JAERI-M
87-066

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May 1987

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編集兼発行　日本原子力研究所
印　　刷　㈱原子力資料サービス

REACTIVITY FOR DT PLASMA WITH BEAM-INDUCED TAIL IN MAGNETIC CONFINEMENT SYSTEM

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(Received April 2, 1987)

The reactivity for a DT plasma in the magnetic confinement system is presented by useful and simple equations. The reactivity for a DT Maxwellian plasma is given as a function of the ion temperature, and that enhanced by the beam-induced deuteron tail is shown as a function of the ion and electron temperatures, the injection energy, and the beam power density. By using these simple expressions, the fusion power multiplication in a beam-heated plasma is studied on the basis of a local analysis. The value of $n_e \tau_E$ to achieve Q = 1 (n_e: electron density, τ_E: gross energy confinement time, Q: fusion power multiplication factor) becomes smaller by $2.7/T^{1.1}$(keV) than that in a Maxwellian plasma without tail component for the plasma temperature of 5keV < T < 8keV.

Keywords; DT Fusion Reactivity, Magnetic Confinement, Beam-Induced Tail, Beam-Heated Plasma, Fusion Power Multiplication

磁気閉じ込め装置中のビーム誘起テールを持つDTプラズマの核反応特性

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（1987年4月2日受理）

磁気閉じ込め装置中のDTプラズマの核反応特性を便利で簡単な式によって表わした。マクスウェル分布するDTプラズマの核反応特性をイオン温度の関数として表わした。二重水素のビーム誘起テールによって増大した核反応特性を，電子温度，イオン温度，ビーム入射エネルギー，ビーム入力密度の関数として表わした。これらの簡単な表式を用いて，ビーム加熱プラズマの局所的核融合増倍について調べた。Q＝1を達成するための$n_e\tau_E$値は（n_e：電子密度，τ_E：全エネルギー閉じ込め時間，Q：核融合出力増倍係数）テール成分のないマクスウェル分布プラズマの$n_e\tau_E$値に比べて，プラズマ温度が5 keV＜T＜8 keVのとき，2.7／$T^{1.1}$（keV）だけ小さくなる。

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1. INTRODUCTION

The purpose of this paper is to present useful and simple equations to express the reactivity for a magnetically confined DT plasma. An equation for the cross section of the DT nuclear reaction is introduced in this section. A method to calculate the reactivity is described in Section 2. In Section 3, the reactivity for a DT Maxwellian plasma is expressed by some simple functions of the ion temperature. The reactivity enhanced by the beam-induced deuteron tail is shown in Section 4. By using these results, the fusion power multiplication is studied in Section 5, and the summary is made in Section 6.

In order to realize the controlled thermonuclear fusion, the DT nuclear reaction

$$\mathrm{D} + \mathrm{T} \rightarrow \underset{(3.5\mathrm{MeV})}{\mathrm{H}_e^4} + \underset{(14.1\mathrm{MeV})}{\mathrm{n}}$$

will be utilized at first because of its large reaction cross section. The value of the cross section, σ, is represented as a function of the relative particle energy. The simple equation for σ at the low energy values has been given theoretically as [1] ,

$$\sigma = \frac{2.19\times10^{7}}{E}\exp\left(\frac{-1399}{\sqrt{E}}\right), \tag{1}$$

where σ is in unit of barn (1 barn = $10^{-28}\mathrm{m}^2$) and the deuteron energy E is in eV. The result of eq.(1) is shown in Fig.1 by a dashed curve. The discrepancy from experimental data, however, is not small even in the energy range of 10-100 keV. By considering the form of eq.(1) basically, the following equation was obtained from the nonlinear least squares fit to the measurement values [2] ;

$$\sigma = \left[\frac{A_2}{1+(A_3E-A_4)^2}+A_5\right] \Big/ E\left[\exp\left(\frac{A_1}{\sqrt{E}}\right)-1\right] \tag{2}$$

$$
\begin{aligned}
A_1 &= 1453 \pm 14 & \sqrt{eV} \\
A_2 &= (5.02 \pm 0.26) \times 10^7 & eV \cdot barn \\
A_3 &= (1.368 \pm 0.045) \times 10^{-5} & /eV \\
A_4 &= 1.076 \pm 0.055 & \\
A_5 &= (4.09 \pm 0.15) \times 10^5 & eV \cdot barn \ .
\end{aligned}
$$

The result using the central value of each coefficient is shown in Fig.1 by a thick curve. Another fitting curve (Fig.2.3 in Ref. [1]) is also shown by a thin curve. Though we cannot judge which curve well fits the measurement values, we use eq.(2) with the central value of each coefficient as a basic cross section in this paper.

2. CALCULATION OF REACTIVITY

The reactivity, $S \equiv \langle \sigma v \rangle$, in a DT plasma is determined by the velocity distribution functions of deuterons, $f_D(\vec{v})$, and of tritons, $f_T(\vec{w})$;

$$S = \iint d\vec{v}\, d\vec{w}\, \sigma(E_r) v_r\, f_D(v)\, f_T(w) \ , \tag{3}$$

where $v_r \equiv |\vec{v} - \vec{w}|$ is the relative speed, E_r is the deuteron energy at v_r ($E_r = m_D v_r^2/2$ with the deuteron mass m_D), and $f_D(\vec{v})$ and $f_T(\vec{w})$ are normalized as

$$\int f_D(\vec{v}) d\vec{v} = \int f_T(\vec{w}) d\vec{w} = 1 \ .$$

The relative speed is expressed by using the angle, β, between $\vec{v}$ and $\vec{w}$;

$$v_r^2 = v^2 - 2vw \cos\beta + w^2 \ , \text{ i.e., } v_r\, dv_r = vw \sin\beta\, d\beta \ .$$

When the velocity distribution of tritons is isotropic, i.e.,

$$A_1 = 1453 \pm 14 \quad \sqrt{eV}$$
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$f_T(\vec{w}) = f_T(w)$, ($w \equiv |\vec{w}|$), we can obtain a function, $G(v)$, depending only on $v \equiv |\vec{v}|$, which we call 'σv-function' averaged by the isotropic triton distribution [3] ,

$$G(v) \equiv \int d\vec{w}\ f_T(w)\ \sigma(E_r)\ v_r$$

$$= 2\pi \int_0^\infty d\vec{w}\ w^2\ f_T(w) \int_0^\pi d\beta\ \sin\beta\ \sigma(E_r)\ v_r$$

$$= \frac{2\pi}{v} \int_0^\infty d\vec{w}\ w\ f_T(w) \int_{|v-w|}^{v+w} dv_r\ v_r^2\ \sigma(E_r)\ . \tag{4}$$

It must be noted that the 'σv-function' is approximated as $G(v) \sim \sigma(E)v$ for $v \gg w_{th}$ ($E = m_D v^2/2$ is the deuteron energy and w_{th} is the triton thermal speed), and the peak position of $G(v)$ is almost the same as that of $\sigma(E)v$. The maximum value of σv for DT reaction is $(\sigma v)_{max} = 1.67\times10^{-21}$ m^3/sec at $v = v_*$ (the deuteron energy, $E_* = m_D v_*^2/2$, is 127keV). The reactivity, S, is then calculated by using this 'σv-function', $G(v)$;

$$S = \int d\vec{v}\ f_D(\vec{v})\ G(v) = 4\pi \int_0^\infty dv\ v^2\ F_{D0}(v)\ G(v)\ , \tag{5}$$

where $F_{D0}(v)$ is the distribution function averaged over the pitch angle, θ,

$$F_{D0}(v) = \frac{1}{2} \int_0^\pi f_D(v,\theta)\ \sin\theta\ d\theta\ .$$

Note that only the angle-averaged distribution function, $F_{D0}(v)$, is needed for calculating the reactivity when $f_T(\vec{w})$ is isotropic.

