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Surface Tearing Modes in Tokamaks

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Surface tearing modes in tokamaks are studied numerically and analytically. The eigenvalue problem is solved to obtain the growth rate and the mode structure. We investigate in detail dependences of the growth rate of the m/n=2/1 resistive MHD modes on the safety factor at the plasma surface, current profile, wall position, and resistivity. The surface tearing mode moves the plasma surface even when the wall is close to the surface. The stability diagram for these modes is presented.

Keywords; Surface Tearing Mode, Tokamak, Resistive MHD Mode

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トカマクにおける表面ティアリングモード

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トカマクにおける表面ティアリングモードについて数値的，解析的に調べた。成長率およびモードの構造を固有値問題として求めた。モード数m＝2，n＝1の抵抗性MHDモードの成長率について詳しく調べ，プラズマ表面の安全係数，電流分布，導体壁の位置，抵抗に対する依存性を明らかにした。導体壁がプラズマ表面に近くなったときでも，表面ティアリングモードによってプラズマ表面は動かされる。これらのモードに関する安定性図表を示した。

目 次 な し

Instabilities driven by the plasma current and the resistivity, such as kink modes and tearing modes, have been considered to cause the current disruption in tokamaks [1-4]. Even if they do not cause the disruption, the interaction between the plasma surface and the limitter is enhanced by the disturbance and the impurity influx is increased in large quantities. The stability diagram for these modes was shown by Hastie et al. [5]. Kurita et al. pointed out that the plasma surface is moved by the "surface tearing mode" as well as by the kink mode [6]. They also showed through the nonlinear simulation the formation of the "vacuum bubble" due to this mode. In the present work, we perform the linear analyses of the "surface tearing mode" in detail and show the unstable region against this mode.

We consider the resistive MHD modes in a circular tokamak with large aspect ratio (major radius R_0, minor radius a, and wall radius b). Effects of the plasma pressure are neglected for simplisity. The basic equations for the present linear-stability analyses are as follows;

$$\gamma\tilde{\psi} = -\frac{B_0}{R_0}F\tilde{\varphi} + \frac{\eta}{\mu_0}\nabla^2\tilde{\psi} , \qquad (1)$$

$$\gamma\nabla\rho\nabla\tilde{\varphi} = \frac{B_0}{R_0\mu_0}F\nabla^2\tilde{\psi} - \frac{m}{r}\frac{dj}{dr}\tilde{\psi} . \qquad (2)$$

where γ is the growth rate, η the resistivity, ρ the mass density, j the current density, m the poloidal mode number, n the toroidal mode number, q the safety factor, and $F \equiv m/q - n$. The value of F becomes zero at the rational surface, $r=r_s$; $q(r_s)=m/n$. Perturbations of the poloidal flux function $\tilde{\psi}$ and the stream function $\tilde{\varphi}$ are defined as $\tilde{B}_r = -im\tilde{\psi}/r$ and $\tilde{v}_r = m\tilde{\varphi}/r$. The convection of the resistivity is not essential because the gradient of the resistivity is small at the rational surface. Equations (1) and (2) are investigated analytically and numerically. In order to solve easily the eigenvalue problem by a numerical calculation, we introduce the "pseudo-vacuum" model; the vacuum is replaced to the low-density and high-resistivity plasma. Radial meshes are accumulated near the rational surface and near the plasma surface, where the skin current flows. The convergence of γ is almost quadratic with respect to the mesh size by using the appropriate mesh accumulation.

Resistive MHD modes in a cylindrical tokamak are classified as follows;

(1) Kink mode is for $r_s>a$ ($q_a<m/n$) and has the mode structure of $\tilde{v}_r(a) \sim -i\tilde{B}_r(a)$. The skin current flows at $r=a$.

(2) Surface tearing mode is for $r_s<a$ ($q_a>m/n$) and has the mode structure of $\tilde{v}_r(a) \sim -i\tilde{B}_r(a)$. The skin current flows at $r=r_s$.

(3) Tearing mode is for $r_s<a$ ($q_a>m/n$) and has the mode structure of $\tilde{v}_r(a) \fallingdotseq 0$. The skin current flows at $r=r_s$.

