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The dependence of stabilizing effect of a conducting shell on a poloidal beta value (β_p) is investigated as to instabilities with low toroidal mode numbers (n = 1 and 2) for a tokamak with a circular cross-section such as JT-60. The n = 1 mode is completely stabilized by the conducting shell which is located at a practically possible position and the critical position of the shell becomes closer to the plasma surface with increasing β_p. The stabilizing effect on the n = 2 mode is remarkable for higher β_p when the shell is placed sufficiently close to the plasma surface but the shell far from the plasma surface has hardly an effect on the stability property of a higher β_p plasma.

It is concluded that critical β of about 2 % is attainable even in a standard circular tokamak such as JT-60 and higher β value is also expected by taking advantage of the closely located conducting shell.

Keywords; Beta-limit, Shell Effect, Kink Mode, Large Tokamak,
Circular Cross-section, ERATO Code

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円形断面大型トカマクのベータ限界

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(1981年12月16日受理)

円形断面トカマクのn＝1，2キンク・モードに対する導体壁の安定化効果のポロイダル・ベータ値（β_P）依存性をERATOコードを使って調べた。n＝1モードは，導体壁の半径がプラズマ半径の1.1倍（$\beta_P = 1.8$の時）ないし1.2倍（$\beta_P = 1.0$の時）程度で完全に安定化される。n＝2モードは導体壁が十分プラズマ表面に近くないと安定化されないが，安定化効果はβ_Pが大きい程著しい。本報告書で用いた平衡状態ではβの限界が2%であったが，JT−60のように導体がプラズマ表面に近い場合には，キンク・モードに対して安定なより高ベータの平衡が期待できる。

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1. Introduction

For a tokamak reactor, stable equilibria with rather high beta values are required, where the beta value (β) is the ratio of a plasma pressure p and a magnetic pressure $B^2/2\mu_o$ ($\beta \equiv 2\mu_o p/B^2$). The beta value is approximately expressed by other equilibrium parameters under an assumption of a uniform current distribution and a large aspect ratio, $\beta = \beta_p(1+E^2)/(2\bar{q}^2A^2)$, where β_p is a poloidal beta value defined by $\beta_p \equiv 8\pi\mu_o p/I^2$, and I, E and A are the total current, the elongation ratio of a cross-section and the aspect ratio of a plasma and $\bar{q}$ ($= \pi R_o B_T(1+E^2)/(\mu_o A^2 I)$, R_o: the major radius, B_T: the toroidal magnetic field) is the safety factor defined for a uniform current plasma. To reach high beta states we may increase β_p or E, or decrease A or $\bar{q}$. Usually the parameters E and A are determined by considering technological constraints through the process of designing a device. In addition to the geometrical parameters such as A and E, there are several parameters which should rather be determined in advance. One of them is the value of safety factor at the magnetic axis (q_o). The lower limit of q_o is determined by a stability criterion for the sawteeth instability[1) and it should roughly be more than unity. Choise of a class of current profile among unlimited possibilities corresponds to the determination of some other parameters in advance such as the value of safety factor at a plasma surface (q_a) and the poloidal beta value (β_p). Therefore, it is very important for the attainment of higher value of beta to find a lowest possible q_a and an optimum β_p value for fixed geometrical parameters and a fixed class of current distribution. The reduction of q_a causes a kink instability when a conducting shell is far from the plasma surface[2). As a shell is placed closer to the plasma surface, stable equilibria with smaller q_a are expected to be attainable and the upper limit of beta values of stable equilibria (the critical beta value, β_c) becomes larger.

The effects of the conducting shell on the kink instability have been investigated by many authors since the early stage of the studies on mhd instabilities. Almost all of these works were carried out by using a sharp-boundary model[3] or cylindrical model[4]. These models are simple and suitable for qualitative analysises but rather differ from a realistic tokamak plasma. To obtain the critical beta value of a given device or to search the stable equilibria with a higher beta value, analysises of realistic (toroidal and diffuse-current) equilibria are required. For this purpose two-dimensional stability code such as the ERATO[5] and PEST[6] is a powerful and indispensable tool. Berger et al.[7] studied the effects of the conducting shell on the mhd behaviors of the Solov'ev equilibrium (a uniform-current one) by using ERATO code. Their results show the instability with the toroidal mode number n = 2 becomes dangerous when the conducting shell is not sufficiently close to a plasma surface.

Berger also paid attention to the destabilizing tendency of the low n mode observed by increasing β_p[8]. In his article it is concluded that the modes responsible for the deterioration of the stability property near the marginal point have a ballooning nature and the equilibrium for $\beta_p = 2$ is advantageous as compared with that for $\beta_p = 1$, from the viewpoint of β-optimization. On the other hand, Bernard et al. showed recently the optimum β_p value is less than unity for the tokamak with a small aspect ratio and noncircular cross-section[9]. This difference may be attributed to the difference of the adopted classes of equilibria and the plasma shape. In any case, however, the stability property is sharply deteriorated by increasing β_p and the critical safety factor is raised considerably.

From the above point of view it is very interesting question whether the stabilizing effect of a conducting shell is weakend or not by increasing

β_p. It is an important problem in the JT-60 program at JAERI because a conducting shell of the JT-60 is placed rather closely to the plasma surface and the stabilizing effect of the shell is expected. Moreover, dependence of the stability on expected β_p has not been checked yet quantitatively.