3. REACTIVITY FOR MAXWELLIAN DT PLASMA

In a thermonuclear fusion reactor of the closed magnetic confinement system, such as a tokamak, the velocity distribution function of the

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3. REACTIVITY FOR MAXWELLIAN DT PLASMA

In a thermonuclear fusion reactor of the closed magnetic confinement system, such as a tokamak, the velocity distribution function of the j-species fuel ion, f_j, is assumed to be almost the Maxwellian, f_j^M;

$$f_j^M(v) = \left(\frac{m_j}{2\pi T_j}\right)^{3/2} \exp\left(\frac{-m_j v^2}{2T_j}\right) , \tag{6}$$

where m_j and T_j are the mass and the temperature, respectively. In the following calculation, the temperatures of deuterons and tritons are assumed to be the same each other, $T_D = T_T = T_i$. The thermonuclear fusion reactivity for a Maxwellian DT plasma, $S_M \equiv \langle \sigma v \rangle_M$, is obtained by inserting $f_j = f_j^M$ to eq.(3). The values of reactivity, S_{M0}, for various values of the temperature, T_i, are numerically calculated by using eqs.(2),(4), and (5). The result is tabulated in Table 1 and shown in Fig.2 by a thick curve, where S_M is in unit of m^3/sec and T_i is in keV. A dashed curve in the figure corresponds to eq.(7) described later, and a thin curve is reproduced from Fig.2.4 in Ref. [1] .

Table 1 Reactivity for Maxwellian DT Plasma

T_i (keV)	S_{M0} (m^3/s)	$S_{M,TY}$ (m^3/s)	T_i (keV)	S_{M0} (m^3/s)	$S_{M,TY}$ (m^3/s)
1	5.46E-27	5.49E-27	20	4.24E-22	4.24E-22
2	2.62E-25	2.60E-25	30	6.65E-22	6.66E-22
3	1.71E-24	1.71E-24	40	8.03E-22	8.00E-22
4	5.57E-24	5.59E-24	50	8.70E-22	8.64E-22
5	1.29E-23	1.29E-23	60	8.98E-22	8.89E-22
6	2.42E-23	2.43E-23	70	9.01E-22	8.93E-22
7	3.97E-23	3.98E-23	80	8.90E-22	8.86E-22
8	5.93E-23	5.94E-23	90	8.72E-22	8.71E-22
9	8.25E-23	8.25E-23	100	8.49E-22	8.53E-22
10	1.09E-22	1.09E-22	150	7.28E-22	7.49E-22
15	2.65E-22	2.65E-22	200	6.28E-22	6.55E-22

Because of the long computation time to integrate eqs.(4) and (5), a simple function of T_i is desirable to give the value of S_M. The dependence of S_M on T_i was analytically found by Gamow. The cross section of eq.(1), $\sigma \sim \exp(-v_\sigma/v)/v^2$, is used, and the double integrations in eq.(4) are replaced by the single integration for simplicity;

$$S_{M,G} \sim \frac{1}{v_{th}^3} \int \exp\left(-\frac{v^2}{2v_{th}^2} - \frac{v_\sigma}{v}\right) v dv \sim \frac{1}{v_{th}^3} \exp\left(-\frac{v_\varepsilon^2}{2v_{th}^2} - \frac{v_\sigma}{v_\varepsilon}\right) v_\varepsilon v_{th} ,$$

where the thermal speed, $v_{th} \equiv \sqrt{T_i/m_D}$, is considered much smaller than the speed for the maximum σ value, $v_\sigma/2$, and the exp term takes the maximum value at $v = v_\varepsilon \equiv (v_\sigma v_t^2)^{1/3}$. Finally one obtains the following equation;

$$S_{M,G} = 3.7 \times 10^{-18}\ T_i^{-2/3} \exp(-20/T_i^{1/3}) , \tag{7}$$

where S_M is in unit of m^3/sec and T_i is in keV. The result of eq.(7) is shown in Fig.2 by a dashed curve. The value of $S_{M,G}$ is about 30% smaller than that of S_{M0} in the region, 5keV < T_i < 20 keV. In order to reduce the error in S_M, $\delta \equiv (S_M/S_{M0}-1) \times 100$ (%), various corrections of the form, $S_M = S_{M,G} \times g$, have been considered. One of the most reliable equation was proposed by Kozlov [4] ;

$$S_{M,K} = 3.37 \times 10^{-18}\ T_i^{-2/3} \exp(-20.4/T_i^{1/3}) \times g$$

$$g = (1 + 0.203 T_i^{0.75}) \Big/ \sqrt{1 + 8.88 \times 10^{-5} T_i^{3.25}} , \tag{8}$$

where the coefficients are given by Hively [5] so that δ becomes small in the wide range of T_i ; δ is within ±10% for 1keV < T < 200keV. Another type of fitting equations have been proposed by Hively [5] :

$$S_{M,5} = \exp(a_1/T_i^r + a_2 + a_3 T_i + a_4 T_i^2 + a_5 T_i^3 + a_6 T_i^4) \tag{9}$$

with $a_1 = -21.38$; $a_2 = -34.41$, $a_3 = -7.101\times10^{-2}$, $a_4 = 1.938\times10^{-4}$, $a_5 = 4.925\times10^{-6}$, $a_6 = -3.984\times10^{-8}$, and r=0.2935.

$$S_{M,5}^{*} = \exp(a_1/T_i^{\,r}+a_2+a_3T_i+a_4T_i^2+a_5T_i^3) \tag{10}$$

with $a_1 = -22.71$, $a_2 = -33.05$, $a_3 = -9.393\times10^{-2}$, $a_4 = 7.994\times10^{-4}$, $a_5 = -3.144\times10^{-6}$, and $r = 0.2747$.

$$S_{M,6} = \exp(a_1/T_i^{\,r}+a_2+a_3T_i^{\,s}) \tag{11}$$

with $a_1 = -71.29$, $a_2 = 52.72$, $a_3 = -37.30$, $r = 0.1396$, and $s = 0.1007$. The errors in the above equations are tabulated in Table 2. One finds in the region, 1keV < T_i < 70keV, that $|\delta|$ in $S_{M,5}$ and $S_{M,5}^{*}$ are less than 2% and $|\delta|$ in $S_{M,6}$ is less than 5%. By taking notice of the relation between $\ln S_M$ and $\ln T_i$ instead of the relation between $\ln S_M$ and T_i, Brunelli found a simple equation [6] ;

$$S_{M,B} = 9.0\times10^{-22}\exp(-0.476\,|\ln(T_i/69)|^{2.25}) \quad , \tag{12}$$

and Hively a little modified this equation so that the error becomes small [7] ;

$$S_{M,H} = 9.131\times10^{-22}\exp(-0.5716\,|\ln(T_i/64.22)|^{2.137}) \quad . \tag{13}$$

The error, $|\delta|$, in $S_{M,H}$ is less than 4% for 1keV < T_i < 100keV. We propose, in the present paper, a new correction of Gamow equation which well fits to S_{M0} in the wide range of T_i, 1keV < T_i < 200keV;

$$S_{M,TY} = S_{M,G}/h \qquad h = \frac{T_i}{37} + \frac{5.45}{3+T_i\{1+(T_i/37.5)^{2.8}\}} , \tag{14}$$

where $S_{M,G}$ is the same one as eq.(7). The result of this equation, $S_{M,TY}$, is tabulated in Table 1 alongside the numerical integration result, $S_{M,0}$. The error, δ, is within ±1% for 1keV < T_i < 100keV.

