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BULK SHIELDING CALCULATION OF HIGH ENERGY ELECTRON ACCELERATORS WITH LINE SOURCE ASSUMPTIONS

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Nobuo SASAMOTO

日本原子力研究所
Japan Atomic Energy Research Institute

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Bulk Shielding Calculation of High Energy Electron Accelerators with Line Source Assumptions

Nobuo SASAMOTO

Department of Reactor Engineering
Tokai Research Establishment
Japan Atomic Energy Research Institute
Tokai-mura, Naka-gun, Ibaraki-ken

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Shielding design calculations for high energy electron accelerators are usually carried out using the point kernel method based on a point source assumption. In order to achieve more realistic and reasonable shielding design of SPring-8, we derive a formula based on a point kernel method for a line source assumption, using a Gauss-Legendre numerical integration method. Assuming a line source of 8 meter long, the dose equivalent value is evaluated to be half of that by a point source assumption.

Keywords: Bulk Shield, High Energy, Electron, Accelerator, Line Source, Point Kernel, Swanson, Jenkins, SPring-8

線状線源を仮定した高エネルギー電子加速器のバルク遮蔽計算

日本原子力研究所東海研究所原子炉工学部
笹本　宣雄

（1990年5月30日受理）

高エネルギー電子加速器の遮蔽設計計算は，通常，点線源に基づく点減衰核法を用いて行われる。現実的で且つ合理的なSpring－8の遮蔽設計を目的として，ガウスールジャンドル数値積分法を用いた，線状線源に基づく点減衰核法の計算式を導出した。Spring－8に対応した8mの長さの線状線源を仮定すると，点線源に基づく線量当量率の約半分の値になることがわかった。

東海研究所：〒319-11　茨城県那珂郡東海村白方字白根2－4

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1. Introduction

At present, bulk shielding calculations of the 8 GeV class synchrotron radiation facility (SPring-8) is in progress at JAERI. In the shielding calculations, we must at first estimate radiation sources, assuming loss rate of accelerating beam. It is generally difficult, however, to estimate beam loss rate exactly because of a lot of problems about accelerators themselves and their operational conditions. Therefore, shielding calculations are usually made on the basis of the conservative beam loss assumptions.

On the other hand, from the viewpoint of reducing construction cost and of efficient utilization of the experimental hall, shield width is eagerly desired to be as thin as possible, so long as it meets shielding design criteria. One of the most important points for beam loss assumption is whether beam loss occurs strictly at one point or within a finite region. Considering the fact that almost all calculational methods available are based on a point source model, one point loss assumption is usually made for conservative estimates.

Except for a rare case of accidental beam loss, however, it is physically quite improbable that particle beam travelling at approximately light speed is lost momentarily at any one point on accelerator tubes. Then, for achieving more realistic and reasonable shielding estimates, we derived a formula of the calculational methods for high energy accelerator shielding based on a line source model, by modifying the current formulas based on a point source model. With these methods, we investigated the effect of the difference between point and line sources on dose equivalent calculations for the configurations which are typical to SPring-8.

2. Calculational Method Based on a Line Source Model

2.1 Modification of Jenkins' Equations

Jenkins' equations based on a point source model are given as follows for photons and neutrons, separately.[1)]

(1) Photons

Jenkins' equation for photons is described as sum of the two decay terms consisting of primary photons generated at the target and of secondary photons generated from high energy neutron interaction with shielding materials.

$$\dot{H} = 3.6\times10^{-14}\cdot J\cdot E_e\cdot\frac{1}{r^2}\left[\frac{133\cdot\exp(-d\cdot\mathrm{cosec}\phi/\lambda)}{(1-0.98\cos\theta)^{1.2}} + \frac{f_1\cdot 0.267\cdot\exp(-d\cdot\mathrm{cosec}\phi/\lambda_1)}{(1-0.72\cos\theta)^2}\right]\ (\mathrm{Sv/h}) \qquad (2.1)$$

$$(30°<\theta<130°)$$

(2) Neutrons

Jenkins' equation for neutrons is expressed as sum of the three decay terms consisting of high energy neutrons, intermediate energy neutrons and giant resonance neutrons,

$$\dot{H} = 3.6\times10^{-14}\cdot J\cdot E_e\cdot\frac{1}{r^2}\times\left[\frac{f_1\cdot\exp(-d\cdot\mathrm{cosec}\phi/\lambda_1)}{(1-0.72\cos\theta)^2} + \frac{f_2\cdot 10\cdot\exp(-d\cdot\mathrm{cosec}\phi/\lambda_3)}{(1-0.75\cos\theta)} + 3.79\cdot Z^{0.73}\cdot\exp(-d\cdot\mathrm{cosec}\phi/\lambda_2)\right]\ (\mathrm{Sv/h}) \qquad (2.2)$$

$$(30°<\theta<130°)$$

where

- J : point beam loss rate (electron/s)
- E_e : electron beam energy (GeV)
- r : distance from beam loss point to estimation point (m)
- d : shield width (cm)
- θ : angle to beam travelling direction (deg)
- ϕ : angle of shield to incident radiation (deg)
- λ_1 : attenuation length of high energy neutron (cm)
- λ_2 : attenuation length of giant resonance neutron (cm)
- λ_3 : attenuation length of intermediate energy neutron (cm)
- λ : attenuation length of photon (cm)

