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POSITION CONTROL CHARACTERISTICS OF A PLASMA COLUMN WITH PROPORTIONAL CONTROL LAW

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After briefly describing a kinetics model for a position-controlled toroidal plasma column in a Tokamak device, the model is applied in analysis of the position control characteristics of a JT-60 plasma column. The influence of the parameters of a proportional controller on the characteristics is examined numerically.

Keywords: JT-60 Tokamak, Plasma Column, Proportional Control, Parameter Study, Position Control.

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比例制御によるプラズマ柱の位置制御特性

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トカマク型プラズマ閉じ込め装置における位置制御されたプラズマ柱の応答解析モデルについて述べ，このモデルに基づき，プラズマ柱の位置制御特性と比例制御系の諸パラメーターとの関係について，ＪＴ－６０の設計値を用いて数値的解析を行った。

CONTENTS

INTRODUCTION

The positional equilibrium of a plasma column is realized as a result of a balance between the expansive force due to the kinetic energy of plasma particles and the magnetic energy of the plasma current, on one hand, and the inward force resulting from the interaction of the plasma current with the vertical field, on the other. Therefore, the plasma column may be maintained at a desired equilibrium position by regulating the virtical field.

The author reported a kinetics model for the horizontal motion of a toroidal plasma column in reference (4) and the inherent characteristics of a JT-60 plasma column motion were given in reference (5). And now, the kinetics model is combined with a model for a position-controller of plasma column, and this model is applied to analyze the position control characteristics of a JT-60 plasma column, and are numerically investigated the influence of the proportional coefficient of the controller, the influence of the time constant of the controller and the influence of the time constant of the verticale field coil, on the characteristics.

1. A KINETICS MODEL FOR CONTROL PERFORMANCE ANALYSIS OF A TROIDAL PLASMA COLUMN

A kinetics model for the horizontal motion of a toroidal plasma column was given in JAERI-M 6292[4] and the model is now combined with a model for a position-controller of plasma column. The equilibrium of a plasma column is defined by the force-balance over the minor radius of the plasma column and also by the force-balance over the major radius. These force-balances depend on the shape of the column cross-section and on the density profile of the plasma current and the shape of the plasma pressure over the column cross-section. It is assumed in the model that the plasma column has a circular cross-section and that the current is distributed homogeneously over the column cross-section and that the asymmetry of distribution of the plasma pressure resulting from the toroidality is small.

1.1 Description of a plasma contining device

The cross-sectional configuration of a plasma contining device is conceptually shown in Fig. 1. A plasma column of minor radius a and major radius R is confined in a conducting case (conductor ℓ) of radius γ_ℓ; on a circle of radius γ_v is located the conductor v, of which the current has a cosinusoidal density profile on the cross-section; and the toroidal coil (conductor t) is assumed to have the form of an axisymmetric toroidal shell with minor radius γ_t. The conductors are assumed to have a circular cross-section of which the centre is located on the torus axis. Subscripts ℓ, v and t are used to signify the variables and the constants in conductor ℓ, v and t respectively; p, in the plasma column, and o in the initial equilibrium state.

1.2 Equation of motion of a toroidal plasma column

A plasma column is maintained in equilibrium with an homogeneous magnetic field transverse to the plane of the torus which is given as below:[1],[2]

$$B_{zo} = \frac{\mu_o}{4\pi} \frac{I_p}{R} \Gamma \tag{1}$$

where $$\Gamma = \ell n \frac{8R}{a} + \beta_\theta + \frac{\ell_i - 3}{2} \tag{2}$$

and $\mu_o \ell_i/4\pi$ is the internal inductance of the plasma column per unit length, and I_p is the plasma current. Motion of the plasma column depends on the total radial force, f, which is written as follows for the deviation of the virtical field from B_{zo} to B_z.

$$f = 2\pi R I_p (B_{zo} - B_z) \tag{3}$$

In the vicinity of the equilibrium, the motion of the plasma column is written as

$$M \frac{d^2R}{dt^2} = \frac{\mu_o}{2} I_p^2 \Gamma - 2\pi R I_p B_z \tag{4}$$

where M is total mass of the plasma column.
B_z is composed of $B_i (i=\ell, v, t)$ produced by the conductor currents I_i $(i = \ell, v, t)$ and the disturbance B_d.

$$B_z = \sum_{i=\ell,v,t,d} B_i \tag{5}$$

where B_i $(i = \ell, t, d)$ are assumed to be homogeneous but B_v to have a positional gradient given below:

$$B_v = B_{vc} \left(1 + m \frac{\delta R}{R_o}\right) \tag{6}$$

where δR is a small displacement of the plasma column from the torus axis, B_{vc} is the field produced by the conductor current I_v at the torus axis, and m is a constant characterizing the positional gradient of the field B_v.

The stability conditions of the plasma column motion are written as satisfying the formula.(4)

$$-\frac{3}{2} < m < 0 \tag{7}$$

The minor radius of the plasma column is assumed to vary with column displacement as follows.

$$a = a_o \left(1 + \alpha \frac{\delta R}{R_o}\right) \tag{8}$$

The value of α is found within the range give below.(4)

$$0 < \alpha < \frac{1}{2} \tag{9}$$

The plasma current I_p varies with the column displacement as

$$I_p = I_{po} \left(1 + n \frac{\delta R}{R_o}\right) \tag{10}$$

where I_{po} is the initial value of the plasma current at the torus axis. The value of n is found within the range given below.[4]

$$-1 < n < 0 \tag{11}$$

The large inductance in series with the plasma implies that the internal inductance ℓ_i and the poloidal beta β_θ remain unchanged, admitting redistribution of the plasma current.[3]

Using equations (5), (6), (8), (10), Eq. (4) is reduced to a linear one in the vicinity of a steady equilibrium state.

$$\left.\begin{aligned}
&M \frac{d^2}{dt^2} \frac{\delta R}{R_o} = \frac{\mu_o}{2} \Gamma_o I_{po}^2 \left[G \frac{\delta R}{R_o} - \sum_{i=\ell, vc, t, d} \frac{\delta B_i}{B_{vco}} \right] \\
&\text{where} \quad \delta B_i = B_i - B_{io} \quad (i = z, \ell, v, t, d) \\
&B_{io} = 0 \qquad (i = \ell, t, d) \\
&\delta B_v = B_{vco}' \, m \frac{\delta R}{R_o} + \delta B_{vc} \\
&\Gamma_o = \ell n \frac{8R_o}{a_o} + \beta_\theta + \frac{\ell_i - 3}{2} \\
&G = \frac{1-\alpha}{\Gamma_o} - 1 + n - m \\
&B_{vco} = \frac{\mu_o \Gamma_o I_{po}}{4\pi R_o}
\end{aligned}\right\} \tag{12}$$

Eq. (12) is a wave-equation with a frequency $\left[\frac{\mu_o \Gamma_o I_{po}^2}{2M} G \right]^{\frac{1}{2}}$ which is found around $10^6 H_z$ for the plasma.
On the other hand, the plasma has the mutual magnetic coupling with the conductors which impose a lower cut-off frequency in the system, and this implies that the left hand side of Eq. (12) can be omitted. So, the equation of motion of the toroidal plasma column is written as

$$G \frac{\delta R}{R_o} = \sum_{i=\ell, vc, t, d} \frac{\delta B_i}{B_{vco}} \tag{13}$$

1.3 Circuit equations

Motion of the plasma column induces a certain amount of current in the conductors and results in appearance of the vertical field to prevent its own motion. Moreover, the conductor currents are mutually affected through magnetic couplings.

Now, some assumptions are made. (1) Each of the conductors has a circular cross-section coaxial with the plasma column; (2) each of the conductor currents has a cosinusoidal density profile on the conductor cross-section, and (3) the plasma current has a homogeneous density profile on the column cross-section.

