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A SIMPLE AND RATIONAL NUMERICAL METHOD OF TWO-PHASE FLOW WITH VOLUME-JUNCTION MODEL,(Ⅰ)

—VERIFICATION CALCULATION IN SATURATED CONDITION—

November 1997

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編集兼発行　日本原子力研究所
印　　　刷　㈱原子力資料サービス

A Simple and Rational Numerical Method of Two-phase Flow with Volume-Junction Model,(I)

— Verification Calculation in Saturated Condition —

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(Received October 2, 1997)

A new numerical method to achieve a rigorous numerical calculation of each phase using a simple explicit method with volume-junction model is proposed. For this purpose, difference equations for numerical use are carefully derived so as to preserve the physical meaning of the basic equations. Specifically, momentum equations for the flow in the volume are newly derived to keep strict conservation of energy within the volume. To prove the validity of the numerical method and of previously proposed basic equations, including the original phase change equations, which were rigorously derived, some numerical calculations were made for each phase independently to examine the correctness of calculated results.

The numerical calculation is advanced by simple integration of an explicitly obtained solution of difference equations without any special treatment. Calculated results of density and specific internal energy of each phase for saturated two-phase blowdown behavior are consistent for two different solution scheme as described below. Further, no accumulation of error in mass or energy was found. These results prove the consistency among basic equations, including phase change equations, and the correctness of numerical calculation method. The two different solution schemes used were:1) solutions of pressure and void fraction in saturated condition were obtained by using mass conservation equation of each phase simultaneously, and 2) fluid properties were calculated directly from mass and energy conservation equation of each phase.

Keywords: Two-phase Flow Analysis Code, Phase Change, Numerical Method

Volume-Junction 法による二相流の単純で合理的な数値解法（Ｉ）
― 飽和状態における検証計算 ―

日本原子力研究所東海研究所原子炉工学部
岡崎　元昭

（1997年10月2日受理）

非定常気液二相流をVolume-Junction法によって気液各相について独立に解析する場合の、単純で合理的な数値解法を提出する。

その要点は、1) 差分形の基礎式を元の偏微分形のもつ物理的意味を保持する形で導いた。2) Volume内エネルギ保存を確実に達成するためVolume内の運動量平衡式を新しく導出した。3) 流れに沿う減圧過程で気液各相に独立に生ずる相変化の理論式を用いた。4) 陽解法による数値積分法により解を求めた。

提出した手法を検証するため、本報では、計算例として減圧により相変化する飽和二相流の圧力、ボイド比変化を求めると共に、気液各相について独立に、質量変化、エネルギ変化を求め、そこから得られる状態量が飽和条件のものであること、質量及びエネルギに対する誤差が蓄積していかないことを確認した。これらの結果から、基礎式と相変化式の整合性を数値的に確認した。

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1. Introduction

In the previous report(1), equation of phase change in the decreasing pressure of two-phase flow were rigorously derived using the first law of thermodynamics with consideration that evaporation from saturated liquid and condensation from saturated vapor takes place independently in each phase. What the equation of the first law of thermodynamics written for an open system generally expresses is described in the APPENDIX II.1. It is shown that the equation of the first law of thermodynamics for the open system can be considered to be one type of an energy equation concerning the state change in the flow and is derived by using the conservation equation of mass and total energy, and a momentum balance equation. So, if three out of the four equations above are given, the other can be derived. Basic equations were presented based upon these phase change equations. The conservation equations of mass and momentum of each phase, and the total energy conservation equation of the two-phase mixture are first derived. Using these basic equations and the phase change equations, the frictional heat terms for the two-phase flow are derived which are the terms contributing to the entropy increase in the relational expression of thermodynamic state change. The energy conservation equation of each phase was derived by combining phase change equations, conservation equations of mass and momentum of each phase, and aiming at consistency with phase change equations in the saturated condition.

The energy equations used for analyzing 2-phase flow in current systems codes such as TRAC or RELAP5, are utilized by eliminating the kinetic energy terms from the total energy equations. However, the energy dissipation terms due to friction in the flow are missing in these energy equations. Moreover the phase change correlations in the codes cannot describe the essential phenomena of all types of phase change

(cf. APPENDIX II.2).

As for the theoretical equation of phase change, E.D. Hughes etc.(4) has proposed the flashing model which presents the equation of the net phase change rate in a two-phase flow mixture. This equation is basically the same as the one proposed by the authors in the previous report(1) as far as the net phase change rate in an isentropic process is concerned. In the previous paper, phase change is considered to take place in each phase independently with the consideration of entropy increase due to friction and external heat in the two-phase flow. In the paper presented by E.D. Hughes etc.

exchanging of heat $q_{g,int}^{\Delta P}$ and $q_{l,int}^{\Delta P}$ in each phase, is recognized(5) in the flashing model. But, it is not related to the phase change which should occur in each phase. Exchanging of heat in each phase is also not considered independently in the energy equation of each phase(6). Only the net heat exchange which is carried by the net phase change in the two-phase flow mixture is considered in the energy equation of each phase. Therefore, some unreasonable results should have been obtained for the thermal-hydraulic behavior of each phase in the saturated condition by using these basic equations of each phase.

Actually, fractions F_{gs}, F_{ls} taking part in flashing are used for controlling the heat exchange in each phase(7). These fractions control only the net phase change rate and do not control the heat exchange rate in each phase since $q_{g,int}^{\Delta P}$ and $q_{l,int}^{\Delta P}$ are not considered in each phase. So, these fractions would not be effective for controlling the state change of each phase.

The general problems in the numerical method with the volume-junction model are outlined as follows. As for the problem of the number of unknowns and the number of equations in the analysis of the saturated condition of two-phase flow, a system of partial differential equations differs from a system of difference equations for a volume-junction model.

In the partial differential equations :

Unknowns : $u_g, u_l, P, \alpha, w_{gl}, w_{lg}$. Total 6

Equations : 2-Mass conservation,2-Momentum balance,2-Energy conservation. Total 6

For the energy equations, either of total energy conservation equation or the equation of the first law of thermodynamics of each phase is available. These equations are consistent with each other among the mass conservation and momentum balance equations. Actually, phase change terms $w_{g,l}$ and $w_{l,g}$ are expressed by the equation of the first law of thermodynamics of each phase, respectively.

In the volume-junction model :

Unknowns : - for Junction flow : $W_{g,J}, W_{l,J}, \dot{M}_{ZE}, \dot{M}_{ZC}$

- for the fluid in Volume : $P, \alpha, \dot{M}_{tE}, \dot{M}_{tC}, W_{g,vol}, W_{l,vol}$. Total 10

Equations : - for Junction flow : 2-Momentum balance, 2-First law of thermodynamics.

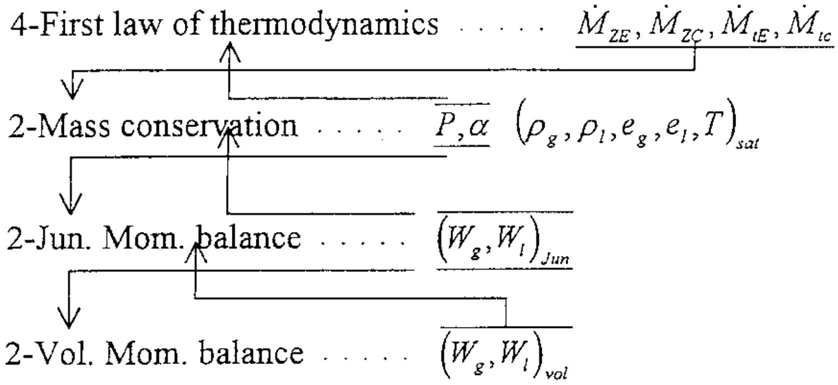
- for the fluid in Volume : 2-Mass conservation, 2-Total energy conservation, 2-First law of thermodynamics. Total 10

The basic equations for the numerical use with volume-junction model should be carefully derived so as to conserve the physical meaning of the original basic equations. Equations of mass conservation and the first law of thermodynamics are easily obtained to satisfy this requirement. For the energy equation, the accounting of incoming and out going energy within the volume must be derived first to meet the physical law. The elimination of kinetic energy terms may be made after that, if necessary, by using the equations of mass conservation and momentum balance, which are formed for the flow in the volume (cf. APPENDIX II.3). The momentum balance equations for the flow in the volume can be obtained by using the equations of mass, total energy and the first law of thermodynamics, because three out of four equations momentum balance equation obtained above, the equations of phase change expressed by the first law of thermodynamics and the conservation equations of total energy become consistent with each other. It means that the kinetic energy in the volume is are given as described before. By defining the flow rate in the volume with the determined by conservation of total energy.

It is also important for correct evaluation of the junction flow rate to determine the flow rate in the volume rigorously, since the momentum change from the flow in volume to the flow at the junction significantly affects the evaluation of junction flow rate in the momentum balance equation.

In order to derive the momentum balance equations for the volume such that they conform to the physical laws of dynamics, the total energy conservation of each phase must be expressed considering phase change and state change in the junction flow. The rate of energy flow into the volume is the same whether the phase change in the junction flow is considered or not. But the influence to the velocity change of each phase in the volume varies because the momentum balance equation in the volume is derived incorrectly if the phase change in the junction flow is not considered (cf. APPENDIX. II.3). Finally, it is very important that the physical meaning of basic equations expressed in the partial differential form, should be strictly preserved when they are converted into difference equations for numerical use with the volume-junction model.

The outline of how the unknowns are solved in the analysis of saturated two-phase flow is explained as follows.



4-First law of thermodynamics ····· $\dot{M}_{ZE}, \dot{M}_{ZC}, \dot{M}_{tE}, \dot{M}_{tc}$

2-Mass conservation ····· P, α $(\rho_g, \rho_l, e_g, e_l, T)_{sat}$

2-Jun. Mom. balance ····· $(W_g, W_l)_{Jun}$

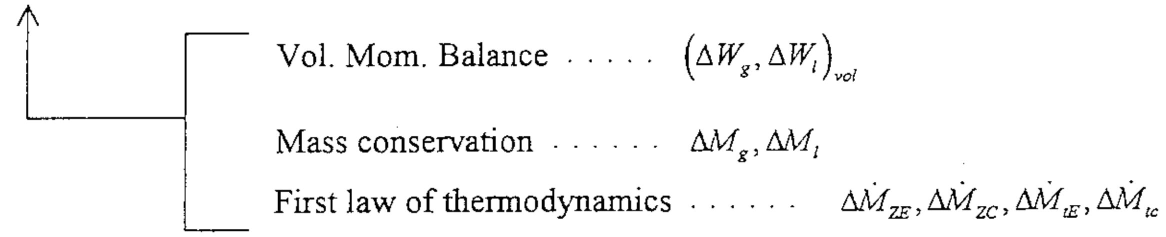
2-Vol. Mom. balance ····· $(W_g, W_l)_{vol}$

The phase change rate in junction flow $\dot{M}_{ZE}$, $\dot{M}_{ZC}$ and in volume $\dot{M}_{tE}$, $\dot{M}_{tC}$ are obtained by the first law of thermodynamics expressed for each phase. Pressure P and void fraction α in the volume are obtained by solving the mass conservation equation of each phase simultaneously. The Junction flow rate is obtained by using the momentum balance equation for the junction flow along with the momentum change from volume to junction, and with the pressure difference between volumes. The volume flow rate is determined by using the momentum balance equation. The updated values are used in the equations designated by arrows.

Saturated states of pressure P such as $(\rho_g, \rho_l, e_g, e_l, T)_{sat}$ are given by using the steam table. On the other hand, the density and the specific internal energy of each phase in the volume can be determined through another way as follows.

Mass conservation of each phase and α ····· ρ_g, ρ_l

Total energy conservation of each phase and M_g, M_l ····· e_g, e_l



Vol. Mom. Balance ····· $(\Delta W_g, \Delta W_l)_{vol}$

Mass conservation ······ $\Delta M_g, \Delta M_l$

First law of thermodynamics ······ $\Delta\dot{M}_{ZE}, \Delta\dot{M}_{ZC}, \Delta\dot{M}_{tE}, \Delta\dot{M}_{tc}$

The mass of each phase M_g, M_l in volume is determined by integrating ΔM_g, ΔM_l, respectively, obtained from the mass conservation equation of each phase. Therefore, density of each phase ρ_g, ρ_l is determined by using void fraction α.