Table 2 Errors $\delta = (S_M/S_{M0}-1)\times 100$ (%)

T_i (keV)	$S_{M,G}$	$S_{M,K}$	$S_{M,5}$	$S_{M,5}^*$	$S_{M,6}$	$S_{M,B}$	$S_{M,H}$	$S_{M,TY}$
1	39.7	2.6	0.6	1.4	-0.3	-20.4	-1.2	0.5
2	13.6	1.0	0.0	-1.1	0.4	-4.4	-0.1	-0.7
3	-1.2	-0.4	1.2	0.1	2.3	3.9	2.6	-0.1
4	-11.0	-1.3	1.5	0.8	2.5	7.3	3.4	0.4
5	-18.3	-2.0	0.9	0.4	1.2	7.4	2.4	0.2
6	-23.2	-1.6	0.6	0.3	0.1	6.9	1.5	0.4
7	-26.7	-0.9	0.2	-0.1	-1.1	5.7	0.4	0.4
8	-29.2	0.0	-0.3	-0.5	-2.3	4.3	-0.8	0.2
9	-30.8	1.3	-0.5	-0.8	-3.2	3.1	-1.7	0.0
10	-32.0	2.4	-0.8	-1.2	-4.0	1.8	-2.6	-0.3
15	-31.0	8.0	0.3	-0.6	-4.4	-0.9	-3.5	0.0
20	-25.3	8.4	1.3	0.0	-3.2	-1.7	-2.7	0.1
30	-7.7	3.6	1.3	0.2	-0.2	-1.3	-0.2	0.1
40	13.7	-0.4	0.0	-0.5	1.1	-0.8	1.3	-0.4
50	37.5	-2.0	-0.3	-0.7	0.8	-0.3	1.9	-0.7
60	62.5	-2.6	0.2	-0.6	-1.0	-0.3	1.5	-1.0
70	88.7	-2.3	1.0	-0.3	-3.7	-0.1	1.0	-0.9
80	115.9	-1.5	-0.3	-0.7	-7.0	0.5	0.3	-0.5
90	143.6	-0.6	-7.3	-3.2	-10.7	0.8	-1.0	-0.1
100	172.0	0.5	-22.4	-8.6	-14.6	0.7	-2.7	0.5
150	317.4	5.6	-99.4	-76.1	-33.8	-5.6	-16.1	2.9
200	463.5	9.4	-100.3	-99.8	-49.7	-17.1	-31.4	4.2

In Table 2, we tabulate the errors in the above fitting equations (7)-(14) for comparison. Since eq.(2) for the cross section contains the ambiguity of a few %, the error in a fitting equation, $\delta \sim S_M/S_{M0}-1$, can be allowable provided that $|\delta| < 5\%$. Therefore, eqs.(9), (10), (11), and (13) given by Hively, and eq.(14) proposed by us are practical to obtain the value of S_M in the temperature range, $1\text{keV} < T_i < 70\text{keV}$.

4. REACTIVITY ENHANCED BY BEAM-INDUCED DEUTERON TAIL

When a plasma is intensely heated by the neutral beam injection and the ICRF wave, the velocity distribution function of ions tends to have a high-energy tail. The reactivity can be enhanced by this tail component which spreads to the large σv region in the energy space [8,9] . In this section, we present a simple expression for the reactivity enhanced by the beam-induced ion tail.

The deuteron neutral beam is injected into a DT plasma (deuteron density, n_D, and triton density, n_T), whose injection energy and deposited power density are $E_b = m_D v_b^2/2$ and P_b, respectively. The distribution function of deuterons is assumed to consist of the Maxwellian bulk with the temperature, T_i, and the beam-induced tail component of which angle-averaged distribution function is described as

$$F_0^{\text{tail}}(v) = \frac{P_b \tau_s}{4\pi n_D E_b h_s} \frac{1}{v^3 + v_c^3/h_s} . \tag{15}$$

The energy confinement time of the tail component, τ_E^{tail}, is assumed to be independent of v and the factor, h_s, denotes $h_s = 1 + \tau_s/2\tau_E^{\text{tail}}$. The slowing-down time of a deuteron by electrons, τ_s, is represented as

$$\tau_s = 2.0\times 10^{19} \frac{T_e^{3/2}}{n_e \ \ln\Lambda} \times \frac{A_D}{Z_D^2} , \tag{16}$$

where τ_s is in unit of sec, the electron temperature, T_e, is in keV, the

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The deuteron neutral beam is injected into a DT plasma (deuteron density, n_D, and triton density, n_T), whose injection energy and deposited power density are $E_b = m_D v_b^2/2$ and P_b, respectively. The distribution function of deuterons is assumed to consist of the Maxwellian bulk with the temperature, T_i, and the beam-induced tail component of which angle-averaged distribution function is described as

$$F_0^{\mathrm{tail}}(v) = \frac{P_b \tau_s}{4\pi n_D E_b h_s} \frac{1}{v^3 + v_c^3/h_s} . \tag{15}$$

The energy confinement time of the tail component, τ_E^{tail}, is assumed to be independent of v and the factor, h_s, denotes $h_s = 1 + \tau_s/2\tau_E^{\mathrm{tail}}$. The slowing-down time of a deuteron by electrons, τ_s, is represented as

$$\tau_s = 2.0\times 10^{19} \frac{T_e^{3/2}}{n_e \ \ln\Lambda} \times \frac{A_D}{Z_D^2} , \tag{16}$$

where τ_s is in unit of sec, the electron temperature, T_e, is in keV, the electron density, n_e, is in m^{-3}, the Coulomb logarithm is given by $\ln\Lambda$ = 39.1 − 1.15 $\log_{10}n_e$ + 2.3 $\log_{10}T_e$, and A_D = 2 and Z_D = 1 denote the mass number and charge number of the deuteron, respectively. The critical energy of a deuteron, $E_c = m_D v_c^2/2$, below which the ion drag exceeds the electron drag, is represented as

$$E_c = 14.8\ T_e \times A_D^{1/3} \left(\sum_j \frac{n_j Z_j^2}{n_e} \frac{A_D}{A_j}\right)^{2/3}, \tag{17}$$

where the summation, $\sum$, is made over all ion species in a plasma, and n_j, Z_j, and A_j denote the density, the charge number, and the mass number of the j-species ion, respectively. The distribution of tritons is Maxwellian with the temperature, T_i, which is the same as that of bulk deuterons. The reactivity is then obtained as

$$S = S_M \left\{ 1 - \frac{P_b \tau_s}{n_D E_b h_s} \frac{1}{3} \ln\left(\frac{h_s v_b^3}{v_c^3} + 1 \right) \right\} + \frac{P_b \tau_s}{n_D E_b h_s} (\sigma v)_{max} \int_0^{v_b} \frac{v^2 \hat{G}(x)\, dv}{v^3 + v_c^3/h_s}, \tag{18}$$

where $\hat{G}(x)$ is the normalized 'σv-function' averaged by the Maxwellian triton distribution, $\hat{G}(x) = G(v)/(\sigma v)_{max}$ ($(\sigma v)_{max} = 1.67\times10^{-21} m^3/sec$), and x is the normalized speed, $x = v/v_*$ ($E_* = m_D v_*^2/2$ = 127keV).

In order to integrate eq.(18) analytically. we propose a simple function for $\hat{G}(x)$,

$$\hat{G}(x) = \hat{G}_k + \hat{g}_k (x-0.97) \quad \text{for} \quad x_k < x < x_{k+1} \quad (k=1,2,3). \tag{19}$$

The function, $\hat{G}(x)$, is zero in the region, $x < x_1$. Coefficients, $\hat{G}_k$ and $\hat{g}_k$, are related with the triton temperature as given in Table 3, where boundaries, x_k, are also given.

