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Scaling of Critical Beta in a Tokamak

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Stability of Tokamak for high-n ballooning mode is studied. Combination of a 2-D transport code and an infinite-n ballooning code is used to optimize the beta value (the ratio of the plasma pressure to the magnetic pressure). We get the functional dependence of the maximum beta as

$$\beta\,(\%) \simeq 7.8\, q_s^{-0.54}\, (A-1)^{-0.76}\, \varepsilon^{1+0.14(q_s-1)} ,$$

here q_s is the safety factor at the plasma surface, A the inverse aspect ratio, ε the ellipticity of the plasma cross-section.

Keywords; Tokamak, Ballooning Mode Instability, Critical Beta Value, Two-Dimensional Tokamak Code, Anomalous Thermal Diffusion

トカマクの臨界ベータ値比例則

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トカマクにおける高モード数のバルーニング・モードの安定性を調べた。二次元輸送コードとバルーニング不安定性コードを用いてベータ値（プラズマ圧力と磁気圧の比）の最適化を行なった。最大ベータ値の比例則は

$$\beta(\%) \simeq 7.8\, q_s^{-0.54} (A-1)^{-0.76} \varepsilon^{1+0.14(q_s-1)}$$

となる。ここで q_s はプラズマ表面における安定係数，A は逆アスペクト比，ε はプラズマ断面の楕円度である。

目次なし

It is desirable from the economic viewpoint that a tokamak fusion reactor is operated in the high beta state. The possible beta value, the ratio of the plasma pressure to the magnetic pressure, may be restricted by MHD instabilities, especially by unstable high-n ballooning modes. If we can heat a plasma up to the second critical beta value much faster than the anomalous diffusion due to unstable ballooning modes, we can obtain a fairly high beta tokamak equilibrium which is stable against ballooning modes over the whole plasma region [1]. When the heating rate is not so much large, the beta is limited by a certain value, i.e., the first critical beta value. In this paper, we study in detail about the dependencies of this value on the safety factor at the plasma surface q_s, the aspect ratio A, and the ellipticity ε.

The basic equations are the combination of the surface averaged transport equations of simplified FCT version

$$\frac{\partial}{\partial t}\, p\left(\frac{\partial V}{\partial \psi}\right)^{5/3} = \left(\frac{\partial V}{\partial \psi}\right)^{3/2} \frac{\partial}{\partial \psi}\left(\chi_p \frac{\partial V}{\partial \psi}\frac{\partial p}{\partial \psi}\right) + S(\psi)\left(\frac{\partial V}{\partial \psi}\right)^{5/3} \qquad (1)$$

$$\frac{\partial}{\partial t}\, q = 0 \;, \qquad (2)$$

and the axisymmetric toroidal MHD equilibrium equation

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi}{\partial R}\right) + \frac{\partial^2 \psi}{\partial Z^2} = -R^2\frac{dp}{d\psi} - T\frac{dT}{d\psi} \qquad (3)$$

where ψ, V, p, S, and T are the poloidal magnetic flux, the volume surrounded by a contour of ψ, the plasma pressure, the pressure source, and the poloidal current flux, respectively. In order to manifest the effect of unstable ballooning modes on the plasma transport, only the anomalous thermal conductivity χ_p due to these instabilities is taken into account and the electric conductivity is assumed infinitely large. We start the computation from the force-free equilibrium with the toroidal current density j_ϕ

$$R\, j_\phi = \alpha_0 \{ 1 - \alpha_1\psi - (1-\alpha_1)\,\psi^4 \} \;, \qquad (4)$$

where ψ is normalized such that ψ=0 at the magnetic axis and ψ=1 at the plasma surface. Parameters α_0 and α_1 are adjusted such that the values of the safety factor at the magnetic axis q_a and at the surface q_s take the prescribed ones.

In the transport equation, we take into account of the pressure source and the anomalous heat conductivity due to unstable high-n ballooning modes, which is zero on stable magnetic surfaces and has the infinitely large value on unstable surfaces. The critical pressure gradient $dp/d\psi|_{crit}$ against the infinitely high-n ballooning mode on a magnetic surface is obtained from the ballooning mode equation described in Ref. [2] with the zero growth rate;

$$\frac{d}{ds}\left(f\frac{d\phi}{ds}\right) + \left(\frac{dp}{d\psi}\right)_{crit} g\,\phi = 0$$

$$f = \frac{1}{R^2B_p}\left[1+ \left(\frac{R^2B_p^2}{B} z\right)^2 \right]$$

$$g = \frac{1}{R^2B_p}\left[\frac{\partial}{\partial\psi}(2p + B^2) - \frac{RB_tB_p}{B} z \frac{\partial B^2}{\partial s} \right]$$

$$z = \int_0^s \frac{ds}{JB_p}\frac{\partial}{\partial\psi}\left(\frac{JB_t}{R}\right)$$

where s is the arc length and J is Jacobian. This equation is solved as the eigenvalue problem by using the finite element method. We start the computation from a very low beta equilibrium with the safety factor at the magnetic axis of unity. This plasma is heated up to higher beta by the pressure source with the profile $(1-\psi^2)^2$. After the plasma becomes unstable localy, the pressure profile near the unstable region is changed by the anomalous diffusion. The beta value still continues to increase by changing the pressure profile. Finally, the whole plasma region becomes marginally stable and the beta value saturates. It must be noted that the final beta value and presuure profile are independent of the source profile.

