JAERI-M
93-115

WALL EFFECT CORRECTION FOR A SMALL CYLINDRICAL PROPORTIONAL COUNTERS

June 1993

Jaroslav PULPAN*

日本原子力研究所
Japan Atomic Energy Research Institute

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編集兼発行　日本原子力研究所
印　　　刷　㈱原子力資料サービス

Wall Effect Correction for a Small Cylindrical Proportional Counters

Jaroslav PULPAN*

Department of Reactor Engineering
Tokai Research Establishment
Japan Atomic Energy Research Institute
Tokai-mura, Naka-gun, Ibaraki-ken

(Received April 27, 1993)

The wall effect correction has been investigated for a small cylindrical proton recoil proportional counter used as a detector for neutron spectroscopy in the energy range of 0.01-1 MeV. The probability that a proton escapes to wall is calculated by the method based on a path length probability function. The probability is applied to calculation of the counter response for the isotropic neutron field. This method is also applied directly to correction for the deposited energy distribution of recoiled protons measured by a hydrogen-argon filled proportional counter. The effect of the correction on the unfolded neutron spectrum is examined.

Keywords: Neutron Spectroscopy, Proportional Counter, Wall Effect

* Institute of Radiation Dosimetry, Czech Academy of Sciences, Prague, Czech Republic

小型円筒比例計数管の壁効果補正

日本原子力研究所東海研究所原子炉工学部
Jaroslav PULPAN*

(1993年4月27日受理)

エネルギー領域0.01－1ＭｅＶの中性子スペクトロメータとして使用される小型円筒反跳陽子比例計数管の壁効果補正の検討を行った。行路長の確率関数に基づく方法により陽子が管壁へ逃れる確率を求め，等方中性子場中での検出器応答の計算に応用した。また，この手法を直接応用して水素とアルゴン封入の比例計数管で測定された反跳陽子の付与エネルギー分布に対する補正を行い，アンフォールディングした中性子スペクトルに対する補正の影響を調べた。

東海研究所：〒319-11　茨城県那珂郡東海村白方字白根２－４
＊　チェコ共和国，プラハ，チェコ科学アカデミー，放射線線量計測研究所

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1. Introduction

The neutron spectrometer system consists of the cylindrical proton-recoil proportional counters (PRC) with different filling[1,2] and associated electronic circuits for continuous mode data acquisition[3] is used for the measurement of neutron spectra in the energy range 0.01-1 MeV for various neutronics experiments at FNS/JAERI. The comparison of the neutron spectra measured by this spectrometer system with results of the transport calculations fully proved its usefulness for fusion blanket and shielding experiments [2,4].

The principle of neutron detection by PRC is simple: a neutron is elastically scattered on a proton (hydrogen nucleus) in the detector volume. The recoiled proton forms ionization in the counter gas. This ionization, measured as pulse height from PRC, is equal to the energy deposited in the PRC by the recoiled proton. The simple relation exists between the incident neutron spectrum and energy distribution of the recoiled protons, if they are fully stopped in the PRC volume.

Oyama[4] suggests possible improvements that could be implemented to this neutron spectrometer system. One of them is an investigation how energy distribution of recoil protons is influenced by fraction of protons that hit the wall of the counter before they are fully stopped. This so called "wall effect" correction is important for extension of the energy range measured by PRC to above 1 MeV.

In this paper the probability of the proton wall escape for two PRCs currently used at FNS[4] is calculated by the method proposed by N.L. Snidow and H.D. Warren[5]. Taking into account protons stopped in the counter wall, the distribution of energy deposited in the PRC volume is derived. This distribution is used for the wall effect correction of the pulse height distribution measured by the cylindrical H_2/Ar PRC. The effect of this correction is examined for the unfolded neutron spectrum in stainless steel shielding experiment[2].

