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A PROPOSAL TO INTRODUCE A TURBULENCE FACTOR INTO A DIFFUSION COEFFICIENT OF A TOKAMAK PLASMA

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A Proposal to Introduce a Turbulence Factor into a Diffusion Coefficient of a Tokamak Plasma

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It is proposed that a diffusion coefficient (D), which is well known to be anomalous in usual Tokamak plasmas as a result of the turbulences caused by the several instabilities, can be expressed with a measure of turbulence, i.e. a so-called "Reynolds number" (R_e), in the following manner,

$$D \sim \left\langle \frac{\omega}{k_\perp^2} \right\rangle R_e^{\frac{1}{R_e + \beta}}$$

where $R_e \sim \left\langle \frac{\gamma}{\omega} \right\rangle$, and $\langle\omega\rangle$, $\langle\gamma\rangle$, $\langle k_\perp\rangle$ are respectively a representative frequency, a growth rate, and a wave number in the radial direction of the responsible instability, β being a numerical factor around $0 \sim 0.5$. introduced for the sake of adjustment caused by unsolved ambiguities.

As sample cases of the instabilities, various types of the drift waves are studied. The proposal is discussed from the viewpoint of a general stochasticity on the basis of the simplified nonlinear oscillator model with use of the mixing-length argument to find the aforementioned relation by a heuristic way. It should be noticed that the proposal is still not definite, including the intrinsic ambiguities resulting from nonlinearity, should be modified by more advanced nonlinear studies in the future.

Keywords: Turbulence, Tokamak, Diffusion Coefficient,
Measure of Turbulence, Plasma

トカマクプラズマにおける拡散係数に乱流度を導入する提案

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(1994年2月1日受理)

各種不安定性の結果、生ずる乱流によってトカマクプラズマの拡散係数は普通、異常輸送状態になっているという事はよく知られているが、この論文では、乱流の尺度によって、その拡散係数(D)が次のようにあらわされる事が提案されている。即ち、

$$D \sim \left\langle \frac{\omega}{k_\perp^2} \right\rangle R_e^{\frac{1}{R_e + \beta}}$$

とし、ここで $R_e \sim \langle \gamma/\omega \rangle$ で、$\langle \omega \rangle$, $\langle \gamma \rangle$, $\langle k_\perp \rangle$ はそれぞれその不安定性の代表的周波数、成長率、そして径方向の波数であり、β は0～0.5程度の数値係数であり、未だ不確定の要因のための補正項である。各種タイプのドリフト波による不安定性が例としてあげてある。この提案は一般的ストカスティスィティの観点から議論され、簡単化された非線型振動子のモデルの上に、混合距離の議論を使って、発見的に上述の関係式を導いた。しかし、この提案はまだ確定的なものではなく、非線型性に内在するあいまいさを含んでおり、将来の進んだ非線型性の研究の結果、改良されるべきものである事を付記しておく。

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1. Introduction

It is a well known fact that the confinement of Tokamak plasmas is not neoclassical but anomalous, resulting from the turbulent state of the plasmas, although the theoretical method to obtain the diffusion coefficients of the fully developed turbulent plasmas is not yet established. Up to the present day, as a means to obtain it in a strong turbulence, only the mixing-length method has been used practically in spite of the insufficiency of the theoretical rigidity, though it is well known that there is a theoretically established quasi-linear method in case of a weak turbulence. But both methods cannot fully express the parameter dependence of a diffusion coefficient in accord with the strength of turbulence from strong to weak continuously. This point is vitally important to understand and control the transport of Tokamak plasmas. Therefore, besides the aforementioned methods, many workers try to tackle with the diffusion problems in certain strength of turbulences, though they cannot succeed to express the diffusion coefficient with use of the strength of turbulence, i.e. "Reynolds number", in the full range of turbulence.

