JAERI-M
89-101

THE EFFECT OF THE VIRTUAL MASS FORCE TERM ON THE STABILITY OF TRANSIENT TWO-PHASE FLOW ANALYSIS

August 1989

Tadashi WATANABE, Masashi HIRANO and Fumiya TANABE

日本原子力研究所
Japan Atomic Energy Research Institute

編集兼発行　日本原子力研究所
印　　刷　㈱高野高速印刷

The Effect of the Virtual Mass Force Term on the Stability of Transient Two-Phase Flow Analysis

Tadashi WATANABE, Masashi HIRANO and Fumiya TANABE

Department of Reactor Safety Research
Tokai Research Establishment
Japan Atomic Energy Research Institute
Tokai-mura, Naka-gun, Ibaraki-ken

(Received July 14, 1989)

The effect of the virtual mass force term on the stability of transient two-phase flow analysis is studied. The objective form of the virtual mass acceleration is used. The virtual mass coefficient is determined from the stability condition of basic equations against infinitesimal high wave-number perturbations. The parameter is chosen so that a reasonable agreement between the analytical and experimental sound speed in two-phase flows can be obtained. A one-dimensional sedimentation problem is simulated by the MINCS code which is a tool for transient two-phase flow analysis. The stability analysis is performed for the numerical procedure. It is shown that calculated results are stabilized so long as the virtual mass coefficient satisfies the stability condition of differential equations.

Keywords: Virtual Mass Force Term, Stability, MINCS, Transient Two-Phase Flow Analysis, Sedimentation

過渡二相流解析の安定性に及ぼす付加質量項の効果

日本原子力研究所東海研究所原子炉安全工学部
渡辺正・平野雅司・田辺文也

（１９８９年７月１４日受理）

過渡二相流解析の安定性に及ぼす付加質量項の効果を検討した。付加質量項として座標変換に対して不変となる形式のものを彩用した。付加質量係数は，微小で高周波の変動に対する基礎方式の安定性の条件から決定した。項中のパラメーターは，二相流中の音速の理論値と実験値が一致するように決定した。数値計算例として一次元の気液置換問題をＭＩＮＣＳコードを用いて解析した。数値解法に対する安定性解析も行なった。その結果，付加質量係数が微分方程式の安定条件を満足している限り，安定な数値解が得られることが明らかとなった。

東海研究所：〒３１９－１１　茨城県那珂那東海村白方字白根２－４

Contents

目　　次

Nomenclature

a, b, e, f	element of matrix
a_{vm}	virtual mass acceleration
A, B, C, E, F	coefficient matrix
c_{tp}	sound speed in two-phase flow
C	$\Delta t/\Delta x(1-e^{-I\theta})$
C_{vm}	virtual mass coefficient
D	vector of dependent variables
f_w	wall friction term
F_{vm}	virtual mass force term
g	gravity
I	imaginary unit
K	wave number ($2\pi/\Lambda$)
M	momentum transfer term
p	pressure
t	time
u	velocity
x	flow direction
X	vector of dependent variables

Greek

α	volume fraction or void fraction
θ	$K\Delta x$
δ	perturbation
Γ	mass transfer rate
φ	perturbation
Φ	field vector
λ	parameter of virtual mass force term
Λ	wavelength
μ	characteristics or eigenvalue
ρ	density

Subscript

c	continuous phase
d	dispersed phase
g	gas phase
i	interface
k	phase index
l	liquid phase

1. Introduction

It is important for safety analyses of nuclear reactors to accurately and efficiently predict transient two-phase flow phenomena under widely ranging conditions. A two-fluid model is considered to be appropriate for the most general and detailed description of two-phase flow since non-equilibrium exchange between phases can be easily included in the analysis. In the basic form of the two-fluid model, each phase is separately described in terms of a set of conservation equations. Interaction terms between two phases appear in the basic field equations as transport terms across interfaces.

The virtual mass force term is one of such transient interaction terms between phasic momentum equations. It contains derivatives of dependent variables which play an important role to determine mathematical characteristics of differential equation system. Andersen et al. [1] have studied the consequences of virtual mass force term on the stability of differential equation system against infinitesimal high wave-number perturbations. They found that the differential equation system was almost stable when the virtual mass coefficient was sufficiently large. The virtual mass acceleration was, however, not objective and there remained some unstable regions even if an extremely large coefficient was used. It has been shown by Drew et al. [2] that the virtual mass force must be objective, that is, the virtual mass acceleration should be invariant under changes of frame of reference. They pointed out that the inclusion of it was able to improve numerical efficiency by changing the nature of eigenvalues of the differential equation system of conservation laws describing the two-phase flow. Afterwards, the combination of the virtual mass force and the lift force has been shown to be objective by Drew and Lahey [3]. They pointed out that neither the virtual mass force nor the lift force was solely objective.

Lahey et al. [4] have numerically studied various nozzle/diffuser flow under steady state conditions. They reported that the inclusion of virtual mass effects into the analysis of two-phase flow would be a physically realistic way to improve numerical stability and efficiency, and to achieve accurate results in many cases of practical concern. The effect of virtual mass on the prediction of critical flow has been investigated by Cheng et al. [5]. The Moby Dick experiment was numerically simulated and the calculated results were compared with the experimental data. They used different values for virtual mass parameters to obtain critical flow in accordance with the experimental conditions. The effect of virtual mass coefficient on the numerical stability has been studied by No and Kazimi [6]. The real characteristics region of the differential equation system as the stability condition was shown in terms of the virtual mass coefficient under some simplification. They recommended that the virtual mass force term be included in all two-fluid models since it could stabilize the differential equation system with real physical effects.

In this paper, the effect of the virtual mass force term on the stability of transient two-phase flow analysis is studied. The objective form of the virtual mass acceleration is used. The virtual mass coefficient is determined from the stability condition of basic equations against infinitesimal high wave-number perturbations. The parameter is chosen so as to obtain a reasonable agreement between the analytical and experimental sound speed in two-phase flows. Numerical simulations are performed on a one-dimensional sedimentation problem by using the MINCS code which is a tool for transient two-phase flow analysis. A stability analysis is carried out for the numerical procedure.

2. Determination of virtual mass force term

2.1 Virtual mass acceleration

The one-dimensional momentum equation for transient two-phase flow is written as

$$\alpha_k \rho_k \frac{D_k u_k}{Dt} = -\alpha_k \frac{\partial p}{\partial x} - f_{wk} + M_{ik} - \alpha_k \rho_k g + (u_i - u_k)\Gamma_k \ , \qquad (1)$$

where α is the volume fraction, ρ the density, p the pressure, g the body force, u the velocity, f_w the wall friction, and Γ and M_i respectively are the interfacial mass and momentum transfer rates. In Eq.(1), the subscript k and i denote phases and the interface, respectively. The interfacial momentum transfer rate M_i contains the standard drag force, investigated by Cheng et al. [5]. The Moby Dick experiment was numerically simulated and the calculated results were compared with the experimental data. They used different values for virtual mass parameters to obtain critical flow in accordance with the experimental conditions. The effect of virtual mass coefficient on the numerical stability has been studied by No and Kazimi [6]. The real characteristics region of the differential equation system as the stability condition was shown in terms of the virtual mass coefficient under some simplification. They recommended that the virtual mass force term be included in all two-fluid models since it could stabilize the differential equation system with real physical effects.

