

DYNAMIC BEHAVIOR AND CONTROL OF PRODUCT ENRICHMENT  
IN A CENTRIFUGE CASCADE  
—A STUDY OF THE PROCESS SIMULATION AS A BASIS OF SAFEGUARDS DESIGN—

May 1989

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mura, Naka-gun, Ibaraki-ken 319-11, Japan.

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編集兼発行 日本原子力研究所  
印 刷 いばらき印刷機

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- A Study of the Process Simulation as a Basis of Safeguards Design -

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(Received April 21, 1989)

It was agreed as a conclusion of the HEXAPARTITE project that a limited frequency unannounced access (LFUA) inspection should be carried out in a centrifuge type enrichment plant as a basic safeguards approach. It might be adopted at a large scale, future commercial enrichment plant, too. Application of the LFUA approach to such a plant, however, should be fully investigated because the plant will have not only a larger capability of enriching uranium 235 but also a more sensitive information to be protected from the commercial and non-proliferation viewpoint. As a part of a design study on the safeguards approach for a model commercial plant, a study of process simulation of the plant has been carried out. This report describes a result of the study.

When a commercial uranium enrichment plant is constructed, a nuisance problem arises; What kind of products should be produced from the plant in order to match a wide range of nuclear fuel enrichment requirements for light-water power reactors. In this report, a reasonable solution to such a problem is investigated.

At first, a transient analysis of start-up for a model centrifuge cascade is made by using the dynamic equations, which were so developed as to be able to accurately compute interstage flow rates and enrichment in a transient state. Then it is investigated how wide in its acceptable

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range the product enrichment can be controlled by regulating cascade characteristic parameters such as cascade cut, recycle flow rate and cascade feed flow rate, and as a result an information about the optimal regulating mode is brought out.

As a result of this study, it has become clear that the specific requirements of a customer are almost fulfilled with only one type of unit cascade system if 10% loss of cascade efficiency is allowed in the plant operation.

Keywords: Safeguards Design, Process Simulation, Uranium Enrichment, Nuclear Fuel, Light-Water Power Reactor, Centrifuge, Cascade, Start-up, Dynamic Equation, Interstage Flow, Product Enrichment, Transient State, Cut, Recycle Flow, Feed Flow, Cascade Efficiency

遠心分離カスケードにおけるプロダクト濃度の挙動とその調整  
— 保障措置設計のためのシミュレーション研究 —

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(1989年4月21日受理)

遠心分離法によるウラン濃縮プラントにどのような保障措置をかけるべきかについては、HEXA-PARTITE プロジェクトにおいて議論が行われ、「頻度限定・無通告」方式による査察を行うことで合意が得られている。大型商用施設に対しても同様の手法が採用されると考えられるが、このような施設では濃縮能力の拡大、機微な情報を保護することの重要性から、適用すべき保障措置の態様については十分な検討が必要とされる。これは、遠心分離法による大型商用ウラン濃縮プラントの保障措置システム設計のために行った、モデル施設の工程シミュレーション研究の結果得られた成果の一部を取りまとめたものである。

商用濃縮プラントを建設する場合に遭遇する悩ましい問題の一つに、軽水炉用燃料製造のために必要とされる広い範囲にわたる濃縮度の低濃縮ウランを得るには、どのような濃度の設計プロダクトで生産すべきかという問題がある。本研究では、この問題に関する有用な対処法について考察する。

まず、モデル遠心分離カスケードの起動時の過渡特性について解析する。ここで用いる動特性方程式は、特に、過渡状態にあるカスケード内流量とその濃度を正確に求めることができるように開発したものである。次に、カスケードの特性パラメーターであるカット、還流流量、カスケードへの原料供給流量を操作することで、どのような範囲までプロダクト濃度を変更させることが可能であるかを調べ、そこから適切な操作モードについての知見を得る。

本研究の結果、顧客からの各種濃縮度の低濃縮ウランの需要は、運転中のカスケード効率の低下を90%まで許容すると、一種類の単位カスケードの構成でほとんどかなえられることが明らかになった。

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## Symbols

- L : Inlet flow rate  
L' : Enriched flow rate  
L'' : Depleted flow rate  
f' : Interstage feed flow rate to the path of enriched flow between stages  
f'' : Interstage feed flow rate to the path of depleted flow between stages  
p' : Interstage removal flow rate from the path of enriched flow between stages  
p'' : Interstage removal flow rate from the path of depleted flow between stages  
N : Enrichment of inlet flow  
N' : Enrichment of enriched flow  
N'' : Enrichment of depleted flow  
n' : Enrichment of the interstage feed flow to the path of enriched flow between stages  
n'' : Enrichment of the interstage feed flow to the path of depleted flow between stages  
h : Holding time for the nuclear material running through the inlet path and centrifuge machines  
h' : Holding time for the nuclear material running through the path of enriched flow  
h'' : Holding time for the nuclear material running through the path of depleted flow  
 $\theta$  : Cut for centrifuge machines  
 $\alpha$  : Head separation factor  
 $\beta$  : Tail separation factor  
j : j-th stage of cascade

## 1. Introduction

Recently, enriching uranium by a laser method [1,2,3] has rapidly been in the spotlight and is gathering high expectations as a subsequent new enrichment technology. On the other hand, in Japan, a commercial enrichment plant is being constructed [6] on a basis of the centrifuge technology [4,5] developed over a long period of time. Since the design product enrichment in an enrichment plant has to be adjusted to the enrichments required by operators of low enriched uranium fuel fabrication facilities for providing LWRs with their nuclear fuel, it is meaningful to discuss about this matter from the viewpoint of efficient operations of a plant at the stage of constructing commercial enrichment plants.

Initial loading fuel or replacement fuel to LWRs, in general, needs to have a wide range of enrichment spectrum, and there is a tendency for the enrichment to shift to higher one for attaining much higher burn-up. One possibility to cope with such situation is to produce LEU with several different enrichments and to blend them properly to attain a suitable enrichment.

In this study, consideration is paid to a method in which the design product enrichment is changed and adjusted to a desired one by regulating characteristic parameters of cascade. From this result the knowledge is obtained on an operation mode suitable for the adjustment of enrichment of low enriched uranium, a separation characteristic required for centrifuge machines, and on basic elements to determine the kind of cascade to be constructed.

## 2. Flow Equations and its Calculation Method for a Transient Cascade

In a cascade dynamics equation, it is often assumed that the flow rate is constant independent from time [7,8,9]. This assumption, however, is incorrect when the flow rate is greatly changed within the cascade by the start-up, shut-down or other operations. Since the variation of flow rate plays an especially important role in the dynamics analysis of centrifuge cascade, it is necessary to use a dynamics equation in which such consideration is fully observed [10]. Here the calculational procedure is described briefly.