With these assumptions, self (L) and mutual (M) inductances are written as

$$\left[\begin{array}{ll} L_i = \frac{\pi^2}{4} \mu_o R_o N_i^2 & (i = \ell, v, t) \\ M_{pi} = -\frac{\pi\mu_o}{2r_i} R_o N_i \delta R & (i = \ell, v, t) \\ M_{ij} = \frac{\pi^2 r_i}{4r_j} \mu_o R_o N_i N_j & (i=\ell, v,\ j=v, t,\ i \neq j) \end{array}\right. \tag{14}$$

where N is number of turns of a conductor concerned with the virtical field, and r is the minor radius of the conductor. With the inductances, a set of circuit equations is written as

$$\left[\begin{array}{l} 0 = R_\ell I_\ell + L_\ell \cdot \frac{dI_\ell}{dt} + \frac{dM_{p\ell}}{dt} \cdot I_p + M_{p\ell} \cdot \frac{dI_p}{dt} + M_{\ell v} \cdot \frac{dI_v}{dt} + M_{\ell t} \cdot \frac{dI_t}{dt} \\ V = R_v I_v + L_v \cdot \frac{dI_v}{dt} + \frac{dM_{pv}}{dt} \cdot I_p + M_{pv} \cdot \frac{dI_p}{dt} + M_{v\ell} \cdot \frac{dI_\ell}{dt} + M_{vt} \cdot \frac{dI_t}{dt} \\ 0 = R_t I_t + L_t \cdot \frac{dI_t}{dt} + \frac{dM_{pt}}{dt} \cdot I_p + M_{pt} \cdot \frac{dI_p}{dt} + M_{t\ell} \cdot \frac{dI_\ell}{dt} + M_{tv} \cdot \frac{dI_v}{dt} \end{array}\right] \tag{15}$$

where V is the voltage applied to the terminal of conductor v, $R_i (i=\ell,v,t)$ are resistances of the conductors, and $I_i (i=\ell,v,t)$ are the conductor currents.

Eq (15) is reduced to a linear one in the vicinity of a steady equilibrium state as below.

$$\begin{cases} 0 = \dfrac{1}{T_\ell}\dfrac{\delta I_\ell}{I_{po}} + \dfrac{d}{dt}\left(\dfrac{\delta I_\ell}{I_{po}}\right) - \dfrac{2R_o}{\pi r_\ell}\dfrac{d}{dt}\left(\dfrac{\delta R}{R_o}\right) + \dfrac{r_\ell N_v}{r_v}\dfrac{d}{dt}\left(\dfrac{\delta I_v}{I_{po}}\right) + \dfrac{r_\ell}{r_t}\dfrac{d}{dt}\left(\dfrac{\delta I_t}{I_{po}}\right) \\[2ex] \dfrac{r_v \Gamma_o}{\pi r_v R_o N_v}\dfrac{\delta V}{V_o} = \dfrac{1}{T_v}\dfrac{\delta I_v}{I_{po}} + \dfrac{d}{dt}\left(\dfrac{\delta I_v}{I_{po}}\right) - \dfrac{2R_o}{\pi r_v N_v}\dfrac{d}{dt}\left(\dfrac{\delta R}{R_o}\right) + \dfrac{r_\ell}{r_v N_v}\dfrac{d}{dt}\left(\dfrac{\delta I_\ell}{I_{po}}\right) + \dfrac{r_v}{r_t N_v}\dfrac{d}{dt}\left(\dfrac{\delta I_t}{I_{po}}\right) \\[2ex] 0 = \dfrac{1}{T_t}\dfrac{\delta I_t}{I_{po}} + \dfrac{d}{dt}\left(\dfrac{\delta I_t}{I_{po}}\right) - \dfrac{2R_o}{\pi r_t}\dfrac{d}{dt}\left(\dfrac{\delta R}{R_o}\right) + \dfrac{r_\ell}{r_t}\dfrac{d}{dt}\left(\dfrac{\delta I_\ell}{I_{po}}\right) + \dfrac{r_v N_v}{r_t}\dfrac{d}{dt}\left(\dfrac{\delta I_v}{I_{po}}\right) \end{cases} \qquad (16)$$

where $T_i = \dfrac{L_i}{R_i}$ $(i = \ell, v, t)$.

On the other hand, the conductor currents are assumed to have a cosinusoidal density profile on their cross-section, and so, each of them produces at the plasma column a homogeneous virtical field of which the normalized density is given as below.

$$\frac{\delta B_i}{B_{vco}} = \frac{\pi R_o N_i}{r_i \Gamma_o}\frac{\delta I_i}{I_{po}} \quad (i = \ell, v, t) \qquad (17)$$

1.4 Equation of a position-controller of a plasma column

The plasma column motion is continuously observed and its position is compared with its reference value. If there were any deviation, the error will be regulated through a position-controller. The position error is used as the activating signal, and the vertical field is regulated by changing the terminal voltage of the conductor v so as the plasma column may be maintained at a desired position.
The control action is written as below.

$$\frac{d}{dt}\left(\frac{\delta V}{V_o}\right) + \frac{1}{T_f}\frac{\delta V}{V_o} = \frac{P}{T_f}\left(\frac{\delta R}{R_o} - c\right) \qquad (18)$$

where c is the setting value of the plasma column position, and the position-controller is characterized by the proportional coefficient P and the first-order time lag$(1+T_f s)^{-1}$.

1.5 A set of equations for control performance analysis of a toroidal plasma column

The equations given above are summarized below.

$$
\left\{
\begin{aligned}
& G\,\frac{\delta R}{R_o} = \sum_{i=\ell,vc,t,d} \frac{\delta B_i}{B_{vco}} \\
& 0 = \frac{1}{T_\ell}\,\frac{\delta I_\ell}{I_{po}} + \frac{d}{dt}\left(\frac{\delta I_\ell}{I_{po}}\right) - \frac{2R_o}{\pi r_\ell}\,\frac{d}{dt}\left(\frac{\delta R}{R_o}\right) + \frac{r_\ell N_v}{r_v}\,\frac{d}{dt}\left(\frac{\delta I_v}{I_{po}}\right) + \frac{r_\ell}{r_t}\,\frac{d}{dt}\left(\frac{\delta I_t}{I_{po}}\right) \\
& \frac{r_v \Gamma_o}{\pi T_v R_o N_v}\,\frac{\delta V}{V_o} = \frac{1}{T_v}\,\frac{\delta I_v}{I_{po}} + \frac{d}{dt}\left(\frac{\delta I_v}{I_{po}}\right) - \frac{2R_o}{\pi r_v N_v}\,\frac{d}{dt}\left(\frac{\delta R}{R_o}\right) + \frac{r_\ell}{r_v N_v}\,\frac{d}{dt}\left(\frac{\delta I_\ell}{I_{po}}\right) + \frac{r_v}{r_t N_v}\,\frac{d}{dt}\left(\frac{\delta I_t}{I_{po}}\right) \\
& 0 = \frac{1}{T_t}\,\frac{\delta I_t}{I_{po}} + \frac{d}{dt}\left(\frac{\delta I_t}{I_{po}}\right) - \frac{2R_o}{\pi r_t}\,\frac{d}{dt}\left(\frac{\delta R}{R_o}\right) + \frac{r_\ell}{r_t}\,\frac{d}{dt}\left(\frac{\delta I_\ell}{I_{po}}\right) + \frac{r_v N_v}{r_t}\,\frac{d}{dt}\left(\frac{\delta I_v}{I_{po}}\right) \\
& \frac{\delta B_i}{B_{vco}} = \frac{\pi R_o N_i}{r_i \Gamma_o}\,\frac{\delta I_i}{I_{po}} \qquad (i = \ell,\ vc,\ t) \\
& \frac{d}{dt}\left(\frac{\delta V}{V_o}\right) + \frac{1}{T_f}\,\frac{\delta V}{V_o} = \frac{P}{T_f}\left(\frac{\delta R}{R_o} - c\right) \\
& \Gamma_o = \ell n\,\frac{8R_o}{a_o} + \beta_\theta + \frac{\ell_i - 3}{2} \\
& G = \frac{1-\alpha}{\Gamma_o} - 1 + n - m
\end{aligned}
\right. \tag{19}
$$

with

$$
\left\{
\begin{aligned}
& -\frac{3}{2} < m < 0 \\
& 0 < \alpha < \frac{1}{2} \\
& -1 < n < 0
\end{aligned}
\right.
$$

2. CONTROL PERFORMANCE CHARACTERISTICS OF A POSITION-CONTROLLED TOROIDAL PLASMA COLUMN

A kinetics model for analyzing the control performance of a toroidal plasma column has been given in the preceding section. And it is applied to a plasma confining device Japan Torus 60 and positional controlability of the plasma column is investigated; especially influence of proportional coefficient of controller, P, influence of time constant of controller, T_f, and influence of time constant of conductor v, T_v, on the control performance. Parameters characterizing the control performance are cut-off frequency, gain margin, phase magin, regelfaktor, logarithmic decrement, damping factor, damping coefficient, maximum gain, frequency for maximum gain, cut-off frequency of an ideal filter, phase lag of an ideal filter, half-rise time and rise time. The results obtained of these parameters are shown in Fig. 2 $\sim$ 10.

