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July 1986

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編集兼発行　日本原子力研究所
印　　刷　㈱高野高速印刷

Derivation of Manipulator Kinematics Based on a Vector Formulation

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(Received July 28, 1986)

This report presents a vector formulation of manipulator kinematics which is different from the homogeneous co-ordinate transformation method now widely used to describe the kinematic relationships between links in a robot manipulator.

The fundamental concept of the present description is to express the motions of individual links as those of spatial vectors in a fixed reference co-ordinate system and to obtain a resultant vector using rotation operators. With this idea applied to a six-link manipulator, the kinematic equations were obtained in the completely same form as those derived by the conventional co-ordinate transformation.

The advantages of this method are :

1) derivation process is easy to understand intuitively.
2) calculation is much simpler than that by co-ordinate transformation.

Keywords : Kinematics, Vector Approach, Rotation Operator, Homogeneous Co-ordinate Transformation

ベクトル定式化に基づくマニプレータ運動学方程式の導出

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（1986年7月28日受理）

本報は，ロボット・マニプレータのリンク間に存在する運動学的関係の記述に対して，今日広く利用されている同次座標変換法と異なるベクトル定式化に基づくマニプレータ運動学方程式を示したものである。この記述の基本となる概念は，個々のリンクの運動を一定の基準座標系における空間ベクトルの運動として表現し，回転演算子を使って合成ベクトルを求めることである。6リンクのマニプレータにこの考え方を適用すると，通常の座標変換により導き出される運動学方程式と全く同一のものが得られた。

本法の利点は，

(1)　導出過程が直感的に理解しやすい，

(2)　計算が座標変換に比べて，より簡単，

の2点にある。

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1. Introductory Remarks

The co-ordinate transformation method is now widely used for the purpose of describing complex relationships between the links of a robot manipulator. The position and orientation of the manipulator end can be determined by execution of successive multiplication of the transformation matrices A from the base of the manipulator to the end, link by link, relating to a fixed reference co-ordinate system.(1),(2) According to this method, a co-ordinate system for each link must be specified in advance. However, its assignment is not unique, but highly dependent on the analyst's intention. The main reason for it is that the essence of the homogeneous transformation method is principally put on relative transformation between the co-ordinate systems.

Viewed in our intuitive understanding of manipulator motion in co-ordinate system, it is generally easier to discuss its motion in a fixed co-ordinate system rather than in a system which moves together with it.

Motivated by this fact, we attempt here a derivation of the kinematic equations from the different angle. The basic concept of the present description is to express the motions of individual links as those of spatial vectors in a fixed reference co-ordinate system and to obtain a resultant vector using rotation operators. Moreover, it is intended to find out the intrinsic nature of kinematic relationships between links in the course of obtaining the equations in question, which may facilitate analytical treatment of the kinematics and dynamics of a robot manipulator.

In the next section, a detailed explanation is given as to how the kinematic equations are obtained.

2. Derivation of Kinematic Equations[3],[4]

The major aim in this section is to introduce a vector formulation for obtaining the kinematic equations of the robot manipulator. The first part of the section is devoted to the notion of the space vector approach based on rotation operators and to its examples applied to simple link mechanisms, while the second part is devoted to an expression of the kinematic relationships for a six-link manipulator. Throughout the process of derivation, the characteristics of this method is demonstrated in comparison with that of the previous one.

To begin with, we have to turn our attention to the point that the movement of individual links can be represented in terms of that of position vectors taken relative to a fixed cartesian space. To show this, we briefly touch upon a fundamental concept of space vectors out of necessity for deriving the kinematic equations.

Consider now a right-handed rectangular coordinate system Σ_0 (x_0, y_0, z_0). By a right-handed system we mean that the coordinate axes are such that if the positive x_0-axis is rotated by 90(deg) to coincide with the y_0-axis, this rotation would advance a right-handed screw along the positive z_0-axis. The unit vectors in the positive directions along the axes from the origin are denoted by i,j and k. If r is a vector with the initial point 0 and the terminal point P(x,y,z), then it is described as :

$$r = xi + yj + zk = (x, y, z)^T \tag{1}$$

where the superscript T means transposition.

The vector r is called the position vector of the point P in a three dimensional space.

Suppose that this space vector r is changed into another vector r^* in terms of a certain transformation (for instance, a rotation), which may be thought of as an "operator" acting on the vector r. This situation may be conveniently written in the form

$$r^* = H\,r \tag{2}$$

where the symbol H is a transformation matrix (usually 3×3 square matrix) of the position vector defined in Cartesian co-ordinates. When expressed in this manner, we can interpret that the vector r is transformed into r^* by the operator H. We shall mention about this in more details.

Referring to Fig.1, let the position vector r rotate by θ in a counter-clockwise direction around any vector a. The resulting vector r^* can be written with the following notation.

$$r^* = \mathrm{Rot}(a,\ \theta)\ r. \tag{3}$$

Rot(a,θ) denotes the general rotation transformation matrix (3×3) corresponding to the operator H.

In this paper, an application of the rotation operator is limited to the special types of vectors from a practical point of view, because normal types of arm of most of existing manipulators usually have rotational, revolute or translational joints with joint axes either perpendicular or parallel to each other. That is to say, these joint axes are considered parallel with any of rectangular reference co-ordinates axes (x_0,y_0 or z_0) connected to the base except for special cases. Thus, a rotation or a revolution around each joint axis is equivalent to that of any of unit vectors i, j and k. This assumption is not only adequate for practical purposes, but also contributive to simplification of calculation of rotation operators.

