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A TRANSPORT MODEL OF OHMICALLY HEATED TOKAMAKS
—"PROFILE CONSISTENCY" REVISITED—

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A Transport Model of Ohmically Heated Tokamaks – "Profile Consistency" Revisited –

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A new transport model of OH-plasmas is proposed. It can well interpret the so-called "Profile Consistency" and makes clear the other important transport issues. The transport treated in this model consists of the contribution from the skin-depth type of electromagnetic turbulence in the inner region and electrostatic drift wave turbulence in the outer plasma with neoclassical ion contribution over the whole area.

In a low density regime the central core is dominated by both the electromagnetic turbulence and the neoclassical ion contribution while the transport of the outer plasma is replaced by the electrostatic drift wave turbulence. In a high density regime, on the other hand, the electromagnetic turbulence region is confined into a narrow region near the axis and almost all the other region is dominated by the neoclassical ion contribution and the electrostatic drift wave turbulence. As a result, this model follows the Neo-Alcator scaling i.e., density-proportional in a low density regime and density-saturated in a high density regime. The critical β-value (β^{cr}) separating these two regions is presented.

Keywords: Transport Model, Ohmic Tokamak, Profile Consistency, Turbulence

オーミィック加熱トカマクの輸送モデル
（プロフィールコンスィスティンシィ再考）

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OH－プラズマの新しい輸送モデルが提案されている。それはいわゆる“Profile Consistency”をうまく説明できるだけでなくその他の重要な輸送問題を明確にすることができた。このモデルにおける輸送はプラズマ内部では表皮型の電磁的乱流で，また外周部では静電型のドリフト波乱流で，かつ，全体領域を新古典論によるイオンの寄与で構成されている。即ち，低密度領域のプラズマでは中心部は電磁的乱流と新古典論によるイオンの寄与で支配されており，外周部は，静電的ドリフト波乱流でとって代わられている。一方，高密度領域のプラズマでは，電磁的乱流の領域は中心軸付近の狭い部分に限定されてしまい，その他の大部分の領域は新古典論によるイオンの寄与と静電的なドリフト波乱流によって支配されている。その結果，このモデルは，Neo-Alcator 則，即ち，低密度領域では密度比例であり，高密度領域では密度飽和である事を裏付けている。この2つの領域を分けている臨界β値（β^{cr}）の表式が示されている。

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Contents

1. Introduction 1
2. Motivation and Presentation of Model 3
3. Concluding Remarks 19
Acknowledgement 20
References 20

目　　次

1. 序　　論 1
2. 動機及びモデルの提出 3
3. 結　　論 19
謝　　辞 20
参考文献 20

1. Introduction

As well known, OH-plasmas have a strong constraint on the electron temperature (T_e) profile, the so called "Profile Consistency" [1] which tells that T_e-profile is resilient to the change of plasma parameters and can be written as follows;

$$T_e(x) = T_e(0)e^{-\alpha_t x^2} \quad (1)$$

where $x = r/a$, and

$$\alpha_t \sim \frac{2}{3}\frac{q_a}{q_o} \quad (2)$$

where the notations obey the usual usage. An important point is that the profile obeys a Gaussian form and the parameter (α_t) is a function of only (q_a/q_o), though the latter relation is not so important because we can easily obtain the relation of Eq.(2) from the relation of

$$I_p = \int JdS \quad (3)$$

with Eq.(1) if we permit that the plasma current density obeys the Spizer-Härm conductivity and the approximation of exp $(-\frac{3}{2}\alpha_t) \ll 1$, which is easily satisfied in the usual experimental condition of $3 \lesssim q_a/q_o \lesssim 7$, is assumed. Thus the key point is Eq.(1). Why does a Gaussian profile emerge over the wide range of parameter changes? There must be some hidden principles leading to Eq(1). Of course, the functional form may not be exactly a Gaussian one that governs the transport, since a Gaussian form was just obtained from the experimental experiences, but it requires a similar form to a Gaussian. Already we have had several works [2-5] trying to interpret the "Profile Consistency". Those are mostly related to the Free Energy Minimum Principle from which the current density profile (J) is deduced as follows,

$$J(x) \sim \frac{J(0)}{\left[1 + \left(\frac{q_a}{q_o} - 1\right) x^2\right]^2} \quad (4)$$

which may be interpreted to be equivalent with Eq. (1), though this form is not completely the same as Eq.(1). Those works, however, despite of its beautiful deduction, never tell us the specific form of heat diffusivity that governs the tokamak transport. If we obey the self-organizing process theory claimed by Kadomtsev [4], the transport may be nonlinearly self-organized and choose the specific type of turbulences in order to make the free energy minimized in accord with the specific plasma parameters. Even though this type of general principle may be imposed on the tokamak turbulence, we eagerly want to define the specific form of heat diffusivity in each case of a specific set of plasma parameters because it will tell us the type of turbulence and its parameter dependence and it gives us the tool to control the confinement. Thus, in this article, we take a different approach which makes us directly pursue the heat diffusivity that enables the T_e-profile take a form of Eq.(1). In the next section, we begin the discussions with presenting the plausible candidates of heat diffusivity and determine the specific form of diffusivity which satisfies the various aspects of "Profile Consistency". In the last section, concluding remarks are given.

2. Motivation and Presentation of Model

As Coppi [1] pointed out that the electromagnetic type of turbulence in which the skin-depth (c/ω_{pe}) plays an important role is suitable for the change of plasma density, we take the electromagnetic type of turbulence the most promising candidate. This type of turbulence was first proposed by Ohkawa [6] who showed the heat diffusivity (χ_e) written to be

$$\chi_e \sim \frac{C^2}{\omega_{pe}^2} \cdot \frac{V_e}{qR}, \tag{5}$$

where the notation obeys the usual usage. Ever since, this type of turbulence has been vigorously studied [7-15] and the diffusivity was fixed into the form of

$$\chi_e \sim \frac{C^2}{\omega_{pe}^2} \cdot \frac{V_e}{qR} \varepsilon^{\alpha_0}, \tag{6}$$

where $\varepsilon = r/R$. The parameter α_0 is around 1 ~2, and differs a little according to the authors. Here we take $\alpha_0 = 1$ as the first try becasue it fits best numerically in the experimental parameters and the case of $\alpha_0 = 2$ is too small to explain experimental data.

