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Keiji TANI, Masanobu SUZUKI*, Shin YAMAMOTO
and Masafumi AZUMI

日 本 原 子 力 研 究 所
Japan Atomic Energy Research Institute

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On the Analysis of Beam Driven Current in a Tokamak

Keiji TANI, Masanobu SUZUKI*, Shin YAMAMOTO and Masafumi AZUMI

Department of Large Tokamak Research
Naka Research Establishment
Japan Atomic Energy Research Institute
Naka-machi, Naka-gun, Ibaraki-ken

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Beam induced plasma current in a tokamak has been analyzed by using an orbit-following Monte-Carlo code. Results indicate that the effect of particle trapping is of great importance. A substantial fraction of fast ion current is induced by the effect of energy diffusion. Taking those important effects into consideration, we have developed a new analytical code in which nummerically derived eigenfunctions of the bounce-averaged, two dimensional Fokker-Planck equation are adopted.

Keywords: Tokamak, Beam Driven Current, Monte-Carlo Code, Particle Trapping, Fokker-Planck equation

トカマクにおけるビーム駆動電流解析

日本原子力研究所那珂研究所臨界プラズマ研究部
谷　啓二・鈴木正信*・山本　新・安積正史

（1988年2月1日受理）

軌道追跡モンテカルロ・コードを用いてトカマク中でのＮＢＩによるビーム駆動電流の解析を行った。その結果，粒子捕捉効果が非常に重要であることが示された。また，エネルギー拡散効果によりかなりの高速イオン電流が誘導されることもわかった。これ等の重要な効果を考慮に入れた解析的手法を用いるコードの開発を行った。新しい解析コードにおいては，バウンス平均化された２次元フォッカープランク方程式を数値的に求めた固有関数で展開することにより解いた。

那珂研究所：〒311－01　茨城県那珂郡那珂町大字向山
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Contents

目　次

1. Introduction

Recent experimental results on H^- ion sources promise that neutral beam injection will be a useful heating method in fusion reactors [1,2]. Consequently, NBI-sustained steady-state tokamak reactors are in the lime-light again reactors [3].

One of the most important investigations of beam sustained tokamaks lies in the beam driven current. In most analytical treatments of beam driven current, fast ion current induced during slowing down of injected beam ions has been estimated by using analytical solutions of the non-bounce averaged Fokker-Planck equation without a energy diffusion term [4,5,6]. For the estimation of fast ion current, however, an accurate knowledge of the distribution function of fast ions is necessary. J.G. Cordey has reported that the effect of particle trapping on the distribution of fast ions in a tokamak is very important [7]. A precise treatment of banana particles requires that the Coulomb collision operators must be bounce averaged. The effect of energy diffusion might be also important in a high temperature plasma. The effect of fast ion losses during slowing down on beam induced current is of great interest. These problems motivated us to estimate the beam driven current by using an orbit-following Monte-Carlo code (hereafter simply referred to as OFMC code) which can describe precisely the behavior of fast ions during slowing down. One of the problems in use of the OFMC code is that it takes a very long CPU time. The OFMC code, however, can provide a lot of important information for the analysis of beam driven current. Taking the important information into consideration, we have developed a new analytical code.

2. Old analytical model

The beam driven current has been estimated by

$$I_B = I_{fi}[1 - F(1-G)] \tag{1}$$

where

I_{fi}: fast ion current,

$F \sim Z_b/Z_{eff}$,

G : the trapped electron correction $\sim 1.4\sqrt{\varepsilon}$

Z_b the charge number of beam ions, Z_{eff} the effective charge number and ε the inverse aspect ratio. The fast ion current I_{fi} can be estimated by

$$I_{fi} = n_b \langle v_\parallel \rangle e Z_b$$

$$= eZ_b \int_0^\infty v^3 dv \int_{-1}^{+1} \eta d\eta \tag{2}$$

where n_b is the beam density, $v_\parallel$ the velocity component parallel to the magnetic field line, v the fast ion velocity, $\eta = v_\parallel / v$ and $f(v,\eta)$ the distribution function of fast ions in steady state. The distribution function f has been solved by J.D.Gaffey [8] from the Fokker-Planck equation. In general, variables of f can be separated as

$$f(v,\eta) = s\tau_s \sum_n a_n(v) c_n(\eta) \tag{3}$$

where s is the volume source of fast ions and τ_s is the slowing down time. In the velocity region $v<v_b$, $a_n(v)$ in eq.(3) is simply given by