The growth rate of surface tearing mode is proportional to $\eta_s^{3/5}$ as the same as that of tearing mode. The mode structure is studied by solving a equation in the discontinuity region;

$$d^2\Phi/dx^2 - x^2\Phi + x = 0 \quad , \tag{3}$$

with boundary conditions, $\Phi \to 0$ for $x \to -\infty$ and $d\Phi/dx = 0$ at $x = x_a$. We put here $x = (r-r_s)/\delta_s$ ($\delta_s^4 = R_0^2\eta\gamma\rho/B_0^2F'^2$) and $\Phi = \sqrt{\eta\rho/\gamma}\tilde{\varphi}/\delta_s\tilde{\psi}$ by assuming $d\tilde{\psi}/dx \sim 0$. The solution to eq.(3) is given by

$$\Phi = C_0\Phi_0 + C_1\Phi_1 - \Phi_3/6 \quad , \tag{4}$$

$$\Phi_k \equiv x^k + a_{4+k}x^{4+k} + \cdots + a_{4m+k}x^{4m+k} + \cdots \quad ,$$

where $a_n = a_{n-4}/(n-1)n$. The values of C_0 and C_1 for $|x_a| < 1$ are approximately given by

$$C_0 = -0.8862 + 0.7396\, x_a^2 \quad , \quad C_1 = 0.5\, x_a^2 \quad . \tag{5}$$

We find from this solution that the surface tearing mode ($\tilde{\varphi}(a) \sim -\tilde{\psi}(a)$) is observed when $a-r_s < \delta_s$.

Numerical results on the m/n=2/1 MHD resistive modes are shown in Figs.1-5. The dependence of the growth rate, $\Gamma \equiv \gamma R_0\sqrt{\mu_0\rho_0}/B_0$, on q_a for

q_0/q_a=0.5 and b/a=2 is shown in Fig.1, where $S \equiv \mu_0 a^2 B_0/\eta R_0 \sqrt{\mu_0 \rho_0}$ and arrows indicate the region of the surface tearing mode. The solid line corresponds to the case of $\eta(r) \propto 1/j(r)$ and the dashed line corresponds to the case of $\eta = const.$. Figure 2 shows Γ vs. q_0/q_a for b/a=2 and S_0=10^6, and Fig.3 shows Γ vs. b/a for q_0/q_a=0.5 and S_0=10^6. For q_a=2.2 in these figures, the mode is the surface tearing mode, which moves the plasma surface even when b/a is close to unity. The dependence of Γ on S_s for q_0/q_a=0.5 and b/a=2 is shown in Fig.4, where Γ is almost proportional to $S_s^{-3/5}$ for both cases of the tearing mode (q_a=3) and the surface tearing mode (q_a=2.05). The unstable region in (q_a , q_0/q_a) space against m/n=2/1 surface tearing mode is shown in Fig.5, where we choose b/a = 2, $j(r) = j_0\{ 1 - (r/a)^\mu \}^\nu$ (ν=2 for $\mu \geqq 2$ and μ=2 for $\nu \geqq 2$), and $\eta(r) \propto 1/j(r)$. The unstable region becomes wide as the resistivity in the region, $r_s \leqq r < a$, increases. Tokamak discharges with $q_a \doteq 2$ have to be operated carefully by taking account of the surface tearing mode.

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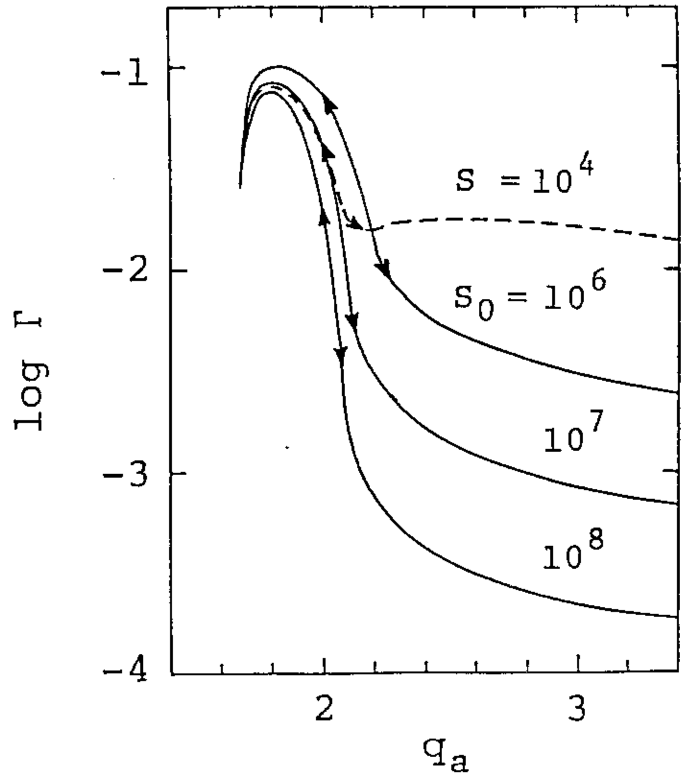

Fig.1 Dependence of growth rate, $\Gamma \equiv \gamma R_0 \sqrt{\mu_0 \rho_0}/B_0$, on q_a for q_0/q_a=0.5 and b/a=2. Arrows indicate the region of surface tearing mode. Solid line corresponds to $\eta(r) \propto 1/j(r)$ and the dashed line to $\eta = const.$.