Table 3 Coefficients in $\hat{G}(x)$ (T_i : keV)

$\hat{G}_1$	$1.06 - 0.058\sqrt{T_i}$	$\hat{g}_1$	$1 / (0.40 + 0.032T_i)$
$\hat{G}_2$	$1.06 - 0.058\sqrt{T_i}$	$\hat{g}_2$	$-1 / (0.91 + 0.016\sqrt{T_i})$
$\hat{G}_3$	0.33	$\hat{g}_3$	-0.11
x_1	$0.97 - \hat{G}_1/\hat{g}_1$		
x_2	0.97		
x_3	$0.97 + (\hat{G}_2 - \hat{G}_3) / (\hat{g}_3 - \hat{g}_2)$		
x_4	3.0		

This approximate function is compared with $\hat{G}(x)$ calculated from eq.(4) for various T_i values in Fig.3. When T_i is much smaller than $E_* = 127$keV, e.g., $T_i < 30$keV, the discrepancy between the approximate function and the real one is quite small except in the region around $x = x_1$. Therefore, this approximate function is useful when the tail spreads beyond the cutoff speed, $x_1 v_*$. The cutoff deuteron energy, $E_1 = x_1{}^2 E_*$, is $E_1 = 38$keV for $T_i = 0$keV, $E_1 = 26$keV for $T_i = 5$keV, and $E_1 = 15$keV for $T_i = 10$keV.

By using eq.(19), we can easily integrate eq.(18),

$$\int_0^{v_b} \frac{v^2 \hat{G}(x)\, dv}{v^3 + v_c{}^3/h_s} = \int_0^{\chi_b} \frac{\chi^2 (\hat{G}_k - 0.97\hat{g}_k + \gamma\hat{g}_k\chi)\, d\chi}{\chi^3 + 1}$$

$$= \Big[\gamma\hat{g}_k\chi + \Big(\frac{\hat{G}_k - 0.97\hat{g}_k}{3} + \frac{\gamma\hat{g}_k}{6} \Big) \ln(\chi^3+1)$$

$$- \frac{\gamma\hat{g}_k}{2} \ln(\chi+1) - \frac{\gamma\hat{g}_k}{\sqrt{3}} \tan^{-1}\Big(\frac{2\chi}{\sqrt{3}} - \frac{1}{\sqrt{3}} \Big) \Big]_0^{\chi_b} , \qquad (20)$$

where $\gamma = h_s{}^{-1/3}\sqrt{E_c/E_*}$, $\chi = h_s{}^{1/3} v/v_c = x h_s{}^{1/3}\sqrt{E_*/E_c}$, and $\chi_b = h_s{}^{1/3}\sqrt{E_b/E_c}$. The dependence of the reactivity on the temperature in a 50:50 DT plasma with the beam-induced deuteron tail is shown in Fig.4. The temperature of each component is the same, $T_e = T_i = T$. The beam power density is chosen constant as $P_b\tau_s/n_e T = 5$ at $T = 5$keV, and the injection energy is (a) $E_b = 50$keV, (b) 100keV, and (c) 200keV. The energy confinement time of the tail deuterons is chosen infinitely large, i.e., $h_s = 1$. A dashed line in the figure denotes S_M, and solid lines are calculated from the integration of eq.(5), where $F_{D0}(v)$ is the solution to the linearlized Fokker-Planck equation [3]. Black circles are obtained from eqs.(18) and (20), which agree well with the solid lines. The error in the results by eqs.(18) and (20) comes from the approximation of $F_0^{tail}(v)$, eq.(15), and of $\hat{G}(x)$, eq.(19), and it becomes large for lower injection beam energy. When E_b is close to T_i, e.g., the case of E_b = 50keV and T_i = 25keV, the profile of the tail component is much different from eq.(15), and when E_b approaches the cutoff energy, $E_1 = x_1^2 E_*$, e.g., the case of E_b = 50keV and E_1 = 34keV for T_i = 2keV, eq.(19) for $\hat{G}(x)$ is insufficient to integrate eq.(18).

Equation (18) with eq.(20) is very useful to obtain the reactivity enhanced by the beam-induced deuteron tail. This result is applied in the next section to study the fusion power multiplication.

5. FUSION POWER MULTIPLICATION

In this section, we study about the fusion power multiplication in a beam-heated plasma on the basis of a local analysis. The fusion power multiplication factor, Q, is defined as, $Q = P_f/P_b$ with $P_f = n_D n_T S E_f$. The released energy per one DT fusion reaction, E_f, is considered to be 17.6MeV.

At first, we optimize the beam energy to maximize the Q value. The increment in the reactivity due to the beam induced tail, $\Delta S = S - S_M$, is proportional to the beam power density, P_b, as described by eq.(18). Therefore, we newly define the beam-fusion multiplication factor, $Q_b = (P_f - P_f^M)/P_b$ with $P_f^M = n_D n_T S_M E_f$, which is independent of P_b but dependent on E_b. Figure 5(a) shows Q_b as a function of E_b for $T_e = T_i = T$ = 10keV in a 50:50 DT plasma ($n_D = n_T = n_e/2$). The profile of $Q_b(E_b)$ is very flat around the peak position, E_b = 215 keV (thick arrow), and the Q_b value lies above 95% of the maximum value in the region, 165keV < E_b < 290keV (between thin arrows). The optimized E_b value (thick curve) and the energy of 95%-$Q_{b,max}$ (thin curves) are given as functions of T in Fig.5(b). The maximum Q_b value is plotted as a function of T in Fig.5(c), which becomes largest at T = 12keV (Q_b = 0.83) and decreases gradually as

T increases. For the case of T_i = 0keV, the optimized E_b value and the energy of 95%-$Q_{b,max}$ are given in Fig.6(b), and the maximum Q_b value is shown in Fig.6(c), as functions of $T = T_e$. The maximum Q_b value for T_i = 0keV continues to increase with the increase of T_e, and it becomes 1.4 at T_e = 30keV. It must be noted that the reactivity, S, for $T_e = T_i = T$ (solid line) is larger than that for $T_e = T$ and T_i = 0keV (dashed line) as shown in Fig.7, where the value of S is obtained at the optimized E_b value for cases of (a) T = 10keV and (b) 20keV. The reactivity, S, increases linearly with the increase of the number density fraction of the beam-induced tail component, N_b, which is proportional to P_b for fixed T value as later given by eq.(23). The value of S at N_b = 0 represents S_M for given T_i value, which is zero for T_i = 0keV (dashed line). Though the derivative, $dS/dN_b \propto Q_b$, of the dashed line is larger than that of the solid line, the solid line lies over the dashed line in the region, $N_b < 0.5$.

In the above analyses, we have given the values of paramerters independently. In order to evaluate the Q value, we now consider a local energy balance equation for a plasma heated by the neutral beam injection;

$$\frac{1}{\tau_E}\left\{\frac{3}{2}n_eT_e + \frac{3}{2}n_TT_i + \frac{3}{2}n_DT_i(1-N_b) + W_b\right\} = P_b - P_r , \qquad (21)$$

where τ_E is the gross energy confinement time at a local position and P_r is the bremsstrahlung power density,

$$P_r = 5.35\times10^{-37}\ n_e^2Z_{eff}\sqrt{T_e} \qquad (22)$$

$$(P_r : \mathrm{watt/m^3} ,\ n_e : \mathrm{m^{-3}} ,\ T_e : \mathrm{keV}) .$$

The effective charge number, Z_{eff}, is defined as $Z_{eff} = \sum n_jZ_j^2/n_e$ ($\sum$ is made over all ion species), and this value is chosen unity in the following calculation. The number density fraction, N_b, and the energy density, W_b, of the beam component are described as

$$N_b = \left(\frac{P_b\tau_s}{n_eE_b}\right) \frac{n_e}{n_D} \frac{1}{3} \ln(\chi_b{}^3+1)\,, \tag{23}$$