$$a_n(v) = \frac{1}{v^3+v_c^3}\left[\frac{v^3}{v_b^3}\,\frac{v_b^3+v_c^3}{v^3+v_c^3}\right]^{\beta\lambda n/3} \tag{4}$$

where v_c is the critical velocity, v_b the injected beam velocity,

$$\lambda_n = n(n+1)$$

$$\beta = \frac{Z_{eff}}{2[Z]},$$

$$[Z] = \frac{m_b}{ln\Lambda_e n_e}\sum_j \frac{Z_j^2 ln\Lambda_j}{m_j},$$

m_j the mass of beam particle and j denotes the j^{th} species of bulk plasma ion. If fast ions are assumed to be stationary (no orbit effect) $c_n(\eta)$ can be simply described by the legendre polynomial functions as

$$c_n = \frac{n}{2} P_n(\eta_0) P_n(\eta) \tag{5}$$

where η_0 is the initial pitch of fast ions.

3. Orbit-following Monte-Carlo code

Our OFMC code consists of four parts;

1) calculation of ionization of neutral beams,
2) simulation of Coulomb collisions,
3) calculation of guiding center orbit, and
4) simulation of charge-exchange reactions of fast ions and reionization of fast neutrals in succession.

Monte-Carlo techniques are adopted to the calculation of beam ionization. Distribution of fast ions produced by the ionization of neutral beams with elliptic cross section can be obtained. Computations for multi beam lines as well as multi energy species of neutral beams can be made with our code.

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The collisional processes of fast ions are also simulated by a technique of Monte-Carlo. The longitudinal and the transverse components of velocity change by Coulomb collision Δv_l and Δv_t are calculated by generating normal random numbers with respective mean values of $<\Delta v_l>$, $<\Delta v_t>$ and mean square deviations $<\Delta v_l^2>$, $<\Delta v_t^2>$. Assuming that the phase of Larmor motion of fast ions is random, Δv_l and Δv_t are converted into velocity components parallel and perpendicular to the magnetic field line $\Delta v_\parallel$ and $\Delta v_\perp$.

Ion trajectories are followed by numerical integration of the standard guiding center equation in an axisymmetric system. A second-order Runge-Kutta method is adopted for the numerical integration. The magnetic field is given by the derivatives of toroidal flux function ψ which is obtained by the widely used MHD equilibrium code "SELENE" [8].

The charge-exchange process is also simulated by a simple hit-or-miss Monte-Carlo method. If a fast ion is once charge exchanged, the orbit of fast neutral (straight flight) is followed until it is reionized by field plasma particles or escapes from the plasma.

In the above described OFMC code, the fast ions produced by the ionization of injected neutral beams are launched from the birth point and are followed until all of them are thermalized. In other words, the time evolution of the distribution function of fast ions derived by the OFMC code is the impulse response. Therefore, the fast ion current in a steady state system can be described by

$$I_{fi} = eZ_b \sum_i w_i \int_0^{\tau_s} v_\parallel^i(t)dt \tag{6}$$

where w_i and $v_\parallel^i(t)$ are the density weight and the time evolution of $v_\parallel$ of i^{th} test particles, respectively.

4. Comparison of calculation results from old analytical model with those from OFMC code

Calculations were made for fast ions produced at a specific point in a plasma $(R,Z) = (4.0, 0.0)$ with $\eta_0 = 1.0$ and beam energy $160\,KeV$. Bulk plasma is assumed to be uniform in space with $n_e = 1.0\times10^{20} m^{-3}$ and $T_e = T_i = 8\,KeV$. Figure 1 shows the spectrum of fast ion current derived the old analytical model without energy diffusion term J_{OAM} and that by OFMC code J_{OFMC}. A substantial fraction of fast ion current is induced by the effect of energy diffusion. Since this fraction increases with v_c/v_b, the effect of energy diffusion becomes very important for the precise treatment of beam driven current in a high temperature plasma.

It must be noted that J_{OFMC} is substantially smaller than J_{OAM} in the low energy range. On the other hand, the distribution function from the old analytical model agrees very well with that from OFMC code in this energy range $E<E_b$. This may implies that the difference between those spectra of fast ion current comes from some orbit effects. Two kinds of orbit effects can be proposed. One is the bounce-averaging effect of $v_\parallel$ on the fast ion trajectory. The other one is the effect of particle trapping on the population of transit particles.