As a first try of this approach, the author[1] previously seeked to obtain the diffusion coefficient (D) in case of a dissipative drift wave turbulence. The results is as follows;

$$D \sim \frac{\rho_s^2 c_s}{r_n} R_e^{\frac{1}{R_e+\beta}} \qquad (1)$$

where, R_e, representing the strength of turbulence, is proposed as

$$R_e \sim \langle k_\theta \rho_s \rangle^{-1} \frac{\nu_{ei} c_s}{r_n \omega_{te}^2 \hat{s}^2} \qquad (2)$$

and the notations obey the usual usage, and β is a numerical factor chosen to be 0.5 in Ref.(1) in order to agree with the result of $D \propto R_e^2$ of the quasi-linear approximation in a weak turbulence limit of $R_e \to 0$.

The equation (1) is obtained heuristically on the basis of the results by the several authors[2)~4)] by arranging the turbulence strength, consistent with the ever-known results of both the quasi-linear limit and the mixing-length estimate.

In this article, the author proposes to extend the form of Eq.(1) of a dissipative drift wave turbulence to a case of a general turbulence caused by the nonlinear wave instabilities in Tokamak plasmas.

Generalization is easily conceivable because the term $\rho_s^2 c_s / r_n$ of Eq. (1) can be easily replaced with $\sim \langle \omega / k_\perp^2 \rangle$ if one can think upon the well-known relation of $\langle \omega \rangle \sim c_s/r_n$, and $\langle k_\perp \rangle \sim \rho_s^{-1}$ in case of the drift wave, so that the diffusion coefficient can be rewritten as

$$D \sim \langle \frac{\omega}{k_{\perp}^{2}} \rangle R_{e}^{\frac{1}{R_e + \delta}} \qquad (3)$$

where R_e may well be chosen to represent the turbulence strength by a general term common to any instability concerned.

In the next section, the serveys on stochasticity and diffusion in relation with nonlinear oscillators are presented to lead us to obtain the explicity form of $R_e \sim \langle \gamma / \omega \rangle$ where $\langle \gamma \rangle$ is a representative growth rate of the instability concerned. In the third section, the specific form of Eq.(3) is presented with regard to the several types of the drift wave instabilities. In the last section, concluding remarks are given.

2. Surveys on Stochasticity and Diffusion

In this section, we focus our attention on the surveys for the meaning of the functional form of R_e^{1/R_e}, and the relationship of $R_e \sim \langle \gamma / \omega \rangle$, which are proposed in the previous section as a crucial term for the diffusions in the nonlinear oscillation systems, specifically in the Tokamak plasma turbulences.

Recently it has been made clear by the development of the studies of the nonlinear caos systems that the basic mechanism of stochasticity is, unexpectedly, rather simple and the criterion (R_e) distinguishing between stochasticity and a stable orbital motion can be given[5) , by using a model of particles in the ensemble of nonlinear oscillators, as a simple relation of

$$R_e \sim \frac{\Delta\omega}{\delta\omega} \gtrsim 1 \tag{4}$$

where $\Delta\omega$ represents the width of a resonance frequency caused by nonlinearity of a specific oscillator, and $\delta\omega$ the neighbouring resonance frequency distance. Therefore, R_e is equivalent with the criterion of overlapping of the resonance frequency spread on the neighbouring resonance frequency spread.

At first, with the use of this parameter (R_e), we begin the discussions on the functional form of R_e^{1/R_e}.

According to Zaslavsky and Chirikov[5)], a correlation losing time (τ_c) which means the time of losing the phase correlation between a particle movement and a wave phase is estimated to be

$$\tau_c \sim \frac{1}{\delta\omega \log R_e} \tag{5}$$

where

$$R_e \sim \frac{\Delta\omega}{\delta\omega} \sim \frac{\omega}{\delta\omega} \tag{6}$$

$$\delta\omega \sim \Delta k \left(v - \frac{d\omega}{dk} \right) \tag{7}$$

and

ω ; a wave frequency concerned

Δk; a representative value of the difference of the neighbouring wave numbers in the spectrum concerned

v ; particle's velocity

k ; a wave number concerned.