Similarly, the internal energy of each phase E_g, E_l in the volume is determined by integrating ΔE_g, ΔE_l, respectively obtained from the total energy conservation equation of each phase. In the calculations of ΔE_g and ΔE_l, using the total energy conservation equations, the calculations of $(\Delta W_g, \Delta W_l)_{vol}$, $(\Delta M_g, \Delta M_l)$ and $(\Delta \dot{M}_{ZE}, \Delta \dot{M}_{ZC}, \Delta \dot{M}_{tE}, \Delta \dot{M}_{tC})$ are made by using equations of momentum balance, mass conservation and of the first law of thermodynamics respectively, and the results are substituted into the total energy conservation equations.

Numerical calculation is advanced by using the thermodynamic states obtained from mass and energy conservation equations of each phase.

Thus, if the thermodynamic states obtained in the two different ways mentioned above coincide with each other, and if the summation of the total energy dealt with in a whole calculation model is conserved at any time during the calculation, the correctness of the calculation with the proposed numerical method will be proved. It means that the basic equations derived for numerical use in the volume-junction model are consistent with each other, and that the changes of thermodynamics state and the flow rates in the volume, and phase change rates in junction flow and in the volume are correctly evaluated in conformity to physical laws. From the results above, it will be further suggested that the equation of the first law of thermodynamics will give a more accurate solution than the equation of total energy conservation for the evaluation of ΔE in the volume because the former estimates the heat itself contributing to the state change whereas the latter estimates through the small difference between large values of an incoming energy and an outgoing energy in the volume.

For achieving rigorous analysis conforming to the physical law even in the volume-junction model as mentioned above, we propose a new numerical method using a simple explicit solution method. In order to prove the validity of the proposed numerical method, including the newly derived difference equations, some numerical calculations are made considering two independent, but saturated phases. The correctness of the results are examined and discussed.

2. Derivation of basic equations for numerical solution with the volume-junction model

In order to make rigorous analyses of the characteristic behavior in the unsteady flows, the basic equations expressed by the partial differential equations should be first transformed to particular total differential equations. However, concerning the unsteady two-phase flow with phase change, this method is not fully developed for practical use. In the reactor safety analysis codes for the thermal hydraulic behaviors in the reactor cooling system, volume-junction method is widely used; where the flow path is divided into several volumes with finite length and the thermodynamics states are determined in the volume by making estimations of the quantities of mass and energy.

Flow rate is determined at the junction between volumes by using equation of motion considering the changes of pressure and momentum in the flow from volume to volume.

In the expression of partial differential equations, a velocity is dealt with as a same value in the momentum balance equation and in the total energy conservation equation. However, in the volume-junction model, the velocity defined for the junction flow by the momentum balance equation is different from the one defined in the volume by total energy conservation equation. The velocity in the volume should be determined individually by the momentum balance equation derived for the volume. Conservation equations of mass and total energy are first given for the volume. Accordingly, using these conservation equations and the equations of the first law of thermodynamics concerning the state change in the volume, momentum balance equation to determine the velocity in the volume can be derived. This idea is the same as used in the derivation of basic equations in the previous report where the total energy conservation equation of each phase is derived by using the conservation equations of mass and momentum, and the equations of the first law of thermodynamics for the open system.

In the case of the volume-junction model, the total energy conservation equation of each phase is first given for the volume. So, the momentum balance equation can be derived in the same way in conformity to physical laws for the flow in the volume. This means that the velocity in the volume is determined so as to satisfy the strict conservation of total energy.

Numerical calculations in this report are focused on verifying the consistency of basic equations system including phase change equations and the correctness of numerical method with the volume-junction model for the saturated condition of two-phase flow which is assumed to occur in decreasing pressure.

We consider a horizontal constant area flow path with no external heat for simplicity and as a volume-junction model, Volume (I, K, L) and Junction (J-1, J, J+1) are defined as shown in Fig. 1.

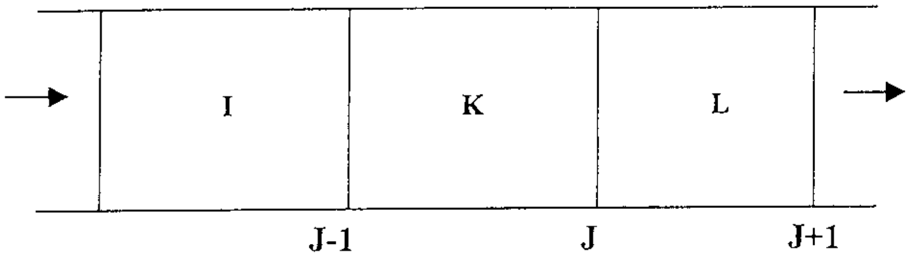


Fig. 1 Control volume for volume-junction model

2.1 Mass and energy conservation equation.

Mass conservation of each phase in the Volume K is expressed as follows by reference to Eqs, (R1) and (R2) in APPENDIX I.

The junction flow rate is given as the departing flow from the volume of upstream. Phase change rate w consists of evaporation and condensation terms presenting positive and negative values, respectively in the expression of partial differential equations.

However, in the volume-junction model, these terms are expressed in positive forms for both evaporation and condensation in the expression of partial difference equations as presented in the section 2.3. Further, phase change should be considered for fluids in the volume by the time dependent term $\dot{M}_{tK}$ and in the junction flow by the space dependent one $\dot{M}_{Z,J-1}$, separately.

$$\frac{\Delta M_{g,K}}{\Delta t} = W_{g,J-1} - W_{g,J} + \left(\dot{M}_{t,K} + \dot{M}_{Z,J-1}\right) \tag{1}$$

$$\frac{\Delta M_{l,K}}{\Delta t} = W_{l,J-1} - W_{l,J} - \left(\dot{M}_{t,K} + \dot{M}_{Z,J-1}\right) \tag{2}$$

Energy conservation equations are similarly expressed as follows so as to preserve the physical meaning of differential equations by reference to Eqs.(R5) and (R6) in APPENDIX I in which each term show a clear physical meaning of the energy conservation in the volume.

Energy carried by phase change flow in Junction J-1 is integrated along the state change from Vol. I to Vol. K.

$$\frac{\Delta}{\Delta t}\left\{M_g\left(e_g+\frac{u_g^2}{2}\right)\right\}_K = W_{g,J-1}\left(h_g+\frac{u_g^2}{2}\right)_I - W_{g,J}\left(h_g+\frac{u_g^2}{2}\right)_K + \dot{M}_{tE,K}\left(h_{gs}+\frac{u_l^2}{2}\right)_K - \dot{M}_{tC,K}\left(h_{ls}+\frac{u_g^2}{2}\right)_K$$
$$+\int_I^K h_{gs}\cdot\Delta\dot{M}_{ZE} + \dot{M}_{ZE,J-1}\left(\frac{u_l^2}{2}\right)_I - \int_I^K h_{ls}\cdot\Delta\dot{M}_{ZC} - \dot{M}_{ZC,J-1}\left(\frac{u_g^2}{2}\right)_I - \left(PV\frac{\Delta\alpha}{\Delta t}\right)_K + \left(W_g\cdot\Delta q'_{Eg}\right)_{J-1} \quad (3)$$

$$\frac{\Delta}{\Delta t}\left\{M_l\left(e_l+\frac{u_l^2}{2}\right)\right\}_K = W_{l,J-1}\left(h_l+\frac{u_l^2}{2}\right)_I - W_{l,J}\left(h_l+\frac{u_l^2}{2}\right)_K + \dot{M}_{tE,K}\left(h_{gs}+\frac{u_l^2}{2}\right)_K + \dot{M}_{tC,K}\left(h_{ls}+\frac{u_g^2}{2}\right)_K$$
$$-\int_I^K h_{gs}\cdot\Delta\dot{M}_{ZE} - \dot{M}_{ZE,J-1}\left(\frac{u_l^2}{2}\right)_I + \int_I^K h_{ls}\cdot\Delta\dot{M}_{ZC} + \dot{M}_{ZC,J-1}\left(\frac{u_g^2}{2}\right)_I + \left(PV\frac{\Delta\alpha}{\Delta t}\right)_K + \left(W_l\cdot\Delta q'_{El}\right)_{J-1} \quad (4)$$

Where, $\left(W_g\cdot\Delta q'_{Eg}\right)_{J-1} = \Delta Q'_{Eg,J-1}$, $\left(W_l\cdot\Delta q'_{El}\right)_{J-1} = \Delta Q'_{El,J-1}$

These terms are presented in Eq.(15).

2.2 . Momentum equations for Junction flow

Spatial momentum change is evaluated from volume to junction. Thermal hydraulic states at junction are defined as follows: the void fraction is used by the one in the upstream volume and the pressure and densities are used by the ones in the downstream volume. The reason is as follows. For evaluating momentum change in a junction flow, the momentum at downstream should be evaluated by using the thermodynamic states of downstream volume basically. However, as for the void fraction, the junction flow rate is strongly controlled by the value of fluid source.

Then, on reference to Eqs.(R3) and (R4) in APPENDIX I

$$\begin{aligned}\Delta Z_K\frac{\Delta W_{g,J}}{\Delta t} &= \left(W_g\cdot u_g\right)_K - \left\{\left(W_{g,J}+\dot{M}_{ZE,J}-\dot{M}_{ZC,J}\right)\cdot u_{g,J} + \left(\dot{M}_{tE}-\dot{M}_{tC}\right)_K\cdot u_{g,K}\right\}\\ &\quad + \dot{M}_{ZE,J}\cdot u_{l,J} - \dot{M}_{ZC,J}\cdot u_{g,J} + \dot{M}_{tE,K}\cdot u_{l,K} - \dot{M}_{tC,K}\cdot u_{g,K}\\ &\quad - \left(\Delta F_{wg}+\Delta F_{gl}\right)_J + A_{g,J}\left(P_K-P_L\right)\\ &= \left(W_g\cdot u_g\right)_K - \left(W_g\cdot u_g\right)_J - \dot{M}_{ZE,J}\cdot\left(u_g-u_l\right)_J - \dot{M}_{tE,K}\cdot\left(u_g-u_l\right)_K\\ &\quad - \left(\Delta F_{wg}+\Delta F_{gl}\right)_J + A_{g,J}\left(P_K-P_I\right) \qquad (5)\end{aligned}$$

Similarly, for the liquid phase, we get

$$\Delta Z_K \frac{\Delta W_{l,J}}{\Delta t} = \left(W_l \cdot u_l\right)_K - \left(W_l \cdot u_l\right)_J + \dot{M}_{ZC,J} \cdot \left(u_g - u_l\right)_J + \dot{M}_{tC,K} \cdot \left(u_g - u_l\right)_K$$
$$- \left(\Delta F_{Wl} + \Delta F_{lg}\right) + A_{l,J}\left(P_K - P_L\right) \qquad (6)$$

2.3. Phase change rate equations

(1) Phase change rate in the junction flow

Phase change rate which arises from each phase in a pressure decreasing flow along flow path is expressed in a form of partial differential equation as follows by reference to Eqs.(21) and (2) of reference [1].