Z : atomic number of target material

f_1 : fall-off factor for high energy neutron source term[2)]

f_2 : fall-off factor for intermediate energy neutron source term[2)]

As is clear from Eqs.(2.1) and (2.2), Jenkins' equations are basically described as sum of the point kernels given below.

$$\dot{H}_{point} = K \cdot J \cdot \frac{1}{r^2} \cdot \frac{\exp(-d \cdot \mathrm{cosec}\theta/\lambda)}{f(\theta)} \quad . \tag{2.3}$$

If a shield surface is parallel to beam travelling direction, $\phi=\theta$, then, Eq.(2.3) is rewritten as,

$$\dot{H}_{point} = K \cdot J \cdot \frac{1}{r^2} \cdot \frac{\exp(-d \cdot \mathrm{cosec}\theta/\lambda)}{f(\theta)} \quad . \tag{2.4}$$

Then we will derive Jenkins' equations for photons and neutrons based on a line source model using Eq.(2.4). Figure 2.1 shows a schematic figure of a line source model. Differential line source segment dz along a beam line is assumed to be a psuedo-point source. Here, source intensity given as dJ/dz is a beam loss rate per unit length of the line source. Differential dose equivalent from the psuedo-point source $d\dot{H}_{line}$ is obtained from Eq.(2.4), a point source model by replacing J with dJ/dz as,

$$d\dot{H}_{line} = K \cdot \frac{dJ}{dz} \cdot \frac{1}{r^2} \cdot \frac{\exp(-d \cdot \mathrm{cosec}\theta/\lambda)}{f(\theta)} \, dz \quad . \tag{2.5}$$

$$(-L/2 \leqq Z \leqq L/2)$$

Here, L is a line source length.

From Fig. 2.1, the following relations are easily obtained.

$r = (a+d)\mathrm{cosec}\theta$

$z = (a+d)\cot\theta$

$dz = (a+d)\mathrm{cosec}^2\theta d\theta$

Substituting the above relations into Eq.(2.5), we can rewrite it as,

$$d\dot{H}_{line} = K \cdot \frac{dJ}{dz} \cdot \frac{1}{a+d} \cdot \frac{\exp(-d \cdot \mathrm{cosec}\theta/\lambda)}{f(\theta)} \, d\theta \quad . \tag{2.6}$$

If we assume that radiation source distributes uniformly along the beam line, we obtain the following relation,

$$\frac{dJ}{dz} = \frac{J}{L} \quad . \tag{2.7}$$

Then, substituting Eq.(2.7) into Eq.(2.6), we can rewrite it as,

$$d\dot{H}_{line} = K\,\frac{J}{L}\cdot\frac{1}{a+d}\cdot\frac{\exp(-d\cdot\mathrm{cosec}\theta/\lambda)}{f(\theta)}\,d\theta\ . \tag{2.8}$$

By integrating Eq.(2.8) over a whole region of the line source from θ_1 to θ_2, we obtain Jenkins' equation for the line source model.

$$\dot{H}_{line} = \frac{K}{a+d}\cdot\frac{J}{L}\int_{\theta_1}^{\theta_2}\frac{\exp(-d\cdot\mathrm{cosec}\theta/\lambda)}{f(\theta)}\,d\theta\ . \tag{2.9}$$

Here, θ is an angle to the beam travelling direction, and θ_1 and θ_2 defined below are the observing angles from the left and right end points of the line source to the estimation point P as depicted in Fig. 2.1, respectively.

$$\theta_1 = \tan^{-1}[(a+d)/(L/2)]$$

$$\theta_2 = \pi - \theta_1$$

Since integration of Eq.(2.9) cannot be performed analytically because of the rather complicated functional forms of $f(\theta)$, it is carried out numerically using Gauss-Legendre integral method.

Thus, Jenkins' equations for photons and neutrons based on a line source model corresponding to Eqs.(2.1) and (2.2) are given as follows.

(1) Photons

$$\dot{H}_{line} = \frac{3.6\times10^{-14}}{a+d}\cdot E_e\cdot\frac{J}{L}$$

$$\times\int_{\theta_1}^{\theta_2}\left[\frac{\exp(-d\cdot\mathrm{cosec}\theta/\lambda)}{(1-0.98\ \cos\theta)^{1.2}} + \frac{f_1\cdot 0.267\cdot\exp(-d\cdot\mathrm{cosec}\theta/\lambda_1)}{(1-0.72\ \cos\theta)^2}\right]d\theta \tag{2.10}$$

(2) Neutrons

$$\dot{H}_{line} = \frac{3.6\times10^{-14}}{a+d}\cdot E_e\cdot\frac{J}{L}$$

$$\times\int_{\theta_1}^{\theta_2}\left[\frac{f_1\cdot\exp(-d\cdot\mathrm{cosec}\theta/\lambda_1)}{(1-0.72\ \cos\theta)^2} + \frac{f_2\cdot 10\cdot\exp(-d\cdot\mathrm{cosec}\theta/\lambda_3)}{(1-0.75\ \cos\theta)}\right.$$

$$\left. + 3.79\ Z^{0.73}\cdot\exp(-d\cdot\mathrm{cosec}\theta/\lambda_2)\right]d\theta\ . \tag{2.11}$$

2.2 Modification of Swanson's equation

Contrary to Jenkins' equation for photons, Swanson's one is described by a single decay term of photons as,[3]

$$\dot{H}_{point} = 1.6\times10^{-13}\cdot J\cdot E\cdot 50\cdot\frac{1}{r^2}\cdot\exp(-d\cdot\mathrm{cosec}\theta/\lambda) \tag{2.12}$$