$$W_b = \left(\frac{P_b\tau_s}{n_eE_b}\right) \frac{n_eE_b}{2} \left(1 - \frac{C_V}{\chi_b{}^2} \right) , \tag{24}$$

with $C_V = \frac{1}{3}\ln(\chi_b{}^3+1) - \ln(\chi_b+1) + \frac{2}{\sqrt{3}}\left\{ \tan^{-1}\left(\frac{2\chi_b}{\sqrt{3}}-\frac{1}{\sqrt{3}}\right) + \tan^{-1}\frac{1}{\sqrt{3}} \right\}$,

where the energy confinement time of the tail deuterons is assumed to be infinitely large, i.e., $h_s = 1$, and $\chi_b = \sqrt{E_b/E_c}$. By introducing the normalized beam-power density, $\hat{P}_b = P_b\tau_s/n_eE_b$, eq.(21) is rewritten as follows;

$$\frac{n_e\tau_s}{n_e\tau_E} \left\{ \frac{3}{2}(T_e+T_i) - \hat{P}_b\frac{T_i}{2}\ln(\chi_b{}^3+1) + \hat{P}_b\frac{E_b}{2}\left(1-\frac{C_V}{\chi_b{}^2}\right) \right\} = \hat{P}_bE_b - \frac{P_r\tau_s}{n_e} . \tag{25}$$

From this equation, one can easily obtain the value of $n_e\tau_E$ by giving T and $P_b/n_e{}^2$ or obtain the value of $P_b/n_e{}^2$ by giving T and $n_e\tau_E$. It must be noted that, in this system, the energy confinement time of the thermal component, $\tau_E{}^{th}$, decreases with the increase of the beam power;

$$\tau_E{}^{th} \sim \frac{W_{th}}{P_b} \sim \tau_E \left(1 - \frac{W_b}{W_{th}+W_b} \right) ,$$

where $W_{th} \sim 3n_e(T_e+T_i)/2$ denotes the energy density of the thermal component. Also note that to obtain the positive value of $P_b/n_e{}^2$ by giving T we should choose $n_e\tau_E$ value larger than a minimum value,

$$(n_e\tau_E)_{\min} = \frac{n_e\tau_s}{2}\left\{ \left(1-\frac{C_V}{\chi_b{}^2}\right) - \frac{T_i}{E_b}\ln(\chi_b{}^3+1) \right\} ,$$

which is about $(n_e\tau_E)_{\min} \sim 2\times10^{19}T_e^{3/2}(\mathrm{keV})/\ln\Lambda$ $\mathrm{m^{-3}sec}$ for $E_b > E_c$.

In the following calculations, we choose E_b = 200keV, which gives almost the same Q value as that given by the optimized E_b, and assume that the temperature of each component is the same, $T_e = T_i = T$, for simplicity. Figure 8(a) shows the relation between P_b/n_e^2 and T in a 50:50 DT plasma for E_b = 200keV and $n_e\tau_E = 5\times10^{19}\text{m}^{-3}\text{sec}$. The density fraction of the tail component, N_b, is also shown by a dashed curve. The temperature is almost proportional to the normalized power density, P_b/n_e^2, while N_b increases nonlinearly with respect to P_b/n_e^2 (see eq.(23)). If we assume the bulk plasma heating only and no tail component ($N_b = 0$), T becomes larger than that for the presence of the tail, which is given by a thin curve in the figure. The dependence of Q on the power for the same parameter is shown in Fig.8(b), where Q value for the case of no tail component ($S = S_M$) is given by a thin curve. The marginal condition, Q = 1, is realized at $P_b/n_e^2 = 7.5\times10^{-35}$ watt·m^3, and this power is 65% of the power required for the case of no tail component. The enhancement of the reactivity by the beam-induced deuteron tail is favourable to achieve the marginal condition.

The fusion power multiplication factor, Q, in the $T - n_e\tau_E$ space is summarized in Figs.9 and 10. The temperature of each component is assumed to be the same, $T_e = T_i = T$ in a 50:50 DT plasma. The diagram of Fig.9 is for the Maxwellian plasma, and that of Fig.10 is for the plasma heated by the deuteron neutral beam injection with E_b = 200keV. A curve in the figure is a constant Q curve. The top curve corresponds to Q = 10, 2nd one to Q = 5, 3rd one to Q = 3, 4th one to Q = 2, 5th one to Q = 1.5, and 6th one (bold curve) corresponds to the marginal condition, Q = 1. Curves below the Q = 1 curve in Fig.9 (or those on the left hand side of the Q = 1 curve in Fig.10) are drawn at intervals of ΔQ = 0.2. From these figures, we find that the condition to achieve Q = 1 can be approximately given as follows;

$$(n_e\tau_E)_M = (T/8)^{-2.2}\times10^{20}\ \text{m}^{-3}\text{sec} \qquad (26)$$

in Fig.9 for 5keV < T < 10keV, and

$$n_e\tau_E = (T/5.4)^{-3.3}\times 10^{20}\ \mathrm{m^{-3}sec} \tag{27}$$

in Fig.10 for 4keV $< T <$ 8keV, where T is in unit of keV. The value of $n_e\tau_E$ is smaller than that of $(n_e\tau_E)_M$ for the same temperature, and the ratio of $n_e\tau_E$ values is simply approximated as $n_e\tau_E/(n_e\tau_E)_M = 2.7/T^{1.1}$(keV) in the region, 5keV $< T <$ 8keV.

6. SUMMARY

We have presented useful and simple equations to express the reactivity for a DT plasma in a magnetic confinement system. The reactivity for the Maxwellian DT plasma, S_M, is given by a function of T_i. We review several functions, which are tabulated together in Table 4, and propose a new expression, $S_{M,TY}$. The relative error in functions, $S_{M,5}$, $S_{M,5}^*$, $S_{M,6}$, $S_{M,H}$, and $S_{M,TY}$, is less than 5%, and these functions are practical to calculate the value of S_M. When a plasma is heated by the deuteron neutral beam injection and the deuteron tail is induced, the reactivity is enhanced by this tail component. We present a simple function of T_e, T_i, E_b, and $P_b/n_D n_e$ to obtain the value of the enhanced reactivity, which is summarized in Table 5.

By using these simple expressions, the fusion power multiplication in a beam-heated plasma has been studied on the basis of a local analysis. Diagrams of the fusion power multiplication factor, Q, in the T-$n_e\tau_E$ space are shown for two cases, i.e., a 50:50 DT Maxwellian plasma without tail component and a 50:50 DT plasma heated by the neutral beam injection with E_b = 200keV. It is found that the value of $n_e\tau_E$ to achieve Q = 1 in the beam-heated plasma becomes smaller by $2.7/T^{1.1}$(keV) than that in the Maxwellian plasma for the plasma temperature of 5keV $< T <$ 8keV. This result has obtained by assuming $T_e = T_i = T$. Detailed studies should be necessary to evaluate the global Q value by considering the spatial effects and the temperature difference, $T_e \neq T_i$.

$$n_e\tau_E = (T/5.4)^{-3.3}\times 10^{20}\ \mathrm{m^{-3}sec} \tag{27}$$

in Fig.10 for 4keV $< T <$ 8keV, where T is in unit of keV. The value of $n_e\tau_E$ is smaller than that of $(n_e\tau_E)_M$ for the same temperature, and the ratio of $n_e\tau_E$ values is simply approximated as $n_e\tau_E/(n_e\tau_E)_M = 2.7/T^{1.1}$(keV) in the region, 5keV $< T <$ 8keV.

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ACKNOWLEDGEMENT

The authors would like to thank Dr. T. Takeda for fluitful suggestion.

