# JAERI-M
# 8238

DEVELOPMENT OF A THREE-DIMENSIONAL
NEUTRON DIFFUSION CODE SERIES BY
LEAKAGE ITERATIVE METHOD

May 1979

Yoshitaka NAITO

日 本 原 子 力 研 究 所
Japan Atomic Energy Research Institute

Development of a Three-Dimensional Neutron Diffusion Code
Series by Leakage Iterative Method

Yoshitaka NAITO

Division of JPDR,
Tokai Research Establishment, JAERI

In nodal and coarse mesh methods, one of the most difficult problems
is to estimate neutron leakages from a subregion because spatial mesh
widths are too wide to evaluate the neutron current at the boundary of the
subregion accurately.  To eliminate this difficulty, a leakage iterative
method is proposed and several computer codes have been developed.
The contents of these codes are briefly shown in this article.

# 漏洩量繰返し法による3次元中性子拡散コード・シリーズの開発

日本原子力研究所東海研究所動力試験炉部

内 藤 俶 孝

　ノード法や粗メッシュ法において最も困難な問題の1つは部分領域からの中性子漏洩量を推定することである。なぜなら，部分領域の境界における中性子流を高精度で求めるには上記計算手法における空間メッシュの巾は広すぎる。この困難さを取除くために，漏洩量繰返し法が提案され，いくつかの計算コードが開発された。この報告書では，これ等の計算コードの内容を簡単に示す。

# Contents

# 目　　　　　次

## 1. Outline of Leakage Iterative Method

In the leakage iterative method (LIM), the reactor is divided into several layers along the z axis and into several rectangular channels across the xy plane as shown in Fig.1. A parallelepiped formed by a channel and a layer is called a block. To start the iterative procedure for solving the diffusion equation, the neutron source and the radial leakage coefficients are assumed as shown in Fig.2. A one-dimensional neutron flux calculation is performed for each channel with the radial leakage coefficient. A two-dimensional neutron flux calculation is then made for each layer with the axial leakage determined from the one-dimensional calculation. The one- and two-dimensional leakage calculations are iterated until the consistency is attained between both. At each step of the iteration the neutron source distribution and the eigenvalue are recalculated. For obtaining the balance of the neutron population within a block, it is important to evaluate the neutron leakage from the block as precisely as possible. For this purpose, a block is subdivided into fine meshes, and the fine-mesh difference approximation method is applied to solve the one- and two-dimensional neutron diffusion equations for each channel and layer, respectively.

The present method has the following characteristics:

1) A fine-mesh difference approximation technique is applied only to the channels and layers. Therefore it is not necessary to calculate the neutron fluxes at all fine-mesh points in the core and thus the computer time is reduced. If the block is a 12 cm cube and the mesh width is 2 cm, the number of fine-mesh points is $6 \times 6 \times 6 = 216$. In the present method, however, the number of mesh points used for the calculations is $6 + (6 \times 6) = 42$, that is about one-fifth of the former. The physical

quantities connecting the channel and layer calculations are only the

neutron leakage and the neutron source, and the fact reduces the computer

memory required.

2)  Since the neutron leakage from a block is calculated by a fine-mesh

difference approximation, the numerical error due to discretization is

minimized.

3)  When only one fine-mesh point is located in each block, this approach

becomes the same as the usual method in a fine-mesh difference approxi-

mation.  In this case, the iterative scheme coresponds to one of the

variants of the Peaceman-Rachford iterative method.  Therefore, it is

possible to establish the condition, under which the consistency is

achieved between the axial and radial leakages in the same manner as

ADI (alternating direction implicit iterative methods of Peaceman and

Rachford).  In addition, it is easy to compare the results with those

obtained from conventional fine-mesh difference approximation methods.

The computer code can thus be used for calculating both the collapsed

flux and the fine-mesh flux.


2.  Fundamental Equation

The iterative process to recalculate the source distribution with

use  of the previously obtained neutron flux is the same as that utilized

in the conventional power-iteration method.  This iterative process is

called as the outer iteration or source iteration, as usual.

The problem with which are concerned here is how to calculate the

neutron flux distribution for a fixed neutron source distribution.

This process is called as the inner iteration.

The fundamental equation to be solved is

quantities connecting the channel and layer calculations are only the neutron leakage and the neutron source, and the fact reduces the computer memory required.

2) Since the neutron leakage from a block is calculated by a fine-mesh difference approximation, the numerical error due to discretization is minimized.

3) When only one fine-mesh point is located in each block, this approach becomes the same as the usual method in a fine-mesh difference approximation. In this case, the iterative scheme coresponds to one of the variants of the Peaceman-Rachford iterative method. Therefore, it is possible to establish the condition, under which the consistency is achieved between the axial and radial leakages in the same manner as ADI (alternating direction implicit iterative methods of Peaceman and Rachford). In addition, it is easy to compare the results with those obtained from conventional fine-mesh difference approximation methods. The computer code can thus be used for calculating both the collapsed flux and the fine-mesh flux.


2. Fundamental Equation

The iterative process to recalculate the source distribution with use of the previously obtained neutron flux is the same as that utilized in the conventional power-iteration method. This iterative process is called as the outer iteration or source iteration, as usual.

The problem with which are concerned here is how to calculate the neutron flux distribution for a fixed neutron source distribution. This process is called as the inner iteration.

The fundamental equation to be solved is

$$\nabla D \nabla \phi - \Sigma_T \phi + \theta = 0 \quad , \tag{1}$$

where $\phi$ is the neutron flux and $\theta$ the neutron source at a fine-mesh point, and D and $\Sigma_T$ are the diffusion coefficient and the macroscopic removal cross section, respectively. Integrating Eq.(1) over a block results in

$$D \int_{\Delta x} \int_{\Delta y} dxdy \int_{\Delta z} \frac{\partial^2 \phi}{\partial z^2} dz$$

$$+ D \int_{\Delta z} dz \int_{\Delta x} \int_{\Delta y} (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) \phi dxdy$$

$$+ \Sigma_T \int_{\Delta x} \int_{\Delta y} \int_{\Delta z} \phi dxdydz + \int_{\Delta x} \int_{\Delta y} \int_{\Delta z} \theta dxdydz = 0 \quad , \tag{2}$$

where $\Delta x$, $\Delta y$, and $\Delta z$ are the widths of the block. Using the following notations:

$$-\ell_z \equiv \frac{\int_{\Delta x} \int_{\Delta y} dxdy \int_{\Delta z} D \frac{\partial^2 \phi}{\partial z^2} dz}{\int_{\Delta x} \int_{\Delta y} \int_{\Delta z} \phi dxdydz} \quad ,$$

$$-\ell_{xy} \equiv \frac{\int_{\Delta x} \int_{\Delta y} \int_{\Delta z} D (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) \phi dxdy}{\int_{\Delta x} \int_{\Delta y} \int_{\Delta z} \phi dxdydz} \quad ,$$

$$\psi_z \equiv \int_{\Delta x} \int_{\Delta y} \phi dxdy \quad , \qquad \psi_{xy} \equiv \int_{\Delta z} \phi dz \quad ,$$

$$\theta_z \equiv \int_{\Delta x} \int_{\Delta y} \theta dxdy \quad , \qquad \theta_{xy} \equiv \int_{\Delta z} \theta dz \quad , \tag{3}$$

Equation (2) is rewritten as

$$\int_{\Delta z} D \frac{\partial^2 \psi_z}{\partial z^2} dz - \int_{\Delta z} (\Sigma_T + \ell_{xy}) \psi_z dz + \int_{\Delta z} \theta dz = 0 \quad , \qquad (4)$$

$$\int_{\Delta x} \int_{\Delta y} D(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) \psi_{xy} \, dxdy - \int_{\Delta x} \int_{\Delta y} (\Sigma_T + \ell_z) \psi_{xy} \, dxdy$$

$$+ \int_{\Delta x} \int_{\Delta y} \theta_{xy} \, dxdy = 0 \quad , \qquad (5)$$

where $\psi_z$ and $\psi_{xy}$ are obtained respectively by solving the following one- and two-dimensional fine-mesh neutron diffusion equations:

$$D \frac{\partial^2 \psi_z}{\partial z^2} - (\Sigma_T + \ell_{xy}) \psi_z + \theta_z = 0 \quad , \qquad (6)$$

$$D(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) \psi_{xy} - (\Sigma_T + \ell_z) \psi_{xy} + \theta_{xy} = 0 \quad . \qquad (7)$$

One-dimensional fine-mesh neutron flux distribution calculations are performed along the channels, and two-dimensional calculations are made over the layers. The axial and radial leakages from each block, $\ell_z$ and $\ell_{xy}$, are obtained as follows:

$$-\ell_z = \frac{\int_{\Delta z} D \frac{\partial^2 \psi_z}{\partial z^2} dz}{\int_{\Delta z} \psi_z \, dz} \quad ,$$

$$-\ell_{xy} = \frac{\int_{\Delta x} \int_{\Delta y} D(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) \psi_{xy} \, dxdy}{\int_{\Delta x} \int_{\Delta y} \psi_{xy} \, dxdy} \quad . \qquad (8)$$

The neutron flux distribution in the core is now determined by solving Eqs.(6) and (7) alternately.

