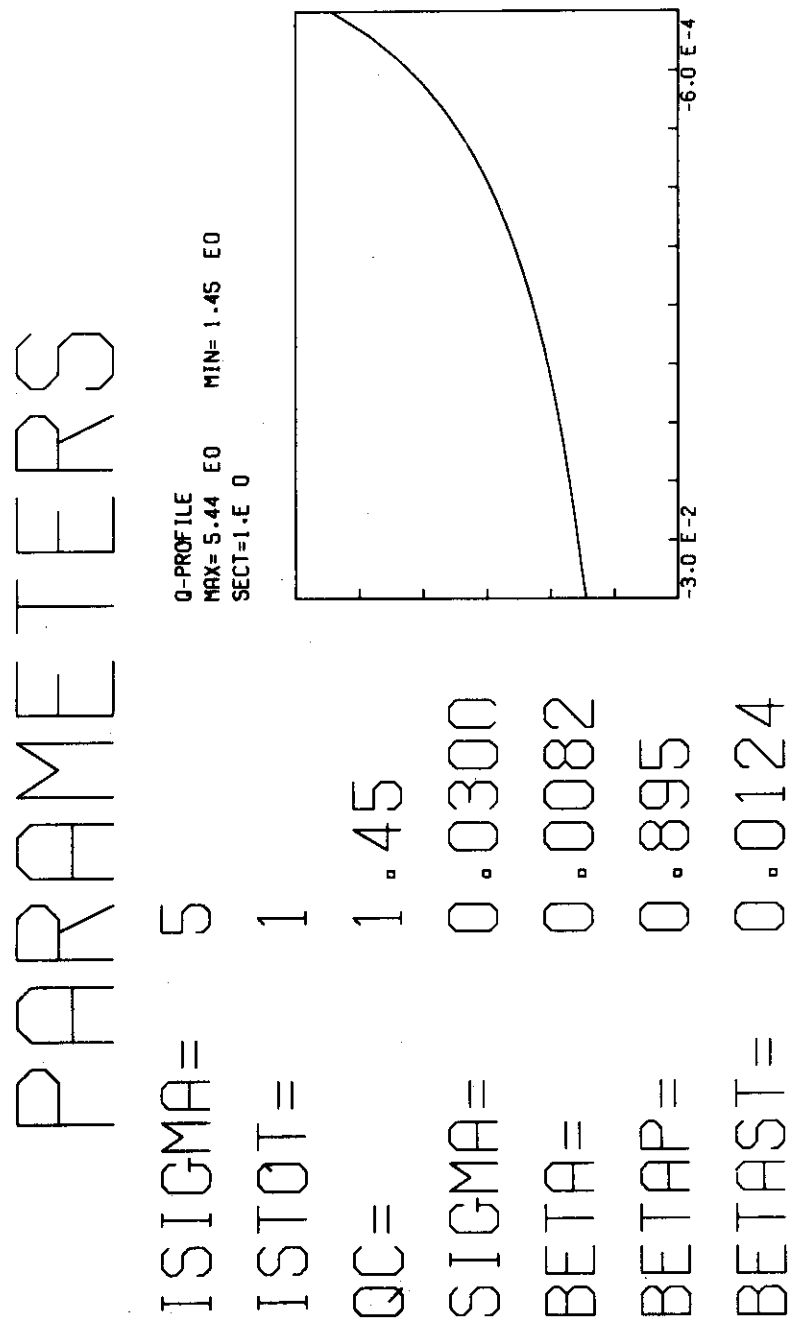

Fig.8 Examples of results for different values of scaling factor σ.
(b) σ=0.030

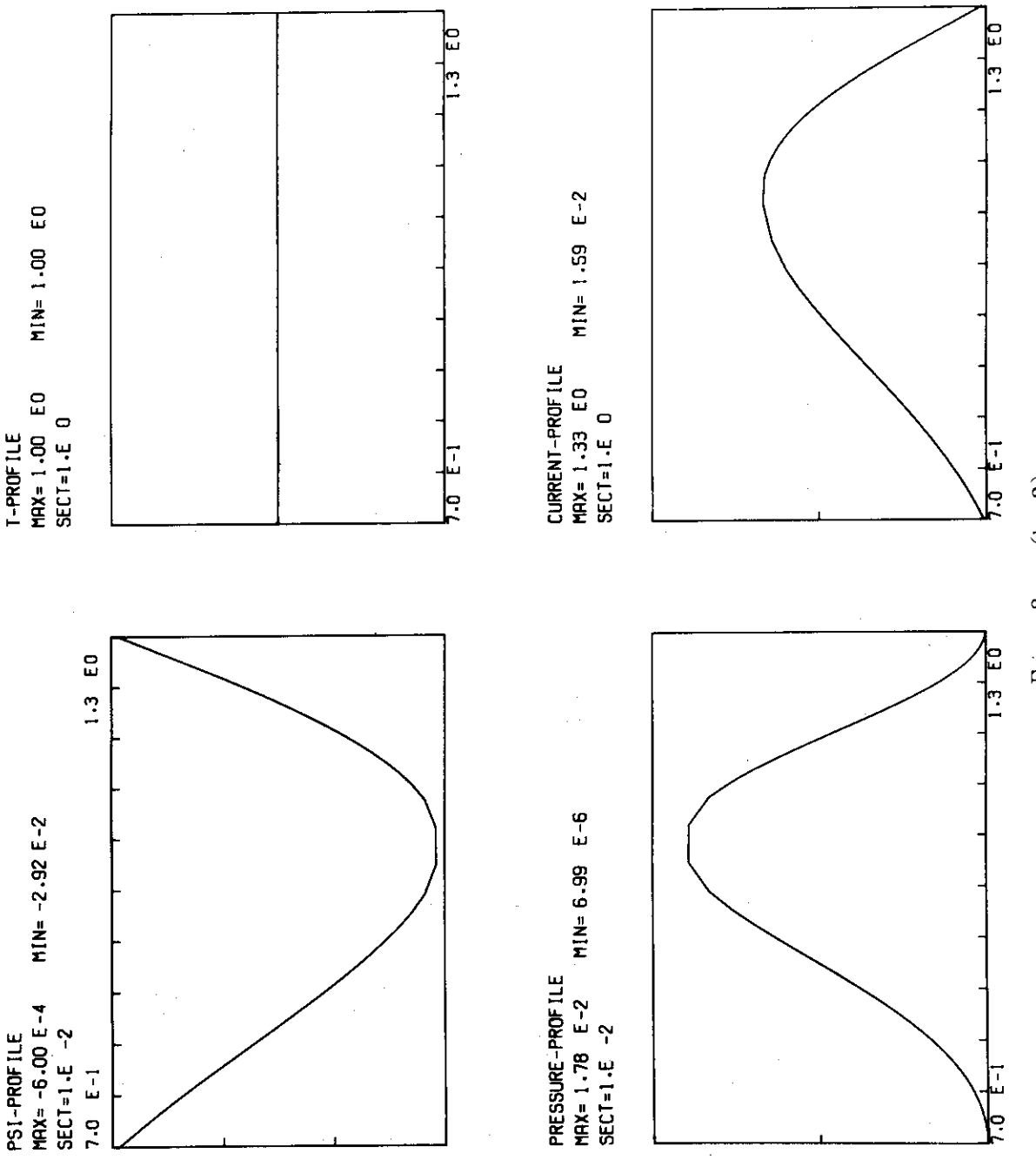
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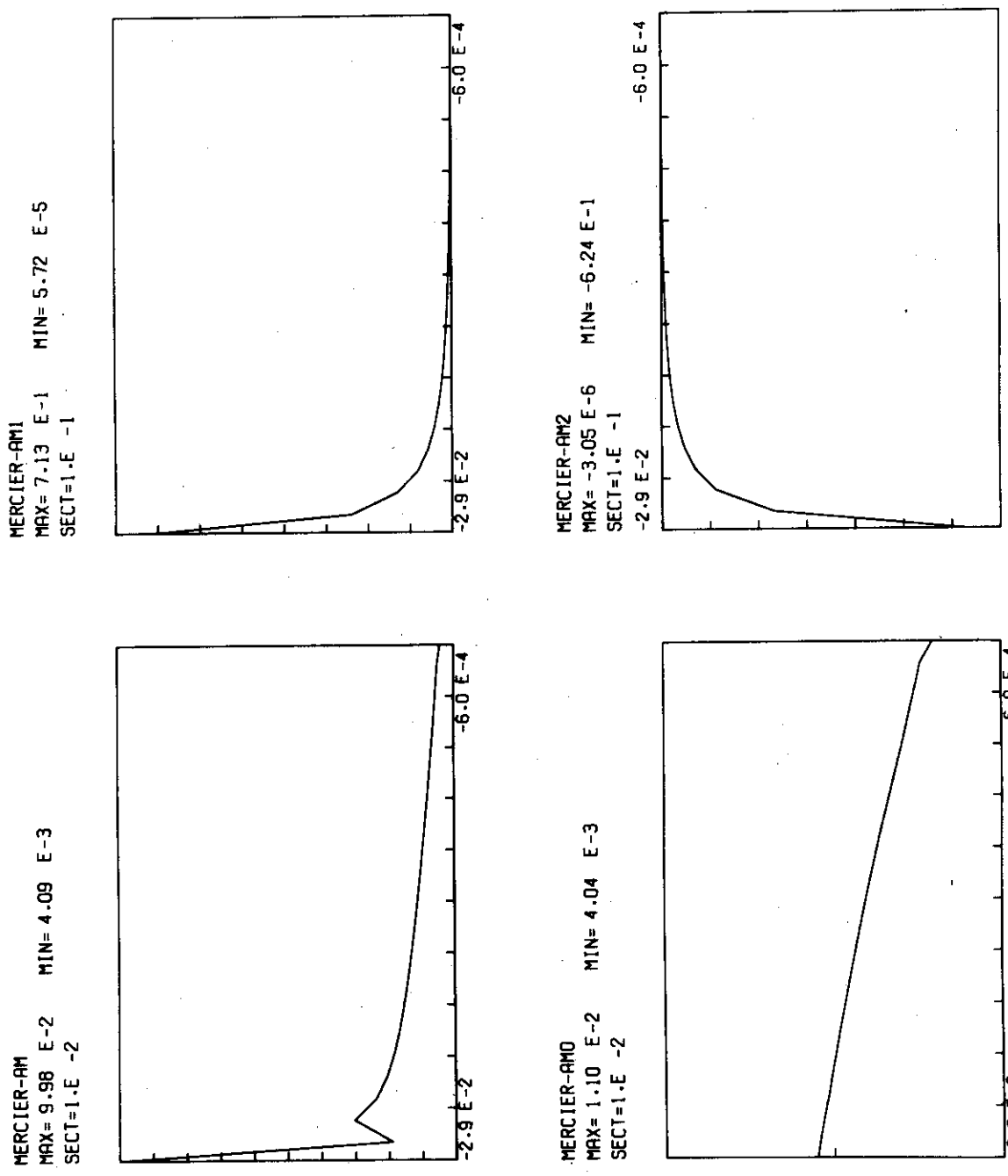
Fig. 8 (b—2)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MERCIER-AM
MAX= 9.98 E-2 MIN= 4.09 E-3
SECT=1.E -2
-6.0 E-4
-2.9 E-2
MERCIER-AM0
MAX= 1.10 E-2 MIN= 4.04 E-3
SECT=1.E -2
-6.0 E-4
-2.9 E-2

