JAERI-M
91-206

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December 1991

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編集兼発行　日本原子力研究所
印　　刷　㈱原子力資料サービス

JAERI-M 91-206

JT-60 Configuration Parameters for Feedback Control Determined by Regression Analysis

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(Received November 8, 1991)

The stepwise regression procedure was applied to obtain measurement formulas for equilibrium parameters used in the feedback control of JT-60. This procedure automatically selects variables necessary for the measurements, and selects a set of variables which are not likely to be picked up by physical considerations. Regression equations with stable and small multicollinearity were obtained and it was experimentally confirmed that the measurement formulas obtained through this procedure were accurate enough to be applicable to the feedback control of plasma configurations in JT-60.

Keywords: JT-60, Feedback, Regression

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回帰解析によるフィードバック制御のための JT－60平衡配位パラメータ検出

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（1991年11月8日受理）

段階的回帰手法を，JT－60においてフィードバック制御に使用される平衡パラメータの検出式を得るために適用した。この方法は，検出に必要な変数を自動的に選択することができるという特長を有しており，結果として得られた変数のセットは物理的考察から推定される変数の組み合わせとはやや異なるものである。しかし，得られた回帰方程式は安定でなおかつ多重共線性が低いという性質を有している。実際，JT－60においてプラズマ配位のフィードバック制御に対して十分な精度で適用可能であることが，実験により確かめられた。

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1. INTRODUCTION

Recently, the statistical approach has been used to determine plasma equilibrium parameters (Hosogane et al., 1986; Braams et al., 1986; van Milligen et al., 1988) or to find some relationships among experimental parameters (Iwata et al., 1987). This approach is very useful especially when desired physical relationships can not be easily obtained from an analytical approach. In the feedback control system of JT-60, the regression functions consisting of twenty-four magnetic signals were used as formulas for determining outer X-point divertor configuration parameters (Hosogane et al., 1986).

However, statistical theory indicates that regression functions consisting of too many variables are usually unstable due to their multicollinearity. To reduce the dimensionality of the regression problems, Braams et al. (1986) have used a principal component analysis. This method finds the principal components among magnetic signals in their linear combinations, but does not select a sub set of magnetic signals. For practical understanding, it may be of interest to perform the regression directly on a sub set of the magnetic signals.

In the field of the statistical research, various mathematical methods have been studied for obtaining regression functions with fewer variables. One of the most reliable method is the stepwise regression procedure of predictor variables (Williams, 1957; Draper and Smith, 1966). This procedure automatically selects magnetic signals according to the strength of the correlation with configuration parameters. In JT-60, the minor modification from an outer X-point divertor configuration to a lower X-point one was performed (Kishimoto et al., 1988) and, in the later case, the measurement formulas for feedback control of plasma position and shape could not be derived from an analytical approach. Therefore, it was necessary to use a statistical approach and the stepwise regression procedure was applied to derive these measurement formulas. It was confirmed in the experiment that this procedure gives the stable regression functions, in the sense that the prediction errors of the plasma parameters are reasonably small, even if measurement errors in the magnetic measurement are taken into account. It is worth noting that the objective of this paper is to show the method of obtaining practical measurement formulas for feedback control, so that the authors do not pay much attention to various detailed problems of regression analysis.

In this paper, the stepwise regression procedure is explained in section 2 and application to the derivation of measurement formulas for the equilibrium parameters is given in section 3. Experimental results of plasma control using the measurement formulas derived by this procedure are described in section 4 and discussion of a more general approach is presented in section 5. The conclusions are summarized in section 6.

2. REGRESSION ANALYSIS WITH THE STEPWISE REGRESSION PROCEDURE OF PREDICTOR VARIABLES

When we estimate a certain variable y, the direct measurement of which is difficult, with several measurement variables $\{x_i\}$ (x_1, x_2, x_3, --- x_r), we can approximate y by means of the general linear equation

$$y = \sum_{i=1}^{r} \beta_i x_i + \varepsilon . \tag{1}$$

This relationship represents the true model in terms of the parameters $\{\beta_i\}$ and the true error ε. Here, we use data in which the averages of predictor variables $\{x_i\}$ and response variable y are set to zero. Using the estimates of $\{\beta_i\}$ and the residual e, we can obtain the following regression equation :

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$$\hat{y} = \sum_{i=1}^{r} b_i x_i, \tag{2}$$

where $\hat{y}$ (= y - e) is the estimated value of y and $\{b_i\}$ are the regression coefficients. The regression coefficients $\{b_i\}$ are determined by minimizing the sum of the squared residuals :

$$E = \sum_{j=1}^{N} e_j^2, \tag{3}$$

where N ($\geq$ r + 1) is the number of sets of the data points. Therefore, the regression coefficients are obtained from the following equation,

$$\mathbf{b} = \{ \mathbf{X}^T\mathbf{X} \}^{-1} \mathbf{X}^T\mathbf{y}, \tag{4}$$

where

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \\ y_N \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} X_{11} & X_{12} & & X_{1r} \\ X_{21} & X_{22} & & X_{2r} \\ & & & \\ X_{N1} & X_{N2} & & X_{Nr} \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ \\ b_r \end{pmatrix}.$$

The mean square of this regression equation is given by

$$S^2 = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 / (N - r - 1), \tag{5}$$

and the estimated variance of the regression coefficient b_i is given by

$$S_{bi} = S\sqrt{C_{ii}} \ , \tag{6}$$

where C_{ii} is the i-th diagonal component of $\{ \mathbf{X}^T\mathbf{X} \}^{-1}$.

In general, the regression equation contains significant and insignificant predictor variables to estimate the values of y. A variable x_i, which is strongly collinear with x_j, puts not only information but also unfavorable noise into the regression equation and the obtained regression equations become unstable. Therefore, it is important to select a suitable set of predictor variables. As a solution for this problem, we adopted the stepwise regression procedure.

If the objective response variables y could be expressed in terms of only one predictor variable x_i, we could write the regression equation as follows,

$$y = b_i x_i + e. \tag{7}$$

The problem is simplified by selecting the most important predictor variable x_i. First, we have to investigate the residual over all predictor variables using Equation (3). Then we can easily select the most correlative variable x_i from the set of n predictor variables, because it is clear that the most correlative variable gives us the least residual. Next, the selected variable has to be checked to see whether it is significant or not. An F-value of the variable provides the evaluated function of probability given by

$$F_i = x_i^2 / C_{ii}. \tag{8}$$

This value follows the F-distribution with degrees of freedom (1, N - r). If the F-value is larger than 2, the obtained variable has a high significant level. Where, F = 2 corresponds to the criterion of 16.7% of F-distribution.

Then we have to pick up the second variable x_j from the remaining n-1 variables. Equation (7) can be rewritten as follows,

$$y = b_i x_i + b_j x_j + e, \tag{9}$$

and the F-value to enter must be investigated for each of remaining variables except x_i. Just as in the first case of x_i , the second variable x_j can be selected by comparing the residuals. At each stage, one adds the predictor variable that leads to the largest reduction in the residual sum of squares (This is equivalent to adding the variable whose regression coefficient has the highest F-value, and also to adding the variable which maximizes the multiple correlation between y and the new regression set). However, one should note that both of the F-values should be checked so that all selected predictor variables have a sufficiently significant level. This is because the variance of b_i and/or b_j may become larger and the variables selected by previous step may be insignificant in the regression equation if the correlation between x_i and x_j is too great. Then the F-value of x_i, as well as x_j, must be checked to see whether x_i is still at a significant level or not. This procedure should be continued from the first predictor variable selection until any one of the F-values becomes smaller than 2 for all predictor variables. Through this stepwise regression procedure, we can obtain a suitable set of predictor variables.

In the original stepwise regression procedure explained by Draper and Smith, a predictor variable included in a previous step would be removed from the regression equation at a later step if the inclusion of a new predictor variable reduced any F-value below the critical value. Therefore the Draper and Smith procedure continued until no further predictor variables were to be removed from and none were to be added to the regression equation.

However, our approach differs from this exhaustive convergence process in that we terminate the stepwise algorithm when any one of the F-values becomes lower than 2. This is done in order to ensure reliability and stability in the regression equation at the expense of the slight improvement in accuracy which would result from the complete stepwise procedure described by Draper and Smith.

3. APPLICATION FOR THE MEASUREMENT FORMULAS OF EQUILIBRIUM PARAMETERS IN JT-60

3.1 Equilibrium parameters, poloidal coil system and magnetic probes

Figure 1 shows a typical lower X-point divertor configuration and the arrangement of magnetic probes in JT-60. The controlled parameters of divertor plasmas by the feedback control system are the horizontal position R_p, the vertical position Z_p and the position of the X-point δ_x, which are controlled mainly by the vertical field coil, the horizontal field coil and divertor coil, respectively. R_p is a geometrical center of the plasma defined as the average of the minimum and maximum horizontal position of the most outer plasma surface. The definition of Z_p is the same as R_p except in the case of divertor configuration. In the divertor configuration, Z_p is the average of the maximum vertical position of the outer most plasma surface and the position of X-point. The plasma current profile is, therefore, not considered in the definition of Rp and Zp. The gaps between the plasma surface and the wall δ_{30}, δ_{52}, δ_{72}, δ_{100} are also important parameters to produce divertor configurations, but these gaps are controlled at suitable values through horizontal and vertical position control of the plasma column. Since there is no shaping coil for producing a quadrupole field, plasma shapes characterized by ellipticity or triangularity are limited within a narrow range.