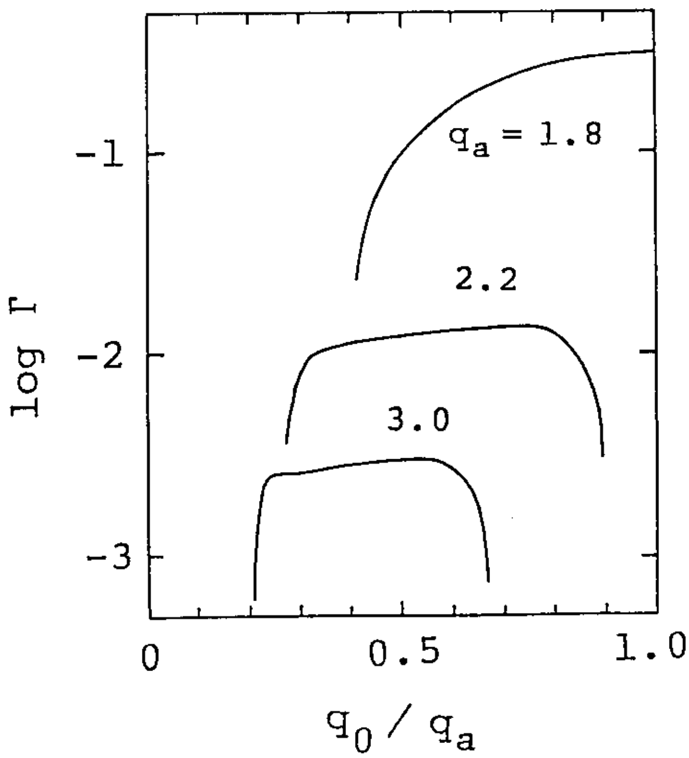

Fig.2 Dependence of Γ on q_0/q_a for b/a=2 and $S_0=10^6$. For q_a=2.2, mode is surface tearing mode.

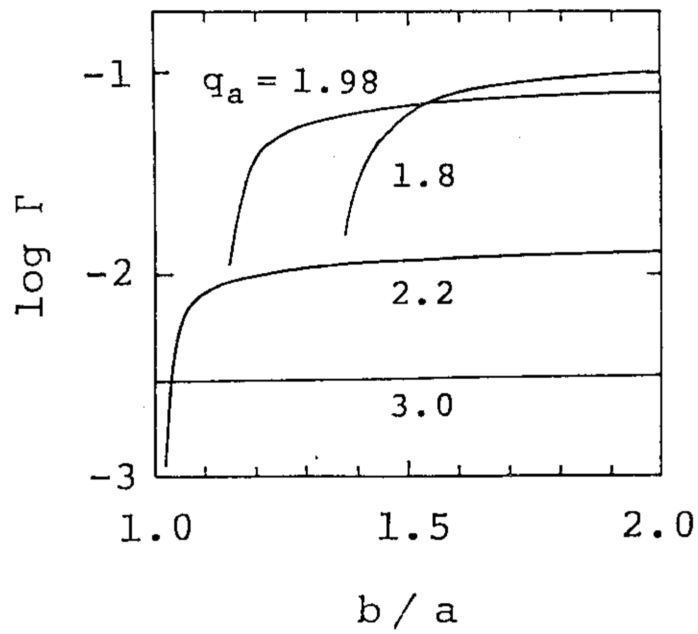

Fig.3 Dependence of Γ on b/a for q_0/q_a=0.5 and $S_0=10^6$. For q_a=2.2, mode is surface tearing mode, which moves plasma surface even when b/a is close to unity.