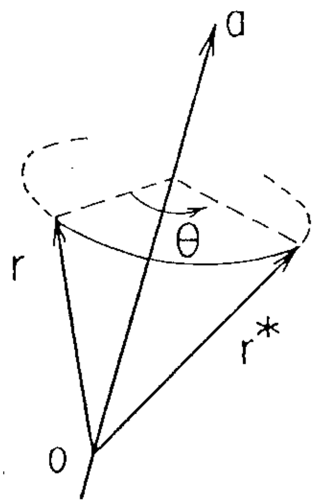


Fig.1 a rotation of a position vector r around any vector a

Now, we shall explain this method by some simple examples.

Figure 2 shows a linkage mechanism with two degrees of freedom. Suppose that the reference co-ordinate system Σ_0 is fixed to the base and having its origin at O. The position and orientation of the respective links are considered with reference to this co-ordinate system. As shown in this figure, the positive direction of revolution or rotation axes coincides with that of unit vectors k and i, respectively. We refer to a directed line segment along each link as a link vector, whose magnitude corresponds to the length of the link.

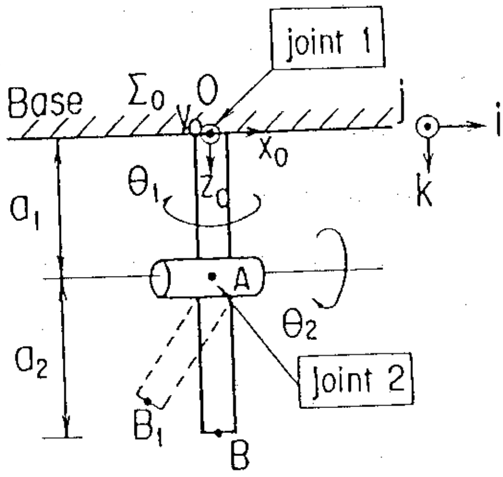


Fig.2 a simple link mechanism with 2-DOF

In order to establish the basic equations, angular displacements are given in turn from a joint 2 near the endpoint B of the open link.

At first, we rotate by θ_2 around the y_0-axis concerning the joint 2 with the consequence that a link vector AB results in a vector AB_1 by the following linear transformation. That is,

$$AB_1 = Rot(i, \theta_2)AB \tag{4}$$

After the end-point B moved to the point B_1 in the space (i.e., vector AB moves to AB_1), a revolution of a vector $OA + AB_1$ ($=OB_1$) by θ_1 around the z_0-axis brings :

$$\begin{aligned} OB_2 &= Rot(k, \theta_1)\ OB_1 \\ &= Rot(k, \theta_1)\ OA + Rot(k, \theta_1)\ AB_1 \\ &= OA + Rot(k, \theta_1)Rot(i, \theta_2)\ AB \end{aligned} \tag{5}$$

where $Rot(k, \theta_1)OA = OA$ is obvious since a vector OA and the unit vector k are in the same direction.

Given the constant link vectors OA and AB in Eq.(5), that is to say,

$$OA = \begin{vmatrix} 0 \\ 0 \\ a_1 \end{vmatrix} \text{ and } AB = \begin{vmatrix} 0 \\ 0 \\ a_2 \end{vmatrix},$$

we obtain the position(x_2 ,y_2 ,z_2) of the link end in the three dimensional space, namely

$$\begin{vmatrix} x_2 \\ y_2 \\ z_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ a_1 \end{vmatrix} + \begin{vmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ a_2 \end{vmatrix} = \begin{vmatrix} a_2 s_1 s_2 \\ -a_2 c_1 s_2 \\ a_1 + a_2 c_2 \end{vmatrix}, \tag{6}$$

Here, the notations s_i, c_i, s_{ij} and c_{ij} mean $\sin\theta_i$, $\cos\theta_i$, $\sin(\theta_i+\theta_j)$, and $\cos(\theta_i+\theta_j)$, respectively.

On the other hand, the orientation tranformation from the base to endpoint is represented by the product of two rotation operators.

(The explanation is given later.)

$$Rot(k,\theta_1)Rot(i,\theta_2). \tag{7}$$

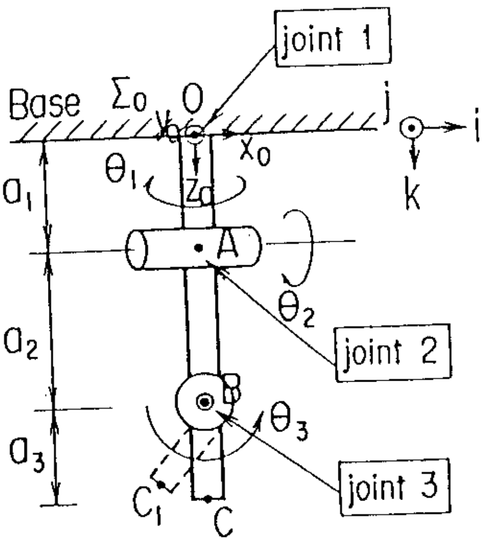


Fig.3 a simple link mechanism with 3-DOF

Next, we proceed to the case of a linkage mechanism with three joints as shown in Fig.3. A rotation (by the angle θ_3 around the y_0-axis, equivalent to the direction of the unit vector j) of the final link BC, produces a space vector BC_1 :