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Table 4 Fitting Equations of Reactivity

$S_M \equiv \langle\sigma v\rangle_M$: m^3/sec (T_i : keV)

Gamow $\quad S_{M,G} = 3.7 \times 10^{-18}\, T_i^{-2/3}\exp(-20/T_i^{1/3})$

Kozlov $\quad S_{M,K} = 3.37\times 10^{-18}\, T_i^{-2/3}\exp(-20.4/T_i^{1/3}) \times g$

$$g = (1 + 0.203T_i^{0.75}) \,/\, \sqrt{1 + 8.88\times 10^{-5}T_i^{3.25}}$$

Hively $\quad S_{M,5} = \exp(a_1/T_i^r + a_2 + a_3T_i + a_4T_i^2 + a_5T_i^3 + a_6T_i^4)$

$$S_{M,5}^* = \exp(a_1/T_i^r + a_2 + a_3T_i + a_4T_i^2 + a_5T_i^3)$$

$$S_{M,6} = \exp(a_1/T_i^r + a_2 + a_3T_i^s)$$

	$S_{M,5}$	$S_{M,5}^*$	$S_{M,6}$
a_1	-21.38	-22.71	-71.29
a_2	-34.41	-33.05	52.72
	(-25.20	-23.84	61.93)#
a_3	-7.101×10^{-2}	-9.393×10^{-2}	-37.30
a_4	1.938×10^{-4}	7.994×10^{-4}	−
a_5	4.925×10^{-6}	-3.144×10^{-6}	−
a_6	-3.984×10^{-8}	−	−
r	0.2935	0.2747	0.1396
s	−	−	0.1007

\# for S_M is in unit of cm^3/sec

Brunelli $\quad S_{M,B} = 9.0 \times 10^{-22} \exp(-0.476\,|\ln(T_i/69)|^{2.25})$

Hively $\quad S_{M,H} = 9.131\times 10^{-22} \exp(-0.5716\,|\ln(T_i/64.22)|^{2.137})$

Present $\quad S_{M,TY} = S_{M,G}/h \qquad h = \dfrac{T_i}{37} + \dfrac{5.45}{3+T_i\{1+(T_i/37.5)^{2.8}\}}$

Table 5 Reactivity Enhanced by Beam-Induced Deuteron Tail

$$S = S_M \left\{ 1 - \frac{P_b \tau_s}{n_D E_b h_s} \frac{1}{3} \ln\left(\frac{h_s v_b^3}{v_c^3} + 1 \right) \right\} + \frac{P_b \tau_s}{n_D E_b h_s} (\sigma v)_{max} \int_0^{v_b} \frac{v^2 \hat{G}(x)\, dv}{v^3 + v_c^3/h_s}$$

$$\int_0^{v_b} \frac{v^2 \hat{G}(x)\, dv}{v^3 + v_c^3/h_s} = \left[\gamma \hat{g}_k \chi + \left(\frac{\hat{G}_k - 0.97\hat{g}_k}{3} + \frac{\gamma \hat{g}_k}{6} \right) \ln(\chi^3+1) - \frac{\gamma \hat{g}_k}{2} \ln(\chi+1) - \frac{\gamma \hat{g}_k}{\sqrt{3}} \tan^{-1}\left(\frac{2\chi}{\sqrt{3}} - \frac{1}{\sqrt{3}} \right) \right]_0^{\chi_b}$$

$h_s = 1 + \tau_s/2\tau_E^{tail}$, $\gamma = h_s^{-1/3}\sqrt{E_c/E_*}$, $\chi = h_s^{1/3} v/v_c = x h_s^{1/3}\sqrt{E_*/E_c}$,

$(\sigma v)_{max} = 1.67 \times 10^{-21} m^3/sec$ at $E_* = m_D v_*^2/2 = 127 keV$

$\tau_s = 2.0 \times 10^{19} \; (T_e^{3/2}/n_e \ln\Lambda) \times (A_D/Z_D^2)$ (τ_s: sec, T_e: keV, n_e: m^{-3}) ,

$E_c = m_D v_c^2/2 = 14.8\, T_e \times A_D^{1/3} \left(\sum_j n_j Z_j^2 A_D / n_e A_j \right)^{2/3}$,

Coefficients in $\hat{G}(x)$ (T_i : keV)

$\hat{G}_1$	$1.06 - 0.058\sqrt{T_i}$	$\hat{g}_1$	$1 / (0.40 + 0.032 T_i)$
$\hat{G}_2$	$1.06 - 0.058\sqrt{T_i}$	$\hat{g}_2$	$-1 / (0.91 + 0.016\sqrt{T_i})$
$\hat{G}_3$	0.33	$\hat{g}_3$	-0.11
x_1	$0.97 - \hat{G}_1/\hat{g}_1$		
x_2	0.97		
x_3	$0.97 + (\hat{G}_2 - \hat{G}_3) / (\hat{g}_3 - \hat{g}_2)$		
x_4	3.0		

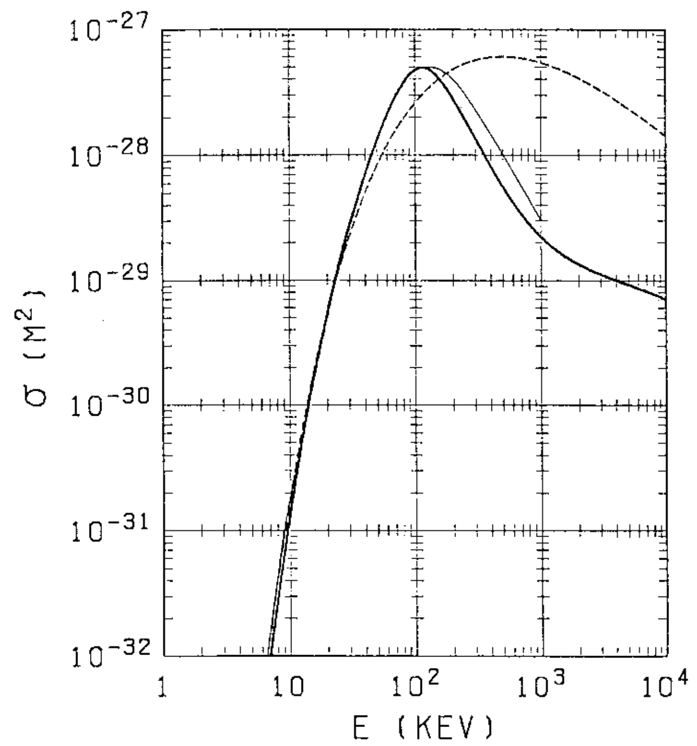

Fig. 1 Cross section of DT fusion reaction, σ, as function of deuteron energy, E. Thick curve corresponds to eq.(2), thin curve to Ref.[1], and dashed one to eq.(1).

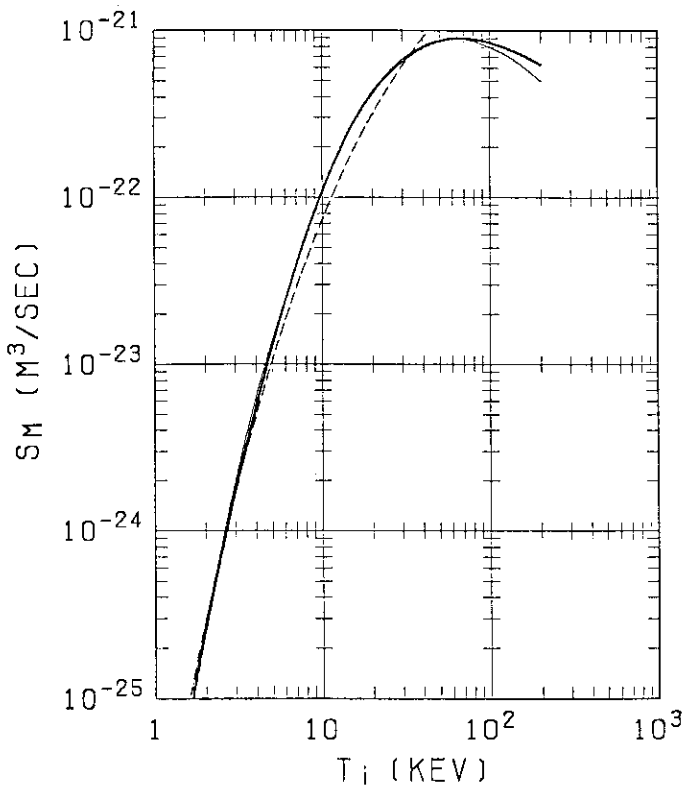

Fig. 2 Reactivity for DT Maxwellian plasma, $S_M = \langle \sigma v \rangle$, as function of ion temperature, $T_i = T_D = T_T$. Thick curve is obtained by use of eq.(2), thin curve corresponds to Ref.[1], and dashed one to eq.(7).