The neutron flux convergence criterion is given by

$$\left| \frac{\int_{\Delta z} \psi_z \, dz - \int_{\Delta x} \int_{\Delta y} \psi_{xy} \, dxdy}{\int_{\Delta z} \psi_z \, dz} \right| < E \quad . \tag{9}$$

If the neutron flux distribution satisfies the above condition, the leakage coefficients also satisfy the following condition, as is readily seen from Eqs.(4), (5) and (8):

$$\left| \ell_{xy}^{new} - \ell_{xy}^{old} \right|$$

$$= \frac{\int_{\Delta x} \int_{\Delta y} \theta_{xy} \, dxdy}{\int_{\Delta x} \int_{\Delta y} \psi_{xy} \, dxdy} \cdot \left| \frac{\int_{\Delta z} \psi_z \, dz - \int_{\Delta x} \int_{\Delta y} \psi_{xy} \, dxdy}{\int_{\Delta z} \psi_z \, dz} \right|$$

$$< \frac{\int_{\Delta x} \int_{\Delta y} \theta_{xy} \, dxdy}{\int_{\Delta x} \int_{\Delta y} \psi_{xy} \, dxdy} \cdot E \quad . \tag{10}$$

The suffix *new* indicates the result of this iteration step and *old* the result of the preceding iteration step. After the convergence criterion (9) is satisfied, the neutron sources, $\theta_{xy}$ and $\theta_z$, are recalculated by using the converged neutron fluxes $\psi_z$ and $\psi_{xy}$. The source iteration process is repeated until the source distribution is converged.

The above explanation gives the general feature of the computer code DIFFUSION-ACE, which is a standard of our code series of LIM. Equation (7) can be solved not only by two-dimensional finite difference approximation method but by finite element method (FEM) or by two one-dimensional calculations as follows,

$$D \frac{\partial^2}{\partial x^2} \psi_x - (\Sigma_T + \ell_y + \ell_z)\psi_x + \theta_x = 0$$

$$D \frac{\partial^2}{\partial y^2} \psi_y - (\Sigma_T + \ell_x + \ell_z)\psi_y + \theta_y = 0$$

$$(7)'$$

Such solution algorithms are also included in our code series.

## 3. Development of Computer Codes with LIM

In Fig.3 and TABLE I are shown the relation between computer codes in our system. Program language used is FORTRAN IV for all codes, and check calculations are done by solving the three energy groups neutron diffusion equation. The contents of these codes are summarized as follows:

(1) DIFFUSION-ACE

A) General calculational flow

This code was developed in 1972 and reported in Nucl. Sci. Eng. 58 (1975)[1]. The general calculational flow diagram is shown in Fig.2. As the figure shows, one-dimensional z axial calculation and two-dimensional x-y layer calculation are repeated alternately until the consistency is attained.

B) Characteristics

1) One- and two-dimensional neutron fluxes are obtained by fine mesh finite difference approximation method (FDM).

2) Neutron flux in reflector region can be calculated analytically and combined with the neutron flux in the core region calculated by FDM. The boundary condition for core calculation is determined by this analytic solution, so that the neutron flux in reflector

$$D \frac{\partial^2}{\partial x^2} \psi_x - (\Sigma_T + \ell_y + \ell_z)\psi_x + \theta_x = 0$$

$$D \frac{\partial^2}{\partial y^2} \Psi_y - (\Sigma_T + \ell_x + \ell_z)\psi_y + \theta_y = 0$$

(7)'

Such solution algorithms are also included in our code series.

## 3. Development of Computer Codes with LIM

In Fig.3 and TABLE I are shown the relation between computer codes in our system. Program language used is FORTRAN IV for all codes, and check calculations are done by solving the three energy groups neutron diffusion equation. The contents of these codes are summarized as follows:

(1) DIFFUSION-ACE

A) General calculational flow

This code was developed in 1972 and reported in Nucl. Sci. Eng. 58 (1975)[1]. The general calculational flow diagram is shown in Fig.2. As the figure shows, one-dimensional z axial calculation and two-dimensional x-y layer calculation are repeated alternately until the consistency is attained.

B) Characteristics

1) One- and two-dimensional neutron fluxes are obtained by fine mesh finite difference approximation method (FDM).

2) Neutron flux in reflector region can be calculated analytically and combined with the neutron flux in the core region calculated by FDM. The boundary condition for core calculation is determined by this analytic solution, so that the neutron flux in reflector

region is not necessary to be calculated.

3) The boundary condition on the surface of a cruciformed control
rod can be given by logarithmic derivative value of the neutron flux.

C) Check calculation

The calculational results by DIFFUSION-ACE are compared in Fig. 4
with those by the CITATION code which adopts the standard fine mesh dif-
ference approximation method. The analysis of JPDR-II (BWR) core
performance was carried out by this code and one of the results was
reported in J. of Nucl. Sci. and Tech[2].

(2) STEADY-ACE

A) General calculational flow

This code was developed in 1973 with the general calculational flow
diagram shown in Fig.5. As the figure shows, this code is made of two
subroutines,a neutron diffusion calculation routine (DIFFUSION-ACE) and
a thermal hydraulics calculation routine (HYDRO-ACE). These two routines
are combined by a subroutine CROSS-ACE. The nuclear group constants for
the diffusion equation are expressed as functions of the thermal
hydraulic condition at each block in the core. The calculation of
thermal power distribution is repeated until the consistency is attained
between thermo-hydraulics and thermal power distribution.

B) Characteristics

1) The subroutine to deal with the neutron diffusion is the same as
DIFFUSION-ACE.

2) The subroutine HYDRO-ACE is for calculating the multi-channel
thermo-hydraulics in a BWR core. So STEADY-ACE is a code for
analysing the core performance of a BWR.

C) Check calculation

The analysis of JPDR-II (BWR) core phenomena was carried out by this program and the some of the results were reported orally at JNS committee in October, 1977.

(3) STEADY-SHIP

A) General calculational flow

This code was developed in 1977 with the general calculational flow diagram shown in Fig.6. This program is developed by modifying STEADY-ACE for calculating the core performance of a ship reactor. The nuclear group constants are expressed as functions of the thermal hydraulic condition as shown in TABLE II. After the gross neutron flux and thermal hydraulics in the core are obtained, a more precise calculation is carried out for the block appointed by a user with a subroutine LOCAL-FINE. For the neutron flux calculation in this block, boundary conditions arround the selected block are obtained by the gross calculation.

B) Characteristics

1) The subroutine to treat the neutron diffusion is named as DIFFUSION-SHIP. In this program, mesh and channel widths are arbitrarily given by input data to evaluate the effect of hetero-geneous structure in a block as shown in Fig.7.

2) The subroutine HYDRO-SHIP is a program to calculate multi-channel thermo-hydraulics in a ship reactor (PWR). So, this code is applied to analysis of a PWR core performance with cruciformed control rods.

3) Taking into consideration of the fine structure in an appointed block as shown in Fig.8, the local peaking factor can be calculated more precisely.

C)  Check calculation

    1)  The power distribution in a ship reactor "MUTSU" is calculated

    and compared with the experimental data.  One example of them is

    given in Fig.9, which shows a good agreement with the experimental

    data.

    2)  The computational results by LOCAL-FINE are compared with those

    by CITATION.  One example is shown in Fig.10, where it is seen the

    results of gross calculation exert influence on the local power peaking.


(4)  WHITE-HORSE

A)  General calculational flow

    The general calculational flow diagram is shown in Fig.11.  This

program was developed in 1975 for a PWR type commercial electric power

plant.  The outline of this code was reported orally at JNS committee

in November, 1975.


B)  Characteristics

    1)  In two-dimensional layer calculations, not only the reflector

    but also the pressure vessel can be taken into consideration.

    Geometry of the pressure vessel is cylindrical and that of the

    core is rectangular, so that some regions are treated by R-$\theta$

    coordinate meshes and the others are by X-Y coordinate meshes.

    2)  This code is programed for analysing the core performance of a

    PWR.  Therefore, critical search routine by a boron concentration and

    thermo-hydraulics calculation routine are included also in this code.


C)  Check calculation

    The check calculation is performed by using the experimental data

of the critical boron contents and power distribution in the IKATA-I (PWR)

of Shikoku electric power company, Inc.

(5)  FEDM

A)  General calculational flow

In this code developed in March, 1978, two-dimensional neutron

diffusion layer calculations are performed by using the finite element

method (FEM) and one-dimensional channel calculations are done by using

the finite difference method.  The general calculational flow diagram

is almost the same as DIFFUSION-ACE.