Among the above relations, Eq.(6) is easily understood to have the same physical meaning with Eq.(4) if one notices of the fact that the width of a resonance frequency of a nonlinear harmonic oscillator is an order of the frequency itself. Here we can estimate Entropy (S) with use of Kolmogorov Entropy (h) and the relation of h $\sim\tau_c^{-1}$ [6] to be

$$S \sim ht \sim \frac{t}{\tau_c} \tag{8}$$

and also a volume ($\Delta\Gamma$) of a phase space can be written to be

$$\Delta\Gamma = e^{s} \sim e^{ht} \sim e^{\frac{t}{\tau_c}} \sim \left(e^{\frac{1}{\omega\tau_c}}\right)^{\omega t} \sim \left(R_e^{\frac{1}{R_e}}\right)^{\omega t} \tag{9}$$

where Eqs.(5) and (6) are used. We interpret this representation as a volume of a phase space increases (or diffuses) with a step of a period of the concerned wave with an amount of R_e^{1/R_e}. Therefore this shows that the diffusion process of the phase space is governed by the functional form of R_e^{1/R_e} in case of a stochastic process of the nonlinear oscillators. It is directly associated with plasma turbulence. It should be noted here that it is enevitable that the diffusion process of a phase space is closely related with the form of R_e^{1/R_e} because Entropy (S) is originally defined in the probability space as

$$S = -\sum_{i=1}^{N} P_i \log P_i \tag{10}$$

where P_i is a probability of the occurrence of i-th phenomenon so that a volume of a phase space can be written as

$$\Delta\Gamma = e^{S} = \prod_{i=1}^{N} W_i^{1/W_i} \tag{11}$$

where $W_i = 1/P_i$. In Eq.(11), $\Delta\Gamma$ is again written by the form of W_i^{1/W_i} though it is a multiplication form of every possible occurrence.

As shown by the aforementioned examples, the form of R_e^{1/R_e} is a fundamentally important function related to all the stochastic processes from a general probabilisitc process to specific nonlinear oscillator processes. Therefore, it will be natural that the function can be extended to be applied to the diffusion processes in the turbulent plasmas which are originally excited by some instabilities and reach the ultimate turbulent "equilibrium" state full of many nonlinear oscillators with the various nonlinear interactions among the many waves in the equilibrium of their deaths and births. But it is intrinsically a too difficult problem to determine R_e exactly and the form of R_e^{1/R_e}

since the process to reach a turbulent state is not a simple interaction between particles and a single nonlinear oscillator but it is the chain reaction of successively emerging new oscillators each of which has each R_i, and this process continues consecutively to the ultimate"equilibrium state". In this article, we only assume R_e as the mean Reynolds number, $\langle R_e \rangle$, which represents these whole processes and try to seek its approximate estimation in case of a turbulent plasma with use of a simplified model of a nonlinear dissipative wave equation evolved by Zaslavsky and Rachko[7]. It should be commented here that the turbulent state whose $\langle R_e \rangle$ is below unity should be allowed since the turbulent state continuously changes according to its R_i which has a various value with the strength of the stochasticity so that a turbulent state with a smaller R_i than $\langle R_e \rangle$ can be possible to exist. Then, we proceed to the estimation of $\langle R_e \rangle$.

Generally speaking, when some plasma parameters grow over a certain threshold, the turbulence starts initially as a growing wave with a certain growth rate, which grows enough to be nonlinearly saturated to decay into other modes with causing another instabilities. As a final result, the system becomes an ensemble of the nonlinear waves with a certain range of spectrum, in which the particles interact with waves nonlinearly and behave stochastically to reach a "steady state", where the wave energies are pumped through free energy caused by certain potentials and dissipated by plasma particles. This nonlinear state is examined by Zaslavsky and Rachko[7] who, using a very simplified model equation in order to depict a kernel structure of the nonlinear dissipative oscillators, obtain a stochasticity criterion,