In the first place, we study the simplest electron transport equation;

$$\frac{1}{r} \frac{d}{dr} \{ -rn_e \chi_e \frac{dT_e}{dr} \} = JE . \tag{7}$$

This equation neglects the electron-ion exchange term (Q_{ei}) which is important in OH-plasma transport. But, for the time being, we take into no account of it in order to make the situaton as simple as possible. Then, Eq.(7) can be solved, with use of Eq.(6), quite easily to be

$$(T_e/T_{e0})^{3/2} = 1 - \beta x , \tag{8}$$

where $x = r/a$, and

$$\beta = \frac{\frac{3}{2\sqrt{2}} m_e^{1/2} \text{ ERaB}}{\frac{C^2 n_e}{\omega_{pe}^2} \mu_0 T_{e0}^{3/2}} . \tag{9}$$

This equation represents a simple straight line of $(T_e/T_{eo})^{3/2}$ against the normalized radius x, and β is a constant, depending on the plasma parameters (E, R, a, B), and the boundary value of electron temperature (T_{eo}). But this result is disappointing in a sense since it much differs from the Gaussian profile, and the gradient at x =0 (T_e' at x=0) is not zero which means that T_e is not smooth at x=0. Despite of these defects, however, we stay around Eq.(6) for the time being since it enables Eq.(7) integrable. The key point of the integrabilty is the term of "qR" in the denominator of Eq.(6), combined with the part of (C^2/ω_{pe}^2) for the elimination of density dependence. Then, the type of Eq.(6), whatever value α_o takes (but we assume $\alpha_o \geqq 0$), can be easily integrated. The condition that the gradient at x=0 be zero requires $\alpha_o<1$. This, however, contradicts the requirement of $1 \leq \alpha_o \leq 2$. [8-15] Therefore, we temporarily return to the original form of heat diffusivity proposed by Ohkawa [6], Eq.(5), adding the constant parameter (Λ),

$$\chi_e \sim \frac{C^2}{\omega_{pe}^2} \cdot \frac{V_e}{qR} \Lambda . \tag{10}$$

With use of Eq.(10), we integrate Eq.(7) to obtain

$$(T_e/T_{eo})^{3/2} = 1 - \frac{\beta}{2\alpha} x^2 , \tag{11}$$

where

$$\alpha = \frac{R}{a} \Lambda . \tag{12}$$

This form satisfies the condition of T_e' (x=0)=0, But it tends to differ from a Gaussian form with increasing a radial position. Moreover if we choose $\Lambda \sim 1$, its thermal conductivity is a little larger than the experimentally estimated value which tells that the preferable order of Λ is $0(10^{-1}) \sim 0(10^{-2})$.

Here, we are betrayed by both of Eq.(6), which matches the ordering of χ_e but not good at x=0 with $T_e' \neq 0$, and Eq.(10), which matches well with T_e' (at x=0)=0 but differs in the ordering. Moreover, both have different forms from a Gaussian. Despite of these obstacles, the author cannot easily discard the form of Eq.(6), becasuse of the great merit of the integrability. Thus, we investigate the form of Eq.(6), disregarding the physical consideration for the time being. In the first place we consider the combination of Eqs.

(6) and (10) with the expectation of each merit of each form preserved. Then we solve Eqs.(7) with the heat diffusivity of

$$\chi_e \sim \frac{C^2}{\omega_{pe}^2} \cdot \frac{V_e}{qR} \quad (\varepsilon + \Lambda) \tag{13}$$

to obtain easily

$$(T_e/T_{eo})^{3/2} = 1 - \beta \ \{x - \alpha \ \log(1 + \frac{x}{\alpha})\} \ . \tag{14}$$

This profile is presented in Fig.1 which tells that the gradient at x=0 is zero and the profile is similar with Eq.(11) near $x \sim 0$ and gradually tends to the straight line of Eq.(8) as the radial position increases. The profile looks more like a Gaussian than the other two (Eqs.(8) and (11)). But near the edge ($x \sim 1$), the dissimilarity enlarges, because the gradient of a Gaussian gradually decreases to zero beyond a certain value of x while Eq.(14) keeps the finite gradient of β. But except for the edge region, similarity is satisfactory in a qualitative sense. Then we proceed to the quantitaive study. At first we investigate the region near the axis ($x \sim 0$). At $x \ll 1$, a Gaussian of Eq.(1) can be approximated to be

$$(T_e/T_{eo})^{3/2} = 1 - \frac{3}{2} \alpha_t x^2 \ , \tag{15}$$

while Eq.(14) can be approximated to be Eq.(11) near the axis. The two equations have the same form completely and if we assume the relation of

$$\frac{3}{2} \alpha_t \sim \frac{\beta}{2\alpha} \ , \tag{16}$$

both equations coincide with each other. But the condition of Eq.(16) cannot be analysed any more unless the physical background of Λ, which is introduced temporarily, is made clear. Therefore, we return to the meaning of Λ.

We start with the examination of electron transport equation (7). To be more exact, we add an electrn-ion energy exchange term (Q_{ei}),

$$Q_{ei} = \frac{3m_e}{M} n_e \ \nu_e \ T_e(1 - \frac{T_i}{T_e}) \ . \tag{17}$$

The resultant equation is

$$\frac{1}{r}\frac{d}{dr}\left(-rn_e\chi_e\frac{dT_e}{dr}\right)=JE-Q_{ei}\ . \qquad (18)$$

As Eq.(17) includes ion temperature (T_i), we have to solve the ion transport equation simultaneously;

$$\frac{1}{r}\frac{d}{dr}\left(-rn_i\chi_i\frac{dT_i}{dr}\right)=Q_{ei}\ . \qquad (19)$$

In these two equations (18) and (19), two new terms (χ_i, Q_{ei}) appear. The Q_{ei} term is a little difficult to treat since it contains the term of $(1-T_i/T_e)$ which is proportional to the difference between T_e and T_i. If T_e is nearly equal to T_i (this often occurs in the high density regime), this term becomes nearly zero, while the other term of $n_e\ \nu_e$ in Eq.(17) becomes very large in the high density regime, (proportional to the square of density). Therefore the Q_{ei} cannot be estimated easily. Next we consider on χ_i. It is evident in the present situation that we should take two kinds of the contribution of ion heat diffusivity, one of which is the contribution from the electromagnetic turbulence corresponding to Eq.(6) of electron side (χ_i^{em}). The other one is the contribution from neoclassical Coulomb Collision (χ_i^{Neo}) (the neoclassical χ_e can be neglected for its smallness). With regard to χ_i^{em} we have no exact knowledge and there have not been any papers treating χ_i with an equal footing with electron side. But if we take into account some works[14, 9, 10] concerning to the ion contribution to this type of turbulence, we can write χ_i^{em} in the following manners,