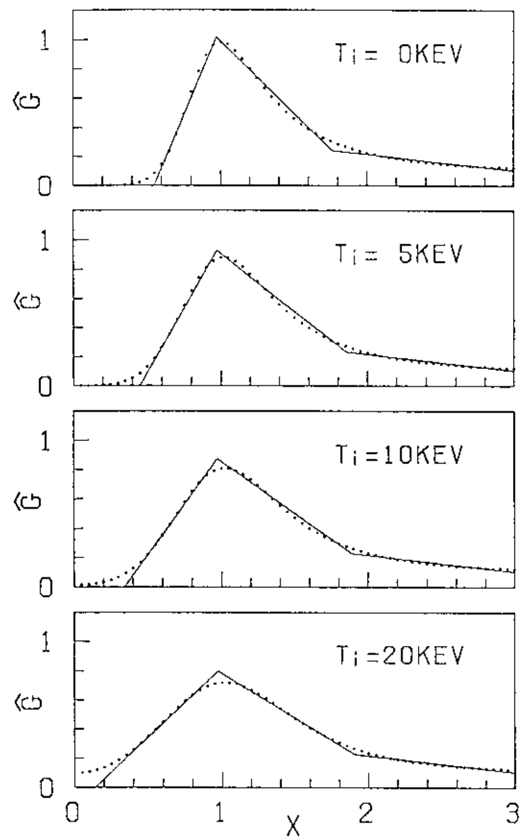

Fig. 3 Normalized '$\sigma\upsilon$-function' averaged by Maxwellian triton distribution, $\hat{G}(x) = G(\upsilon)/(\sigma\upsilon)_{max}$, for various T_i value. Deuteron speed is normalized as $x = \upsilon/\upsilon_*$ ($E_* = m_D\upsilon_*^2/2 = 127$keV). $\hat{G}(x)$ is approximated by linear functions (solid line), and exact '$\sigma\upsilon$-function' is shown by dotted curve.

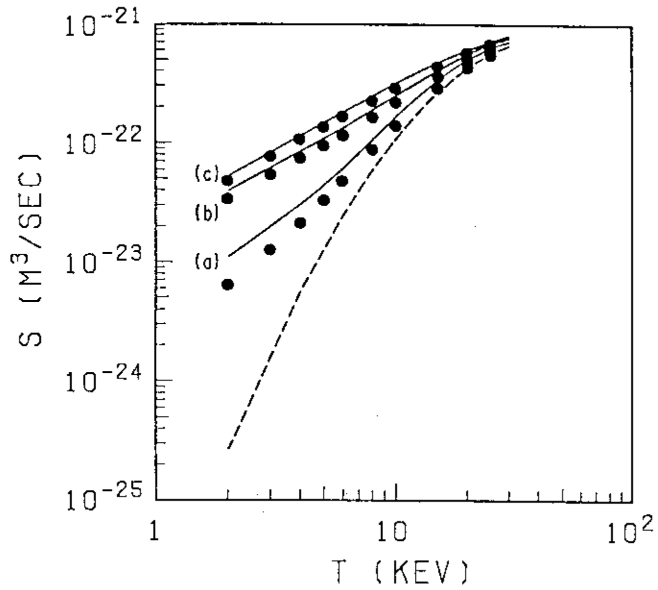

Fig. 4 Reactivity, S, enhanced by beam-induced deuteron tail in 50:50 DT plasma ($T_e = T_i = T$). Deposited beam power density is chosen constant as $P_b\tau_s/n_eT = 5$ at $T = 5$keV, and injection energy is (a) $E_b = 50$keV, (b) 100keV, and (c) 200keV. Reactivity obtained from exact numerical calculation is given by solid curve, and that obtained from eqs.(18) and (20) is plotted by black circle. Dashed curve corresponds to S_M.

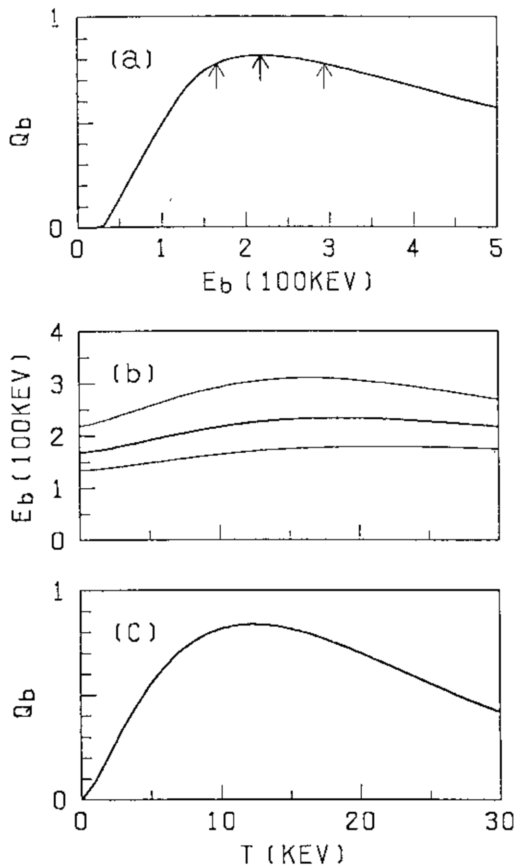

Fig. 5 Beam-fusion multiplication factor, Q_b, (a) as function of E_b for $T_e = T_i = 10$keV. Q_b takes its maximum value at E_b of thick arrow and 95%-$Q_{b,max}$ at E_b of thin arrows. E_b for $Q_{b,max}$ (thick curve) and for 95%-$Q_{b,max}$ (thin curve) are shown as functions of $T = T_e = T_i$ in (b). Maximum Q_b value is plotted in (c).

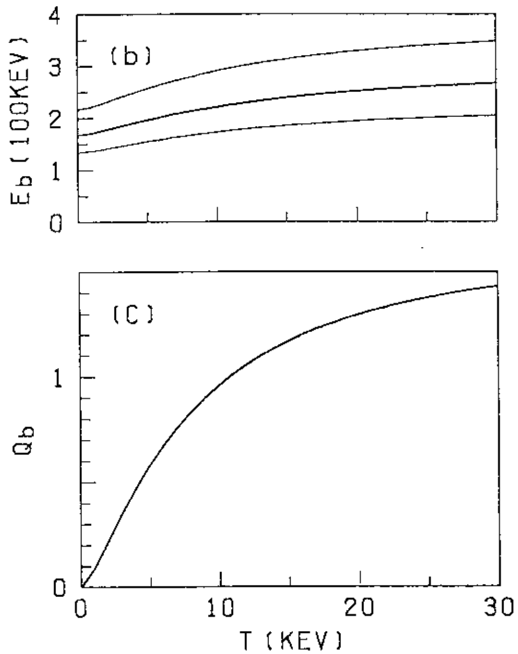

Fig. 6 Beam-fusion multiplication factor, Q_b, for $T_e = T$ and $T_i = 0$keV. E_b for $Q_{b,max}$ (thick curve) and for 95%-$Q_{b,max}$ (thin curves) are shown as functions of $T = T_e$ in (b). Maximum Q_b value is plotted in (c), which is larger than that in Fig.5 ($T = T_e = T_i$).