In JT-60, there is no flux loop which is usually used for feedback control of non-circular plasmas. Instead, two types of magnetic probe, T-probes and N-probes, are installed in order to measure the tangential and normal components of the poloidal magnetic field.

3.2 Derivation of the measurement formulas

Regression analysis using the stepwise regression procedure is applied to determine the measurement formulas for equilibrium parameters R_p, Z_p, δ_x,

δ_{100} etc. in the feedback control of JT-60. To obtain relationships among the equilibrium parameters, magnetic signals and coil currents, an MHD equilibrium database was generated, because no other database existed before the initiation of lower X-point divertor experiments. This database is composed of the equilibrium parameters, magnetic probe signals, coil currents, plasma current etc. for various divertor and limiter configurations. The equilibrium configurations are calculated with the up-down asymmetry free-boundary MHD equilibrium code modified from SELENE 40 (Azumi et al., 1980). The database consists of an adequate sample of the equilibrium configurations in the operational region of JT-60. Figures 2(a) and (b) show all data points in the R_p-Z_p space.

The number of simulated equilibria is 49 for the limiter configuration with excitation of the divertor coil ($i_d \neq 0$), and 173 for the divertor configuration. Although these numbers of equilibria seem to be relatively low, they may be expected to be sufficient, because the operational region of JT-60 is relatively narrow. Of course, a large database is obviously an advantage in obtaining accurate results. However, it should be stressed that the information does not increase linearly with the size of the database and, as described later, we have gotten satisfactory results with the numbers cited above.

In JT-60, the following signals can be used for the raw variables : twelve magnetic signals B_{tk}, B_{nk} (k=1,6) (tangential and normal components of poloidal magnetic field, respectively), the divertor coil current I_d, and the plasma current I_p. The vertical and horizontal coil currents have not been used because the shielding effect of the vacuum vessel would not be negligible compared with their fast variations. From these 14 variables, we generated the following 24 predictor variables x_i in the sense of the regression analysis described in section 2;

$$x_i \; (i = 1, 24) : \frac{B_{tk}}{I_p}, \frac{B_{nk}}{I_p}, \frac{B_{tk} I_d}{I_p^2}, \frac{B_{nk} I_d}{I_p^2} \quad (k = 1, 6) .$$

Here, the variables normalized by the plasma current are used because the shape of the plasma boundary is independent of the plasma current. These 24 predictor variables are the same as the variables used in the case of outer X-point operation of JT-60 (Hosogane et al., 1986). With these predictor variables, we sought suitable regression equations for the equilibrium parameters R_p, Z_p, δ_x, δ_{100} etc. through the stepwise regression procedure. Although we relate these quantities to predictor variables empirically at first, more generalized predictor relations were tested with both simulation and experimental data. Results will be presented for comparison in the Discussion section.

The stepwise regression procedure selects successively the most significant variables for the regression equation. However, it should be noted that the database used in the regression analysis here was generated by calculations, and does not contain any measurement error in each of the magnetic probe signals, the divertor coil current and plasma current. The influence of the measurement errors tends to increase with the number of the variables taken into the regression equation. Therefore the choice of the variables should also be influenced by the estimation of the error due to the measurement errors. The following form is used to estimate the influence of the measurement errors as an auxiliary reference:

$$\varepsilon_m = \sum_{i=1}^{k} \left| b_i \langle x_i y \rangle \times 0.02 \right| .$$

This form corresponds to the sum of absolute error for every predictor variable term in the regression equation. Here, $\langle x_i y \rangle$ is the covariance of x_i and y. We assumed 2% error for each predictor variable based on prior experience. The sum of absolute error corresponds to the worst case under that assumption. The number of predictor variables was determined by comparing the decrease of residual and the increase of predicted error from the view point of feedback control.

An example of the regression analysis for the plasma horizontal position R_p is described as follows. Figure 3 shows the standard deviation S, the prediction errors ε_m and the minimum F-value of the regression coefficients for each step as a function of the number of selected predictor variables N_x. The standard deviation S rapidly decreases until $N_x = 4$. This result indicates that the first four predictor variables make the largest contribution to R_p. The decreasing rate of S becomes lower when N_x exceeds five, but all F-values are still larger than 2 until N_x=10. On the other hand, it is noted that the minimum F-value changes up and down as a function of N_x. In fact, all F-values again become larger than 2 after the appearance of a F-value less than 2 in the case of N_x=15. Therefore, we look at the prediction error as a figure of merit. We can determine a suitable N_x by comparing the changes of residual and predicted error as mentioned above. It is noted that the analytical formula gives only a rough approximation to the simulated prediction error. Consequently, it seems sensible to use no more than 10 variables in the regression equation.

Figure 4 shows the relationship between the plasma positions from the MHD database, R_p^{MHD}, and those estimated from the regression equation obtained using the above procedure, R_p^{REG}. The value of R_p^{REG} coincides reasonably (±2%) with R_p^{MHD}, and satisfactory results for Z_p, δ_x, δ_{100} etc. are also obtained as shown in Figs. 4(b) and (c). Therefore, these regression equations were applied to the feedback control system of JT-60 for real time plasma parameter recovery.

A good feature of this procedure is that it automatically selects magnetic signals necessary to measure equilibrium parameters from existing sensors. A given set of sensors is often quite different from those which might be chosen from physical considerations. Moreover, we can easily revise the regression equation when any of the magnetic sensors in use are faulty. This is one of the most advantageous points of this procedure. The result of δ_x is a good example to demonstrate the previous feature. Corresponding to the fact that the X-point is controlled by the divertor coil, one might expect that the magnetic probe signals near the divertor region would be selected as the predictor variables. The set of magnetic signals chosen by the stepwise procedure for prediction of δ_x is shown in Fig. 5. The probe located at the top of vacuum vessel would be unlikely to be chosen from physical considerations alone.

4. EXPERIMENTAL RESULTS

Figure 6 shows time traces of the equilibrium parameters R_p, Z_p, δ_x and δ_{100} in a typical divertor discharge. The divertor configuration is produced from 1.5 s to 4.5 s in the discharge. Solid lines show the time trace calculated from the measurement formulas obtained from the stepwise regression procedure using the calculated database. Open circles show equilibrium parameters at every 0.2 s calculated from the magnetic fitting code (FBI code which is based on all of available magnetic signals, refer S. Tsuji et al., 1986 and D.W. Swain & G.H. Neilson, 1982). In general, the equilibrium parameters have some offset values. The offsets for R_p, Z_p, δ_x and δ_{100} observed in the experiment are about 3.6 cm, 5.6 cm, 1.8 cm and 1.2 cm, respectively. The errors expected from ε_m are 2.3 cm, 1.8 cm, 0.63 cm and 1.2 cm for R_p, Z_p, δ_x and δ_{100}, respectively. The offset values of the experiment are 2-3 times as large as those of the expected errors. The offset might be partly due to the magnetic probes, and partly due to the difference in the accuracy of the FBI code and the MHD equilibrium code.

We can obtain a new database of equilibrium parameters calculated from the FBI code from actual experimental data. The offsets, which are the difference between the average values given by the FBI code and the regression equation, can be reduced through regression analysis using the new database. In other words, we can derive the modified regression equation using the FBI database to demonstrate the previous feature. Corresponding to the fact that the X-point is controlled by the divertor coil, one might expect that the magnetic probe signals near the divertor region would be selected as the predictor variables. The set of magnetic signals chosen by the stepwise procedure for prediction of δ_x is shown in Fig. 5. The probe located at the top of vacuum vessel would be unlikely to be chosen from physical considerations alone.

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We can obtain a new database of equilibrium parameters calculated from the FBI code from actual experimental data. The offsets, which are the difference between the average values given by the FBI code and the regression equation, can be reduced through regression analysis using the new database. In other words, we can derive the modified regression equation using the FBI database which includes the measurement errors. In this way, after the FBI database was created once, the simulated database not needed. Figures 7(a), (b) and (c) show the comparisons of R_p^{REG}, Z_p^{REG}, δ_p^{REG} obtained from the new regression equation using the FBI database and those calculated from the FBI code. The number of equilibria in the new data set is 440. The database contains equilibrium configurations for various discharge conditions and discharge phases, including limiter and divertor discharges, ohmic and NBI discharges, current ramp-up, shift of divertor to limiter configurations, current flat-top and current ramp-down phases. As shown in these figures, the offsets are completely removed. The variances are very small (less than 1 cm for each). The time traces after this procedure are shown by dashed lines in Fig. 6 and good agreement with the FBI code is obtained. We therefore used these new measurement formulas for feedback control instead of the old measurement formulas after the initiation of experiments.