In this paper we investigate the relation between the stabilizing effect on the low n (n=1,2) modes and the values of β_p in the various range of current profiles (from a flat one to a peaky one) for a circular cross-section plasma such as JT-60 by using the ERATO code. The high n ballooning mode is analyzed by using BOREAS code[10]. In section 2 the mhd equilibria and parameters used in our investigations are shown and in section 3 the computational results are shown. The results are summarized and discussed in the last section.

2. MHD equilibria

In an axisymmetric toroidal plasma, mhd equilibria are expressed by the Grad-Shafranov equation in a cylindrical coordinate system (r,z,ϕ),

$$\Delta^*\psi = - r\mu_o j_\phi(r,\psi) \quad , \tag{1}$$

$$j_\phi = r\frac{dp}{d\psi} + \frac{1}{r}T\frac{dT}{d\psi} \quad , \tag{2}$$

$$\Delta^* \equiv r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \quad , \tag{3}$$

where ψ, p and T are a poloidal flux function, a plasma pressure, and a toroidal field function respectively. The function p and T depends only on ψ. These functions are not determined in the framework of the equilibrium theories and are determined by considering transport processes[11). When we are concerned with mhd equilibria or mhd stabilities, we usually prepare the functional form of p and T a priori. Many classes of functions have been proposed especially for the studies on high beta equilibria and their stabilities[12-14). We choose the following forms of functions[12),

$$p = \beta_J\, p_o \{(\psi-\psi_b) - \frac{\alpha}{2}[(\psi-\psi_m)^2 - (\psi_b-\psi_m)^2] + \frac{\gamma}{L+1}\{(\psi-\psi_m)^{L+1} - (\psi_b-\psi_m)^{L+1}\} \quad , \qquad L \geq 4 \quad , \tag{4}$$

$$\gamma = -[1 + \alpha(\psi_m-\psi_b)]/(\psi_b-\psi_m)^L \quad , \tag{5}$$

$$T\frac{dT}{d\psi} = (1/\beta_J - 1)\, R_o^2 \frac{dp}{d\psi} \quad , \tag{6}$$

where α is related directly to the half width of the current profile, β_J is the approximate value of β_p, and Eq.(5) denotes that there is no skin current, i.e. $\frac{dp}{d\psi} = 0$ at a plasma surface, $\psi = \psi_b$. In our calculation p_o is automatically determined by the condition that the safety factor at the magnetic axis is fixed. The position $r = R_o$ is the one of the magnetic axis.

We adopt the circular poloidal cross-section of a plasma with the minor radius a_p and the conducting shell is placed on a concentric circle (radius : a_w) with the plasma surface, and the ratio Λ ($\equiv a_w/a_p$) is used as the parameter which represents the position of the conducting shell. Parameters used in our calculation and relating parameters of JT-60 are summarized in Tables 1 and 2, respectively.

3. Numerical Results

Using equilibria shown in §2, we calculated the growth rates of unstable modes versus q_a for various values of Λ by fixing q_o to unity (Figs.1-4). The frequency ω is normalized by the toroidal Alfvén frequency at the magnetic axis. Comparing Figs.1 and 3 with Figs.2 and 4 in the respective order, we find that both n = 1 and n = 2 modes for $\beta_p = 1.8$ are more unstable than those for $\beta_p = 1$ in the case of $\Lambda = \infty$. The n = 2 internal mode, however, is more stable for $\beta_p = 1.8$, that is, both the growth rate and the critical q_a of the n = 2 internal mode are smaller than those for $\beta_p = 1$. This behavior is quite different from that for $\Lambda = \infty$.

For more detailed understanding of the stabilizing effect of a conducting shell, we show the growth rates of n = 1 and n = 2 modes versus Λ in Figs.5-8. Figs.5 and 6 show that the n = 1 kink modes for both $\beta_p = 1$ and $\beta_p = 1.8$ are completely stabilized by the shell and the critical Λ is smaller for $\beta_p = 1.8$ ($\Lambda_c = 1.1$) than for $\beta_p = 1$ ($\Lambda_c = 1.2$). On the other hand, Figs.7 and 8 show that the n = 2 internal modes remain unstable, and the stabilizing effect of the shell is stronger for $\beta_p = 1.8$ when the shell is placed near the plasma surface.

As for the high n ballooning mode, we calculated the minimum eigenvalues defined by Eqs.(4)-(6) in the reference 15 by using BOREAS code[10). Results are presented in Fig.9. This figure shows our equilibria with β_p=1.8 are unstable against the high n ballooning mode for all q_a's.

Above results are summarized as a relation between β_c and n in Fig.10. This relation shows that the n = 1 kink mode is the most dangerous one in the case of $\Lambda \gg 1$, and the high n ballooning mode is the most dangerous in the case of $\Lambda \sim 1$ irrespective of β_p values.

4. Conclusions and Discussions

The n=1 kink mode is strongly stabilized by a conducting shell, but the stabilizing effect on the n=2 mode is weak except for a nearly fixed boundary ($\Lambda \gtrsim 1$) in both β_p=1 and β_p=1.8 cases. For $\Lambda \gtrsim 1$ the stabilizing effect on n=2 mode is more remarkable for higher β_p than for lower β_p and the critical q_a is smaller for higher β_p. It presumably results from the fact that high mode number components due to the deformation of magnetic surfaces which localize near the plasma surface appear more remarkably in the case of β_p=1.8 than β_p=1.0. Stabilizing effect on n=2 mode for $\Lambda \gtrsim 1$ is quite different from that for large Λ. The critical q_a for large Λ increases as β_p increases, as shown by Berger[8)] and also by our results, but it is not the case for Λ=1.0. Therefore, the critical β value relating the low n modes can be increased by increasing β_p value and locating the conducting shell very close to the plasma surface.