2. Relation between Neutron and Recoil-Proton Spectra

Since n-p scattering is isotropic for considered energy region neutron spectrum $\Phi(E)$ can be determined from the following relation [1] (assuming proton and neutron masses to be equal):

$$\Phi(E) = \frac{1}{N} \cdot \frac{E}{\sigma(E)} \cdot \left\{ \frac{dD(E_p)}{dE_p} \right\}_{E_p = E} \quad , \tag{1}$$

where
E : neutron energy,
N : the number of the H atoms in the PRC volume,
$\sigma(E)$: n-p scattering cross section,
$D(E_p)$: energy distribution of recoiled protons.

The energy distribution of recoiled protons, $D(E_p)$, is equal to the measured pulse height distribution only if all protons are fully stopped in the PRC volume. When the number of protons stopped in the PRC wall is not negligible, the measured energy distribution in PRC (i.e., pulse height spectrum) should be corrected for this effect. Wall escape probability is thereby an important parameter giving applicable energy range of the PRC.

3. Wall Escape Probability

The probability of the wall escape (i.e., the probability that a proton hits the wall before it stops completely) depends on the proton track length, the dimensions and shape of counter.

The proton track length for a given initial proton energy is defined by the proton range R(E). The range of proton in the volume of PRC filled with the mixture of H_2 and Ar gas can be expressed by Bragg rule[6] as:

$$R(E) = \left(\frac{P_H}{R_H(E)} + \frac{P_{Ar}}{R_{Ar}(E)} \right)^{-1} \quad , \tag{2}$$

where P_H and P_{Ar} are the partial pressures of hydrogen and argon, respectively, and $R_H(E)$ and $R_{Ar}(E)$ are the ranges of protons in 1 kg/cm^2 of hydrogen and argon, respectively.

Proton ranges as a function of proton energy are plotted in Fig. 1 for two proportional counters: the first is filled by 5.6 kg/cm^2 of hydrogen (H_2-

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Proton ranges as a function of proton energy are plotted in Fig. 1 for two proportional counters: the first is filled by 5.6 kg/cm^2 of hydrogen (H_2-

PRC) and the second by the mixture of the 2.98 kg/cm^2 of hydrogen and 2.98 kg/cm^2 of argon (H_2/Ar -PRC). Tabulated data from reference [6] were used for range of protons $R_H(E)$ and $R_{Ar}(E)$ in hydrogen and in argon.

When the PRC is irradiated with an isotropic neutron field, proton tracks start from any points inside the cylinder into random direction. A useful tool for description of such tracks is the path length probability function derived in by Snidow and Warren[5]. The path length probability density function f(x) defines the probability f(x) dx that the length of random path stays within interval (x,x+dx). The path length probability distribution function F(x) corresponding to f(x) is thereby equal to the probability that path length is shorter than x.

The analytical expressions for the functions f(x) and F(x), derived in reference 5, contain integrals that cannot be expressed in a closed form with terms of elementary integrals. Instead of repeating these long formulae from reference 5 the source program in FORTRAN77 for calculation of both functions f(x) and F(x) is listed in the Appendix. The expansion of the intergrals into elliptic functions is replaced by numerical integration. Figure 2 shows both functions, f(x) and F(x), for cylindrical PRC with radius of R=0.95 cm and length of L=12.7 cm calculated by this program.

Combining definition of F(x) and $R(E_p)$ the wall escape probability can be easily calculated as the probability that path length is longer than range:

$$P_W = 1 - F\left(R(E_p)\right). \tag{3}$$

The wall escape probabilities for H_2-PRC and H2/Ar-PRC are plotted in Fig.3 as a function of proton energy. The wall escape probability is less than 5% for the H_2-PRC detector used for neutron spectrum measurements below 150 keV. This value is surely below the statistical uncertainty of the pulse height measurements.

On the other hand, in the H_2/Ar proportional counter, 40% of recoiled protons with initial energy 1 MeV are stopped in the counter wall. Since protons with such energy are detected when this PRC is used for neutron spectrum measurement at the neutron energy up to 1 MeV, the influence of the wall effect to the neutron response of this PRC will be analyzed in detail in the next sections.