5. DISCUSSIONS

5.1 Stability against new data out of the range in the original database

Features of the unstable regression formula may appear when new data, which are not considered in the original database, are appended or when measurement errors cannot be neglected. The former case is studied using the database and the results are shown in Fig. 8. To check the stability of regression, the regression function is derived from equilibrium data in the range of $0.1 < \beta_p < 0.25$, although the range of β_p in the original database is 0.1 to 1.2. Two regression formulas composed of seven variables and twenty-four variables are derived for R_p. All F-values are larger than 2 in the case of seven variables, but some F-values are smaller than 2 in the case of twenty-four variables. First, the obtained formulas were tested on the partial data of the original database (data of which includes the measurement errors. In this way, after the FBI database was created once, the simulated database not needed. Figures 7(a), (b) and (c) show the comparisons of R_p^{REG}, Z_p^{REG}, δ_p^{REG} obtained from the new regression equation using the FBI database and those calculated from the FBI code. The number of equilibria in the new data set is 440. The database contains equilibrium configurations for various discharge conditions and discharge phases, including limiter and divertor discharges, ohmic and NBI discharges, current ramp-up, shift of divertor to limiter configurations, current flat-top and current ramp-down phases. As shown in these figures, the offsets are completely removed. The variances are very small (less than 1 cm for each). The time traces after this procedure are shown by dashed lines in Fig. 6 and good agreement with the FBI code is obtained. We therefore used these new measurement formulas for feedback control instead of the old measurement formulas after the initiation of experiments.

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$0.1 < \beta_p < 0.2$). Figures 8(a) and (b) show the good agreement in both cases because data in this case are in the expected region. Next, the two regression equations were applied to the case of the whole database. Figures 8(c) and (d) correspond to the case of seven variables and twenty-four variables, respectively. It is seen that the regression equation of twenty-four variables is more sensitive than that of seven variables against the variation of β_p in the database. This result could be explained as the influence of the extension of estimated standard error for some of the regression coefficients. Thus, omitting variables which have F-values of less than 2 is very important in order to keep the regression equation stable.

5.2 Effect of measurement errors

Next, we discuss the case where the measurement error cannot be neglected. The sensitivity to measurement errors is compared for two measurement formulas of R_p derived through the stepwise regression procedure from the simulated database, composed of six variables and thirteen variables. Figures 9(a) and (b) show the relationship of R_p^{REG} measured by those formulas and R_p^{FBI} calculated from the FBI code for equilibrium configurations produced in the experiment. As shown in Fig. 9(a), the calculated R_p^{REG}'s obtained from six variables coincide fairly well with those obtained from the FBI code. However, for the measurement formula with thirteen variables, the calculated R_p^{REG}'s scatter against those obtained from FBI code. This difference could be thought to come from the stability of the regression functions to measurement errors. The property of the coincidence for R_p^{REG} as a function of the number of predictor variables N_x is shown in Fig.9(c), where the residual of R_p^{REG} was adjusted to remove the offset. It is found from this figure that the residual of R_p^{REG} is small when N_x is less than 10, but it increases with increasing N_x in the region of $N_x >$ 10. This behavior suggests that the influence of measurement errors becomes significant for $N_x > 10$. This is a good example of the appearance of instability in the regression equation because Figure 9.(c) has properties very similar to the superimposing of the predicted error ε_m and the residual S shown in Fig.3.

5.3 In the case of full quadratic model

In section 3.2, 24 predictor variables were introduced in a similar way from experience with outer X-point divertor operation in JT-60. These variables are a subset of the quadratic model which were produced from the available magnetic signals. Since the concrete set of the quadratic model includes a subset of it, the concrete set seems advantageous to get accurate results. If the computing speed of feedback controller were improved, the regression equation derived from the concrete set of it become applicable to real feedback system. In fact, the computing speed has been improved continuously and steadily for recent years. Therefore, it is very interesting to investigate the accuracy and stability of the regression equation derived from the concrete set of the quadratic model.

The predictor variables of the quadratic model used in the regression analysis are as follows.

$$x_i(i=1,15) : \frac{B_{tk}}{I_p}, \frac{B_{nk}}{I_p}, \frac{I_d}{I_p}, \frac{I_h}{I_p}, \frac{I_v}{I_p}, (k=1,6)$$

$$x_i(i=16,135) : ((x_j x_k, j=k,15), k=1,15)$$

Two extra poloidal coil currents, horizontal and vertical field coil currents, are adopted to improve the accuracy of fitting results. One may worry that the horizontal and/or vertical field coil currents seem to be risky because of those signals may be shielded in the higher frequency region with the vacuum vessel. It must be true, but all of the predictor variables selected in the regression equation would not have those current signals. And the improvement of the sensing accuracy, that is to suppress the fluctuation of controlled parameter, supports the stablility of feedback system. Therefore, the controllability of feedback system is an another problem essentially.

Figure 10 to 12 show the differences of the regression property between the case of using the partial quadratic model and the case of full quadratic model. Here, the used regression coefficients are obtained from MHD database not from FBI database. In the case of sensing the plasma horizontal position Rp, the observed minimum residual, after its offset removed, decreased to 8.2 mm by 7 variables from 9 mm by 6 variables. It may be slight improvement but the observed residual S^{OBS} has been significantly improved especially in the case of larger number of the predictor variables. It is one of the intertesting differences, because the property of multicollinearity has changed. In the present control system of JT-60, the application of the full quadratic model is impossible. However these results imply that it may be one of the future options. In the both cases of sensing the plasma vertical position Zp and the height of X-point Xp, the regression property were roughly similar with Rp as shown in Fig11-12.

The feedback control computer in JT-60 does not have capability to calculate the predictor variables of the concrete quadratic model. Therefore a subset of it is used for lower divertor experiment in JT-60. We thus simulated the property of the agreement between the simulated MHD database and the experimentally derived FBI database and obtained satisfactory results for all of the measurement signals.

6. CONCLUSIONS

The stepwise regression analysis procedure is useful for deriving measurement formulas for estimation of equilibrium parameters. This procedure automatically selects variables necessary for the measurements, and selects a set of variables which are not likely to be picked up by physical considerations alone. It has been demonstrated by simulation that a sufficiently accurate and stable an another problem essentially.

Figure 10 to 12 show the differences of the regression property between the case of using the partial quadratic model and the case of full quadratic model. Here, the used regression coefficients are obtained from MHD database not from FBI database. In the case of sensing the plasma horizontal position Rp, the observed minimum residual, after its offset removed, decreased to 8.2 mm by 7 variables from 9 mm by 6 variables. It may be slight improvement but the observed residual S^{OBS} has been significantly improved especially in the case of larger number of the predictor variables. It is one of the intertesting differences, because the property of multicollinearity has changed. In the present control system of JT-60, the application of the full quadratic model is impossible. However these results imply that it may be one of the future options. In the both cases of sensing the plasma vertical position Zp and the height of X-point Xp, the regression property were roughly similar with Rp as shown in Fig11-12.

The feedback control computer in JT-60 does not have capability to calculate the predictor variables of the concrete quadratic model. Therefore a subset of it is used for lower divertor experiment in JT-60. We thus simulated the property of the agreement between the simulated MHD database and the experimentally derived FBI database and obtained satisfactory results for all of the measurement signals.

6. CONCLUSIONS

The stepwise regression analysis procedure is useful for deriving measurement formulas for estimation of equilibrium parameters. This procedure automatically selects variables necessary for the measurements, and selects a set of variables which are not likely to be picked up by physical considerations alone. It has been demonstrated by simulation that a sufficiently accurate and stable representation of the equilibrium parameters as a function of the magnetic and current measurements was obtained. A clear advantage of such a representation is the small calculation time, so that it is in principle suitable for feedback control.

It was experimentally confirmed that the measurement formulas obtained through this procedure were accurate enough to be applicable to the feedback control of plasma configurations in JT-60.

Acknowledgements

The authors would like to thank Drs. K. Kurihara, T. Kimura and other members of Control Group for their useful discussions and arrangement of the feedback control system. They also wish to express their gratitude to Dr. D. Humphreys for his useful comments and to Drs. Y. Shimomura and H. Shirakata for continued encouragement and support.