Fig. 8 (b－3)

PARAMETERS

ISIGMA= 5

ISTOT= 2

QC= 1.36

SIGMA= 0.0320

BETA= 0.0093

BETAP= 0.895

BETAST= 0.0141

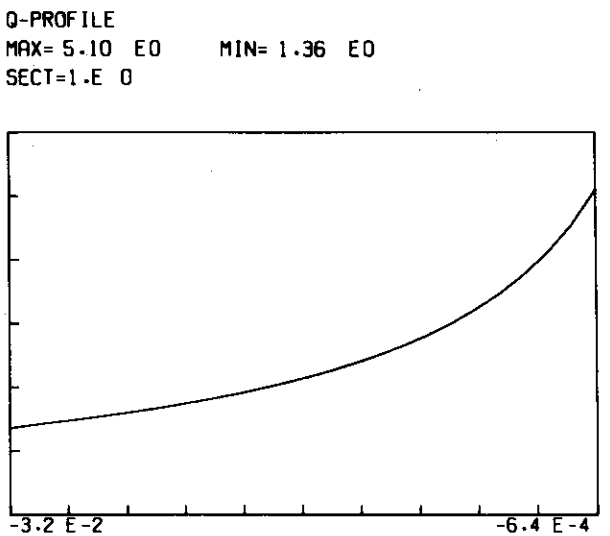

Fig.8 Examples of results for different values of scaling factor σ.

(c) σ=0.032

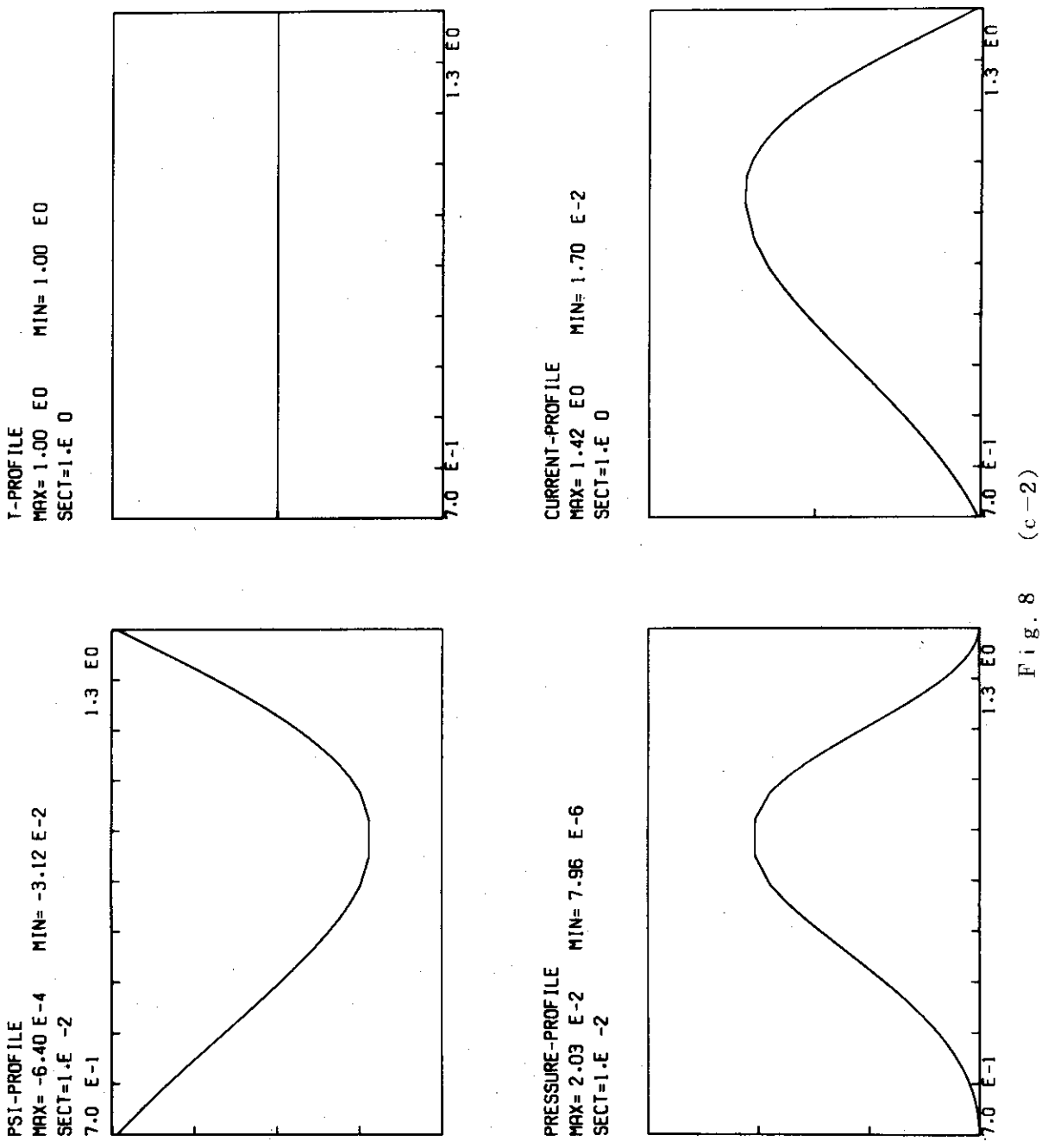
T-PROFILE
MAX= 1.00 E0 MIN= 1.00 E0
SECT=1.E 0
7.0 E-1
1.3 E0
CURRENT-PROFILE
MAX= 1.42 E0 MIN= 1.70 E-2
SECT=1.E 0
7.0 E-1
1.3 E0
PSI-PROFILE
MAX= -6.40 E-4 MIN= -3.12 E-2
SECT=1.E -2
7.0 E-1
1.3 E0
PRESSURE-PROFILE
MAX= 2.03 E-2 MIN= 7.96 E-6
SECT=1.E -2
7.0 E-1
1.3 E0

Fig. 8 (c－2)

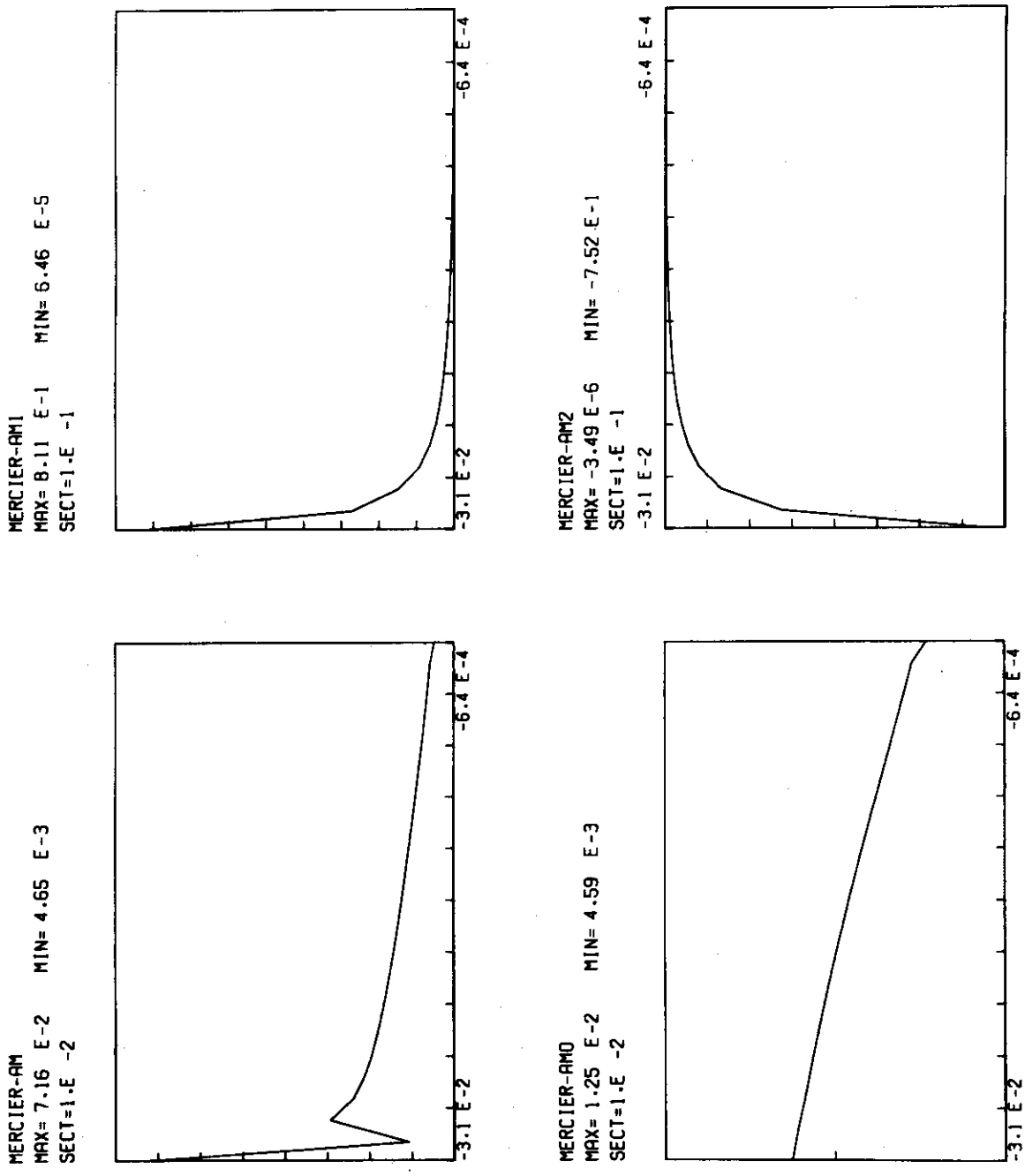
MERCIER-AM1
MAX= 8.11 E-1 MIN= 6.46 E-5
SECT=1.E -1
-6.4 E-4
-3.1 E-2
MERCIER-AM2
MAX= -3.49 E-6 MIN= -7.52 E-1
SECT=1.E -1
-6.4 E-4
-3.1 E-2
MERCIER-AM
MAX= 7.16 E-2 MIN= 4.65 E-3
SECT=1.E -2
-6.4 E-4
-3.1 E-2
MERCIER-AM0
MAX= 1.25 E-2 MIN= 4.59 E-3
SECT=1.E -2
-6.4 E-4
-3.1 E-2