Using the same procedure for Jenkins' equation, Swanson's equation based on a line source model is obtained as follows,

$$\dot{H}_{line} = \frac{1.6\times10^{-13}}{a+d}\cdot\frac{J}{L}\cdot E\cdot 50\times\int_{\theta_1}^{\theta_2}\exp(-d\cdot\mathrm{cosec}\theta/\lambda)d\theta\ . \tag{2.13}$$

From Eqs.(2.1), (2.2), (2.10), (2.11), (2.12) and (2.13), we can formulate ratios between the dose equivalent values based on a line source and a point source models,

$$\frac{\dot{H}_{line}}{\dot{H}_{point}} = \frac{a+d}{L}\cdot\frac{\int_{\theta_1}^{\theta_2}f(\theta)d\theta}{f(\theta_0)}\quad , \tag{2.14}$$

where

$\theta_0 = \pi/2$ (rad)

$\theta_1 = \tan^{-1}\{(a+b)/(L/2)\}$ (rad)

$\theta_2 = \pi - \theta_1$ (rad)

Further, $f(\theta)$ is expressed as below according to each equation.

For Jenkins' Eq.(photon)

$$f(\theta) = \frac{133\cdot\exp(-d\cdot\mathrm{cosec}\theta/\lambda)}{(1-0.98\cos\theta)^{1.2}} + \frac{f_1\cdot 0.267\cdot\exp(-d\cdot\mathrm{cosec}\theta/\lambda)}{(1-0.72\cos\theta)^2} \tag{2.15}$$

For Jenkins' Eq.(neutron)

$$f(\theta) = \frac{f_1\cdot\exp(-d\cdot\mathrm{cosec}\theta/\lambda_1)}{(1-0.72\cos\theta)^2} + \frac{f_2\cdot 10\cdot\exp(-d\cdot\mathrm{cosec}\theta/\lambda_3)}{(1-0.75\cos\theta)} + 3.79\ Z^{0.73}\cdot\exp(-d\cdot\mathrm{cosec}\theta/\lambda_2) \tag{2.16}$$

For Swanson's Eq.(photon)

$$f(\theta) = \exp(-d\cdot\mathrm{cosec}\theta/\lambda) \tag{2.17}$$

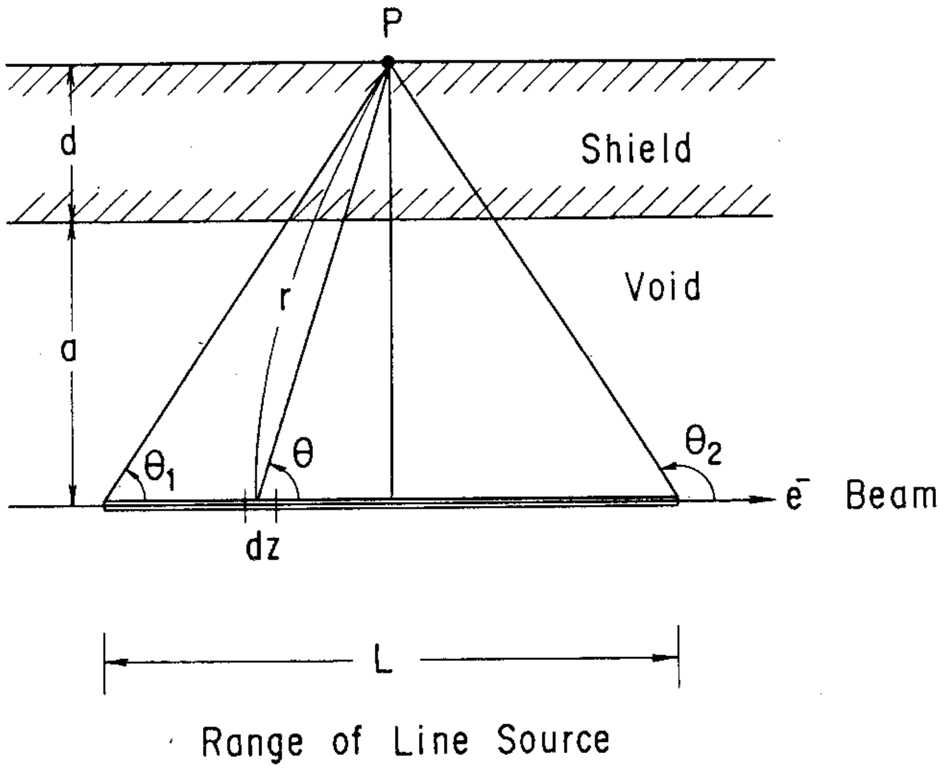

Fig. 2.1 Configuration of a line source model.

3. Profiles of Dose Equivalent Ratios

Equation (2.14) is a function of parameters, /d/, /a/ and /E/, denoting a shield thickness, a distance from beam duct to an inner surface of the shield and electron beam energy, respectively. It is important to make clear functional behavior of Eq.(2.14) with /d/, /a/ and /E/, before the equation is applied to practical shielding calculations. For this purpose, we obtain as reference data, dose equivalent values with Eq.(2.14) with /d/ = 150 cm, /a/ = 250 cm and /E/ = 8 GeV, which are considered to be typical to SPring-8.