$$\chi_i^{em}\sim\alpha_{ii}\chi_e^{em}\sim\ \alpha_{ii}\frac{C^2}{\omega_{pe}^2}\cdot\frac{V_e}{qR}\quad\varepsilon^{\alpha_o}\ , \qquad (20)$$

where α_{ii} is introduced as an ion factor assuming the order of O(1). As for χ_i^{Neo}, we only consult with one of many works to adopt a complete plateau χ_i for its simplicity (later we add the effect of collisions),

$$\chi_i^{Neo}\sim\ \frac{\rho_i^2\ V_i}{R}q\ . \qquad (21)$$

Thus, we may have a resultant χ_i as follows ;

$$\chi_i \sim \chi_i^{em} + \chi_i^{Neo}$$

$$\sim \alpha_{i1} \frac{C^2}{\omega_{pe}^2} \cdot \frac{V_e}{qR} \varepsilon + \frac{\rho_i^2 V_i}{R} q , \qquad (22)$$

where α_0 is taken to be unity. As for Eq.(22) the basic question should be noticed here whether the addition of turbulent χ_i^{em} and neoclassical χ_i^{Neo} can be justified. Addition may be right if the interaction between Coulomb collision and turbulent collision is too small to produce a cross-term like χ_i^{em-Neo}. But the problem should be considered on the rigid theoretical basis when the nonlinear turbulence behavior is fully made clear. In this article, therefore, we adopt Eq.(22) which seems a priori natural. Then we return to Eqs.(18) and (19) to solve. These two equatins, however, have two unknowns (T_e and T_i) and a delicate term (Q_{ei}). We eliminate Q_{ei}, by adding the two equations to obtain

$$- \frac{1}{r} \cdot \frac{d}{dr} [rn \{\chi_e \frac{dT_e}{dr} + \chi_i \frac{dT_i}{dr} \}] = JE . \qquad (23)$$

For the preference of simplicity, we assume

$$T_i = \tau T_e , \qquad (24)$$

where τ is assumed independent of the radial position in order to obtain an approximate T_e-profile for the first place. After these preparations, Eq.(23) becomes electron transport equation of

$$- \frac{1}{r} \cdot \frac{d}{dr} [rn(\chi_e + \tau \chi_i) \frac{dT_e}{dr}] = JE , \qquad (25)$$

where

$$\chi_e + \tau \chi_i = (1 + \tau \alpha_{i1}) \frac{C^2}{\omega_{pe}^2} \cdot \frac{V_e}{qR} \varepsilon + \tau^{5/2} \frac{\rho_e^2 C_e}{R} q . \qquad (26)$$

With the use of the relation of

$$\rho_s \frac{\omega_{pe}}{C} = \sqrt{\beta_e \frac{M}{m_e}} , \qquad (27)$$

we can rewrite the second term of Eq.(26) as

$$\tau^{5/2}\frac{\rho_s^2 C_s}{R}q=\tau^{5/2}\frac{C^2}{\omega_{pe}^2}\cdot\frac{V_e}{qR}\left(\beta_e q^2\sqrt{\frac{M}{m_e}}\right) \quad (28).$$

Equation (26) can be rewritten to be

$$\chi_e+\tau\chi_i=(1+\tau\alpha_{i1})\frac{C^2}{\omega_{pe}^2}\cdot\frac{V_e}{qR}(\varepsilon+\Lambda_i)\,, \quad (29)$$

where

$$\Lambda_i=\frac{\tau^{5/2}}{1+\tau\alpha_{i1}}\beta_e q^2\sqrt{\frac{M}{m_e}}\ . \quad (30)$$

Eqs.(25) and (29) have the completely same form with Eqs.(7) and (13) whose solution we already have for Λ=constant. In the case of Eqs.(25) and (29), the condition of Λ_i=constant depends on the term ($\beta_e q^2$) in Eq.(30). Fortunately, the relation of $\beta_e q^2$=constant holds in the natural current profile theory [2-4] of Kadomtsev [4], although this relation is only justified when the total energy is determined by the magnetic and thermal stored energies. Moreover, reflecting upon the usual experimental results, we see that β_e is a decreasing function of x and q^2 an increasing function so that the multiplied ($\beta_e q^2$) is expected to be a very weak function of x. However, near the edge region where the other energy sources and sinks exist it cannot be applied.Therefore, we can regard Λ_i a constant in the inner region ($x\lesssim\frac{1}{2}\sim\frac{2}{3}$).

Thus, the physical background of Λ in Eq.(13) is the contribution from a neoclassical ion diffusivity (but this conclusion is to be modified afterwards). The numerical order suggested by Eq.(30) is, though it depends on β_e-value, $O(10^{-1})\sim O(10^{-2})$ in usual experimental conditions. This agrees well with the aforementioned implication. Then at this point, we can evaluate Eq.(16).

We rewrite β of Eq.(9), where we attach here the factor $(1+\alpha\tau_{i1})$ in the denominator in accord with Eq.(29), using only the universal relations, to obtain

$$\beta = \frac{a^2}{\frac{C_{s0}\rho_{s0}^2 q_0}{R}} \cdot \frac{V_p I_p}{\frac{3}{2} nT_e V} \cdot \frac{\int_0^r JdS}{J_0 \pi r^2} \cdot \frac{T_{e0} q_0}{T_e q} \cdot$$

$$\frac{R}{a} \left[\frac{q_a}{q_0} \beta_e q^2 \left(\frac{M}{m_e} \right)^{1/2} \right] \cdot \frac{9}{8} (1 + \tau \alpha_{it})^{-1} \qquad (31)$$

where V_p is the one-turn voltage, V is the plasma volume, and the suffix "0" means the value at x=0. Therefore, with use of $\alpha = (R/a)\ \Lambda_i$, and Eq.(30), we obtain

$$\frac{\beta}{2\alpha} = \underbrace{\frac{a^2}{\tau^{5/2} \frac{C_{s0}\rho_{s0}^2 q_0}{R}}}_{①} \cdot \underbrace{\frac{V_p I_p}{\frac{3}{2} nT_e V}}_{②} \cdot \underbrace{\frac{\int_0^r JdS}{J_0 \pi r^2}}_{③} \cdot \underbrace{\frac{T_{e0} q_0}{T_e q}}_{④} \cdot \underbrace{\frac{9}{16}}_{⑤} \cdot \underbrace{\frac{q_a}{q_0}}_{⑥} \qquad (32)$$

where the term ① represents $a^2/\chi(x=0)$ which can be $\sim \tau_E$ and the second term ② is evidently $\sim \tau_E^{-1}$ from the definition and the terms③∼⑤ are nearly unity. Therefore Eq.(32) can be order of the last term ⑥ which means $(3/2)\ \alpha_t$;

$$\frac{\beta}{2\alpha} \sim \frac{q_a}{q_0} \sim \frac{3}{2} \alpha_t \ . \qquad (33)$$

Although this estimation cannot present an exact proof of Eq.(16) since the terms (①, ②, ③, ④, and ⑤) is order O(1), namely the same order with q_a/q_0, the order of $(\beta/2\alpha)$ is found consistent with the order of (q_a/q_0).