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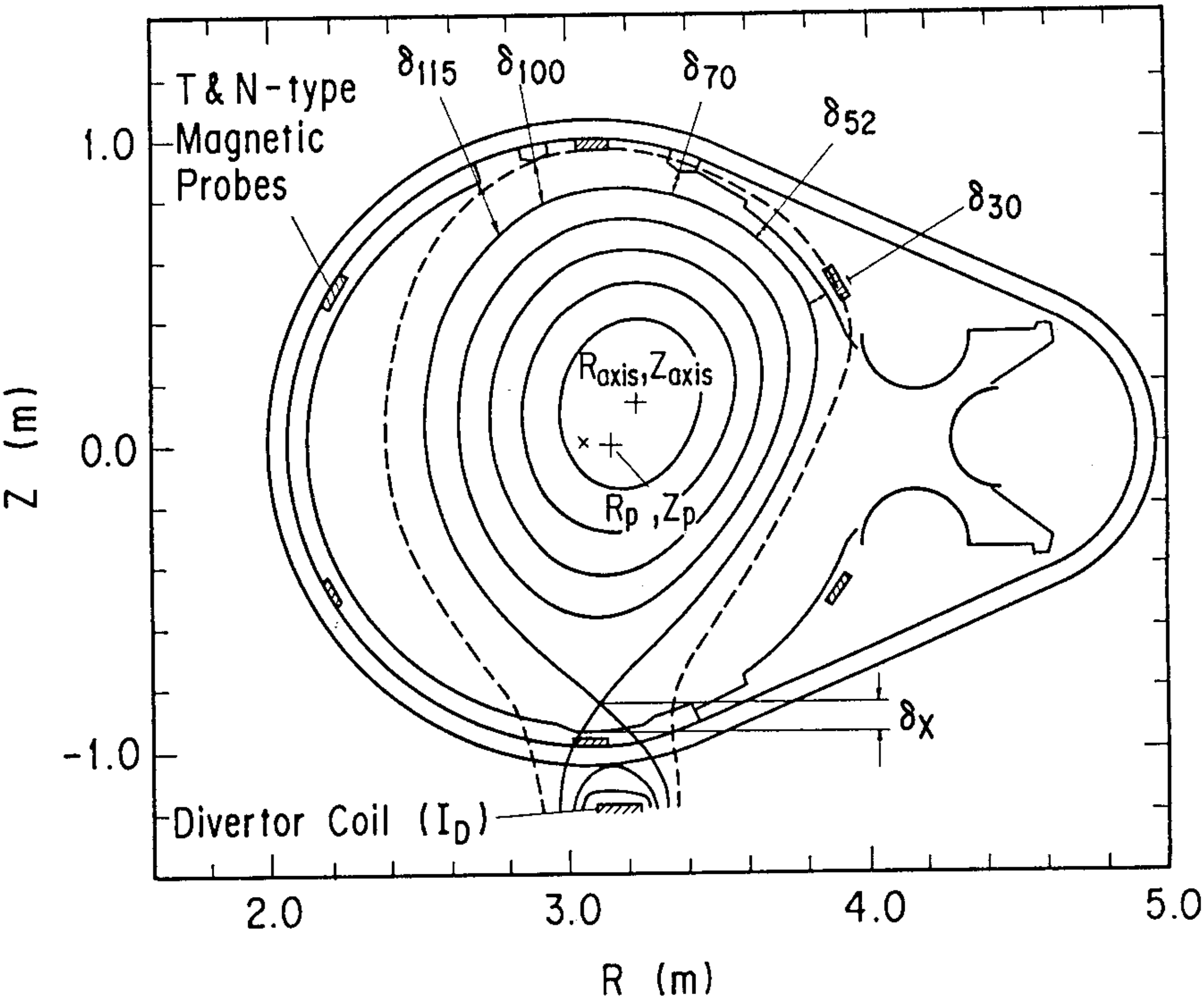


Fig. 1 Equilibrium parameters of JT-60 lower X-point divertor plasma and arrangement of magnetic probes. T and N-type magnetic probes are arranged for measuring the tangential and normal components of poloidal magnetic field, respectively.

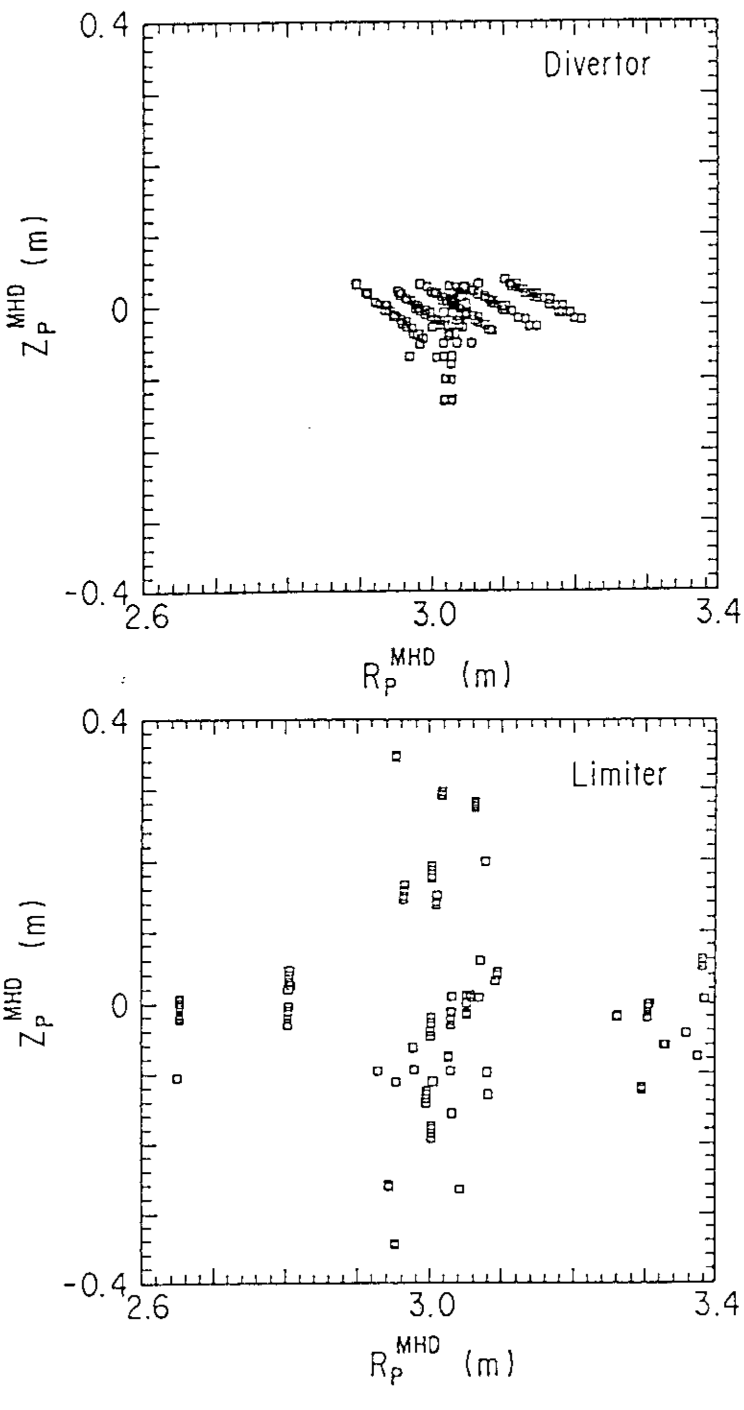


Fig. 2 Data points of the MHD database in Rp-Zp space. The data of limiter configurations include the case in which the divertor coil current is excited but the configuration is a limiter one. Calculated condition is that the internal inductance of plasma is 1 and β_p is from 0.1 to 1.2.