Figures 3.1 through 3.3 give profiles of calculated dose equivalent ratios as a function of L, a length of the line source, being based on Jenkins' equations for photons(Jenkins<γ>) and neutrons(Jenkins<n>) and a Swanson's one for photons, respectively. Here, Eq.(2.14) cannot be defined at L = 0 cm. This difficulty, however, can be avoided, because a line source with L = 0 cm is equivalent to a point source, then Eq.(2.14) can always be defined as 1.0. Jenkins<γ> and Jenkins<n> equations give actually the same profiles. This is because the Jenkins<γ> includes a term of secondary photons from high energy neutron interation with matters, as well as a term of primary photons from target materials and because in the present case the decay behavior of photons within a shield is dominated by high energy neutrons. Jenkins<γ> and Jenkins<n> equations give profiles whose magnitudes of each material are not exactly in order with atomic numbers. On the other side, Swanson's equation gives more rapidly decreasing profile than those by Jenkins' equations. Besides their magnitudes are exactly in order with atomic numbers.

Since we consider Figs. 3.1 through 3.3 as reference data, the following calculational results by Eq.(2.14) with different parameter values are given as ratios with corresponding reference data. Preliminary shielding calculations indicate that shield thickness ranges from 100 cm to about 200 cm at the synchrotron and the storage ring where a line source is to be assumed. So we selected /d/ values as 100 cm, 150 cm and 200 cm. Figure 3.4 presents calculational results as a function of /d/ for the fixed line source of L = 10 m, with /a/ = 250 cm and /E/ = 8 GeV. Jenkins' equations present quite different profiles from various materials, some are monotonically increasing and others are decreasing. Contrary to Jenkins' equations, Swanson's equation presents a weak dependence on /a/ for all materials of interest, slowly and monotonically decreasing with /d/.

As for /a/ dependence, the functions $f(\theta)$ corresponding to every equations in Eq.(2.14) are apparently independent of /a/ and its dependence is described only by a term of (a+d)/L in Eq.(2.14). From the fundamental layout of the synchrotron and the storage ring of SPring-8, we selected /a/ values as 200 cm, 250 cm and 300 cm. Figure 3.5 is the graphs presenting functional variation of Eq.(2.14) as a function of /a/, in the case of L = 10 m, with /d/ = 150 cm and /E/ = 8 GeV. All of them show the same linearly increasing profiles with /a/.

Electrons having the energy of about 1 GeV are injected to the synchrotron and they are accelerated there up to 8 GeV. Then, they are injected to the storage ring. So only electrons of 1 GeV and 8 GeV are needed to be taken into account. Therefore, we employed /E/ as 0.5, 1.6 and 8 GeV. Figure 3.6 shows calculational results as a function of /E/ at L = 10 m for the configuration of /a/ = 250 cm and /d/ = 150 cm. While Swanson's equation does not show /E/ dependence because $S(\theta)$ in Eq.(2.17) is independent of /E/, Jenkins' equations show /E/ dependence through parameters f_1 and f_2, the fall-off factors for high energy and intermediate energy neutron sources. Very weak /E/ dependence is observed above 1.5 GeV, but rapid increase with /E/ is seen below there.

In the shielding calculations of SPring-8, we assumed line sources of 8 meter long for the synchrotron and those of 7 meter long for the storage ring. Since our design calculations are based on Jenkins' equations, photons and neutrons are considered separately. Then we employed, for the line source of 8 meter long, photon dose equivalent ratios of 0.45 and 0.5 for ordinary and heavy concretes, respectively. Similarly, neutron dose equivalent ratios of 0.58 and 0.52 are employed. As is seen from Figs. 3.1 and 3.2, there observed practically no differences on the ratios between the line sources of 7 meter long and 8 meter long. That is to say, by assuming a line source, we obtain dose equivalent values approximately a half of those for a point source model.

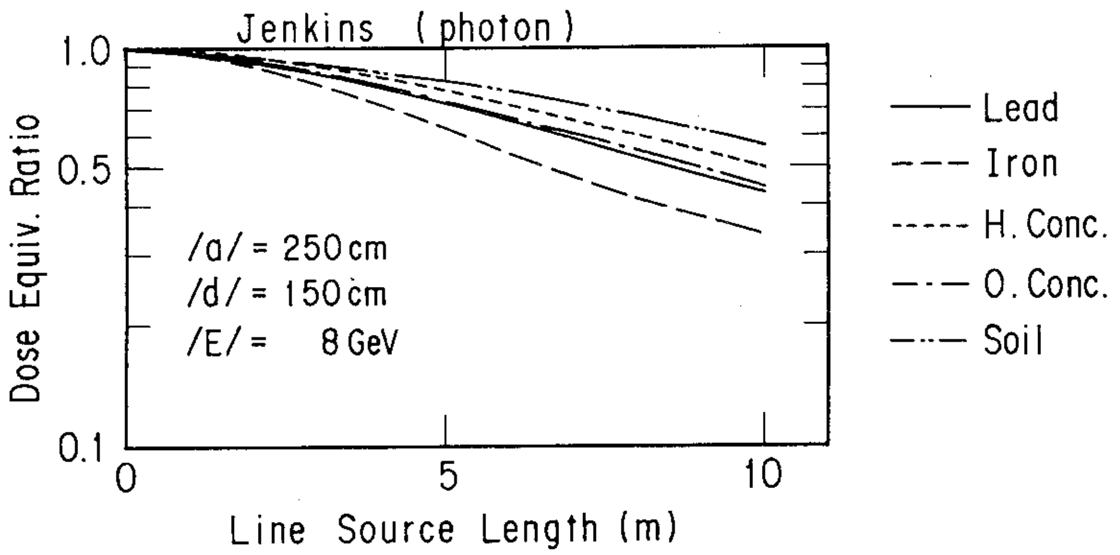

Fig. 3.1 Ratio of dose equivalent values by Jenkins' equation for photons, based on a line source model against those based on a point one.