Fig. 1 shows a generalized flow diagram where there are removal/feed of nuclear material from/to an intermediate stage. For each stage there are removal,  $p'j$  (or  $p''j$ ), and feed,  $f'j$  (or  $f''j$ ), of nuclear material from/to the path of enriched flow (or depleted flow) between stages, and  $Bj$  and  $Cj$



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are a junction of the flows with identical or different enrichment.  $A_j$  is a junction of the flows with different enrichment. Since the flows,  $p'^j$  and  $f'^j$  (or  $p''^j$  and  $f''^j$ ), in each path may be used to connect cascades or to construct a recycle flow, this cascade model makes it possible to analyze the dynamics of the cascade with more complicated flow paths. In the figure, a recycle flow is, as an example, shown which connects the flow,  $p'^{j-1}$ , from the path of enriched flow of the  $(j-1)$ -th stage with the flow,  $f''^j$ , to the path of depleted flow of the  $j$ -th stage.

For the cascade with such flows, the basic equation for the inlet flow rate,  $L_j$ , in the  $j$ -th stage is described as follows:

$$L_j(t) = L'_j(t + h_j + h'_j) + L''_j(t + h_j + h''_j) \quad (1)$$

$$\theta_j = L'_j(t + h_j + h'_j) / L_j(t) \quad (2)$$

$$L_j(t) = L'_{j-1}(t) + L'_{j+1}(t) + f'_{j-1} + f''_{j+1} - p'_{j-1} - p''_{j+1} \quad (3)$$

Here it is assumed that it is possible to omit the second order differential terms with regard to time and that the cut and the holding time are constant independent from changes of the flow rate. Then, the dynamics equation for the enriched flow rate,  $L'_j$ , is described as follows:

For stages from the upper most stage in the enriching section ( $j=S$ ) to the first stage,

$$\begin{aligned} -L'_S(t) + L'_{j-1}(t) - L'_j(t) + \sum_{i=j-1}^{S-1} (f'_i - p'_i) + \sum_{i=j+1}^S (f''_i - p''_i) \\ \approx \sum_{i=j}^S Q_i \frac{dL'_i(t)}{dt} \quad (4) \\ (j=S, \dots, 2, 1) \end{aligned}$$

For stages from the feed stage ( $j=0$ ) to the  $(-(B-1))$ -th stage,

$$\sum_{\ell=1}^k F_{\ell} L'_S(t) + L'_{j-1}(t) - L'_j(t) + \sum_{i=j-1}^{S-1} (f'_i - p'_i) + \sum_{i=j+1}^S (f'_i - p'_i) \approx \sum_{i=j}^S Q_i \frac{dL'_i(t)}{dt} \quad (5)$$

$$[ j = 0, -1, -2, \dots, -(B-1) ]$$

For the bottom stage of the stripping section ( $j=-B$ ),

$$\sum_{\ell=1}^k F_{\ell} L'_S(t) - L'_{-B}(t) + \sum_{i=-B}^{S-1} (f'_i - p'_i) + \sum_{i=-(B-1)}^S (f'_i - p'_i) \approx \sum_{i=-B}^S Q_i \frac{dL'_i(t)}{dt} \quad (6)$$

where,

$$L'_j(t) = G_j(t) - L'_j(t) + (h'_j - h''_j) (K_j - 1) \frac{dL'_j(t)}{dt} \quad (7)$$

$$G_j(t) = \frac{1}{\theta_j} \cdot L'_j(t) \quad (8)$$

$$K_j = \frac{1}{\theta_j} \quad (9)$$

$$Q_j = h'_j - h''_j + (h_j + h''_j) K_j \quad (10)$$

Here,  $L'_S(t)$  and  $L''_{-B}(t)$  are corresponding to the product flow rate,  $P(t)$ , and the tail flow rate,  $W(t)$ , respectively. In order to solve these equations, they are converted to difference equations as shown in Appendix. Likewise, similar difference equations are obtained for the inlet flow rate,  $L'_j(t)$ , and for the depleted flow rate,  $L''_j(t)$ . These flow rates, however, can be solved in principle using the numerical solutions of  $L'_j(t)$  and  $L'_j(t + \Delta t)$ .

In general, variation of the inlet flow rate at each stage of the cascade causes the changes of processed flow rate for each centrifuge machine set up at that stage. Therefore, depending on the fluctuation of this processed flow rate, the separation characteristics [11] of the centrifuge machine should be calculated at the same time for the separation factors,  $\alpha_j(t)$  and  $\beta_j(t)$  at the time,  $t$ . Then, using these  $\alpha_j(t)$ ,  $\beta_j(t)$ ,  $L_j(t)$ ,  $L'_j(t)$ , and  $L''_j(t)$ , the time dependent changes of enrichment are calculated. Here, the dynamics equation for the enrichment,  $N'_j(t)$ , in the enriched flow at the  $j$ -th stage can be easily derived in the same way as for the flow rate,  $L'_j(t)$  [12]. Furthermore, it has been devised that the cascade feed flow rate,  $F$ , and the flow rates of  $p'_j$ ,  $p''_j$ ,  $f'_j$  and  $f''_j$  can be given separately in the computer code (CCS-I: Centrifuge Cascade Simulation - I) the authors have developed.

### 3. Accurate Product Enrichment Profile at a Cascade Start-up

In a commercial enrichment plant, quite a lot of unit cascades are constructed, each of which is a minimum unit of cascade with small separative work unit (SWU), and the enriched uranium is produced by operating these unit cascades in parallel. As an example of this unit cascade, Fig. 2 shows the cascade of 20 ton SWU/y. The number of model centrifuge machines for each stage is proportional to the size of the segment described in the figure and both the enriching section and the stripping section are composed of about the same number (1430) of centrifuge machines. The cascade, however, is not an ideal cascade in a strict meaning because the cut for each stage is assumed to be identical to that of centrifuge machines. Below, a transient characteristic of this unit cascade is investigated at the initial feeding phase (start-up) [13].

According to a given start-up procedure, the cascade is supplied with natural uranium and after 60 minutes the designed feed flow rate of  $F=26.19$  ton U/y is reached. Fig. 3 shows the changes of inlet flow rate at each stage for such start-up operation. The figure displays a way of transferring the feed material step by step among stages from the feed stage to the top stage of the enriching section on one hand and to the bottom stage of the stripping section on the other hand. The removal of tails from this cascade starts about 50 minutes after the start-up of feeding and the product is produced a little later than the tails because of a greater number of stages in the enriching section. At the stationary state, the product flow rate reaches to  $P=4.13$  ton U/y.

On the other hand, corresponding to such variation of inlet flow rates the gradient of enrichment within the cascade is changed rapidly. Fig. 4

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shows such changes to the elapsed time as a parameter. At the stationary state ( $t = \infty$ ), the enrichment of enriched flow at the top stage of the enriching section,  $N'_{12}$  ( $=N_p$ ) accurately reaches to the design value of 3.345%. Here it should be noted that the gradient of enrichment is shown by a dotted line ( $t=200$  min.) if the inlet flow rate to each stage is assumed to be always constant even at the time of start-up. It is quite clear that there is a big difference if this gradient is compared with the corresponding solid line which is the result of the numerical calculations carried out under the accurate assumption. It is noted that this fact has become clear only by accurately calculating time dependent flow rates and by simultaneously solving the equations for enrichment. In this sense, the calculation model adopted here would more accurately simulate the dynamics of a real cascade.