This critical value is mainly dependent on the plasma shape and the q profile. Figures 1 and 2 show the critical beta value β and the critical poloidal beta value β_J, respectively, as functions of q_s, A and ε. The critical β decreases with increasing q_s and A (Figs. 1 a, b and c), while β_J increases almost linearly with increasing q_s and A (Figs. 2 a, b and c). The beta value depends almost linearly on the ellipticity in the range $0.8 < \varepsilon < 2.0$ (Fig. 1 d) , while β_J has the peak near $\varepsilon = 1.1$ (Fig. 2 d). From these results, we get the functional dependence of β on q_s, A and ε as

$$\beta(\%) \simeq 7.8\, q_s^{-0.54} (A-1)^{-0.76} \varepsilon^{1+0.14(q_s-1)} .$$

This relation is much different from the well-known one, $\beta \sim 1/q_s{}^2A$. This is caused mainly by the effect of the shear, as is seen in Fig.3, where the safety factor q, the local shear $s = rq'/q$ and the corresponding marginally stable pressure p are plotted for $q_s = 2$ (dashed line) and $q_s = 4$ (solid line).

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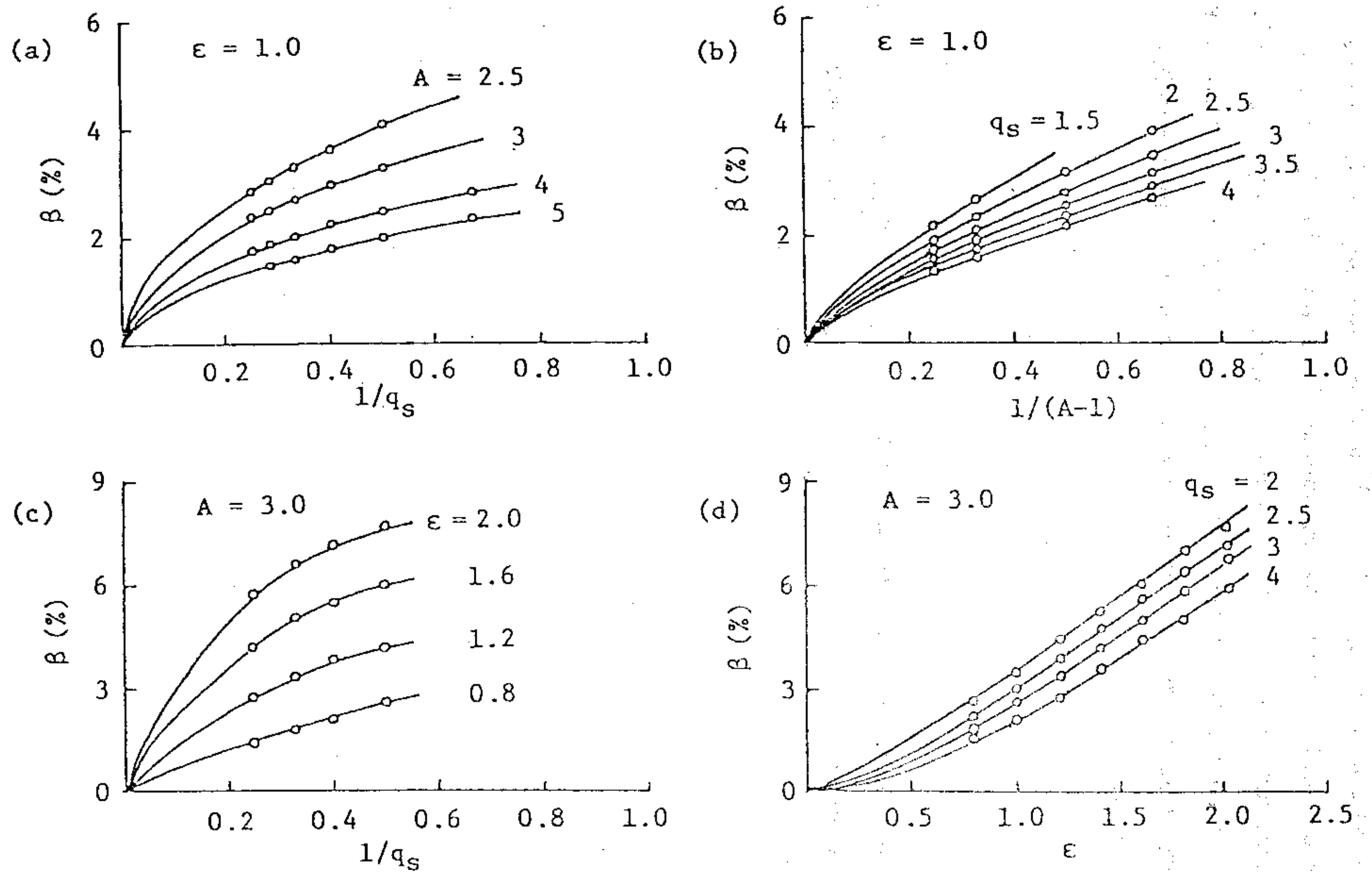