Fig. 8 (c－3)

PARAMETERS

ISIGMA= 5
ISTOT= 3
QC= 1.28
SIGMA= 0.0340
BETA= 0.0105
BETAP= 0.895
BETAST= 0.0159

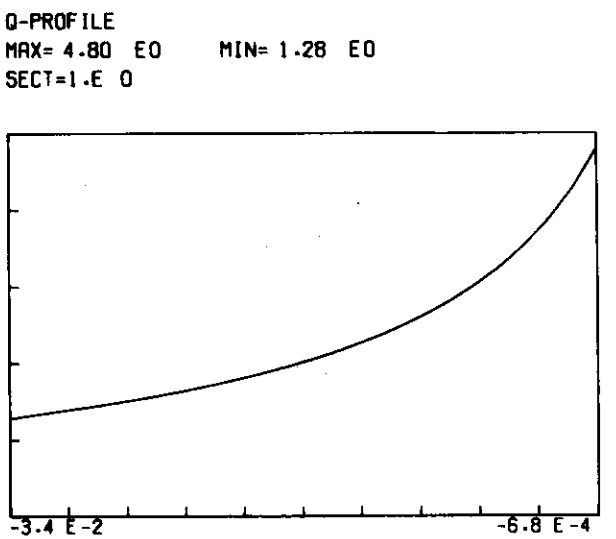

Fig.8 Examples of results for different values of scaling factor σ.
(d) σ=0.034

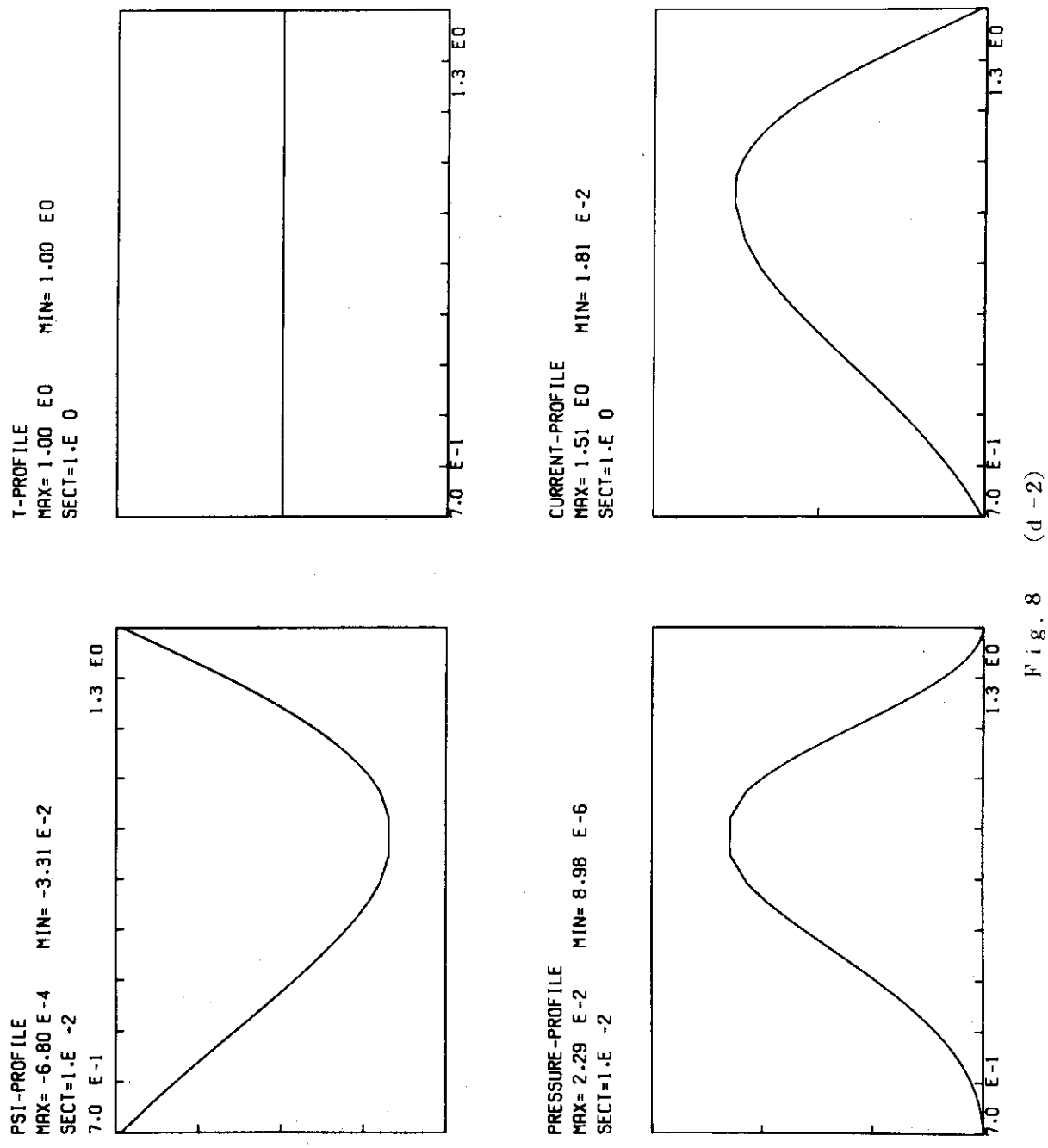
T-PROFILE
MAX= 1.00 E0 MIN= 1.00 E0
SECT=1.E 0
7.0 E-1
1.3 E0
CURRENT-PROFILE
MAX= 1.51 E0 MIN= 1.81 E-2
SECT=1.E 0
7.0 E-1
1.3 E0
PSI-PROFILE
MAX= -6.80 E-4 MIN= -3.31 E-2
SECT=1.E -2
7.0 E-1
1.3 E0
PRESSURE-PROFILE
MAX= 2.29 E-2 MIN= 8.98 E-6
SECT=1.E -2
7.0 E-1
1.3 E0

Fig. 8 (d－2)

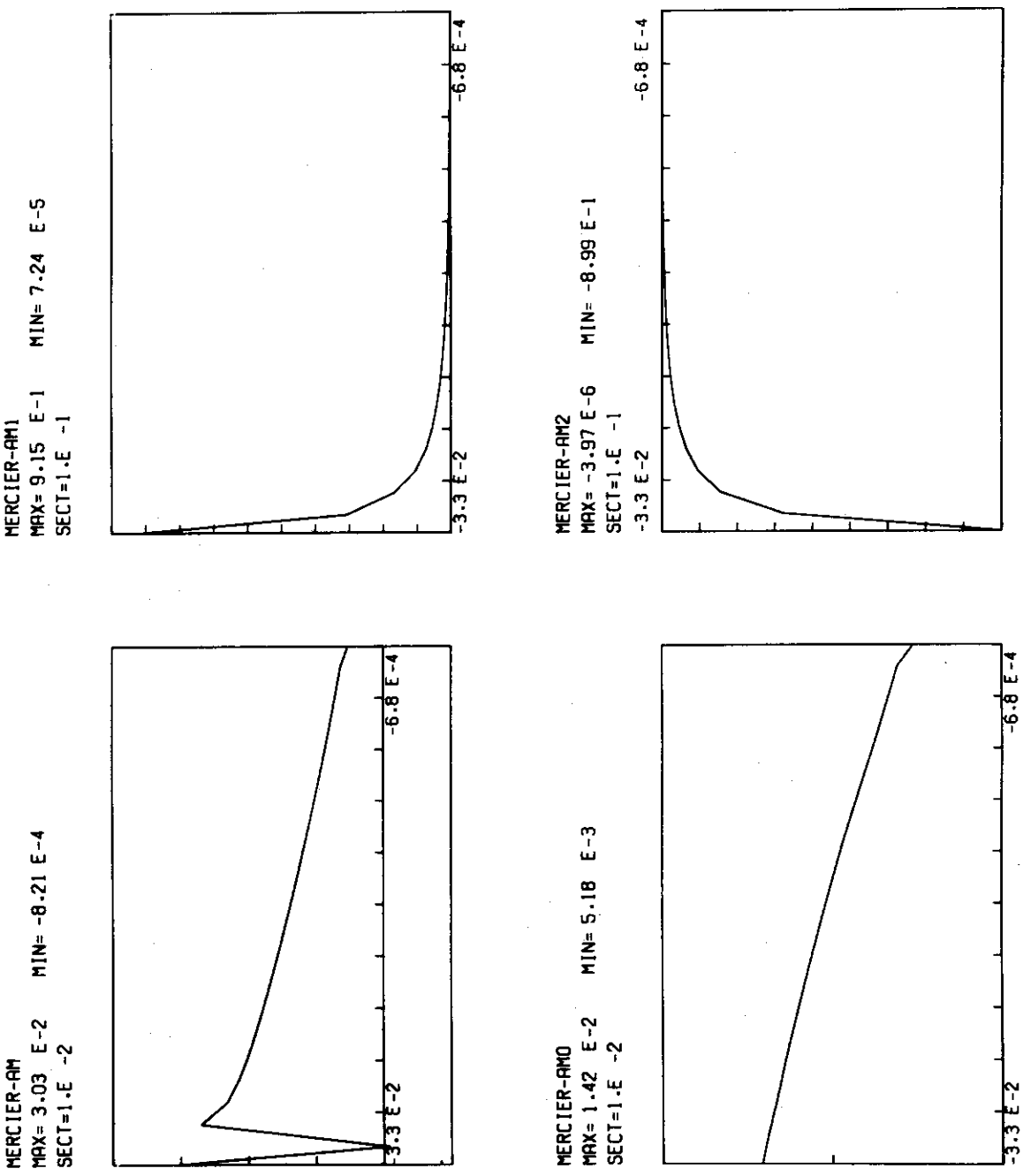
MERCIER-AM1
MAX= 9.15 E-1 MIN= 7.24 E-5
SECT=1.E -1
-6.8 E-4
-3.3 E-2
MERCIER-AM2
MAX= -3.97 E-6 MIN= -8.99 E-1
SECT=1.E -1
-6.8 E-4
-3.3 E-2
MERCIER-AM
MAX= 3.03 E-2 MIN= -8.21 E-4
SECT=1.E -2
-6.8 E-4
-3.3 E-2
MERCIER-AM0
MAX= 1.42 E-2 MIN= 5.18 E-3
SECT=1.E -2
-6.8 E-4
-3.3 E-2

Fig. 8 (d－3)

PARAMETERS

ISIGMA= 5
ISTOT= 4
QC= 1.21
SIGMA= 0.0360
BETA= 0.0117
BETAP= 0.895
BETAST= 0.0178

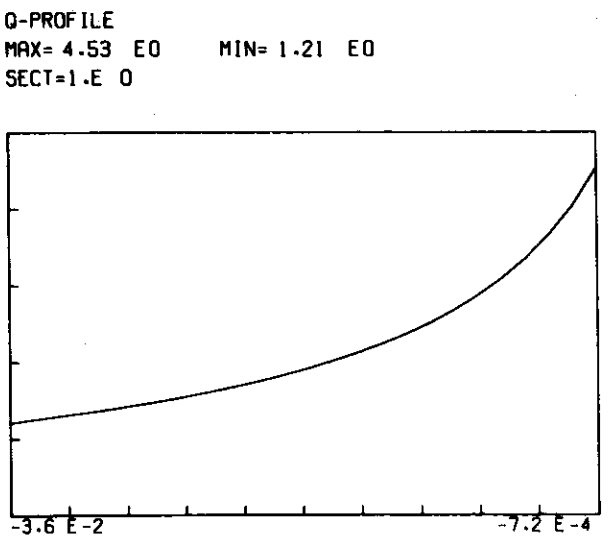

Fig.8 Examples of results for different values of scaling factor σ.