To sum up the arguments here, we can say that if we set $T_i = \tau T_e$, the ion and electron transport equations merge into the same form with Eq.(7) of electron transport, and with ion heat diffusivity (χ_i) from the electromagnetic turbulence and the neoclassical coulomb collision, we have the same profile of Eq.(14). This result is very interesting. Then we proceed to examine the ion transport in which we must consider Q_{ei}. The left hand side of ion transport Eq.(19) is, with account of Eq.(22), substantially the same with that of electron transport Eq.(7). But the right hand sides of Eqs.(19) and (7) are utterly different. Therefore, we have to examine the behavior of Q_{ei} in the right hand side of Eq.(19). When taking out the important factors in Q_{ei}, we can write Q_{ei} as

$$Q_{ei} \propto \frac{n_e^2}{T_e^{1/2}} (1-T_i/T_e) \ , \qquad (34)$$

where we extract the dependences of n_e and T_e in ν_e. In this relation, a strong dependence of n_e(square of n_e) is contained. Therefore we need to investigate the particle transport which determines the density profile

$$\frac{1}{r} \frac{d}{dr} (r\Gamma) = S_p \ , \qquad (35)$$

where Γ is a particle flux and S_p is a particle source.

If we limit our discussions in the inner region of plasma ($x \lesssim \frac{1}{2} \sim \frac{2}{3}$), we can approximate as $S_p \sim 0$ which is easily conceivable in the usual OH-experiments. On this account, Eq.(35) can easily reach the relation of

$$\Gamma \sim 0 \ , \qquad (36)$$

which means that the outward flow (Γ_{out}) must be equilibrating with the inward flow (Γ_{in}), namely,

$$\Gamma = \Gamma_{out} - \Gamma_{in} \sim 0 \ . \qquad (37)$$

Therefore, if we define Γ_{out} with a diffusion coefficient (D) and the density gradient, as often done, by

$$\Gamma_{out} = -D \frac{dn}{dr} \qquad (38)$$

then we must consider the balanced inward flux whose existence is very important as pointed out by Coppi [1]. He and his co-worker proposed [17] the inward flow resulting from ion-temperature gradient and many other workers [18] proposed several types of the inward flow. Neglecting the neoclassical Ware pinch effect (which is too small in our situation considered here), we assume that the inward flux is proportional to electron temperature gradient, and can be written to be

$$\Gamma_{in} = -\delta D \cdot \frac{n_e}{T_e} \cdot \frac{dT_e}{dr} \tag{39}$$

where δ is assumed a positive constant. We should have taken into account the contribution from ion temperature gradient and some other contribution from turbulences. But those are still open questions now, thus it should be rather regarded in Eq.(39) that those unknown elements are all packed into δ. The assumption of δ=constant may be too bold to take. Thus we limit our discussions in a qualitative sense for obtaining only a crude physical prospect. On these assumptins, we obtain the relation at once,

$$\frac{n_e}{n_{eo}} = \left(\frac{T_e}{T_{eo}} \right)^{\delta} \tag{40}$$

Then we return to the relation of (34). With use of Eq.(40), Q_{ei} becomes

$$Q_{ei} \propto T_e^{2\delta - \frac{1}{2}} (1-\tau) \tag{41}$$

When $\delta=1$ (which means $n_e \propto T_e$), the right hand side is proportional to $Te^{3/2}$ and, if a thermal conductivity obeys Spitzer-Härm relation, this term (Q_{ei}) substancially has a same parameter dependence with JE. This means Eq.(19) has a same structure with Eq.(7). Upon this point, we recall "Profile Consistency" on density claimed by Coppi [1], who says that when n_e is low, the density profile becomes flat and behaves like $n_e \propto T_e^{\delta}$ ($\delta \ll 1$), and as n_e becomes higher, the profile changes to a similar form with T_e-profile, that is, $n_e \propto T_e$ (which corresponds to $\delta=1$). With regard to T_i-profile, "profile consistency" never gives any mentions, and our arguments developed above cannot be moved ahead any more because of the intricacy of Q_{ei}.

Here we reconsider the several assumptions adopted so far. In the the first place, the assumption of $T_i = \tau T_e$ is only employed for the simplicity of modelling and this assumption does not contradict with the actual experimental results. But it is not needed in the numerical calculations [18]. Second, we neglect the various loss terms (radiation, ionization, charge exchange etc.) in the transport equations. This corresponds to that those terms are assumed much smaller than JE or Q_{ei} in OH-plasmas. This situation also does not contradict with actual experimental results. Those are also to be included in the numerical caculations [18]. In the third place, the assumption of a complete plateau is adopted for the neoclassical ion heat diffusivity. If we take into account the collision effect in χ_i^{Neo}, we add into Eq.(21) the factor $K_2\ \nu_{*i}$ in the following manners,

$$\chi_i^{Neo} \sim K_2 \nu_{*i} \frac{\rho_i^2 \nu_i}{R} q \tag{42}$$

where ν_{*i} is an effective ion collisionality and K_2 is a numerical coefficient introduced by Hinton and Hazeltine [19] and modified by Chang and Hinton [20] later.

This factor is order of O(1) but changes relatively largely near the axis in the usual plasma parameters. Therefore, we should rewrite Eq.(30) as follows ;

$$\Lambda_i = \frac{\tau^{5/2}}{1+\tau\alpha_i} K_2 \nu_{*i}\ \beta_e q^2 \sqrt{\frac{M}{m_e}} \tag{43}$$

and it cannot be treated a constant near the axis. In this case, however, with taking into account that $K_2 \nu_{*i}$ is a decreasing function near the axis and ε is an increasing function, we can redefine Λ_i to be

$$\Lambda_i = \tau \alpha_{i1} \varepsilon + K_2 \nu_{*i} \tau^{5/2} \beta_e q^2 \sqrt{\frac{M}{m_e}} \tag{44}$$

which can be a weak function of x. And $(\chi_e + \tau\chi_i)$ is also modified to be

$$\chi_e + \tau\chi_i = \frac{C^2}{\omega_{pe}^2} \cdot \frac{V_e}{qR} \ (\varepsilon + \Lambda_i) \tag{45}$$

where Λ_i is Eq.(44).