Evaporation rate from liquid phase :

$$\left[w_{gl}\right]_Z = -W_l \frac{1}{r}\left[\left\{\left(\frac{dh_l}{dp}\right)_{Sat} - \frac{1}{\rho_l}\right\}\frac{\partial P}{\partial Z} - \frac{\partial q_{TPl}}{\partial Z}\right] \qquad (7)$$

Condensation rate from gas phase :

$$\left[w_{lg}\right]_Z = -W_g \frac{1}{r}\left[\left\{\left(\frac{dh_g}{dp}\right)_{Sat} - \frac{1}{\rho_l}\right\}\frac{\partial P}{\partial Z} - \frac{\partial q_{TPg}}{\partial Z}\right] \qquad (8)$$

Where

$$\left.\begin{aligned} dq_{TPl} &= dq_{El}^{0} + dq_{fl} \\ dq_{TPg} &= dq_{Eg}^{0} + dq_{fg} \end{aligned}\right\} \qquad (9)$$

Frictional heat which arises in each phase is

$$\left.\begin{aligned} dq_{fl} &= dq_{fWl} + dq_{fil} + dq_{fpcl} \\ dq_{fg} &= dq_{fWg} + dq_{fig} + dq_{fpcg} \end{aligned}\right\} \qquad (10)$$

To apply these equations for the volume-junction model, we have to consider one more type of frictional heat which is peculiar to this model. That is the mixing frictional heat which arises in the volume because the velocity in the upstream volume is different from the one in the downstream volume. Consequently all frictional heat terms in the volume-junction model are expressed as follows :

$$\left.\begin{aligned} \Delta q_{fl} &= \Delta q_{fWl} + \Delta q_{fil} + \Delta q_{fpcl} + \Delta q_{ml} \\ \Delta q_{fg} &= \Delta q_{fWg} + \Delta q_{fig} + dq_{fpcg} + \Delta q_{mg} \end{aligned}\right\} \qquad (11)$$

For the flow from Vol. I to Vol. K, each term of right hand side of Eq.(11) is expressed as follows by referring to Eqs. (55)～(57) of reference [1].

$$\left.\begin{aligned} W_l \cdot \Delta q_{fwl} &= u_l \cdot \Delta F_{wl} \\ W_g \cdot \Delta q_{fwg} &= u_g \cdot \Delta F_{wg} \end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} W_l \cdot \Delta q_{fil} &= u_l \cdot \Delta F_{lg} + \Delta Q'_{El} \\ W_g \cdot \Delta q_{fig} &= u_g \cdot \Delta F_{gl} + \Delta Q'_{Eg} \end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} W_l \cdot \Delta q_{fpcl} &= \Delta \dot{M}_{ZC} \cdot \frac{1}{2}\left(u_{g,I} - u_{l,K}\right)^2 \\ W_g \cdot \Delta q_{fpcg} &= \Delta \dot{M}_{ZE} \cdot \frac{1}{2}\left(u_{l,I} - u_{g,K}\right)^2 \end{aligned}\right\} \tag{14}$$

Where $\Delta Q'_{El}$ and $\Delta Q'_{Eg}$ in Eq.(13) is expressed as follows referring Eq. (58) of reference [1].

$$\begin{aligned} \Delta Q'_{Eg} &= \Delta F_{gl} \cdot \left\{ \eta\left(u_g - u_l\right) - u_g \right\} \\ \Delta Q'_{El} &= -\Delta Q'_{Eg} \end{aligned} \tag{15}$$

Further, Δq_{mg} and Δq_{ml} are mixing frictional heat which arise due to the mixing of upper fluids in the Vol. K. So, similar to Eq.(14), these terms are expressed as follows.

$$\begin{aligned} \Delta q_{ml} &= \frac{1}{2}\left(u_{lk} - u_{l,J}\right)^2 \\ \Delta q_{mg} &= \frac{1}{2}\left(u_{g,k} - u_{g,J}\right)^2 \end{aligned} \tag{16}$$

We apply Eqs. (7) and (8) to the flow from Vol. I to Vol. K, dividing the difference pressure (P_I - P_K) into n equal parts. Then, in a small part of arbitrary number i, the phase change rate in each phase is expressed as follows.

$$r \cdot \Delta \dot{M}_{ZE}^{(i)} = -\left[W_l \left\{ \left(\Delta h_l - \frac{\Delta P}{\rho_l} \right) - \Delta q_{TPl} \right\} \right]_{J-1}^{(i)} \tag{7A}$$

$$r \cdot \Delta \dot{M}_{ZC}^{(i)} = \left[W_g \left\{ \left(\Delta h_g - \frac{\Delta P}{\rho_g} \right) - \Delta q_{TPg} \right\} \right]_{J-1}^{(i)} \tag{8A}$$

Terms, Δq_{TPg} and Δq_{TPl} are substituted by Eqs. (11)～(16), after making definition of thermal hydraulic states as follows. For the calculation of frictional heat Δq_{fw} and Δq_{fi}, the states are used by the ones defined at Junction J-1, and flow rates $W_{g,J-1}$ and $W_{l,J-1}$ are assumed to be constant from Vol. I to Vol. K. Frictional heat caused by phase change Δq_{fPC} is given by Eqs. (14) using the phase change rate at i-th ΔP considering the change of velocity from Vol. I to Vol. K. Further, for the calculation of mixing frictional heat in Vol. K, we consider that flow rate of each phase coming into Vol. K is $(W_l - \dot{M}_{ZE})_{J-1}$ for liquid phase and $(W_g - \dot{M}_{ZC})_{J-1}$ for gas phase.

Then, the frictional heat contributes to phase change rate at i-th ΔP is expressed as follows.

$$(W_l \cdot \Delta q_{TPl})_{J-1}^{(i)} = \left\{u_l\left(\Delta F_{Wl} + \Delta F_{lg}\right) + \Delta Q'_{El} + W_l \cdot \Delta q_{ml}\right\}_{J-1} \cdot \frac{1}{n} - \left(\Delta q_{ml} \cdot \Delta\dot{M}_{ZE} - C_1 \cdot \Delta\dot{M}_{ZC}\right)_{J-1}^{(i)} \tag{17}$$

$$(W_g \cdot \Delta q_{TPg})_{J-1}^{(i)} = \left\{u_g\left(\Delta F_{Wg} + \Delta F_{gl}\right) + \Delta Q'_{Eg} + W_g \cdot \Delta q_{mg}\right\}_{J-1} \cdot \frac{1}{n} - \left(\Delta q_{mg} \cdot \Delta\dot{M}_{ZC} - C_2 \cdot \Delta\dot{M}_{ZE}\right)_{J-1}^{(i)} \tag{18}$$

Where

$$\left.\begin{aligned} C_1 &= \frac{1}{2}\left(u_{g,J} - u_{l,K}\right)^2 \\ C_2 &= \frac{1}{2}\left(u_{l,J} - u_{g,K}\right)^2 \end{aligned}\right\} \tag{19}$$

Substituting Eqs. (17),(18) into Eqs. (7A),(8A), phase change rate in a junction flow along flow path can be solved simultaneously. We get

$$\Delta\dot{M}_{ZE}^{(i)} = \frac{1}{y_5}\left(y_2 y_3 + c_1 y_1\right) \tag{20}$$

$$\Delta\dot{M}_{ZC}^{(i)} = \frac{1}{y_5}\left(y_1 y_4 + c_2 y_2\right) \tag{21}$$

Where

$$\left.\begin{aligned} y_1 &= \left(W_g H_1\right)^{(i)} - \left\{u_g\left(\Delta F_{Wg} + \Delta F_{gl}\right) + \Delta Q'_{Eq} + W_g \cdot \Delta q_{mg}\right\}_{J-1} \cdot \frac{1}{n} \\ y_2 &= -\left(W_l H_2\right)^{(i)} + \left\{u_l\left(\Delta F_{Wl} + \Delta F_{lg}\right) + \Delta Q'_{El} + W_l \cdot \Delta q_{ml}\right\}_{J-1} \cdot \frac{1}{n} \\ y_3 &= r^{(i)} - \Delta q_{mg} \\ y_4 &= r^{(i)} + \Delta q_{ml} \\ y_5 &= y_3 y_4 + c_1 c_2 \\ H_1^{(i)} &= \left(\Delta h_g - \frac{\Delta P}{\rho_g}\right)^{(i)} \\ H_2^{(i)} &= \left(\Delta h_l - \frac{\Delta P}{\rho_l}\right) \end{aligned}\right\} \tag{22}$$

Consequently we get

$$\left.\begin{aligned}\dot{M}_{ZE,J-1} &= \sum_{i=1}^{n} \Delta\dot{M}_{ZE}^{(i)}\\ \dot{M}_{ZC,J-1} &= \sum_{i=1}^{n} \Delta\dot{M}_{ZC}^{(i)}\end{aligned}\right\} \tag{23}$$

(2) Phase change rate in the volume

Phase change rate which arises from each phase in the volume caused by pressure decreasing with time is expressed in a form of partial differential equation as follows by reference to Eqs. (19) and (20) of reference [1].

$$\left[W_{gl}\right]_t = \frac{-\alpha_l \rho_l A}{r}\left[\left\{\left(\frac{dh_l}{dp}\right)_{Sat} - \frac{1}{\rho_l}\right\}\frac{\partial P}{\partial t} - \frac{\partial q_{TPl}}{\partial t}\right] \tag{24}$$

$$\left[W_{lg}\right]_t = \frac{-\alpha_g \rho_g A}{r}\left[\left\{\left(\frac{dh_g}{dp}\right)_{Sat} - \frac{1}{\rho_g}\right\}\frac{\partial P}{\partial t} - \frac{\partial q_{TPg}}{\partial t}\right] \tag{25}$$

In Eqs.(24) and (25), external heat terms contributing to phase change are contained which are not contained in reference [1]. This is because, in the volume-junction model, the phase change with respect to time and space should be evaluated in the volume and in the junction, respectively. Therefore, the time dependent frictional heat term due to phase change occurred in each phase in the volume is given as follows.

$$\left.\begin{aligned}\alpha_l \rho_l A\,\frac{\partial q_{TPl}}{\partial t} &= \left[w_{lg}\right]_t \frac{1}{2}\left(u_g - u_l\right)^2\\ \alpha_g \rho_g A\,\frac{\partial q_{TPg}}{\partial t} &= \left[w_{gl}\right]_t \frac{1}{2}\left(u_g - u_l\right)^2\end{aligned}\right\} \tag{26}$$

We apply Eqs (24) and (25) to the phase change rate in Vol. K. Then we get

$$\dot{M}_{tE,K} = \left(EP\frac{\Delta P}{\Delta t} + \frac{M_l}{r}\frac{\Delta q_{TPl}}{\Delta t}\right)_K \tag{24A}$$

$$\dot{M}_{tC,K} = \left(CP\frac{\Delta P}{\Delta t} - \frac{M_g}{r}\frac{\Delta q_{TPg}}{\Delta t}\right)_K \tag{25A}$$

Where,

$$\left.\begin{aligned}EP_K &= \left[\frac{-M_l}{r}\left\{\left(\frac{dh_l}{dP}\right)_{Sat} - \frac{1}{\rho_l}\right\}\right]_K\\ CP_K &= \left[\frac{M_g}{r}\left\{\left(\frac{dh_g}{dP}\right)_{Sat} - \frac{1}{\rho_g}\right\}\right]_K\end{aligned}\right\} \tag{27}$$

$$\left.\begin{aligned} M_l \frac{\Delta q_{TPl}}{\Delta t} &= C_3 \cdot \dot{M}_{tC,K} \\ M_g \frac{\Delta q_{TPg}}{\Delta t} &= C_3 \cdot \dot{M}_{tE,K} \end{aligned}\right\} \tag{28}$$

$$C_3 = \frac{1}{2}\left(u_g - u_l\right)_K^2 \tag{29}$$

Substituting Eqs (24A) and (25A) into Eqs (28) and (29), phase change rate in Vol. K can be solved simultaneously. Consequently, we get

$$\dot{M}_{tE,K} = \frac{y_7}{y_6}\frac{\Delta P}{\Delta t} \tag{30}$$

$$\dot{M}_{tC,K} = -\frac{y_8}{y_6}\frac{\Delta P}{\Delta t} \tag{31}$$

Where

$$\left.\begin{aligned} y_6 &= 1 + \left(\frac{C_3}{r}\right)^2 \\ y_7 &= EP_K + \frac{C_3}{r} \cdot CP_K \\ y_8 &= \frac{C_3}{r} \cdot EP_K - CP_K \end{aligned}\right\} \tag{32}$$

2.4. Derivation of Momentum Equations for the flow in the volume

Momentum equations in the volume are derived by using the conservation equations of mass and energy, and phase change equations (based on the first law of thermodynamics).