2.1 Values of parameters

Parameters required for analyzing the kinetics of a position-controlled plasma column are listed in Table 1, where the values concern JT-60. Major radius, Ro, and minor radius, a_o, of the plasma column and minor radii, r_i ($i = \ell, v, t$), and equivalent numbers of turns, N_i ($i = \ell, v, t$), of conductors, are known from the design of the device. The value of beta, β_θ, may be expected to be 0.1 but might reach to 1.0. The internal inductance of the plasma column, ℓ_i, is expected to be 0.9 but may be found in the range between 0.5 and 1.5. Speaking of the boundary values of ℓ_i, 0.5 is given for a parabolic density profile of the plasma current and 1.5 for a homogeneous one. As described already, m, α and n, which characterize positional gradients of the virtical field, minor radius of the plasma column and plasma current, respectively, are found in the ranges below:

$$-\frac{3}{2} < m < 0, \qquad 0 < \alpha < \frac{1}{2}, \qquad -1 < n < 0.$$

Γ_o, defined by Eq.(12), depends on β_θ and ℓ_i, and G, defined by Eq. (12), depends on m, α, n. The influence of these parameters on the characteristics of plasma column motion was precisely investigated and the results state in JAERI-M 6508[5] adequancy of giving 2.2 to Γ_o and -0.8 to G. And also, the influence of the intrinsic time constants of conductors,

T_ℓ, T_v and T_t, on the characteristics were investigated and it is said in the previous report that uncertainty of T_t does not much influence the characteristics in the range beyond 0.6 H_z, uncertainty in T_v affects a little the characteristics, and uncertainty in T_ℓ does not influence the characteristics in the range below 4 H_z but affects them slightly in the range beyond it. With the results, in the control frequency range, T_t may be set at 5.0 sec, T_v at one of three T_vs (0.25, 0.5, 1.0 (sec.)) and T_ℓ at 0.05 sec. The time constant of controller, T_f, is set at one of three T_fs (0.025, 0.05, 0.1 (sec.)) and the proportional coefficient of controller, P, is set at various values and the influence of P on the control performance is parametrically investigated.

2.2 Influence of proportional coefficient, P, time constants, T_f and T_v, on the cut-off frequency of the system

The gain characteristics of loop transfer function cross zero decibel line at the cut-off frequency, which concerns the response time of the system. The cut-off frequency of the position-controlled plasma column is given in Figs. 2 ∿ 10 as the function of proportional coefficient P. It is seen in the figures that, regardless of the value of T_v or T_f, or of both, the cut-off frequency increases with increase of the value of P. With a smaller value of T_v or T_f, or of both, the cut-off frequency becomes higher and is affected more by the value of T_v than the one of T_f.

2.3 Influence of proportional coefficient, P, time constants, T_f and T_v, on gain margin and phase margin

The system becomes unstable if the characteristic equation of the system has roots of which the real parts are positive. The stability of the system is now estimated with gain margin and phase margin.

It is seen in Figs. 2 ∿ 10 that, regardless of the value of T_v or T_f, or of both, the loop gain of the system increases with increase of the value of P and the gain margin decreases. With a larger value of T_v or T_f, or of both, the gain margin increases slightly and is affected more by the value of T_v than the one of T_f.
The phase margin decreases also with increase of the value of P, regardless of the value of T_v or T_f, or of both. In the cases for the same T_v value, the phase margin becomes smaller with a larger value of T_f but, in the cases for the same T_f value, the phase margin becomes larger with

a larger value of T_v, and is affected more by the value of T_v than the one of T_f.

2.4 Influence of proportional coefficient, P, time constants, T_f and T_v, on the regelfaktor

The off-set of a controlled variable defines the quality of the control and is estimated with the regelfaktor which is a steady state value of a controlled variable for unit step change of the setting value. The regelfaktor does not depend on the time constants of the system.

It is seen in Fig. 2 that the regelfaktor decreases with increase of the value of P and it is 0.01 for the P value of 80. This implies that the off-set of the system is practically removed by the proportional control.

2.5 Influence of proportional coefficient, P, time constants, T_f and T_v, on logarithmic decrement, damping factor and damping coefficient

The steady state error, which is considered in the preceeding section, is desired to be as small as possible, and also the system must reach the steady state as fast as possible. And, for estimating the transient characteristics of the system, the attenuation of the transient components of the response is discussed.

The transient response characteristics of the system is discussed with the characteristic equation of the system, and the information on the time response is given by the roots of the characteristic equation. The attenuation of the response is now estimated with the logarithmic decrement which is the absolute value of the smallest real part of the roots of the characteristic equation.
It is seen in Figs. 2 $\sim$ 10 that, regardless of the value of T_v or T_f, or of both, the logarithmic decrement increases with increase of the value of P, that is, the transient component of the response dampes out in shorter time. In the cases for the same T_v value, the logarithmic decrement decreases slightly for a larger value of T_f and, in the cases for the same T_f value, it increases slightly for a larger value of T_v.

The logarithmic decrement gives the information on the transient characteristics of the real parts of the roots of characteristic equation but not the information of the characteristics of the imaginary parts of the roots. On the other hand, the damping factor, which is defined as the ratio of real part of a root to its imaginary part, gives the information on the transient characteristics of both real and imaginary part of the roots.

It is seen in Figs. 1∿9 that, regardless of the value of T_v or T_f, or of both, the damping factor decreases with increase of the value of P, that is, the transient response becomes more oscillatory. In the cases for the same T_v value, the damping factor increases with increase of the T_f value but, in the cases for the same T_f value, it decreases with increase of the T_v value.

The damping coefficient is defined as $K/\ 1+K^2$ where K is the damping factor, and gives also the information on the transient characteristics of both real and imaginary part of the roots of the system characteristic equation.

It is seen in Figs. 2∿10 that, regardless of the value of T_v or T_f, or of both, the damping coefficient decreases with increase of the value of P, that is, the transient response becomes more oscillatory. In the cases of the same T_v value, the damping coefficient becomes larger with a larger value of T_f but, in the cases of the same T_f value, it becomes smaller with a larger T_v value. The adoption of the damping factor or the damping coefficient depends on their resolving power of the attenuation.

2.6 Influence of proportional coefficient, P, time constants, T_f and T_v, on the maximum gain point

The gain of the system has, at a frequency, the maximum value which depends on the proportional coefficient and also the time constants. The augmentation of the gain characteristics improves the time response of the system.

It is seen in Figs. 2∿10 that, regardless of the value of T_v or T_f, or of both, the maximum value of gain increases with increase of the value of P, and, in the cases for the same T_v value, it increases with increase of the value of T_f but, in the cases for the same T_f value, it decreases with increase of the value of T_v.

Considering the frequency at which the gain of the system has the maximum value, the frequency becomes higher with increase of the value of P regardless of the value of T_v or T_f, or of both, and, in the cases of the same T_v value, it becomes lower with a larger T_f value and, in the cases of the same T_f value, it also becomes lower with a larger T_v

value. This is explained by the fact that a larger time constant brings a lower cut-off frequency in the system.

2.7 Influence of proportional coefficient, P, time constants, T_f and T_v, on the characteristics of an ideal filter of the system

The transient characteristics of the system can be estimated with those of an ideal filter for the system. The frequency characteristics of an ideal filter are expressed as below.

$$W(j2\pi f) = \begin{cases} e^{-j(\psi_o/f_o)f} & : 0 \leq f \leq f_o \\ 0 & : f_o \leq f \end{cases}$$

where f_o is a cut-off frequency and ψ_o is a phase lag at f_o.

It is seen in Figs. 2∿10 that, regardless of the value of T_v or T_f, or of both, the cut-off frequency of an ideal filter increases with increase of the value of P and, in the cases for the same T_v value, it becomes lower for a larger T_f value and, in the cases for the same T_f value, it becomes also lower for a larger T_v value. Considering the phase lag at the cut-off frequency, it increases with increase of the value of P regardless of the value of T_v or T_f, or of both, and, in the cases for the same T_v value, it becomes larger for a larger T_f value but, in the cases for the same T_f value, it becomes smaller for a larger T_v value.