At the start-up of the cascade, product of higher enrichment (<20%) than the designed one is temporarily produced. Fig. 5 shows the amount of product and its average enrichment. Here the dotted line shows an approach to the stationary value of the flow rate of product from the top stage of the cascade, and the solid line gives the integrated amount of product corresponding to the elapsed time.

On the other hand, from the figure it is understood that the product enrichment is homogenized and averaged at such a collection stage as the cold trap and is retrieved into a cylinder as normal low enriched uranium, because a time-dependent behavior of the average product enrichments given at several points of time on the solid line shows this fact. For example, the average product enrichment is 5.9% and the amount of product is about 3000 gU at the time when the product flow becomes nearly stationary ( $t=600$  min.).

#### 4. Change of a Design Product Enrichment by regulating Cascade Parameters

It is possible to produce a product which has enrichment different from the design by regulating characteristic parameters of the cascade [14,15]. Consideration is paid here to have a product with the desired enrichment by regulating the cascade cut, a recycle flow rate or the feed flow rate. Firstly, the changes of inlet flow distribution within the cascade are investigated for the case where the designed cut for each stage of the cascade is changed simultaneously to another cut. As a mode of changes of the cascade cut, typical three cases are selected, i.e., changes for all stages of the cascade (CASE-I), changes for all stages of the enriching section (CASE-II), and changes for all stages of the stripping section (CASE-III). Fig. 6 shows the inlet flow distributions for the CASE-I where the designed cut is changed to  $\theta = 0.44, 0.5$  and  $0.53$ . It is shown here that

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the inlet flows distribute linearly against the stage numbers for the case of the cut being 0.5 for all stages of the cascade because the enriched flow rate is identical to the depleted flow rate for each stage. For a smaller cut than 0.5, a reduction rate of total inlet flow rate within the enriching section is larger than that of the stripping section. Fig. 7 shows the changes of total inlet flow rate within the cascade for such changes of the cut. In this figure, however, the total inlet flow rate and the cut are normalized by the corresponding operational values. While in CASE-III the total inlet flow rate is increased when the cut is increased, in CASE-I the total inlet flow rate turns out to be maximum when the cut,  $\theta^*$ , is 1.085. In CASE-II the total inlet flow rate is not so much changed by the cut,  $\theta^*$ .

Fig. 8 shows an effect on the product enrichment of the changes of total inlet flow rate within the cascade. In general, the product enrichment becomes high if the cut is decreased. Especially this tendency is remarkable for CASE-I and CASE-II. If a feature of the inlet flow distribution given in Fig. 6 is taken into consideration, it is understood that this shows the separation of two-component gas has further stepped forward in the enriching section. If the cut regulation is used for adjusting the product enrichment, CASE-III is superior among three cases because the control would be easy due to slow changes against the cut regulation.

Secondly, changes of the product enrichment are investigated for the case of recycle flow where a part of the enriched flow rate at the  $j$ -th stage,  $p^j$ , is connected with  $f^{j-1}$ , feed flow rate to the path of enriched flow at the adjacent  $(j-1)$ -th stage. Here a ratio of  $p^j$  to  $L^j$  is defined as the recycle flow rate. Relationship between this recycle flow rate and the product enrichment is shown in Fig. 9 for each of the stages in the enriching section ( $j=3, 7, 9, \text{ and } 11$ ). This figure shows that the product enrichment is more gradually changed at a stage near the top one in the enriching section. It also shows that the product enrichment is rapidly increased if the recycle flow rate is increased at any stages. In this case, however, the product enrichment may be limited because of the mechanism for separation in a centrifuge machine.

The cut regulation and the recycle flow regulation described above are a special method for adjusting the product enrichment, not usually applied to a normal enrichment plant. Against these two methods, the easiest possible is to control the feed flow rate to the cascade. Fig. 10 shows the dependency on the feed flow rate of the product enrichment for this case. The figure shows that the product enrichment is gradually increased when the feed flow rate is decreased.



## 5. Investigations of Cascade Efficiency and an Adjustable Range of Product Enrichment

In the previous section, changes of the product enrichment are investigated against the control of one of three parameters. In any case, the separative work units of the cascade are decreased because of the departure from the optimal operational conditions. Namely the efficiency of the cascade is decreased. For example, as shown in Fig. 10, the efficiency declines if the feed flow rate is changed from the optimal flow rate, while the efficiency is 100% at the normal feed flow rate.

If the cascade efficiency is permitted to decrease to 90% in operation, a curve of the cascade efficiency shown in Fig. 10 gives evidence to the possibility of adjusting the product enrichment from the lower limit of 2.66% to the upper limit of 3.83%. It is understood that the smaller is the decrease of separation power of a centrifuge machine against the change of feed flow rate, the wider range of enrichment adjustment is possible. Fig. 11 shows the relationship between an adjustable range of product enrichment and the cascade efficiency. It shows that in CASE-III, where the cut is regulated at all stages in the stripping section, the widest range (Region I) of product enrichment is possible among three cases of parameter control, while a high efficiency of the cascade may be maintained. On the other hand, in the case of control of the recycle flow rate of enriched flow, it is possible to adjust the product enrichment within the range (Region II) from the designed enrichment,  $N_p = 3.345\%$ , to the upper limit of 3.70%, under the condition of more than 90% cascade efficiency. This adjustable range, however, is included in that of the corresponding feed flow rate regulation (Region III).

Furthermore, under the condition of more than 95% cascade efficiency in operation, an adjustable range of product enrichment is extremely narrow except for CASE-III. Especially, in the case of control of the feed flow rate, a separate unit cascade should be constructed for producing a product with lower enrichment than the designed enrichment. In this way, only if a high cascade capacity is permitted for the plant operation, then types of unit cascades should be increased in order to provide customers with a variety of product enrichments. Therefore, the final decision on how to construct the cascade system as a whole plant should be made based on this fact as well as the blending loss [16], etc.

## 6. Conclusion

The result of the study is summarized as follows:

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The result of the study is summarized as follows:

- (1) Flow equation for a cascade in a non-stationary state has been derived. As a result, it becomes easier to do a design calculation for a cascade with an arbitrary cut for each stage.
- (2) If there are substantial changes in the flow rates within a cascade, the usual constant-flow-rate model cannot give an accurate prediction of dynamic behavior of nuclear material in the cascade. The calculation model developed in this study to cope with such changes of flow rates can give more realistic numerical solution, which has been shown in a case study applied to the start-up operation of the cascade.
- (3) In this study, three methods have been investigated, in each of which a specific parameter of cascade is controlled. In the case of cut regulations, the cut regulation in the stripping section is suitable among three modes for adjusting the production enrichment for low enriched uranium. Besides, it is possible to obtain a wide range of product enrichment with a high cascade efficiency.
- (4) In the case of control of the recycle flow, recycle operations at the top stage in the enriching section is suitable for adjusting the product enrichment for low enriched uranium, compared with other stages, because a range of flow rate control is wide and a change of enrichment is gradual.
- (5) The most realistic method for adjusting the product enrichment for low enriched uranium is to control the feed flow rate to the cascade. In this case, important factors to decide the type of cascades to be constructed for each kind of production are an acceptable efficiency of cascade, a feature of separation for the centrifuge machine, and a spectrum of enrichment to be covered by the cascade, etc.