Another interesting point in the comparison of the results from those two methods lies in the effect of particle loss during slowing down. The effect of finite banana size on the distribution of beam driven current is also of great interest. Calculation results in FER are shown in Fig.2. We cannot find out any significant difference between those results with and without charge-exchange loss. This is because that charge-exchange loss usually occurs when fast ions slow down to the energy range $E < 50KeV$ where the contribution of fast ions to the ion current becomes very small.

In order to clarify the effect of finite banana size ρ_B on the spatial distribution of beam induced current, calculations with and without banana size were made. Results are also shown in Fig.2. What is evident form Fig.2 is that the effect of finite banana size on the distribution of beam driven current is not essential.

5. Improved analytical model

OFMC code is a very useful tool to analyze the slowing down process of fast ions. A problem in the use of OFMC code is that it takes a very long CPU time. For example, it takes about 2 hours to analyze the beam driven current in FER for 2000 test particles (results are shown in Fig.2). On the other hand, there are some problems in the old analytical treatment as is discussed in the last section. This prompt us to develop an improved analytical model code. We have found some important effects from the comparison of the calculation results derived by the OFMC code with those by the old analytical treatment. Three kinds of effects are taken into consideration in the new analytical model.

5.1 Energy diffusion

The expression of the energy diffusion term derived by J.D.Gaffey [9] is employed in the velocity region $v > v_b$, that is,

$$a_n(v) = \frac{1}{v_b^3 + v_c^3} \exp\left[-g(v)\right] \qquad (v > v_b) \tag{7}$$

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where

$$g(v)=\frac{2\,(\,1+v_c^3/v_b^3\,)}{T_e/E_b+T_i/E_b v_c^3/v_b^3}$$

5.2 Bounce average of $v_\parallel$

The fast ion current can be calculated by integrating $v_\parallel$ with weight of the distribution function over v and $v_\parallel$. It must be noted that in most of old analytical treatments fast ions are assumed to be stationary at their birth point throughout the slowing down time. In a tokamak plasma, however, every particle makes its own particular bounce motion along a magnetic field line. Therefore, in order to calculate the fast ion current, we must integrate the bounce averaged $v_\parallel$, that is, $<v_\parallel> = H(v_\parallel^*/v)$ where $v_\parallel^*$ is the $v_\parallel$ defined in the midplane outer side of the torus.

The H function is shown in Fig.3 by the red curve. The green line is the current weight function of an old analytical model.

5.3 Effect of particle trapping

Fast ions produced by a tangential co-injection of neutral beams undergo pitch angle scattering, and some of them diffuse into the banana trap region. On the way of diffusive motion crossing the banana-transit boundary, particles can easily get into the transit region of negative pitch. This is because the transition probabilities of banana particles into co and counter transit particles are approximately 0.5 : 0.5. Therefore, the pitch angle scattering is effectively enhanced in the presence of banana trapping region. J.G.Cordey has analyzed this effect [7].

The bounce averaged differential equation for c_n in eq.(3) can be written as

$$\frac{1}{R(\eta,\eta_t)}\frac{d}{d\eta}\left[(1-\eta^2)Q(\eta,\eta_t)\frac{dc_n}{d\eta}\right]+\lambda_n c_n=0 \tag{8}$$

where

$$R(\eta,\eta_t)=\frac{2}{\pi}K[(\frac{\eta_t}{\eta})^2]\eta,$$

$$Q(\eta,\eta_t)=\frac{2}{\pi}E[(\frac{\eta_t}{\eta})^2],$$

K: the complete elliptic integral of the first kind,

E: the complete elliptic integral of the second kind,

and η_t is the pitch of barely trapped particle. It must be noted that

$$\lim_{\eta\to\eta_t}R(\eta,\eta_t)=\infty.$$

Assuming $R\sim 1.0$, $Q\sim 1.0$, Cordey solved the differential eq.(8) and derived analytical eigenfunctions

$$c_n(\eta)=P_{\nu n}(\eta) \tag{9}$$

where $P_{\nu n}(\eta)$ is the legendre function and ν_n is given by the boundary condition

$$P_{\nu n}(\eta)=0.$$

For the precise treatment of distribution function of fast ions, we take the effect of barely trapped particle whose bounce time $\tau_b=\infty$ into consideration. We derive numerically the eigenfunctions for R and Q with exact expressions in eq.(8) by adopting a variational method. That is, we expand c_n in a series of