4. Distribution of the Deposited Energy

The response of PRC for protons with initial energy E_p (i.e., the distribution of energy E_d deposited in the detector volume) is given as [5]:

$$g(E_d,E_p) = f\left(R(E_p) - R(E_p - E_d)\right)\frac{dR(E_p - E_d)}{d(E_p - E_d)} + \delta(E_p - E_d).F\left(R(E_p)\right) \quad for \;\; Ed < Ep$$

$$g(E_d,E_p) = 0 \qquad for \;\; Ed > Ep, \qquad (4)$$

where δ denotes Dirac function.

The first term of the eq. (4) is the contribution of protons whose initial energy E_p is higher than actually deposited energy E_d, and the second term is a fraction of the fully stopped protons.

Since the energy distribution of protons from n-p scattering is uniform, the equation (4) can be used for calculation of the PRC response to monoenergetic neutrons with energy E_n. Therefore, the neutron response is written as:

$$r(E_d,E_n) = \frac{N.\sigma(E_n)}{E_n}.\int_{E_d}^{E_n} g(E_d,E_p).dE_p. \qquad (5)$$

The first term defines distribution of the initial proton energies E_p for given neutron energy E_n as uniform one (from 0 to E_n), and the integration represents a wall effect correction. Above formula describes a deformation of an "ideal" (rectangular) PRC response that is assumed for the derivation of the eq. (1). It is noticed [6] that the response to the highest neutron energy measured by the PRC should have edge in the pulse height (PH) spectrum.

Integration of the equation (5) for H_2/Ar PRC gives responses close to the rectangular shape for low energy neutrons (see Fig. 4), but the deformation of the edge can be observed for energies above 0.4 MeV. In contrast, the responses above 1 MeV do not have any edge.

Equation (4) also describes the relation between the measured distribution of pulse heights $M(E_d)$ and the energy spectrum of recoiled protons $D(E_p)$:

$$M(E_d) = \int_{E_d}^{\infty} g(E_d, E_p).D(E_p).dE_p. \tag{6}$$

When the measured PH spectrum $M(E_d)$, the spectrum of the recoiled protons $D(E_p)$ and the proton response $g(E_d,E_p)$ are approximated by vectors M_i, D_j and a matrix. $G_{i,j}$, respectively, the wall effect correction can be written down as a set of linear equations:

$$M_i = \sum_{j=1}^{n} G_{i,j}.D_j \qquad i = 1..m \quad , \tag{7}$$

where

n is the number of the pulse heights bins and m is the number of the proton energy groups.

The solution of equation (7), based on the measured PH distribution M_i, gives distribution of recoiled protons, D_j, in PRC. The recoil proton spectrum D_j corrected by this way can be used in eq. (1) for neutron spectrum calculation, even in the case that the wall escape is not negligible. The program WALL was written to calculate the matrix, G_{ij}, according to the eq. (4) and to solve a set of linear equations (7) by the method described in reference 7. The input data for program WALL are the counter dimensions, the pressure of the filling and the measured distribution M_i of energies deposited in PRC (i.e. energy-calibrated pulse height spectrum). The output is the spectrum of recoiled protons, D_j, corrected for wall effect.

The wall effect correction was calculated for several sample spectra measured in the stainless steel assembly irradiated by T-d neutrons[2]. Although the wall effect correction generally depends on the neutron spectrum shape, similar results are obtained for all of them. Figure 5 presents the typical result from the program WALL, i.e., the corrected and uncorrected PRC responses to neutron spectra in the experimental assembly. The wall escape affects only the PH spectrum above 0.5 MeV. The part of the spectra above 1 MeV is not used for the neutron spectra unfolding, because the wall escape at this region is too high (also counting statistics is poor at this region). The comparison of the calculated neutron spectra, when uncorrected and corrected PH spectra are used in eq.(1), is more important than comparison of the PH spectra. Figure 6 shows the neutron spectra calculated from the corrected and uncorrected PRC responses. The wall effect correction changes neutron spectrum slightly at lower energy region. This is due to the shape of the PRC response function. The edge deformation of the responses due to

is compensated by contribution from the responses for the higher energy neutron. While the neutron spectrum above 700 keV is overestimated by 15% in the case of calculation without the wall effect correction. Because the wall escape has high probability for this energy. It is notable that the wall escape depends on the neutron spectrum shape, but the above mentioned example shows magnitude of the wall escape correction.