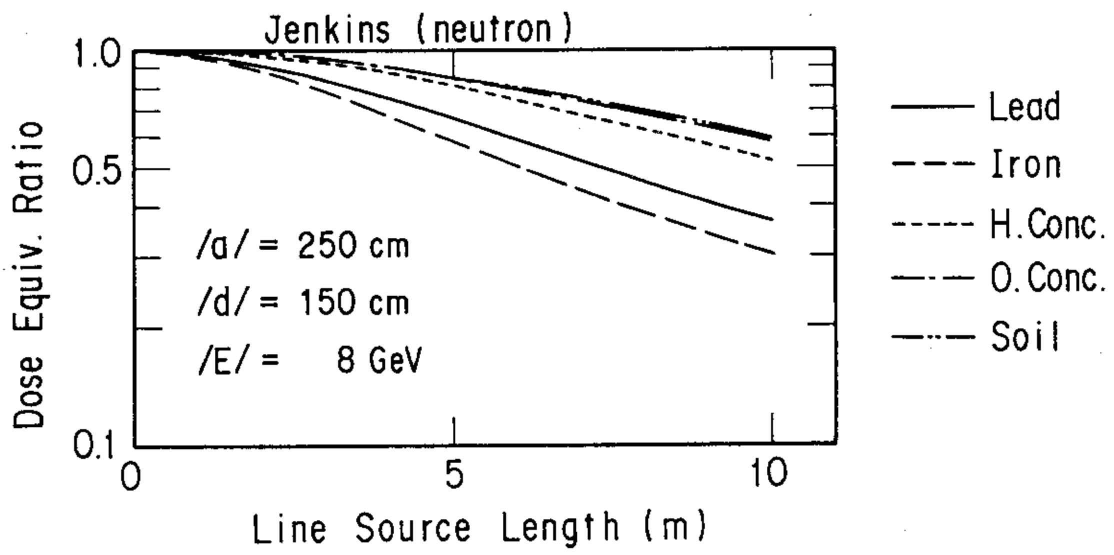

Fig. 3.2 Ratio of dose equivalent values by Jenkins' equation for neutrons, based on a line source model against those based on a point one.

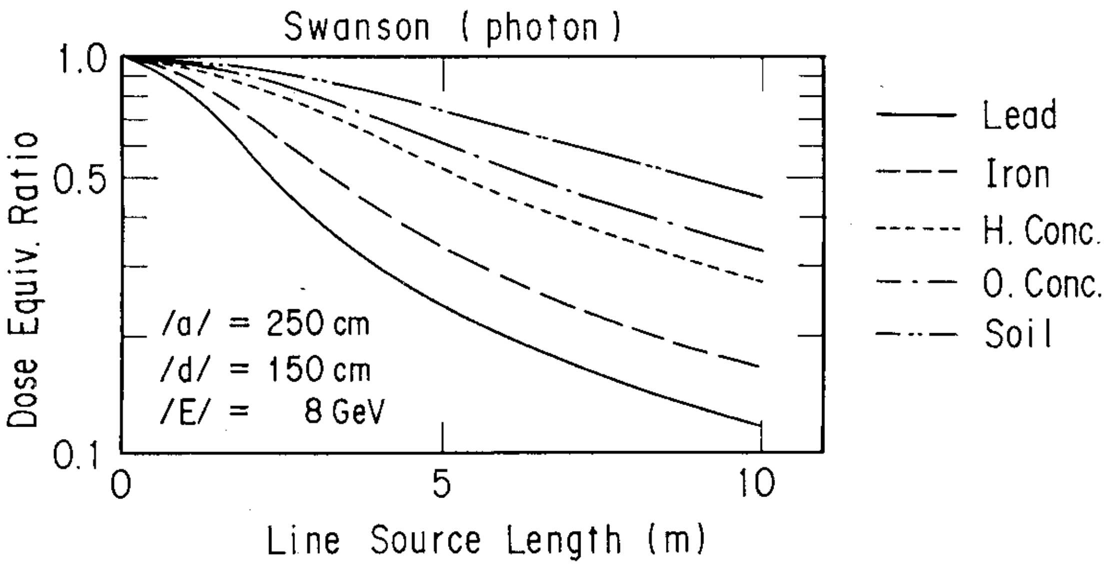

Fig. 3.3 Ratio of dose equivalent values by Swanson's equation for photons, based on a line source model against those based on a point one.