$$c_n(\eta)=\sum_m a_{mn}(1-\eta)^{2m-1}. \tag{10}$$

Then, eigenvalues λ_n are given as solutions of the following equation

$$|\alpha_{mn}+\lambda\beta_{mn}|=0 \ , \tag{11}$$

where

$$\alpha_{mn} = \int_0^{1-\eta_t} \{1-(\eta+\eta_t)^2\} E(k)\ (2m-1)(2m-2)\eta^{2m+2n-4} d\eta$$

$$+ \int_0^{1-\eta_t} \left[-2(\eta+\eta_t)\ E(k) - \{1-(\eta+\eta_t)^2\} \frac{dE(k)}{dk} \frac{2\eta_t}{(\eta+\eta_t)^3} \right] (2m-1)\eta^{2m+2n-3} d\eta$$

$$\beta_{mn} = \int_0^{1-\eta_t} K(k)(\eta+\eta_t)\eta^{2m-2n-2} d\eta$$

$$k = \frac{\eta_t^2}{(\eta+\eta_t)^2}$$

With these eigenvalues λ_n and the boundary condition,

$$c_n(1) = \sum_m (1-\eta_t)^{2m-1} = 1.0 \ ,$$

α_{mn} are easily obtained.

6. Calculation results from improved analytical model

Calculations are made for parameters appropriate to JT-60 which are summarized in Table 1. Fast ions are launched from a specific point $R = 4.0$ in the midplane. Banana size of fast ions are assumed to be zero. Beam driven current given by Gaffey's model with energy diffusion term (old analytical treatment) is shown in Fig.4 by the dotted curve as a function of initial pitch angle. The dashed curve is the one given by Cordey's analytical expression of eigenfunctions with energy diffusion term by Gaffey. The closed circles are the results from OFMC code. Results from Cordey's model agree very well with those from OFMC code near the tangential injection angel (pitch angle $\sim 90°$. For injection at a pitch angle near the banana region, however, one can find the difference of 10~30 between the results of OFMC code and Cordey's model. The solid

where

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$$\beta_{mn} = \int_0^{1-\eta_t} K(k)\,(\eta+\eta_t)\eta^{2m+2n-2} d\eta$$

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curve in Fig.4 shows the results from the improved analytical model described in the last section. One can see that the results from the improved analytical code agree very well with those from OFMC code in the wide range of initial pitch angle.

7. Conclusions

Beam induced plasma current has been analyzed by using an orbit-following Monte-Carlo code. Results have been compared with those from an old analytical model. Comparison of those results indicates that the effect of particle trapping as well as the effect of energy diffusion are very important for the precise analysis of beam driven current. Taking those important effects into consideration, an improved analytical code has been developed. Calculation results from the new code agree very well with those from the OFMC code.

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Table 1 Calculation parameters

major radius	R_t	=	3.03 m
minor radius	a	=	0.95 m
plasma density	n_e	=	1×10^{20} m^{-3} (uniform)
plasma temperature	T_e	=	8 keV (uniform)
			$T_i = T_e$
effective Z	Z_{eff}	=	2.0 (uniform)
impurity		:	oxygen