Fig. 1 Critical beta values β (%) as functions of (a) the safety factor at the plasma surface q_s for the aspect ratio $A = 2.5, 3, 4, 5$ and the ellipticity $\varepsilon = 1$, (b) A for $q_s = 1.5, 2, 2.5, 3, 3.5, 4$ and $\varepsilon = 1$, (c) q_s for $A = 3$ and $\varepsilon = 0.8, 1.2, 1.6, 2$, and of (d) ε for $A = 3$ and $q_s = 2, 2.5, 3, 4$. The safety factor at the axis is chosen to be $q_a = 1$.

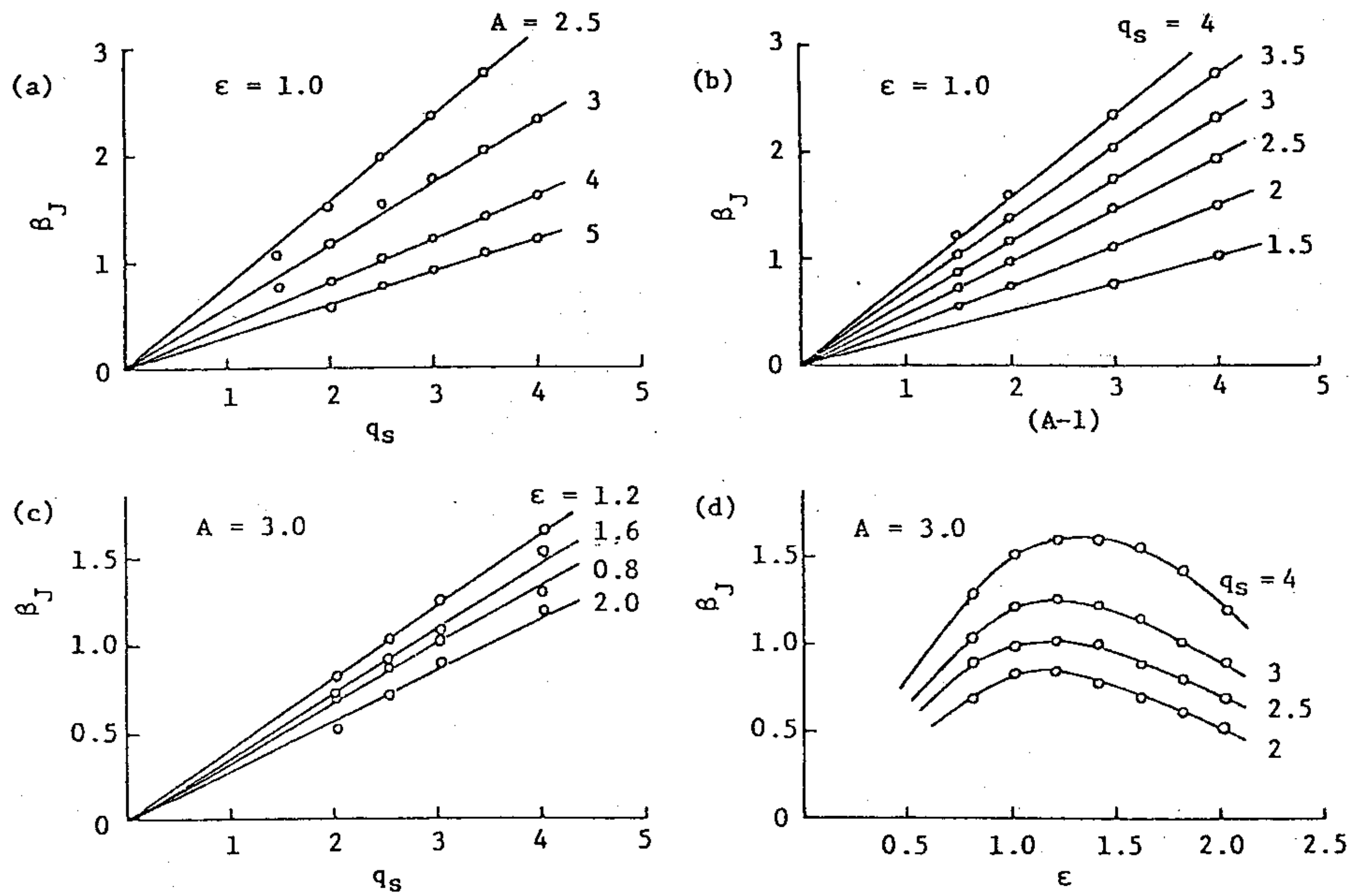