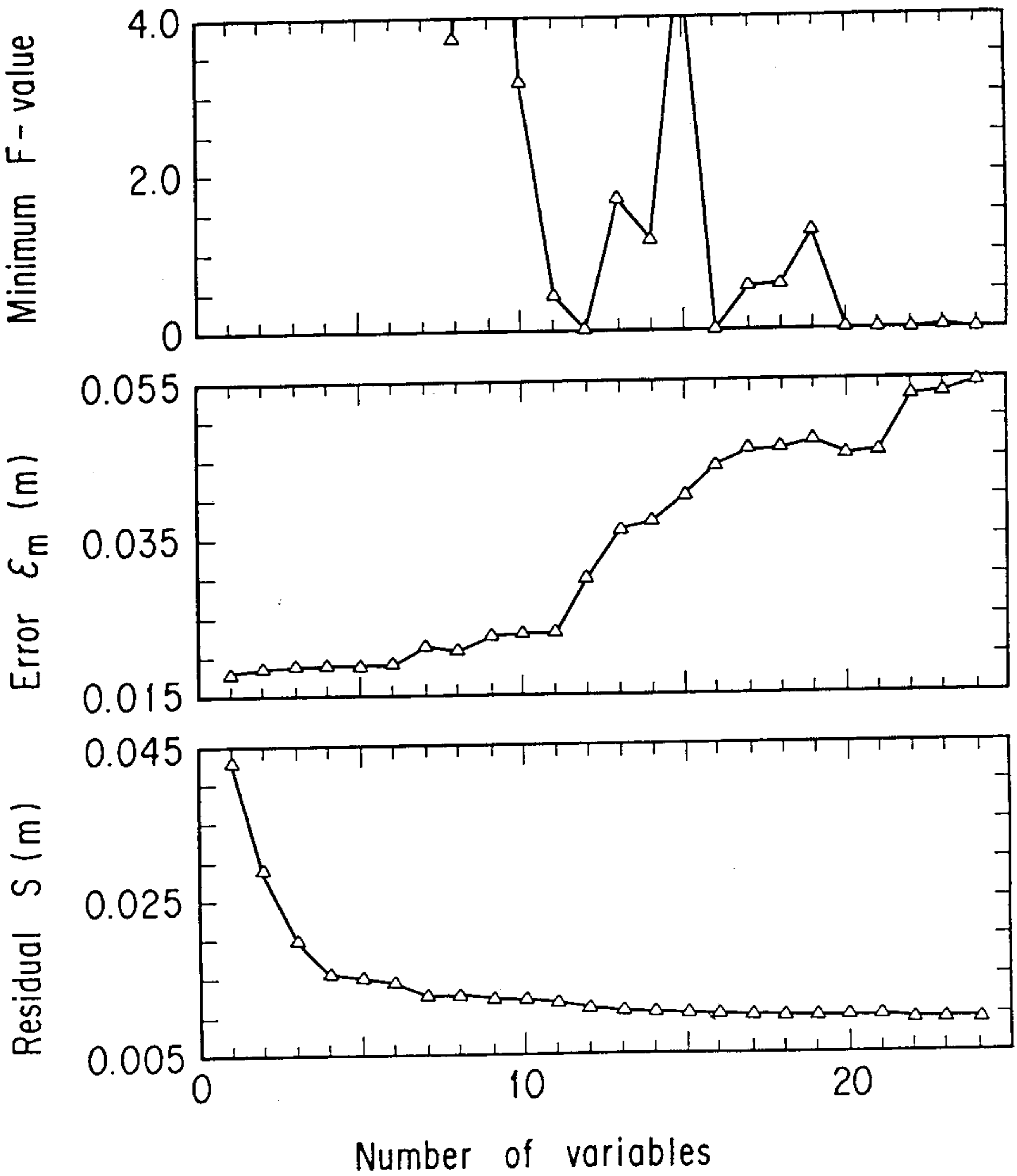
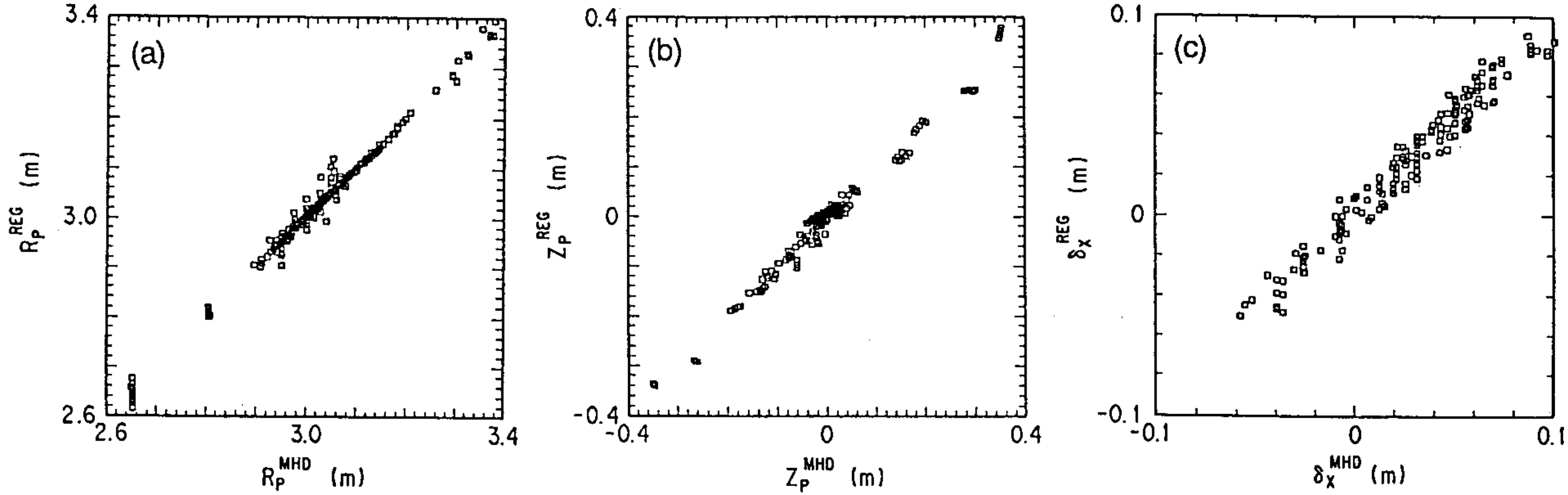


Fig. 3 Standard deviation S, error caused by 2% measurement errors ε_m and the minimum F-value of coefficient bj as a function of variables.



(a) Horizontal position R_p (b) Vertical position Z_p (c) Height of X-point δ_x

Fig. 4 Relations of equilibrium parameters of the MHD database and those calculated with regression equations.

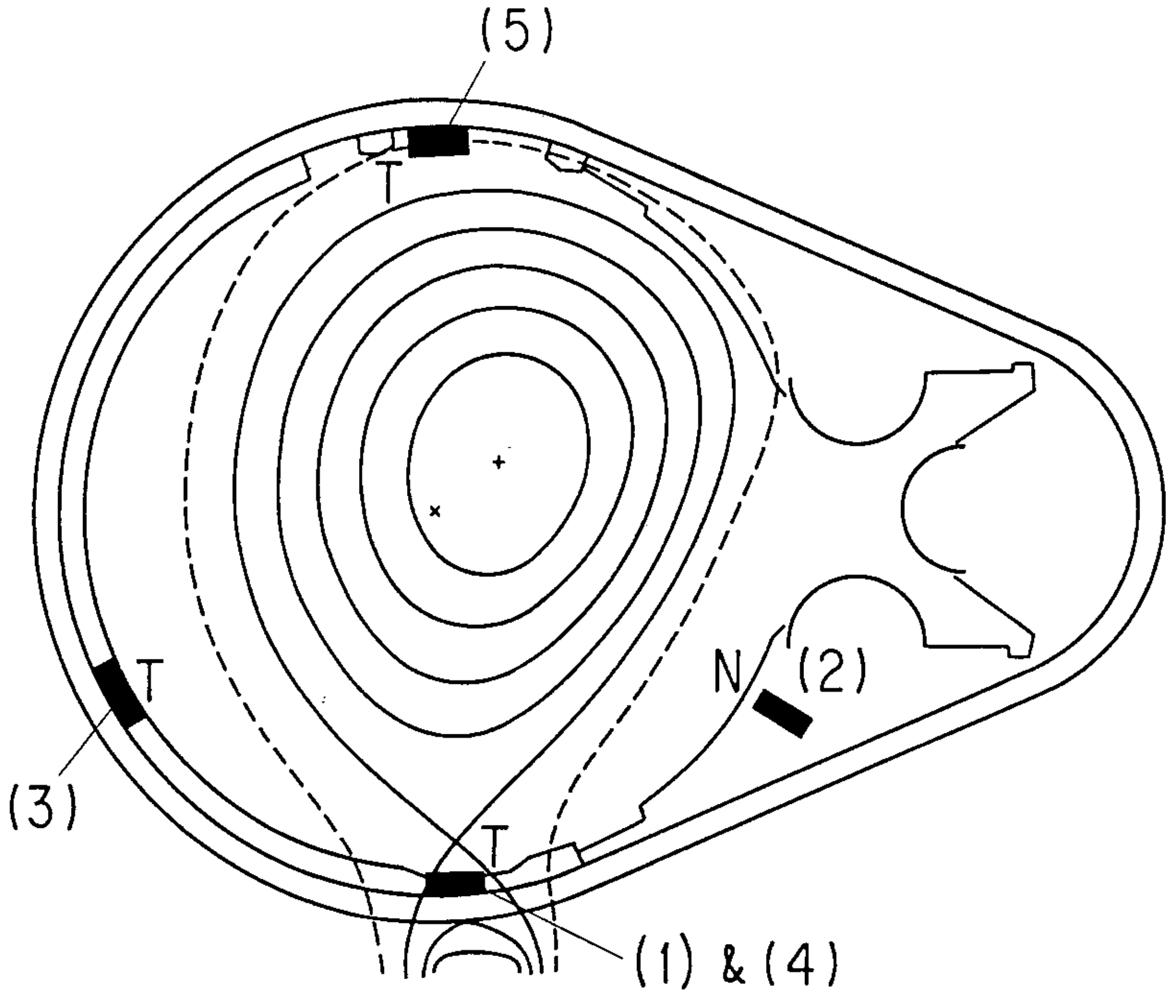


Fig. 5 Example of magnetic probes selected by the stepwise regression procedure for the measurement of δ_x. The numbers stand for the order of selection through this procedure. The bracket stands for the products of magnetic probe signals and divertor coil current $(B_t/I_p)(I_d/I_p)$ or $(B_n/I_p)(I_d/I_p)$.