B)  Characteristics

1)  Due to two-dimensional FEM layer calculations, this code is

powerful to analyse the core performance with complex geometry.

C)  Check calculation

The check calculation is performed by using the experimental data

of the high flux reactor (HFR) of Kyoto University.  An example for the

neutron flux calculation is shown in Fig.12.

(6)  JUMP-FDM

A)  General calculational flow

In the above mentioned five codes, the layer calculation is performed

by the two-dimensional finite difference or finite element method.  In

this code, however, the calculation is done by repeatedly using the

one-dimensional finite difference method.  The fundamental treatment of

the neutron diffusion equation is shown in Appendix A-I.  By this

treatment, coarse mesh points are combined with each other by one-dimensional

fine mesh finite difference calculations.  The general calculational

flow is shown in Fig.13.

B) Characteristics

1) Since coarse mesh points are combined with each other by the one-dimensional fine mesh finite difference scheme, the fundermental difference equation easily corresponds with the coarse mesh seven points difference equation. That is, with the exception of the coefficients of flux vector, the calculational method is almost the same as the coarse mesh defference method.

2) As shown in A-I, if the neutron source distribution in a block is assumed to be fixed, the coefficients of flux vector do not change with each iteration step. In this case the one-dimensional calculation between coarse mesh points must be done only once before performing the coarse mesh calculation.

3) As coarse mesh points are combined with one-dimensional fine meshes, the intervals between coarse mesh points can be selected much larger than the node length used in the ordinary node method.

4) Since the three-dimensional block geometry is expressed only three one-dimensional lines, it is not appropriate to perform the core calculation with complex geometry.

Acknowledgment

B) Characteristics

1) Since coarse mesh points are combined with each other by the one-dimensional fine mesh finite difference scheme, the fundermental difference equation easily corresponds with the coarse mesh seven points difference equation. That is, with the exception of the coefficients of flux vector, the calculational method is almost the same as the coarse mesh defference method.

2) As shown in A-I, if the neutron source distribution in a block is assumed to be fixed, the coefficients of flux vector do not change with each iteration step. In this case the one-dimensional calculation between coarse mesh points must be done only once before performing the coarse mesh calculation.

3) As coarse mesh points are combined with one-dimensional fine meshes, the intervals between coarse mesh points can be selected much larger than the node length used in the ordinary node method.

4) Since the three-dimensional block geometry is expressed only three one-dimensional lines, it is not appropriate to perform the core calculation with complex geometry.

Acknowledgment

# References

(1) NAITO, Y., et al.: A leakage iterative method for solving the three-dimensional neutron diffusion equation, Nucl. Sci. Eng., 58, 182 (1975).

(2) NAITO, Y.,: Moderator temperature coefficient in BWR core, J. of Nucl. Sci. and Tech., 14, 826 (1977).

Appendix A-1

Fundamental Treatment of Neutron Diffision Equation in JUMP-FDM

A reactor is divided into many blocks which are called coarse meshes. If intervals between coarse mesh points are too wide to express neutron diffusion by difference equation, the intervals must be divided into finer meshes as shown in Fig.A1. The neutron leakage from a block is obtained by using fine mesh flux $\psi_B$ arround the block boundary.

The one-dimensional fine mesh difference equation between coarse mesh points I and I + 1, is expressed as follows:

$$
\begin{bmatrix}
r_1 & -\ell_1 & & & & \\
-\ell_1 & r_2 & -\ell_2 & & & \\
& \ddots & \ddots & \ddots & & \\
& & -\ell_{p-1} & r_p & -\ell_p & \\
& & & \ddots & \ddots & \\
& & & & -\ell_{k-1} & r_k
\end{bmatrix}
\begin{Bmatrix}
\psi_1 \\
\psi_2 \\
\vdots \\
\vdots \\
\psi_p \\
\vdots \\
\psi_k
\end{Bmatrix}
=
\begin{Bmatrix}
S_1 + \ell_0 \Phi^I \\
S_2 \\
\vdots \\
\vdots \\
S_p \\
\vdots \\
S_k + \ell_k \Phi^{I+1}
\end{Bmatrix}
\tag{1}
$$

where

$$
\left.
\begin{aligned}
\ell_p &\equiv 2 \cdot \frac{D_p \cdot D_{p+1}}{D_p \cdot \Delta X_{p+1} + D_{p+1} \cdot \Delta X_p} \quad, \\[2mm]
\ell_0 &\equiv 2 \cdot \frac{D_1}{\Delta X_1} \quad, \qquad \ell_k \equiv 2 \cdot \frac{D_k}{\Delta X_k} \quad, \\[2mm]
r_p &\equiv (\Sigma_{ap} + D_p B^2) * \Delta X_p + \ell_{p-1} + \ell_p \quad.
\end{aligned}
\right\}
\tag{2}
$$

In the above expressions, $\Phi^I$ is the neutron flux at the center point in a block I and $\Delta X_p$ is a fine mesh width.

The neutron source vector in Eq.(1) is divided into three terms as follows:

$$
\vec{S} = \begin{Bmatrix} S_1' \\ S_2' \\ \vdots \\ \vdots \\ \vdots \\ S_p' \\ \vdots \\ S_k' \end{Bmatrix} S_{I+1}^I + \begin{Bmatrix} \ell_0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \Phi^I + \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ \vdots \\ \ell_k \end{Bmatrix} \Phi^{I+1}
$$

$$
= \vec{S}^E \cdot S_{I+1}^I + \vec{S}^I \cdot \Phi^I + \vec{S}^{I+1} \Phi^{I+1} \quad . \tag{3}
$$

Now, three vectors $\vec{\psi}^E$, $\vec{\psi}^I$ and $\vec{\psi}^{I+1}$ are defined as solutions of the following equations:

$$
M\vec{\psi}^E = \vec{S}^E \quad , \qquad M\vec{\psi}^I = \vec{S}^I \quad , \qquad M\vec{\psi}^{I+1} = \vec{S}^{I+1} \quad , \tag{4}
$$

where M is the matrix shown in Eq.(1). With using above three vectors, the one-dimensional fine mesh neutron flux $\vec{\psi}$ is expressed as follows:

$$
\vec{\psi} = \vec{\psi}^E \cdot S_{I+1}^I + \vec{\psi}^I \cdot \Phi^I + \vec{\psi}^{I+1} \cdot \Phi^{I+1} \quad . \tag{5}
$$

The neutron current between blocks is expressed with the fine mesh neutron flux near the boundary,

$$J(I|I+1) = \ell_B(I|I+1) * \{\psi_B(I|I+1) - \psi_{B+1}(I|I+1)\} * A(I|I+1) \quad , \tag{6}$$

where $A(I|I+1)$ is the boundary area between the blocks I and I+1. The neutron current J can be divided into three terms, $J^E$, $J^I$ and $J^{I+1}$:

$$\left.\begin{aligned}
J^E(I|I+1) &= \ell_B(I|I+1) * \{\psi_B^{\ E}(I|I+1) - \psi_{B+1}^{E}(I|I+1)\} * A(I|I+1) \\[2mm]
J^I(I|I+1) &= \ell_B(I|I+1) * \{\psi_B^{\ I}(I|I+1) - \psi_{B+1}^{I}(I|I+1)\} * A(I|I+1) \\[2mm]
J^{I+1}(I|I+1) &= \ell_B(I|I+1) * \{\psi_B^{\ I+1}(I|I+1) - \psi_{B+1}^{I+1}(I|I+1)\} * A(I|I+1) \\[2mm]
J(I|I+1) &= J^E(I|I+1)S_{I+1}^I + J^I(I|I+1)\Phi^I + J^{I+1}(I|I+1)\Phi^{I+1} \quad .
\end{aligned}\right\} \tag{7}$$

In the same manner as for the neutron current, the neutron absorption in a block is expressed as follows:

$$\left.\begin{aligned}
P^E(I|I+1) &= \underset{p=1,B}{\text{Sum}} \ \psi_p^{\ E} \cdot \Sigma_{ap} V_p \\[2mm]
P^I(I|I+1) &= \underset{p=1,B}{\text{Sum}} \ \psi_p^{\ I} \cdot \Sigma_{ap} V_p \\[2mm]
P^{I+1}(I|I+1) &= \underset{p=1,B}{\text{Sum}} \ \psi_p^{\ I+1} \cdot \Sigma_{ap} V_p
\end{aligned}\right\} \quad , \tag{8}$$

$$P(I|I+1) = P^E(I|I+1) \cdot S_{I+1}^I + P^I(I|I+1)\Phi^I + P^{I+1}(I|I+1)\Phi^{I+1} \quad . \tag{9}$$