In our results the high n ballooning mode is more unstable for higher β_p. This is because that the pressure gradient is larger for higher β_p notwithstanding the difference of q-profile is very slight (Figs.11 and 12).

In summary we can expect to attain a plasma with $\beta_c \geq 2\%$ in JT-60 for small β_p ($\beta_p \sim 1$). Moreover, as a conducting wall is located very close to the plasma surface in this device and the finite-n effect is also expected to stabilize the high-n ballooning instability, higher critical β will be attained for higher β_p plasma with a different current profile.

Acknowledgements

The authors are very grateful to Dr. M. Tanaka for fruitful discussions. They also would like to acknowledge Drs. S. Mori and Y. Obata for their continuing encouragements.

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In summary we can expect to attain a plasma with $\beta_c \gtrsim 2\%$ in JT-60 for small β_p ($\beta_p \sim 1$). Moreover, as a conducting wall is located very close to the plasma surface in this device and the finite-n effect is also expected to stabilize the high-n ballooning instability, higher critical β will be attained for higher β_p plasma with a different current profile.

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Table 1

Parameters for the calculations

Aspect ratio	$A = R_o/a_p = 3$
Poloidal cross-section	Circular
q_o	$q_o = 1$
q_a	$q_a = 1.9 - 6.0$
β_p	$\beta_p \simeq 1 \quad (\beta_J = 1)$
	$\beta_p \simeq 1.8 \; (\beta_J = 1.65)$
Shell positions	$\Lambda = 1 - \infty$

Table 2

Parameters of the JT-60 tokamak

Major radius	$R_o = 3.03$ m
Minor radius	$a_p = 0.95$ m
Poloidal cross-section	Circular
Toroidal field strength	$B_t = 4.5$ T
Plasma current	$I_p = 2.6$ MA
Wall position	
Mo liner	$a_l = 0.97$ m
Vacuum vessel	$a_v = 1.00$ m

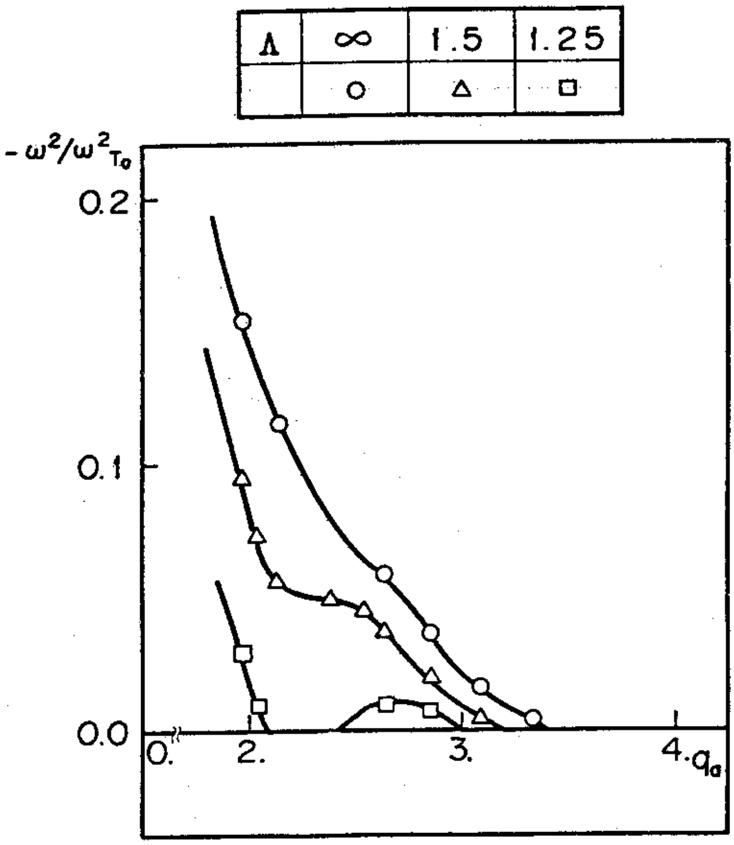

Fig. 1 : The square of the growth rate for n = 1 and β_p = 1.0.