The following problems should be solved in a future study: i.e., although it is difficult to predict theoretically the relationship between a feed flow rate and a circulating flow rate within a centrifuge machine, separation factor, or holding time because of a complicated gas dynamics, the calculation model should be improved or modified so as to give a genuine value to these parameters. It is, of course, the most idealistic to modify the calculation model directly using the data obtained in an experimental centrifuge plant. Furthermore, if a pulse operation is carried out so as to positively use the sharp increase of the product enrichment at the start-up operation, then a higher enrichment will be obtained as a product. Some theoretical study is also expected with regard to the control of this periodic, pulsed operation in the near future.

## Acknowledgment

This study has been carried out as a part of the design study of a safeguards system for a large scale uranium enrichment facility. The whole study has been guided by his valuable advice of Dr. M. Hirata, Establishment Director of Oarai Research Establishment, Japan Atomic Energy Research Institute, and been supported extensively by Professor R. Kiyose, Tokai University. The authors are greatly thankful both to Dr. Hirata and Professor Kiyose.

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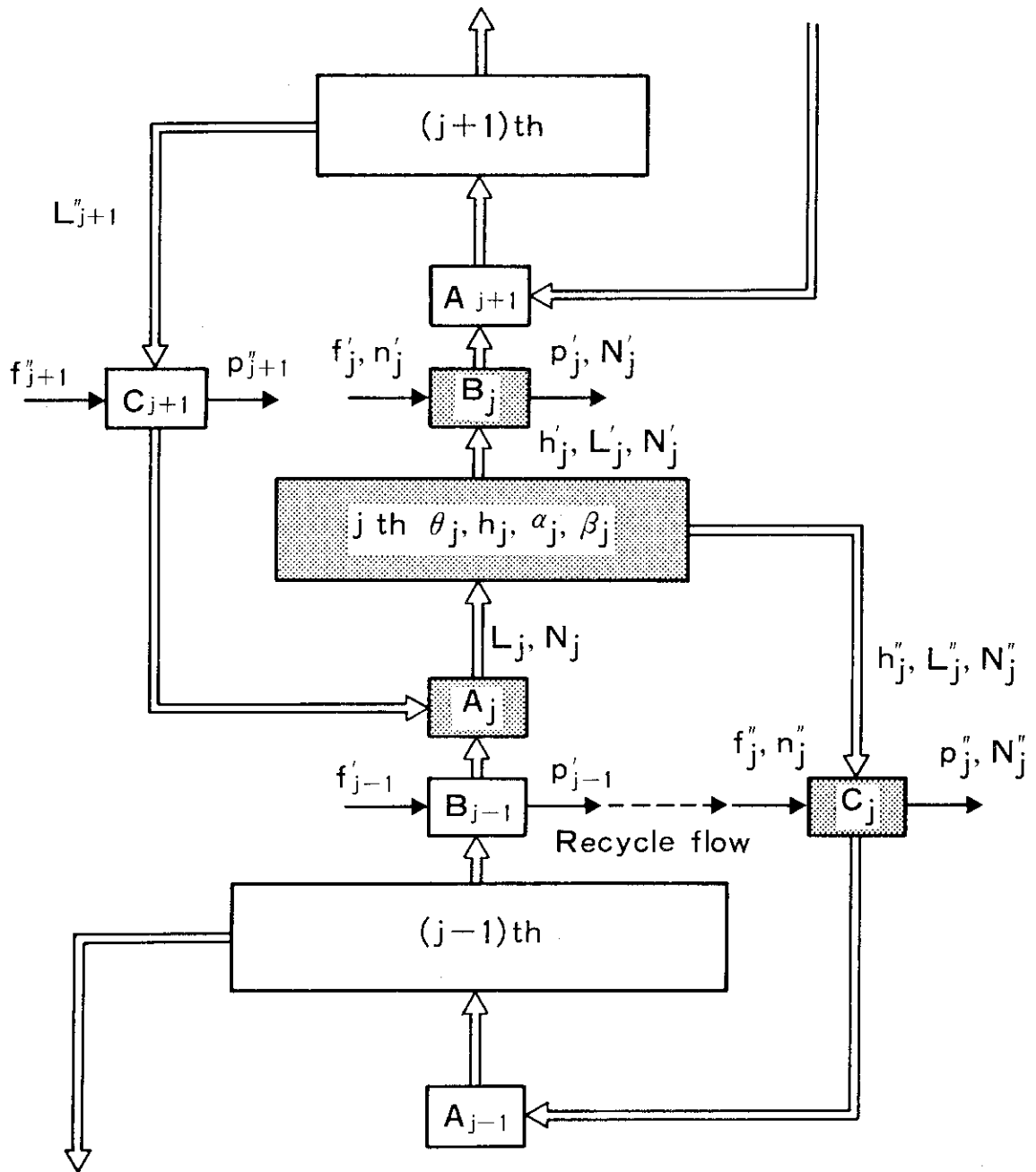