5. Conclusions

The wall escape probability was calculated for two proton-recoil proportional counters currently operated at the FNS facility in JAERI for fusion neutronics measurements. When H_2-Ar counter is used for measurement of the neutron spectra up to 1 MeV considerable fraction of recoiled protons are not fully stopped in the detector volume. The wall effect correction described in this work should be applied in this case. The influence of wall effect correction on the resulting neutron spectrum was proved to be quite small for low energy neutrons but it can reach 15% for neutrons above 700 keV.

The present work demonstrated that the wall escape does not represent any practical problem for proportional counters the currently used at FNS/JAERI. The described method and the developed program will be useful for testing of the PRC with another gas filling aimed at extending the energy range.

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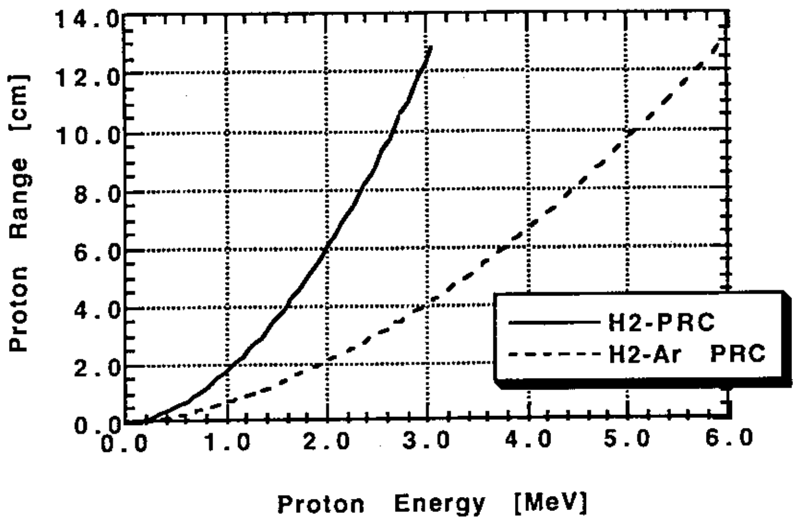

Fig.1 Proton range for two PRC

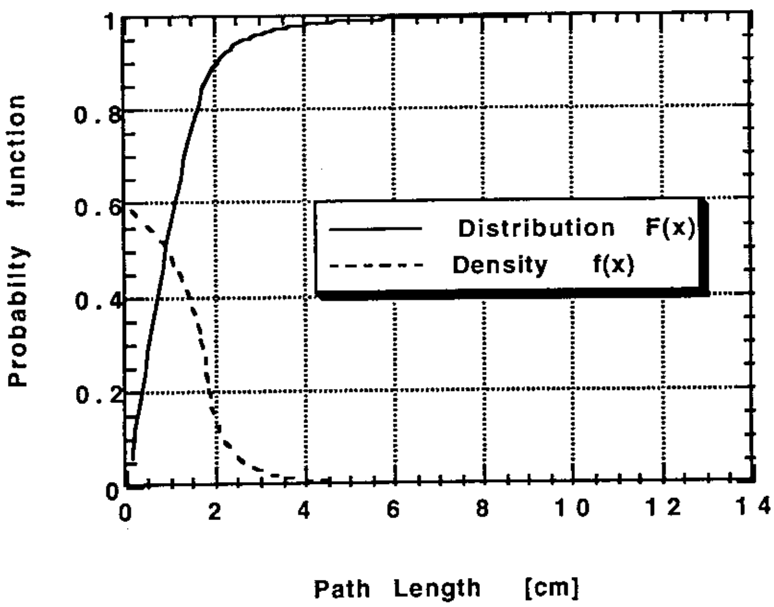

Fig.2 Path length probability functions f(x) and F(X) for cylinder of R=0.95 cm L=12.7 cm