2.8 Influence of proportional coefficient, P, time constants, T_f and T_v, on half-rise time and rise time

The transient characteristics of the system can be estimated with half-rise time and rise time which are obtained by using the concept of ideal filter. The controlled variable reaches the maximum value for the first time at the rise time and the derivative of the variable has the maximum value at the half-rise time.

It is seen in Figs. 2∿10 that, regardless of the value of T_v or T_f, or of both, half-rise time and rise time decrease with increase of the value of P, that is, the system responses more quickly to the control demand with a larger P value. In the cases for the same T_v value, half-rise time and rise time become longer for a larger T_f value and also becomes longer for a larger T_v value in the cases for the same T_f value. This is explained by the fact that a larger time constant brings a lower cut-off frequency in the system.

3. SUMMARY AND CONCLUSION

The control performance characteristics of a position-controlled JT-60 plasma column have been considered; especially the influence of the proportional coefficient of controller, the influence of the time constant of controller and the influence of the intrinsic time constant of a conductor, on the characteristics.

Cut-off frequency, gain margin and phase margin concern the frequency characteristics of the system, with which frequency response and stability of the system have been estimated. Increasing the P value, the cut-off frequency of the system increases and the system responds to the demand of higher frequency component, but gain and phase margin decrease. The regelfaktor concerns the steady state of the system and the off-set of the controlled variable decreases with increase of the P value. Logarithmic decrement, damping factor and damping coefficient concern the transient characteristics of the system. The logarithmic decrement offers informations of the attenuation of response but not of the oscillatory component of response. On the other hand, damping factor and damping coefficient give information on the attenuation of the response of oscillatory component. Increasing the P value, the logarithmic decrement increases and this fact might imply that the transient state damps out in shorter time, but damping factor and damping coefficient decrease with increase of the P value, which says that the transient response becomes more oscillatory with increase of the P value. The augmentation of the gain characteristics brings in the system higher cut-off frequency and so the maximum gain point concerns the time response of the system. Increasing the P value, both maximum gain and frequency of the maximum gain point increase. The transient characteristics of the system is estimated with an ideal filter for the system. Increasing the P value, both cut-off frequency and phase lag increase, and both half-rise time and rise time decrease, that is, the response time becomes shoter.

Concerning the influence of the values of T_v and T_f on the control performance characteristics, the response of the system is more affected by the T_v value than T_f. It is because the inverse value of T_v is in the vicinity of the control frequency but the inverse value of T_f is ten times higher than the frequency. And the response of the system becomes worse with increase of the value of T_v or T_f, or of both.

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Table 1 List of notations

Notation	Value	Complement
R_o	3.0	Major radius of plasma column
a_o	1.0	Minor radius of plasma column
r_ℓ	1.2	Minor radius of conductor ℓ
r_v	1.4	Minor radius of conductor v
r_t	1.8	Minor radius of conductor t
N_ℓ	1	Equivalent number of turns of conductor ℓ
N_v	64	Equivalent number of turns of conductor v
N_t	1	Equivalent number of turns of conductor t
T_ℓ	0.05	Intrinsic time constant of conductor ℓ
T_v	0.25,0.5,1.0	Intrinsic time constant of conductor v
T_t	5.0	Intrinsic time constant of conductor t
β_θ	0.1	Value of poloidal beta
ℓ_i	0.9	Internal inductance of plasma column
m	$-1.5 < \quad < 0$	Positional gradient of vertical field
α	$0 < \quad < 0.5$	Positional gradient of minor radius
n	$-1 < \quad < 0$	Positional gradient of plasma current
Γ_o	2.2	$\Gamma_o = \ell n \frac{8R_o}{a_o} + \beta_\theta + \frac{\ell_i - 3}{2}$
G	-0.8	$G = \frac{1-\alpha}{\Gamma_o} - 1 + n - m$
P		Proportional coefficient of controller
T_f	0.025,0.05,0.1	Time constant of controller
C		Setting point of controller

