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Shell model calculation of alpha-formation factor for use in pre-equilibrium alpha emission

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Alpha formation factor $F_{l,m}$ introduced by Iwamoto and Harada, which represents the probability to form the alpha with l particles above the Fermi level and m particles below, is calculated based on the shell model. The result shows a close similarity to the Iwamoto and Harada's calculation which is obtained by use of the Fermi-gas model. This offers a foundation to their approximation and the insensitivity of $F_{l,m}$ against the underlying model provides an approval to use $F_{l,m}$ in the pre-equilibrium calculation.

Keywords : Pre-equilibrium Alpha Emission, Alpha Formation Factor, Shell Model Calculation

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前平衡状態からのアルファ粒子放出に用いるアルファ形成因子の殻模型計算

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岩本，原田の模型で導入されたアルファ型成因子$F_{\ell,m}$はフェルミ準位より上のℓ個の粒子とフェルミ準位より下のm個の粒子よりアルファ粒子を形成する確率を示すものであるが，これを殻模型に基づき計算した。その結果は，フェルミガス模型に基づいた岩本－原田の計算結果と非常に良く一致した。これにより彼等が用いた近以が基礎づけられたと同時に，$F_{\ell,m}$が原子核の模型にあまり依存せずに求まることから，これを前平衡状態からのアルファ粒子放出の計算に用いる模型の有用性が示された。

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1. Introduction

The emission of the composite particle in the pre-equilibrium model contained an ambiguity in its formulation.[1] A model proposed by Iwamoto and Harada[2] is intended to generalize the exciton model so as to include the effect of the intrinsic state of the emitted particle. The numerical calculation based on this model[2] gives a good fit to several (p, α) reactions. In this model, they introduced the alpha formation factor $F_{l,m}(\varepsilon)$, which stands for the probability that the alpha particle of energy ε is composed of l particles above the Fermi level and m particles below it. They calculated this quantity quasi-classically with the Fermi-gas model. Because this $F_{l,m}$ plays an important role in their formalism and also the final result depends much on the value of it, we think it is worthy to calculate $F_{l,m}(\varepsilon)$ full quantum mechanically with the shell model in order to check the validity of the model used in Ref.2.

In Ref.2, the quantity $F_{l,m}$ is defined in Eq.(3.17). The condition of the alpha formation in the nuclear surface is expressed by Eq.(3.14) in which the c.m. coordinate of the alpha is fixed at the surface of the residual nuclei. This "surface" condition , however, affects only the overall normalization of $F_{l,m}$ and the functional form is invariant with this condition[2]. Thus we calculate the quantity without this condition and compare it with the value of Fig.4 of Ref.2. The quantity which has such a character is expressed by $f_{l,m}(\varepsilon)$ given in Eq.(2.17) of Ref.2. In the following, we will calculate this quantity based on the shell model. Because $F_{l,m}$ and $f_{l,m}$ has the same functional form, we will identify them and use the symbol $F_{l,m}$ in this paper. The explicit form of the $F_{l,m}$ is given as

$$F_{l,m}(\varepsilon) = \sum_{\substack{e_i > \varepsilon_F (i=1,..l) \\ e_j \leq \varepsilon_F (i=l+1,..4)}} |\langle \psi_\alpha \chi^{(\varepsilon)}(\vec{R}) | \phi_1 \phi_2 \phi_3 \phi_4 \rangle|^2 . \qquad (1)$$

2. Method of Calculation

The intrinsic wave function of α-particle is assumed as in Ref.2,

$$\psi_\alpha = \frac{1}{\sqrt{8}}\left(\frac{\nu}{\pi}\right)^{9/4}\exp\{-\frac{\nu}{2}(\xi_1{}^2+\xi_2{}^2+\xi_3{}^2)\}\ , \tag{2}$$

where

$$\vec{\xi}_1 = \vec{r}_1-\vec{r}_2,\quad \vec{\xi}_2 = \vec{r}_3-\vec{r}_4,\quad \vec{\xi}_3 = \frac{\vec{r}_1+\vec{r}_2}{2} - \frac{\vec{r}_3+\vec{r}_4}{2}\ ,$$

$$\nu = \frac{m\omega}{\hbar}\ .$$

An α-particle state can be written as

$$|\ \psi_\alpha \chi_{NL}^{(\varepsilon)}(\vec{R})\ >\ , \tag{3}$$

where

$$\vec{R} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4}{4}\ ,$$

and $\chi_{NL}^{(\varepsilon)}(\vec{R})$ represents the center-of-mass wave function of α-particle having an energy ε . Here N and L stand for the radial quantum number and the angular momentum, respectively. Now we expand the α-particle state in terms of four particle shell model wave function $\{\phi_{nl}(r_i)\}^4$. The $F_{l,m}$ function in the shell model can be written in the form