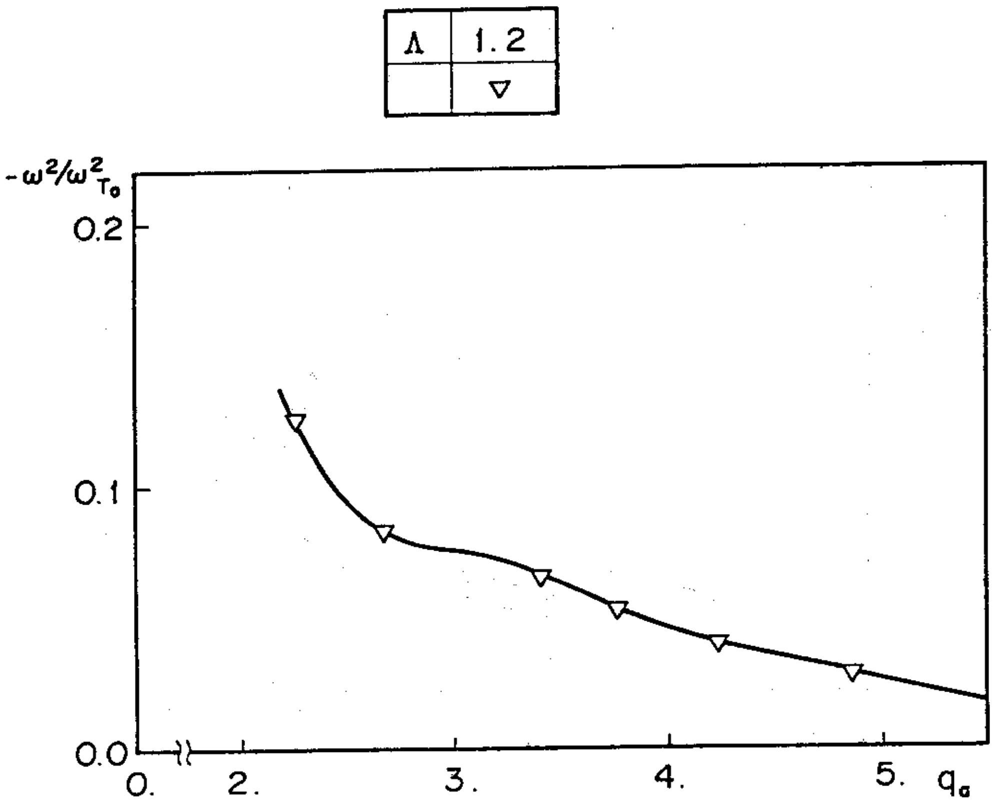

Fig. 2 : The square of the growth rate for n = 1 and β_p = 1.8.

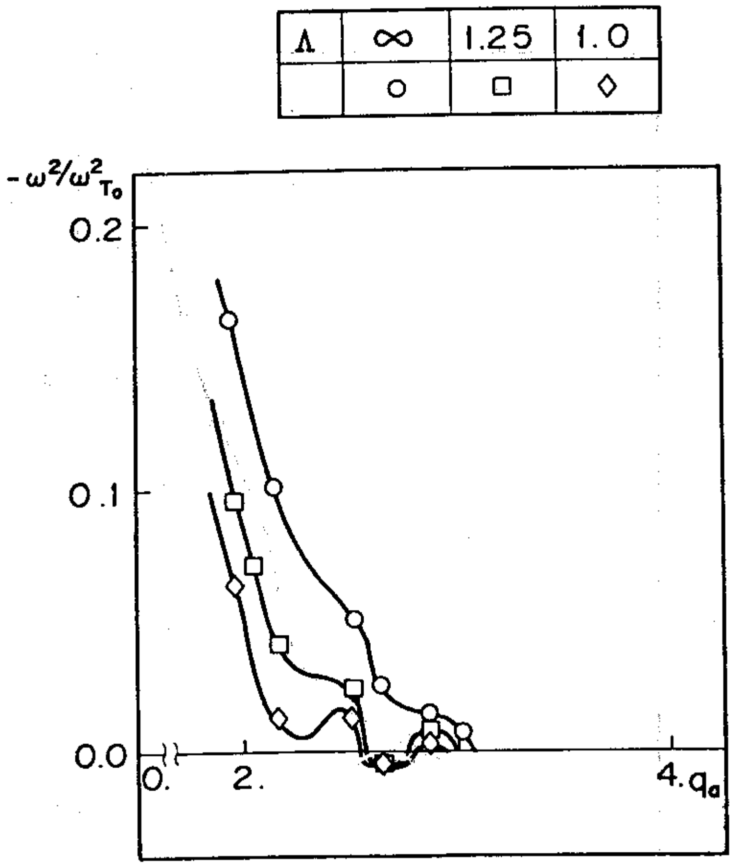

Fig. 3 : The square of the growth rate for n = 2 and β_p = 1.0.