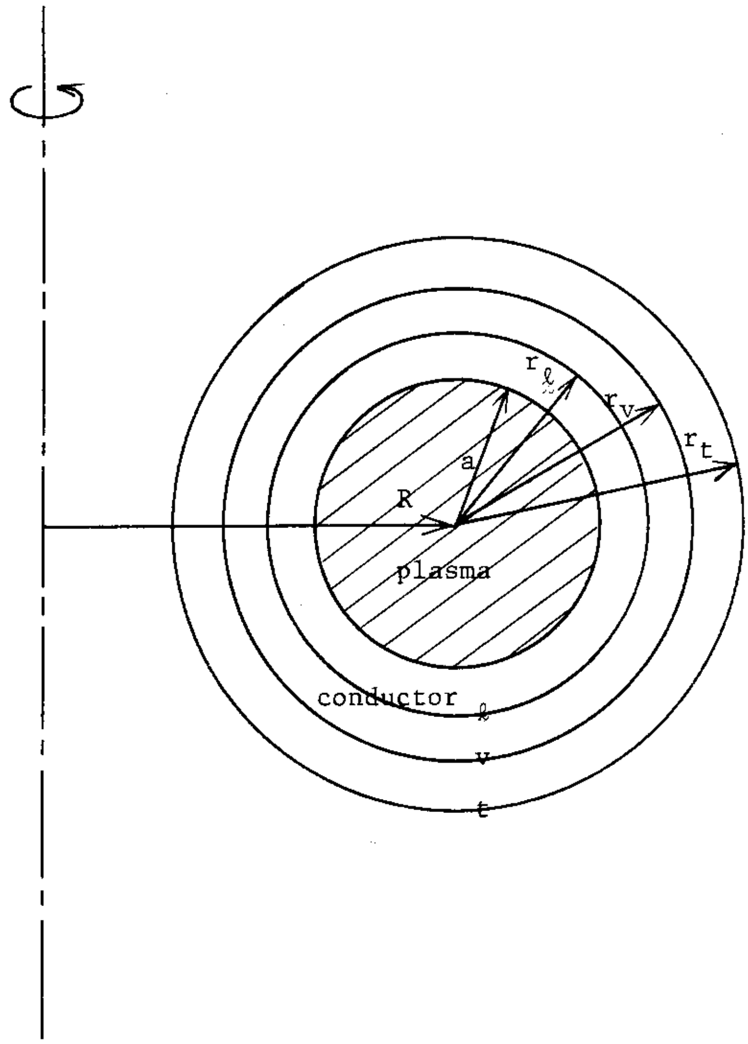


Fig. 1 Cross-sectional contiguration of the device

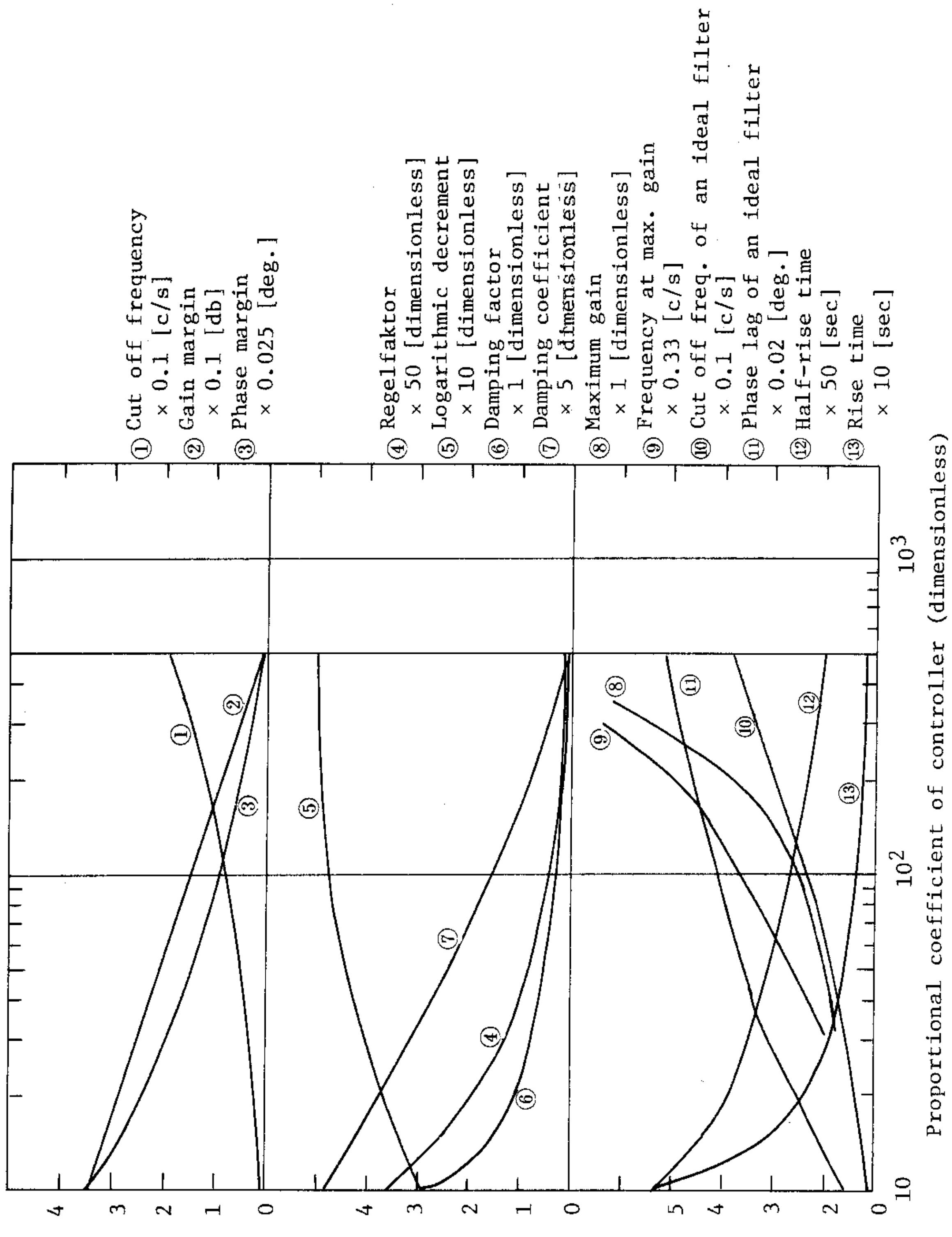


Fig. 2 Control performance characteristics

actual value = ordinate × scale modulus (given on right hand side)

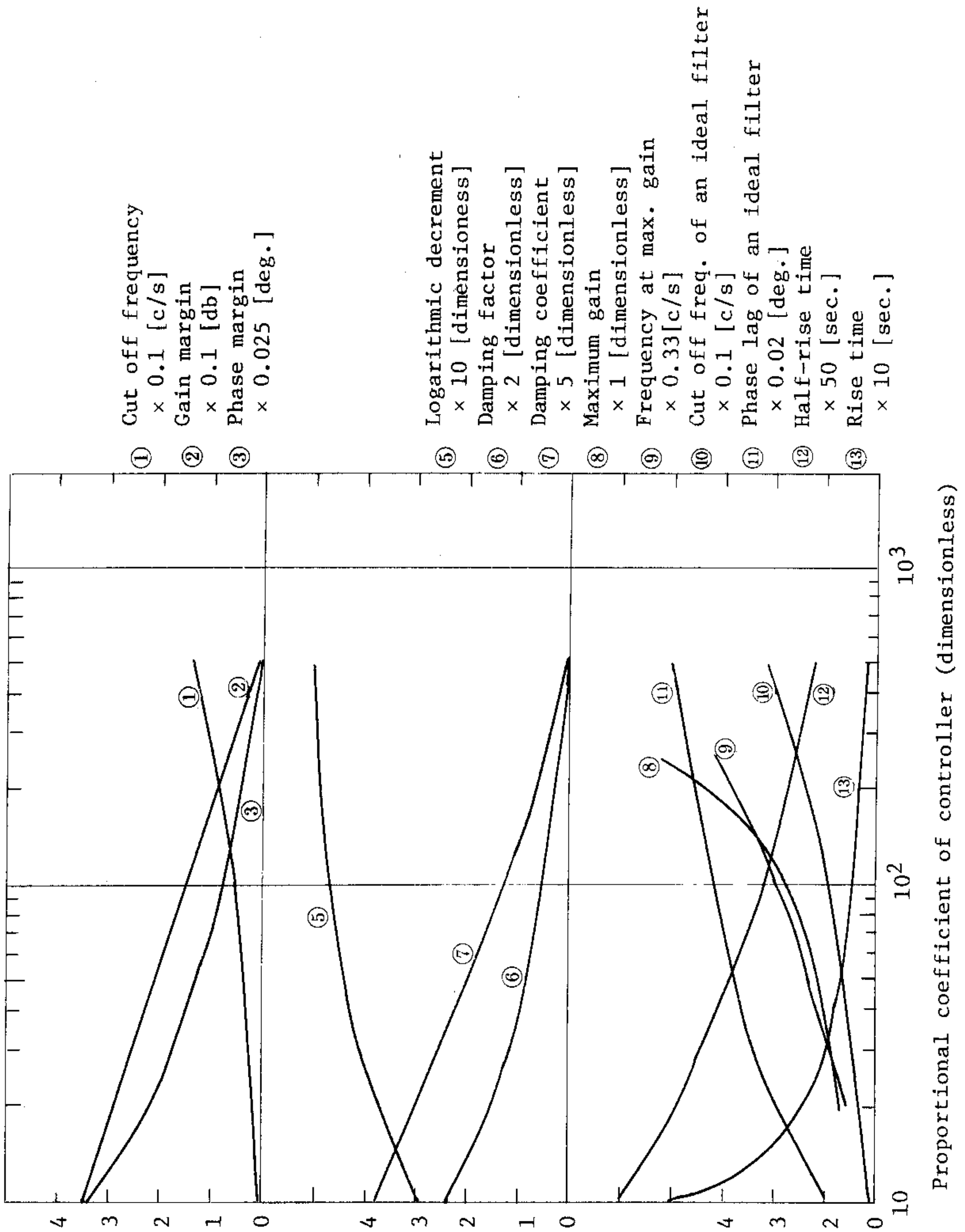


Fig. 3 Control performance characteristics

actual value = ordinate × scale modulus (given on right hand side)