$$BC_1 = Rot(j,\theta_3)\ BC. \tag{8}$$

Then, a rotation and a revolution of the remaining θ_2 and θ_1 bring the following results. namely,

$$AC_1 = AB + BC_1,$$

$$\begin{aligned} AC_2 &= Rot(i,\theta_2)\ AC_1 \\ &= Rot(i,\theta_2)\ AB + Rot(i,\theta_2)Rot(j,\theta_3)\ BC, \end{aligned} \tag{9}$$

$$OC_2 = OA + AC_2,$$

$$\begin{aligned} OC_3 &= Rot(k,\theta_1)\ OC_2 \\ &= Rot(k,\theta_1)\ OA + Rot(k,\theta_1)Rot(i,\theta_2)\ AB + \\ &\quad Rot(k,\theta_1)Rot(i,\theta_2)Rot(j,\theta_3)\ BC. \end{aligned} \tag{10}$$

From the vector OC_3, we can obtain the location at the end of the link in the fixed space Σ_0.

In a similar manner as was described in these two examples , we can apply this inductive method to a six-link manipulator given in Fig.4. The derivation is started with a rotation expression of the sixth joint. By this rotation around the y_0-axis by θ_6 , a link vector EF representing the final link is moved to EF_1 in the space.

$$EF_1 = Rot(j, \theta_6)EF. \tag{11}$$

Proceeding to the 5th rotation axis, we take DF_1 as a vector sum of DE and EF_1 caused by above rotation. A transformation of this vector DF_1 holds that:

$$\begin{aligned} DF_1 &= DE + EF_1, \\ DF_2 &= Rot(i, \theta_5)\, DF_1 \\ &= Rot(i, \theta_5)(\, DE + EF_1 \,) \\ &= Rot(i, \theta_5)\{DE + Rot(j, \theta_6)EF\} \\ &= Rot(i, \theta_5)DE + Rot(i, \theta_5)Rot(j, \theta_6)EF. \end{aligned} \tag{12}$$

That is, an expression was given here which describes the end-point of the manipulator moving from the point F to F_2 in terms of angular displacements of θ_6 and θ_5.

In succession, a revolution (by θ_4 around the z_0-axis) of the joint 4 yields a new vector CF_3, which is established using a revolution transformation of the sum of the 4-th link vector CD and DF_2 obtained above.

$$\begin{aligned} CF_2 &= CD + DF_2 \\ CF_3 &= Rot(k, \theta_4)\, CF_2 = Rot(k, \theta_4)(\, CD + DF_2 \,) \\ &= Rot(k, \theta_4)\, CD + Rot(k, \theta_4)Rot(i, \theta_5)DE + \\ &\quad Rot(k, \theta_4)Rot(i, \theta_5)Rot(j, \theta_6)EF. \end{aligned} \tag{13}$$