In the fourth place, we assumed that the energy flow is conductive and neglected the convective contribution $T_e\Gamma$ or $T_i\Gamma$. It should be justified when we consider only the inner region where $\Gamma \sim 0$ is assured in the usual situations. But in the outer region, our model should be also modified not only by this Γ-effect, but some other effects resulting from the changes of plasma parameters.

Then let's enter in the discussions on the outer region. Our profile suggested by the model is shown in Fig.1, only dissimlar with a Gaussian in the outer region. Is the electromagnetic turbulence still dominant in the outer region? It is not, because the condition of $k_{\parallel}\lambda_e > 1$ ($k_{\parallel}$: a wave number along the magnetic field, λ_e : electron mean free path) for the electromagnetic turbulence, is not satisfied in the outer region in case of usual plasma parameters. In case of $k_{\parallel}\lambda_e < 1$, the main turbulence is replaced by the electrostatic type of turbulence, i.e. drift wave type turbulence is most conceivable. As a representative of this type of turbulence, we take the dissipative drift wave turbulence which is most plausible and has a large effect. The author [21, 22] has already proposed the form of the diffusion coefficient (D) of this type of turbulence ;

$$D \sim \frac{\rho_s^2 C_s}{r_n} R_e^{\frac{1}{R_e + \beta_o}} \qquad (46)$$

where β_o is a numerical factor around 0～0.5 and R_e is introduced as a turbulence factor and written to be

$$R_e \sim \langle k_\theta \rho_s \rangle^{-1} \frac{C_s}{r_n} \cdot \frac{\nu_e q^2 R^2}{V_e^2 \hat{S}^2} \qquad (47)$$

in case of a dissipative drift wave turbulence. This type of diffusion coefficient is an increasing function of x in the usual plasma parameters, that is, negligibly small in the inner region and grows swiftly larger as the radial position comes nearer the edge. This fact implies that the gradient of T_e-profile flattens more than the one shown in Fig.1 in the outer region and the profile becomes resembling with a Gaussian in the entire region since the difference between the form of Eq.(14) and a Gaussian in the outer region can be easily fulfilled by the addition of a large χ_e in the outer region in the energy transport equation of Eq.(7). But in the outer region, we should take into account the other various effects such as ionization loss, radiation loss, or convective loss, since the main term of JE itself becomes small and some other effects cannot be neglected relatively. Thus any more qualitative analysis cannot be done in case of the near-edge region. Therefore the numerical calculation should be done. In this article, in order to limit the purpose only to ascertain the justificalion of the model, the situation is set to be as simple as possible. Thus we consider only electron energy transport and particle transport, neglecting ion transport by assuming $T_i = \tau T_e$ and solve Eq.(35) and the folowing equations,

$$\frac{1}{r} \frac{d}{dr} (rQ_e) = JE - Q_{ei} - W_R - W_I \qquad (48)$$

where

$$Q_e = -n_e \chi_e \frac{\partial T_e}{\partial r} + T_e \Gamma \qquad (49)$$

$$\chi_e = \frac{C^2}{\omega_{pe}^2} \frac{V_e}{qR} \varepsilon + \frac{\rho_s^2 C_s}{r_n} R_e^{\frac{1}{R_e + \beta_o}} \qquad (50)$$

and W_R is assumed bremstrahlung radiation loss and W_I is an ionization loss. As for S_p, we take

$$S_p = n_e n_g <\sigma V>_{ion} \qquad (51)$$

where $<\sigma V>_{ion}$ is a cross-section of ionization and n_g is a neutral gas density, assumed to be accumulated in the e-folding length near the edge. As for Γ, we take

$$\Gamma = -D \frac{dn_e}{dr} + \delta D \cdot \frac{n_e}{T_e} \cdot \frac{dT_e}{dr} \qquad (52)$$

where D is assumed equal to χ_e and δ is taken to be a parameter. As an example, we take the Alcator parameters [1] and show the results in Fig.2 where the profile is compared with a Gaussian and found very similar with it. In the case of Fig.2, we put $\delta=0.4$, although the change of this parameter affects the n_e-profile, while T_e-profile remains unchanged. For the comparison, we present the result in Fig.3 which is computed with the neglection of the second term of Eq.(50). Both figures show simply that the second term of Eq.(50) makes the profile more similar to a Gaussian than Fig.1 in the outer region.

Up to this point, our model is approximately fulfilled in the entire region and the total χ_e should be written to be

$$\chi_e \sim \frac{C^2}{\omega_{pe}^2}\cdot\frac{V_e}{qR}\ \varepsilon + \frac{\rho_s^2 C_s}{r_n}\ R_e^{\frac{1}{R_e+\beta_o}} + \chi_e^{Neo} \tag{53}$$

where the neoclassical electron contribution is included only for formality. And the corresponding χ_i should have the form of

$$\chi_i \sim \alpha_{i1}\ \frac{C^2}{\omega_{pe}^2}\cdot\frac{V_e}{qR}\ \varepsilon + \alpha_{i2}\ \frac{\rho_s^2 C_s}{r_n}\ R_e^{\frac{1}{R_e+\beta_o}} + \chi_i^{Neo} \tag{54}$$

where parameters (α_{i1}, α_{i2}) of order O(1) cannot be helped to be attached. Thus, the effective χ_e in the energy transport equation (25) is

$$\chi_e^{eff} = \chi_e + \tau\chi_i$$

$$\sim \frac{C^2}{\omega_{pe}^2}\cdot\frac{V_e}{qR}\ (\varepsilon+\Lambda_i) + (1+\tau\alpha_{i2})\ \frac{\rho_s^2 C_s}{r_n}\ R_e^{\frac{1}{R_e+\beta_o}} \tag{55}$$

$$\sim \frac{C^2}{\omega_{pe}^2}\cdot\frac{V_e}{qR}\ \left\{\varepsilon+\Lambda_i+(1+\tau\alpha_{i2})\ \frac{R}{qr_n}\ R_e^{\frac{1}{R_e+\beta_o}}\right\} \tag{55$'$}$$

where

$$\Lambda_i = \tau\alpha_{i1}\varepsilon + K_2\nu_{*i}\tau^{5/2}\ \beta_e q^2\sqrt{\frac{M}{m_e}} \tag{56}$$