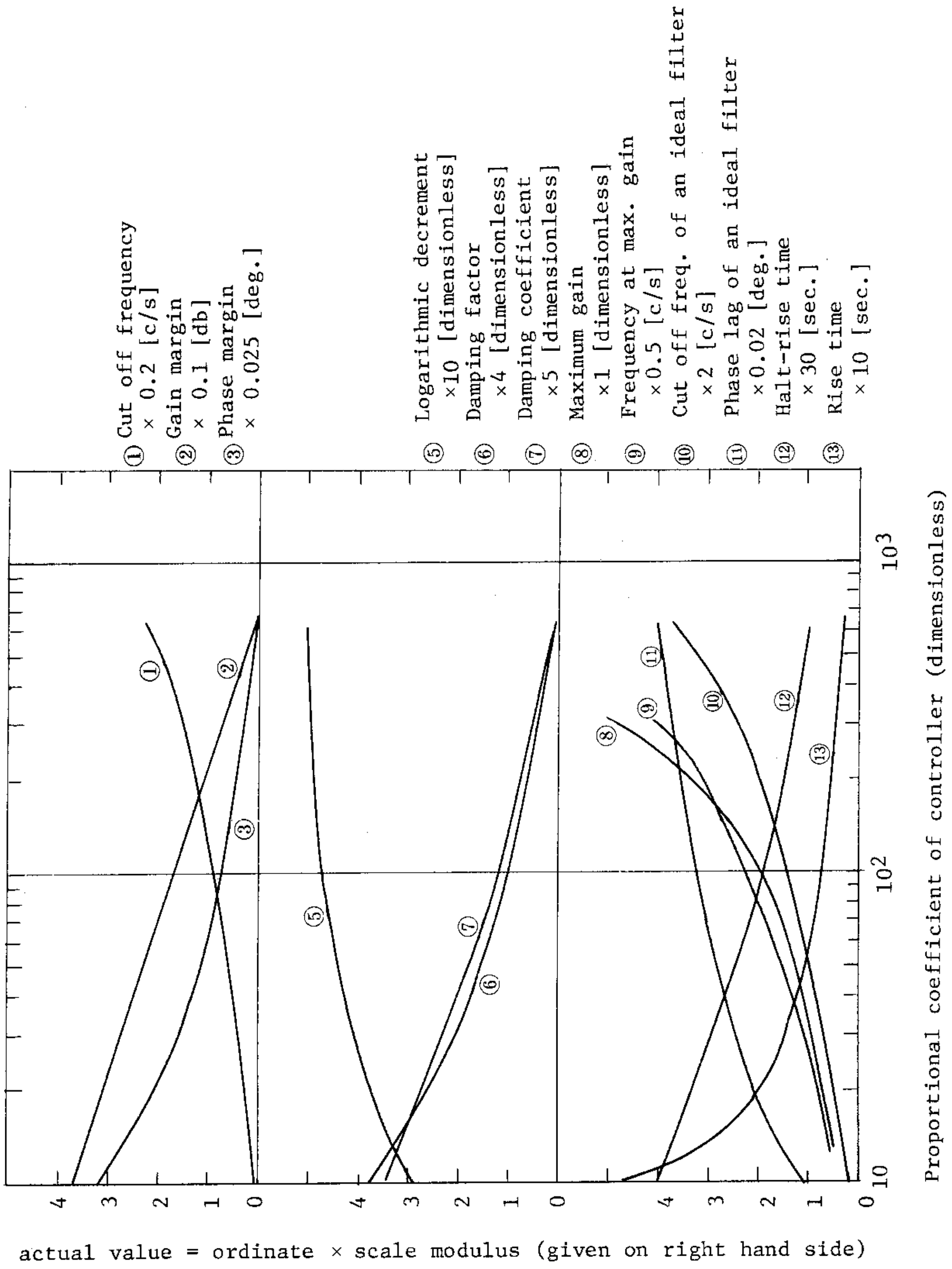


Fig. 4 Control performance characteristics

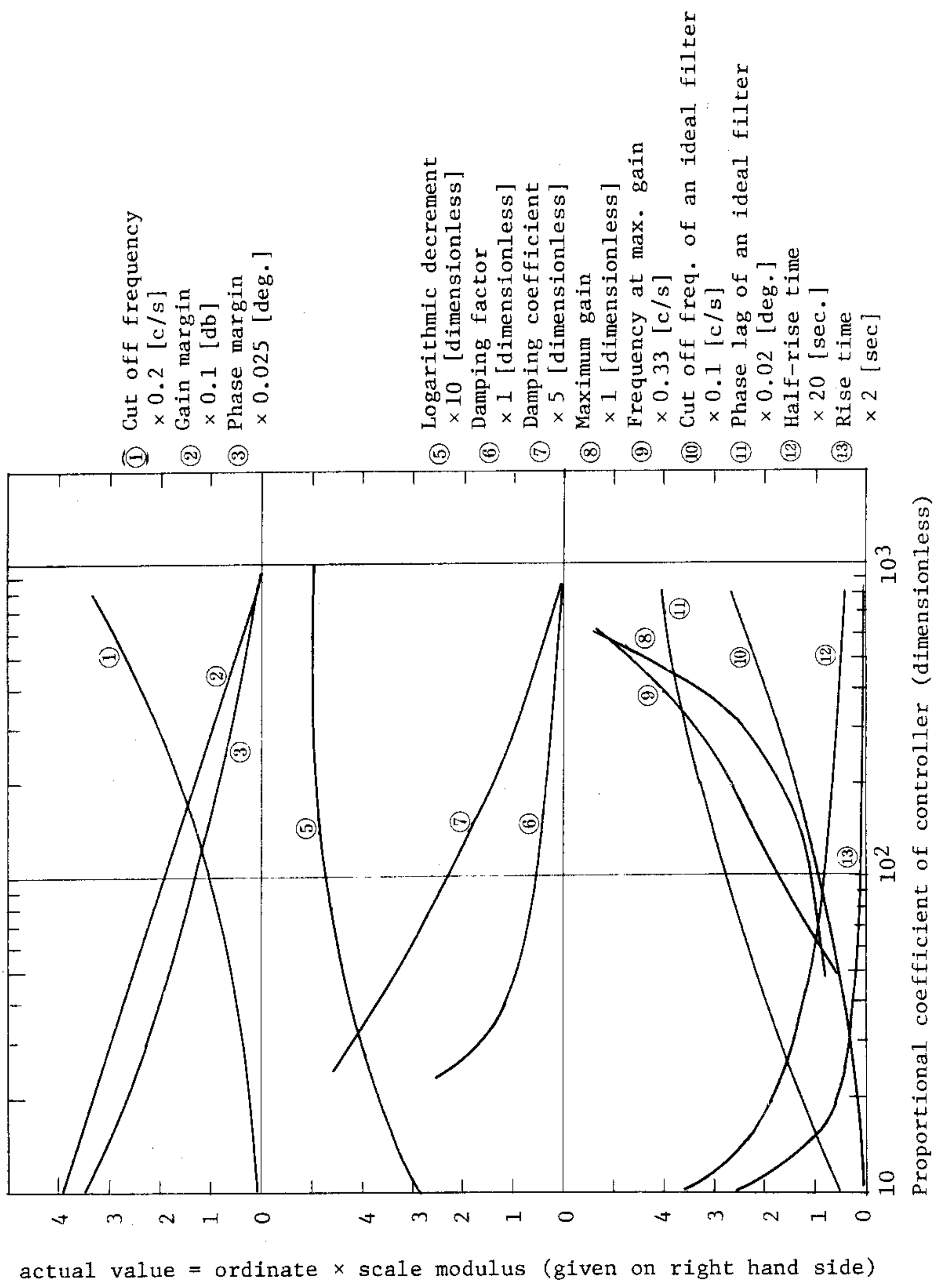


Fig. 5 Control performance characteristics