In this paper, the effect of the virtual mass force term on the stability of transient two-phase flow analysis is studied. The objective form of the virtual mass acceleration is used. The virtual mass coefficient is determined from the stability condition of basic equations against infinitesimal high wave-number perturbations. The parameter is chosen so as to obtain a reasonable agreement between the analytical and experimental sound speed in two-phase flows. Numerical simulations are performed on a one-dimensional sedimentation problem by using the MINCS code which is a tool for transient two-phase flow analysis. A stability analysis is carried out for the numerical procedure.

2. Determination of virtual mass force term

2.1 Virtual mass acceleration

The one-dimensional momentum equation for transient two-phase flow is written as

$$\alpha_k \rho_k \frac{D_k u_k}{Dt} = -\alpha_k \frac{\partial p}{\partial x} - f_{wk} + M_{ik} - \alpha_k \rho_k g + (u_i - u_k)\Gamma_k \ , \tag{1}$$

where α is the volume fraction, ρ the density, p the pressure, g the body force, u the velocity, f_w the wall friction, and Γ and M_i respectively are the interfacial mass and momentum transfer rates. In Eq.(1), the subscript k and i denote phases and the interface, respectively. The interfacial momentum transfer rate M_i contains the standard drag force, the virtual mass force (M_{vm}), the Basset force, the lift force and so on [7].

The virtual mass force term can be expressed as [2]

$$M_{vm} = \alpha_d F_{vm} \doteq \alpha_d \rho_c C_{vm} a_{vm} \ , \qquad (2)$$

where α_d is the volume fraction of dispersed phase, ρ_c is the density of continuous phase, C_{vm} is the virtual mass coefficient, and the objective virtual mass acceleration a_{vm} is given by

$$a_{vm} = \frac{\partial (u_d - u_c)}{\partial t} + u_d \frac{\partial (u_d - u_c)}{\partial x} + \left[(\lambda - 2)(u_d - u_c)\frac{\partial u_d}{\partial x} + (1-\lambda)(u_d - u_c)\frac{\partial u_c}{\partial x} \right] \ , \qquad (3)$$

where λ is the parameter, and subscripts d and c respectively refer to the dispersed and continuous phases. The lift force is not considered. Equation (3) was derived by Drew et al.[2] under the assumption that the virtual mass force should be objective. They considered a frame which moves relative to another one in a rigid body motion. The change of frame can be expressed by a time dependent vector which expresses a translation and a time dependent tensor which expresses a rigid body rotation. They derived a general expression of the objective accelertion, shown in Eq.(3), where the expression in one frame can be expressed by that in another frame multptied by the rotation tensor.

If we consider not only the virtual mass force but also the lift force, we can obtain the another form of the virtual mass acceleration. In this case, while neither the virtual mass force nor the lift force can not be solely objective, their combination is objective. In the one-dimensional formulation, the interfacial momentum transfer term contains no lift force, and the virtual mass acceleration is [3]

$$a_{vm} = \left[\left(\frac{\partial u_d}{\partial x} + u_d \frac{\partial u_d}{\partial x} \right) - \left(\frac{\partial u_c}{\partial x} + u_c \frac{\partial u_c}{\partial x} \right) \right] \ . \qquad (4)$$

In this study, we use the objective form of Eq.(3), since Eq.(4) does not contribute to the stability of basic equation system as shown in the following section. The virtual mass coefficient C_{vm} and the parameter λ thus have to be determined in a practical use of Eqs.(2) and (3).

2.2 Virtual mass coefficient

We consider the stability of basic differential equations against infinitesimal high wave-number perturbations. The governing equations for the two-fluid model consist of six conservation equations for mass, momentum and energy for both phases. The energy equations are, however, neglected since the characteristics corresponding to the energy equations are the gas and liquid velocities [1]. The momentum equation (1) and the following mass equation are therefore considered:

$$\frac{\partial(\alpha_h\rho_h)}{\partial t}+\frac{\partial(\alpha_h\rho_h u_h)}{\partial x} = \Gamma_h \ , \tag{5}$$

where the variation of cross-sectional area in the flow direction is neglected.

The mass equation for both phases and the sum and the difference of two momentum equations are rearranged in one matrix equation:

$$A\frac{\partial\Phi}{\partial t} + B\frac{\partial\Phi}{\partial x} + C\Phi + D = 0 \ , \tag{6}$$

where the field vector Φ contains the four dependent variables; 4 x 4 coefficient matrix A, B and C, and 4-component vector D are all real functions of Φ. We assume that an infinitesimal sharp perturbation φ is introduced into a solution of Eq.(6). The linearized equation for disturbances are then obtained:

$$A\frac{\partial\varphi}{\partial t} + B\frac{\partial\varphi}{\partial x} = 0 \ , \tag{7}$$

where φ is definéd as the column vector of independent variables:

$$\varphi = (\ \delta\alpha \ , \ \delta p \ , \ \delta u_g \ , \ \delta u_l \ )^T \ . \tag{8}$$

In the above equations, δ denotes the perturbation, and A and B respectively are the coefficient matrices:

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix} , \qquad (9)$$

where

$$a_{11} = \rho_g ,$$

$$a_{12} = \frac{\alpha}{c_g^2} ,$$

$$a_{21} = -\rho_l ,$$

$$a_{22} = \frac{(1-\alpha)}{c_l^2} ,$$

$$a_{33} = \alpha \rho_g ,$$

$$a_{34} = (1-\alpha)\rho_l ,$$

$$a_{43} = (1-\alpha)\rho_g + \rho_l C_{vm}$$

and

$$a_{44} = -\{(1-\alpha)\rho_l + \rho_l C_{vm}\} ,$$

and

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 \\ b_{21} & b_{22} & 0 & b_{24} \\ 0 & 1 & b_{33} & b_{34} \\ 0 & 0 & b_{43} & b_{44} \end{bmatrix} , \qquad (10)$$

where

$$b_{11} = \rho_g u_g ,$$

$$b_{12} = \frac{\alpha u_g}{c_g^2} ,$$

$$b_{13} = \alpha \rho_g ,$$

$$b_{21} = -\rho_l u_l ,$$

$$b_{22}=\frac{(1-\alpha)u_l}{c_l^2} \ ,$$

$$b_{24}=(1-\alpha)\rho_l \ ,$$

$$b_{33}=\alpha\rho_g u_g \ ,$$

$$b_{34}=(1-\alpha)\rho_l u_l \ ,$$

$$b_{43}=\{(1-\alpha)\rho_g+\rho_l C_{vm}(\lambda-1)\}u_g-\rho_l C_{vm}(\lambda-2)u_l$$

and

$$b_{44}=-\{(1-\alpha)+C_{vm}(1-\lambda)\}\rho_l u_l-\rho_l C_{vm}\lambda u_g$$

In the above equations, the variables for the dispersed and continuous phases respectively are simply displaced by those for the gas and liquid phases. The sound speed in gas and liquid phases c_g and c_l are respectively

$$\frac{1}{c_g^2} = (\frac{\partial\rho_g}{\partial p})_s \quad \text{and} \quad \frac{1}{c_l^2} = (\frac{\partial\rho_l}{\partial p})_s \ . \qquad (11)$$

The characteristics μ of Eq.(7) are defined by

$$| \ \mu A - B \ | = 0 \ . \qquad (12)$$

We assume that the sound speed is sufficiently large in comparison with the velocities. In order for Eq.(12) to have real characteristics, the following equation must be satisfied:

$$C_{vm}^2\{\lambda^2+4(1-\lambda)(1-\alpha)\} + 4C_{vm}(1-\lambda)\alpha(1-\alpha)^2(1-\frac{\rho_g}{\rho_l})$$

$$- 4\alpha(1-\alpha)^3(\frac{\rho_g}{\rho_l}) \geqq 0 \ . \qquad (13)$$

We can find the two-phase region is entirely unstable if the basic equations contain no virtual mass force term. The above equation is the stability condition based on the objective virtual mass acceleration of Eq.(3).