Fig. 2 Critical poloidal beta values β_J measured by the total plasma current, as functions of q_s, A and ε.

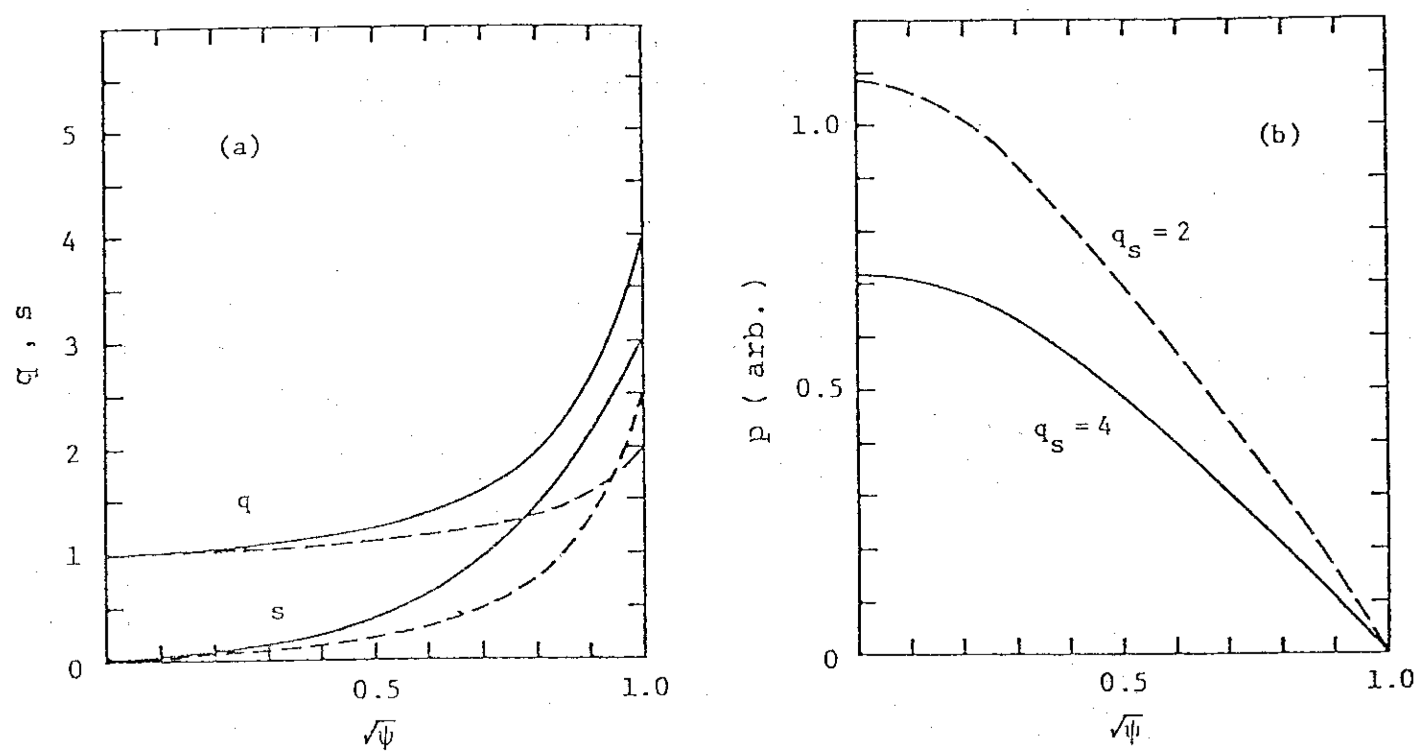

Fig. 3 Profiles of (a) the safety factor q and the shear parameter $s = rq'/q$ and of (b) the pressure marginally stable against ballooning modes, as functions of $\sqrt{\psi}$. The solid curves correspond to the case of $q_s = 4$ and the broken curves to the case of $q_s = 2$.