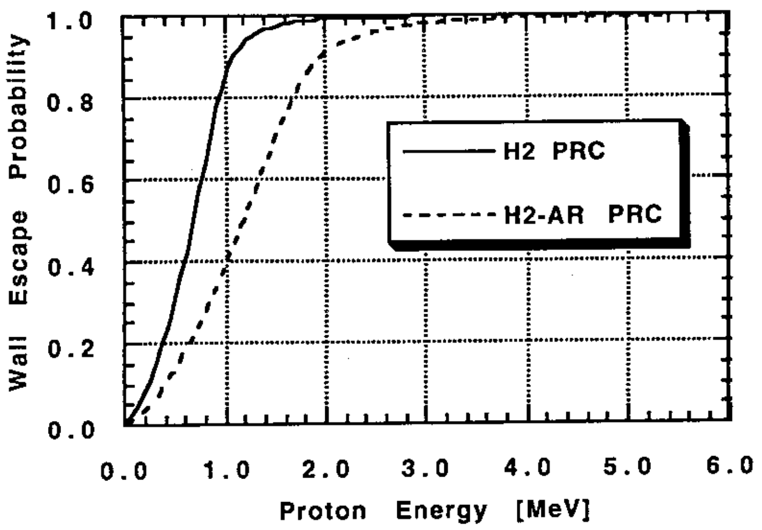

Fig.3 Proton wall escape probability for two PRC

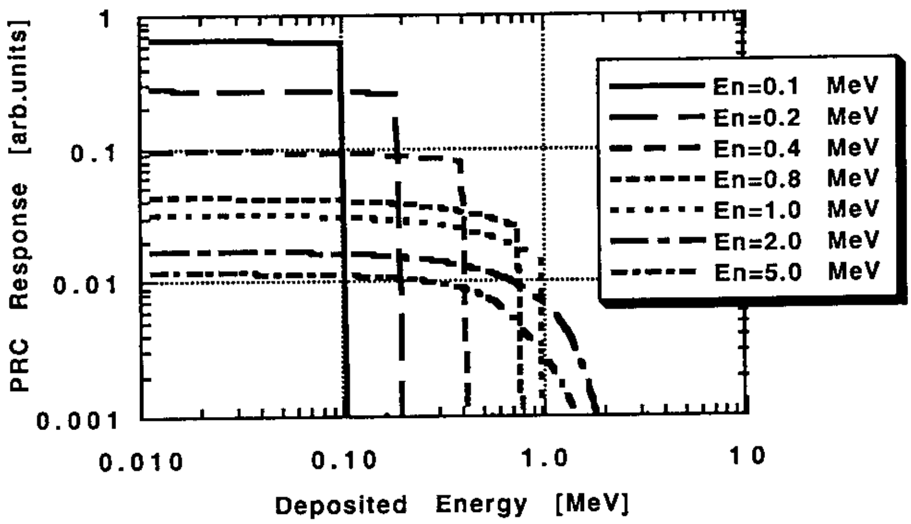

Fig. 4 Responses of the H_2-Ar PRC for neutrons with several energies

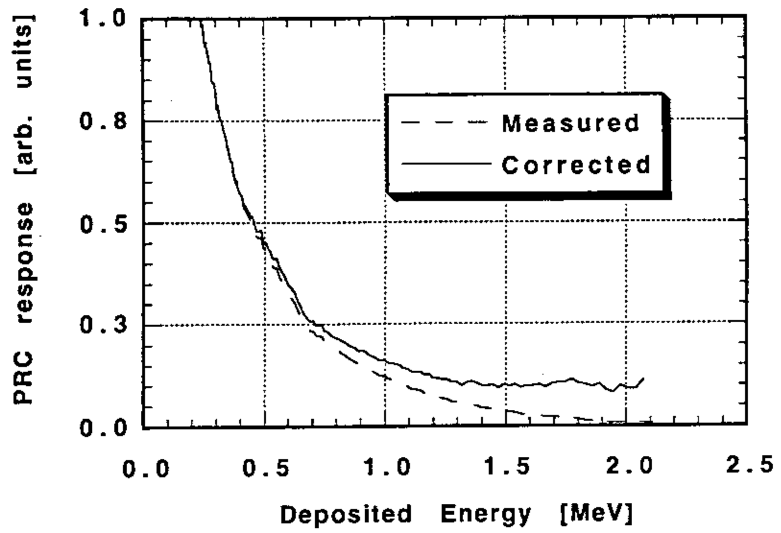

Fig. 5 Corrected and measured H_2-Ar PRC response in the assembly irradiated by D-T neutrons