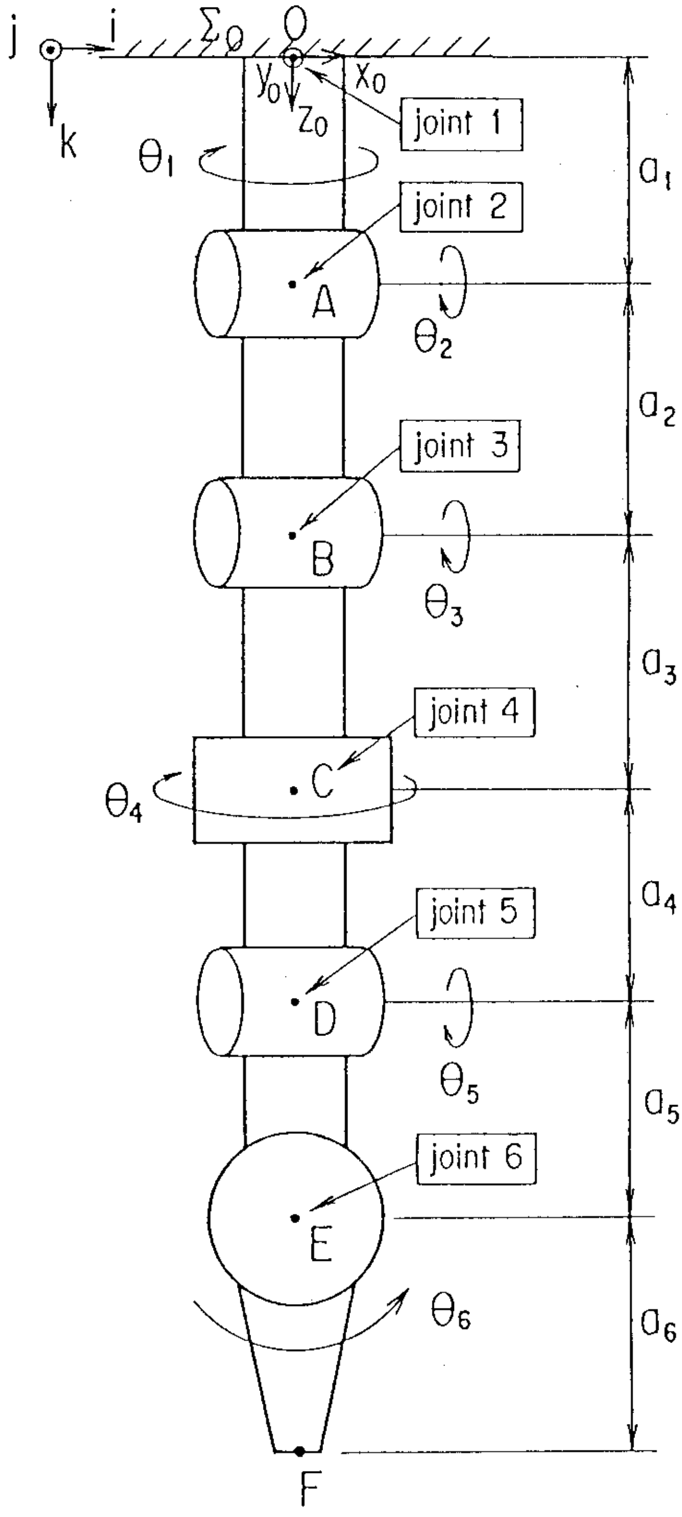


Fig.4 Schematics of a six-link manipulator

Since CD is clearly parallel to the unit vector k, it is possible to describe as $Rot(k,\theta_4)\,CD = CD$. As such, a vector CF_3 offers the location of the hand, given three joint angles θ_6, θ_5 and θ_4.

Furthermore, continuing similar operation for the joint axis located at the point B, we can obtain a new vector BF_4, which is made by a rotation (by the angle θ_3) around the x_0-axis.

$$BF_3 = BC + CF_3,$$

$$\begin{aligned} BF_4 &= Rot(i,\theta_3)\,BF_3 = Rot(i,\theta_3)(\,BC + CF_3\,) \\ &= Rot(i,\theta_3)\,BC + Rot(i,\theta_3)\,CD + \\ &\quad Rot(i,\theta_3)Rot(k,\theta_4)Rot(i,\theta_5)DE + \\ &\quad Rot(i,\theta_3)Rot(k,\theta_4)Rot(i,\theta_5)Rot(j,\theta_6)EF. \end{aligned} \tag{14}$$

Up to here, we gave four angular displacements for deriving the transformation equations. For the remaining two, the position of the hand arising from a displacement of the joint angle θ_2 is first expressed in terms of the following space vector AF_5.

$$AF_4 = AB + BF_4,$$

$$\begin{aligned} AF_5 &= Rot(i,\theta_2)\,AF_4 = Rot(i,\theta_2)(\,AB + BF_4\,) \\ &= Rot(i,\theta_2)\,AB + Rot(i,\theta_2)Rot(i,\theta_3)\,BC + \\ &\quad Rot(i,\theta_2)Rot(i,\theta_3)\,CD + \\ &\quad Rot(i,\theta_2)Rot(i,\theta_3)Rot(k,\theta_4)Rot(i,\theta_5)DE + \\ &\quad Rot(i,\theta_2)Rot(i,\theta_3)Rot(k,\theta_4)Rot(i,\theta_5)Rot(j,\theta_6)EF. \end{aligned} \tag{15}$$

In this case, the product $Rot(i,\theta_2)Rot(i,\theta_3)$ of two rotation operators is simplified by $Rot(i,\theta_2 + \theta_3)$ because the manipulator joint axes NO.2 and NO.3 are parallel to each other as seen in Fig.4.

Finally, by a revolution (by θ_1 around the z_0-axis) of the first link connected to the supporting base, we can obtain the position vector OF_6 describing the co-ordinate of the hand with respect to Σ_0.

$$OF_5 = OA + AF_5,$$

$$\begin{aligned} OF_6 &= Rot(k,\theta_1)\ OF_5 = Rot(k,\theta_1)(\ OA + AF_5\ ) \\ &= Rot(k,\theta_1)\ OA + Rot(k,\theta_1)Rot(i,\theta_2)\ AB + \\ &\quad Rot(k,\theta_1)Rot(i,\theta_2)Rot(i,\theta_3)\ BC + \\ &\quad Rot(k,\theta_1)Rot(i,\theta_2)Rot(i,\theta_3)\ CD + \\ &\quad Rot(k,\theta_1)Rot(i,\theta_2)Rot(i,\theta_3)Rot(k,\theta_4)Rot(i,\theta_5)DE + \\ &\quad Rot(k,\theta_1)Rot(i,\theta_2)Rot(i,\theta_3)Rot(k,\theta_4)Rot(i,\theta_5)Rot(j,\theta_6)EF \\ &= R_1 + R_2 + R_3 + R_4 + R_5 + R_6. \end{aligned} \tag{16}$$

Where

$$R_1 = Rot(k,\theta_1)\ OA = OA, \tag{17}$$

$$R_2 = Rot(k,\theta_1)Rot(i,\theta_2)\ AB, \tag{18}$$

$$\begin{aligned} R_3 &= Rot(k,\theta_1)Rot(i,\theta_2)Rot(i,\theta_3)\ BC \\ &= Rot(k,\theta_1)Rot(i,\theta_2 + \theta_3)\ BC, \end{aligned} \tag{19}$$

$$\begin{aligned} R_4 &= Rot(k,\theta_1)Rot(i,\theta_2)Rot(i,\theta_3)Rot(k,\theta_4)\ CD \\ &= Rot(k,\theta_1)Rot(i,\theta_2 + \theta_3)\ CD, \end{aligned} \tag{20}$$

$$\begin{aligned} R_5 &= Rot(k,\theta_1)Rot(i,\theta_2)Rot(i,\theta_3)Rot(k,\theta_4)Rot(i,\theta_5)DE \\ &= Rot(k,\theta_1)Rot(i,\theta_2 + \theta_3)Rot(k,\theta_4)Rot(i,\theta_5)DE, \end{aligned} \tag{21}$$

$$\begin{aligned} R_6 &= Rot(k,\theta_1)Rot(i,\theta_2)Rot(i,\theta_3)Rot(k,\theta_4)Rot(i,\theta_5)Rot(j,\theta_6)EF \\ &= Rot(k,\theta_1)Rot(i,\theta_2 + \theta_3)Rot(k,\theta_4)Rot(i,\theta_5)Rot(j,\theta_6)EF. \end{aligned} \tag{22}$$

Now, concrete forms of R_i (i=1,2,...,6) will be given.