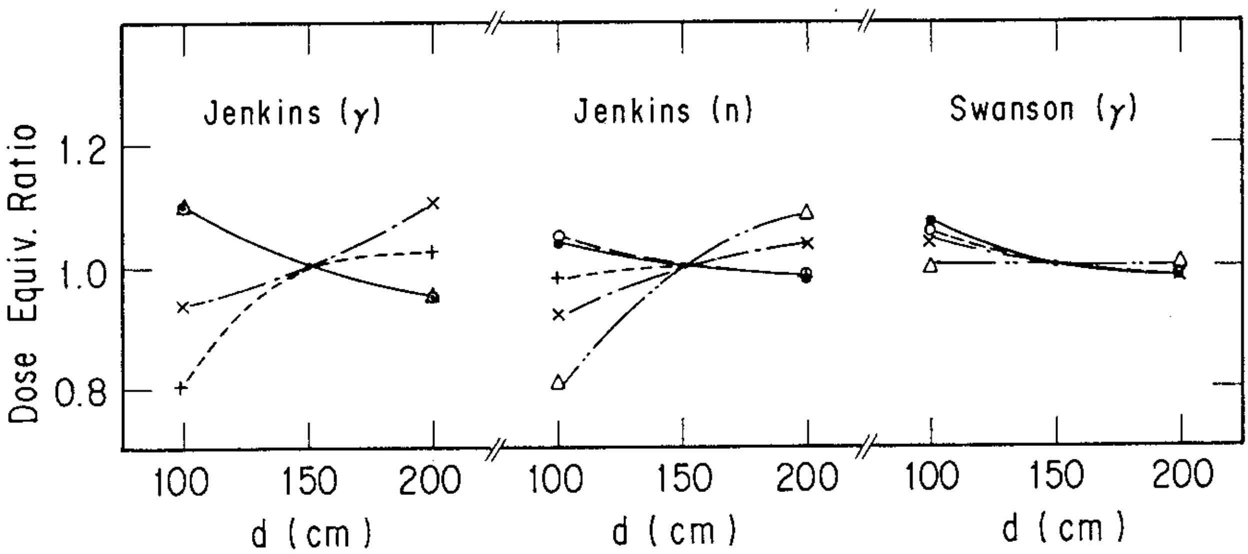

Fig. 3.4 Dose equivalent ratios for L = 10 meter long as a function of /d/, a shield thickness, against the reference data obtained under the conditions of /a/ = 250 cm, /d/ = 150 cm and /E/ = 8 GeV. Symbols in the figure denote as,
● : Lead, ○ : Iron, + : Heavy Concrete,
× : Ordinary Concrete, Δ : Soil.

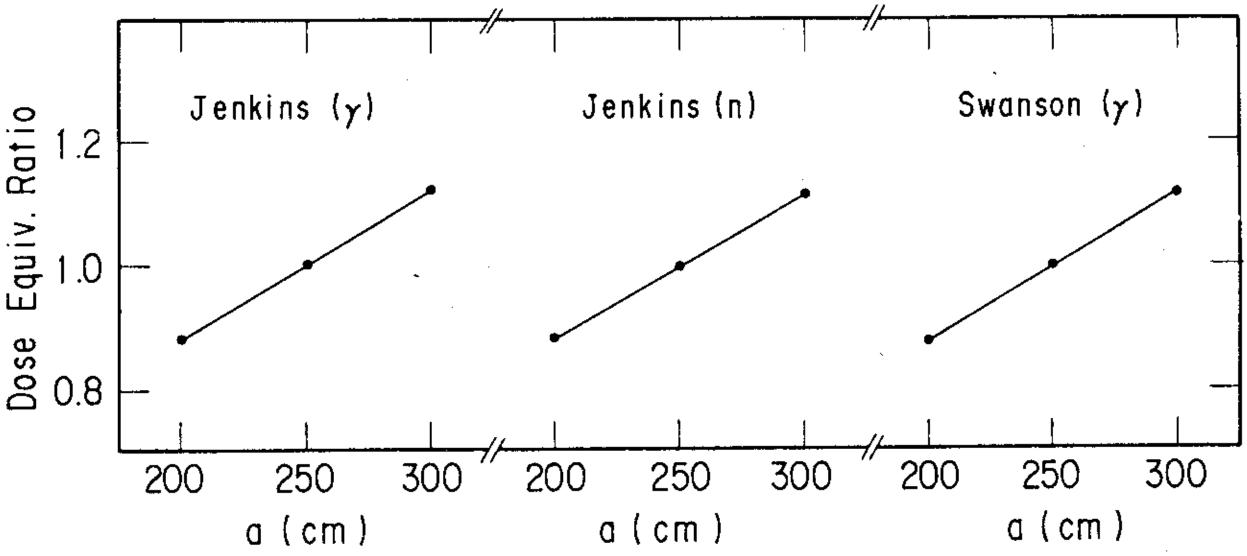

Fig. 3.5 Dose equivalent ratios for L = 10 meter long as a fucntion of /a/, a distance from source to inner surface of a shield, against the reference data obtained under the conditions of /a/ = 250 cm, /d/ = 150 cm and /E/ = 8 GeV. No differences are observed with shield materials.