With the above notations, the neutron balance equation in the block I is expressed as

$$\{J^{E,I+1} + J^{E,J-1} + J^{E,J+1} + J^{E,J-1} + J^{E,k+1} + J^{E,k-1}\}S(I,J,k \; ; \; g)$$

$$+ \{P^{E,I+1} + P^{E,I-1} + P^{E,J+1} + P^{E,J-1} + P^{E,k+1} + P^{E,k-1}\}S(I,J,k \; ; \; g)$$

$$+ [\{J^{I,I+1} + J^{I,I-1} + J^{J,J+1} + J^{J,J-1} + J^{k,k+1} + J^{k,k-1}\}$$

$$+ \{P^{I,I+1} + P^{I,I-1} + P^{J,J+1} + P^{J,J-1} + P^{k,k+1} + P^{k,k-1}\}]\phi(I,J,k \; ; \; g)$$

$$+ \{J^{I+1,I+1} + P^{I+1,I+1}\}*\phi(I+1,J,k \; ; \; g)$$

$$+ \{J^{I-1,I-1} + P^{I-1,I-1}\}*\phi(I-1,J,k \; ; \; g)$$

$$+ \{J^{J+1,J+1} + P^{J+1,J+1}\}*\phi(I,J+1,k \; ; \; g)$$

$$+ \{J^{J-1,J-1} + P^{J-1,J-1}\}*\phi(I,J-1,k \; ; \; g)$$

$$+ \{J^{k+1,k+1} + P^{k+1,k+1}\}*\phi(I,J,k+1 \; ; \; g)$$

$$+ \{J^{k-1,k-1} + P^{k-1,k-1}\}*\phi(I,J,k-1 \; ; \; g)$$

$$= \text{Source.} \tag{10}$$

In the above expression, $J^E(I|I+1)$ is written as $J^{E,I+1}$ and the other terms are expressed in the same manner. Equation (10) is a seven point coarse mesh equation. The coefficients of the neutron flux depend only on the calculational system but do not change with iteration steps. These values are obtained by solving only once Eq.(4). The values of $J^E$ and $P^E$ depend on the source distribution in a block, and they must be obtained at every outer iteration steps. If the source distribution in a block is assumed to be fixed, however, also these values do not change with iteration steps. This suggests the possibility of developing a high speed calculation method.

TABLE I    Summary of a code series by LIM

| NAME | Purpose | Property | Author |
|---|---|---|---|
| DIFFUSION-ACE | Three dimensional neutron diffusion | JAERI | Y. Naito, M. Maekawa, K. Shibuya |
| STEADY-ACE | BWR core performance for JPDR | JAERI | Y. Naito, M. Maekawa, K. Abe |
| WHITE-HORSE | PWR core performance for IKATA-I | Shikoku Electric Power Company INC. | I. Tsujimoto, K. Kawanishi et al. |
| STEADY-SHIP | Ship reactor core performance for MUTSU | Japan Nuclear Ship Development Agency | Y. Naito, M. Itagaki, Y. Tokuno et al. |
| FEDM | High flux reactor core performance for HFR of Kyoto University | Kyoto University | Y. Naito, M. Hayashi, S. Tsuruta et al. |
| JUMP-FDM | Three dimensional neutron diffusion for LWR | JAERI and Central Research Institute of Electric Power Industry | Y. Naito, T. Matsumura |

## TABLE II  The nuclear group constants fitting formulas

$$T_f = e_0 + e_1 * P + e_2 * P^2 + T_m$$

$$T_m = T_m - T_m^{STD}$$

$$\sqrt{T_f} = \sqrt{T_f} - \sqrt{T_f^{STD}}$$

$$1/D = d_0 + d_1 * T_m + d_2 * T_m^2$$

$$\Sigma_R = r_0 + r_1 * T_m + r_2 * T_m^2$$

$$(\delta\Sigma_a)_{xe} = \frac{P}{b_0 + b_1 * P}$$

$$(\delta\Sigma_a)_{DOP} = a_0 + a_1 * \sqrt{T_f} + a_2 * (\sqrt{T_f})^2$$

$$\Sigma_a = f_0 + f_1 * T_m + f_2 * T_m^2 + F * \{a_0 + a_1 * \sqrt{T_f} + a_2 * (\sqrt{T_f})^2\}$$

$$+ G * \{P / (b_0 + b_1 * P)\}$$

$$\nu\Sigma_f = h_0 + h_1 * T_m + h_2 * T_m^2 + G * \{P / (c_0 + c_1 * P)\}$$

$$G = \delta (g - 3) , \qquad F = \delta (g - 2)$$
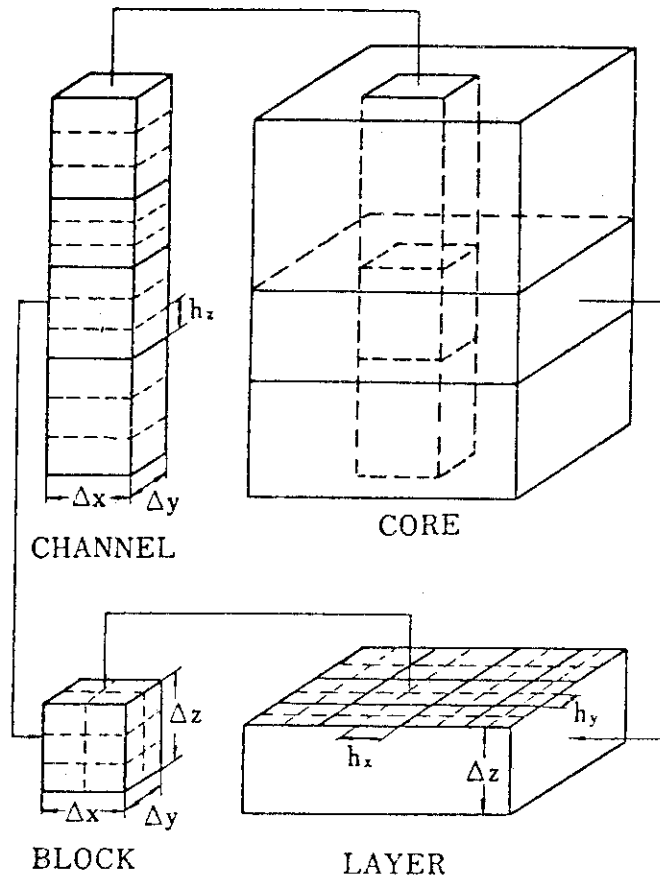
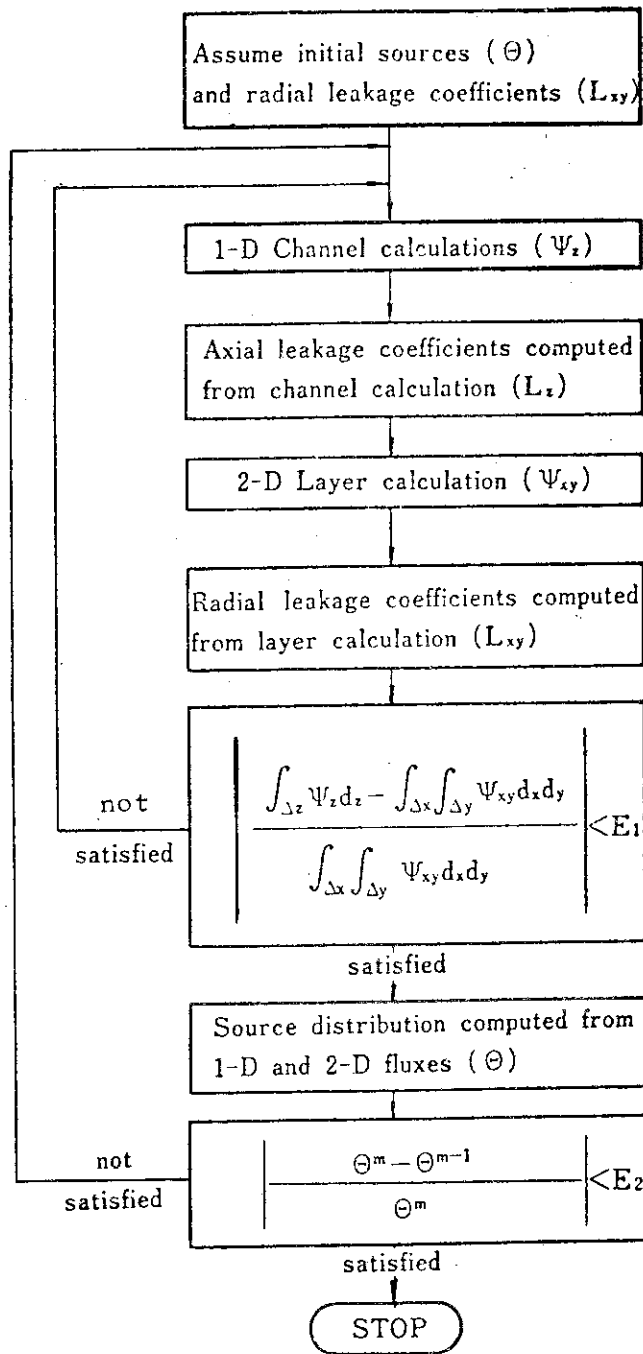Fig. 1   Configuration of Channels, Layers, and Blocks.

```
┌─────────────────────────────────────┐
│ Assume initial sources ( Θ )        │
│ and radial leakage coefficients (Lₓy)│
└─────────────────────────────────────┘
            │
┌─────────────────────────────────────┐
│ 1-D Channel calculations ( Ψ_z )    │
└─────────────────────────────────────┘
            │
┌─────────────────────────────────────┐
│ Axial leakage coefficients computed │
│ from channel calculation (L_z)      │
└─────────────────────────────────────┘
            │
┌─────────────────────────────────────┐
│ 2-D Layer calculation ( Ψ_{xy} )    │
└─────────────────────────────────────┘
            │
┌─────────────────────────────────────┐
│ Radial leakage coefficients computed│
│ from layer calculation (Lₓy)        │
└─────────────────────────────────────┘
            │
┌─────────────────────────────────────┐
│                                     │
│   | ∫_Δz Ψ_z dz − ∫_Δx ∫_Δy Ψ_{xy} dx dy |   │
│   | ───────────────────────────── | < E₁  │
│   |   ∫_Δx ∫_Δy Ψ_{xy} dx dy       |   │
│                                     │
└─────────────────────────────────────┘
```