R_1 is a vector produced by means of a revolution θ_1 of the first link vector OA. (see Fig. 5.a)

$$R_1 = \begin{vmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ a_1 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ a_1 \end{vmatrix} \tag{17'}$$

R_2 is a vector produced by means of a revolution θ_1 and a rotation θ_2 of the second link vector AB. (see Fig. 5.a)

$$R_2 = \begin{vmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ a_2 \end{vmatrix} = \begin{vmatrix} a_2 s_1 s_2 \\ -a_2 c_1 s_2 \\ a_2 c_2 \end{vmatrix} \qquad (18)'$$

R_3 is a vector produced by means of a revolution θ_1 and rotations θ_2 and θ_3 of the third link vector BC. (see Fig. 5.a)

$$R_3 = \begin{vmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ a_3 \end{vmatrix} = \begin{vmatrix} a_3 s_1 s_{23} \\ -a_3 c_1 s_{23} \\ a_3 c_{23} \end{vmatrix} \qquad (19)'$$

R_4 is a vector produced by means of a revolution θ_1 ; rotations θ_2 and θ_3 ; and a revolution θ_4 of the fourth link vector CD. (see Fig. 5.a)

$$R_4 = \begin{vmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{vmatrix} \begin{vmatrix} c_4 & -s_4 & 0 \\ s_4 & c_4 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ a_4 \end{vmatrix} = \begin{vmatrix} a_4 s_1 s_{23} \\ -a_4 c_1 s_{23} \\ a_4 c_{23} \end{vmatrix} \qquad (20)'$$

R_5 is a vector produced by means of a revolution θ_1; rotations θ_2 and θ_3 ; a revolution θ_4 and a rotation θ_5 of the fifth link vector DE.

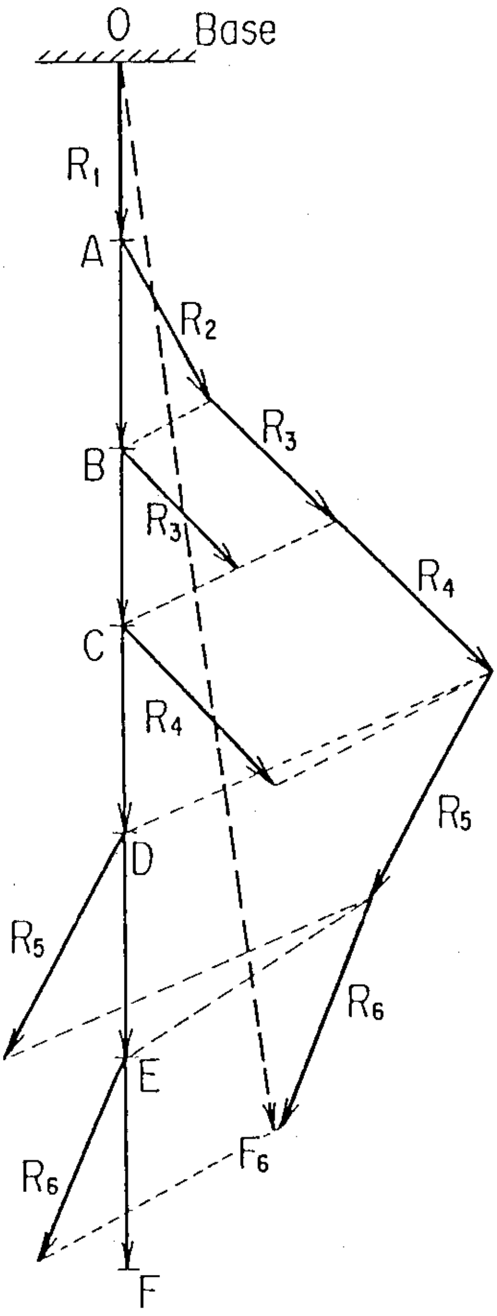
$$R_5 = \begin{vmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{vmatrix} \begin{vmatrix} c_4 & -s_4 & 0 \\ s_4 & c_4 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_5 & -s_5 \\ 0 & s_5 & c_5 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ a_5 \end{vmatrix}$$

$$= \begin{vmatrix} a_5 s_5 (c_1 s_4 + s_1 c_4 c_{23}) + a_5 s_1 s_{23} c_5 \\ a_5 s_5 (s_1 s_4 - c_1 c_4 c_{23}) - a_5 c_1 c_5 s_{23} \\ a_5 (c_5 c_{23} - c_4 s_5 s_{23}) \end{vmatrix} \qquad (21)'$$

R_6 is a vector produced by means of a revolution θ_1 ; rotations θ_2 and θ_3 ; a revolution θ_4 and rotations θ_5 and θ_6 of the final link vector EF.