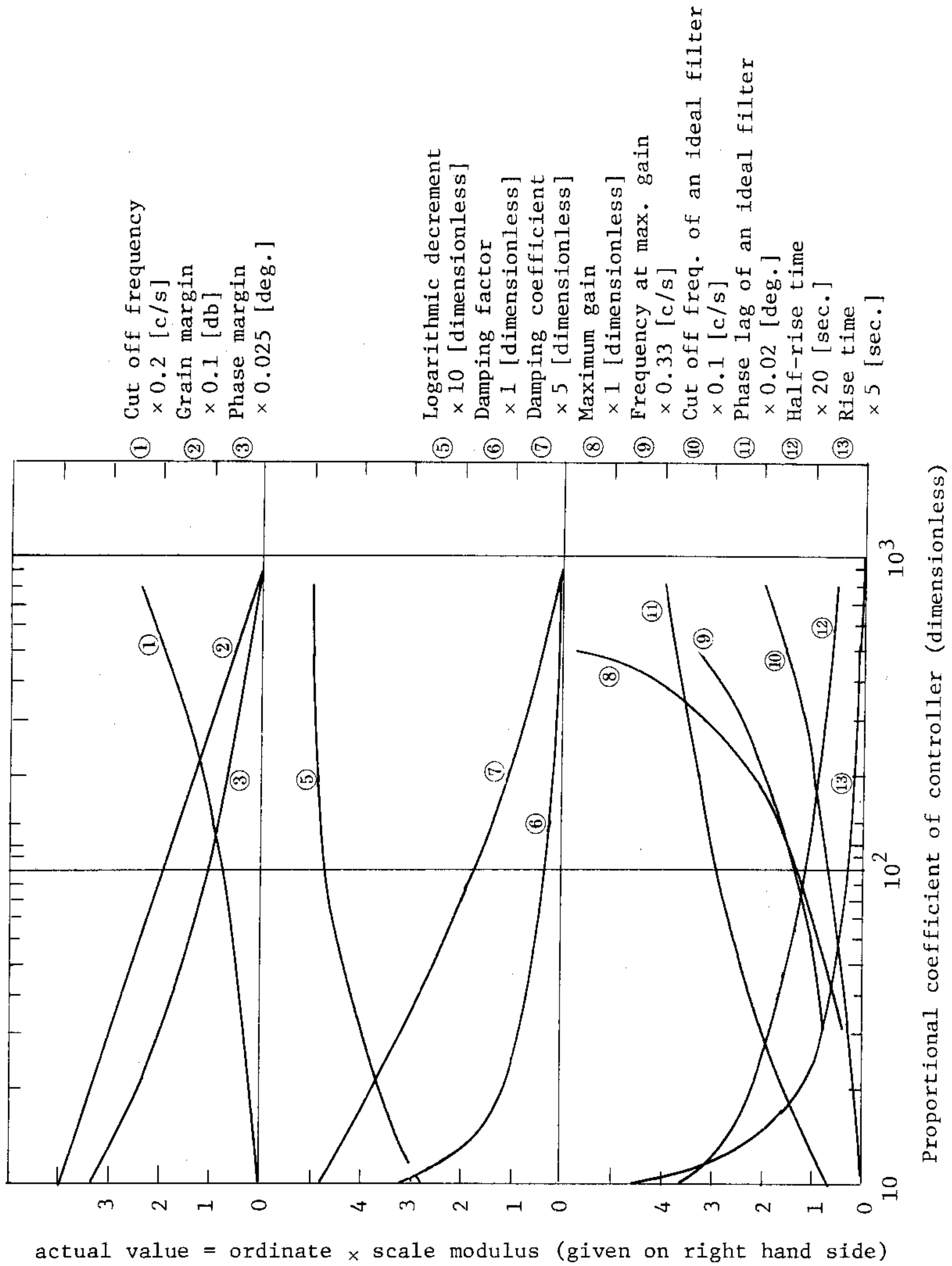
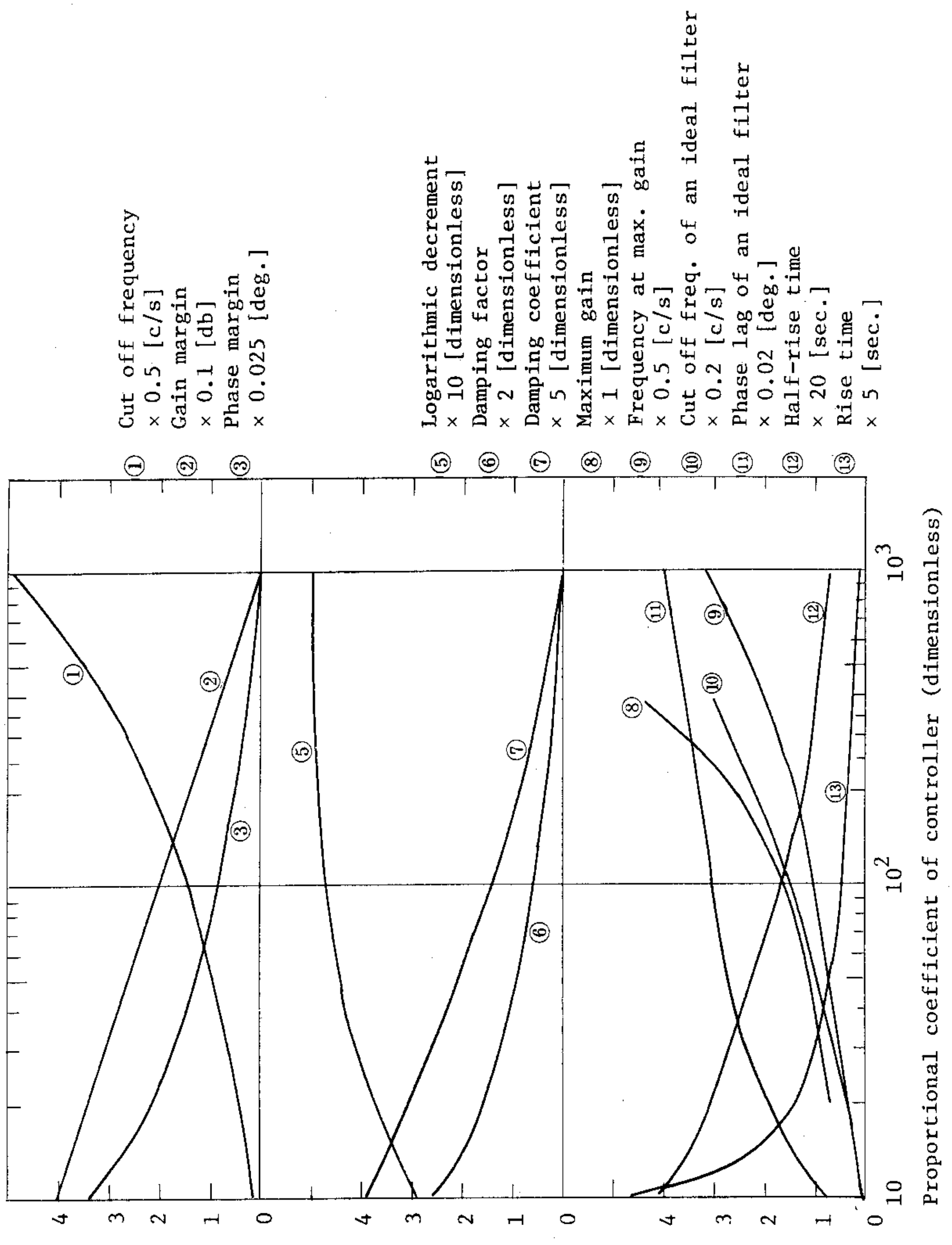


Fig. 6 Control performance characteristics



actual value = ordinate × scale modulus (given on right hand side)