(e) σ=0.036

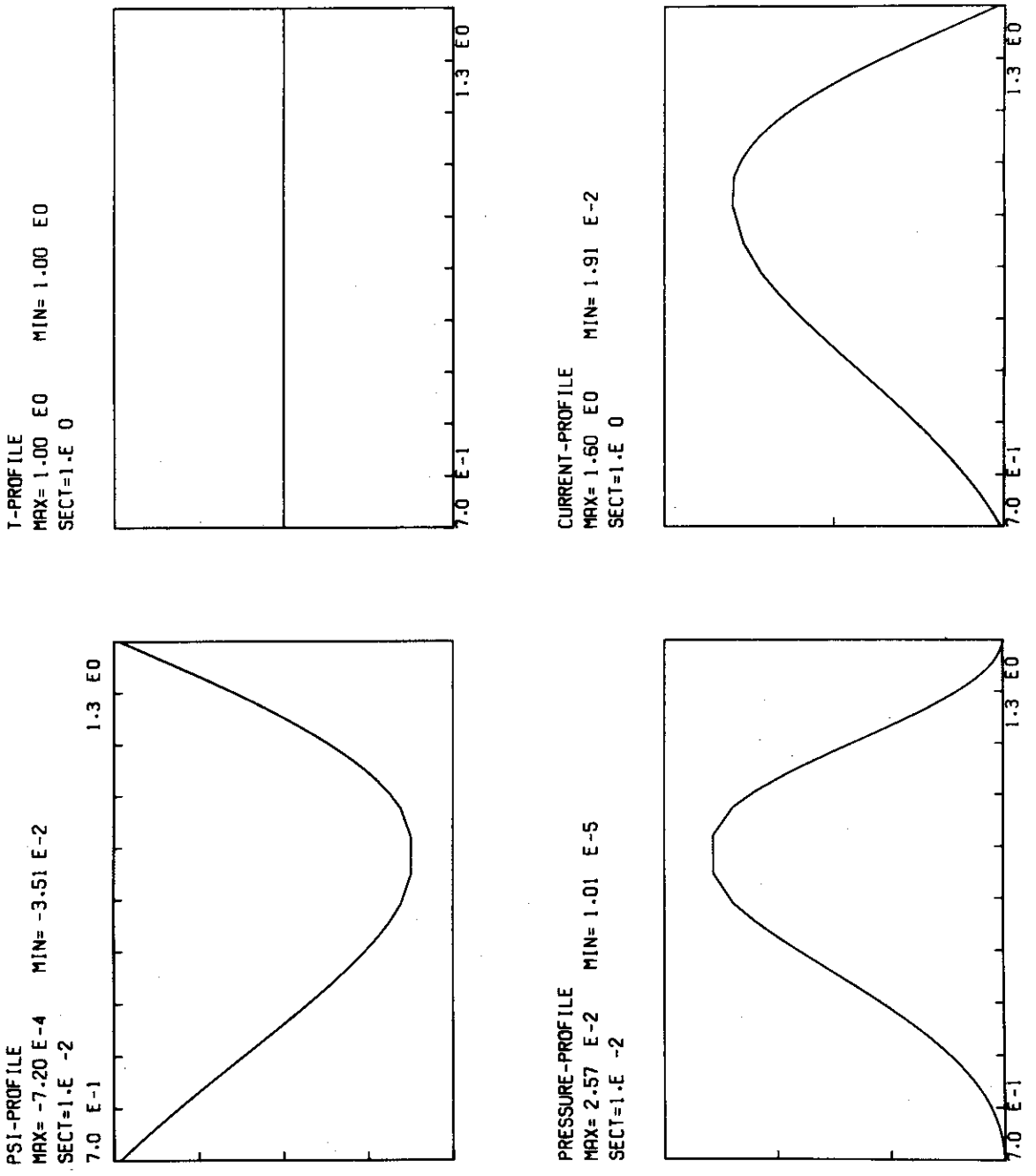
T-PROFILE
MAX= 1.00 E0 MIN= 1.00 E0
SECT=1.E 0
7.0 E-1
1.3 E0
CURRENT-PROFILE
MAX= 1.60 E0 MIN= 1.91 E-2
SECT=1.E 0
7.0 E-1
1.3 E0
PSI-PROFILE
MAX= -7.20 E-4 MIN= -3.51 E-2
SECT=1.E -2
7.0 E-1
1.3 E0
PRESSURE-PROFILE
MAX= 2.57 E-2 MIN= 1.01 E-5
SECT=1.E -2
7.0 E-1
1.3 E0

Fig. 8 (e－2)

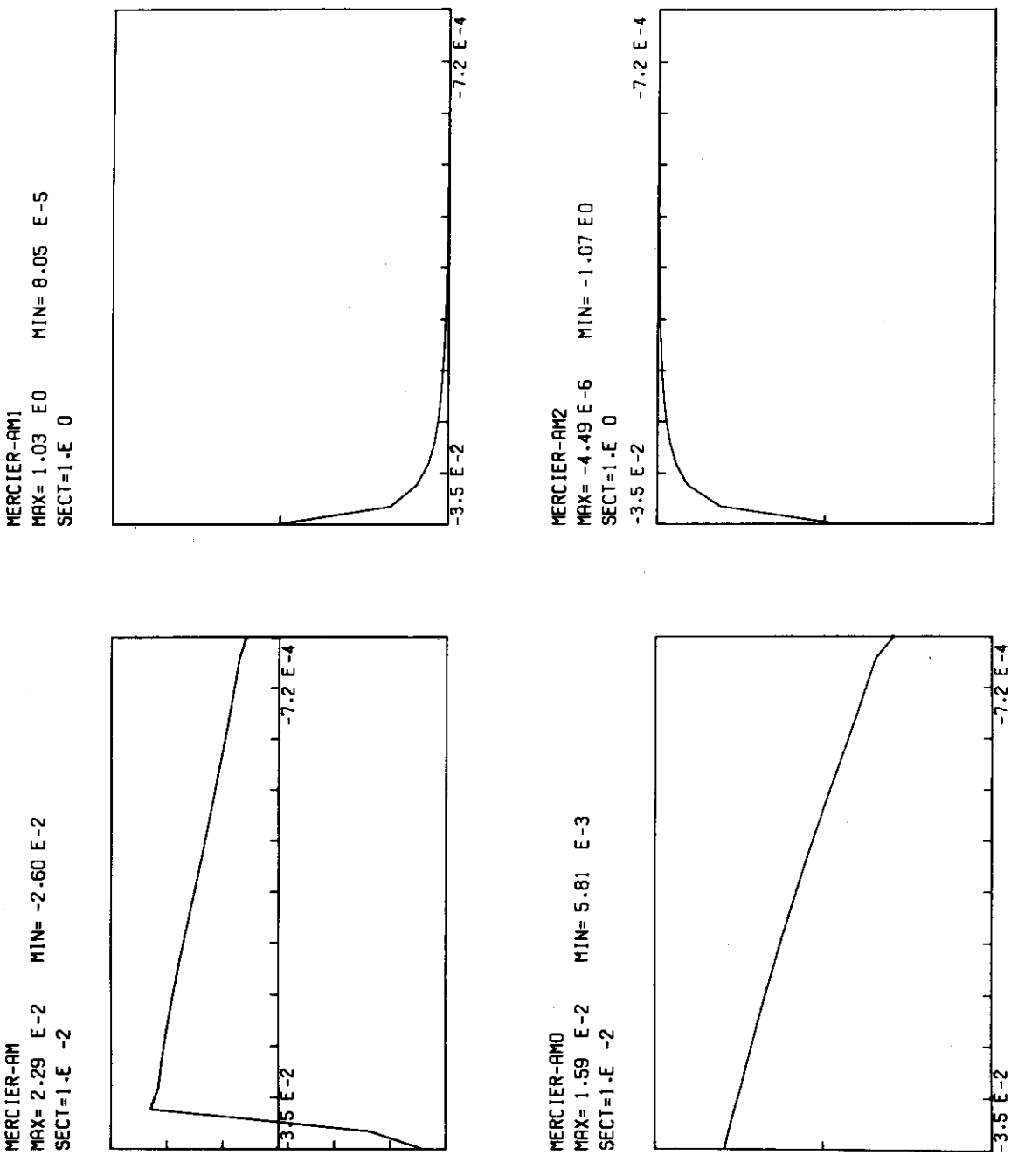
MERCIER-AM1
MAX= 1.03 E0
MIN= 8.05 E-5
SECT=1.E 0
-3.5 E-2
-7.2 E-4
MERCIER-AM2
MAX= -4.49 E-6
MIN= -1.07 E0
SECT=1.E 0
-3.5 E-2
-7.2 E-4
MERCIER-AM
MAX= 2.29 E-2
MIN= -2.60 E-2
SECT=1.E -2
-3.5 E-2
-7.2 E-4
MERCIER-AM0
MAX= 1.59 E-2
MIN= 5.81 E-3
SECT=1.E -2
-3.5 E-2
-7.2 E-4

Fig. 8 (e－3)