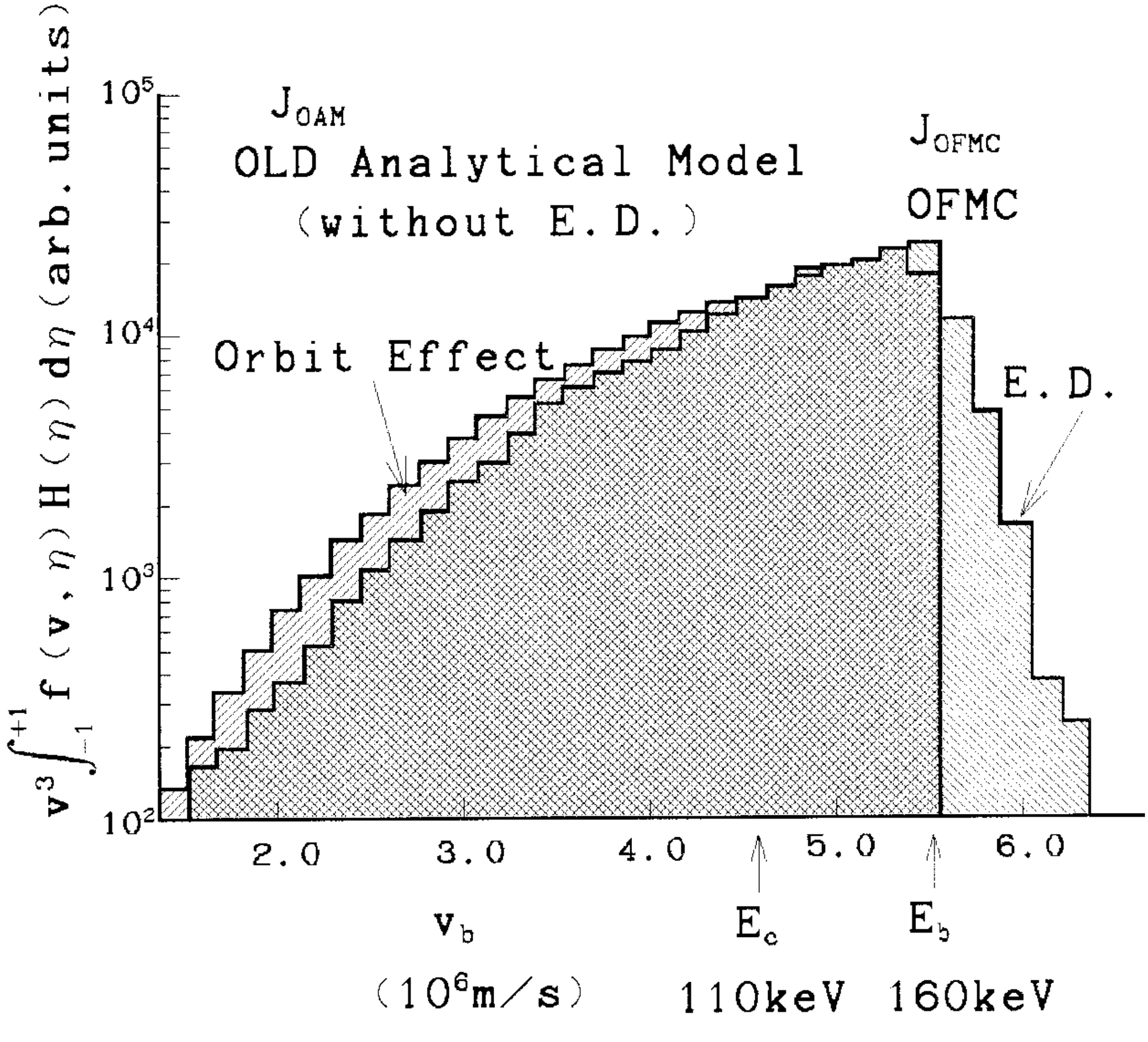
Figure captions

Fig.1 Velocity spectrum of fast ion current. J_{OAM} is the spectrum derived by the old analytical model without energy diffusion effect, J_{OFMC} is the one from OFMC code. Beam energy $E_b = 160\,keV$, $n_e = 1.0\times10^{20}m^{-3}$ and $T_e = T_i = 8\,keV$.

Fig.2 Calculation results by OFMC code for FER current drive efficiency with and without charge exchange process and beam driven current profiles with and without banana width.

Fig.3 Schema for bounce average of $v_{\parallel}$.

Fig.4 Beam driven current v.s. initial pitch angle of fast ions. The dotted curve is derived by Gaffey's model (old analytical treatment), the dashed curve by Cordey's model with analytical eigenfunctions and the solid curve by the improved analytical model with numerical eigenfunctions. Effect of energy diffusion on fast ion current is taken into consideration in all of those analytical treatments. Results from OFMC code are shown by the closed circles.



J_{OAM}
OLD Analytical Model
(without E.D.)
J_{OFMC}
OFMC
Orbit Effect
E.D.
10^5
10^4
10^3
10^2
$v^3\int_{-1}^{+1} f(v,\eta) H(\eta) d\eta$ (arb. units)
2.0
3.0
4.0
5.0
6.0
v_b
$(10^6 m/s)$
E_c
110keV
E_b
160keV

Fig. 1

FER

$E_b=500keV$, $Z_{eff}=2.2$, $R_t=5.4m$,

$\langle T_e\rangle=20keV$, $\langle n_e\rangle=7\times10^{19}m^{-3}$

efficiency MA/MW	cx-off		cx-on
	$\rho_B=0$	$\rho_B\neq0$	$\rho_B\neq0$
OFMC	0.123	0.130	0.125