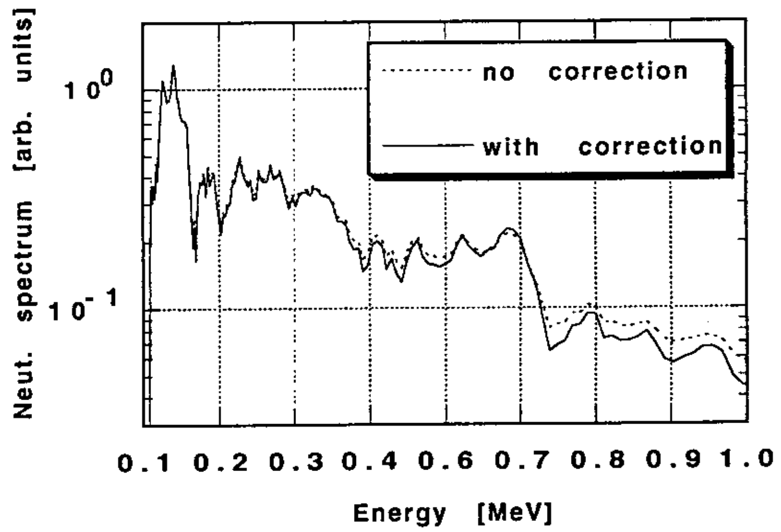

Fig. 6 Comparision of the neutron spectra with and without wall effect correction