First, we change the left hand side of energy equation of gas phase (3) as follows.

$$\frac{\Delta}{\Delta t}\left\{M_g\left(e_g + \frac{u_g^2}{2}\right)\right\}_K = \left\{h_g \frac{\Delta M_g}{\Delta t} + M_g \frac{\Delta h_g}{\Delta t} - V\frac{\Delta}{\Delta t}\left(\alpha_g P\right) + \left(\Delta Z \cdot u_g\right)\frac{\Delta W_g}{\Delta t} - \left(\frac{u_g^2}{2}\right)\frac{\Delta M_g}{\Delta t}\right\}_K \tag{33}$$

Next, substituting Eqs.(27) and (28) into Eq.(25A) and after rearranging, we get

$$\left(M_g \frac{\Delta h_g}{\Delta t}\right)_K = \left(r \cdot \dot{M}_{tC} + \alpha_g V \frac{\Delta P}{\Delta t} + C_3 \cdot \dot{M}_{tE}\right)_K \tag{34}$$

Now, substituting Eq.(34) and mass conservation equation (1) into Eq.(33) and after rearranging, we get

$$\frac{\Delta}{\Delta t}\left\{M_g\left(e_g+\frac{u_g^2}{2}\right)\right\}_K=\left(h_g-\frac{u_g^2}{2}\right)_K\left\{W_{g,J-1}-W_{g,J}+\left(\dot{M}_{tE}-\dot{M}_{tC}\right)_K+\left(\dot{M}_{ZE}-\dot{M}_{ZC}\right)_{J-1}\right\}$$
$$+\left(r\cdot\dot{M}_{tC}-PV\frac{\Delta\alpha_g}{\Delta t}+\Delta Z\cdot u_g\frac{\Delta W_g}{\Delta t}+C_3\dot{M}_{tE}\right)_K \quad (35)$$

For the left hand side of energy equation of liquid phase (4), similarly to the gas phase, we get

$$\frac{\Delta}{\Delta t}\left\{M_l\left(e_l+\frac{u_l^2}{2}\right)\right\}_K=\left(h_l-\frac{u_l^2}{2}\right)_K\left\{W_{l,J-1}-W_{l,J}-\left(\dot{M}_{tE}-\dot{M}_{tC}\right)_K-\left(\dot{M}_{ZE}-\dot{M}_{ZC}\right)_{J-1}\right\}$$
$$-\left(r\cdot\dot{M}_{tE}-PV\frac{\Delta\alpha_g}{\Delta t}-\Delta Z\cdot u_l\frac{\Delta W_l}{\Delta t}+C_3\dot{M}_{tC}\right)_K \quad (36)$$

Subsequently, substituting Eq.(35) into the left hand side of Eq.(3), and after rearranging we get

$$\left(\Delta Z\cdot u_g\frac{\Delta W_g}{\Delta t}\right)_K=W_{g,J-1}\left(h_{g,I}-h_{g,K}\right)+W_{g,J-1}\frac{1}{2}\left(u_{g,K}^2+u_{g,I}^2\right)-W_{g,J}\cdot u_{g,K}^2+\left(\dot{M}_{tE}\cdot u_g\cdot u_l\right)_K$$
$$-\left(\dot{M}_{tC}\cdot u_g^2\right)_K+\sum_{i=1}^{n}\left(h_g\cdot\Delta\dot{M}_{ZE}\right)_{J-1}^{(i)}-h_{g,K}\cdot\dot{M}_{ZE,J-1}+\frac{1}{2}\left(u_{l,J}^2+u_{g,K}^2\right)\cdot\dot{M}_{ZE,J-1}$$
$$-\sum_{i=1}^{n}\left(h_l\cdot\Delta\dot{M}_{ZC}\right)_{J-1}+h_{g,K}\cdot\dot{M}_{ZC,J-1}-\frac{1}{2}\left(u_{g,I}^2+u_{g,K}^2\right)\cdot\dot{M}_{ZC,J-1}+\Delta Q'_{Eg,j-1} \quad (37)$$

Though we can calculate the flow change in Vol. K using Eq.(37), this equation can be changed to the simpler one which expresses clearer physical meaning as a momentum equation.

The first term of the right hand side of Eq.(37) is expressed as follows.
Dividing Eq.(8A) by $W_{g,J-1}^{(i)}$ and after rearranging, we get

$$\Delta h_{g,J-1}^{(i)}=\left(\Delta q_{TPg}+\frac{\Delta P}{\rho_g}+r\cdot\frac{\Delta\dot{M}_{ZC}}{W_g}\right)_{J-1}^{(i)} \quad (8B)$$

The term $\Delta q_{TPg,J-1}^{(i)}$ is gotten from Eq.(18), dividing by $W_{g,J-1}^{(i)}$. Substituting this term into Eq.(8B) and integrating from i=1 to n (this means the integral from P_I to P_K of $h_{g,J-1}$), we get the equation of ($h_{g,K}$-$h_{g,I}$). After multiplying by $W_{g,J-1}$, we get

$$\begin{aligned}
W_{g,J-1}\left(h_{g,K}-h_{g,I}\right) &= \left\{u_g\left(\Delta F_{Wg}+\Delta F_{gl}\right)+\Delta Q'_{Eg}+W_g\cdot\Delta q_{mg}\right\}_{J-1}\cdot\frac{W_{g,J-1}}{n}\sum_{i=1}^{n}\frac{1}{W^{(i)}_{g,J-1}} \\
&\quad +C_2\cdot W_{g,J-1}\sum_{i=1}^{n}\left(\frac{\Delta\dot{M}_{ZE}}{W_g}\right)^{(i)}_{J-1}-W_{g,J-1}\cdot\Delta q_{mg}\sum_{i=1}^{n}\left(\frac{\Delta\dot{M}_{ZC}}{W_g}\right)^{(i)}_{J-1}+W_{g,J-1}\sum_{i=1}^{n}\left(\frac{\Delta P}{\rho_g}\right)^{(i)}_{J-1} \\
&\quad +W_{g,J-1}\sum_{i=1}^{n}\left(\frac{r\cdot\Delta\dot{M}_{ZC}}{W_g}\right)^{(i)}_{j-1}
\end{aligned}\tag{38}$$

Substituting Eq.(38) into the first term of right hand side of Eq.(37) and after dividing by $u_{g,K}$, we get the momentum equation for the gas flow in the volume K.

$$\begin{aligned}
\left(\Delta Z\cdot\frac{\Delta W_g}{\Delta t}\right)_K &= \left\{\frac{1}{2}\frac{\left(1-B_1\right)}{u_{g,K}}\left(u_{g,I}-u_{g,K}\right)^2+u_{g,I}\right\}W_{g,J-1}-W_{g,J}\cdot u_{g,K}+\left(\dot{M}_{tE}\cdot u_l\right)_K-\left(\dot{M}_{tC}\cdot u_g\right)_K \\
&\quad +\left\{\frac{1}{2}\frac{\left(\dot{M}_{ZE,J-1}-B_2\right)}{u_{hg,K}}\left(u_{l,I}-u_{g,K}\right)^2+\dot{M}_{ZE,J-1}\cdot u_{l,I}\right\}-\left\{\frac{1}{2}\frac{\left(\dot{M}_{ZC,J-1}-B_3\right)}{u_{g,K}}\left(u_{g,I}-u_{g,K}\right)^2+\dot{M}_{ZC,J-1}\cdot u_{g,I}\right\} \\
&\quad +\left\{\sum_{i=1}^{n}\left(h_g\cdot\Delta\dot{M}_{ZE}\right)^{(i)}_{J-1}-h_{g,K}\cdot\dot{M}_{ZE,J-1}\right\}\frac{1}{u_{g,K}}-\Bigg\{\sum_{i=1}^{n}\left(h_l\cdot\Delta\dot{M}_{ZC}\right)^{(i)}_{J-1}-h_{g,K}\cdot\dot{M}_{ZC,J-1} \\
&\quad +W_{g,J-1}\sum_{i=1}^{n}\left(r\frac{\Delta\dot{M}_{ZC}}{W_g}\right)^{(i)}_{J-1}\Bigg\}\frac{1}{u_{g,K}}-B_1\left(\Delta F_{Wg}+\Delta F_{gl}\right)_{J-1}\frac{u_{g,J-1}}{u_{g,K}} \\
&\quad +\frac{\left(1-B_1\right)}{u_{g,K}}\cdot\Delta Q'_{Eg,j-1}-\frac{W_{g,J-1}}{u_{g,K}}\sum_{i=1}^{n}\left(\frac{\Delta P}{\rho_g}\right)^{(i)}_{J-1}
\end{aligned}\tag{39}$$

Similarly, for the liquid phase, we get the momentum equation for the liquid flow in the volume K.

$$\begin{aligned}
\left(\Delta Z\cdot\frac{\Delta W_l}{\Delta t}\right)_K &= \left\{\frac{1}{2}\frac{\left(1-B_4\right)}{u_{l,K}}\left(u_{l,I}-u_{l,K}\right)^2+u_{l,I}\right\}W_{l,J-1}-W_{l,J}\cdot u_{l,K}-\left(\dot{M}_{tE}\cdot u_l\right)_K+\left(\dot{M}_{tC}\cdot u_g\right)_K \\
&\quad +\left\{\frac{1}{2}\frac{\left(\dot{M}_{ZC,J-1}-B_5\right)}{u_{l,K}}\left(u_{g,I}-u_{l,K}\right)^2+\dot{M}_{ZC,J-1}\cdot u_{g,I}\right\}-\left\{\frac{1}{2}\frac{\left(\dot{M}_{ZE,J-1}-B_6\right)}{u_{l,K}}\left(u_{l,I}-u_{l,K}\right)^2+\dot{M}_{ZE,J-1}\cdot u_{l,I}\right\} \\
&\quad +\left\{\sum_{i=1}^{n}\left(h_l\cdot\Delta\dot{M}_{ZC}\right)^{(i)}_{J-1}-h_{l,K}\cdot\dot{M}_{ZC,J-1}\right\}\frac{1}{u_{l,K}}-\Bigg\{\sum_{i=1}^{n}\left(h_g\cdot\Delta\dot{M}_{ZE}\right)^{(i)}_{J-1}-h_{l,K}\cdot\dot{M}_{ZE,J-1} \\
&\quad -W_{l,J-1}\sum_{i=1}^{n}\left(r\frac{\Delta\dot{M}_{ZE}}{W_l}\right)^{(i)}_{J-1}\Bigg\}\cdot\frac{1}{u_{l,K}}-B_4\left(\Delta F_{Wl}+\Delta F_{lg}\right)_{J-1}\frac{u_{l,J-1}}{u_{l,K}}+\frac{\left(1-B_4\right)}{u_{l,K}}\cdot\Delta Q'_{El,j-1}-\frac{W_{l,J-1}}{u_{l,K}}\sum_{i=1}^{n}\left(\frac{\Delta P}{\rho_l}\right)^{(i)}_{J-1}
\end{aligned}\tag{40}$$

Where,

$$\left.\begin{aligned} B_1 &= \frac{W_{g,J-1}}{n}\sum_{i=1}^{n}\left(\frac{1}{W_g}\right)_{J-1}^{(i)} , & B_4 &= \frac{W_{l,J-1}}{n}\sum_{i=1}^{n}\left(\frac{1}{W_l}\right)_{J-1}^{(i)} \\ B_2 &= W_{g,J-1}\sum_{i=1}^{n}\left(\frac{\Delta\dot{M}_{ZE}}{W_g}\right)_{J-1}^{(i)} , & B_5 &= W_{l,J-1}\sum_{i=1}^{n}\left(\frac{\Delta\dot{M}_{ZC}}{W_l}\right)_{J-1}^{(i)} \\ B_3 &= W_{g,J-1}\sum_{i=1}^{n}\left(\frac{\Delta\dot{M}_{ZC}}{W_g}\right)_{J-1}^{(i)} , & B_6 &= W_{l,J-1}\sum_{i=1}^{n}\left(\frac{\Delta\dot{M}_{ZE}}{W_l}\right)_{J-1}^{(i)} \end{aligned}\right\} \qquad (41)$$

3. Numerical solution

Using the basic equations for numerical model as described in chapter 2, solution of two-phase flow variables is obtained as follows. Numerical calculation of two-phase flow behavior is advanced by simple integration of explicitly derived solution using the Eulerian method without any special treatment of obtained variables for attaining consistencies between them.

3.1. Solution of pressure and void fraction

The mass of each phase in a volume V is expressed as follows.

$$M_a = \alpha_a \rho_a V \qquad (a = g, l) \tag{42}$$

The total differential form of Eq. (42) is

$$\Delta M_a = \left(\alpha_a^* \cdot \Delta\rho_a + \rho_a \cdot \Delta\alpha_a\right)V \tag{43}$$

Where $\alpha_a^* = \alpha_a + \Delta\alpha_a \qquad (a = g, l)$

Equation (43) is a completely conserved equation of mass.

In this report, numerical calculations are made aiming at the verification of the validity of the numerical method only for the saturated two-phase flow conditions. Therefore, the density of each phase is expressed as ρ =f(P).