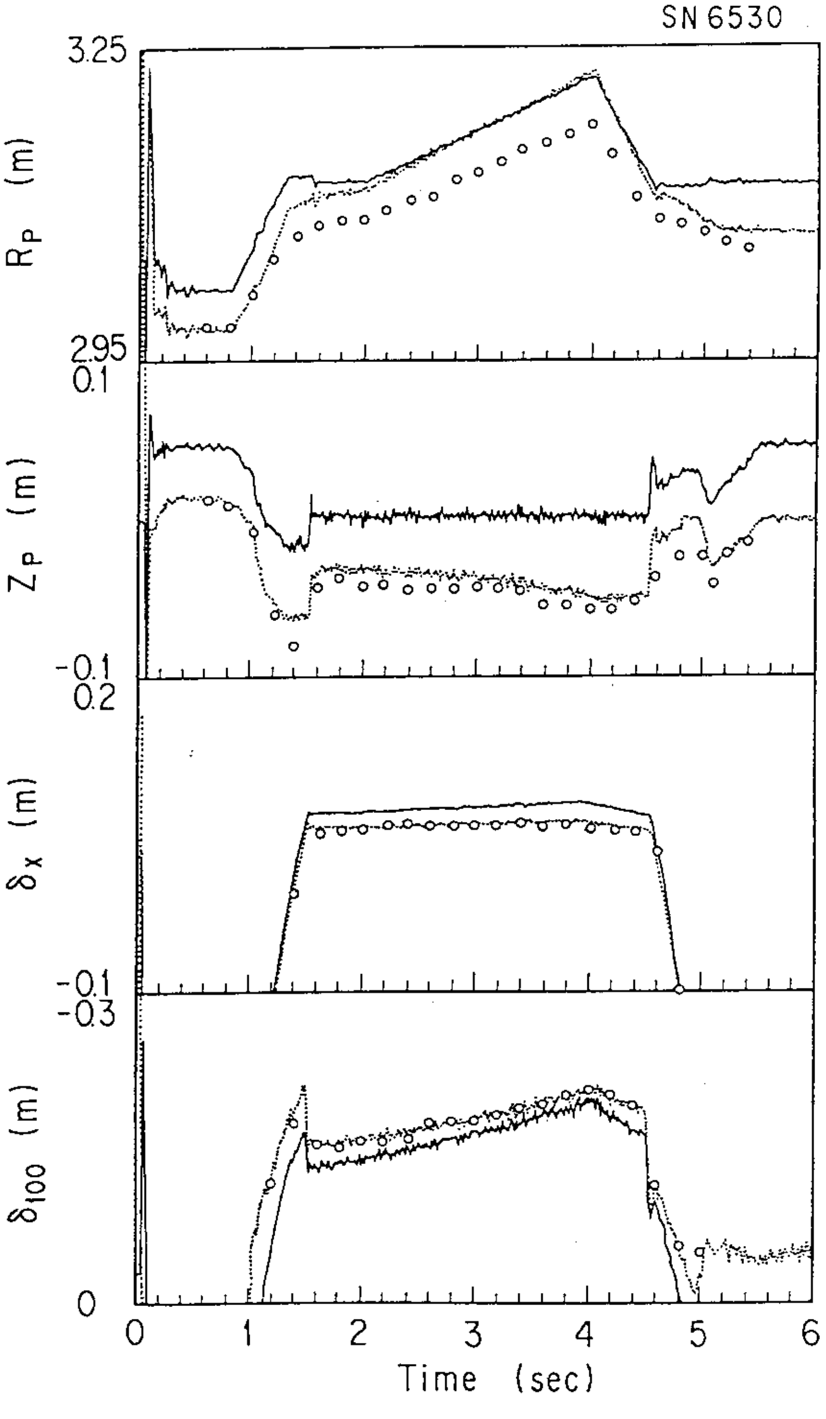
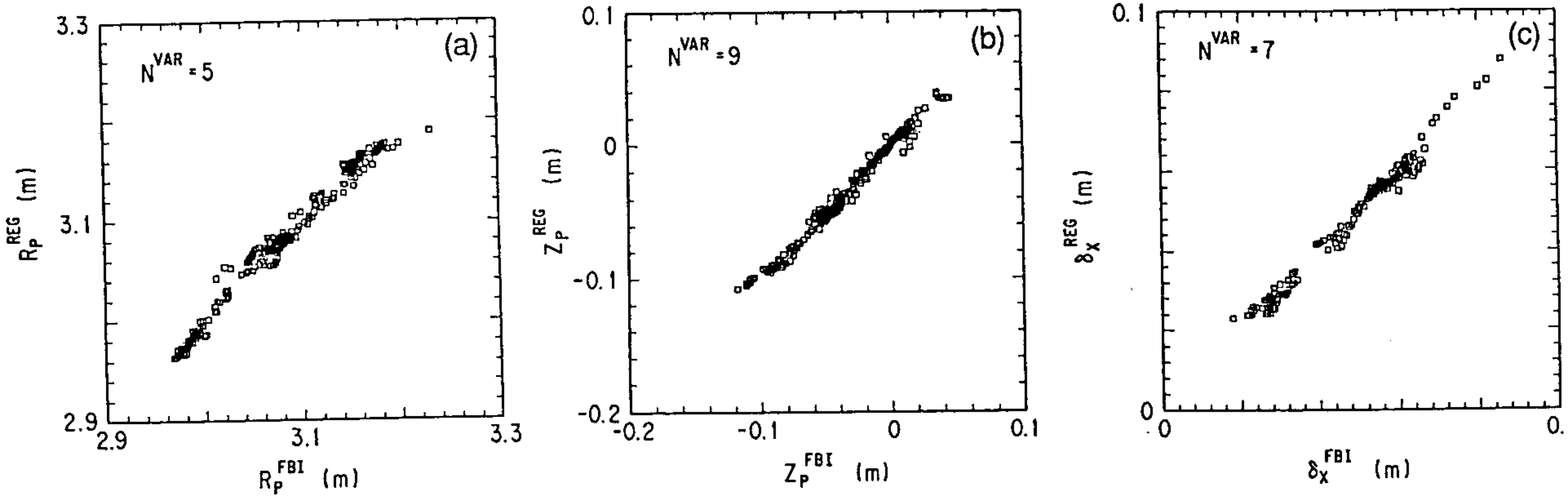


Fig. 6 Comparison of equilibrium parameters calculated with the regression equations (solid lines) and those calculated with the FBI code (open circles) for a typical divertor discharge. The divertor configuration is generated from 1.5 s to 4.5 s. Dashed lines stand for the results calculated with the regression equations obtained through regression analysis using the discharge database.



(a) Horizontal position R_p

(b) Vertical position Z_p

(c) Height of X-point δ_x

Fig. 7 Comparison of equilibrium parameters calculated from the FBI code and those obtained from regressions for experimental data.