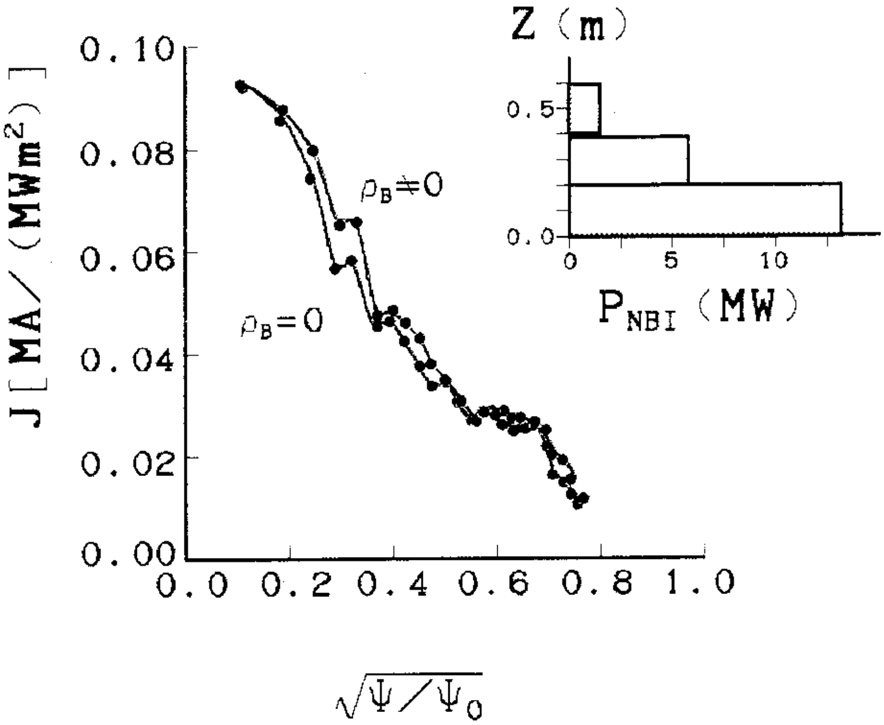


Fig. 2

Bounce average of $\eta = \frac{v_{\parallel}}{v}$

$$I_{fi} = eZ_b \int_0^{\infty} v^3 dv \int_{-1}^{+1} f(v, \eta) H(\eta) d\eta$$

$$H(\eta) = \frac{1}{v} \frac{v_{\parallel} dt}{dt}$$