On the contrary, if we use the virtual mass acceleration of Eq.(4), a slightly different stability condition is obtained:

$$-\alpha(1-\alpha)\{\rho_l{}^2C_{vm}^2 + (1-\alpha)(\rho_l^2+\rho_g\rho_l)C_{vm} + (1-\alpha)^2\rho_g\rho_l\} \geqq 0 \ . \tag{14}$$

The coefficient C_{vm} must be negative if Eq.(14) is to be satisfied. That is, the basic equation system is always unstable against the infinitesimal high wave-number perturbations when the virtual mass acceleration of Eq.(4) is used. In other words, the form of Eq.(4) does not contribute to the stability of basic equations. For this reason, we use the form of Eq.(3) in this study.

The real characteristics region of Eq.(13), that is, the stable region of basic equations is shown in Fig.1. In Fig.1, values of 4.28 (kg/m^3) and 999.2 (kg/m^3), which are the air and liquid densities at the condition of 0.355 MPa and 288.9 K, are chosen for ρ_g and ρ_l. The basic equation system is stable if we use a large value as the virtual mass coefficient. We have to pay attention, however, for selecting the coefficient since the virtual mass force acts as the transient drag force. The general value of coefficient is not well known at present, so we use the minimum value in this study so as to clearly see the effect of virtual mass force on the stability. We can obtain the minimum value of coefficient from the equality of Eq.(13) if the parameter λ is determined.

2.3 Parameter in virtual mass acceleration

The parameter λ is determined so as to obtain a reasonable agreement between the analytical and experimental sound speed in two-phase flows. The sound speed in two-phase flows c_{tp} is obtained from Eq.(7) as a function of virtual mass coefficient:

$$c_{tp}^2 = \frac{(1-\alpha)\{(1-\alpha)\rho_g+\alpha\rho_l\}+\rho_lC_{vm}}{[(1-\alpha)\rho_g\rho_l+\{\alpha\rho_g+(1-\alpha)\rho_l\}\rho_lC_{vm}]}\left[\frac{(1-\alpha)}{\rho_lc_l^2}+\frac{\alpha}{\rho_gc_g^2}\right]^{-1} . \tag{15}$$

If the virtual mass effect is negligibly small in comparison with other terms in Eq.(15'), the sound speed is the same as the velocity of compressibility wave in stratified flow derived by Wallis [8].

The sound speed is shown in Fig.2 along with the experimental results obtained by Martin and Padmanabhan [9] for the air-water case and by Grolmes and Fauske [10] for the steam-water case. It is found that the reasonable agreement between analytical and experimental results are obtained when the parameter λ is chosen to be 2. The constant value of 2 is, therefore, assumed for λ and hence the minimum virtual mass coefficient is

$$C_{vm} = \frac{1}{2}(1-\alpha)^2(1-\frac{\rho_g}{\rho_l})+\frac{1}{2}[(1-\alpha)^3\{(1-\alpha)(1-\frac{\rho_g}{\rho_l})^2+4(\frac{\rho_g}{\rho_l})\}]^{1/2} \tag{16}$$

from Eq.(13).

In summary, the basic equations are stable against infinitesimal high wave-number perturbations and the analytical sound speed is in good agreement with the experimental data by using the virtual mass coefficient defined by Eq.(16).

3. One-dimensional sedimentation

3.1 Description of the problem

The effect of virtual mass force term defined by Eqs.(2), (3) and (16) is evaluated through the numerical simulation of a one-dimensional sedimentation problem. This problem is based on the numerical benchmark test of International Two-Phase Flow Fundamentals [11]. The basic conservation equations with virtual mass force term are solved by using the finite difference method. In this study, the MINCS code is used [12]. The numerical method of MINCS is based on the implicit upwind scheme with the staggered mesh system. The input model to MINCS is shown in Fig.3. The water phase initially rests above the air phase in a 2 meter long vertical pipe, then it falls down due to gravity. The same values used in Fig.1 are assumed for the initial conditions: 0.355 MPa and 288.9 K. Mesh cells are equally spaced and values of 40, 50 and 80 are chosen for the number of cells. The time step width is 0.0005 sec for the 40-cell case, 0.0004 sec for the 50-cell case and 0.00025 sec for the 80-cell case. The ratio $\Delta t/\Delta x$ is, therefore, always constant of 0.01. The frictional terms are all neglected so that we can clearly see the effect of virtual mass force term on the calculated results.

and by Grolmes and Fauske [10] for the steam-water case. It is found that the reasonable agreement between analytical and experimental results are obtained when the parameter λ is chosen to be 2. The constant value of 2 is, therefore, assumed for λ and hence the minimum virtual mass coefficient is

$$C_{vm} = \frac{1}{2}(1-\alpha)^2(1-\frac{\rho_g}{\rho_l})+\frac{1}{2}[(1-\alpha)^3\{(1-\alpha)(1-\frac{\rho_g}{\rho_l})^2+4(\frac{\rho_g}{\rho_l})\}]^{1/2} \quad (16)$$

from Eq.(13).

In summary, the basic equations are stable against infinitesimal high wave-number perturbations and the analytical sound speed is in good agreement with the experimental data by using the virtual mass coefficient defined by Eq.(16).

3. One-dimensional sedimentation

3.1 Description of the problem

The effect of virtual mass force term defined by Eqs.(2), (3) and (16) is evaluated through the numerical simulation of a one-dimensional sedimentation problem. This problem is based on the numerical benchmark test of International Two-Phase Flow Fundamentals [11]. The basic conservation equations with virtual mass force term are solved by using the finite difference method. In this study, the MINCS code is used [12]. The numerical method of MINCS is based on the implicit upwind scheme with the staggered mesh system. The input model to MINCS is shown in Fig.3. The water phase initially rests above the air phase in a 2 meter long vertical pipe, then it falls down due to gravity. The same values used in Fig.1 are assumed for the initial conditions: 0.355 MPa and 288.9 K. Mesh cells are equally spaced and values of 40, 50 and 80 are chosen for the number of cells. The time step width is 0.0005 sec for the 40-cell case, 0.0004 sec for the 50-cell case and 0.00025 sec for the 80-cell case. The ratio $\Delta t/\Delta x$ is, therefore, always constant of 0.01. The frictional terms are all neglected so that we can clearly see the effect of virtual mass force term on the calculated results.