$$\left| \frac{\int_{\Delta z} \Psi_z \, dz - \int_{\Delta x}\int_{\Delta y} \Psi_{xy} \, dx \, dy}{\int_{\Delta x}\int_{\Delta y} \Psi_{xy} \, dx \, dy} \right| < E_1$$

not satisfied

satisfied

```
┌─────────────────────────────────────┐
│ Source distribution computed from   │
│ 1-D and 2-D fluxes ( Θ )            │
└─────────────────────────────────────┘
            │
┌─────────────────────────────────────┐
│   | Θᵐ − Θᵐ⁻¹ |                     │
│   | ───────── | < E₂                │
│   |    Θᵐ     |                     │
└─────────────────────────────────────┘
```

$$\left| \frac{\Theta^m - \Theta^{m-1}}{\Theta^m} \right| < E_2$$

not satisfied

satisfied

( STOP )

Fig. 2    Schematic diagram of the Leakage Iterative Method.
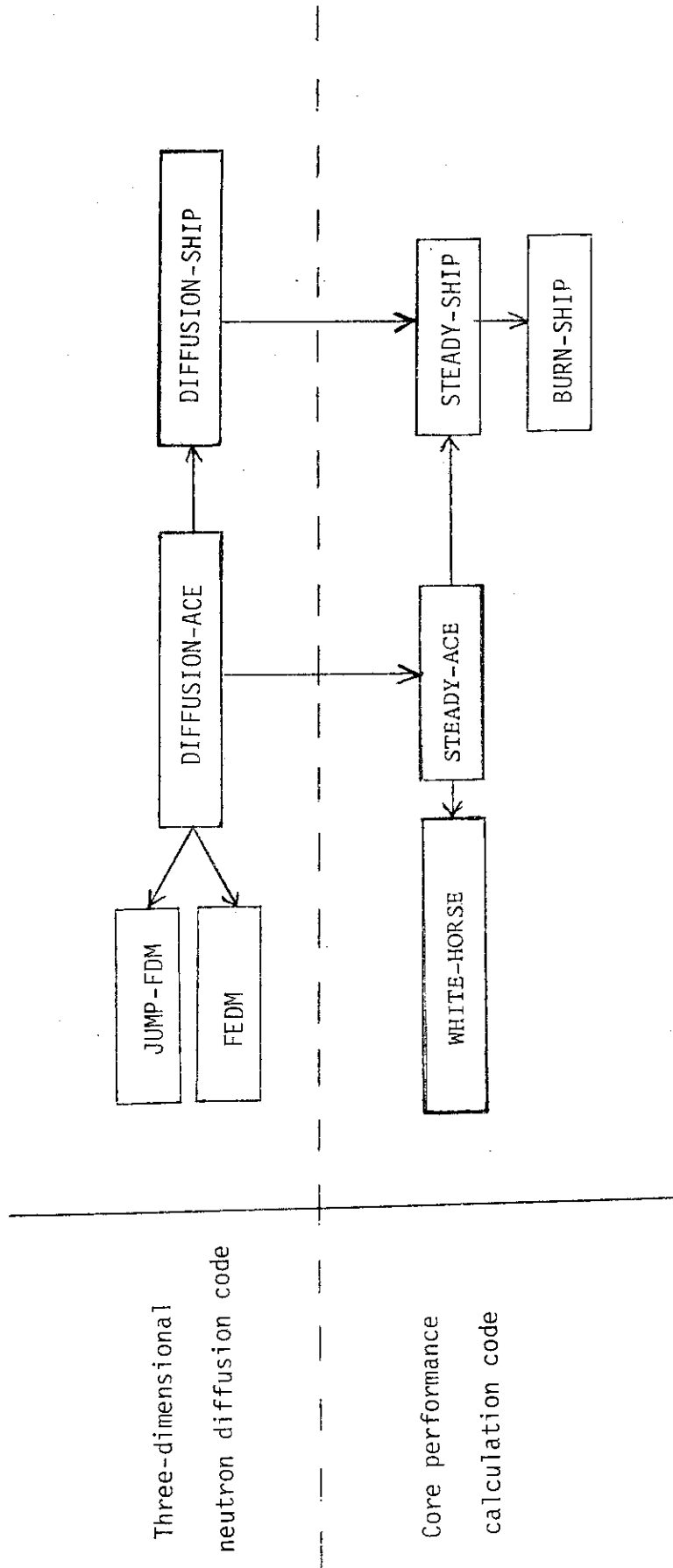
Fig. 3  A computer code series by leakage iterative method
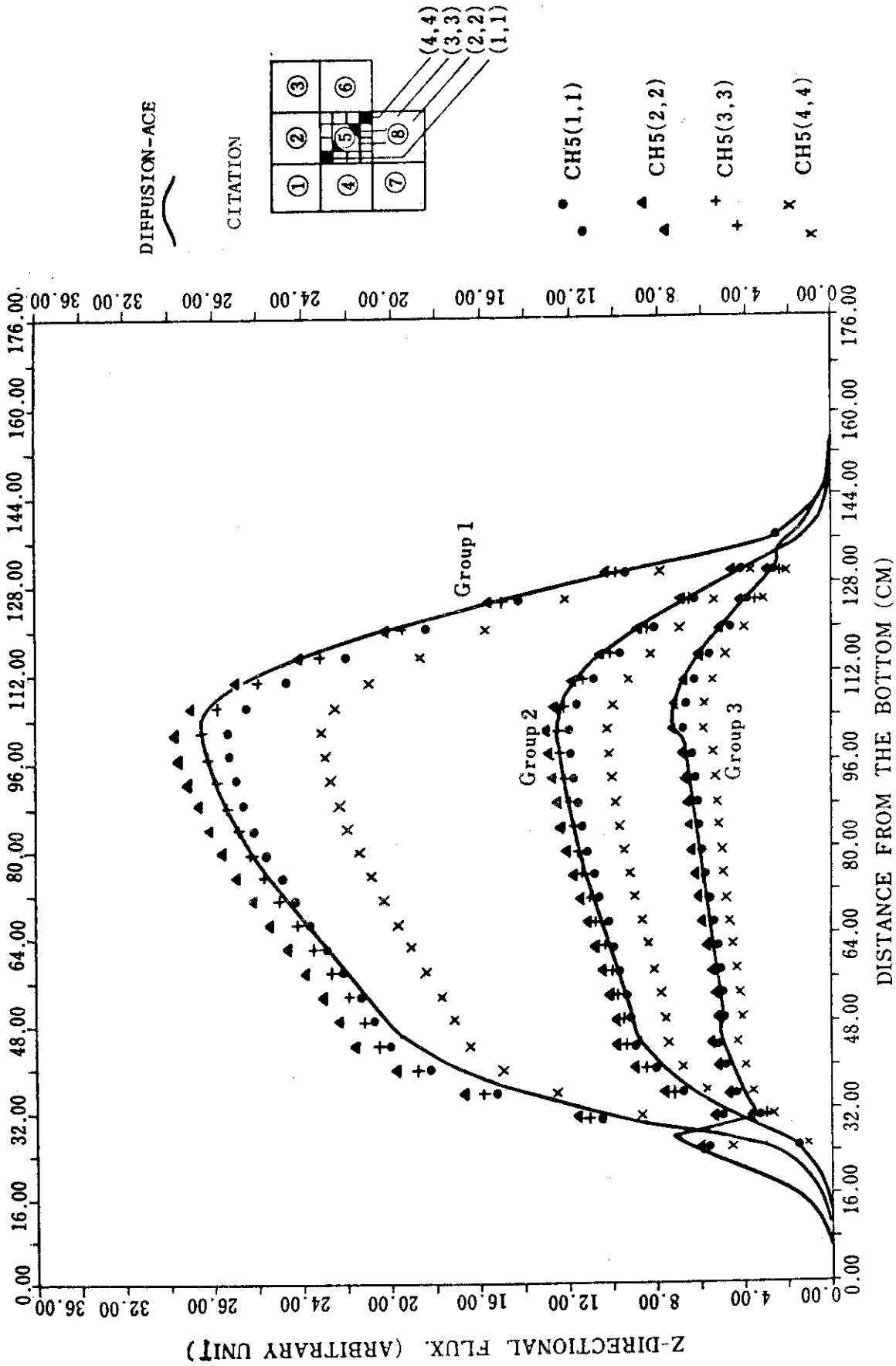
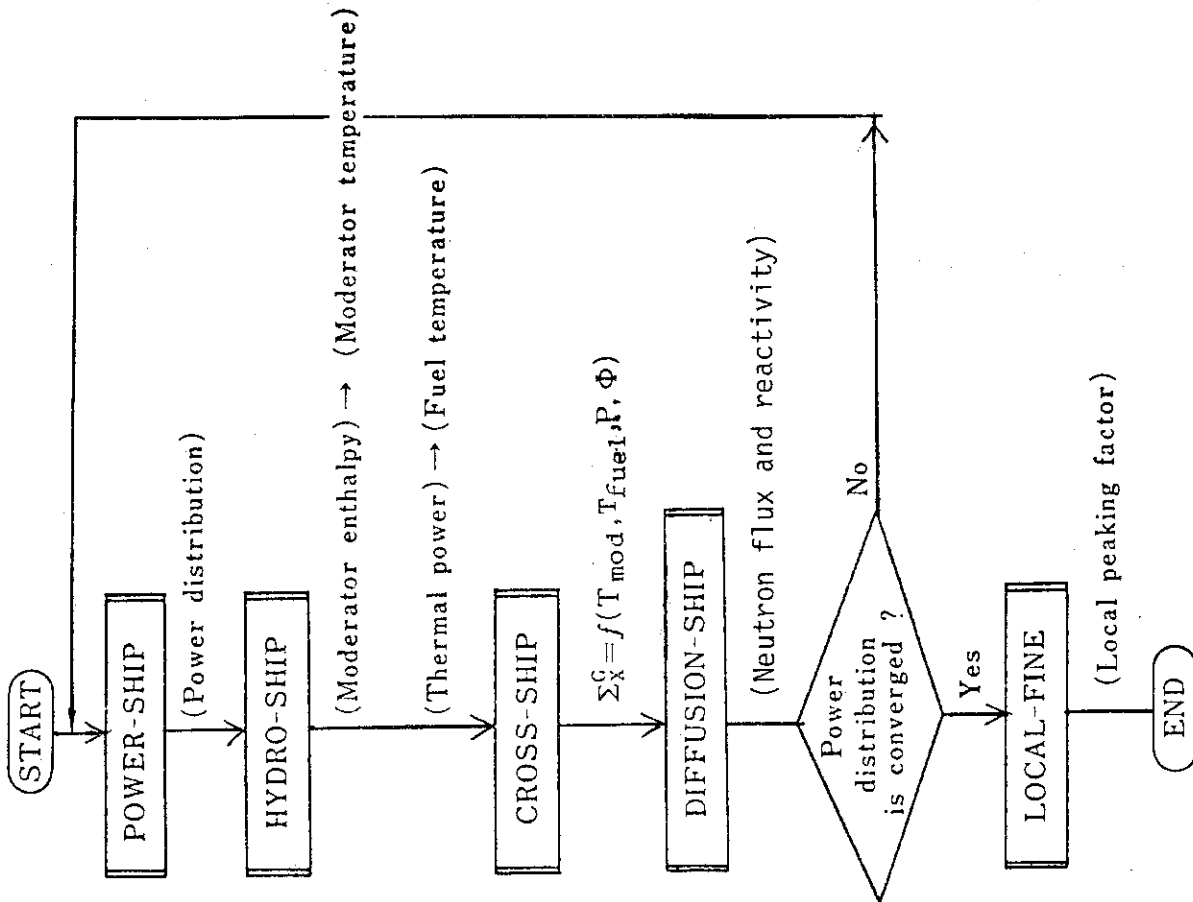Fig. 4    Comparison of Z-Directional Flux.