$$F_{l,m}^{(L)}(\varepsilon) = \tag{4}$$
$$\Sigma\ |\ <\psi_\alpha \chi_{NL}^{(\varepsilon)}(\vec{R})\ |\ \phi_{n_1l_1}\phi_{n_2l_2}(L_{12})\phi_{n_3l_3}\phi_{n_4l_4}(L_{34});[4],L>\ |^2,$$

where the summation is made over the four single-particle orbits and the intermediate angular momenta L_{12} and L_{34} under the restriction that l particles are above the Fermi level and m particles below (l+m=4). As the single-particle wave function $\phi_{n_il_i}$, we use the harmonic oscillator wave function which has the single-particle energy $(\tilde{N}_i+3/2)\hbar\omega$, where $\tilde{N}_i = 2n_i+l_i$ is the oscillator quanta.

If $\tilde{N}_F$ is the oscillator quanta of the nucleon at the Fermi surface, the condition of the summation in Eq.(4) can be expressed as

$$\tilde{N}_i > \tilde{N}_F \qquad (i=1,..1) ,$$
$$\tilde{N}_j \le \tilde{N}_F \qquad (j=1+1,..4) . \tag{5}$$

By using an assumption that the spin and the isospin of the α-state have S = T = 0, it is easily found that the orbital part of four particle state should have permutation symmetry [4] , and then only the orbital part of the wave function is specified in Eq.(4). Furthermore, the assumption that the oscillator parameter of the α-particle is the same as that of the parent nucleus leads to the well known oscillator rule[3)]

$$\sum_{i=1}^{4} (\tilde{N}_i + \frac{3}{2})\hbar\omega = \frac{9}{2}\hbar\omega + (2N + L + \frac{3}{2})\hbar\omega . \tag{6}$$

In the r.h.s. of Eq.(6), the first term represents an intrinsic energy of the α -particle and the second the center-of-mass of the alpha

$$\varepsilon = (2N + L + \frac{3}{2}) \hbar\omega . \tag{7}$$

One sees from Eqs.(6) and (7) that the value of ε is solely determined if the four particle configuration is given.

The explicit form of the overlap integral in Eq.(4) is given as follows;

$$< \psi_\alpha \chi_{NL}(\vec{R}) \mid \phi_{n_1 l_1}\phi_{n_2 l_2}(L_{12})\phi_{n_3 l_3}\phi_{n_4 l_4}(L_{34}) \; ; [4],L > =$$
$$(l_1 0 l_2 0 \mid L_{12}0)\,(l_3 0 l_4 0 \mid L_{34}0)\,(L_{12}0L_{34}0 \mid L0) \times$$
$$\prod_{i=1}^{4} \sqrt{\frac{[l_i]2^{n_i+l_i}}{n_i!(2n_i+2l_i+1)!!}}\,(-)^{\frac{\Sigma l_i - L}{2}}\,\frac{1}{2^{2N+L}}\sqrt{\frac{N!(2N+2L+1)!!}{[L]2^{N+L}}} . \tag{8}$$

This expression is independent of the value of the oscillator parameter ν .

3. Results and Discussions

The calculated $F_{1,m}(\varepsilon)$ functions are shown in Figs. 1 and 2 for even $\tilde{N}$ and odd $\tilde{N}$, where $\tilde{N}=2N+L=\Sigma N_i$.

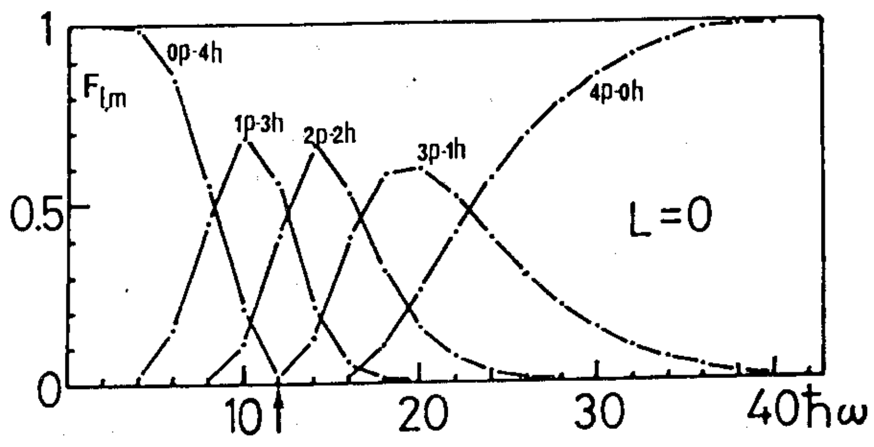

Fig.1 Calculated α-particle formation factor $F_{1,m}$ for L=0 as a function of the α energy in unit of $\hbar\omega$. In the abscissa, the position of the Fermi energy is indicated by an arrow. It is calculated for mass number A=80 and $\hbar\omega$ =9.53 MeV.