3.2 Numerical stability

The stability analysis for the numerical procedure is performed based on Neumann's method [13]. Three equations are considered, namely, the two mass conservation equations and the difference of two momentum equations. For simplicity, the incompressible and countercurrent flow is assumed. We suppose that the infinitesimal perturbation is superposed on the solution of discretized equations. The discretized equations are linearized, and then the perturbations in the difference terms are substituted by components of Fourier series [14], we obtain

$$EX^{n+1} = FX^{n} \tag{17}$$

In the above equation, X is the vector composed of $\delta\alpha_j$, $\delta u_{g,j}$ and $\delta u_{l,j}$, where δ indicates the perturbation, and the subscript j and the superscript n respectively denote the spatial mesh number and the time step. The coefficient matrices E and F are described as

$$E = \begin{bmatrix} e_{11} & e_{12} & 0 \\ e_{21} & 0 & e_{23} \\ 0 & e_{32} & e_{33} \end{bmatrix}, \tag{18}$$

where

$$e_{11} = 1 + Cu_g \ ,$$
$$e_{12} = C\alpha \ ,$$
$$e_{21} = 1 + Cu_l e^{I\theta} \ ,$$
$$e_{23} = -C(1-\alpha) \ ,$$
$$e_{32} = \{(1-\alpha)\rho_g + \rho_l C_{vm}\}(1 + Cu_g)$$

and

$$e_{33} = -[\{(1-\alpha)\rho_l - \rho_l C_{vm}\}(1 + Cu_l e^{I\theta}) + 2\rho_l C_{vm}(1 + Cu_g C)] \ ,$$

and

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & f_{32} & f_{33} \end{bmatrix}, \tag{19}$$

where

$$f_{32}=\{(1-\alpha)\rho_g+\rho_l C_{vm}\}$$

and

$$f_{33}=-\{(1-\alpha)\rho_l+\rho_l C_{vm}\}\ .$$

In the above equations, α, u_g and u_l are the unperturbed values and the variables C and θ are defined as

$$C = \Delta t/\Delta x(1-e^{-I\theta})\ ,$$

and

$$\theta = K\Delta x\ ,$$

where I is the imaginary unit, K the wave number defined as $(2\pi/\Lambda)$ and Λ the wavelength of perturbation.

By calculating the eigenvalue of amplification matrix $E^{-1}F$, we can find whether or not the infinitesimal perturbation grows. The eigenvalue μ is defined as the root of the parabolic equation:

$$\{(1-\alpha)e_{11}e_{32}-\alpha e_{21}e_{33}\}\mu^2 - \{(1-\alpha)(e_{32}+e_{11}f_{32})-\alpha(e_{33}+e_{21}f_{33})\}\mu + \{(1-\alpha)f_{32}-\alpha f_{33}\} = 0\ . \tag{20}$$

The finite difference scheme is stable against infinitesimal perturbations if the maximum absolute value of eigenvalue - also known as the spectrum radius - is smaller than unity [13]:

$$|\mu| < 1\ . \tag{21}$$

3.3 Calculated results and discussion

The void fraction transients in the cell one mesh upper from the midplane are shown in Figs.4 and 5. The calculated results without the virtual mass force term are shown in Fig.4 and those with it in Fig.5. We can find that the water is gradually replaced by the air.

The instability grows and unrealistic results are obtained as the number of mesh cells increases as shown in Fig.4. Stable results however are obtained as shown in Fig.5 even if 80 cells are used. It is found that the numerical solutions are stabilized by the virtual mass force term. The instability in the numerical results appears to be corresponding to the ill-posedness of the basic equations.

The maximum absolute values of eigenvalue of the discretized equation system are shown in Fig.6 in terms of the dimensionless wavelength ($\Lambda/\Delta x$). In this problem, the void fraction is varied from 0.0 to 1.0 and, though the flow directions are different, the maximum velocities are about 5 m/s for both phases. Hence values of 5 m/s and -5 m/s are assumed for the unperturbed velocities of air and water, and 0.5 for the unperturbed void fraction in Fig.6. The eigenvalue is larger than unity in some region of wavelength for the case without the virtual mass force term, while smaller than unity for the case with it. The same results are obtained for eigenvalues even if other values are assumed for the unperturbed variables. We can confirm that the discretized equation system and hence the numerical solutions are stabilized by introducing the virtual mass force term.

At the next step, we study the stability of numerical solution when the virtual mass coefficient is smaller than that defined by Eq.(16) and can not satisfy the stability condition. The virtual mass coefficient obtained by Eq.(16) is multiplied by 1.0, 0.7, 0.5 and 0.3. The basic equations are unstable when the factors of 0.7, 0.5 and 0.3 is multiplied to the virtual mass coefficient since the coefficient defined by Eq.(16) is the minimum which is needed to stabilize the basic equations. The void fraction transients in the cell one mesh lower from the midplane are shown in Fig.7. The number of mesh cells is 50. We can find that tha air is gradually replaced by the water. The smaller the virtual mass coefficient, the larger the spike in the void fraction transient. The eigenvalues of the discretized equation system are shown in Fig.8. The eigenvalues exceed unity for the case with the multiplication factors of 0.7, 0.5 and 0.3, while smaller than unity for the case with 1.0. The stability of discretized equation system corresponds to the stability of differential equation system.

The stability is further examined when the virtual mass coefficient is constant. Values of 0.5, 0.3, 0.1 and 0.05 are chosen for the constant.

The basic equations are partially stable by these coefficients from Fig.1. The overall void profiles at 0.3 *sec* for the 50-cell case are shown in Fig.9. It is found that the smaller the coefficient, the larger the instability. The eigenvalues are shown in Fig.10. A value of 0.1 is assumed for the void fraction in Fig.10. The eigenvalues exceed unity for all cases, and become larger as the coefficients become smaller. The instability of discretized equation system is in good agreement with the instability of differential equation system.

The stability of the differential equation system can be described by the condition defined by Eq.(13) while that of the discretized equation system by Eq.(21). The stability of numerical results is determined by the eigenvalue of the amplification matrix of the discretized equation system and it does not always agree with the stability of differential equation system [14]. The virtual mass force term however can stabilize the discretized equation system so long as the virtual mass coefficient satisfies the stability condition of basic equations.

4. Conclusions

The effect of the virtual mass force term on the stability of transient two-phase flow analysis has been studied. The objective form of virtual mass acceleration which was derived by Drew et al. was used. The virtual mass coefficient was determined from the stability condition of basic differential equations against infinitesimal high wave-number perturbations. The parameter was determined so as to obtain the reasonable agreement between the analytical and experimental sound speed in two-phase flows. The one-dimensional sedimentation problem was simulated by using the finite difference method. The stability analysis based on Neumann's method was also performed for the implicit upwind scheme with the staggered mesh system. It has been found that the virtual mass force term stabilizes the discretized equation system so long as the virtual mass coefficient satisfys the stability condition of differential équations.

The basic equations are partially stable by these coefficients from Fig.1. The overall void profiles at 0.3 *sec* for the 50-cell case are shown in Fig.9. It is found that the smaller the coefficient, the larger the instability. The eigenvalues are shown in Fig.10. A value of 0.1 is assumed for the void fraction in Fig.10. The eigenvalues exceed unity for all cases, and become larger as the coefficients become smaller. The instability of discretized equation system is in good agreement with the instability of differential equation system.

The stability of the differential equation system can be described by the condition defined by Eq.(13) while that of the discretized equation system by Eq.(21). The stability of numerical results is determined by the eigenvalue of the amplification matrix of the discretized equation system and it does not always agree with the stability of differential equation system [14]. The virtual mass force term however can stabilize the discretized equation system so long as the virtual mass coefficient satisfies the stability condition of basic equations.