By obtaining these relations, at first we can consider the case of a low density plasma in which $\beta_i q^2$ ($\beta_e q^2$ is replaced by $\beta_i q^2$ from the consideration of the origin of neoclassical ion contribution) is very small. In other words,

$$K_2 \nu_{*i} \beta_i q^2 \sqrt{\frac{M}{m_e}} \ll \langle \varepsilon \rangle \tag{57}$$

where $\langle \ \rangle$ means the average of $\varepsilon = r/R$, and $\langle \varepsilon \rangle \sim \frac{1}{2}(a/R)$. In this case the second term of Eq.(55) (the electrostatic turbulence) is very small in the inner region ($x < \frac{1}{2} \sim \frac{2}{3}$) since a low density plasma is usually a high electron temperature plasma and R_e is very small. Thus χ_e^{eff} in the inner region is determined by the ε-term in Eq.(55). Therefore, as it can be said that the stored energy is approximately determined by the energy in the inner region, ($x < \sim \frac{2}{3}$), the effective energy confinement time $\langle \tau_E \rangle$ can be approximated to be

$$\langle \tau_E \rangle \sim \frac{a^2}{\chi_e^{eff}} \sim \frac{a^2}{\langle (1+\alpha_{i1}\tau) \frac{C^2}{\omega_{pe}^2} \frac{V_e}{qR} \varepsilon \rangle} \propto \bar{n}_e a R^2 \langle q \rangle \tag{58}$$

which leads to the Neo-Alcator scaling of τ_E. In case of a high density plasma of a large $\beta_i q^2$, on the contrary, i.e.

$$K_2 \nu_{*i} \beta_i q^2 \sqrt{\frac{M}{m_e}} \gg \langle \varepsilon \rangle . \tag{59}$$

χ_e^{eff} is determined both by the neoclassical contribution of Λ_i-term (Λ_i^{Neo}) and the electrostatic term, thus $\langle \tau_E \rangle$ can be written to be

$$\langle \tau_E \rangle \sim \frac{a^2}{\chi_e^{eff}} \sim \frac{a^2}{\langle \frac{C^2}{\omega_{pe}^2} \cdot \frac{V_e}{qR} \Lambda_i^{Neo} + (1+\tau\alpha_{i2}) \frac{\rho_s^2 C_s}{r_n} R_e \frac{1}{R_e + \beta_o} \rangle} \tag{60}$$

In this case, we cannot neglect the second term in Eq.(55) since a high density plasma is usually tantamount to a low electron temperature plasma so R_e is not so low to neglect. With use of the relation (27), we can rewrite Eq. (60) into

$$\langle\tau_E\rangle \sim \frac{a^2}{\chi_e^{eff}}$$

$$\sim \frac{a^2}{\langle \frac{C^2}{\omega_{pe}^2}\cdot\frac{V_e}{qR}\ \beta_e q^2\sqrt{\frac{M}{m_e}}\ \{\tau^{5/2} K_2 \nu_{*i} + (1+\tau\alpha_{i2})\frac{R}{qr_n} R_e^{\frac{1}{R_e+\beta_o}}\}\rangle} \tag{61}$$

where { } -part in the denominator is very difficult to treat since the turbulence part ($R_e^{\frac{1}{R_e+\beta_o}}$) is usually a rapidly increasing function of x until R_e reaches ~1. But it should be noticed here that the { } - part is possible to be settled to be a moderate function of x since $K_2\nu_{*i}$ is usually a decreasing function while the R_e-term is an increasing function.
Thus it can be said that $\langle\tau_E\rangle$ becomes independent of a density since the density dependence of (C^2/ω_{pe}^2) and $(\beta_i q^2)$ in the denominator is cancelled out. $\langle\tau_E\rangle$ in Eq.(61) is smaller than the one determined only by χ_i^{Neo}, the first term in the denominator, by the amount of the second term in the denominator, i.e, the contribution from the electrostatic drift wave turbulence. Therefore the results implied by Eq.(58) and (61) can interpret well the experimental results [23, 24] suggesting Alcator-Scaling. Also we can see the criterion separating the density dependence of $\langle\tau_E\rangle$ scaling in the above arguments. The criterion is

$$K_2\nu_{*i}\beta_i q^2\sqrt{\frac{M}{m_e}} \gtrsim \langle\varepsilon\rangle \tag{62}$$

namely

$$\beta_i^{cr} \sim \frac{\langle\varepsilon\rangle\sqrt{\frac{m_e}{M}}}{q^2 K_2\nu_{*i}} \tag{63}$$

If we assume the usual experimental fact of $q_o \sim 1$, and $\langle\varepsilon\rangle \sim \frac{1}{2}$ a/R, then the criterion becomes

$$\beta_{io}^{cr} \sim \frac{1}{2}\frac{a}{R}\sqrt{\frac{m_e}{M}}\Big/(K_2\nu_{*i}) \tag{64}$$

This result is very interesting, especialy with respect to the mass-number dependence though this result may be modified if the electrostatic turbulence makes a larger contribution than the neoclassical ion in case of a high density regime. Thus the criterion should be written more exactly in the following manners ;

$$\langle \varepsilon \rangle \gtrsim \frac{1}{1+\tau \alpha_{i1}} \{ K_2 \nu_{*i} \tau^{5/2} \beta_a q^2 \sqrt{\frac{M}{m_a}} + (1+\tau \alpha_{i2}) \frac{R}{q r_a} R_e^{\frac{1}{R_a+\beta_a}} \} \tag{65}$$

3. Concluding Remarks

Summarizing the arguments discussed in the previous section, we can say that the dominant turbulence is the electromagnetic one in the inner region and the electrostatic one in the outer region with the neoclassical ion contribution prevailing in the whole region. The boundary of the dominant turbulence between the electromagnetic and the electrostatic one is changed by the plasma parameters, especially the density. In case of a low density plasma, the dominant turbulence becomes the electromagnetic one, prevailing beyond $x \gtrsim \frac{1}{2} \sim \frac{2}{3}$, and the $\langle \tau_E \rangle$-scaling shows the n_e-proportionality. In case of a high density plasma, on the other hand, the dominant turbulence becomes the electrostatic one, with the electromagnetic one confined in the narrow region near the axis, and the $\langle \tau_E \rangle$-scaling becomes independent of n_e. With regard to the T_e-profile, we can reach Eq.(14) which is similar with a Gaussian form if the region is limited within $x < \frac{1}{2} \sim \frac{2}{3}$, and if we take into account the electrostatic contribution in the outer region, we can have the similar profile to a Gaussian by the numerical calculation. With regard to the n_e-profile, we only indicate the importance of the inward flow, yet to prove the claim by "Profile Consistency" as to the n_e-profile, for which the understanding of the particle transport is needed. As for the T_i-profile, we have to perform a numerical calculation for the intricacy of Q_{ei}-term, whose results, as well as other profiles, are to be done in the near future. The most important point of our model is the combination of the electromagnetic and electrostatic turbulences with the neoclassical ion effect. These intriguingly ingenious combinations may explain some other recent transport problems as well as "profile consistency". In the last place it should be noted that we develop the discussions by the informal way of starting with a simple model by adding some complexities one by one to reach a final appearance since the author want to envision the kernel part of the model as clearly as possible though the more rigid formulation can not be helped to be sacrificed and is to be left in the future work, in which the formal formulation is to be employed and the framework is to be more rigidly fixed and numerically examined.