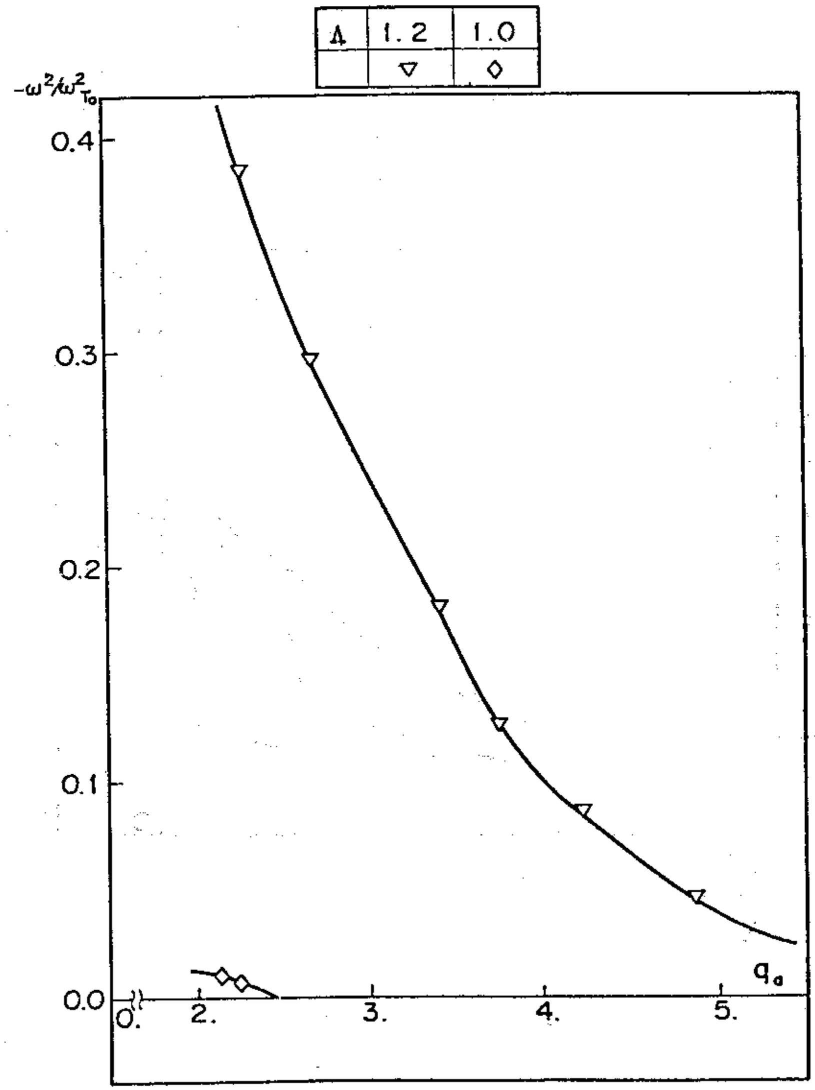

Fig. 4 : The square of the growth rate for n = 2 and β_p = 1.8.

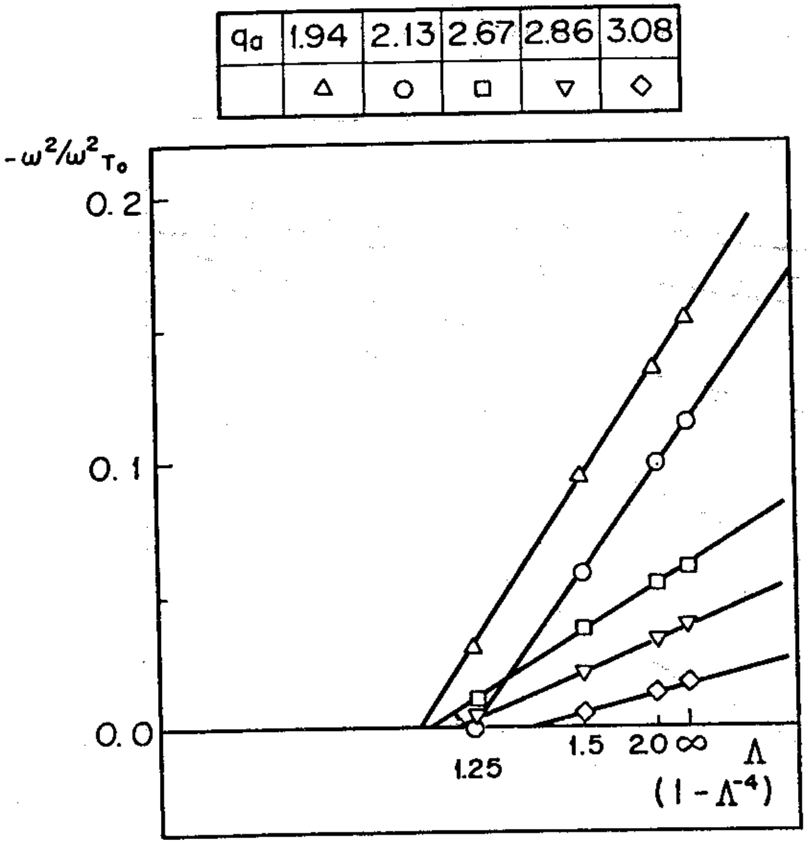

Fig. 5 : The square of the growth rates versus Λ for n = 1 and β_p = 1.0.