Then,

$$\left.\begin{aligned} \Delta\rho_g &= R_{1s} \cdot \Delta P \\ \Delta\rho_l &= R_{3S} \cdot \Delta P \end{aligned}\right\} \tag{44}$$

Where,

$$\left.\begin{aligned} R_{1S} &= \left(\frac{d\rho_g}{dP}\right)_{sat} \\ R_{3S} &= \left(\frac{d\rho_l}{dP}\right)_{sat} \end{aligned}\right\} \tag{45}$$

Solution of pressure and void fraction can be gotten only by using the mass conservation equation of each phase in this case. Now we consider for the volume K.

Rewrite the mass conservation equation of each phase (1) and (2) as follows :

$$\Delta M_{g,K} = \left\{W_{g,J-1} - W_{g,J} + \left(\dot{M}_{ZE} - \dot{M}_{ZC}\right)_{J-1} + \left(\dot{M}_{tE} - \dot{M}_{tC}\right)_K\right\}\Delta t \tag{1A}$$

$$\Delta M_{l,K} = \left\{W_{l,J-1} - W_{l,J} - \left(\dot{M}_{ZE} - \dot{M}_{ZC}\right)_{J-1} - \left(\dot{M}_{tE} - \dot{M}_{tC}\right)_K\right\}\Delta t \tag{2A}$$

After substituting Eqs.(30) and (31) into the terms $\dot{M}_{tE}$, $\dot{M}_{tC}$ of Eqs.(1A) and (2A), upon eliminating by using Eqs.(43), (44) and (45), we have simultaneous equations for ΔP, $\Delta\alpha$ as follows.

$$y_9 \Delta P + \rho_g \Delta\alpha_g = \frac{RHSCG}{V}\Delta t \tag{46}$$

$$y_{10} \Delta P - \rho_l \Delta\alpha_g = \frac{RHSCL}{V}\Delta t \tag{47}$$

Where,

$$\left.\begin{aligned}
y_9 &= \alpha_g^* R_{1S} - \frac{y_7 + y_8}{y_6}\frac{1}{V} \\
y_{10} &= \alpha_l^* R_{3S} + \frac{y_7 + y_8}{y_6}\frac{1}{V} \\
RHSCG &= W_{g,J-1} - W_{g,J} + \left(\dot{M}_{ZE} - \dot{M}_{ZC}\right)_{J-1} \\
RHSCL &= W_{l,J-1} - W_{l,J} - \left(\dot{M}_{ZE} - \dot{M}_{ZC}\right)_{J-1}
\end{aligned}\right\} \tag{48}$$

The solution of simultaneous equations (46) and (47) are as follows

$$\Delta P = \frac{RHSCL \cdot \rho_g + RHSCG \cdot \rho_l}{y_{11}}\Delta t \tag{49}$$

$$\Delta\alpha = \frac{RHSCG \cdot y_{10} + RHSCL \cdot y_9}{y_{11}}\Delta t \tag{50}$$

Where

$$y_{11} = \left(y_{10}\rho_g + y_9\rho_l\right)V \tag{51}$$

Then, we get a new time step value P(n+1), α(n+1) by the Eulerian method, using the old time step value P(n), α (n) and the small difference ΔP, $\Delta\alpha$ as follows.

$$P(n+1) = P(n) + \Delta P \tag{52}$$

$$\alpha(n+1) = \alpha(n) + \Delta\alpha \tag{53}$$

Further, the phase change rates which occur in the junction flow and in the volume are respectively evaluated by using Eqs.(20), (21) and (23) and Eqs.(30) and (31). We can get the saturated value of densities (ρ_{gs}, ρ_{ls}) and specific internal energies (e_{gs}, e_{ls}), with the pressure from the steam table.

3.2. Evaluation of flow rates, masses and internal energies

The flow rates in the junction and in the volume are respectively evaluated by Eqs.(5) and (6) and Eqs.(39) and (40).

For the evaluation of velocities in the junction, thermal hydraulic states are defined as stated in chapter 2.2.

Wall friction force is determined by the equation of conventional from as follows:

$$\Delta F_{Wa} = f_{Wa} A_a \frac{L}{d_{ha}} \frac{\rho_a}{2} u_a^2 \qquad (a = g, l) \tag{54}$$

Interphase friction force is determined by the equation of an empirical from as follows :

$$\Delta F_{gl} = f_{gl} \cdot A_{gl} \cdot V \cdot \rho_C \left(u_g - u_l\right)^2 \tag{55}$$

The details of these frictional terms are omitted since the quantitative evaluation method of these terms has little to do with the aim of this report.

The density of each phase (ρ_g, ρ_l) is determined by using Eq.(42).

Further, the specific internal energy of each phase is evaluated by using energy equations as follows

First, the internal energy in the volume K is defined as

$$E_a = \left(M_a e_a\right)_K \qquad (a = g, l) \tag{56}$$

Second, the change of the kinetic energy in the volume K is expressed as

$$\frac{\Delta}{\Delta t}\left(M_a \frac{u_a^2}{2}\right)_K = \left(\Delta Z \cdot u_a\right)_K \frac{\Delta W_{a,K}}{\Delta t} - \frac{1}{2} u_{a,K}^2 \frac{\Delta M_{a,K}}{\Delta t} \qquad (a = g, l) \tag{57}$$

Substituting Eqs.(56) and (57) into energy Eqs.(3) and (4), we can get ΔE_a(a=g, l). In this procedure, the terms ΔM_a are evaluated by Eqs.(1A) and (2A), and the enthalpy of each phase is determined by using pressure and the specific internal energy which is obtained from the energy equations as described in this section.

Mass M_a and internal energy E_a in the volume are determined by simple integration of ΔM_a and ΔE_a respectively, same as in the pressure or void fraction.

Consequently densities ρ_g, ρ_l and specific internal energies e_g, e_l are determined by Eq.(42) and Eq.(56), respectively.

Thus, examination can be made to see if ρ_g, ρ_l are consistent with ρ_{gs}, ρ_{ls} and if e_g, e_l are consistent with e_{gs}, e_{ls}.

3.3. Evaluation of Temperatures

Although the consistency of basic equations including the phase change equations would be verified by the results obtained in the previous calculations, if the temperature change evaluated by a total differential equation which is expressed as a function of the mass and the internal energy of each phase agrees with the saturated value of pressure, the validity of the numerical method would be confirmed further.

So, if we suppose $T_a = f\left(M_g, M_l, E_g, E_l\right) \qquad (a = g, l)$

Total differential equation of temperature is

$$\Delta T_a = \frac{\partial T_a}{\partial M_g}\Delta M_g + \frac{\partial T_a}{\partial M_l}\Delta M_l + \frac{\partial T_a}{\partial E_g}\Delta E_g + \frac{\partial T_a}{\partial E_l}\Delta E_l \tag{58}$$

If we take $\qquad T_a = f(P_a, e_a) \qquad (a = g, l)$,

then,

$$\left.\begin{aligned}
&\frac{\partial T_l}{\partial M_g} = \left(\frac{\partial T_l}{\partial P}\right)_{e_l} \cdot \frac{\partial P}{\partial M_g} \quad , \quad \frac{\partial T_g}{\partial M_l} = \left(\frac{\partial T_g}{\partial P}\right)_{e_g} \cdot \frac{\partial P}{\partial M_l} \\
&\frac{\partial T_l}{\partial E_g} = \left(\frac{\partial T_l}{\partial P}\right)_{e_l} \cdot \frac{\partial P}{\partial E_g} \quad , \quad \frac{\partial T_g}{\partial E_l} = \left(\frac{\partial T_g}{\partial P}\right)_{e_g} \cdot \frac{\partial P}{\partial E_l} \\
&\frac{\partial T_a}{\partial M_a} = \left(\frac{\partial T_a}{\partial P}\right)_{e_a} \left\{\frac{\partial P}{\partial M_a} + \left(\frac{\partial P}{\partial e_a}\right)_{T_a} \frac{e_a}{M_a}\right\} \\
&\frac{\partial T_a}{\partial E_a} = \left(\frac{\partial T_a}{\partial P}\right)_{e_a} \left\{\frac{\partial P}{\partial E_a} - \left(\frac{\partial P}{\partial e_a}\right)_{T_a} \frac{1}{M_a}\right\}
\end{aligned}\right\} \tag{59}$$

By using of Eqs. (42) and (56), we get

$$\left.\begin{aligned}
&\frac{\partial P}{\partial M_a} = \frac{\rho_a}{VD}\left\{1 + \frac{e}{\rho}\left(\frac{\partial \rho}{\partial e}\right)_P\right\}_a \\
&\frac{\partial P}{\partial E_a} = \frac{-1}{VD}\frac{\rho_b}{\rho_a}\left(\frac{\partial \rho_a}{\partial e_a}\right)_P \qquad \qquad (a = g, l \to b = l, g) \\
&D = \alpha_g \rho_l \left(\frac{\partial \rho_g}{\partial P}\right)_{e_g} + \alpha_l \rho_g \left(\frac{\partial \rho_l}{\partial P}\right)_{e_l} \\
&\rho_a = f(P, e_a)
\end{aligned}\right\} \tag{60}$$

From the general expressions of thermodynamics we get

$$\left.\begin{aligned}
\left(\frac{\partial T}{\partial P}\right)_e &= \frac{\kappa P - T\beta}{P\beta - \rho c_P} \\
\left(\frac{\partial \rho}{\partial e}\right)_P &= \frac{\rho^2\beta}{P\beta - \rho c_P} \\
\left(\frac{\partial \rho}{\partial P}\right)_e &= \frac{c_v\kappa\rho^2}{\rho c_P - P\beta} \\
\left(\frac{\partial P}{\partial e}\right)_T &= \frac{\rho}{P\kappa - T\beta}
\end{aligned}\right\} \qquad (61)$$

Thus, using the Eqs(58)～(61) and the terms of ΔM_a, ΔE_a obtained in the previous section we can estimate the ΔT_a.

4. Numerical calculation results

Some examples of numerical calculation results which verify the validity of the method proposed in this study are shown as follows.

Calculations are made for the blowdown process of high pressure two-phase fluids which occurs in a horizontal pipe with one end is fully opened to atmosphere.

Initial conditions for the calculation of blowdown of saturated steam-water two-phase flow are set as : pressure is 2.6 MPa and the cases of void fraction are 0.95 and 0.3. Fraction η of interphase frictional heat distributed to gas phase in Eq.(15) is assumed to be 1.0 in all cases.

Test pipe dimension is about 73mm diameter and about 4.1m length and the pipe is divided into eight sections of equal control volume for the numerical calculation by the volume-junction method.

Control volume of number 8 has the open end to atmosphere and number 1 has a closed end.

The calculated results for initial void fraction α_0=0.95 are shown in Fig.2～Fig.12 and for α_0=0.3 are in Fig.13～Fig.23, respectively. In these figures, the number in the parenthesis shows the number of the control volume.

The discharge flow rate at the open plane is evaluated as an inertia flow in these cases.

As for the discharge flow rate considering the occurrence of two-phase critical flow, we will study in the later report dealing also the cases of lower initial void fraction where thermodynamically non-equilibrium states take place caused by oscillatory pressure change in the early phase of blowdown. Calculated behaviors of two-phase flow in the case of α_0=0.95 are shown in Fig.2 for pressure, in Fig.3 for void fraction and Fig.4～Fig.7 for velocities.

These results indicate that two-phase flow is developing from the open end to upstream induced by blowdown showing reasonable values on the whole. Pressure in the test pipe decreased to almost atmospheric pressure after 0.1 seconds after the initiation of blowdown. The main purpose of this report is focused on the verification that the calculated results have no inconsistency between variables and no accumulation of calculation errors in the advancement of calculation.

First, the changes of densities are shown in Fig.8～Fig.9. Densities are obtained in two different ways using different variables obtained in the numerical calculation.

One is defined from the mass and void fraction which are obtained by using mass conservation equation and the solution of simultaneous equations concerning pressure and void fraction as described in the chapter 3. Another is defined as a saturated density of pressure. The former is ρ_g, ρ_l and the latter is ρ_{gs}, ρ_{ls}, respectively. These two kinds of densities agree very exactly with each other for both gas and liquid as shown in Fig.8～Fig.9.

Next, the changes of specific internal energies are shown in Fig.10～Fig.11. One specific internal energy of each phase e_g, e_l is respectively defined by using mass and energy conservation equations and the solution of void fraction obtained as described in the chapter 3. Another specific internal energy of each phase e_{gs}, e_{ls} is defined as the saturated one of pressure. These two kinds of specific internal energies agree very exactly with each other for those of gas and liquid as shown in Fig.10～Fig.11.