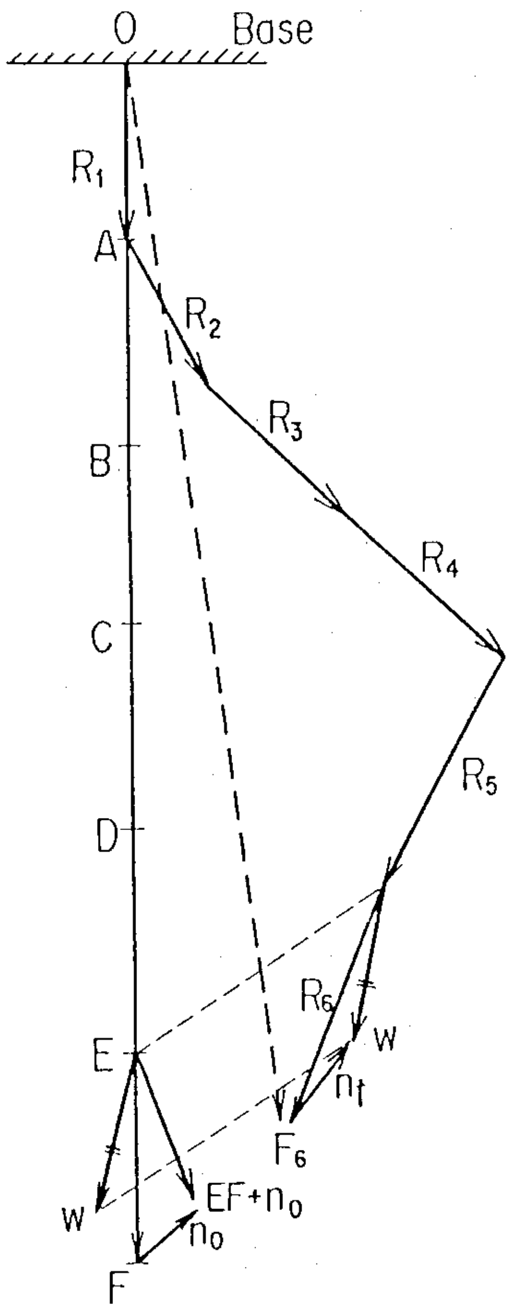
$$R_6 = \begin{vmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{vmatrix} \begin{vmatrix} c_4 & -s_4 & 0 \\ s_4 & c_4 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_5 & -s_5 \\ 0 & s_5 & c_5 \end{vmatrix} \begin{vmatrix} c_6 & 0 & s_6 \\ 0 & 1 & 0 \\ -s_6 & 0 & c_6 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ a_6 \end{vmatrix}$$

$$= \begin{vmatrix} a_6 s_6 (c_1 c_4 - s_1 c_{23} s_4) + a_6 c_6 (s_5 (c_1 s_4 + s_1 c_4 c_{23}) + s_1 c_5 s_{23}) \\ a_6 s_6 (s_1 c_4 + c_1 c_{23} s_4) - a_6 c_6 (s_5 (-s_1 s_4 + c_1 c_4 c_{23}) + c_1 c_5 s_{23}) \\ a_6 (s_4 s_{23} s_6 + c_5 c_6 c_{23} - c_4 c_6 s_5 s_{23}) \end{vmatrix} \qquad (22)'$$



$$\overrightarrow{OF_6} = \sum_{i=1}^{6} R_i$$

Fig.5.a A resultant vector OF_6 produced as the sum of six link vectors



$$W = Rot(k_1, \Theta_1) Rot(i, \Theta_2 + \Theta_3) Rot(k, \Theta_4) \times Rot(i, \Theta_5) Rot(j, \Theta_6) \{EF + n_0\}$$

Fig.5.b Orientation vectors at the initial and terminal points

Hence, OF_6 is expressed as follows.

$$OF_6 = R_1 + R_2 + R_3 + R_4 + R_5 + R_6$$

$$= \begin{vmatrix} a_6s_6(c_1c_4-s_1c_{23}s_4)+a_6c_6(s_5(c_1s_4+s_1c_4c_{23})+s_1c_5s_{23})+ \\ a_5s_5(s_1c_{23}c_4+c_1s_4)+a_5c_5s_1s_{23}+(a_3+a_4)s_1s_{23}+a_2s_1s_2 \\ a_6s_6(s_1c_4+c_1c_{23}s_4)-a_6c_6(s_5(-s_1s_4+c_1c_4c_{23})+c_1c_5s_{23})+ \\ a_5s_5(s_1s_4-c_1c_4c_{23})-a_5c_1c_5s_{23}-(a_3+a_4)c_1s_{23}-a_2c_1s_2 \\ a_6(s_4s_{23}s_6+c_5c_6c_{23}-c_4c_6s_5s_{23})+a_5(c_5c_{23}-c_4s_5s_{23})+ \\ (a_3+a_4)c_{23}+a_2c_2+a_1 \end{vmatrix} \quad (16)'$$

In this way, we have obtained a resultant vector OF_6, issuing from the origin O in the reference co-ordinate system Σ_0, as the sum of six spatial vectors R_1, $R_2, \ldots, R_6$, each of which indicates the position made by rotational or revolute motions of all joints located before its own link.