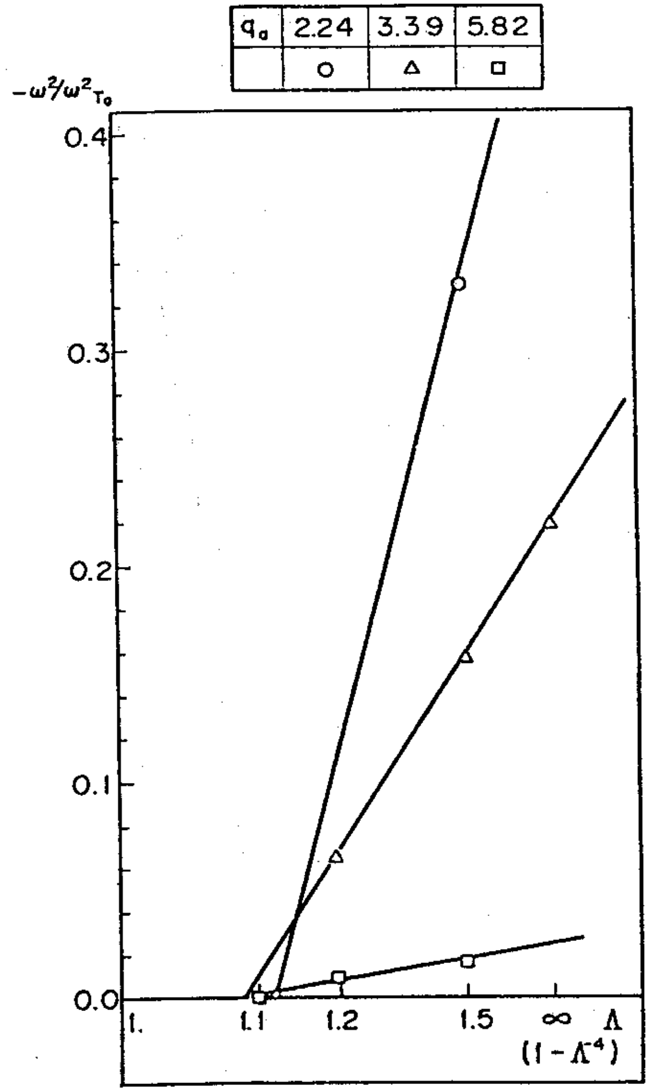

Fig. 6 : The square of the growth rates versus Λ for n = 1 and β_p = 1.8.

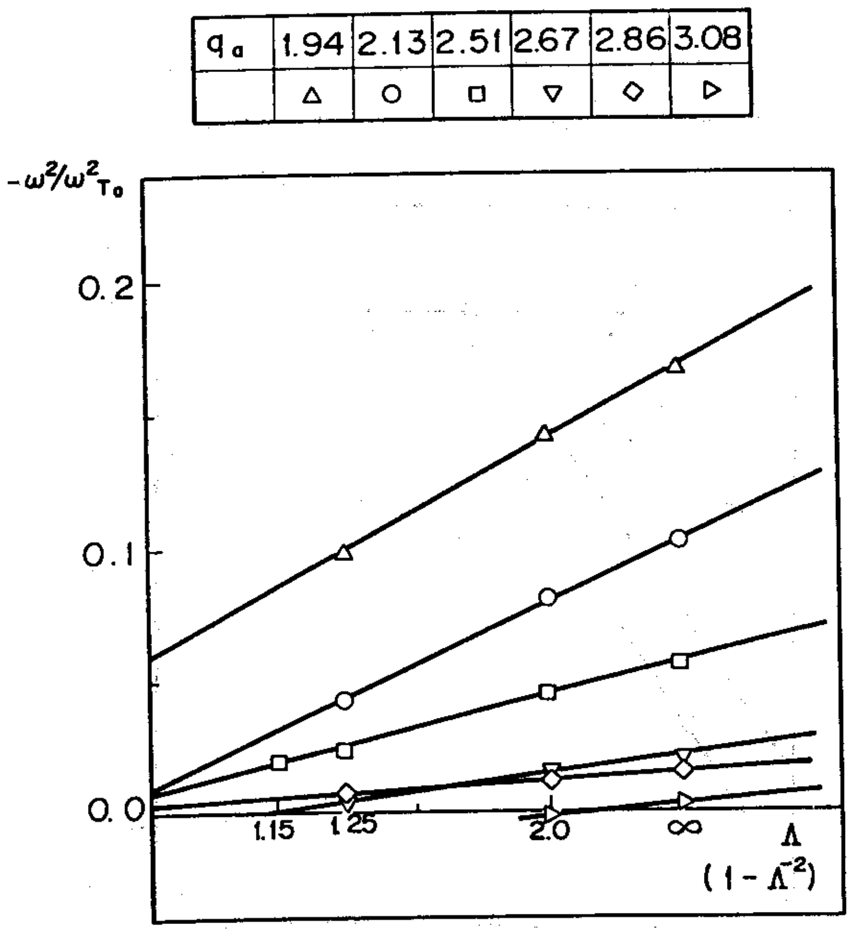

Fig. 7 : The square of the growth rates versus Λ for n = 2 and β_p = 1.0.