Fig. 1 Generalized flow diagram near by the j-th stage in a cascade

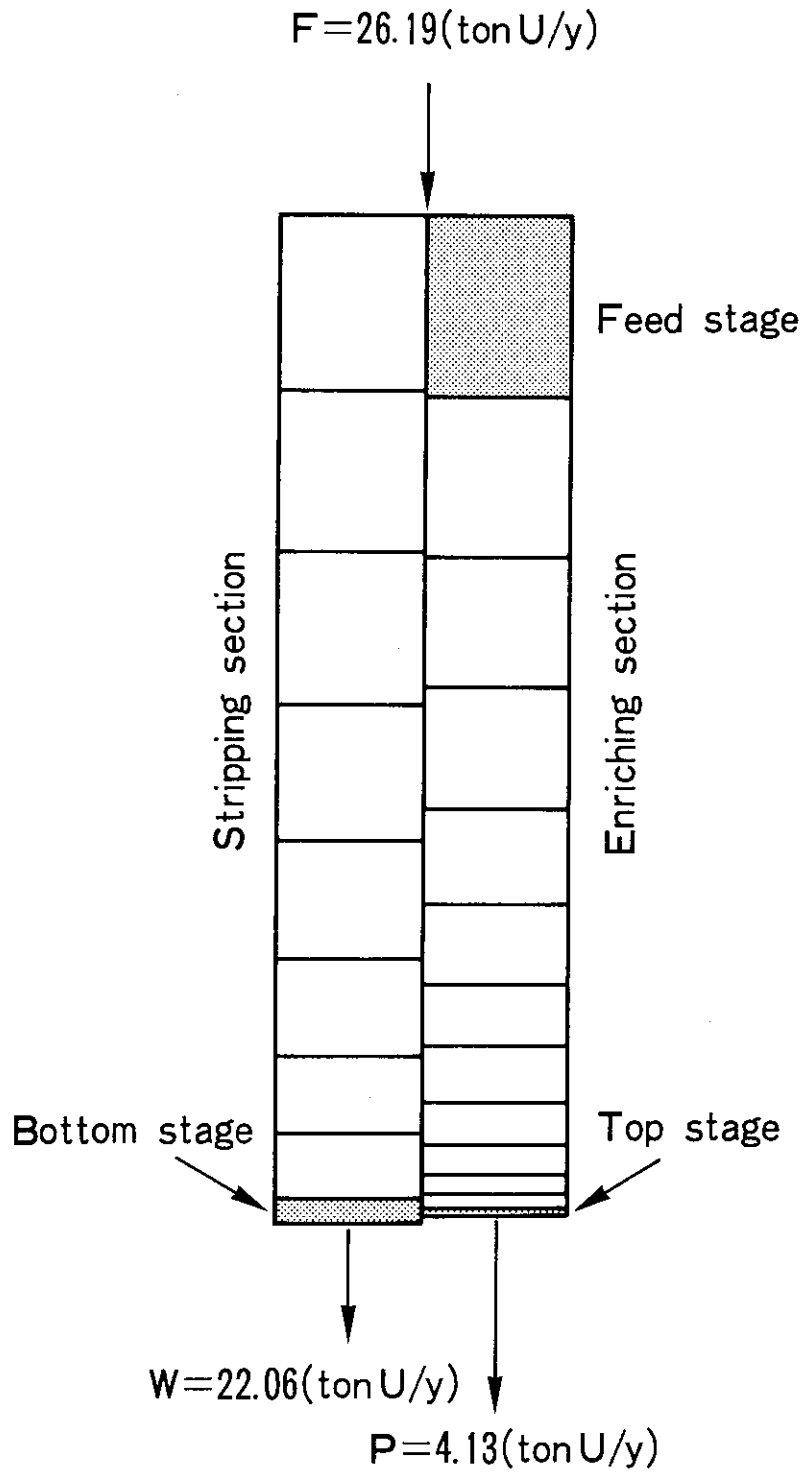


Fig. 2 Example of a unit cascade composed of model  $\text{UF}_6$  gas centrifuges

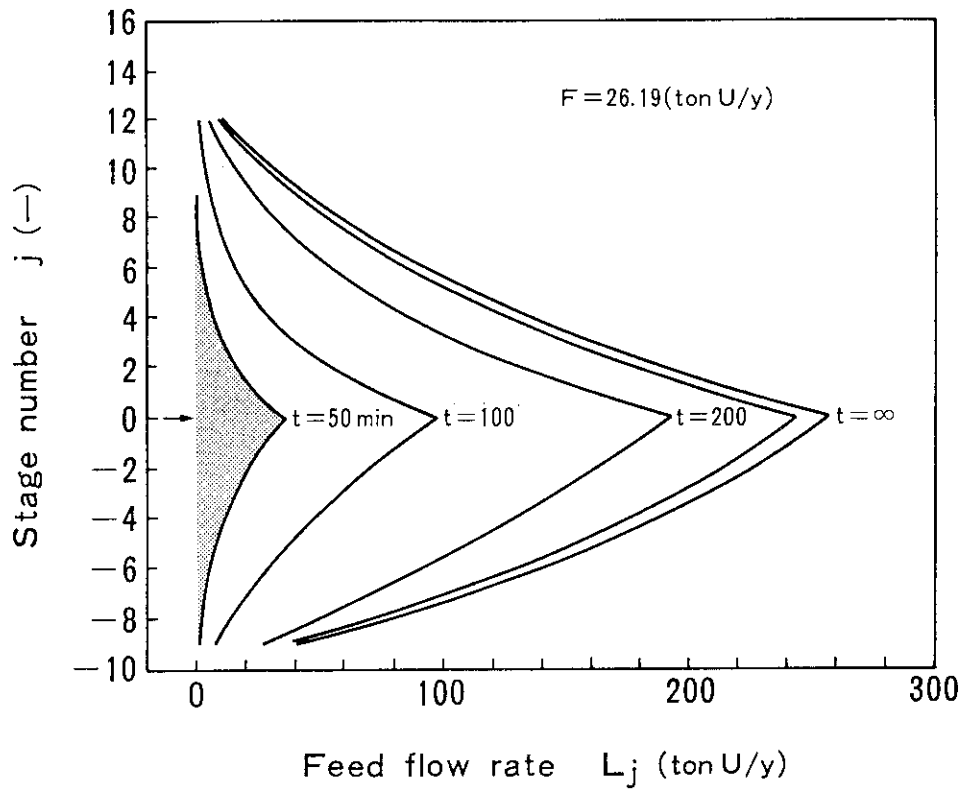


Fig. 3 Feed flow rate profile at each stage after material feeding

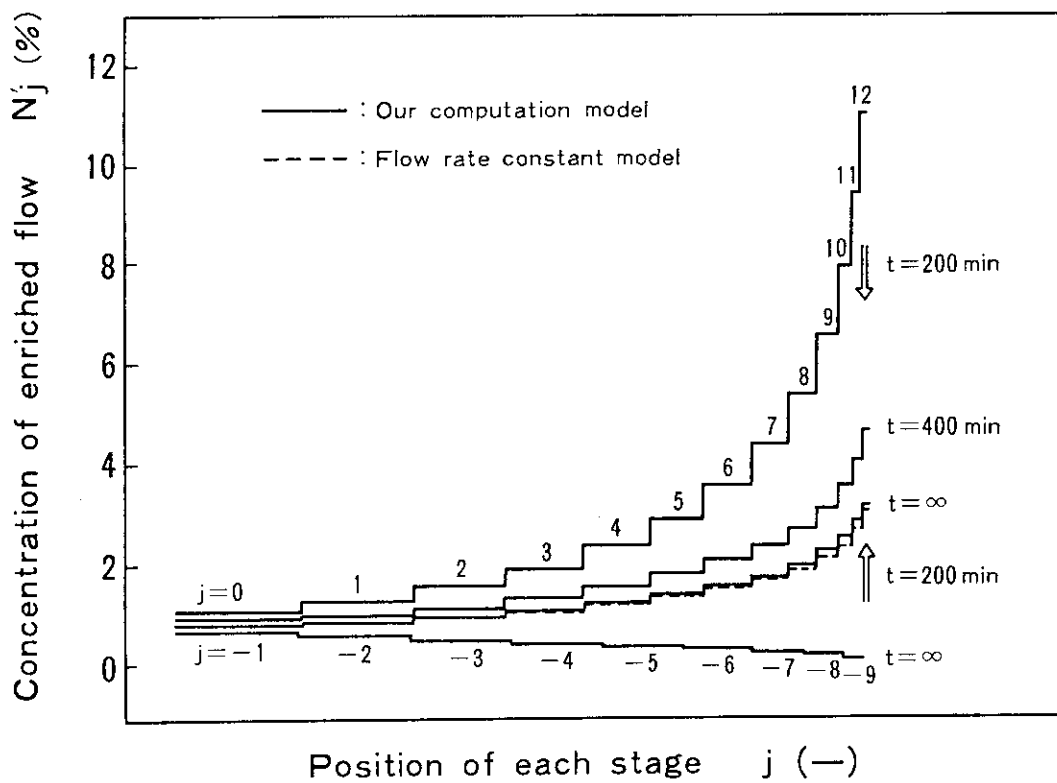


Fig. 4 Transitional profile of concentration gradient in a centrifuge cascade at start-up