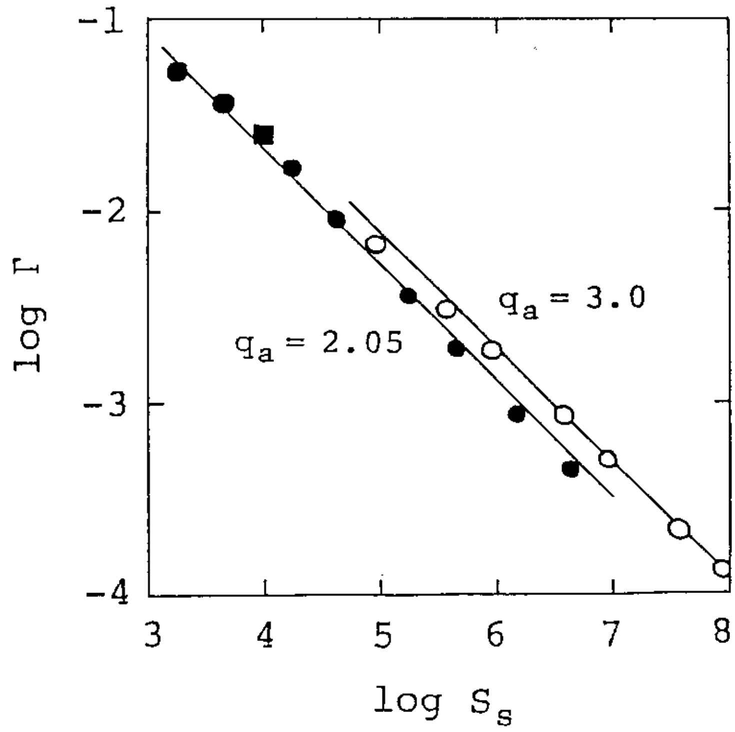

Fig.4 Dependence of Γ on S_s for q_0/q_a=0.5 and b/a=2. Γ is almost proportional to $S_s^{-3/5}$ for both cases of tearing mode (q_a=3) and surface tearing mode (q_a=2.05).

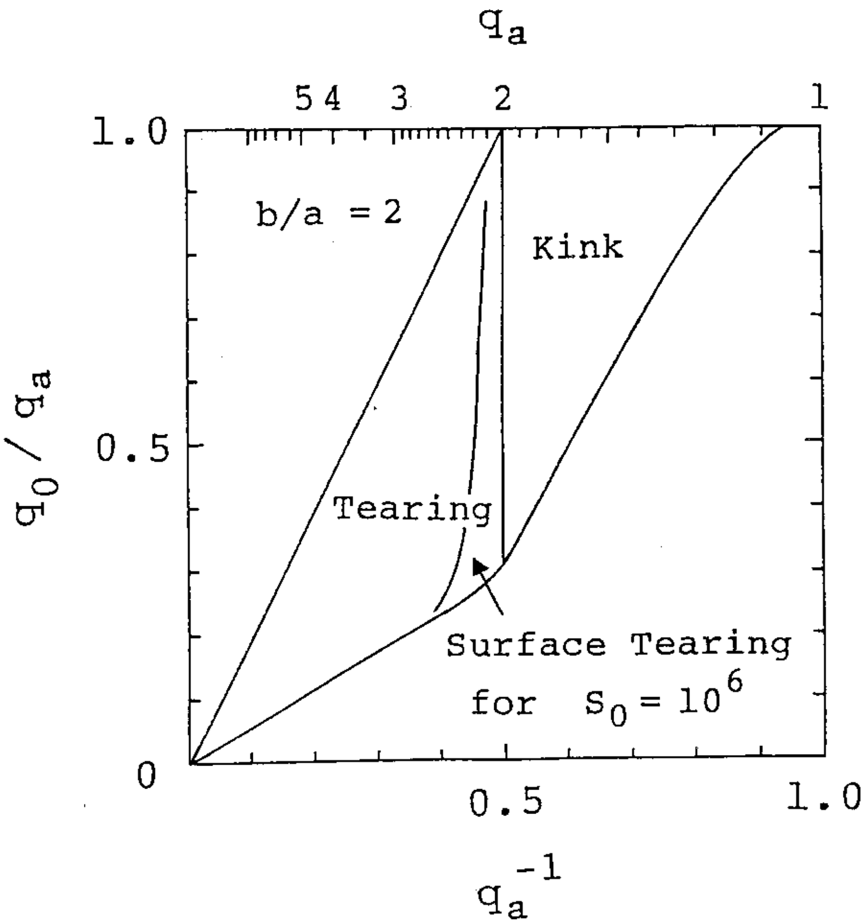

Fig.5 Stability diagram for m/n=2/1 resistive MHD modes in a tokamak with b/a=2. Unstable region against surface tearing mode is shown for $S_0=10^6$. This region becomes wide as resistivity in the region, $r_s \leqq r < a$, increases.