Fig. 7 Control performance characteristics

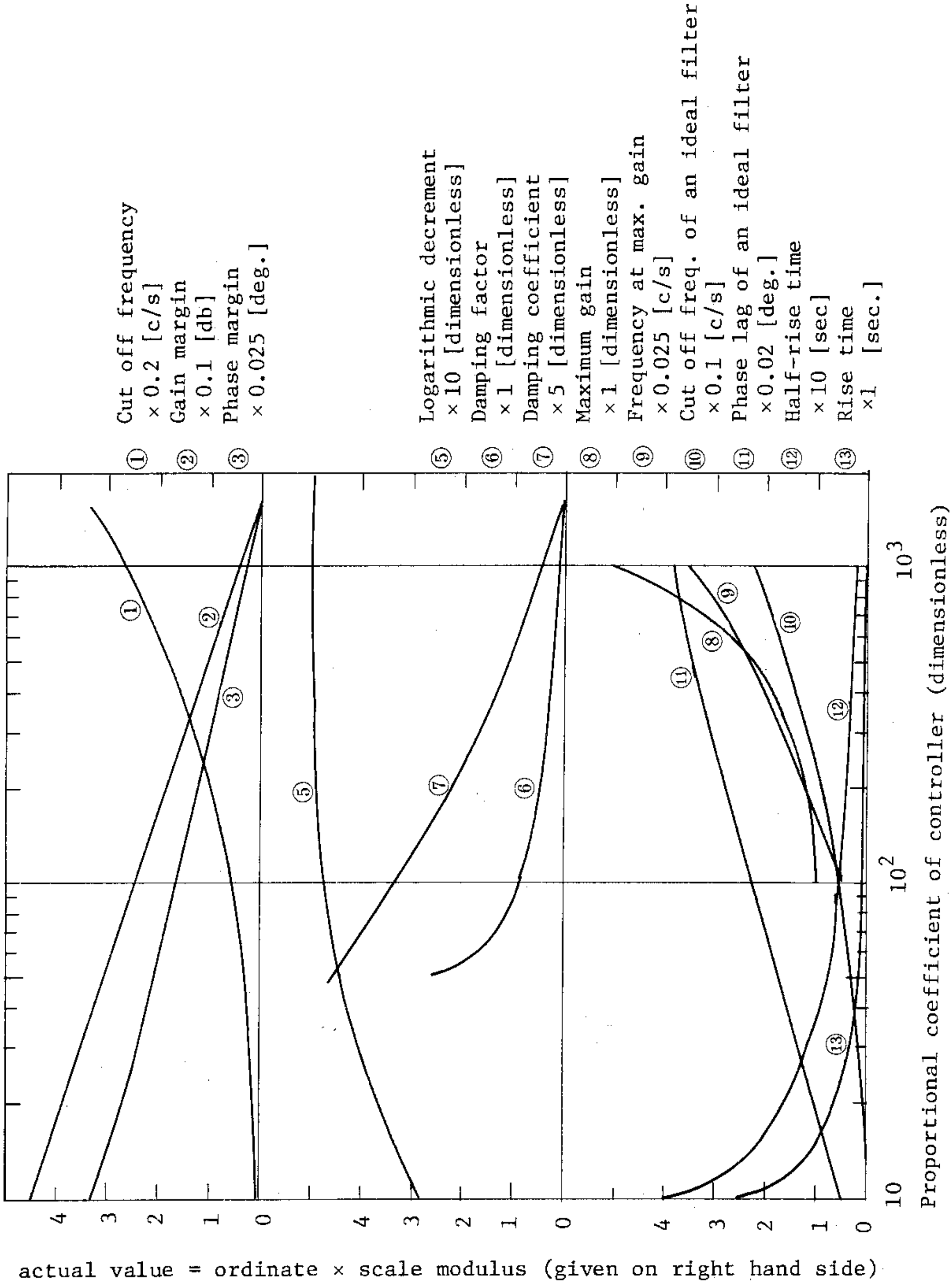
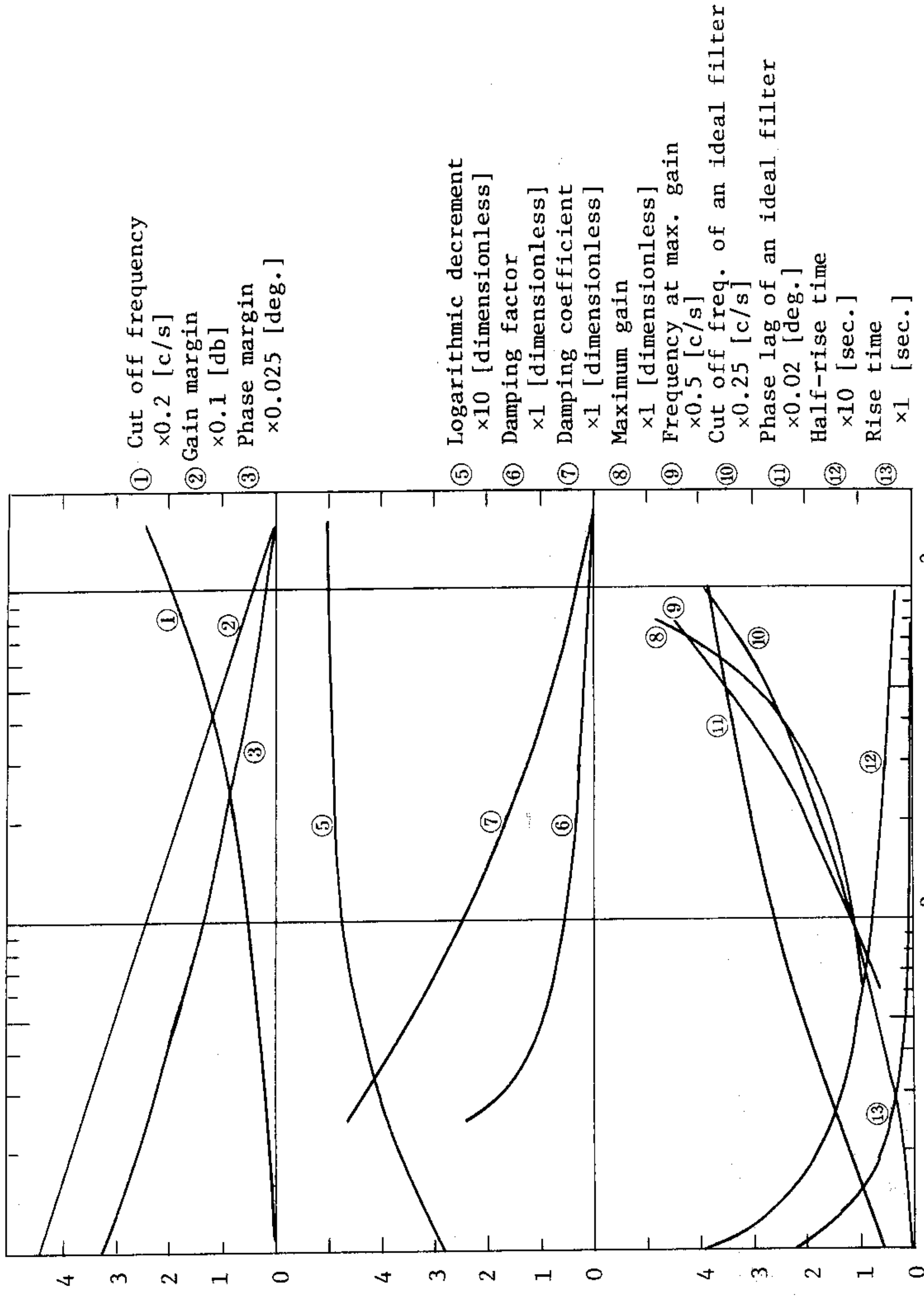


Fig. 8 Control performance characteristics



actual value = ordinate × scale modulus (given on right hand side)

Fig. 9 Control performance characteristics

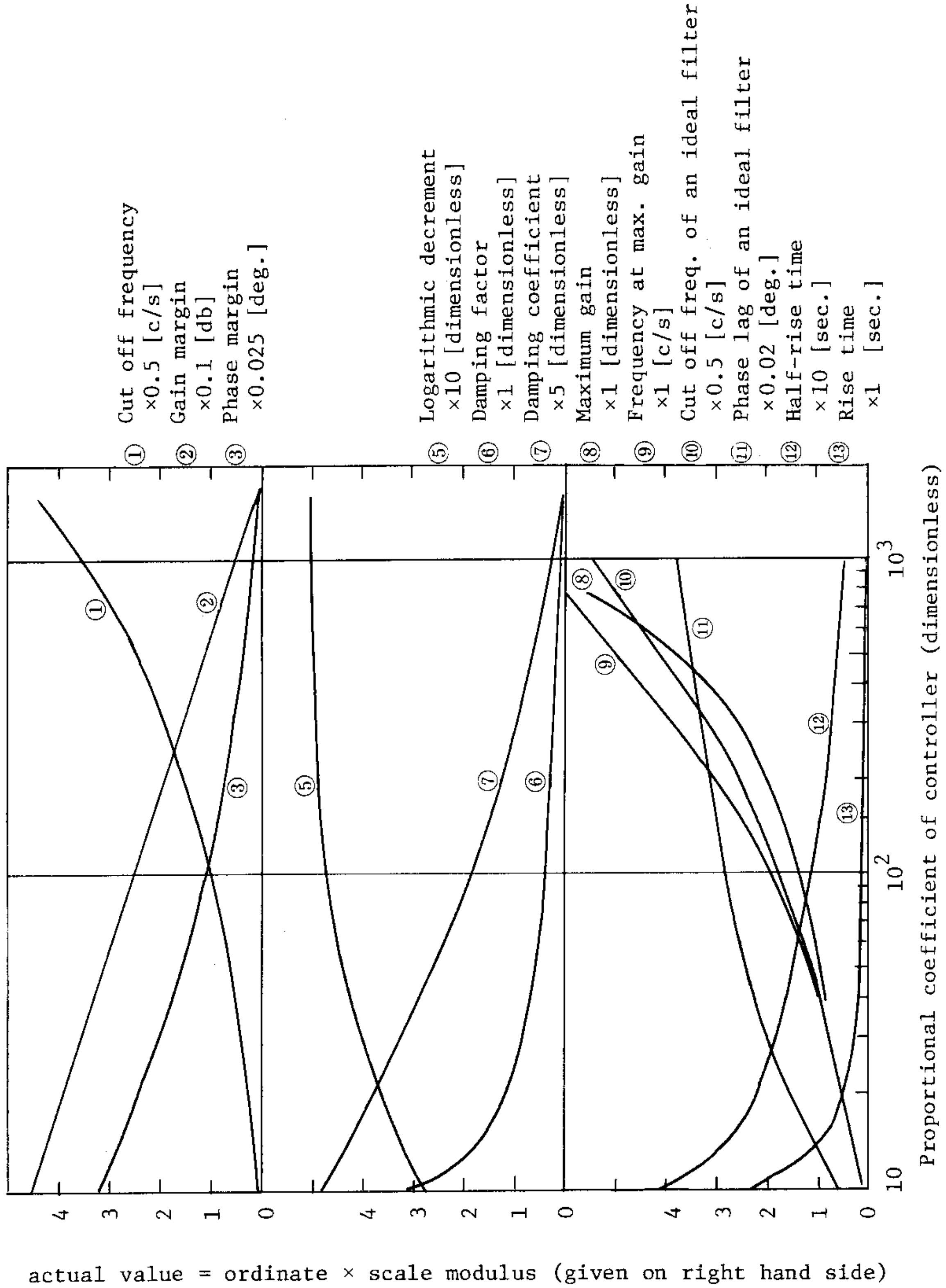


Fig. 10 Control performance characteristics