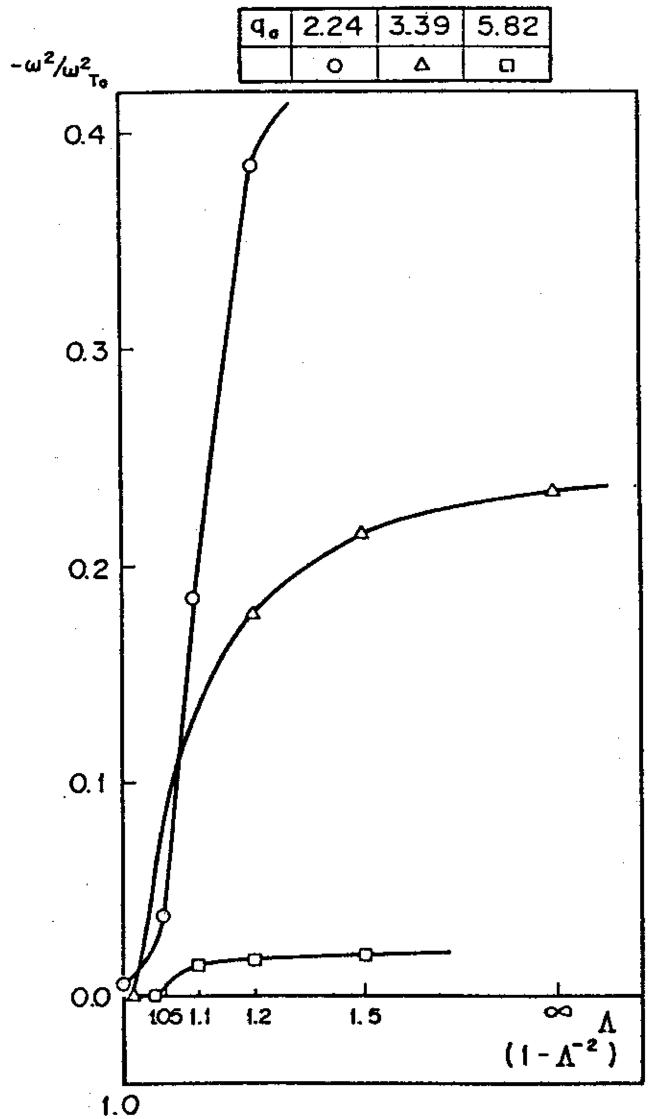

Fig. 8 : The square of the growth rates versus Λ for n = 2 and β_p = 1.8.

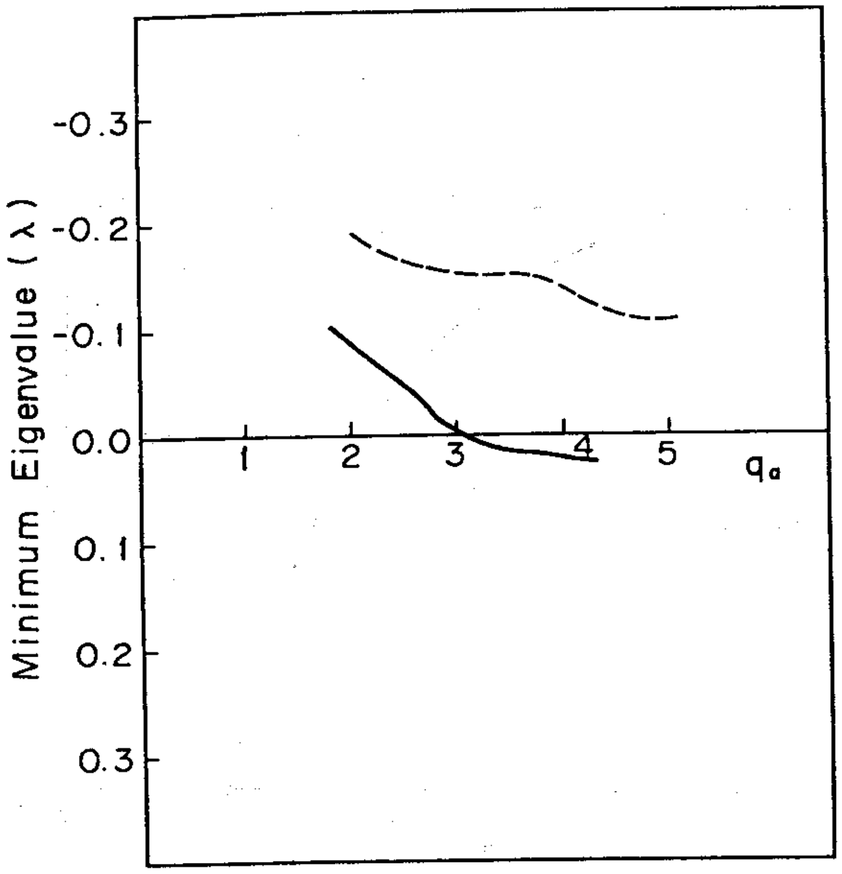

Fig. 9 : The minimum eigenvalue of the high n ballooning mode versus q_a for β_p = 1.0 (solid line) and β_p = 1.8 (dashed line).