4. Conclusions

The effect of the virtual mass force term on the stability of transient two-phase flow analysis has been studied. The objective form of virtual mass acceleration which was derived by Drew et al. was used. The virtual mass coefficient was determined from the stability condition of basic differential equations against infinitesimal high wave-number perturbations. The parameter was determined so as to obtain the reasonable agreement between the analytical and experimental sound speed in two-phase flows. The one-dimensional sedimentation problem was simulated by using the finite difference method. The stability analysis based on Neumann's method was also performed for the implicit upwind scheme with the staggered mesh system. It has been found that the virtual mass force term stabilizes the discretized equation system so long as the virtual mass coefficient satisfys the stability condition of differential équations.

References

[1] P.S.Andersen, P.Astrup and O.Rathmann,
Characteristics of a one-dimensional two-fluid model for two-phase flow.
A study of added mass effects, NORHAV-D-017(1976)

[2] D.Drew, L.Cheng and R.T.Lahey,Jr.,
The analysis of virtual mass effects in two-phase flow,
Int. J. Multiphase Flow, 5(1979)233-242.

[3] D.Drew and R.T.Lahey,Jr.,
The virtual mass and lift force on a sphere in rotating and straining inviscid Flow,
Int. J. Multiphase Flow, 13(1987)113-121.

[4] R.T.Lahey,Jr., L.Y.Cheng, D.A.Drew and J.E.Flaherty,
The effect of virtual mass on the numerical stability of accelerating two-phase flow,
Int. J. Multiphase Flow 6(1980)281-294.

[5] L.Cheng, R.T.Lahey,Jr. and D.A.Drew,
The effect of virtual mass on the prediction of critical flow, in: Transient Two-Phase Flow:
Proc. 3rd CSNI Specialist Meeting
(Hemisphere, Washington, 1983) p.323-340.

[6] H.C.No and M.S.Kazimi, On the formulation of the virtual mass term in two fluid models, Nucl. Engrg. Des. 95(1986)163-170.

[7] M.Ishii and K.Mishima, Two-fluid model and hydrodynamic constitutive relations, Nucl. Engrg. Des. 82(1984) 107-126.

[8] G.B.Wallis, One-Dimensional Two-Phase Flow,
(McGraw-Hill, New York, 1969)p.143.

[9] C.S.Martin and M.Padmanabhan, Pressure pulse propagation in two-component slug flow,
Trans. ASME 101(1979)44-52.

[10] M.A.Grolmes and H.K.Fauske, Propagation characteristics of compression and rarefaction pressure pulses in one-component vapor-liquid mixtures, Nucl. Engrg. Des. 11(1969)137-142.

[11] G.F.Hewitt, et al., Multiphase Science and Technology, Vol.3 (Hemisphere, Washington, 1987) p.474-476.

[12] M.Akimoto et al., Development of transient two-phase flow analyzer: MINCS, in: Proc. 2nd Int. Topical Meeting on Nuclear Power Plant Thermal-Hydraulics and Operations, Tokyo, April 1986, Vol.1, p.72-79.

[13] P.J.Roache, Computation Fluid Dynamics, (Hermosa, Albuquerque, 1976) p.42-45.

[14] T.Watanabe and M.Hirano, Numerical stability of void waves in stratified two-phase flow, in: Proc. Japan-U.S. Seminar on Two-Phase Flow Dynamics, Ohtsu, 15-20 July 1988, F.4.

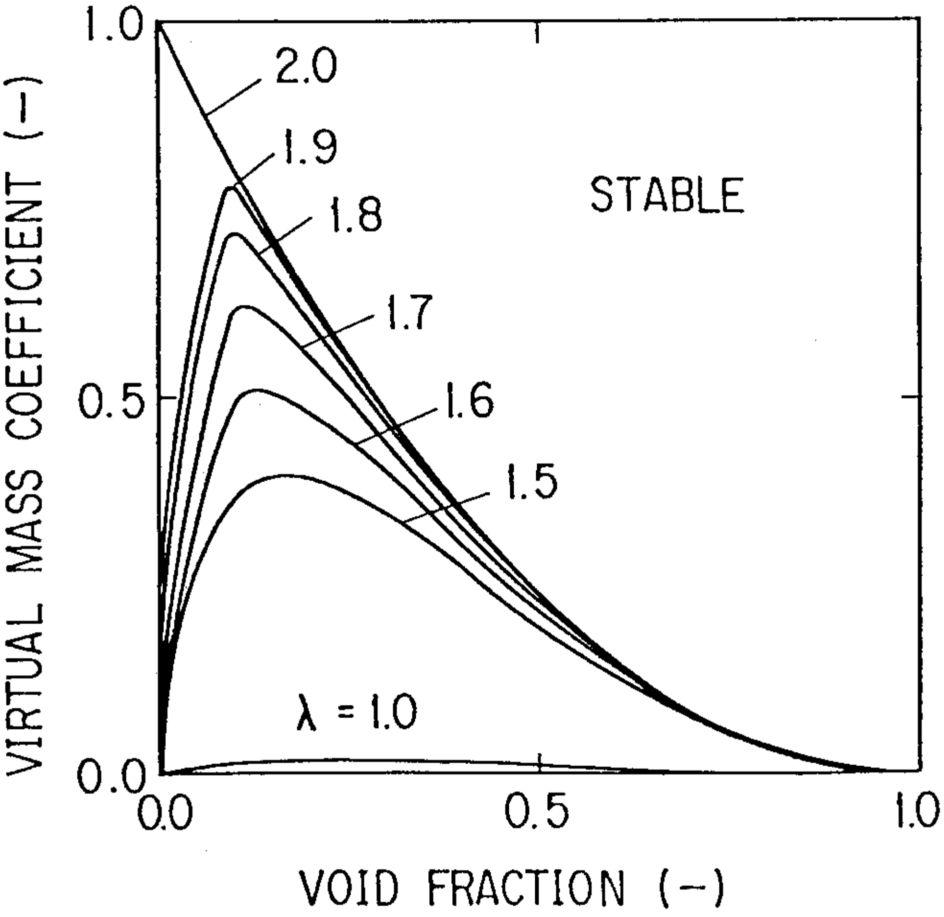
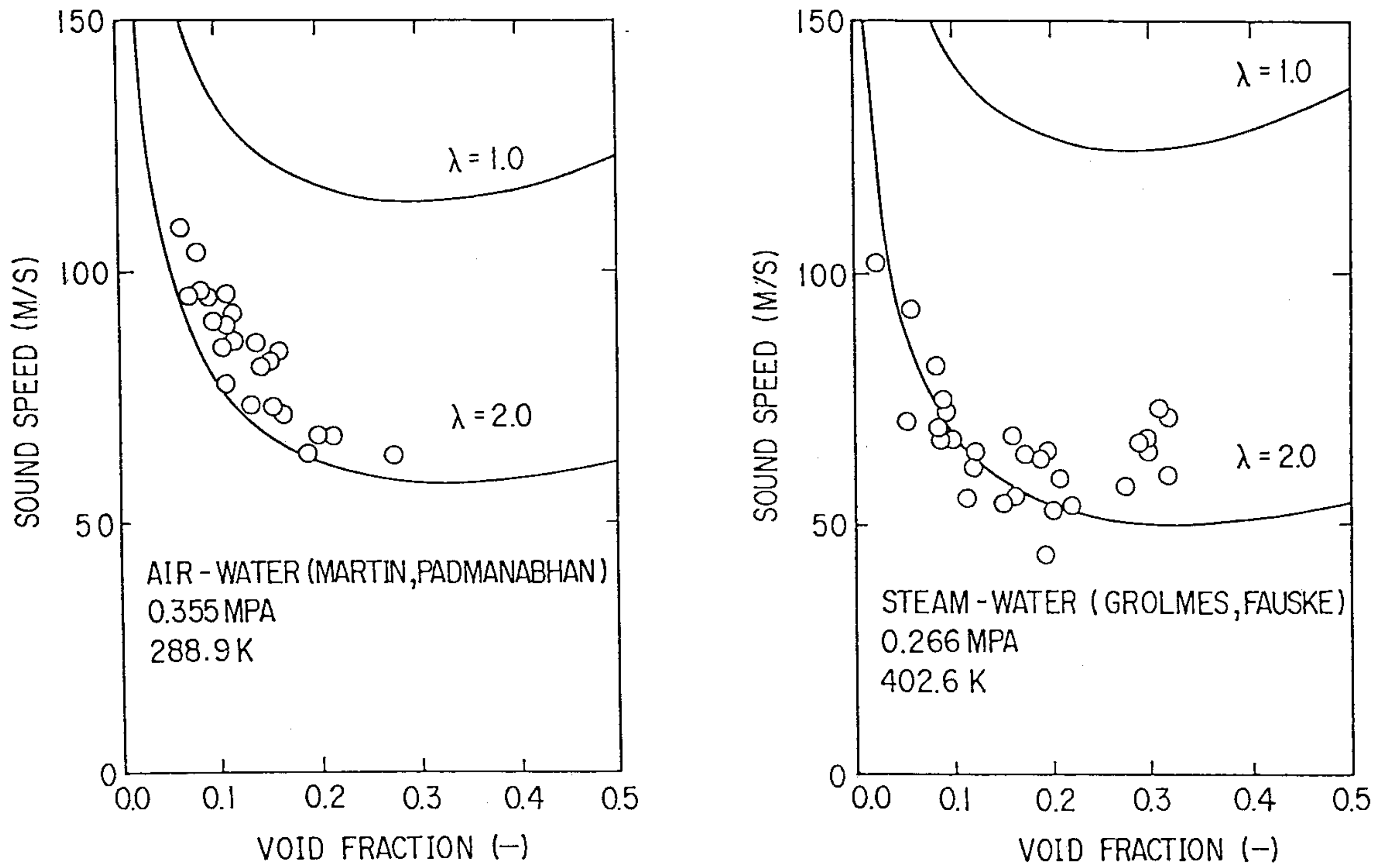