APPENDICES

A1. The main program and COMMON variables

A1.1 The main program

```
C
C       *** MAIN PROGRAM (ELM=AAA) ***
C
C       ************************************************************
C       *                                                          *
C       *                .S. .E. .L. .E. .N. .E. .I. .O.           *
C       *                 .   .   .   .   .   .   .   .            *
C       *                                                          *
C       *                                                          *
C       *     M H D   E Q U I L I B R I U M   A N A L Y S I S      *
C       *                                                          *
C       *         -- ITERATION FOR POSITIONS OF CONTOURS --        *
C       *         -- MAY  1, 1978,.A=TT & TT, PTL, DTFR, JAERI --  *
C       *         -- JULY 6, 1978,.A=TT & TT, PTL, DTFR, JAERI --  *
C       *                                                          *
C       *         -- FIXED BOUNDARY PLASMA                         *
C       *         -- DORY TROYON EQUILIBRIUM                       *
C       *                                                          *
C       ************************************************************
C
C
C             ..... STRUCTURE OF THE PROGRAM .....
C                   ----------------------
C
C
C             00,                       MAIN(ELM=AAA)
C                01                     ,INPUT
C                02                     ,NUMBER
C                03                     ,INTMSH
C                 3,1                   ,,LINTP0
C                 3,2                   ,,LINTP3
C                04                     ,ABCDCL
C                05                     ,ITERAT
C                 5,1                   ,,INLOOP
C                  ,1,1                 ,,,MATGEN
C                    ,1,1               ,,,,MATELM
C                      ,1,1             ,,,,,INTGRL
C                        ,1,1           ,,,,,,FUNC
C                          ,1,1         ,,,,,,FUNCF1
C                   ,1,2                ,,,LINE@ (ELM=LINE@)
C                     ,2,1              ,,,,ALDLT(ELM=LINE@)
C                     ,2,2              ,,,,LYV  (ELM=LINE@)
C                     ,2,3              ,,,,DYV  (ELM=LINE@)
C                     ,2,4              ,,,,LTXW (ELM=LINE@)
C                   ,1,3                ,,MAGAXS
C                 5,2                   ,,CORDXY
C                  ,2,1                 ,,,MAGAXS
C                  ,2,2                 ,,,CNTSTR
C                    ,2,1               ,,,,LINTP0
C                    ,2,2               ,,,,LINTP2
C                   ,2,3                ,,,ELMSCH
C                   ,2,4                ,,,CNTMSH
C                   ,2,5                ,,,LINTP0
C                   ,2,6                ,,,LINTP3
C                 5,3                   ,,OLDNEW
C                06                     ,STBLTY
C                 6,1                   ,,BTYPE
C                  ,1,1                 ,,,CENELM
C                    ,1,1               ,,,,PT (ELM=FUNCF1)
C                 6,2                   ,,CDETYP
C                  ,2,1                 ,,,INTPOL(ELM=DIFF)
C                 6,3                   ,,MERCIR
C                  ,3,1                 ,,,DIFF
C                07                     ,OUTPUT
C
C       ************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
C
C           ** MATRIX SIZE **
C
      COMMON /ICMSIZ/ MATSIZ,IVECSZ,NODSIZ,NELMSZ
      DIMENSION AL(30000),X(1700)
C
      MATSIZ=30000
      IVECSZ=1700
      NODSIZ=1700
      NELMSZ=1650
C
C       ************************************************************
C           ** 0000 **      START
 1000 CONTINUE
C
C           ** 01 **        INPUT
      CALL INPUT(IEND)
      IF(IEND.NE.0) STOP '     ***** END OF CALCULATION...AAA *****'
C
C           ** 02 **        NUMBERING
      CALL NUMBER
C
C           ** 03 **        INITIAL MESH GENERATION
      CALL INTMSH
C
C           ** 04 **        GEOMETRIC FACTOR ABCD CALCULATION
      CALL ABCDCL
C
C           ** 05 **        ITERATION LOOP
      CALL ITERAT(AL,X,NDIM,NWID)
  900 CONTINUE
C
C           ** 06 **        QUANTITIES FOR STABILITY CALCULATION
      CALL STBLTY
C
C           ** 07 **        OUTPUT
      CALL OUTPUT(LOOPSG)
C
C
      IF(LOOPSG.NE.0) GO TO 900
      GO TO 1000
      END
```

AAA00100 AAA00200 AAA00300 AAA00400 AAA00500 AAA00600 AAA00700 AAA00800 AAA00900 AAA01000 AAA01100 AAA01200 AAA01300 AAA01400 AAA01500 AAA01600 AAA01700 AAA01800 AAA01900 AAA02000 AAA02100 AAA02200 AAA02300 AAA02400 AAA02500 AAA02600 AAA02700 AAA02800 AAA02900 AAA03000 AAA03100 AAA03200 AAA03300 AAA03400 AAA03500 AAA03600 AAA03700 AAA03800 AAA03900 AAA04000 AAA04100 AAA04200 AAA04300 AAA04400 AAA04500 AAA04600 AAA04700 AAA04800 AAA04900 AAA05000 AAA05100 AAA05200 AAA05300 AAA05400 AAA05500 AAA05600 AAA05700 AAA05800 AAA05900 AAA06000 AAA06100 AAA06200 AAA06300 AAA06400 AAA06500 AAA06600 AAA06700 AAA06800 AAA06900 AAA07000 AAA07100 AAA07200 AAA07300 AAA07400 AAA07500 AAA07600 AAA07700 AAA07800 AAA07900 AAA08000 AAA08100 AAA08200 AAA08300 AAA08400 AAA08500 AAA08600 AAA08700 AAA08800 AAA08900 AAA09000 AAA09100 AAA09200 AAA09300 AAA09400 AAA09500 AAA09600 AAA09700 AAA09800 AAA09900 AAA10000 AAA10100 AAA10200 AAA10300 AAA10400 AAA10500 AAA10600 AAA10700 AAA10800 AAA10900 AAA11000 AAA11100 AAA11200 AAA11300

A1.2 COMMON variables

COMMON BLOCKS (INTEGER)

Label of COMMON BLOCKS / Name of SUBROUTINES	ICMSIZ	ICMOUT	ICMNOD	ICMMSH	ICMFIL	ICMITR	ICMKND	ICMOPT	ICMBND	ICMSCL
MAIN	O									
ABCDCL		O								
BTYPE		O	O	O	O					
CDETYP		O		O	O					
CENELM										
CNTMSH				O						
CNTSTR		O	O	O						
CORDXY		O	O	O		O	O			
DIFF										
ELMSCH			O	O						
FUNC										
FUNCF1										
INLOOP			O	O	O	O	O			
INPUT		O		O	O			O	O	O
INTGRL										
INTMSH		O	O	O		O		O		
ITERAT				O		O				
LINEQ										
LINTP										
MAGAXS			O	O						
MATELM										
MATGEN		O	O	O		O	O			
MERCIR				O						
NUMBER		O	O	O			O			
OLDNEW			O							
OUTPUT	O	O	O	O	O					O

COMMON BLOCKS (REAL)

Label of COMMON BLOCKS / Name of SUBROUTINES	RCMABC	RCMCOR	RCMSCL	RCMGSS	RCMCDE	RCMPQT	RCMBET	RCMLTT	RCMVUP	RCMMSH	RCMLNG	RCMBND	RCMTHE	RCMPLA	RCMLNG	RCMMER
MAIN																
ABCDCL	O															
BTYPE		O	O	O												
CDETYP			O	O	O	O	O	O	O							
CENELM	O															
CNTMSH																
CNTSTR		O								O						
CORDXY		O		O							O	O	O			
DIFF																
ELMSCH		O														
FUNC	O			O												
FUNCF1			O	O										O		
INLOOP		O		O						O						
INPUT			O	O						O		O		O	O	
INTGRL																
INTMSH		O		O						O		O	O		O	
ITERAT				O						O						
LINTP																
MAGAXS		O		O												
MATELM	O															
MATGEN		O														
MERCIR					O	O		O								O
NUMBER																
OLDNEW		O														
OUTPUT		O	O	O	O		O	O				O		O		O

COMMON variables

*Integer

/ICMSIZ/	Memory size
	MATSIZ,IVECSZ,NODSIZ,NELMSZ
/ICMFIL/	File data
	NFOUT,NFEQU,NFST1,NFST2,NFST3
/ICMNOD/	F.E.M. data
	NODELM(1650,4),NODBOD(150),NODSYM(200),
	MAXNOD,MAXELM,MAXBOD,MAXSYM,MAXVAR
/ICMMSH/	F.E.M. and iteration data
	NDIV,NDIV1,MDIV,MDIV1,IDPS(100),
	ITRMXI,ITRMXO,ICHNGO
/ICMKND/	F.E.M. data
	NODKND(1700),IDIF
/ICMBND/	F.E.M. data (boundary shape)
	ISHAPE,NPNT
/ICMITR/	Iteration data
	ITRIN,ITROUT
/ICMSCL/	Scaling data
	ISIGMA,ISTOT
/ICMOUT/	Output control
	IOUT(20)
/ICMOPT/	Option data
	ITITLE(18),IOPT(20)

*Real

/RCMABC/	F.E.M. data
	A3(3),B3(3),C3(3),A4(4),B4(4),C4(4),D4(4)
/RCMLNG/	F.E.M. data
	RDLN(101)
/RCMTHE/	F.E.M. data
	DCOSTT(250)
/RCMBND/	F.E.M. data (boundary shape)
	RC,ASP,ELL,DLT,RBND(250),ZBND(250)

/RCMMSH/	F.E.M. and iteration data
	PLYR(100),EPSINM,EPSOUM,EPSIN,EPSOU
/RCMGSS/	Iteration data
	ALAM,DAXIS,PSMX,AXOLD
/RCMPLA/	Plasma data
	PPARA(10),TPARA(10)
/RCMCOR/	Mesh data
	PASN(1700),PCAL(1700),ROLD(1700),ZOLD(1700),
	RNEW(1700),ZNEW(1700),RBOD(150),ZBOD(150),
	RTMP(150),ZTMP(150)
/RCMPQT/	Solution
	OPS(100),OT(100),OPR(100)
/RCMSCL/	Scaling factor
	SIGMA,DSIGMA
/RCMCDE/	Surface quantities and volume integrals
	OCHI(1700),OQ(100),OC1(100),OC2(100),OC3(100),
	OC4(100),OC5(100),OIP,OIP2,OIB2,OIV,OII,QC
/RCMVUP/	Surface quantities
	VPR(100),UPR(100),VPP(100),UPP(100),SSS(100)
/RCMBET/	Beta values
	BETA,BETAP,BETAST
/RCMMER/	Mercier criterion
	AM(100),AM0(100),AM1(100),AM2(100)
/RCMPLT/	Data for plotting
	PSPLT(200),PRPLT(200),TTPLT(200),
	CUPLT(200),RRPLT(200)