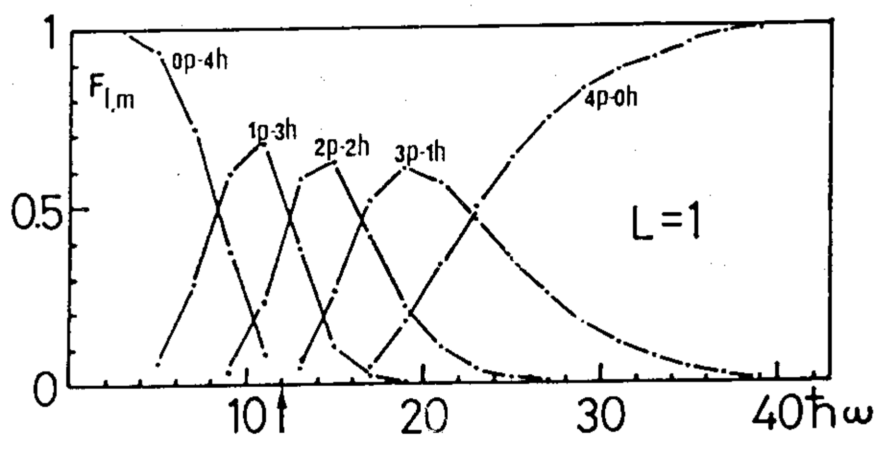

Fig.2 Calculated α-particle formation factor $F_{1,m}$ for L=1. Others are the same as in Fig.1.

In this calculation, $\tilde{N}_F = 3$ is assumed and it corresponds to an L-S closed nucleus (f-p closed nucleus) with A=80. The abscissae of these figures are energies in unit of $\hbar\omega$, and the actual value of the energy is given only through an empirical formula,

$$\hbar\omega = 41\ A^{-1/3}\ \text{MeV} .$$

Although the component $F_{0,4}$ is also plotted, it has no effect on the α-emission because the α-particle has negative energy in this case. The characteristic feature of these figures is that the $F_{1,m}$ functions do not depend on the angular momentum L. This fact can be explained by using the relation developed in the work of Ichimura et al[4]. It is shown there that $F_{1,m}$ has no dependence on L and is determined only by $\tilde{N}$ due to the SU(3) scalar character of the transformation from the four-body shell model state to the α-cluster state. Thus Fig.1 for L=0 and Fig.2 for L=1 are complementary to each other. In the Fermi-gas model, of course, there exists no information about the angular momentum. In our quantum mechanical calculation, it has no dependence on L, which provides a suitable circumstance to compare the two different calculations.

The most remarkable thing is that our $F_{1,m}$ functions displayed in Figs.1 and 2 closely resemble to Fig.4 of Ref.2. In particular, by making their Fermi energies equal to each other, one obtains almost the same values for $F_{1,3}$ and $F_{2,2}$ components, which have leading effects on the high energy α-emission in the model of Ref.2 as is shown in their Figs.11 and 12. Since the quantity $F_{1,m}$ represents the α-formation probability multiplied by the number of various particle states having a definite α energy, it may depend strongly on the level density formula. In the present calculation, we used the harmonic oscillator model. In this model, the level density of the single-particle state is proportional to $e^2/(\hbar\omega)^3$ for $\tilde{N}_i >> 1$. The number of states for four particle system increases in proportion to ε^{11} for fixed c.m. state where ε is the c.m. energy of the α-particle. On the other hand, the Fermi-gas model yields the single-particle level density proportional to $e^{1/2}$. The number of state of four particle system in this model is proportional to ε^5. If we assume a constant single-particle level density, the formula of Ericson[5] tells us that the number of state of four particle system is proportional to ε^3. In the treatment of Ref.2, no explicit use of the level density formula is made. There, the product of the level density and the square of the overlap matrix is obtained by calculating the phase space volume under some conditions. The fact that the $F_{1,m}$ calculated and is shown in Fig.4 in Ref.2 is very similar to the present result tells the following things: First, the plausibility of $F_{1,m}$ calculated semi-classically in Ref.2 is demonstrated. In addition, they used an approximation to replace the momentum and the position vectors by the mean square values. The validity of this approximation is also supported with the present calculation. The second thing to be stressed here is that in spite of the fact that the number of states is strongly model dependent, the quantity $F_{1,m}$ is rather insensitive to the underlying single-particle model. This insensitivity is desirable for the application of $F_{1,m}$ to the pre-equilibrium model calculation as was done in Ref.2.

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