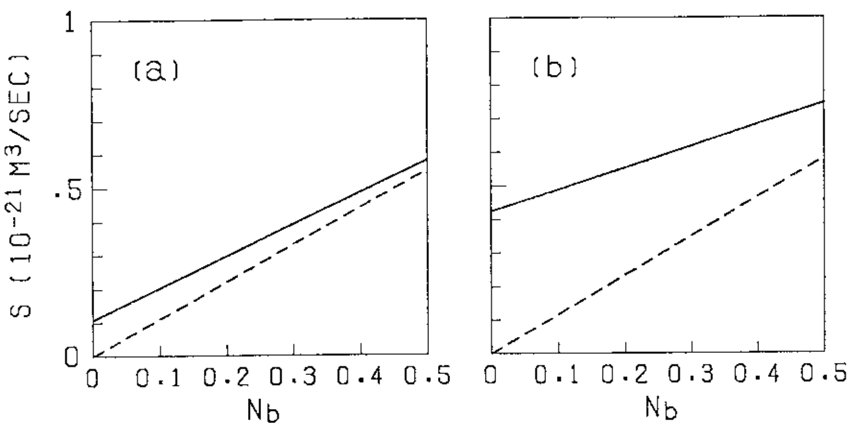

Fig. 7 Dependence of reactivity, S, on density fraction of beam-induced tail component, $N_b \propto P_b$, for (a) T = 10keV and (b) T = 20keV. Value of S for $T_e = T_i = T$ (solid line) is larger than that for $T_e = T$ and $T_i = 0$keV (dashed line).

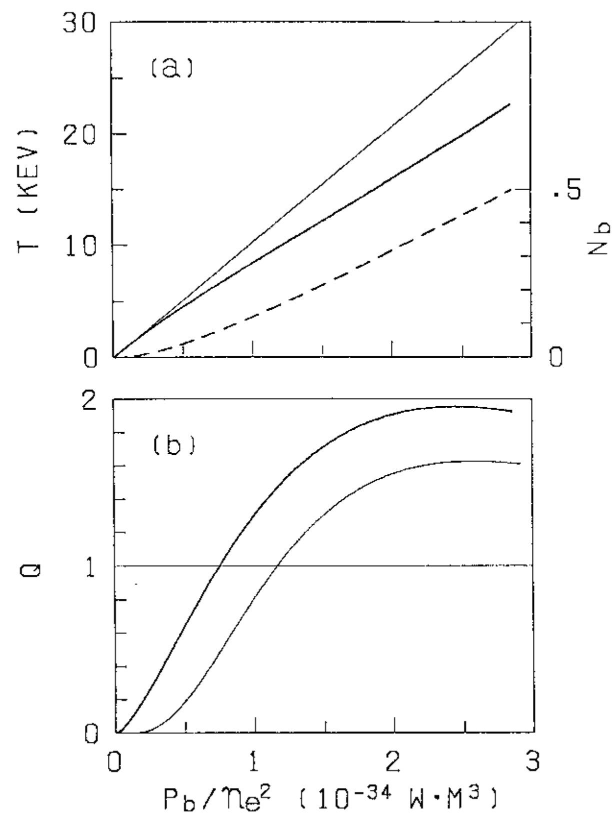

Fig. 8 Relation between temperature, $T = T_e = T_i$, and normalized power density, P_b/n_e^2, in 50:50 DT plasma for $E_b = 200$keV and $n_e\tau_E = 5 \times 10^{19} m^{-3}$sec. T is almost proportional to P_b/n_e^2 (solid curve), while N_b is nonlinearly increases with P_b/n_e^2 (dashed curve) as shown in (a). Case of bulk plasma heating without tail component is shown by thin curve for comparison. Fusion multiplication factor, Q, is enhanced by beam-induced tail component as shown in (b).

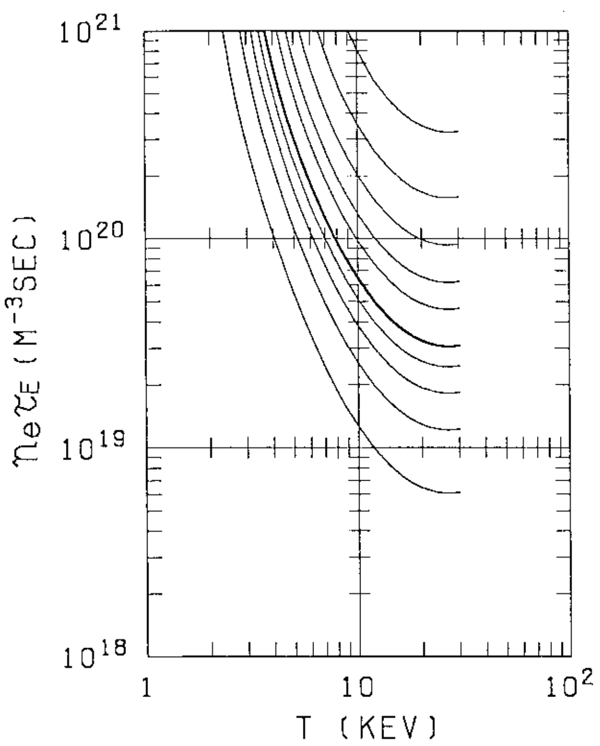

Fig. 9 Fusion multiplication factor, Q, in $T - n_e\tau_E$ space in 50:50 DT Maxwellian plasma ($T_e = T_i = T$). Top equi-Q-curve corresponds to Q = 10, 2nd one to Q = 5, 3rd one to Q = 3, 4th one to Q = 2, 5th one to Q = 1.5, and 6th one (bold curve) to Q = 1. Curves below the Q = 1 curve are drawn at intervals of $\Delta Q = 0.2$.

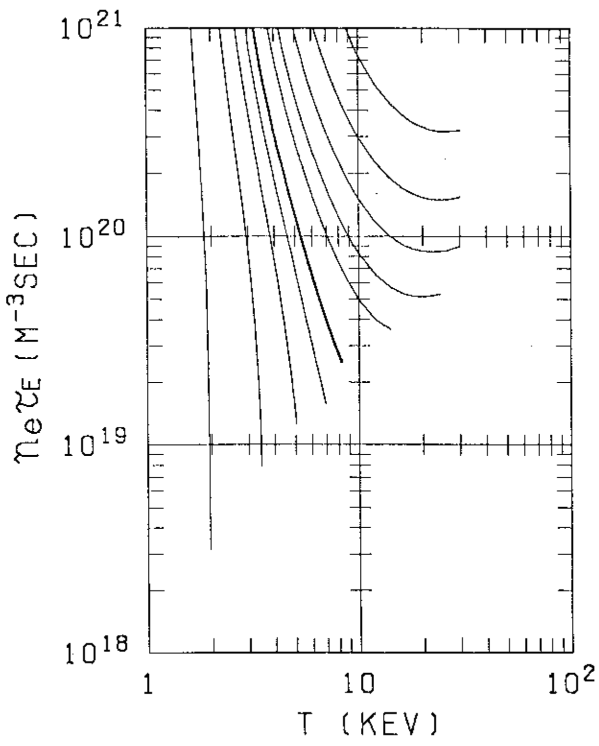

Fig. 10 Fusion multiplication factor, Q, in $T - n_e\tau_E$ space in 50:50 DT plasma ($T_e = T_i = T$) heated by neutral beam injection with $E_b = 200$keV. Q value for each equi-Q-curve is the same as that in Fig.9.