A2. Job control statement

```
 1     ¥NC RSEL1,0205  ,   P,0C,3T,5W,2,   C35
 2     ¥USER   J2636       ,
 3 VM ¥GJOB 3112636,TA,TAKEDA,021,12,SMF=CLS
 4 VM ¥LIBE,L J2636,SELDATA,((SEL10D40)),SYSOUT=CLS
 5 VM ¥HFORT J2636,SELENE10,SYSOUT=CLS
 6 VM ¥HLIEDRUN SYSOUT=CLS
 7     ¥FD READ,FILE=(CATLG,J2636,SELDATA(SEL10D40))
 8 VM ¥TPDISK F77
 9 VM ¥TPDISK F90
10 VM ¥TPDISK F91
11 VM ¥TPDISK F92
12 VM ¥TPDISK F93
13 VM ¥HRUN EFNAME=J2636,GRAPHEBB,NAME=2,SYSOUT=CLS
14 VM ¥GCOM35
15 VM ¥TPDISK F77
16     ¥JEND
```

* remarks

J2636.SELDATA.....Input data file

J2636.SELENE10.....Source program file

J2636.GRAPHEBB.....EB file of a graphic output program

A3. Input data format

A3.1 Subroutine "INPUT"

```
      SUBROUTINE INPUT(IEND)                                            INP00100
C                                                                       INP00200
C                                                                       INP00300
C        *********************************                              INP00400
C        * R E A D   I N P U T   D A T A *                              INP00500
C        *********************************                              INP00600
C                                                                       INP00700
C                                                                       INP00800
C     ** PART 1 **      TITLE                                           INP00900
C         ITITLE(18)                                                    INP01000
C     ** PART 2 **      OPTION SET                                      INP01100
C         IOPT(20)                                                      INP01200
C     ** PART 3 **      PLASMA PARAMETERS                               INP01300
C         PPARA(10)                                                     INP01400
C         TPARA(10)                                                     INP01500
C     ** PART 4 **      BOUNDARY SHAPE                                  INP01600
C         ISHAPE                                                        INP01700
C        ISHAPE=1.....ELLIPSE                                           INP01800
C         RC,ASP,ELL                                                    INP01900
C        ISHAPE=2.....EXTENDED SOLOVEV                                  INP02000
C         RC,ASP,ELL,DLT,80.00,RSCT                                     INP02100
C        ISHAPE=3.....ARBITRARY SHAPE                                   INP02200
C         NPNT,RBND(NPNT),ZBND(NPNT)                                    INP02300
C     ** PART 5 **      MESH AND ITERATION DATA                         INP02400
C         NDIV,MDIV                                                     INP02500
C         IDPS(NDIV).....DELTA PSI                                      INP02600
C         ITRMXI,.....,MAX ITR OF INNER LOOP                            INP02700
C         ITRMXO.....,MAX ITR OF OUTER LOOP                             INP02800
C         ICHNGO,.....,THE 1ST OUTER ITR TO MAKE CONT MESH              INP02900
C         ISIGMA.....,MAX NO OF SIGMA LOOP                              INP03000
C     ** PART 6 **      MESH DATA                                       INP03100
C         IDLN                                                          INP03200
C     ** PART 7 **      STOPPING PARAMETER FOR ITR                      INP03300
C         EPSINM,.....,INNER LOOP                                       INP03400
C         EPSOUM,.....,OUTER LOOP                                       INP03500
C     ** PART 8 **      MISCEL                                          INP03600
C         ALAM,.....,EIGENVAL                                           INP03700
C         DAXIS,.....,POSITION OF AXIS                                  INP03800
C         PSMX                                                          INP03900
C         SIGMA                                                         INP04000
C         DSIGMA                                                        INP04100
C     ** PART 9 **      OUTPUT OPTION                                   INP04200
C         IOUT(20)                                                      INP04300
C            IOUT(1)=1,.....,NUMBER                                     INP04400
C            IOUT(2)=1,.....,INTMSH                                     INP04500
C            IOUT(3)=1,.....,ABCDCL                                     INP04600
C            IOUT(4)=1,.....,MATGEN                                     INP04700
C            IOUT(5)=1,.....,CNTSTR                                     INP04800
C            IOUT(6)=1,.....,CORDXY                                     INP04900
C            IOUT(7)=1,.....,BTYPE                                      INP05000
C            IOUT(8)=1,.....,CDETYP                                     INP05100
C            IOUT(9)=1,.....,OUTPUT                                     INP05200
C     ------------------------------------------                        INP05300
C                                                                       INP05400
C                                                                       INP05500
      IMPLICIT REAL*8(A-H,O-Z)                                          INP05600
      COMMON /ICMOPT/ ITITLE(18),IOPT(20)                               INP05700
      COMMON /RCMPLA/ PPARA(10),TPARA(10)                               INP05800
      COMMON /ICMBND/ ISHAPE,NPNT                                       INP05900
      COMMON /RCMBND/ RC,ASP,ELL,DLT,RBND(250),ZBND(250)                INP06000
      COMMON /ICMMSH/ NDIV,NDIV1,MDIV,MDIV1,IDPS(100),                  INP06100
     1                ITRMXI,ITRMXO,ICHNGO                              INP06200
      COMMON /RCMMSH/ PLYR(100),EPSINM,EPSOUM,EPSIN,EPSOU               INP06300
      COMMON /RCMLNG/ RDLN(101)                                         INP06400
      COMMON /RCMGSS/ ALAM,DAXIS,PSMX,AXOLD                             INP06500
      COMMON /ICMFIL/ NFOUT,NFE0U,NFST1,NFST2,NFST3                     INP06600
      COMMON /RCMSCL/ SIGMA,DSIGMA                                      INP06700
      COMMON /ICMSCL/ ISIGMA,ISTOT                                      INP06800
      COMMON /ICMOUT/ IOUT(20)                                          INP06900
      DIMENSION IDLN(100)                                               INP07000
C                                                                       INP07100
C                                                                       INP07200
      NAMELIST /INOPT / IOPT                                            INP07300
      NAMELIST /RNPLA1/ PPARA                                           INP07400
      NAMELIST /RNPLA2/ TPARA                                           INP07500
      NAMELIST /INBND / ISHAPE,NPNT                                     INP07600
      NAMELIST /RNBND1/ RC,ASP,ELL,DLT                                  INP07700
      NAMELIST /RNBND2/ RBND                                            INP07800
      NAMELIST /RNBND3/ ZBND                                            INP07900
      NAMELIST /INMSH1/ NDIV,MDIV                                       INP08000
      NAMELIST /INMSH2/ IDPS                                            INP08100
      NAMELIST /INMSH3/ ITRMXI,ITRMXO,ICHNGO,ISIGMA                     INP08200
      NAMELIST /INMSH4/ IDLN                                            INP08300
      NAMELIST /RNMSH1/ EPSINM,EPSOUM                                   INP08400
      NAMELIST /RNGSS / ALAM,DAXIS,PSMX,SIGMA,DSIGMA                    INP08500
      NAMELIST /INOUT / IOUT                                            INP08600
C                                                                       INP08700
C                                                                       INP08800
C                                                                       INP08900
      READ (5,50,END=100) (ITITLE(I),I=1,18)                            INP09000
   50 FORMAT(18A4)                                                      INP09100
      GO TO 200                                                         INP09200
  100 CONTINUE                                                          INP09300
      IEND=1                                                            INP09400
      RETURN                                                            INP09500
  200 CONTINUE                                                          INP09600
      IEND=0                                                            INP09700
C                                                                       INP09800
C        ** DEFAULT VALUES ** CLEAR STORAGE **                          INP09900
C                                                                       INP10000
      DO 300 I=1,20                                                     INP10100
      IOPT(I)=0                                                         INP10200
  300 CONTINUE                                                          INP10300
      DO 400 I=1,10                                                     INP10400
      PPARA(I)=0.0                                                      INP10500
      TPARA(I)=0.0                                                      INP10600
  400 CONTINUE                                                          INP10700
      ISHAPE=1                                                          INP10800
      NPNT=1                                                            INP10900
      RC=3.0                                                            INP11000
      ASP=3.0                                                           INP11100
      ELL=1.0                                                           INP11200
                                                                        INP11300
                                                                        INP11400
```

```
      DLT=0.0                                                           INP11500
      DO 500 I=1,250                                                    INP11600
      RBND(I)=0.0                                                       INP11700
      ZBND(I)=0.0                                                       INP11800
  500 CONTINUE                                                          INP11900
      IDPSO=0                                                           INP12000
      DO 600 I=1,100                                                    INP12100
      IDPSN=I**2                                                        INP12200
      IDPS(I)=IDPSN-IDPSO                                               INP12300
      IDLN(I)=1                                                         INP12400
      IDPSO=IDPSN                                                       INP12500
  600 CONTINUE                                                          INP12600
      DO 700 I=1,20                                                     INP12700
      IOUT(I)=0                                                         INP12800
  700 CONTINUE                                                          INP12900
      NFOUT=77                                                          INP13000
      NFEOU=90                                                          INP13100
      NFST1=91                                                          INP13200
      NFST2=92                                                          INP13300
      NFST3=93                                                          INP13400
      ICHNGO=3                                                          INP13500
      ITRMXI=10                                                         INP13600
      ITRMXO=5                                                          INP13700
      EPSINM=1.0D-6                                                     INP13800
      EPSOUM=1.0D-6                                                     INP13900
      ISIGMA=1                                                          INP14000
      ISTOT =0                                                          INP14100
      ALAM=1.0                                                          INP14200
      DAXIS=0.0                                                         INP14300
      PSMX=1.0                                                          INP14400
      SIGMA=0.01                                                        INP14500
      DSIGMA=0.01                                                       INP14600
C                                                                       INP14700
C                                                                       INP14800
C                                                                       INP14900
      READ (5,INOPT)                                                    INP15000
      READ (5,RNPLA1)                                                   INP15100
      READ (5,RNPLA2)                                                   INP15200
      READ (5,INBND)                                                    INP15300
      IF(ISHAPE.GE.3) GO TO 1100                                        INP15400
      READ (5,RNBND1)                                                   INP15500
      GO TO 1200                                                        INP15600
 1100 READ (5,RNBND2)                                                   INP15700
      READ (5,RNBND3)                                                   INP15800
 1200 READ (5,INMSH1)                                                   INP15900
      READ (5,INMSH2)                                                   INP16000
      READ (5,INMSH3)                                                   INP16100
      READ (5,INMSH4)                                                   INP16200
      READ (5,RNMSH1)                                                   INP16300
      READ (5,RNGSS)                                                    INP16400
      READ (5,INOUT)                                                    INP16500
      NDIV1=NDIV+1                                                      INP16600
      MDIV1=MDIV+1                                                      INP16700
      DLTOT=0.0                                                         INP16800
      DO 2000 J=1,NDIV                                                  INP16900
      DLTOT=DLTOT+IDLN(J)                                               INP17000
 2000 CONTINUE                                                          INP17100
      RDLN(1)=0.0                                                       INP17200
      DO 2100 J=2,NDIV1                                                 INP17300
      RDLN(J)=RDLN(J-1)+IDLN(J)/DLTOT                                   INP17400
 2100 CONTINUE                                                          INP17500
      IF(DAXIS.LE.0.0) DAXIS=RC                                         INP17600
C                                                                       INP17700
C         ** WRITE INPUT DATA **                                        INP17800
C                                                                       INP17900
C                                                                       INP18000
      WRITE(6,6000) (ITITLE(I),I=1,18)                                  INP18100
 6000 FORMAT(1H1////1H ,10X,72(1H=)//1H ,10X,18A4//                     INP18200
     1       1H ,10X,72(1H=)////)                                       INP18300
C                                                                       INP18400
C                                                                       INP18500
      WRITE(6,6001) (IOPT(I),I=1,20)                                    INP18600
      WRITE(6,6002) (PPARA(I),I=1,10)                                   INP18700
      WRITE(6,6003) (TPARA(I),I=1,10)                                   INP18800
      WRITE(6,6004) ISHAPE,NPNT                                         INP18900
      WRITE(6,6005) RC,ASP,ELL,DLT                                      INP19000
      WRITE(6,6006) (RBND(I),I=1,NPNT)                                  INP19100
      WRITE(6,6007) (ZBND(I),I=1,NPNT)                                  INP19200
      WRITE(6,6008) NDIV,MDIV                                           INP19300
      WRITE(6,6009) (IDPS(I),I=1,NDIV)                                  INP19400
      WRITE(6,6010) (IDLN(I),I=1,NDIV)                                  INP19500
      WRITE(6,6011) ITRMXI,ITRMXO,ICHNGO,ISIGMA                         INP19600
      WRITE(6,6013) EPSINM,EPSOUM                                       INP19700
      WRITE(6,6014) ALAM,DAXIS,PSMX                                     INP19800
      WRITE(6,6015) (IOUT(I),I=1,20)                                    INP19900
      WRITE(6,6017) SIGMA,DSIGMA                                        INP20000
C                                                                       INP20100
C                                                                       INP20200
C         ** OUTPUT FORMAT **                                           INP20300
C                                                                       INP20400
 6001 FORMAT(1H0,4X,'IOPT=',20I5)                                       INP20500
 6002 FORMAT(1H0,4X,'PPARA=',10E12.5)                                   INP20600
 6003 FORMAT(1H0,4X,'TPARA=',10E12.5)                                   INP20700
 6004 FORMAT(1H0,4X,'ISHAPE=',I3,2X,'NPNT=',I3)                         INP20800
 6005 FORMAT(1H0,4X,'RC=',E12.5,2X,'ASP=',E12.5,2X,'ELL=',E12.5,2X,     INP20900
     1       'DLT=',E12.5                                               INP21000
     2       )                                                          INP21100
 6006 FORMAT(1H0,4X,'RBND=',10E12.5/(10X,10E12.5))                      INP21200
 6007 FORMAT(1H0,4X,'ZBND=',10E12.5/(10X,10E12.5))                      INP21300
 6008 FORMAT(1H0,4X,'NDIV=',I5,2X,'MDIV=',I5)                           INP21400
 6009 FORMAT(1H0,4X,'IDPS=',20I5/(10X,20I5))                            INP21500
 6011 FORMAT(1H0,4X,'ITRMAX(INLOOP) =',I5,2X,                           INP21600
     1       'ITRMAX(OUTLOOP)=',I5,2X,'ICHNGO=',I5,2X,'ISIGMA=',I5)     INP21700
 6013 FORMAT(1H0,4X,'EPSINM=',E12.5,2X,'EPSOUM=',E12.5)                 INP21800
 6014 FORMAT(1H0,4X,'ALAM=',E12.5,2X,'DAXIS=',E12.5,2X,'PSMX=',E12.5)   INP21900
 6015 FORMAT(1H0,4X,'IOUT=',20I5)                                       INP22000
 6016 FORMAT(1H0,4X,'IDLN=',20I5/(10X,20I5))                            INP22100
 6017 FORMAT(1H0,4X,'SIGMA=',E12.5,2X,'DSIGMA=',E12.5)                  INP22200
C                                                                       INP22300
C                                                                       INP22400
      RETURN                                                            INP22500
      END                                                               INP22600
```