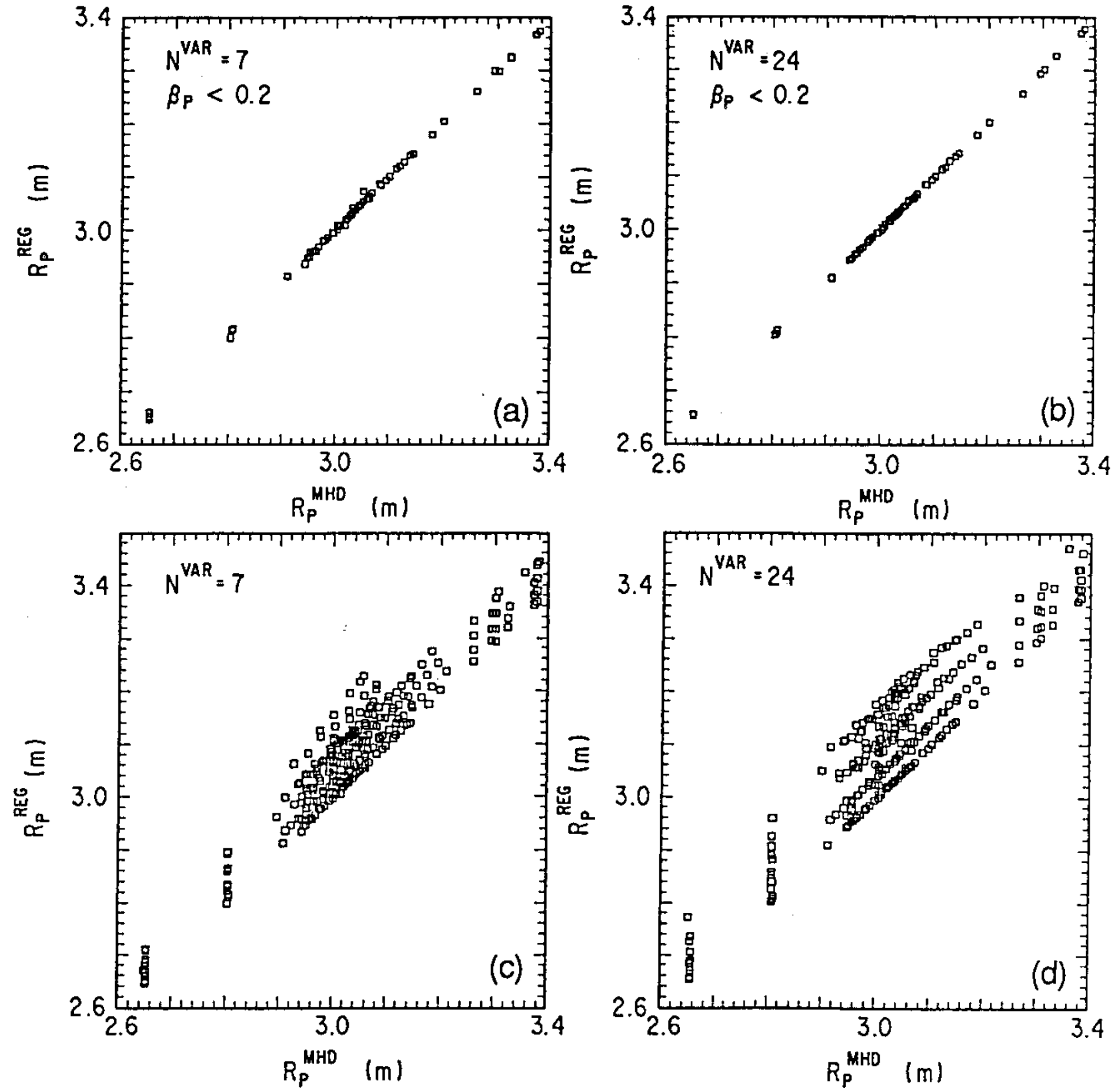


Fig. 8 Comparison of stable and unstable regression equations.
(a) Fitting results to the partial data for the seven variables case.
(b) Fitting results to the partial data for the twenty-four variables case.
(c) Fitting results to the whole data for the seven variables case.
(d) Fitting results to the whole data for the twenty-four variables case.

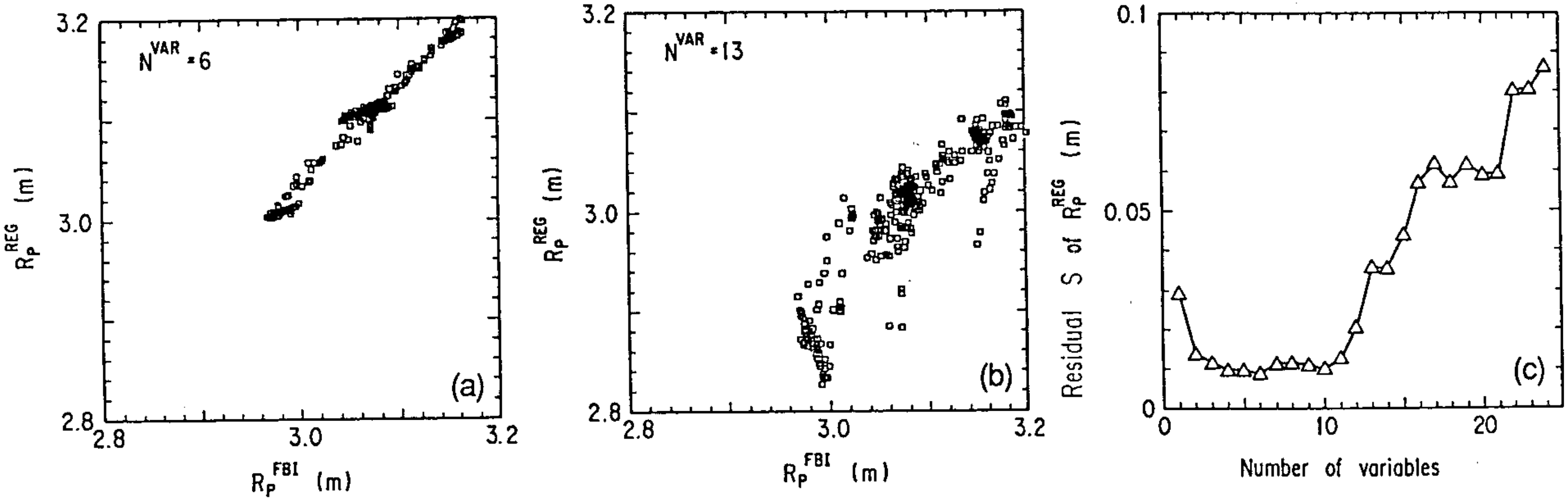
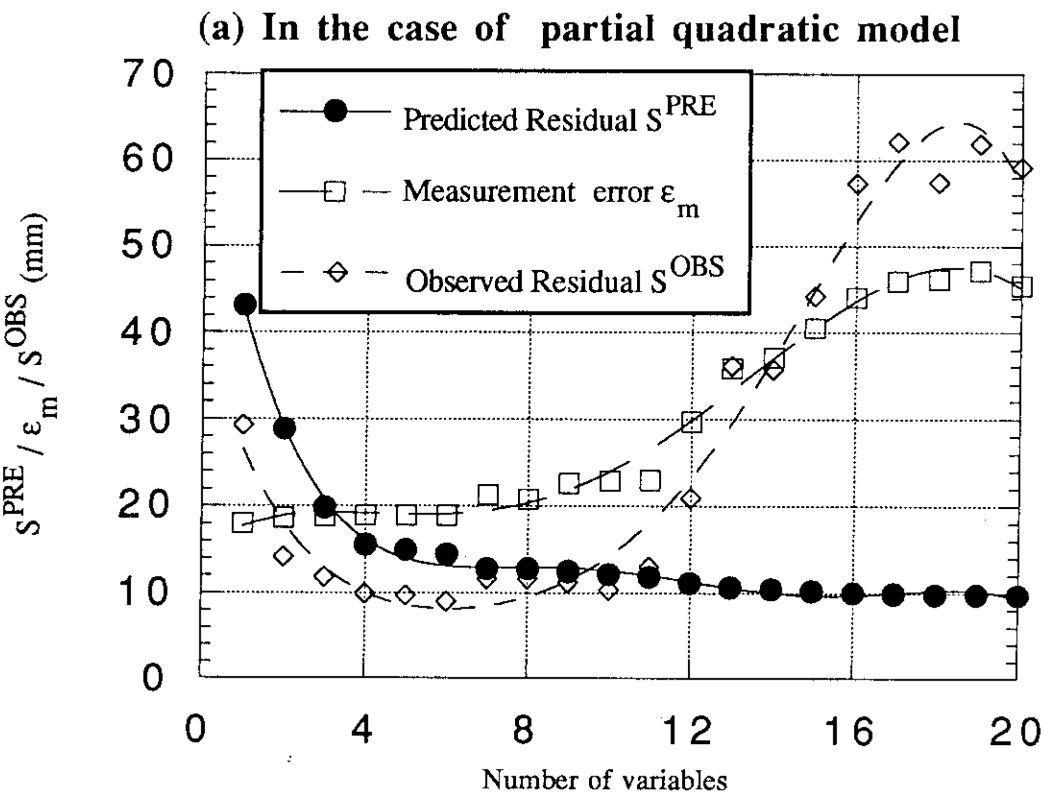


Fig. 9 Appearance of multicollinearity and/or influence of measured errors.
(a) Fitting result by stable regression equation.
(b) Fitting result by unstable regression equation.
(c) Residual of R_p^{REG} as a function of increment of variables.



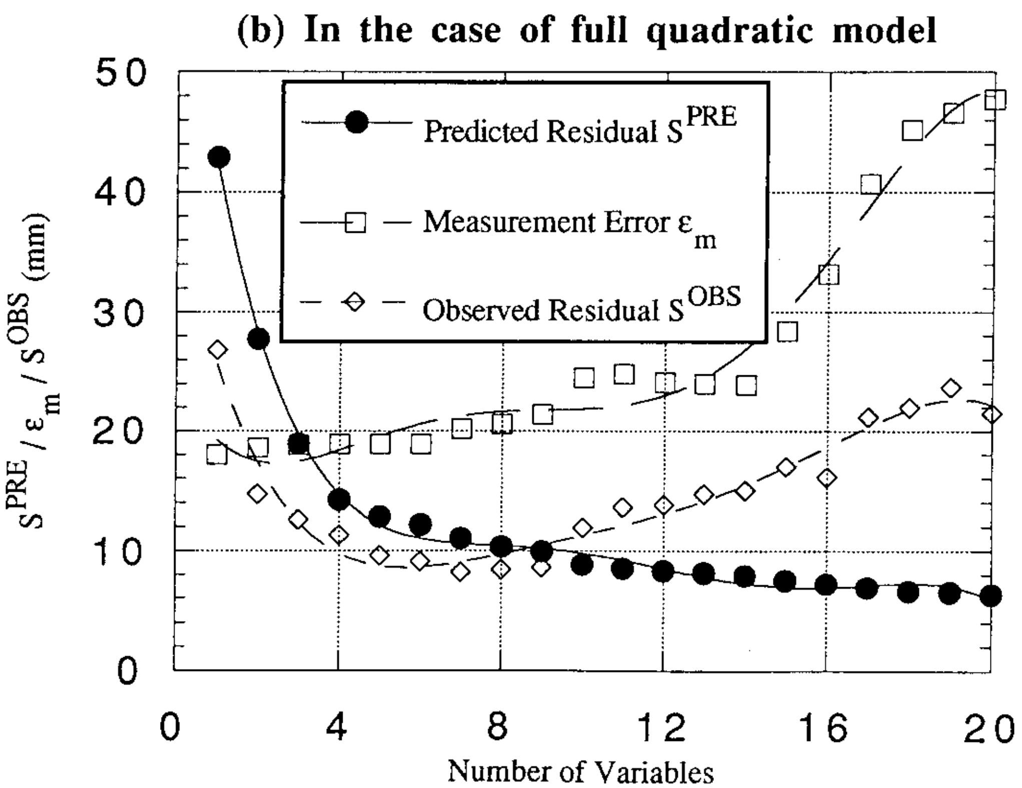


Fig. 10 Coincidence for Rp by partial quadratic model and by full quadratic model.