$$R_e = K\mu \qquad (12)$$

where $K = \varepsilon\alpha\Omega$, $\mu = (1 - e^{-\Gamma})/\Gamma$, $\Gamma = \gamma T$, $\alpha = (d\omega/dI)(I_0/\omega_0)$, $\Omega = \omega_0 T$, $T^{-1} = (d\omega_k/dk)\Delta k$, and ω : a frequency of a primary wave, ω_0: a linearly obtained frequency of a primary wave, I : an intensity of a primary wave, I_0 : a nonlinearly saturated intensity of a primary wave, γ : a linearly obtained growth rate of a primary wave, ε : a parameter defining the strength of nonlinear interaction between a primary wave and other excited modes, ω_k : a representative wave frequency of other excited modes, k : a wave number of other excited modes, Δk : a representative wave number gap of neighbouring wave frequencies of other excited modes. With Eq.(12) they figure out the numerical results by changing the parameters (ε , α , Ω and Γ) to reach the conclusion that a small γ enlarges the region of strange attractors and makes the system similar to the stochasitc structure of non-dissipative Hamiltonian system while a large γ makes the system prone to a stable limited motion. But, this estimation should be premature because it treats the parameters (ε , α , Ω and Γ) independently for the preference for the simplification of the circumstances. Actually the parameters (ε , α) depends on γ , ω_0 and T. If we take a more careful look into the structures of ε and α , we can easily obtain the crude estimates of $\varepsilon \sim \gamma T$ and $\alpha \sim \gamma/\omega_0$. Therefore, Eq.(12) can be rewritten to be

$$R_e \sim \Gamma^2, \qquad \Gamma \ll 1 \qquad (13)$$

$$\sim \Gamma, \qquad \Gamma \gtrsim 1 \qquad (14)$$

which endorses our common knowledge of a strong turbulence with a large γ. If the relation of $\Gamma = (\gamma/\omega_0)\cdot \omega_0 T$ is taken into account, Eq.(14) can be interpreted simply as a multiplication of $\omega_0 T$ which is equivalent with the criterion of the nonlinear oscillators of Eq.(4), and γ/ω_0 which newly appears in the dissipative case. Here we can easily speculate that $\omega_0 T$ must be an increasing function of (γ/ω_0) since the nonlinear overlap should become wilder as a primary wave's growth rate becomes larger. But, unfortunately, we cannot write down the function since we have only the poor knowledge of this problem for the sake of intricacy and complexity of nonlinearity. Here we only satisfy ourselves with rewriting Eqs.(13) and (14) with the knowledge of the above arguments as

$$R_e \sim \left(\frac{\gamma}{\omega_0}\right)^f \qquad (15)$$

where f is a certain postive number, and probably a decreasing function of (γ/ω_0) because of both forms of Eq.(13) in case of a small (γ/ω_0) and Eq.(14) in case of a large (γ/ω_0)

Next, we proceed to compare the results with the mixing length approximation, according to which, the diffusion coefficient is often written as

$$D \sim \frac{\gamma}{k_\perp^2}, \qquad (\gamma \sim \omega_0) \qquad (16)$$

$$\sim \frac{\gamma^2}{\omega_0 k_\perp^2}, \qquad (\gamma \ll \omega_0) \qquad (17)$$

As it is obvious from the basis of the mixing length argument that the primary term of the diffusion coefficient is $\omega_0/k_\perp^2$, we pull out the term of $(\omega_0/k_\perp^2)$ intentionally from Eqs.(16) and (17), having respectively,

$$D \sim \frac{\omega_0}{k_\perp^2}\left(\frac{\gamma}{\omega_0}\right), \qquad (\gamma \sim \omega_0) \qquad (18)$$

$$\sim \frac{\omega_0}{k_\perp^2}\left(\frac{\gamma}{\omega_0}\right)^2, \qquad (\gamma \ll \omega_0) \qquad (19)$$

It can be interpreted as the particles diffuses with a spread of a wave length $\langle 1/k_\perp^2 \rangle$ within the interval of a period of wave frequency (ω_0) accompanying the factor of $(\gamma/\omega_0)^{1\sim2}$. Therefore, the turbulence factor should be a term of $(\gamma/\omega_0)^{1\sim2}$ which endorses the relation of Eq.(15) though the functional form of R_e^{1/R_e} does not appear in Eqs.(18) and (19).