Now, we turn to a determination of the orientation at the hand (i.e., at the end-point of the manipulator). Let the orientation be n_0 initially at the point F. When the initial point F is moved to F_6 by the angular displacements of each joint, we define its orientation (at the point F_6) with a vector n_t. Referring to Fig.5.b, the relation including the position vector OF_6 and orientation vector n_t holds :

$$OF_6 + n_t = R_1 + R_2 + R_3 + R_4 + R_5 + \mathrm{Rot}(k,\theta_1)\mathrm{Rot}(i,\theta_2+\theta_3)\mathrm{Rot}(k,\theta_4)\mathrm{Rot}(i,\theta_5)\mathrm{Rot}(j,\theta_6)(EF + n_0) \quad (23)$$

Using Eqs.(16) through (23), we obtain the following description with respect to the orientation :

$$n_t = \mathrm{Rot}(k,\theta_1)\mathrm{Rot}(i,\theta_2+\theta_3)\mathrm{Rot}(k,\theta_4)\mathrm{Rot}(i,\theta_5)\mathrm{Rot}(j,\theta_6)\, n_0$$

$$= \begin{vmatrix} n_x & o_x & a_x \\ n_y & o_y & a_z \\ n_z & o_z & a_z \end{vmatrix} n_0 \quad (24)$$

where

$$
\begin{aligned}
n_x &= c_6(c_1c_4-s_1s_4c_{23})-s_6(s_4s_5c_1+s_1s_5c_4c_{23}+s_1s_{23}c_5) \\
n_y &= c_6(s_1c_4+c_1c_{23}s_4)+s_6(c_1c_5s_{23}-s_1s_4s_5+c_1c_4c_{23}s_5) \\
n_z &= s_4s_{23}c_6-s_6(c_5c_{23}-c_4s_5s_{23}) \\
o_x &= -c_5(c_1s_4+s_1c_4c_{23})+s_1s_5s_{23} \\
o_y &= c_5(-s_1s_4+c_1c_4c_{23})-c_1s_5s_{23} \\
o_z &= c_4c_5s_{23}+s_5c_{23} \\
a_x &= s_6(c_1c_4-s_1s_4c_{23})+c_6(s_4s_5c_1+s_1s_5c_4c_{23}+s_1s_{23}c_5) \\
a_y &= s_6(s_1c_4+c_1c_{23}s_4)-c_6(c_1c_5s_{23}-s_1s_4s_5+c_1c_4c_{23}s_5) \\
a_z &= s_4s_{23}s_6+c_6(c_5c_{23}-c_4s_5s_{23})
\end{aligned}
\tag{25}
$$

As a result, the product of the above five rotation operators Rot(k,θ_1), Rot($i,\theta_2+\theta_3$), Rot(k,θ_4)Rot(i,θ_5)and Rot(j,θ_6) expresses the orientation at the end-point with respet to the reference co-ordinates.
For instance, suppose that n_0 is the unit vector k with a description of $n_0=(0,0,1)^T$. Substituting it into Eq.(24), we obtain n_t in the form of $(a_x,a_y,a_z)^T$, which means the direction cosine of the hand related to the z_0-axis. Likewise, $n_t=(o_x,o_y,o_z)^T$ and $(n_x,n_y,n_z)^T$ are indicative of the direction cosines relating to y_0 and x_0 axes, respectively.

In closing this section, the results derived by means of the co-ordinate transformation method are given to compare with those by the present method. Referring to Fig.6 with the link co-ordinate systems, the location and orientation at the end-point of the manipulator with respect to the base co-ordinate are represented by the components of matrix T_6.

$$
\begin{aligned}
T_6 &= A_1A_2A_3A_4A_5A_6 \\
&= \begin{vmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ 0 & 0 & 0 & 1 \end{vmatrix}
\end{aligned}
\tag{26}
$$