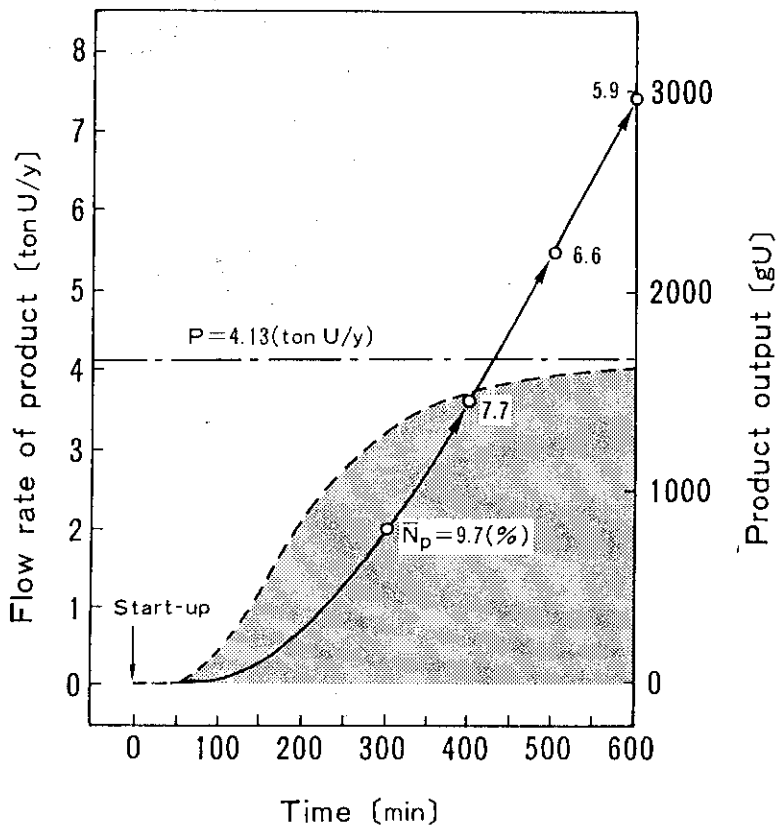


Fig. 5 Momentary change of average product enrichment and its product output

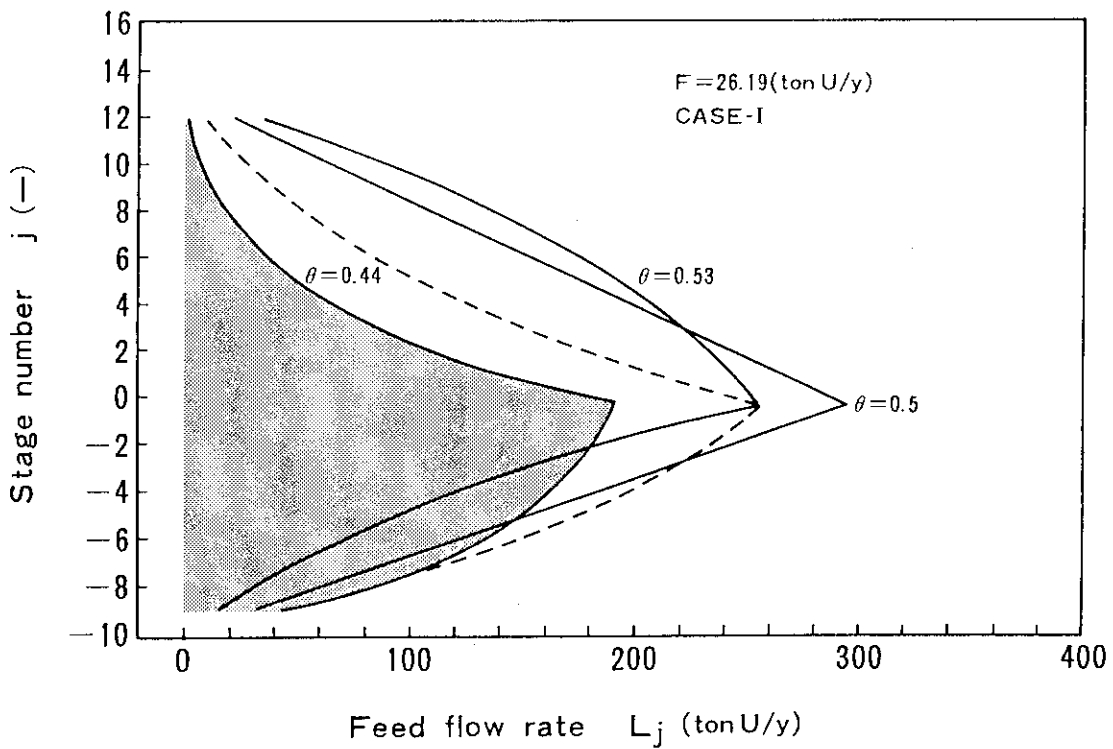


Fig. 6 Flow shape in a unit cascade resulting from cut regulation in all stages

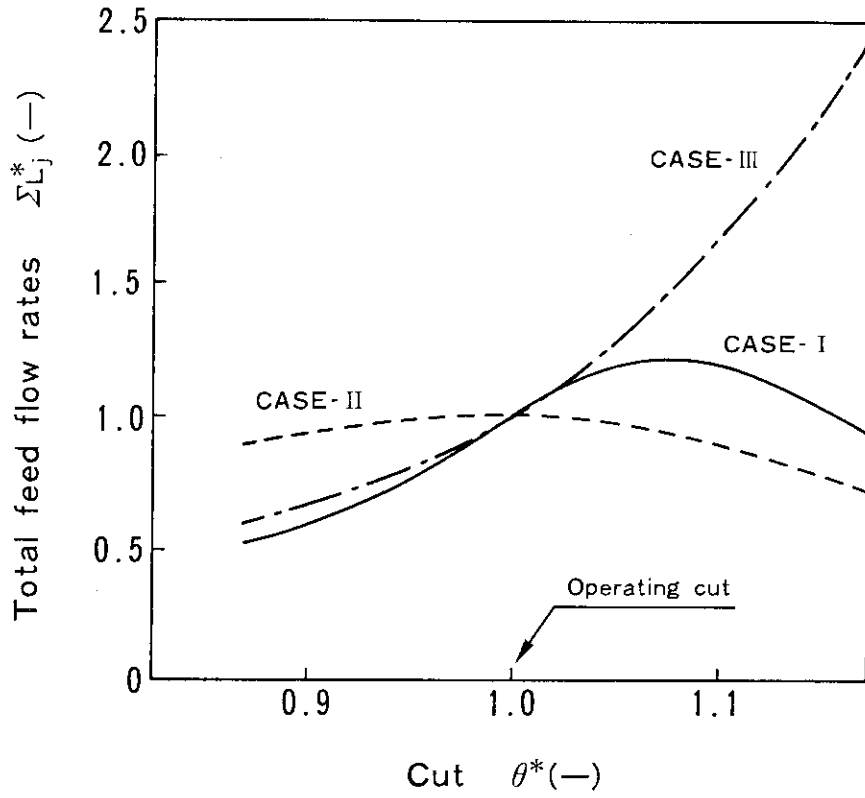


Fig. 7 Change of total feed flow rates vs. cut regulation