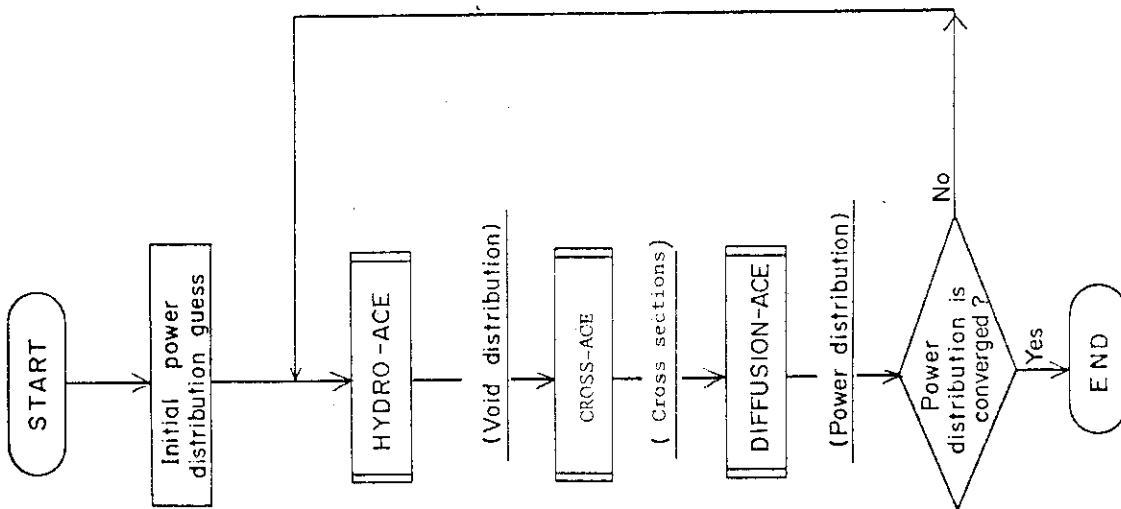Fig. 6   Schematic diagram of the STEADY-SHIP calculation



Fig. 5   Schematic diagram of STEADY-ACE
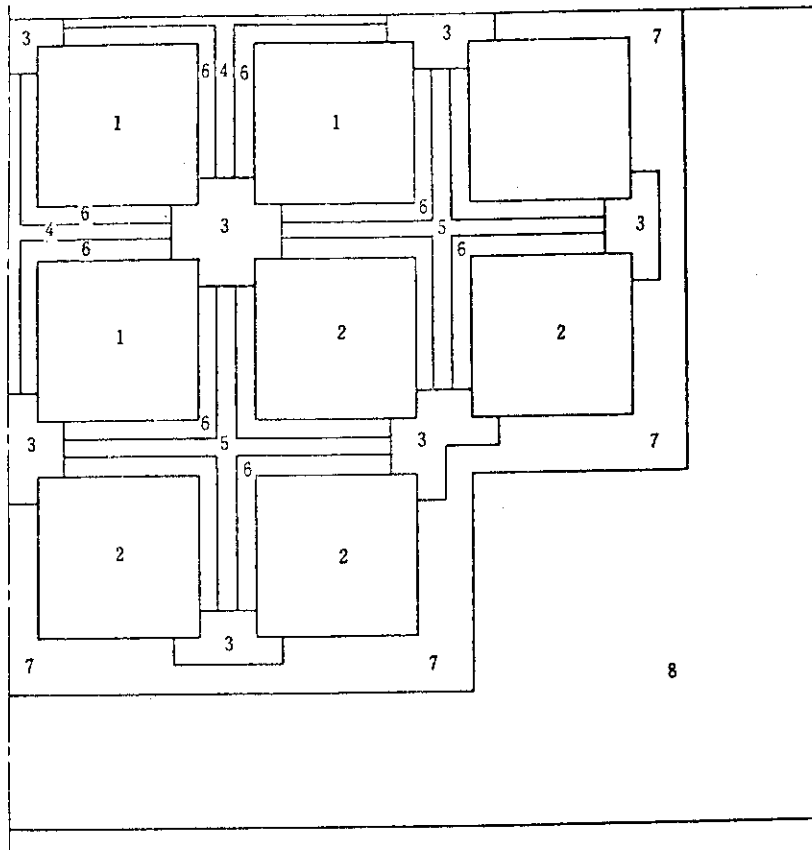
1. 3.24 w/o Fuel Cell
   and BPR Region.
2. 4.44 w/o Fuel Cell
   and BPR Region.
3. Zr-2 Filler, Assembly Can Box
   and Water Homogenized Region.
4. G1, G2 Group Control Rod (C/R)
   or Zr-2 Follower Region.
5. G3, G4 Group C/R
   or Zr-2 Follower Region.