A3.2 An example of an input data set

```
    TEST CALCULATION, DORY-TYPE
*INOPT *END
*RNPLA1 PPARA=1,50,0,7,4,0 *END
*RNPLA2 *END
*INBND ISHAPE=2 *END
*RNBND1 RC=1,,ASP=3,3,ELL=1,7,DLT=0,0 *END
*INMSH1 NDIV=30,MDIV=30 *END
*INMSH2 IDPS=40*1, *END
*INMSH3 ITRMXI=8,ITRMXO=5,ICHNGO=3 *END
*INMSH4 IDLN=40*1, *END
*RNMSH1 EPSINM=1,0D-5,EPSOUM=1,0D-5 *END
*RNGSS SIGMA=0,04 *END
*INOUT *END
```

A4. An example of line printer output

```
**********************************************************************

    TEST CALCULATION, DORY-TYPE

**********************************************************************


IOPT=   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0

PPARA= 0,15000D+01 0,70000D+00 0,40000D+01 0,0          0,0          0,0          0,0          0,0          0,0          0,0

TPARA= 0,0         0,0         0,0         0,0         0,0         0,0         0,0         0,0         0,0         0,0

ISHAPE=  2  NPNT=  1

RC= 0,10000D+01  ASP= 0,33000D+01  ELL= 0,17000D+01  DLT= 0,0

RBND= 0,0

ZBND= 0,0

NDIV=  30  MDIV=  30

IDPS=   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
        1   1   1   1   1   1   1   1   1   1   1

IDLN=   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
        1   1   1   1   1   1   1   1   1   1   1

ITRMAX(INLOOP) =    8  ITRMAX(OUTLOOP)=    5  ICHNGD=    3  ISIGMA=    1

EPSINM= 0,10000D-04  EPSOUM= 0,10000D-04

ALAM= 0,10000D+01  DAXIS= 0,10000D+01  PSMX= 0,10000D+01

IOUT=   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0

 SIGMA= 0,40000D-01  DSIGMA= 0,10000D-01
 ITROUT=   1 ITRIN=   1(   8) EPSIN= 0,98521D+00( 0,10000D-04) ALAM= 0,67608D+02
 ITROUT=   1 ITRIN=   2(   8) EPSIN= 0,15858D+00( 0,10000D-04) ALAM= 0,58354D+02
 ITROUT=   1 ITRIN=   3(   8) EPSIN= 0,85001D-02( 0,10000D-04) ALAM= 0,57862D+02
 ITROUT=   1 ITRIN=   4(   8) EPSIN= 0,56972D-03( 0,10000D-04) ALAM= 0,57829D+02
 ITROUT=   1 ITRIN=   5(   8) EPSIN= 0,16887D-04( 0,10000D-04) ALAM= 0,57830D+02
 ITROUT=   1 ITRIN=   6(   8) EPSIN= 0,26202D-04( 0,10000D-04) ALAM= 0,57832D+02
 ITROUT=   1 ITRIN=   7(   8) EPSIN= 0,11585D-04( 0,10000D-04) ALAM= 0,57833D+02
 ITROUT=   1 ITRIN=   8(   8) EPSIN= 0,38714D-05( 0,10000D-04) ALAM= 0,57833D+02
*ITROUT=   1(   5)             EPSOUT= 0,21870D-01( 0,10000D-04)                    DAXIS= 0,10219D+01 **IFIN=   0
 ITROUT=   2 ITRIN=   1(   8) EPSIN= 0,15683D+00( 0,10000D-04) ALAM= 0,68590D+02
 ITROUT=   2 ITRIN=   2(   8) EPSIN= 0,14611D+00( 0,10000D-04) ALAM= 0,59846D+02
 ITROUT=   2 ITRIN=   3(   8) EPSIN= 0,81147D-02( 0,10000D-04) ALAM= 0,59364D+02
 ITROUT=   2 ITRIN=   4(   8) EPSIN= 0,65501D-03( 0,10000D-04) ALAM= 0,59326D+02
 ITROUT=   2 ITRIN=   5(   8) EPSIN= 0,23466D-04( 0,10000D-04) ALAM= 0,59324D+02
 ITROUT=   2 ITRIN=   6(   8) EPSIN= 0,16173D-04( 0,10000D-04) ALAM= 0,59325D+02
 ITROUT=   2 ITRIN=   7(   8) EPSIN= 0,71304D-05( 0,10000D-04) ALAM= 0,59325D+02
*ITROUT=   2(   5)             EPSOUT= 0,36253D-03( 0,10000D-04)                    DAXIS= 0,10222D+01 **IFIN=   0
 ITROUT=   3 ITRIN=   1(   8) EPSIN= 0,13538D+00( 0,10000D-04) ALAM= 0,68615D+02
 ITROUT=   3 ITRIN=   2(   8) EPSIN= 0,14595D+00( 0,10000D-04) ALAM= 0,59875D+02
 ITROUT=   3 ITRIN=   3(   8) EPSIN= 0,81028D-02( 0,10000D-04) ALAM= 0,59394D+02
 ITROUT=   3 ITRIN=   4(   8) EPSIN= 0,65275D-03( 0,10000D-04) ALAM= 0,59355D+02
 ITROUT=   3 ITRIN=   5(   8) EPSIN= 0,23753D-04( 0,10000D-04) ALAM= 0,59354D+02
 ITROUT=   3 ITRIN=   6(   8) EPSIN= 0,16027D-04( 0,10000D-04) ALAM= 0,59355D+02
 ITROUT=   3 ITRIN=   7(   8) EPSIN= 0,70759D-05( 0,10000D-04) ALAM= 0,59355D+02
*ITROUT=   3(   5)             EPSOUT= 0,10751D-03( 0,10000D-04)                    DAXIS= 0,10223D+01 **IFIN=   1
 ITROUT=   4 ITRIN=   1(   8) EPSIN= 0,82257D-02( 0,10000D-04) ALAM= 0,58871D+02
 ITROUT=   4 ITRIN=   2(   8) EPSIN= 0,95324D-02( 0,10000D-04) ALAM= 0,59199D+02
 ITROUT=   4 ITRIN=   3(   8) EPSIN= 0,44970D-04( 0,10000D-04) ALAM= 0,59201D+02
 ITROUT=   4 ITRIN=   4(   8) EPSIN= 0,10008D-04( 0,10000D-04) ALAM= 0,59201D+02
 ITROUT=   4 ITRIN=   5(   8) EPSIN= 0,31379D-05( 0,10000D-04) ALAM= 0,59200D+02
*ITROUT=   4(   5)             EPSOUT= 0,68720D-03( 0,10000D-04)                    DAXIS= 0,10217D+01 **IFIN=   1
 ITROUT=   5 ITRIN=   1(   8) EPSIN= 0,37270D-02( 0,10000D-04) ALAM= 0,59422D+02
 ITROUT=   5 ITRIN=   2(   8) EPSIN= 0,52138D-02( 0,10000D-04) ALAM= 0,59232D+02
 ITROUT=   5 ITRIN=   3(   8) EPSIN= 0,24410D-04( 0,10000D-04) ALAM= 0,59233D+02
 ITROUT=   5 ITRIN=   4(   8) EPSIN= 0,30385D-05( 0,10000D-04) ALAM= 0,59233D+02
*ITROUT=   5(   5)             EPSOUT= 0,42814D-03( 0,10000D-04)                    DAXIS= 0,10221D+01 **IFIN=   2

  @C= 0,12916D+01

  $ = 0,13148D+01 0,13498D+01 0,13851D+01 0,14239D+01 0,14644D+01 0,15070D+01 0,15519D+01 0,15994D+01 0,16497D+01 0,17031D+01
      0,17600D+01 0,18207D+01 0,18856D+01 0,19554D+01 0,20306D+01 0,21119D+01 0,22004D+01 0,22969D+01 0,24028D+01 0,25197D+01
      0,26495D+01 0,27946D+01 0,29583D+01 0,31446D+01 0,33588D+01 0,36086D+01 0,39043D+01 0,42617D+01 0,47056D+01 0,52797D+01

  C1= 0,16401D+05 0,44038D+04 0,22646D+04 0,16155D+04 0,12937D+04 0,11033D+04 0,97996D+03 0,89570D+03 0,83662D+03 0,79501D+03
      0,76632D+03 0,74773D+03 0,73730D+03 0,73457D+03 0,73838D+03 0,74874D+03 0,76577D+03 0,78995D+03 0,82210D+03 0,86347D+03
      0,91586D+03 0,98188D+03 0,10653D+04 0,11715D+04 0,13087D+04 0,14899D+04 0,17362D+04 0,20849D+04 0,26076D+04 0,34626D+04