Fig. 1 Stable region for infinitesimal higi wave-number perturbations



(a) Air-water case

(b) Steam-water case

Fig. 2 Sound speed in two-phase flows

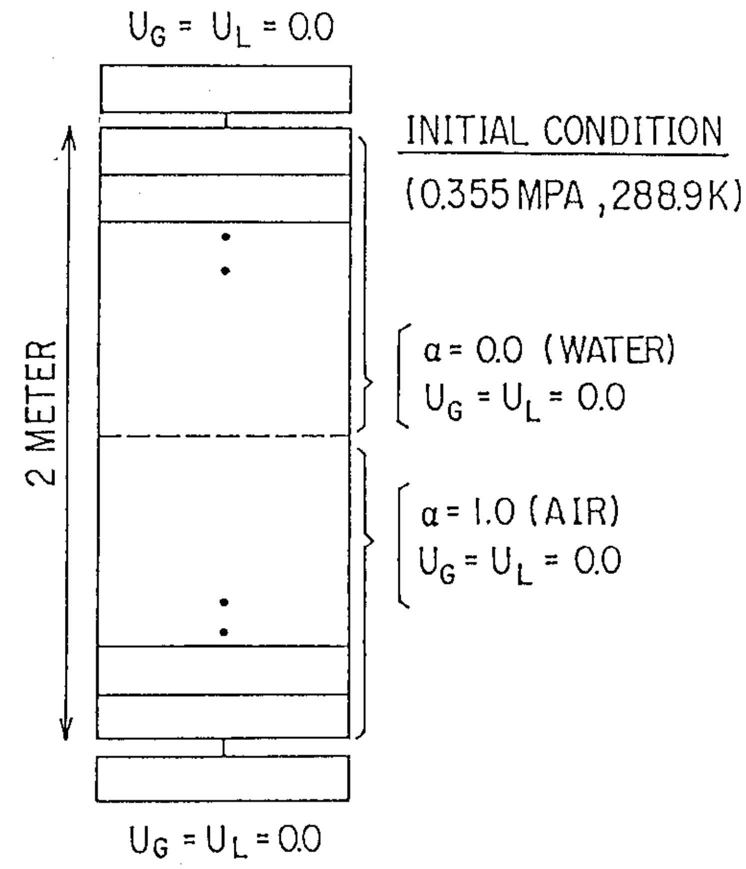


Fig. 3 Input model for sedimentation

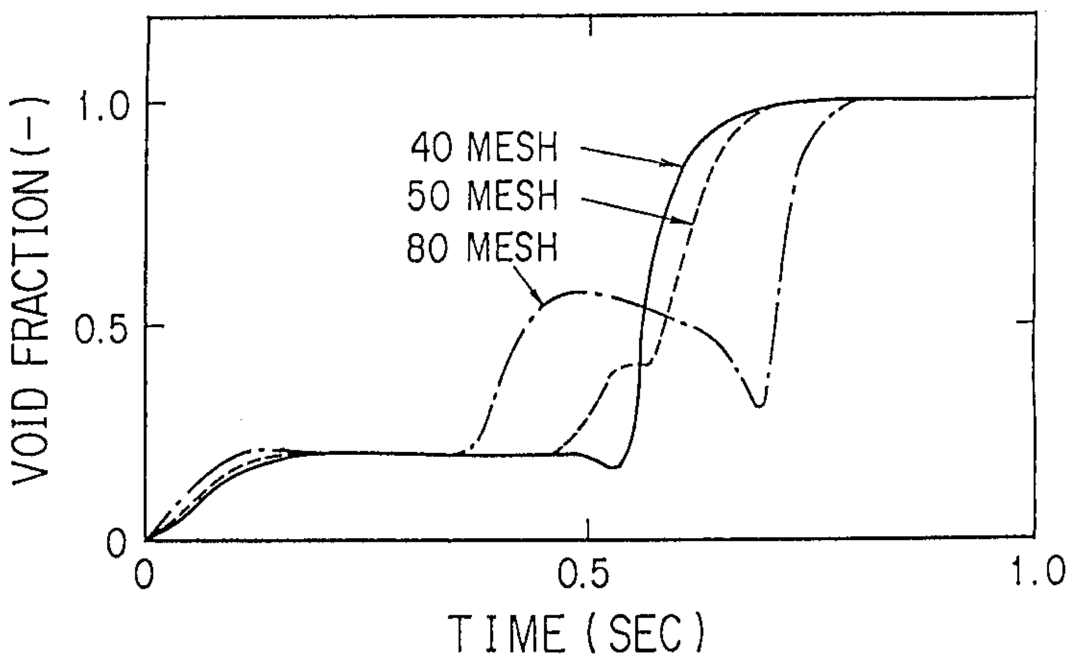