Appendix

Listing of the program for calculation of the functions f(x) and F(x)

```
C TCPCSU.FOR          6/92            JPS s.r.o. MSF IBM PC   JAERI
C=========================================================
C test of the following subroutine
C       SETCOM(R,H)
C   and functions
C FD  - path length probability distribution for cylinder REAL*8
C F    - path length probability density for cylinder     REAL*8
C---------
        PARAMETER   (N=126)! Max. length of the energy scales
        REAL         RAD    ! Counter radius [cm]
        REAL         H      !lenght of active part of the chamber
C
        REAL*8         FD,F,XMAX,SUM
        REAL*4         X
        EXTERNAL FD,F
C============================================================
        WRITE (*,*) 'Insert a detector radius'
        READ (*,*) Rad
        WRITE (*,*) 'Insert a detector length'
        READ (*,*) H
        CALL SETCOM(RAD,H)
        OPEN   (UNIT=1,FILE='FTST')
        OPEN   (UNIT=2,FILE='FDTST')
C
        XMAX=DSQRT(H**2+4*RAD**2)
        DX=XMAX/N
        DO 20 I=2,N
        X=(I-1)*DX
        G=FD(X)
        FF=F(X)
111     FORMAT(6F10.5)
        WRITE (1,*) X,FF
        WRITE (2,*) X,G,G-SUM
 20     CONTINUE
        CLOSE (UNIT= 1)
        CLOSE (UNIT= 2)
C Check integral F(X)
        CALL DQG32(0.D0,XMAX,F,SUM)
        WRITE(*,*) 'INT.=',SUM
        CLOSE (UNIT= 1)
        STOP
        END
C============================================================
        SUBROUTINE SETCOM(R4,H4)
C============================================================
C  only set commons used for F(X) and FD(X) calculations
C  convert real*4 to real*8 used for F and FD calculation
C
        IMPLICIT        REAL*8 (A-H,O-Z)
        REAL*4 R4,H4
```

```
      COMMON /DIMENS/  A,        ! RADIUS
     *                 RL,       ! LENGTH OF CIRCULAR CYLINDER
     *                 ALF1,     ! ALFA=.5*RL/A ALF1=1/ALFA
     *                 PIA,      ! PIA=1/A/PI
     *                 RK        ! RN=X/RK => RN=2*A
      COMMON /CYLDIM/ R,
     2                 H,
     3                 XMAX,R2,RR,XX,V
      DATA PI/3.141592653589/
C
      R=R4
      H=H4
      XMAX=DSQRT(H**2+4*R**2)
      R2=2*R
      RR=R*R
      V=PI*RR*H4
C
      RL=H4
      A=R4
      PIA=1/A/PI
      ALF1=2*A/RL
      RETURN
      END
C=============================================================
      REAL*8  FUNCTION F(X)
C=============================================================
C Path length probability density
C
      IMPLICIT         REAL*8 (A-H,O-Z)
      REAL*4         X
      EXTERNAL CORE
      COMMON /DIMENS/  A,        ! RADIUS
     *                 RL,       ! LENGTH OF CIRCULAR CYLINDER
     *                 ALF1,     ! ALFA=.5*RL/A ALF1=1/ALFA
     *                 PIA,      ! PIA=1/A/PI
     *                 RK        ! RN=X/RK => RN=2*A
      COMMON /CYLDIM/ R,
     2                 H,
     3                 XMAX,R2,RR,XX,V
C INTEGRAL BOUNDARY & INICIALIZATION
      IF (X.GE.XMAX) THEN
      F=0
      RETURN
      ENDIF
      XX=X
      RK=.5*XX/A
      IF ( XX.NE.0)   THEN
      RM=RL/XX
      RN=1/RK
      ELSE
      RM=1.
      RN=1.
      END IF
      IF ( RM.GT.1.) RM=1.
      IF ( RN.GT.1.) RN=1.
      XL=ACOS (RM)
      XU=ASIN (RN)
      CALL DQGVF(XL,XU,CORE,SUM)
```

```
      F=SUM
      RETURN
      END

      REAL*8    FUNCTION CORE (TH)
      IMPLICIT         REAL*8 (A-H,O-Z)
      COMMON /DIMENS/  A,       ! RADIUS
     *                RL,       ! LENGTH OF CIRCULAR CYLINDER
     *                ALF1,     ! ALFA=.5*RL/A ALF1=1/ALFA
     *                PIA,      ! PIA=1/A/PI
     *                RK        ! RN=X/RK => RN=2*A
C
      C=COS(TH)
      S=SIN(TH)
      S2=S**2
      SK=RK*S
      SK2=SQRT(ABS(1-SK**2))
      CORE=(2*PIA*S2*SK2)-(3*RK*PIA*ALF1*S2*C*SK2)+
     *      (ALF1/2/A*S*C)-(PIA*ALF1*C*S*ASIN(SK))
      RETURN
      END