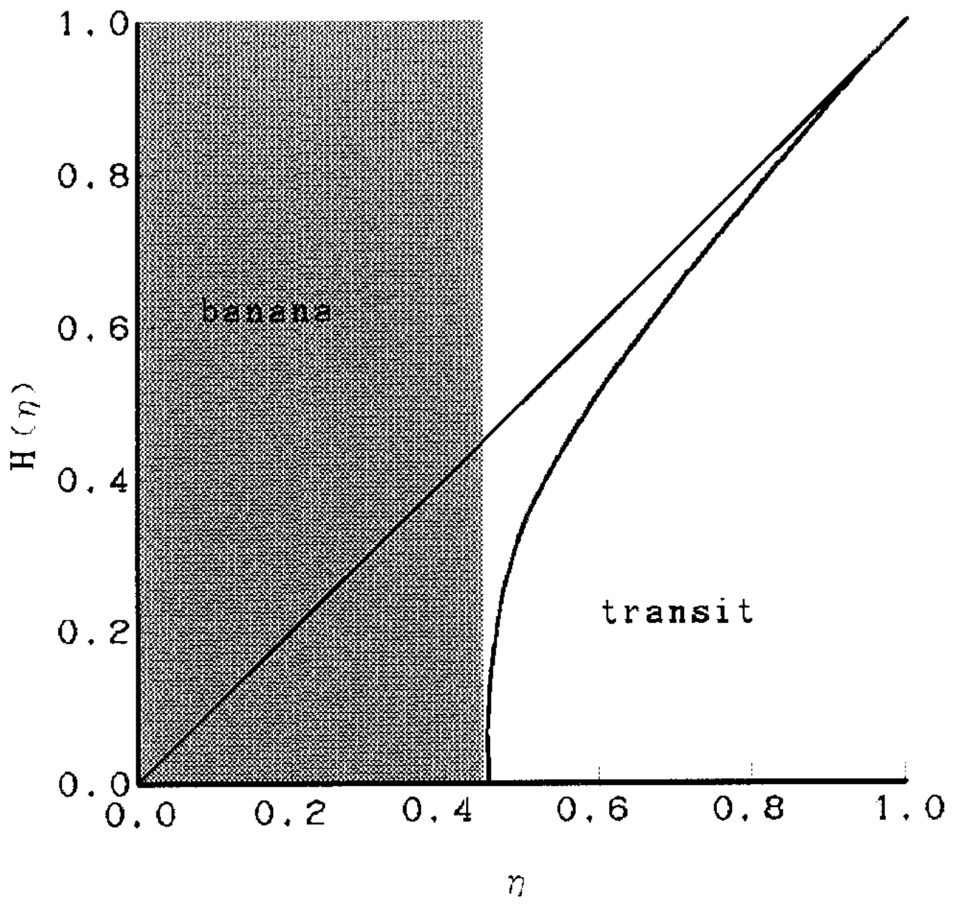
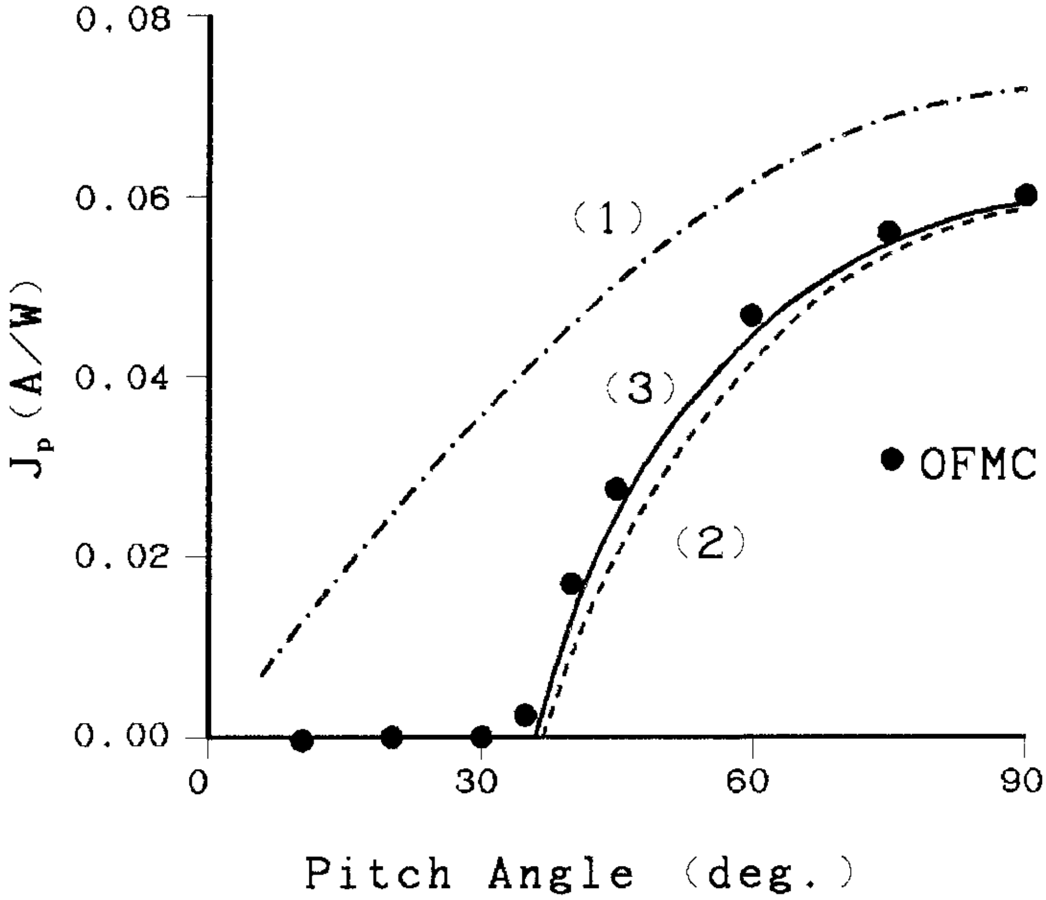


Fig. 3



(1) Gaffey model

(2) Cordey model with E.D.

(analytical eigen function)

(3) Numerical eigen function
effect of barely trapped ions

Fig. 4