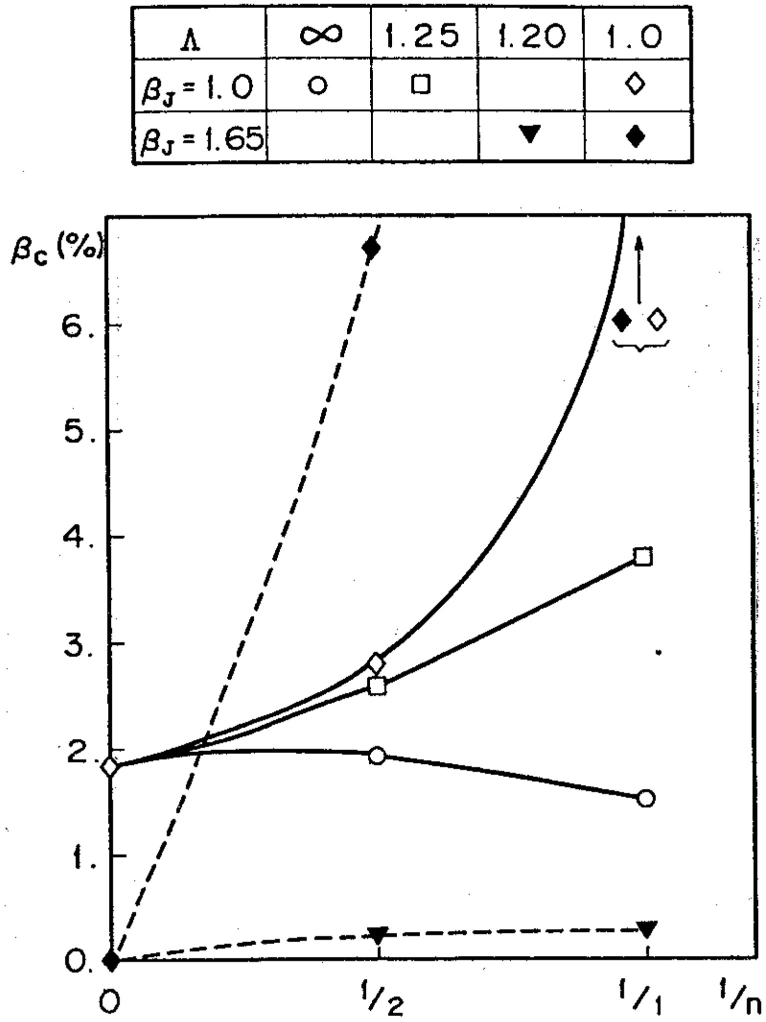

Fig.10 : The critical beta value versus 1/n.

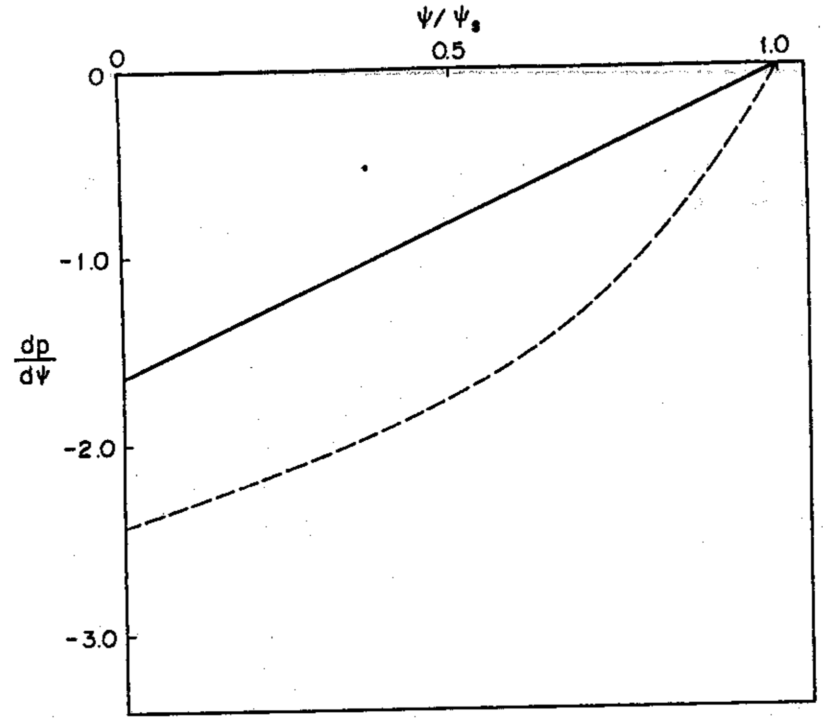

Fig.11 : The pressure gradients versus ψ for β_p = 1.0 (solid line) and β_p = 1.8 (dashed line).

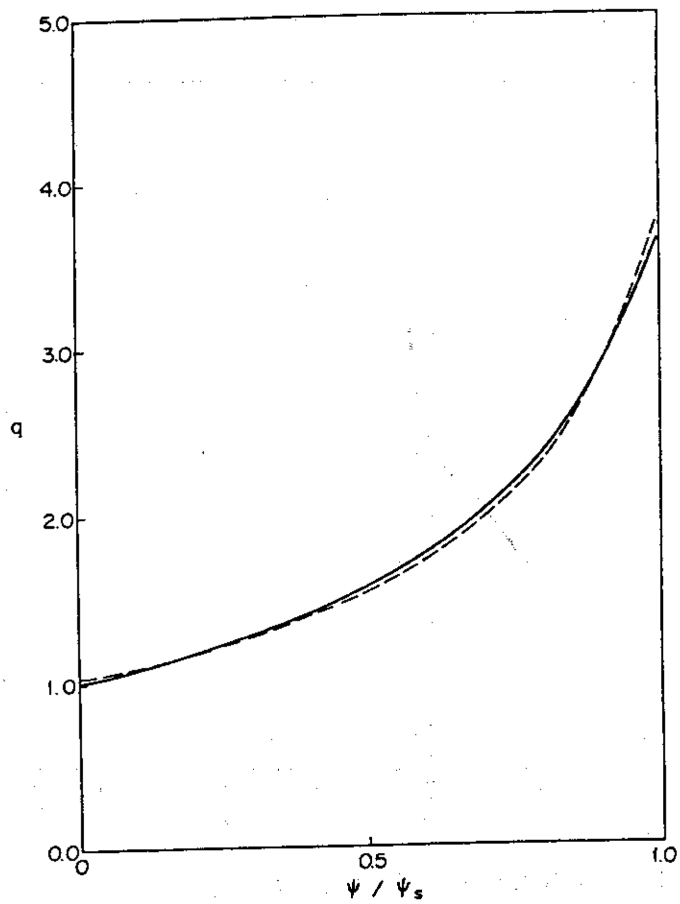

Fig.12 : The q-profile versus ψ for β_p = 1.0 (solid line) and β_p = 1.8 (dashed line).