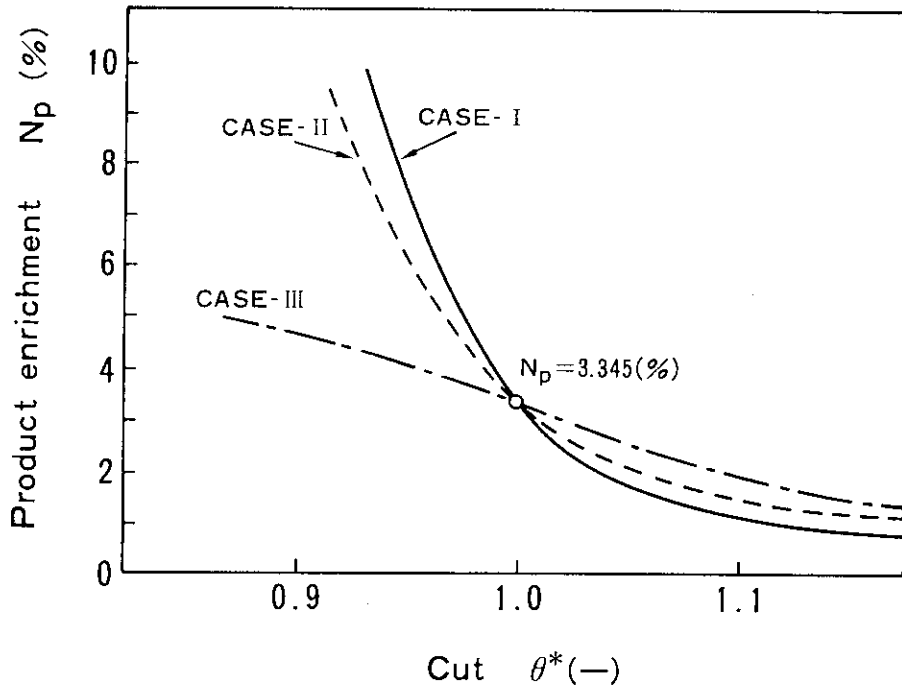


Fig. 8 Effect of cut regulation on product enrichment

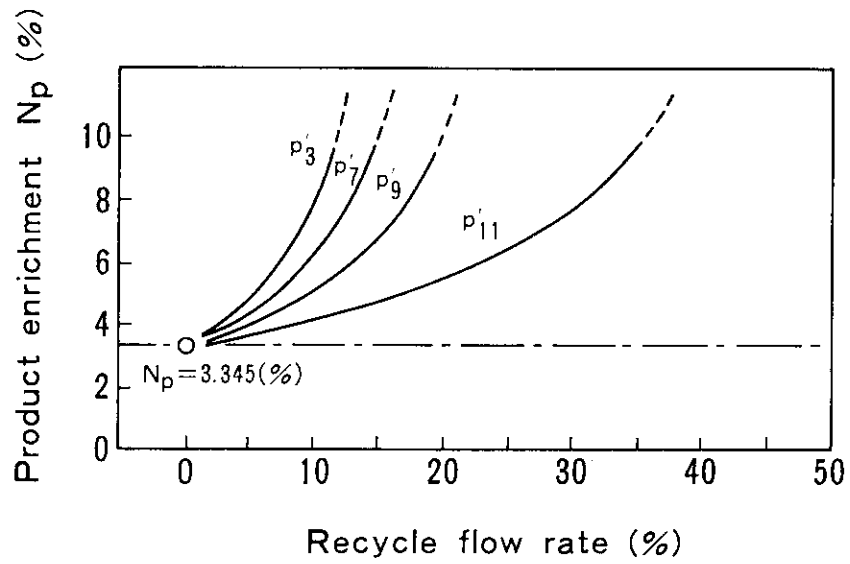


Fig. 9 Difference in enrichment characteristics among recycle stages in enriching section

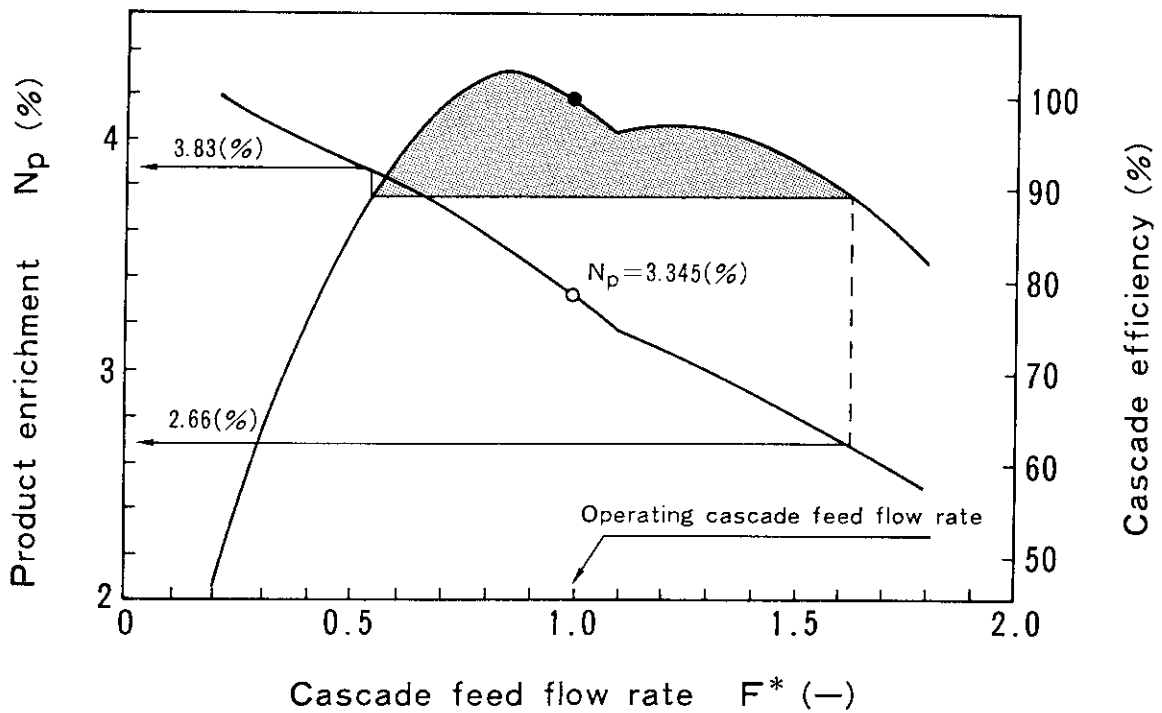
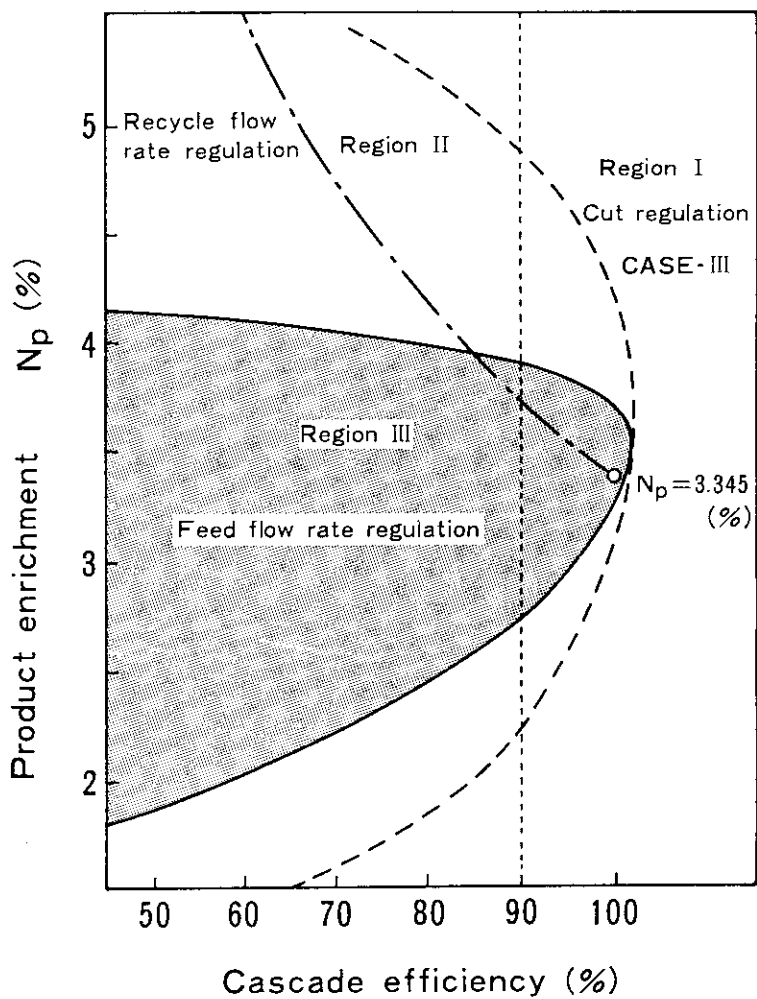