The verification to be achieved in this report would be completed through the above mentioned examination. However the further examination can be made for the temperature of each phase to make certain of the consistency between the variables. Three kinds of temperature are compared: one is defined as a saturated one, T_s of pressure, the second and the third one are evaluated as the integrated effects of mass and energy change of each phase by using Eq. (58) for the each phase.

These three kinds of temperatures agree very exactly with one another as shown in Fig.12.

For the case of α_0=0.3, same comparisons as in the case of α_0=0.95 are made for the verification of calculated results. Two-phase flow behaviors are shown in Fig.13 for pressure, in Fig.14 for void fraction and Fig.15～Fig.18 for velocities. These results show the similar behavior as in the case of α_0=0.95, however it takes about 0.39 seconds for pressure in the test pipe to decrease to atmospheric pressure after initiation of blowdown. The comparisons for densities are shown in Fig.15～Fig.20, for specific internal energies are in Fig.21～Fig.22 and for temperatures are in Fig.23. These comparisons show that no inconsistency between variables existed, in the numerical calculations for saturated two-phase blowdown behaviors.

Conservation of mass and energy in the calculation of flow behavior from volume to volume is checked as follows.

The mass and the energy in the test pipe are reduced by the progress of blowdown, but the total ones including the integrated values of discharged ones should be conserved at any time. The change of total mass of each phase is shown in Fig.24 for the case ofα_0=0.3. The half of total mass(TOTM)of gas and liquid is shown in the same figure. The fraction of change of TOTM is about 1.5×10^{-5}. The change of total energy of each phase is shown in Fig. 25 in the same manner as Fig.24. The fraction of change of the half of total energy (TOTE) is about 4×10^{-4}. These results show that no accumulation of error in mass or energy was found. It is considered that the energy transportation caused by the phase change in the flow from volume to volume is also correctly evaluated in the calculation of thermodynamical state change.

5. Conclusions

(1) A new numerical method is proposed that will achieve a rigorous numerical calculation of each phase using a simple explicit method with a volume-junction model.

(2) Difference equations for use in a volume-junction numerical model are carefully derived from the partial differential equations proposed in a previous report so as to preserve the physical meaning of the original equations. For this purpose, momentum equations for the flow rate in the volume are derived and used with the momentum equations for junction flow.

(3) Numerical calculations are made for the depressurization of a saturated two-phase mixture. The calculation is advanced by simple integration of an explicitly derived solution which uses the Eulerian method without any special treatment of obtained variables.

(4) Through the examination of the calculated results, it was concluded that no inconsistency between variables existed, and calculational error did not accumulate.

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References

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(5) ibid. Eq.(II.5-21a), Eq.(II.5-21b)

(6) ibid. Eq.(II.5-39), Eq.(II.5-40)

(7) ibid. Eq.(II.5-34b), Eq.(II.5-34c)

(8) e.g. LANDAU and LIFSHITZ "Fluid Mechanics". § 9.1 Flow of a viscous gas in a pipe. Eq.(91.2), Pergamon Press Ltd. (1959)

[NONENCLATURE]

A	:	Flow area
c_p	:	Isobaric specific heat capacity
c_v	:	Isochoric specific heat capacity
d_h	:	Hydraulic diameter
E	:	Internal energy
e	:	Specific internal energy
ΔF	:	Frictional force acting on the fluid in a volume
F_{gs}	:	Fraction of vapor phase at saturation (used in the reference (4))
F_{ls}	:	Fraction of liquid phase at saturation (used in the reference (4))
h	:	Specific enthalpy
L	:	Length of Volume
M	:	Mass
$\dot{M}_{tK}$	:	Time dependent total phase change rate in a Vol.K. $(\dot{M}_{t,K} = \dot{M}_{tE,K} - \dot{M}_{tC,K})$
$\dot{M}_{Z,J-1}$	:	Space dependent total phase change rate in the flow of Junction J-1. $(\dot{M}_{Z,J-1} = \dot{M}_{ZE,J-1} - \dot{M}_{ZC,J-1})$
$\dot{M}_{tE}$	:	Evaporation rate due to pressure decreasing in a volume
$\dot{M}_{tC}$	:	Condensation rate due to pressure decreasing in a volume
$\dot{M}_{ZE}$	:	Evaporation rate due to pressure decreasing in a junction flow
$\dot{M}_{ZC}$	:	Condensation rate due to pressure decreasing in a junction flow
P	:	Pressure
q_{ig}, q_{il}	:	Heat transfer rate to gas and liquid phase, respectively in a unit volume of two-phase flow, due to phase change. (used in TRAC and RELAP5codes.)
$\Delta q_{g,int}^{\Delta P}$	:	Energy exchange of vapor phase due to flashing (used in the reference (4))
$\Delta q_{l,int}^{\Delta P}$	:	Energy exchange of liquid phase due to flashing (used in the reference (4))
Δq_{Eq}^{0}	:	External heat added to a unit mass of gas in a volume
Δq_{El}^{0}	:	External heat added to a unit mass of liquid in a volume
$\Delta q_{Eq}', \Delta q_{El}'$	:	Terms which define distribution of interphase frictional heat to

gas or liquid phase

Δq_{fg} : Frictional heat added to a unit mass of gas phase in a junction flow

Δq_{fl} : Frictional heat added to a unit mass of liquid phase in a junction flow

Δq_{fig} : Of Δq_{fg}, interphase frictional heat

Δq_{fil} : Of Δq_{fl}, interphase frictional heat

Δq_{fpcg} : Of Δq_{fg}, frictional heat caused by phase change due to different velocities between phases

Δq_{fpcl} : Of Δq_{fl}, frictional heat caused by phase change due to different velocities between phases

Δq_{mg} : Mixing frictional heat in a gas phase caused by incoming fluid flow to a volume

Δq_{ml} : Mixing frictional heat in a liquid phase caused by incoming fluid flow to a volume

Δq_{TPg} : Heat added to a unit mass of a gas phase

Δq_{TPl} : Heat added to a unit mass of a liquid phase

r : Latent heat

T : Temperature

t : Time

TOTML : Total mass of liquid including discharged one

TOTMG : Total mass of gas including discharged one

TOTM : (TOTML+TOTMG)/2

TOTEL : Total internal energy of liquid including discharged one

TOTEG : Total internal energy of gas including discharged one

TOTE : (TOTEL+TOTEG)/2

u : Velocity

v : Specific volume

V : Volume

W : Mass flow rate

w : Phase change rate in a unit length of flow

α : Void fraction

β : Coefficient of thermal expansion

Γ : Total phase change rate in a unit volume of two-phase flow.

(used in TRAC and RELAP5 codes)

ρ : Density

κ : Isothermal compressibility

η : Fraction of interphase frictional heat distributed to gas phase

(subscript)

c : Continuous phase

g : Gas phase

l : Liquid phase

$g+l$: Summation of gas and liquid phase

gs : Saturated state of gas

ls : Saturated state of liquid

s : Saturated

t : Time

Z : Spatial

Wg : Effect of wall to gas

Wl : Effect of wall to liquid

gl : Gas phase affected by liquid phase

lg : Liquid phase affected by gas phase

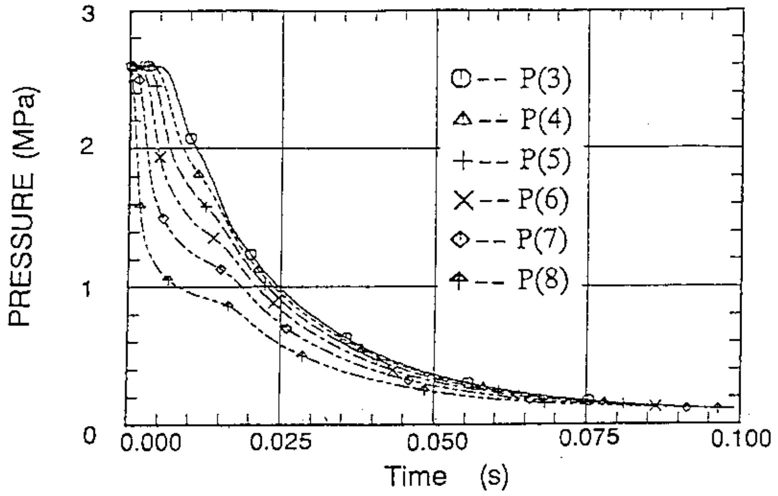


Fig.2 Pressure in Volume 3~8 (α_0=0.95)

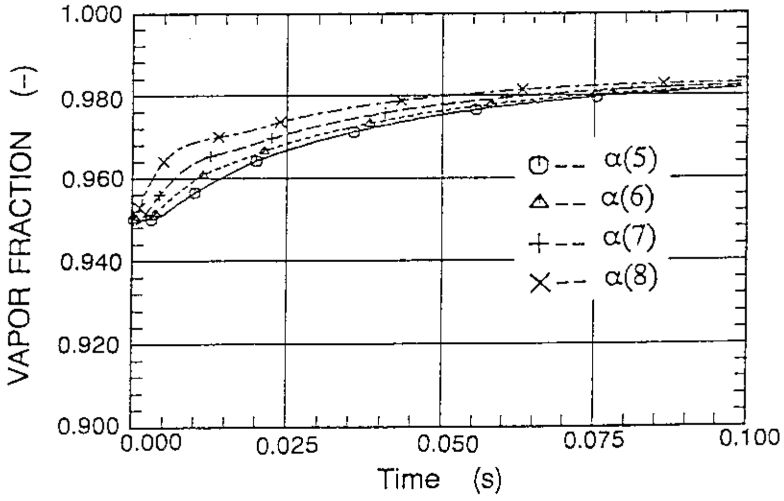


Fig.3 Void fraction in Volume 5~8 (α_0=0.95)

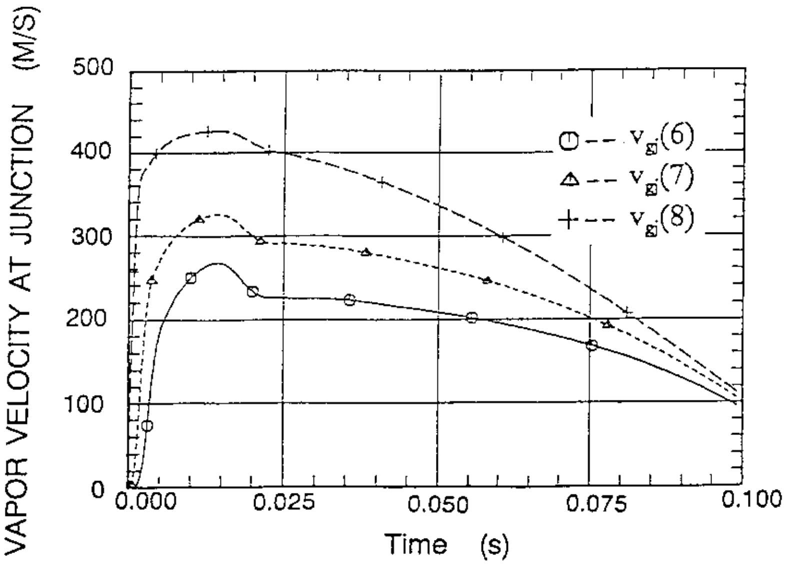


Fig.4 Vapor velocity at Junction 6~8 (α_0=0.95)

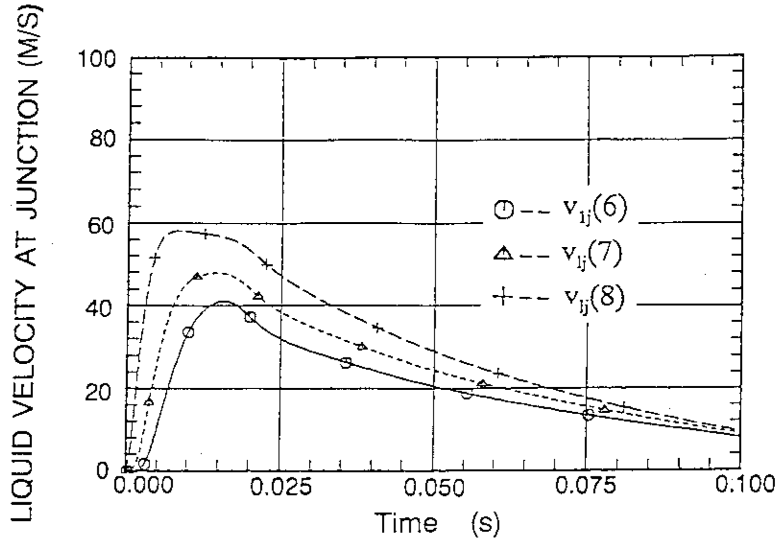


Fig.5　　Liquid velocity at Junction 6~8 (α_0=0.95)