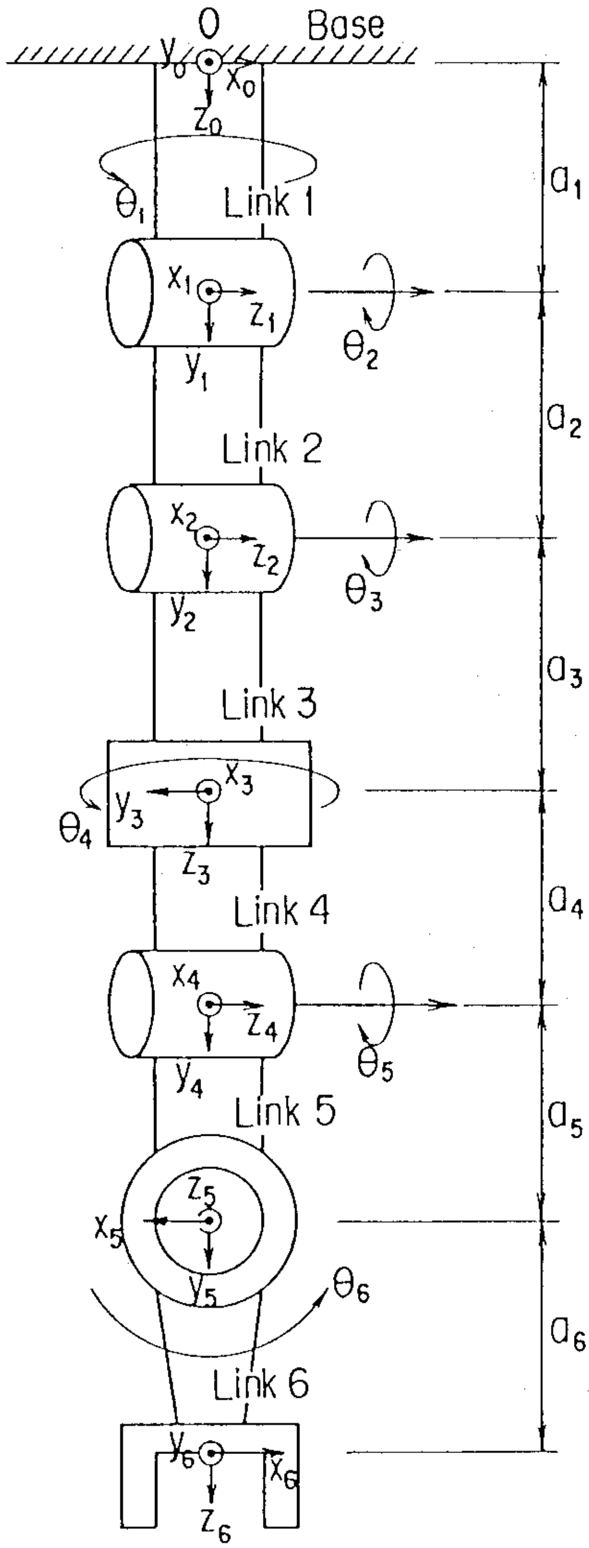


Fig.6 Link co-ordinate systems specified in the homogeneous co-ordinate method

where,

$$A_1 = Rot(z_0, \theta_1+\pi/2)Trans(0,0,a_1)Rot(x_0, \pi/2)$$
$$A_2 = Rot(z_1, \theta_2)Trans(0,a_2,0)$$
$$A_3 = Rot(z_2, \theta_3)Trans(0,a_3,0)Rot(x_2, -\pi/2)$$
$$A_{4'} = Rot(z_3, \theta_4)Trans(0,0,a_4)Rot(x_3, \pi/2) \quad (27)$$
$$A_5 = Rot(z_4, \theta_5)Trans(0,a_5,0)Rot(y_4, \pi/2)$$
$$A_6 = Rot(z_5, \theta_6)Trans(0,a_6,0)Rot(z_5, \pi)Rot(x_5, \pi/2)$$

$$t_{11} = -c_6(s_1s_4c_{23}-c_1c_4)-s_6(s_1s_5c_{23}c_4+c_1s_4s_5+s_1s_{23}c_5)$$
$$t_{21} = c_6(c_1c_{23}s_4+s_1c_4)+s_6(c_1c_{23}c_4s_5-s_1s_4s_5+c_1c_5s_{23})$$
$$t_{31} = s_4s_{23}c_6-s_6(c_5c_{23}-c_4s_{23}s_5)$$
$$t_{12} = -c_5(s_1c_{23}c_4+c_1s_4)+s_1s_{23}s_5$$
$$t_{22} = c_5(c_1c_{23}c_4-s_1s_4)-c_1s_{23}s_5$$
$$t_{32} = c_4c_5s_{23}+c_{23}s_5$$
$$t_{13} = -s_6(s_1s_4c_{23}-c_1c_4)+c_6(s_1s_5c_{23}c_4+c_1s_4s_5+s_1s_{23}c_5) \quad (28)$$
$$t_{23} = s_6(c_1c_{23}s_4+s_1c_4)-c_6(c_1c_{23}c_4s_5-s_1s_4s_5+c_1c_5s_{23})$$
$$t_{33} = s_4s_{23}s_6+c_6(c_5c_{23}-c_4s_{23}s_5)$$
$$t_{14} = a_6s_6(c_1c_4-s_1c_{23}s_4)+a_6c_6(s_5(c_1s_4+s_1c_4c_{23})+s_1c_5s_{23})+ a_5s_5(s_1c_{23}c_4+c_1s_4)+a_5c_5s_1s_{23}+(a_3+a_4)s_1s_{23}+a_2s_1s_2$$
$$t_{24} = a_6s_6(s_1c_4+c_1c_{23}s_4)-a_6c_6(s_5(-s_1s_4+c_1c_4c_{23})+c_1c_5s_{23})+ a_5s_5(s_1s_4-c_1c_4c_{23})-a_5c_1c_5s_{23}-(a_3+a_4)c_1s_{23}-a_2c_1s_2$$
$$t_{34} = a_6(s_4s_{23}s_6+c_5c_6c_{23}-c_4c_6s_5s_{23})+a_5(c_5c_{23}-c_4s_5s_{23})+ (a_3+a_4)c_{23}+a_2c_2+a_1$$

Comparisons made between Eqs.(16) through (25) and (28) resulted in the complete agreement of the kinematic representation. Accordingly, the present approach is also useful to derive the kinematic equations of a manipulator.

3. Concluding Remarks

In place of the traditional co-ordinate transformation method, a formulation based on the space vectors was introduced here to derive the kinematic relationships between links. The present method, when applied to an articulated manipulator kinematics, is effective only when rotation or revolution axes of individual joints are either perpendicular or parallel to any of the reference coordinate axes x_0, y_0 and z_0. Of course, it is possible to establish an expression for an arbitrary arm configuration at its home position, but the computation becomes more or less cumbersome since the general rotation transformation around any vector is needed.

The basic features of the present method are :

(1) Derivation process is easy to understand intuitively.

(2) An aspect of the motion of each link made by angular displacements can be illustrated clearly.

(3) The calculation is comparatively simple. (for orthogonal types of joint structures)

(4) No assignment of the co-ordinate system on each link is required.

Acknowledgement

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