6. Assembly Can Box, Water
   and C/R Clad Homogenized
   Region.
7. SUS Buffle.
8. Water Reflector, Core Tank
   and Thermal Shroud
   Homogenized Region.

Fig. 7   The Full Core Three Dimensional Model. (Horizontal Section)
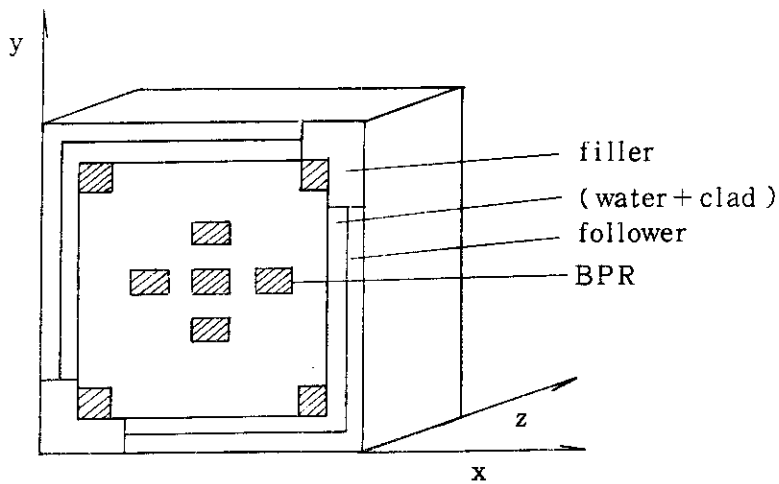


Fig. 8   Calculational model for LOCAL-FINE

Fig. 10　Power distribution in an assembly
　　　　　calculated by the LOCAL-FINE code



Fig. 9　Relative power distribution
　　　　in the critical core
　　　　of the ship reactor MUTSU
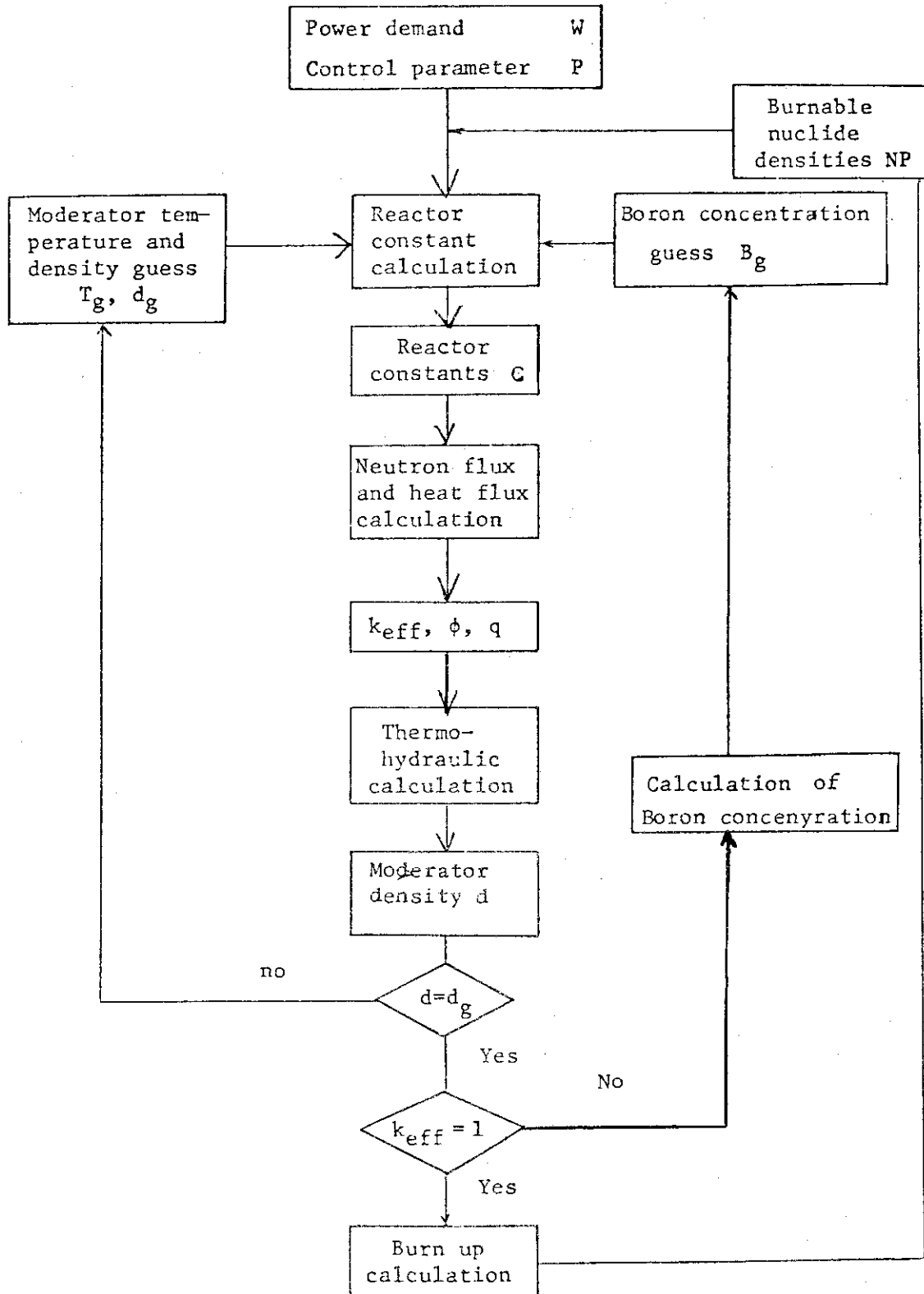
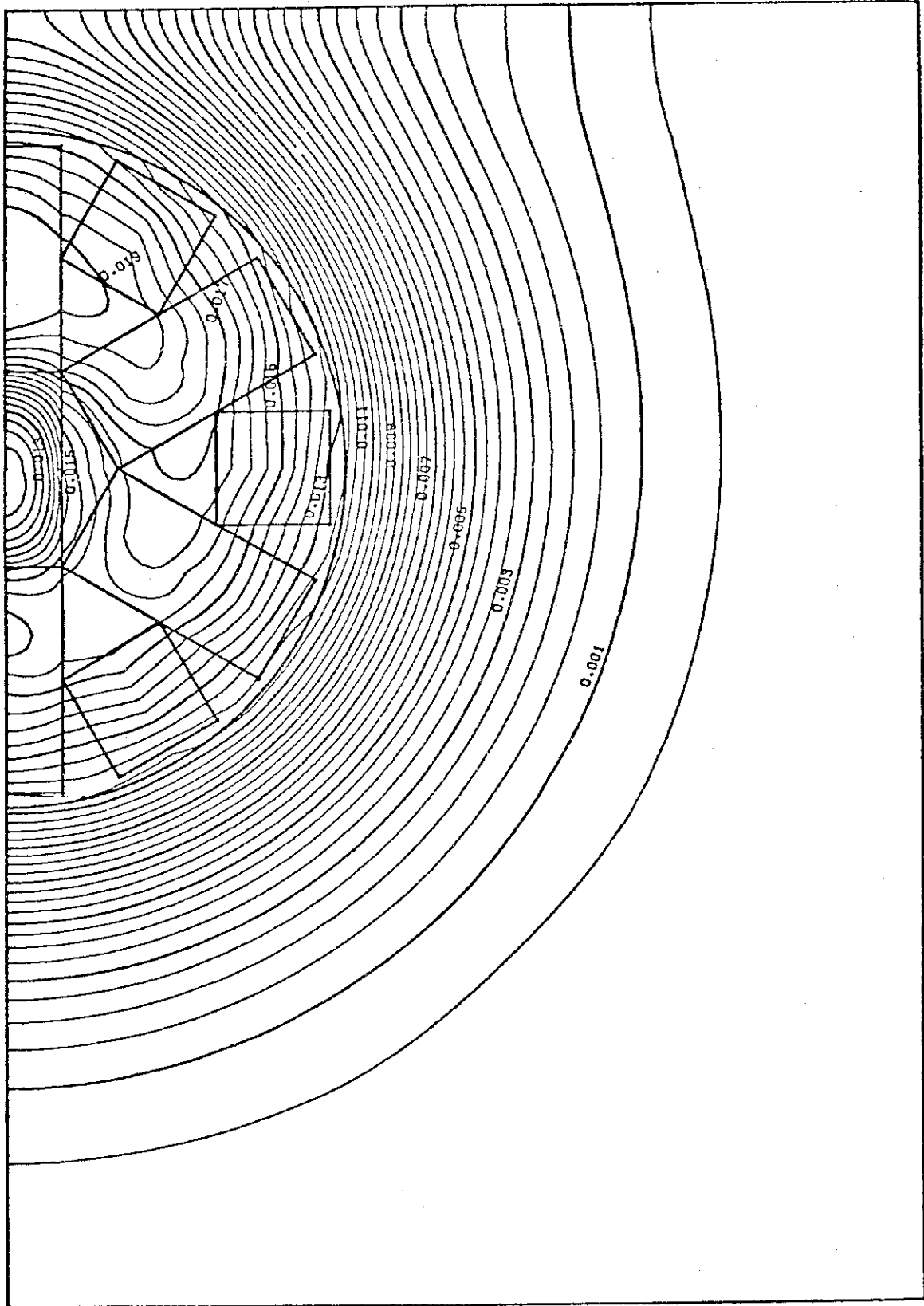Fig. 11  Schematic diagram of WHITE-HORSE