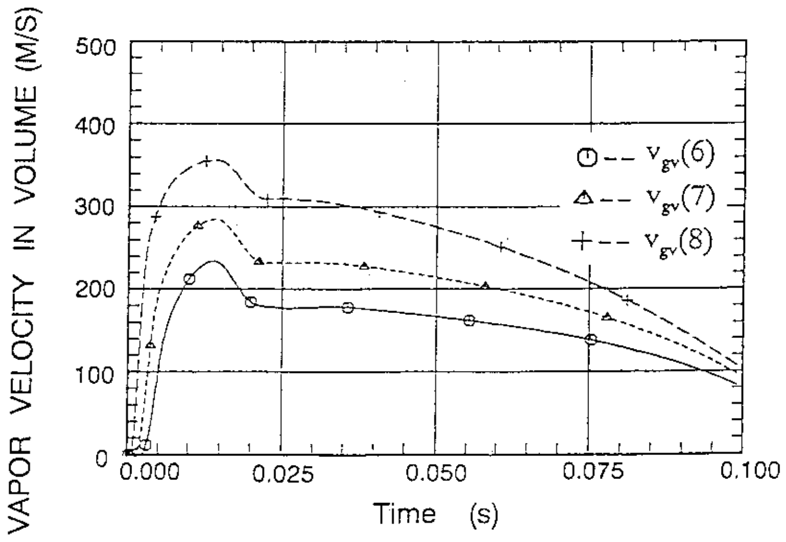


Fig.6　　Vapor velocity in Volume 6~8 (α_0=0.95)

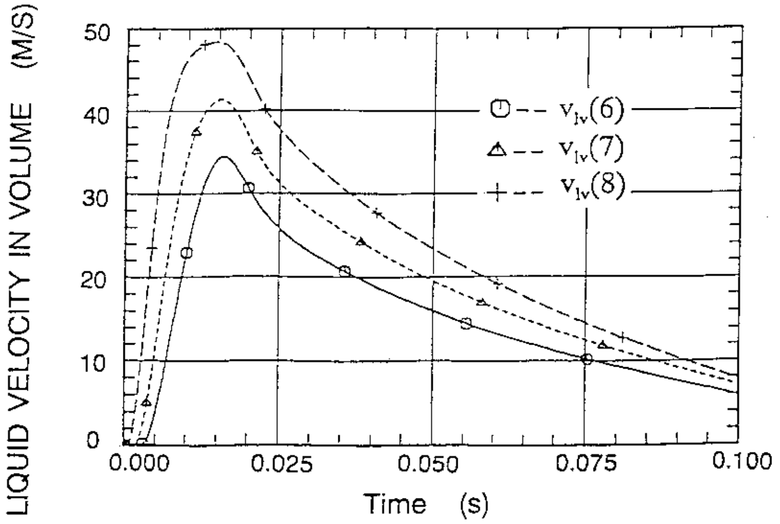


Fig.7　　Liquid velocity in Volume 6~8 (α_0=0.95)

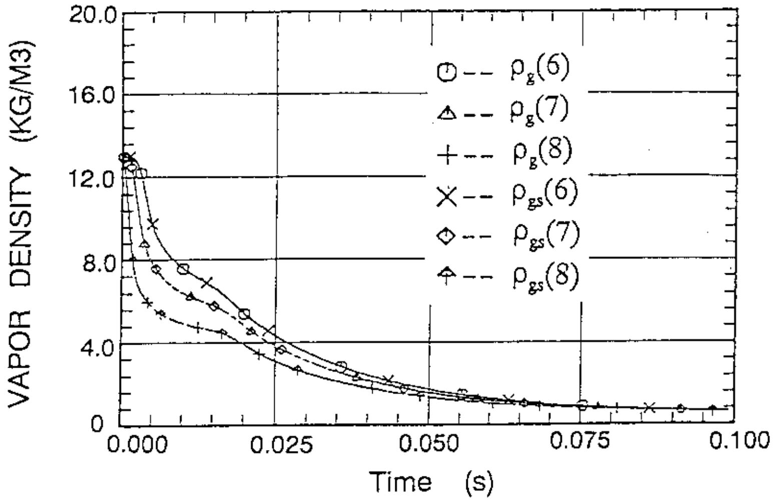


Fig.8 Comparison of vapor densities, ρ_g and ρ_{gs} in Volume 6~8 (α_0=0.95)

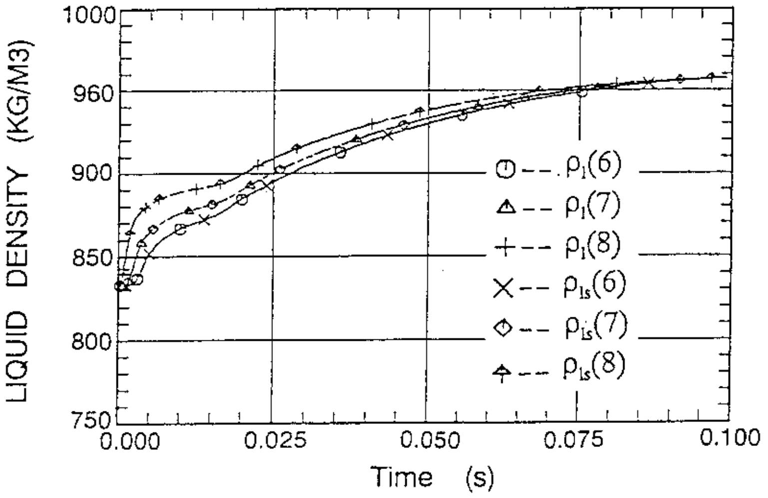


Fig.9 Comparison of liquid densities, ρ_l and ρ_{ls} in Volume 6~8 (α_0=0.95)

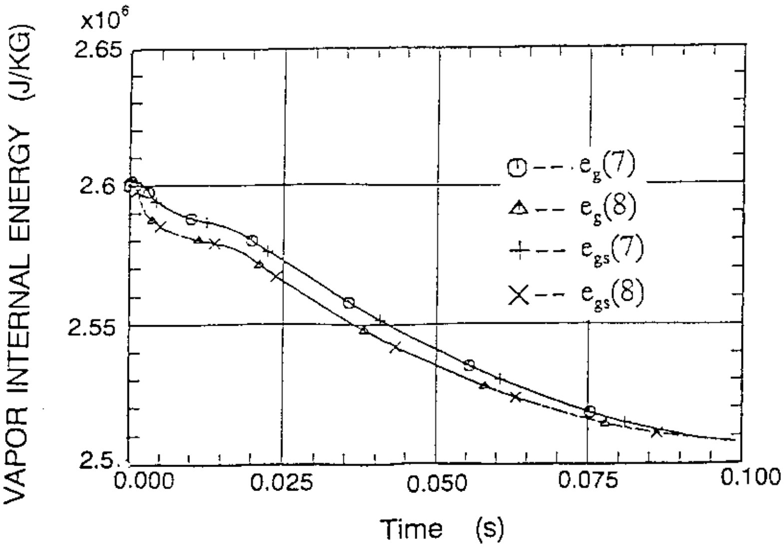


Fig.10 Comparison of vapor specific internal energies, e_g and e_{gs} in Volume 7~8 (α_0=0.95)

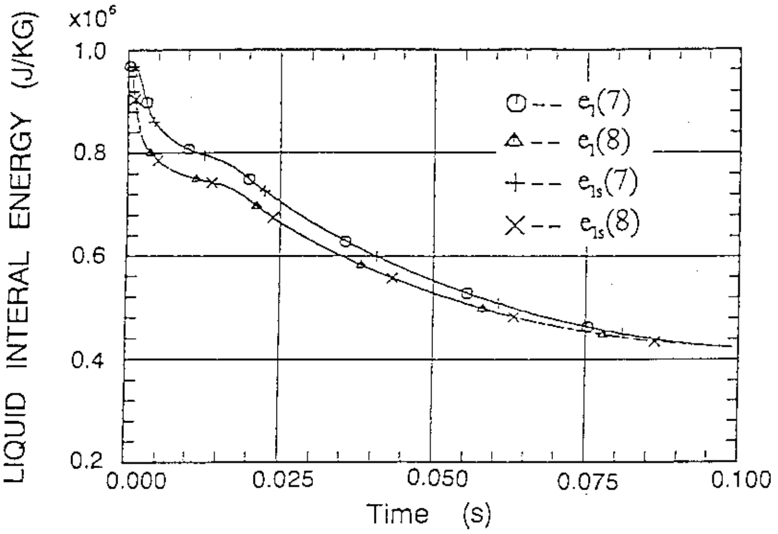


Fig.11 Comparison of liquid specific internal energies, e_l and e_{ls} in Volume 7~8 (α_0=0.95)

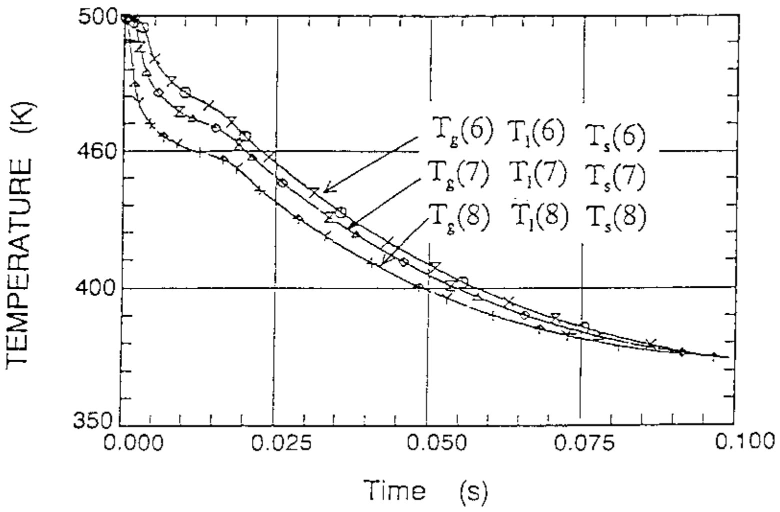


Fig.12 Comparison of temperatures in Volume 6~8 (α_0=0.95)

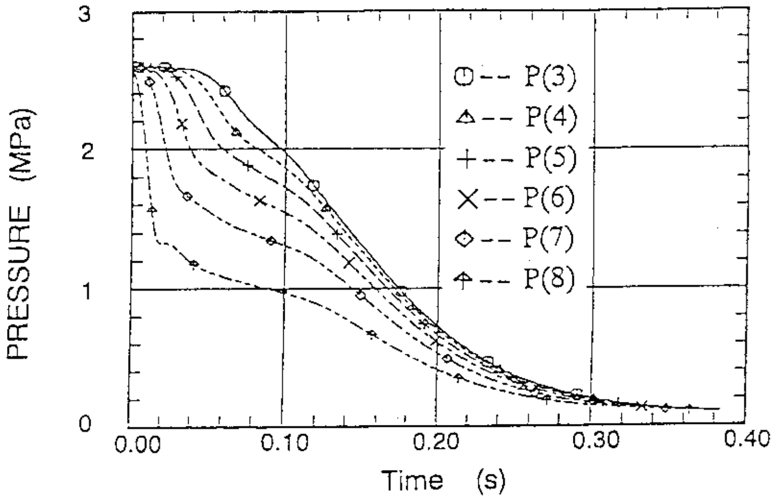


Fig. 13 Pressure in Volume 3~8 (α_0=0.3)