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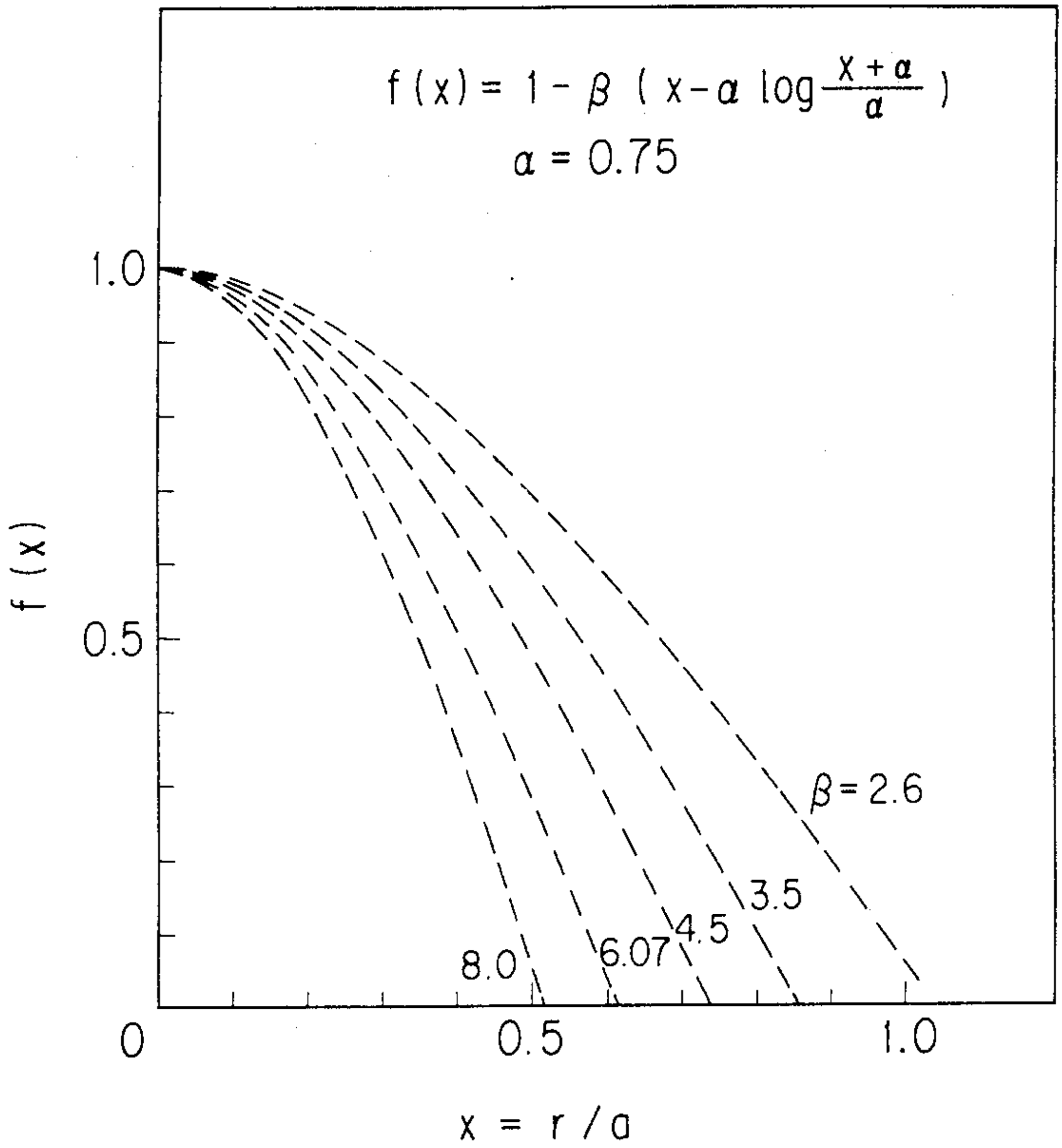

Fig. 1 ; $(T_e/T_\infty)^{3/2} = 1 - \beta \{x - \alpha \log (1 + \frac{x}{\alpha})\}$, $\beta = 2.6, 3.5, 4.5, 6.07, 8.0$ and $\alpha = 0.75$