There have been no works referring to the functional form of R_e except Ref.(1) in which the author investigates the power part of R_e and discovers that it depends on R_e itself and it is a decreasing function of R_e in case of of a dissipative drift wave turbulence. The power part of Eqs.(18) and (19) also shows a decreasing function of (γ/ω_0). Gang et al.[8] dubs the part of $(\gamma/\omega_0)^2$ as "a weak turbulent factor" in case of a trapped electron drift mode. The exact form of R_e may be impossible to write down nor the functional form of R_e because R_e itself contains the nonlinearity which requires consecutive renormalization processes for the determination. Therefore the ultimate form as a result of the chain reactions may not be expressed by elementary functions.

In this article, therefore, we only confine ourselves into the purpose of determining the diffusion coefficient which is sufficiently usable, in the practical transport problems. Upon the basis of that principle, we adopt the form of

$$D \sim \langle \frac{\omega_0}{k_\perp^2} \rangle R_e^{\frac{1}{R_e+\beta}} \qquad (20)$$

where $R_e = \langle \gamma/\omega_0 \rangle$, and β is introduced to adjust the unknown ambiguities. The value β may be chosen as 0.5 since it gives the term of $1/(R_e + \beta)$ equal to 2 in the limit of $R_e \to 0$ in order to match with the quasi-linear approximation.

As for the relation of $R_e = \langle \gamma/\omega_0 \rangle$, it should be referred to the so-called "iδ" model[11] which is often used to treat the nonlinear interactions between particles and waves where the phase shift between the particle fluctuations and the nonlinear waves is packed into this "iδ" expression. This δ-value is estimated from the dissipative part of electron propagator as

$$\delta \sim \frac{\gamma}{\omega_0} \qquad (21)$$

in case of $\gamma/\omega_0 \ll 1$ in the various nonlinear studies[12]. As this "δ" is a measure of nonlinearity, it is natural that Reynolds number presented in Eq.(20) should be replaced with this δ if we consider the situation more generally. But the estimation of δ in general cases is not availavle at present.

There are several works which reinforce Eq.(20). Horton[9] presents the numerical result of a collisionless drift wave that the diffusion coefficent has a peak at $R_e \sim 1$ and rather decreases as R_e becomes larger than~ 1. Also Waltz and Domingues[10] suggest in case of a dissipative trapped electron mode that the term of (ω_*/ν_{eff}) where ω_* is a diamagnetic electron frequency and $\nu_{eff} = \nu_{ei}/\varepsilon$ should be replaced by $(\omega_*/\nu_{eff})^{\alpha}$ where $\alpha = 0 \sim 1$.

In the next section, we present the examples of specific instabilities.

3. Examples : Diffusion Coefficients of Several Types of Instabilities

In this section we present the explicit form of R_e of each well-known instability.

i) Collisionless drift wave

$$R_e \sim \langle \frac{\omega_*}{k_\parallel v_e} \rangle \tag{22}$$

ii) Collisional drift wave

$$R_e \sim \langle \frac{\omega_* \nu_{ei}}{k_\parallel^2 \ v_e^2} \rangle \tag{23}$$

iii) Collisional trapped electron drift wave

$$Re \sim \varepsilon^{1/2} \eta_e \frac{\omega_*}{\nu_{eff}} \; [13] \tag{24}$$

iv) Collisionless trapped electron drift wave

$$R_e \sim \varepsilon^{1/2} \eta_e \left(\frac{R_o}{r_n G}\right)^{3/2} \left(\frac{R_0}{r_n G} - \frac{3}{2}\right) \exp\left(-\frac{R_0}{r_n G}\right) \; [14] \tag{25}$$

4. Concluding Remarks

The presented form is not a final one. Maybe some advanced modification should be needed. The completeness is not near at hand but is to be arrived in the end of the full understanding of nonlinear physics. But this article tries to presents the first attempt to parameterize the turbulent factor in the explicit form by using a general term.

With regard to the practical usage, we had better pay a careful attention to determining $\langle k_\parallel \rangle$, $\langle k_\perp \rangle$, and $\langle k_\theta \rangle$. But the general characteristics is determined by the functional form of $R_e^{1/R_e+\beta}$ and it is not so greatly changeable, so that Eq.(20) can be used without affecting a general behavior of transport problems.

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