Fig. 4 Void fraction transients without virtual mass

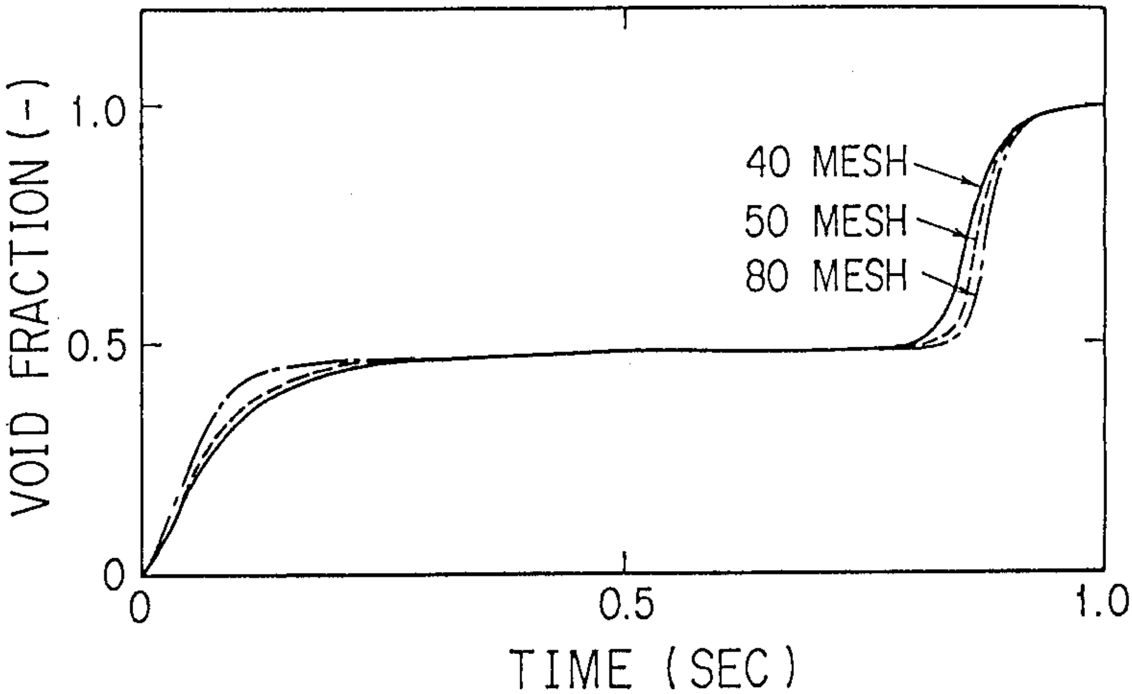


Fig. 5 Void fraction transients with virtual mass

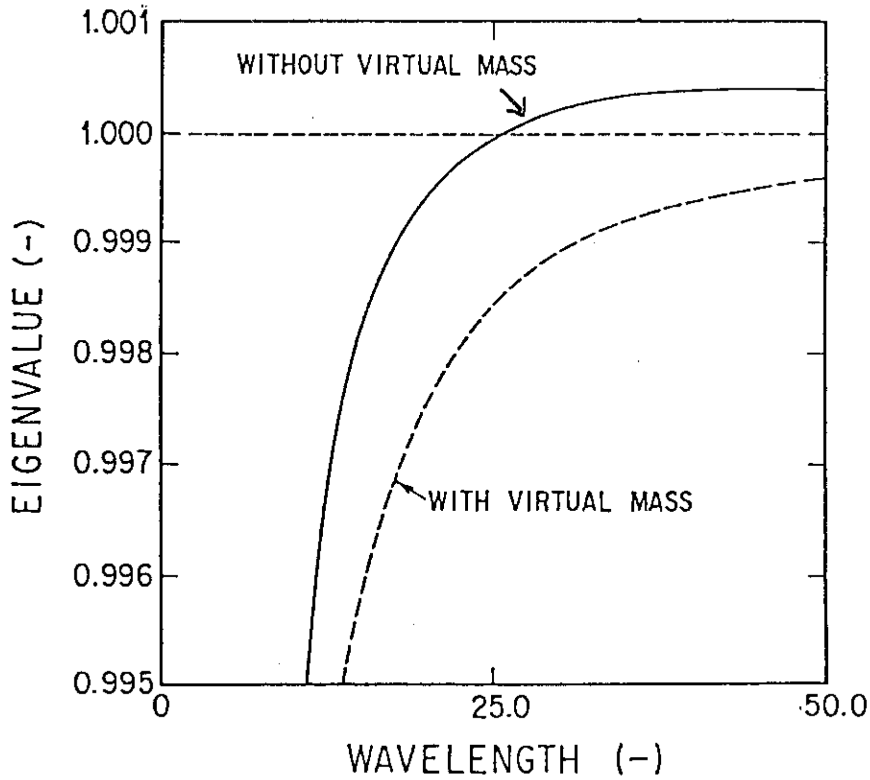


Fig. 6 Maximum absolute values of the eigenvalues

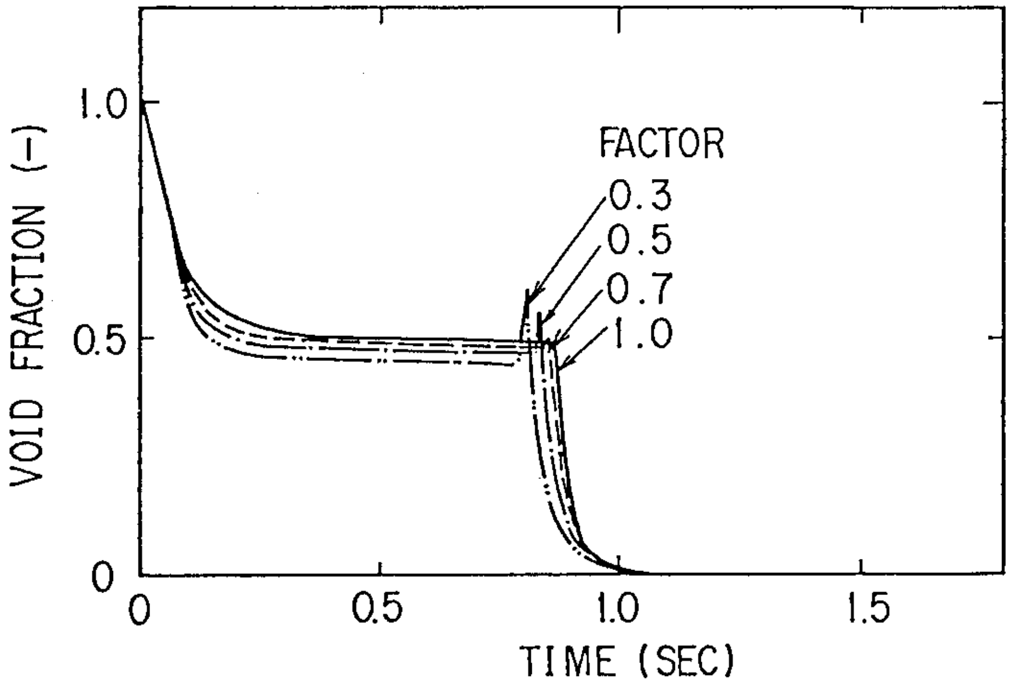


Fig. 7 Void fraction transients with factored coefficients