  C2= 0,15698D+05 0,42630D+04 0,22241D+04 0,16083D+04 0,13058D+04 0,11294D+04 0,10175D+04 0,94373D+03 0,89478D+03 0,86337D+03
      0,84535D+03 0,83819D+03 0,84044D+03 0,85135D+03 0,87070D+03 0,89873D+03 0,93607D+03 0,98384D+03 0,10437D+04 0,11179D+04
      0,12099D+04 0,13240D+04 0,14669D+04 0,16478D+04 0,18810D+04 0,21882D+04 0,26051D+04 0,31943D+04 0,40750D+04 0,55079D+04

  C3= 0,17145D+05 0,45654D+04 0,23229D+04 0,16404D+04 0,13003D+04 0,10975D+04 0,96433D+03 0,87175D+03 0,80508D+03 0,75617D+03
      0,72017D+03 0,69406D+03 0,67589D+03 0,66441D+03 0,65885D+03 0,65879D+03 0,66410D+03 0,67490D+03 0,69159D+03 0,71487D+03
      0,74584D+03 0,78609D+03 0,83797D+03 0,90497D+03 0,99235D+03 0,11084D+04 0,12667D+04 0,14912D+04 0,18283D+04 0,23798D+04

  C4= 0,90096D+01 0,91370D+01 0,92680D+01 0,93955D+01 0,95265D+01 0,96622D+01 0,98029D+01 0,99493D+01 0,10102D+02 0,10261D+02
      0,10428D+02 0,10603D+02 0,10787D+02 0,10982D+02 0,11189D+02 0,11410D+02 0,11646D+02 0,11900D+02 0,12174D+02 0,12473D+02
      0,12801D+02 0,13162D+02 0,13565D+02 0,14018D+02 0,14534D+02 0,15128D+02 0,15826D+02 0,16662D+02 0,17695D+02 0,19027D+02

  C5= 0,47348D-02 0,18953D-01 0,38051D-01 0,56001D-01 0,73536D-01 0,90780D-01 0,10776D+00 0,12449D+00 0,14095D+00 0,15716D+00
      0,17308D+00 0,18872D+00 0,20404D+00 0,21905D+00 0,23370D+00 0,24797D+00 0,26184D+00 0,27527D+00 0,28821D+00 0,30063D+00
      0,31247D+00 0,32366D+00 0,33414D+00 0,34382D+00 0,35262D+00 0,36040D+00 0,36705D+00 0,37240D+00 0,37624D+00 0,37831D+00

  OPS=-0,39297D-01-0,38237D-01-0,36849D-01-0,35497D-01-0,34153D-01-0,32811D-01-0,31470D-01-0,30130D-01-0,28790D-01-0,27451D-01
      -0,26112D-01-0,24772D-01-0,23433D-01-0,22094D-01-0,20756D-01-0,19417D-01-0,18078D-01-0,16740D-01-0,15401D-01-0,14062D-01
      -0,12723D-01-0,11384D-01-0,10045D-01-0,87060D-02-0,73669D-02-0,60277D-02-0,46886D-02-0,33496D-02-0,20110D-02-0,67085D-03

  OT = 0,10006D+01 0,10014D+01 0,10025D+01 0,10036D+01 0,10046D+01 0,10055D+01 0,10065D+01 0,10074D+01 0,10083D+01 0,10092D+01
       0,10100D+01 0,10108D+01 0,10116D+01 0,10123D+01 0,10131D+01 0,10138D+01 0,10144D+01 0,10150D+01 0,10156D+01 0,10162D+01
       0,10167D+01 0,10172D+01 0,10176D+01 0,10180D+01 0,10184D+01 0,10186D+01 0,10189D+01 0,10191D+01 0,10192D+01 0,10193D+01

  VPR= 0,86782D+01 0,87859D+01 0,89267D+01 0,90865D+01 0,92518D+01 0,94240D+01 0,96039D+01 0,97920D+01 0,99894D+01 0,10197D+02
       0,10415D+02 0,10646D+02 0,10891D+02 0,11151D+02 0,11429D+02 0,11726D+02 0,12046D+02 0,12392D+02 0,12769D+02 0,13181D+02
       0,13634D+02 0,14137D+02 0,14699D+02 0,15334D+02 0,16060D+02 0,16901D+02 0,17892D+02 0,19086D+02 0,20568D+02 0,22491D+02

  UPR= 0,83269D+01 0,84691D+01 0,86807D+01 0,89147D+01 0,91593D+01 0,94166D+01 0,96883D+01 0,99756D+01 0,10280D+02 0,10604D+02
       0,10949D+02 0,11317D+02 0,11712D+02 0,12136D+02 0,12594D+02 0,13090D+02 0,13629D+02 0,14218D+02 0,14865D+02 0,15579D+02
       0,16374D+02 0,17263D+02 0,18266D+02 0,19408D+02 0,20724D+02 0,22258D+02 0,24077D+02 0,26276D+02 0,29009D+02 0,32546D+02

  IP= 0,83413D-02  IP2= 0,26344D-03  IB2= 0,71114D+00  IV= 0,54252D+00  II=-0,38666D+00

  BETA= 0,23459D-01  BETAP= 0,14022D+01  BETAST= 0,33622D-01

  @DP= 0,36337D+02 0,29720D+02 0,27069D+02 0,29427D+02 0,30951D+02 0,32637D+02 0,34480D+02 0,36494D+02 0,38710D+02 0,41158D+02
       0,43876D+02 0,46906D+02 0,50307D+02 0,54136D+02 0,58472D+02 0,63410D+02 0,69074D+02 0,75612D+02 0,83216D+02 0,92129D+02
       0,10268D+03 0,11532D+03 0,13065D+03 0,14955D+03 0,17324D+03 0,20366D+03 0,24388D+03 0,29925D+03 0,37997D+03 0,47684D+03

  ** MERCIER CRITERION **
```

M0	M1	M2	M
0.10345D-01	0.32072D+01	-0.36355D+01	-0.41796D+00
0.12071D-01	0.11099D+01	-0.12311D+01	-0.10914D+00
0.14293D-01	0.61665D+00	-0.57482D+00	0.56121D-01
0.15184D-01	0.40076D+00	-0.36716D+00	0.48788D-01
0.15076D-01	0.29165D+00	-0.26279D+00	0.43943D-01
0.15008D-01	0.22588D+00	-0.19984D+00	0.41046D-01
0.14954D-01	0.18184D+00	-0.15782D+00	0.38971D-01
0.14911D-01	0.15028D+00	-0.12746D+00	0.37360D-01
0.14884D-01	0.12659D+00	-0.10547D+00	0.36001D-01
0.14874D-01	0.10815D+00	-0.88155D-01	0.34870D-01
0.14884D-01	0.93387D-01	-0.74392D-01	0.33880D-01
0.14916D-01	0.81283D-01	-0.63208D-01	0.32990D-01
0.14969D-01	0.71168D-01	-0.53953D-01	0.32184D-01
0.15046D-01	0.62573D-01	-0.46176D-01	0.31442D-01
0.15144D-01	0.55156D-01	-0.39553D-01	0.30747D-01
0.15265D-01	0.48671D-01	-0.33849D-01	0.30087D-01
0.15411D-01	0.42935D-01	-0.28889D-01	0.29457D-01
0.15579D-01	0.37804D-01	-0.24538D-01	0.28844D-01
0.15769D-01	0.33165D-01	-0.20697D-01	0.28236D-01
0.15977D-01	0.28934D-01	-0.17288D-01	0.27623D-01
0.16201D-01	0.25048D-01	-0.14251D-01	0.26998D-01
0.16444D-01	0.21462D-01	-0.11545D-01	0.26361D-01
0.16704D-01	0.18135D-01	-0.91382D-02	0.25701D-01
0.16979D-01	0.15036D-01	-0.70093D-02	0.25006D-01
0.17265D-01	0.12148D-01	-0.51472D-02	0.24265D-01
0.17566D-01	0.94602D-02	-0.35493D-02	0.23476D-01
0.17895D-01	0.69727D-02	-0.22215D-02	0.22646D-01
0.18286D-01	0.46918D-02	-0.11781D-02	0.21799D-01
0.18820D-01	0.26270D-02	-0.44275D-03	0.21005D-01
0.17336D-01	0.81760D-03	-0.57124D-04	0.18096D-01

```
** MATRIX SIZE **
     MATSIZ=   28800(  30000)  MAXVAR=    900(   1700)
     MAXNOD=     931(   1700)  MAXELM=    900(   1650)
     NFOUT= 77
     NFEQU= 90
     NFST1= 91
     NFST2= 92
     NFST3= 93
```