Fig. 10 Dependence of product enrichment on cascade feed flow rate



Region I : Cut regulation in stripping section for CASE-III  
 Region II : Recycle flow rate regulation at 11 th stage  
 Region III : Cascade feed flow rate regulation

Fig. 11 Relation between cascade efficiency and adjustable range of product enrichment

Appendix Difference Equation on an Enriched Flow at each Stage of a Cascade

[Enriching section]

$$L'_S(t+\Delta t) \simeq L'_S(t) + \frac{1}{(h_S+h'_S)K_S} \left[ L'_{S-1}(t) - G_S(t) + f'_{S-1} - p'_{S-1} \right] \Delta t$$

(j = S) (A 1)

$$L'_j(t+\Delta t) \simeq L'_j(t) + \frac{1}{(h_j+h'_j)K_j} \left[ \left\{ -L'_S(t) + L'_{j-1}(t) - G_j(t) \right. \right. \\ \left. \left. + L'_j(t) + \sum_{i=j-1}^{S-1} (f'_i - p'_i) + \sum_{i=j+1}^S (f''_i - p''_i) \right\} \Delta t \right. \\ \left. - \sum_{i=j+1}^S Q_i \left\{ L'_i(t+\Delta t) - L'_i(t) \right\} \right]$$

(A 2)

[ j = 1, 2, ..., (S-1) ]

[Feed stage]

$$L'_0(t+\Delta t) \simeq L'_0(t) + \frac{1}{(h_0+h'_0)K_0} \left[ \left\{ \sum_{\ell=1}^k F_\ell(t) - L'_S(t) + L'_{-1}(t) \right. \right. \\ \left. \left. - G_0(t) + L'_0(t) + \sum_{i=-1}^{S-1} (f'_i - p'_i) + \sum_{i=1}^S (f''_i - p''_i) \right\} \Delta t \right. \\ \left. - \sum_{i=1}^S Q_i \left\{ L'_i(t+\Delta t) - L'_i(t) \right\} \right]$$

(A 3)

(j = 0)

[Stripping section]

$$\begin{aligned}
 L'_{-1}(t+\Delta t) \simeq & L'_{-1}(t) + \frac{1}{(h_{-1}+h'_{-1})K_{-1}} \left[ \left\{ \sum_{\ell=1}^k F_{\ell}(t) - L'_S(t) + L'_{-2}(t) \right. \right. \\
 & - G_{-1}(t) + L'_{-1}(t) + \sum_{i=-2}^{S-1} (f'_i - p'_i) + \sum_{i=0}^S (f''_i - p''_i) \left. \right\} \Delta t \\
 & \left. - \sum_{i=0}^S Q_i \left\{ L'_i(t+\Delta t) - L'_i(t) \right\} \right] \quad (A4) \\
 & (j = -1)
 \end{aligned}$$

$$\begin{aligned}
 L'_j(t+\Delta t) \simeq & L'_j(t) + \frac{1}{(h_j+h'_j)K_j} \left[ \left\{ \sum_{\ell=1}^k F_{\ell}(t) - L'_S(t) + L'_{j-1}(t) - G_j(t) \right. \right. \\
 & + L'_j(t) + \sum_{i=j-1}^{S-1} (f'_i - p'_i) + \sum_{i=j+1}^S (f''_i - p''_i) \left. \right\} \Delta t - \sum_{i=0}^S Q_i \times \\
 & \left. \left\{ L'_i(t+\Delta t) - L'_i(t) \right\} - \sum_{i=j+1}^{-1} Q_i \left\{ L'_i(t+\Delta t) - L'_i(t) \right\} \right] \quad (A5) \\
 & [j = -2, -3, \dots, -(B-1)]
 \end{aligned}$$

$$\begin{aligned}
 L'_{-B}(t+\Delta t) \simeq & L'_{-B}(t) + \frac{1}{(h_{-B}+h'_{-B})K_{-B}} \left[ \left\{ \sum_{\ell=1}^k F_{\ell}(t) - L'_S(t) - G_{-B}(t) \right. \right. \\
 & + L'_{-B}(t) + \sum_{i=-B}^{S-1} (f'_i - p'_i) + \sum_{i=-(B-1)}^S (f''_i - p''_i) \left. \right\} \Delta t \\
 & - \sum_{i=0}^S Q_i \left\{ L'_i(t+\Delta t) - L'_i(t) \right\} - \sum_{i=-(B-1)}^{-1} Q_i \left\{ L'_i(t+\Delta t) \right. \\
 & \left. - L'_i(t) \right\} \left. \right] \quad (A6) \\
 & (j = -B)
 \end{aligned}$$

where,  $\Delta t$  is a time interval for numerical calculations.