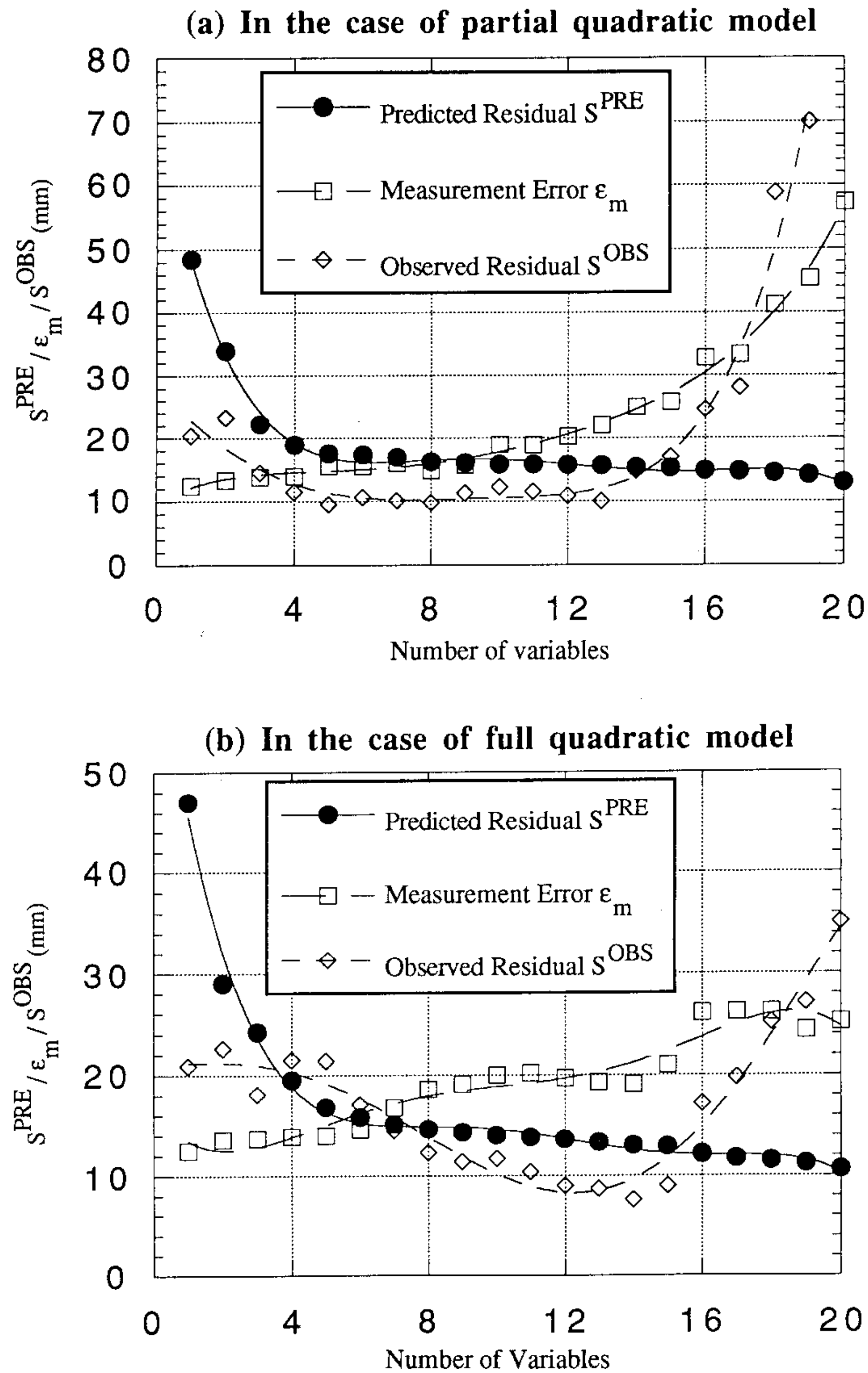


Fig. 11 Coincidence for Zp by partial quadratic model and by full quadratic model.

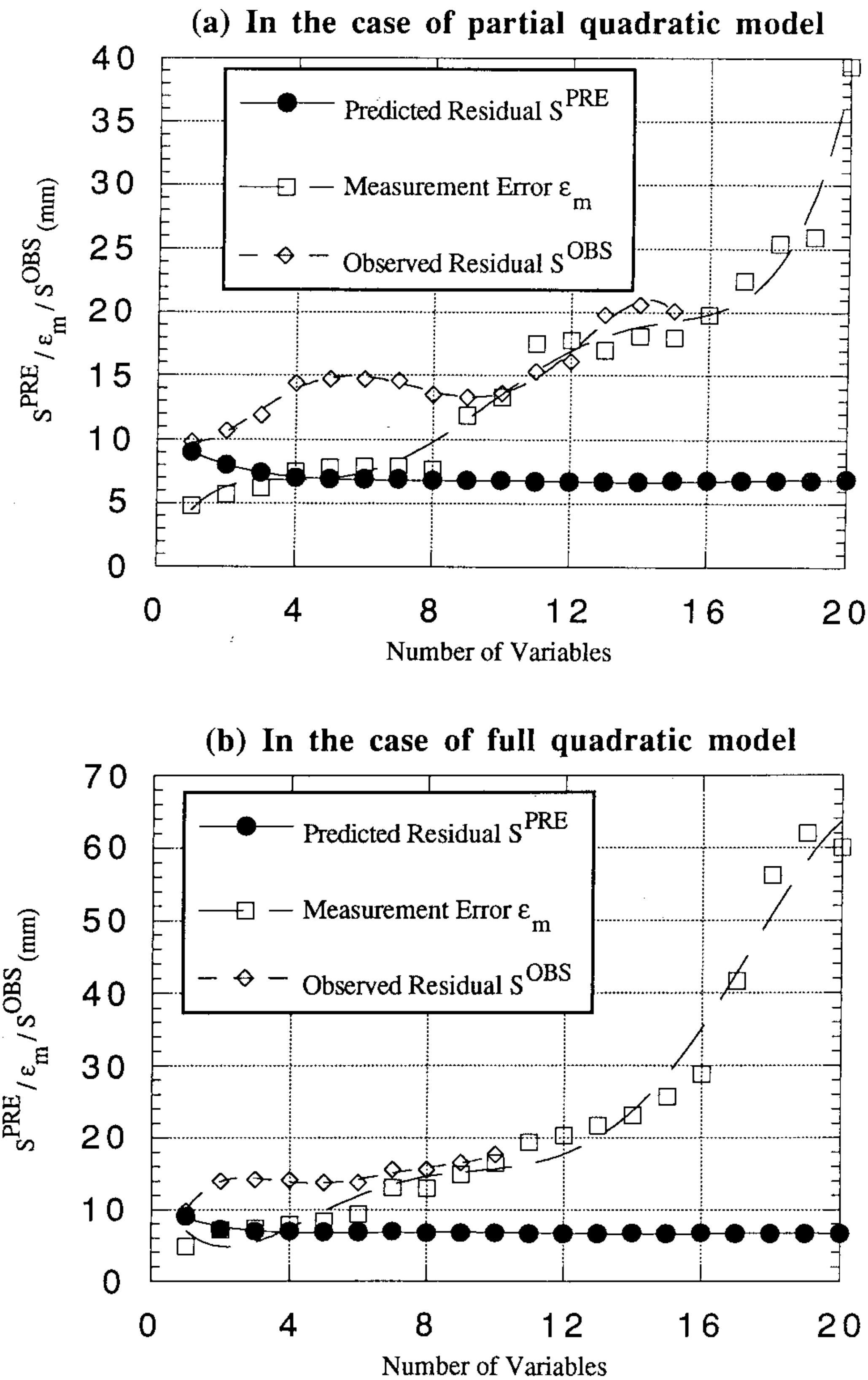


Fig. 12 Coincidence for Xp by partial quadratic model and by full quadratic model.