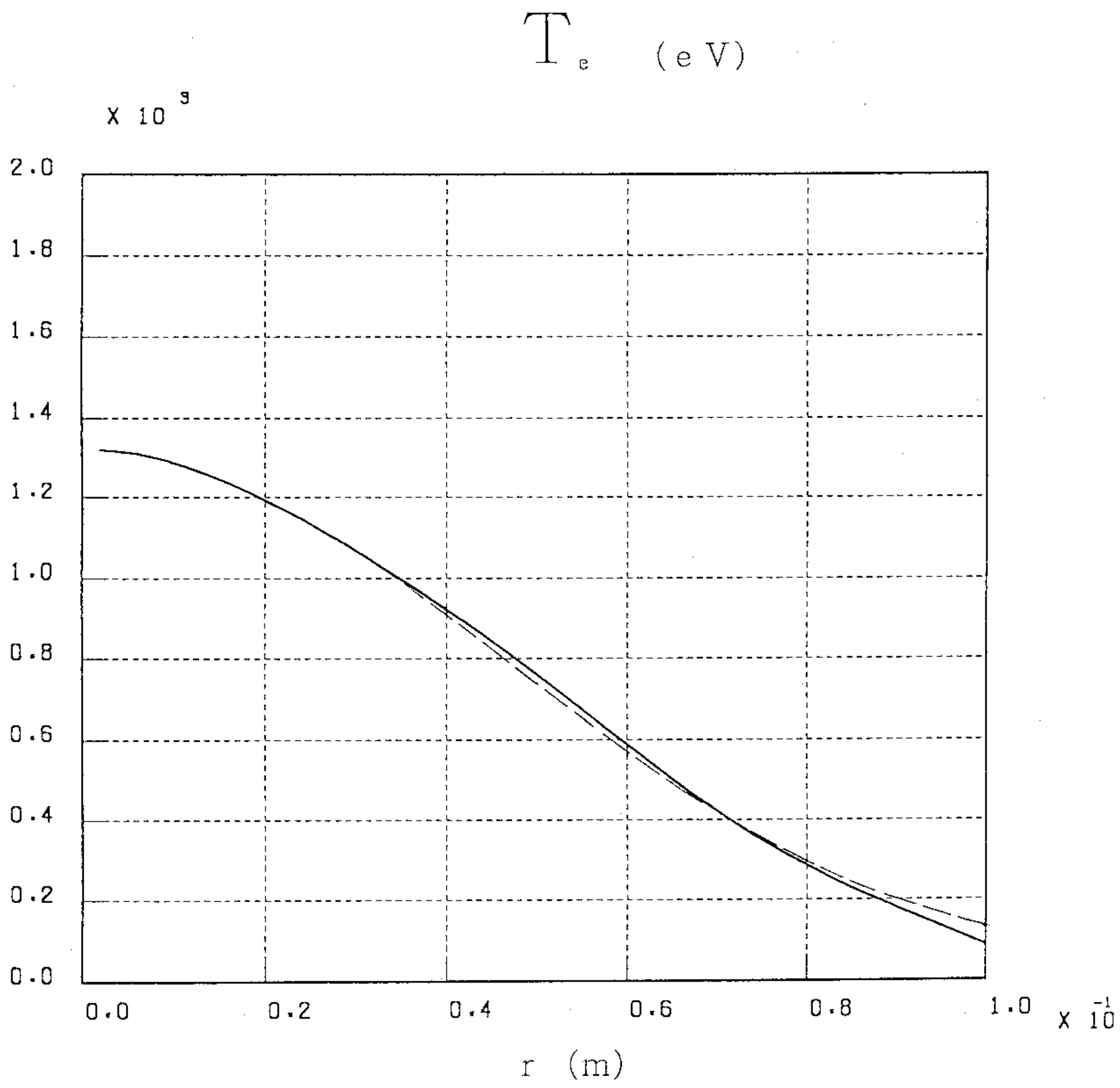

Fig. 2 ; T_e-profile computed from Eqs. (48) ～ (52) and (35) with the initial data of $T_{e0} = 1.32 \times 10^3$ eV, $n_{e0} = 1.9 \times 10^{20}$ m^{-3}, a = 0.1 m, R = 0.54 m, E = 0.7 V/m, B = 6T, $\tau = 0.9$ and $\delta = 0.4$. A broken line represents Eq. (1) with $\alpha_t = 2.31$.

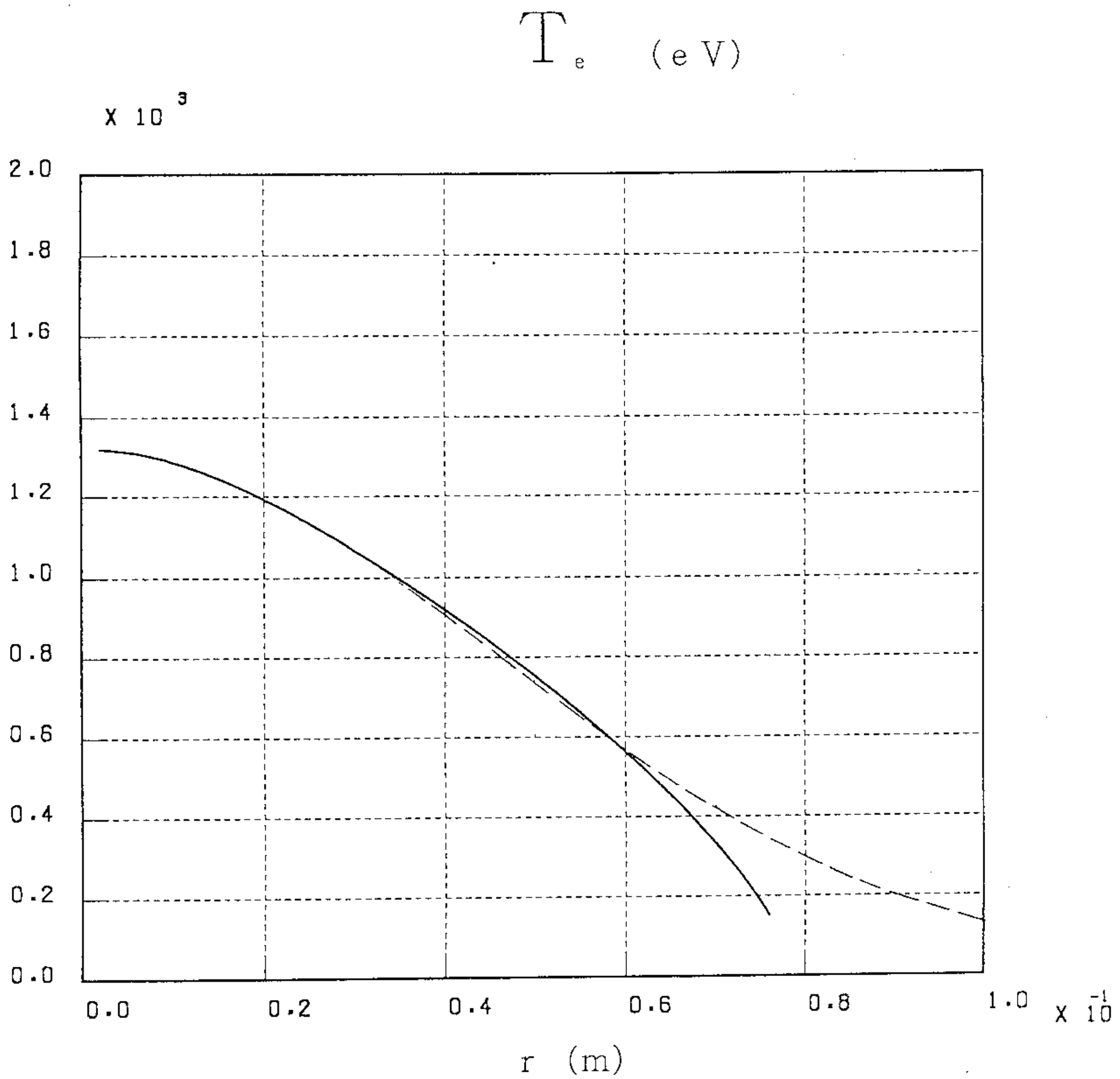

Fig. 3 ; T_e-profile computed from Eqs. (48) ～ (52) and (35) with the same initial data with the case of Fig. 2 except for the elimination of the second term of Eq. (50) . A broken line represents Eq. (1) with $\alpha_t = 2.31$.