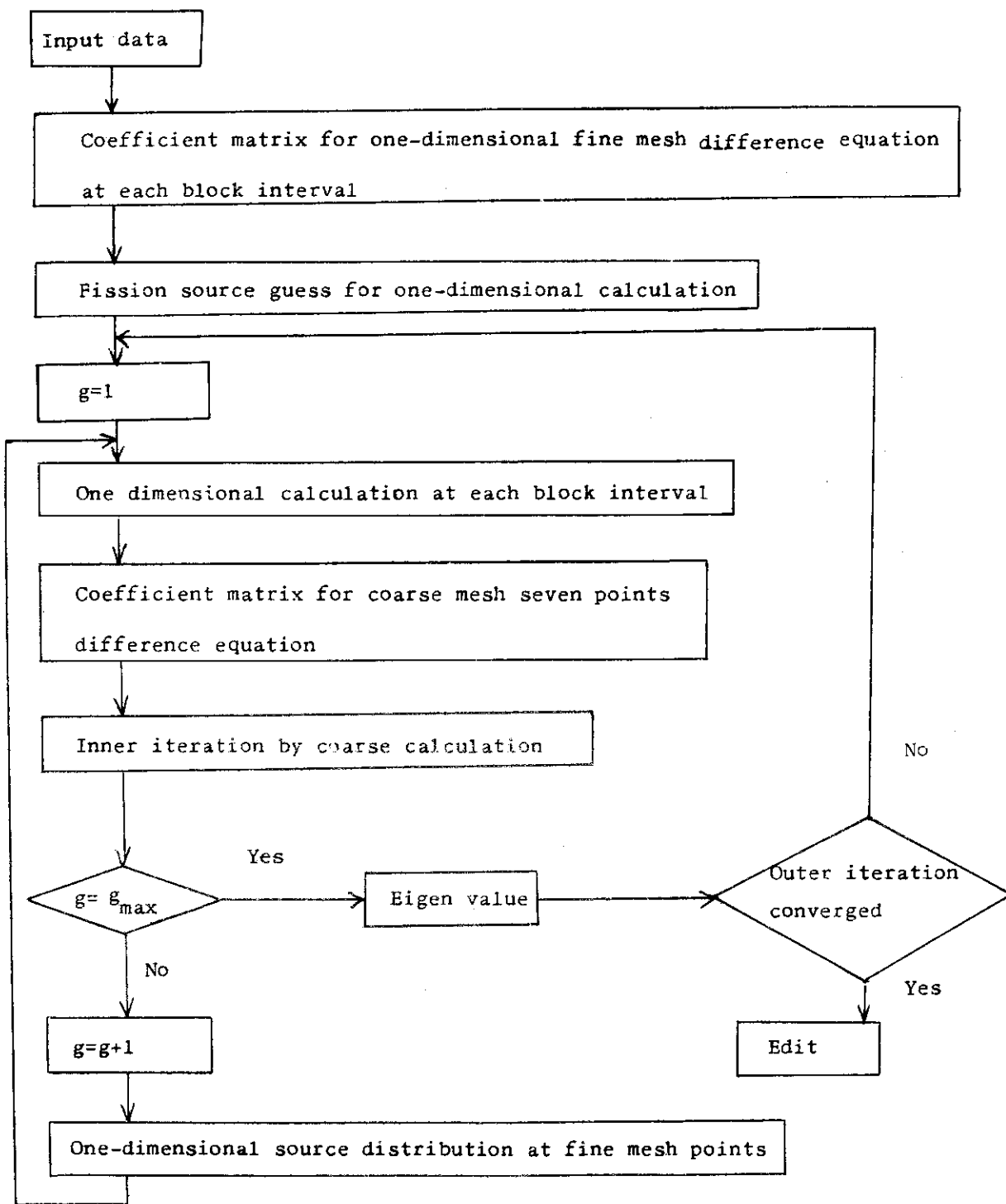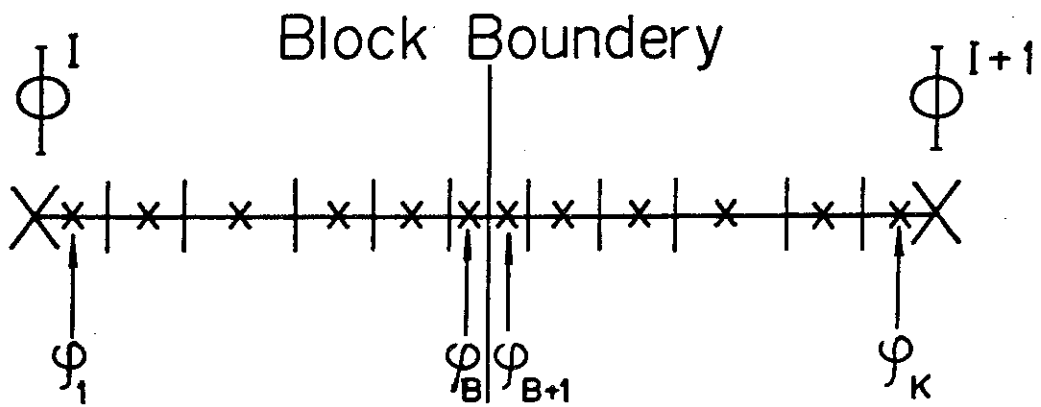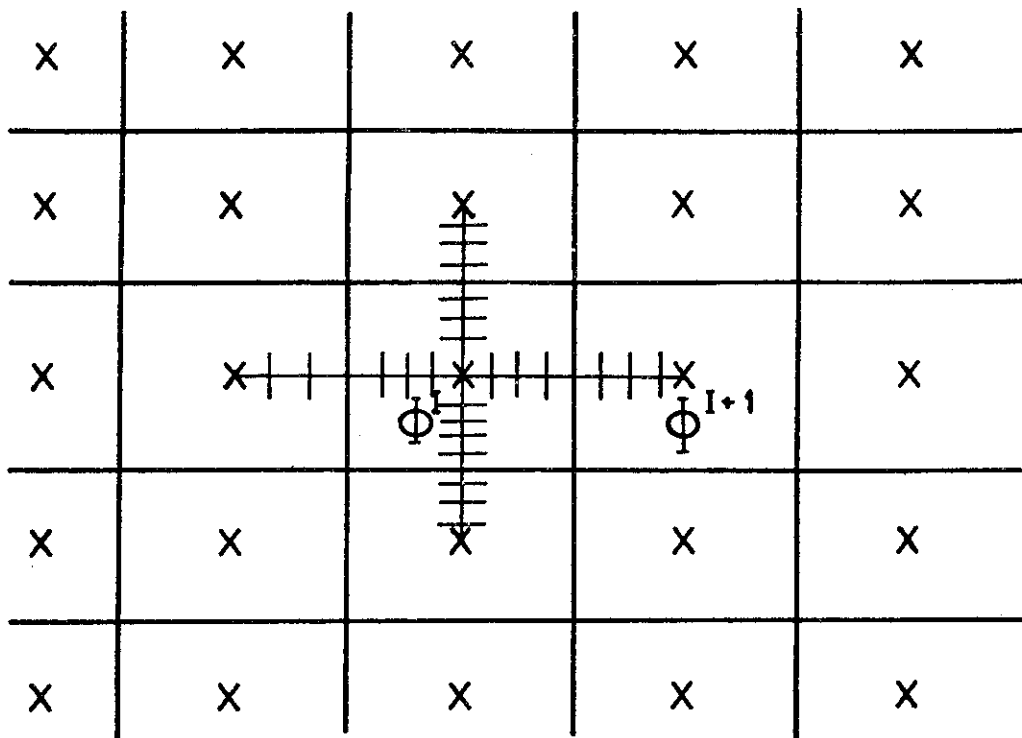Fig. 12　Neutron flux distribution in HFR calculated by FEDM

Fig.13    Schematic diagram of JUMP-FDM

Fig.Al    Mesh description in JUMP-FDM