C===========================================================
      REAL*8  FUNCTION FD(X)
C===========================================================
C Path length probability distribution
C
      IMPLICIT          REAL*8 (A-H,O-Z)
      REAL*4          X
      EXTERNAL VFD
      COMMON /CYLDIM/ R,
     2                H,
     3                XMAX,R2,RR,XX,V
      DATA PI/3.141592653589/
      IF(X.GT.XMAX) THEN
      FD=1
      ELSE
      THMIN=0
      IF (X.GT.H) THMIN=DACOS(H/X)
      THMAX=PI/2
      IF (X.GT.R2) THMAX=DASIN(R2/X)
      XX=X
      CALL DQGVFD(THMIN,THMAX,VFD,SUM)
      FD=1-SUM/V
      ENDIF
      RETURN
      END
C
      REAL*8          FUNCTION VFD(TH)
      IMPLICIT         REAL*8 (A-H,O-Z)
      COMMON /CYLDIM/ R,
     2                H,
     3                XMAX,R2,RR,XX,V
      DATA PI/3.141592653589/
C
      STH=DSIN(TH)
      T=XX*STH/R2
      VFD=(H-XX*DCOS(TH))*RR*(PI-2*T*DSQRT(1-T*T)-2*DASIN(T))
      VFD=VFD*STH
```

```
        RETURN
        END
C
C         SUBROUTINE DQG32
C         PURPOSE
C       TO COMPUTE INTEGRAL(FCT(X), SUMMED OVER X FROM XL TO XU)
C         USAGE
C       CALL DQG32 (XL,XU,FCT,Y)
C       PARAMETER FCT REQUIRES AN EXTERNAL STATEMENT
C
C         DESCRIPTION OF PARAMETERS
C  XL   - DOUBLE PRECISION LOWER BOUND OF THE INTERVAL.
C  XU   - DOUBLE PRECISION UPPER BOUND OF THE INTERVAL.
C  FCT  - THE NAME OF AN EXTERNAL DOUBLE PRECISION FUNCTION
C                 SUBPROGRAM USED.
C  Y      - THE RESULTING DOUBLE PRECISION INTEGRAL VALUE.
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C THE EXTERNAL DOUBLE PRECISION FUNCTION SUBPROGRAM FCT(X)
C MUST BE FURNISHED BY THE USER.
C
C METHOD
C EVALUATION IS DONE BY MEANS OF 32-POINT GAUSS
C QUADRATURE FORMULA, WHICH INTEGRATES POLYNOMIALS UP
C TO DEGREE 63 EXACTLY. FOR REFERENCE, SEE
C  V.I.KRYLOV, APPROXIMATE CALCULATION OF INTEGRALS,
C  MACMILLAN, NEW YORK/LONDON, 1962,
C   PP.100-111 AND 337-340.
C
      SUBROUTINE DQG32(XL,XU,FCT,Y)
C
        REAL*4 A,C
      DOUBLE PRECISION XL,XU,Y,B,FCT
C
      A=.5D0*(XU+XL)
      B=XU-XL
      C=.4986319309247407 8D0*B
      Y=.3509305004735048 3D-2*(FCT(A+C)+FCT(A-C))
      C=.4928057557726341 7D0*B
      Y=Y+.8137197365452835D-2*(FCT(A+C)+FCT(A-C))
      C=.4823811277937532 2D0*B
      Y=Y+.1269603265463103 0D-1*(FCT(A+C)+FCT(A-C))
      C=.4674530379688698 4D0*B
      Y=Y+.1713693145651071 7D-1*(FCT(A+C)+FCT(A-C))
      C=.4481605778830260 6D0*B
      Y=Y+.2141794901111334 0D-1*(FCT(A+C)+FCT(A-C))
      C=.4246838068662849 9D0*B
      Y=Y+.2549902963118808 8D-1*(FCT(A+C)+FCT(A-C))
      C=.3972418979839712 0D0*B
      Y=Y+.2934204673926777 4D-1*(FCT(A+C)+FCT(A-C))
      C=.3660910593701448 4D0*B
      Y=Y+.3291111138818092 3D-1*(FCT(A+C)+FCT(A-C))
      C=.3315221334651076 0D0*B
      Y=Y+.3617289705442425 3D-1*(FCT(A+C)+FCT(A-C))
      C=.2938578786203811 6D0*B
      Y=Y+.3909694789353515 3D-1*(FCT(A+C)+FCT(A-C))
      C=.2534499544661147 0D0*B
      Y=Y+.4165596211347337 8D-1*(FCT(A+C)+FCT(A-C))
      C=.2106756380653176 7D0*B
```

```
      Y=Y+.43826046502201906D-1*(FCT(A+C)+FCT(A-C))
      C=.16593430114106382D0*B
      Y=Y+.45586939347881942D-1*(FCT(A+C)+FCT(A-C))
      C=.11964368112606854D0*B
      Y=Y+.46922199540402283D-1*(FCT(A+C)+FCT(A-C))
      C=.72235980791398250-1*B
      Y=Y+.47819360039637430D-1*(FCT(A+C)+FCT(A-C))
      C=.24153832843869158D-1*B
      Y=B*(Y+.48270044257363900D-1*(FCT(A+C)+FCT(A-C)))
      RETURN
      END
C============================================================
C renamed DQG32 - to allow anothere copy of it
      SUBROUTINE DQGVF(XL,XU,FCT,Y)
C============================================================
...                 ! the same function as above
C
C============================================================
C renamed DQG32 - to allow anothere copy of it
      SUBROUTINE DQGVFD(XL,XU,FCT,Y)
...                 ! the same function as above
```