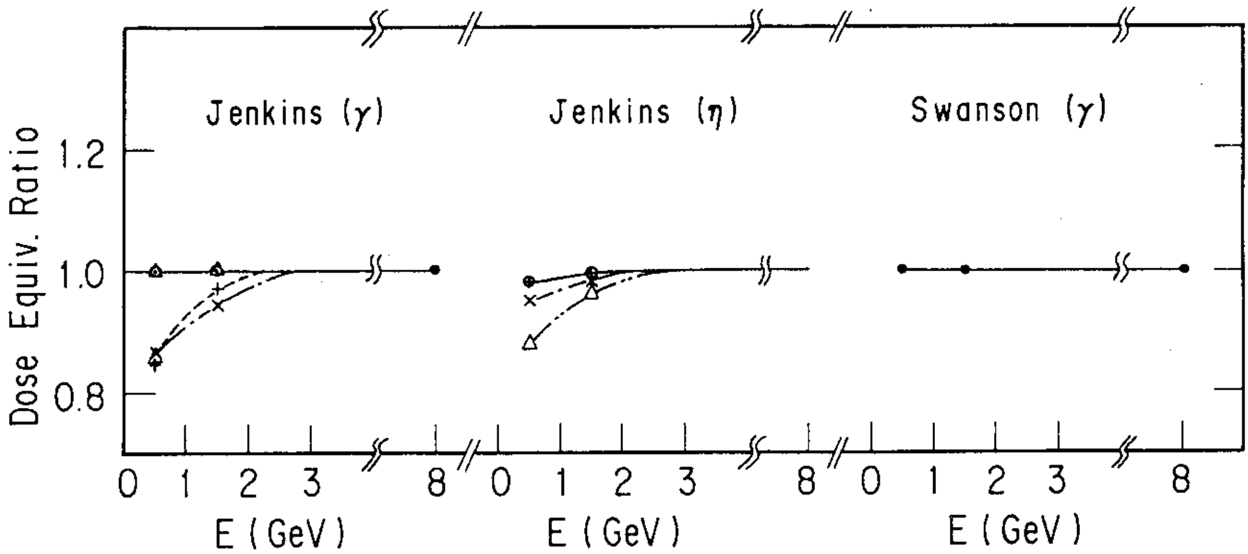

Fig. 3.6 Dose equivalent ratios for L = 10 meter long as a function of /E/, an electron energy, against the reference data obtained under the conditions of /a/ = 250 cm, /d/ = 150 cm and /E/ = 8 GeV. Symbols in the figure denote as,
● : Lead, ○ : Iron, + : Heavy Concrete,
× : Ordinary Concrete, △ : Soil

4. Accuracy Estimation of Empirical Formula[4)]

An empirical formula is usually recommended to be used, which gives a relation between dose equivalent values based on models of a line source and a point source. It is meaningful to estimate accuracy of the empirical formula and so we compared it with the theoretical formula of Eq.(2.14) to find out its deviation and its application limits.

According to the empirical formula, effective source intensity for a line source J_{eff} is given as,

$$J_{eff} = L_{eff} \times \frac{dJ}{dL} \qquad (4.1)$$

$$L_{eff} = 1.9 \times \left(\frac{d}{\lambda}\right)^{-0.4} \cdot \left(a + \frac{d}{\rho}\right) \qquad (4.2)$$

where

- dJ/dL : beam loss rate per unit length of line source
- d : shield width (g/cm^2)
- λ : attenuation length (g/cm^2)
- ρ : material density (g/cm^3)
- a : distance from a beam duct to an inner surface of shield (cm)

Assuming that beam loss distributes uniformly along a line source, we can rewrite dJ/dL as,

$$\frac{dJ}{dL} = \frac{J_0}{L} \qquad (4.3)$$

Here, J_0 and L indicate total beam loss rate within a line source and length of a line source, respectively. Therefore we rewrite Eq.(4.1) as,

$$J_{eff} = L_{eff} \cdot \frac{J_0}{L} \qquad (4.4)$$

Then we obtain

$$\frac{J_{eff}}{J_0} = \frac{L_{eff}}{L} = \frac{1.9 \times \left(\frac{d}{\lambda}\right)^{-0.4} \cdot \left(a + \frac{d}{\rho}\right)}{L} \qquad (4.5)$$

The left side of Eq.(4.5) describes a normalized effective line source intensity, giving a ratio of dose equivalent values based on a line source model to that on a point source model.

For the configuration with /a/ = 250 cm and /d/ = 150 cm of ordinary concrete, dose equivalent ratios were calculated using the empirical formula, Jenkins' equations and Swanson's one and their results were compared with one another in Fig. 4.1. At L > /a/ + /d/, the empirical formula approaches asymptotically to Swanson's equation, but always underestimates Jenkins' ones. From this figure, it is apparent that the empirical formula is derived on the basis of Swanson's equation. At L < /a/ + /d/, however, the empirical formula increases rapidly with decreased of L, leading to divergence at L = 0. The deviations of the empirical formula from Swanson's equation are more explicitly given in Fig. 4.2 for the five shielding materials of interest. They all decrease monotonically with L at L > /a/ + /d/ to converged values depending on the materials. It is seen from the figure that overestimate of the empirical formula to the theoretical one at L > /a/ + /d/ is at most 17% which may be acceptable in actual shielding design calculations. Similarly, a lot of calculations were performed with a variety of combinations of /a/ and /d/, making clear that the maximum deviation of the empirical formula of photons for high energy accelerators with the accuracy of 20%, for the calculations with a single point kernel type equation, such as Swanson's equation. Besides, it is worth noticing that application of the empirical formula should be limited to source length L > /a/ + /d/.