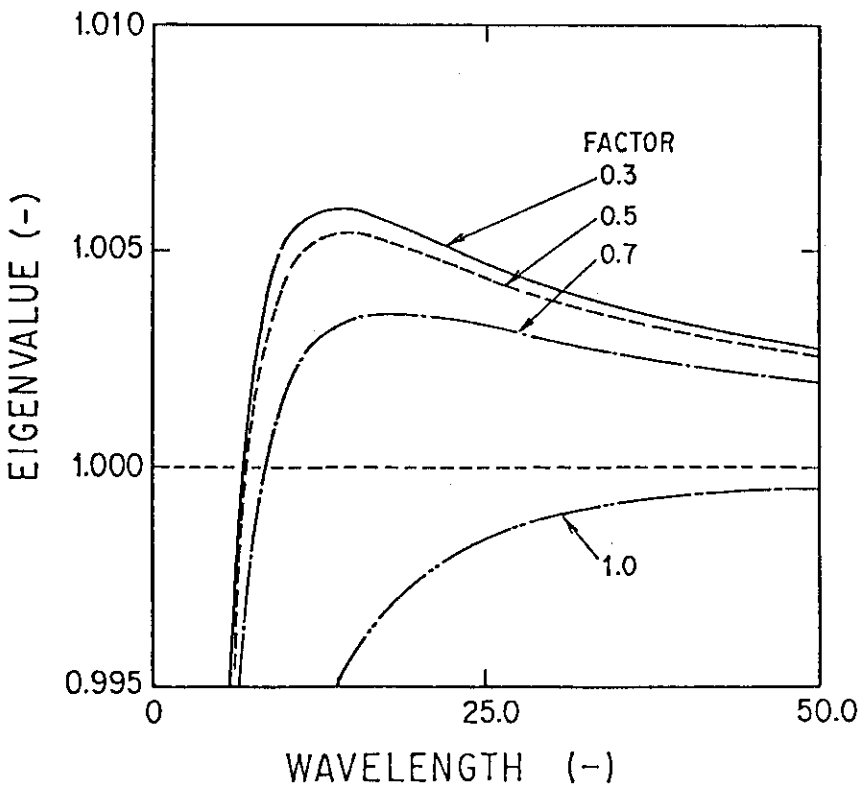


Fig. 8 Maximum absolute values of the eigenvalues

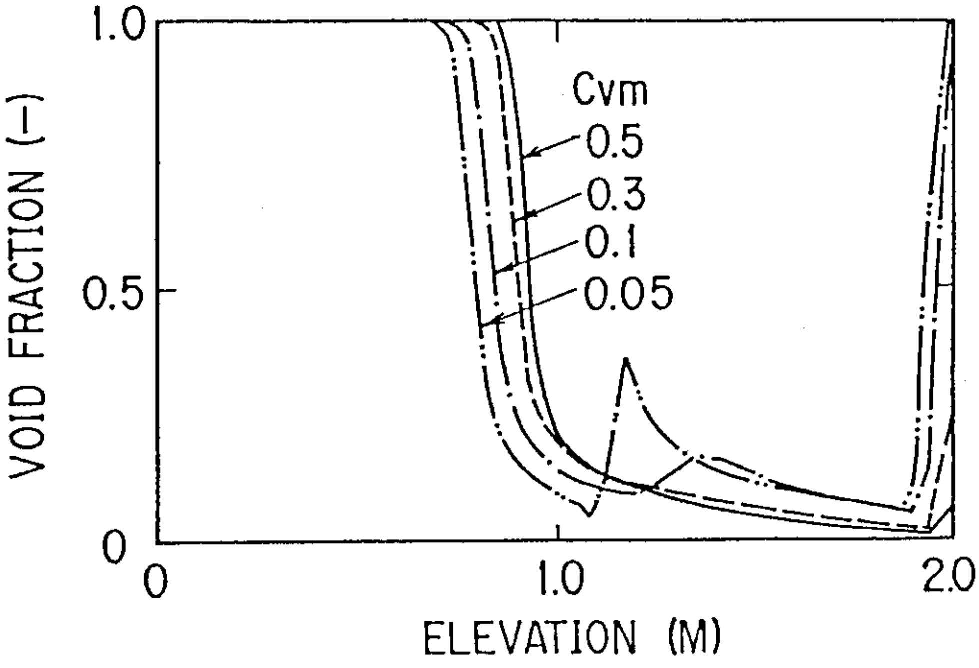


Fig. 9 Void fraction transients with constant coefficients

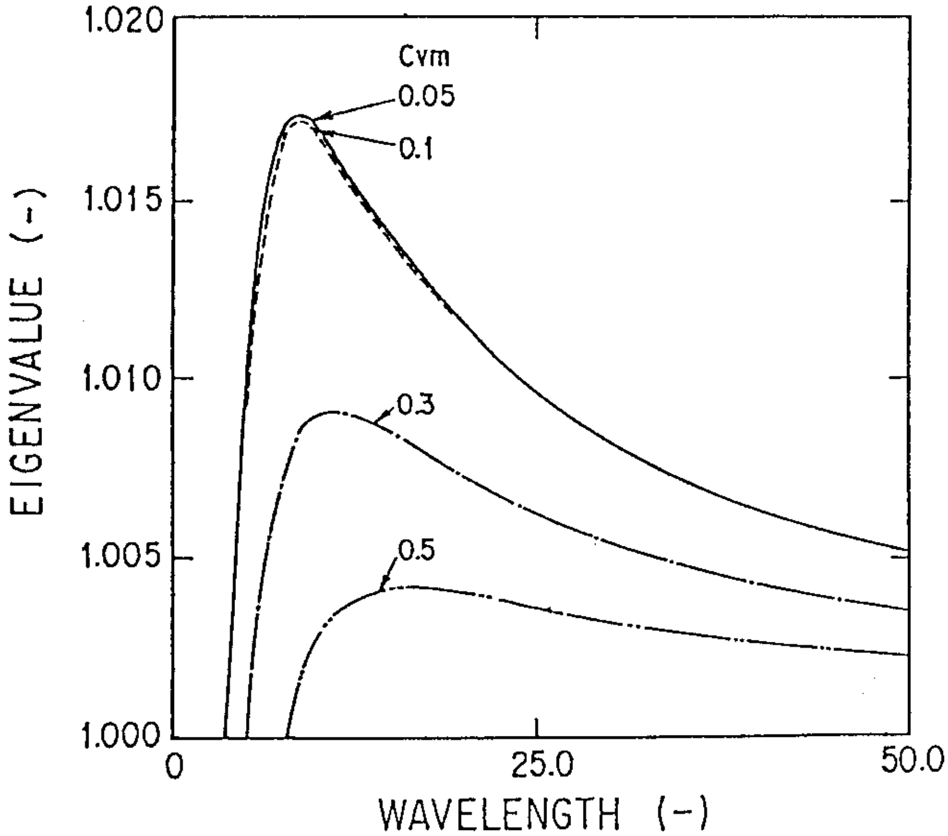


Fig. 10 Maximum absolute values of the eigenvalues