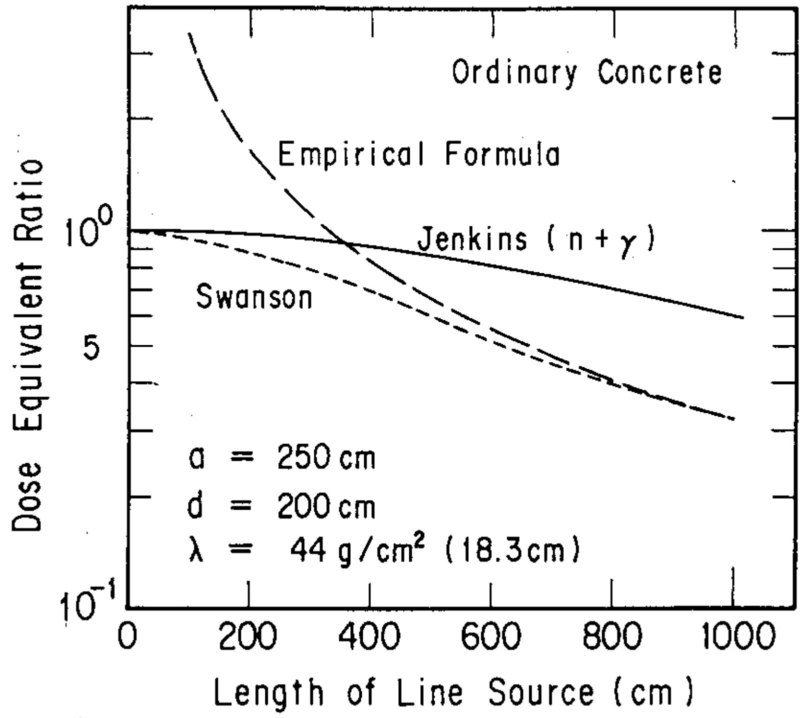

Fig. 4.1 Comparison of the dose equivalent ratios for ordinary concrete by the empirical formula with the theoretical ones.

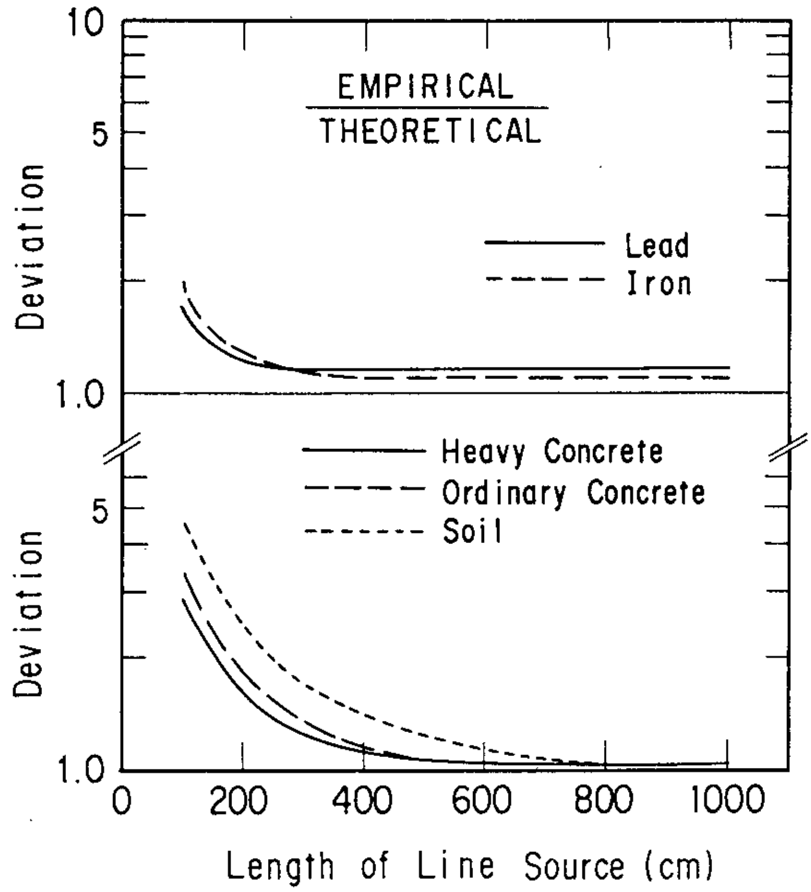

Fig. 4.2 Deviation of the dose equivalent ratios by the empirical formula from the theoretical ones.

5. Summary

Aiming at realistic and reasonable shielding design of SPring-8, Jenkins' equations and Swanson's one based on a line source model were derived using Gauss-Legendre numerical integration method. The ratio of the dose equivalent value from a line source model to that from a point source was defined as a function of the line source length L, and its functional behavior was investigated. The ratio based on Swanson's equation is seen to decrease more rapidly than those based on Jenkins' equations for photons and neutrons which show quite a similar tendency throughout the whole region of L. The ratio based on a Swanson's equation reduces to half the ones based on Jenkins' equations at L = 10 meter long.

When applying Jenkins' equations to the shielding design of SPring-8, we found that the dose equivalent value from a line source model would be estimated to be nearly half the value from a point source model, for the line source of 7 ~ 8 meter long along the synchrotron or the storage ring, originated from 8 GeV electrons.

For the actual shielding design study of accelerators, we are recommended to use a formula which transforms the dose values from a point source model to that from a line source model. It was revealed through comparison of the formula with our theoretical work that the present formula has been derived on the basis of the calculations with Swanson's equation, always underestimating the values based on Jenkins' equations. The formula should be adopted to design studies under the condition of $L > /a/ + /d/$, that is, the line source length should be longer than the distance from the source point to the estimation point. The deviation of the formula from our theoretical evaluation for the SPring-8 is found to be at most 20%, which might well be acceptable to the actual shielding design of accelerators.

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