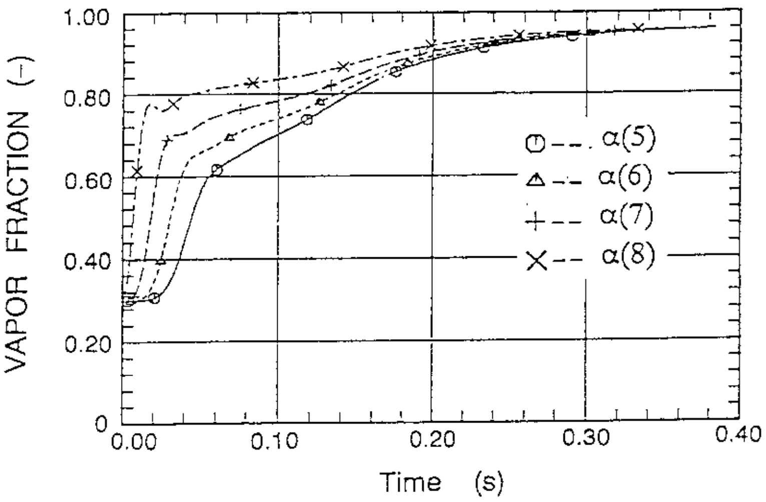


Fig. 14 Void fraction in Volume 5~8 (α_0=0.3)

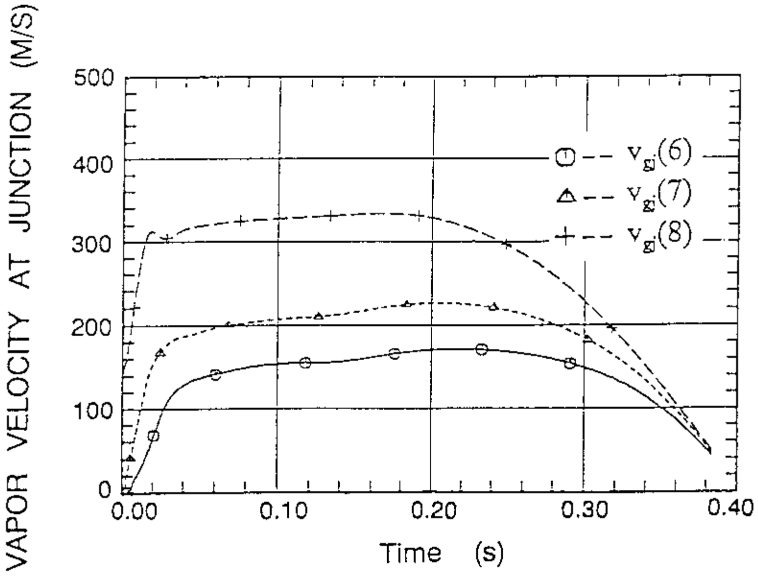


Fig. 15 Vapor velocity at Junction 6~8 (α_0=0.3)

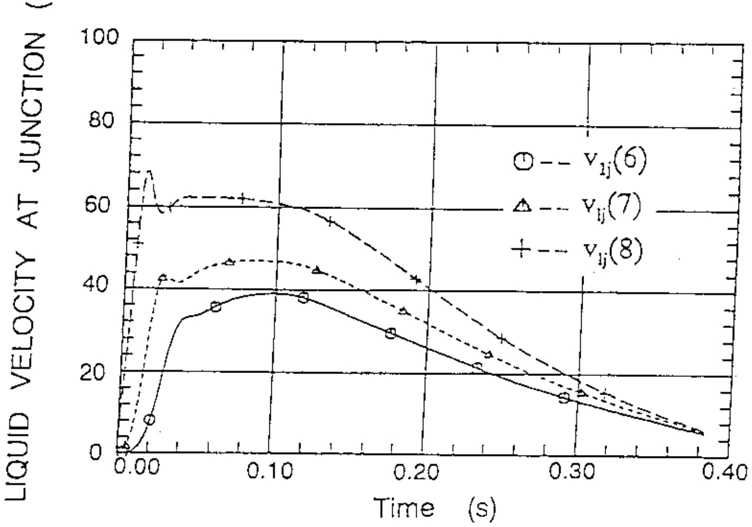


Fig. 16 Liquid velocity at Junction 6~8 (α_0=0.3)

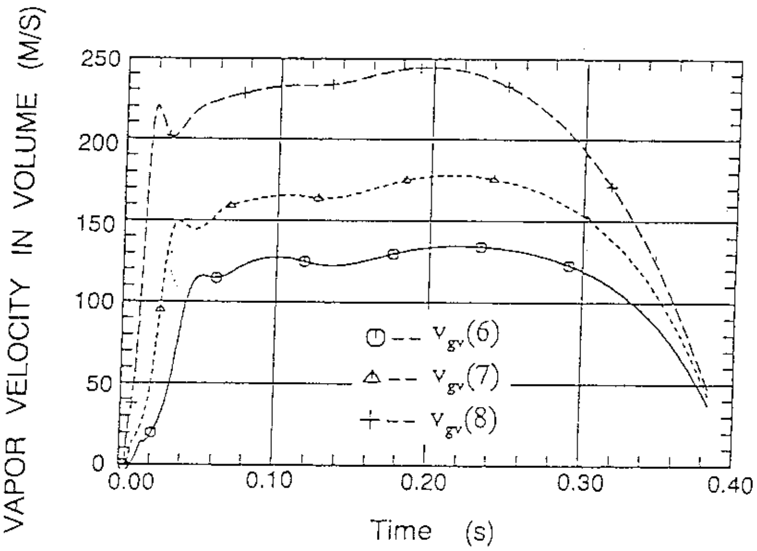


Fig. 17 Vapor velocity in Volume 6~8 (α_0=0.3)

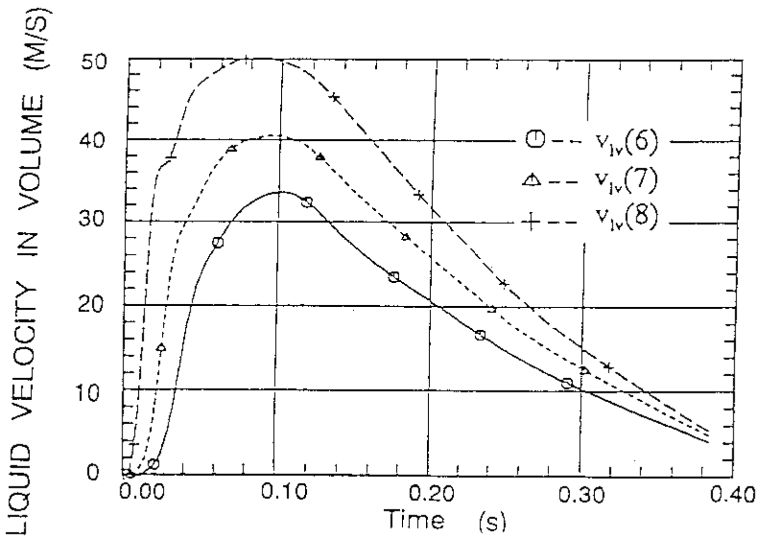


Fig. 18 Liquid velocity in Volume 6~8 (α_0=0.3)

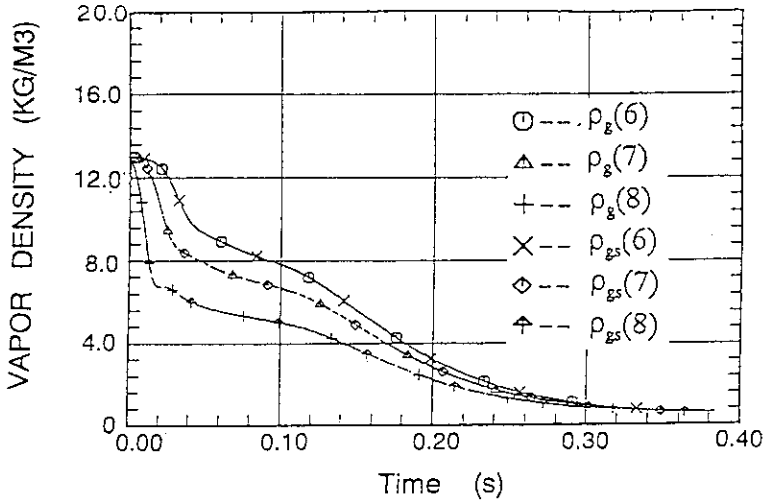


Fig.19 Comparison of vapor densities, ρ_g and ρ_{gs} in Volume 6~8 (α_0=0.3)

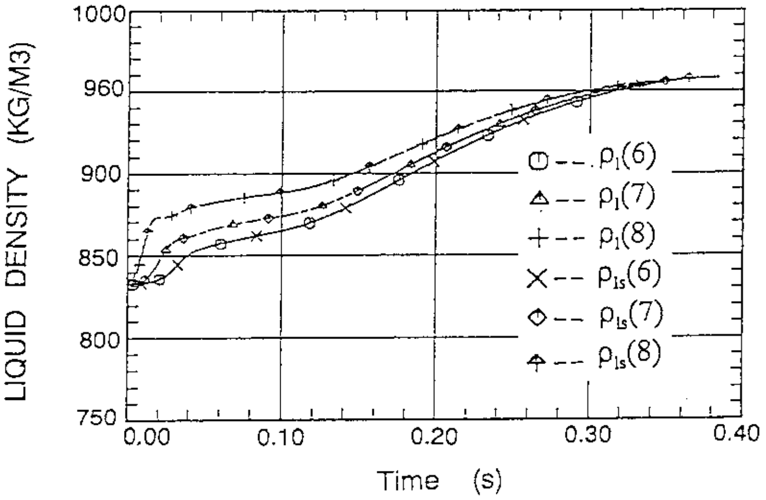


Fig.20 Comparison of liquid densities, ρ_l and ρ_{ls} in Volume 6~8 (α_0=0.3)

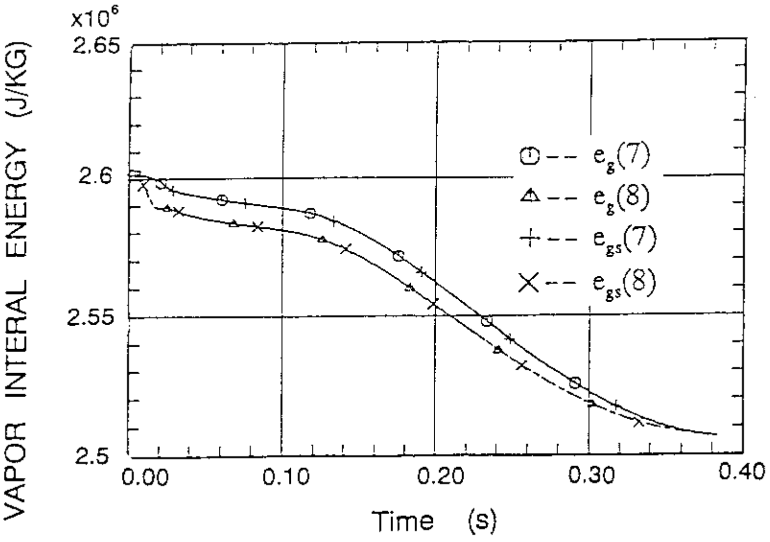


Fig.21 Comparison of vapor specific internal energies, e_g and e_{gs} in Volume 7~8 (α_0=0.3)

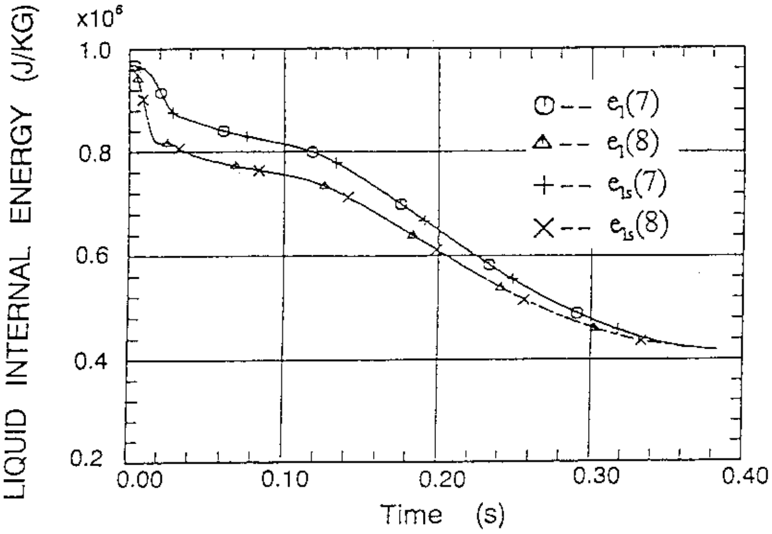


Fig.22 Comparison of liquid specific internal energies, e_l and e_{ls} in Volume 7~8 (α_0=0.3)

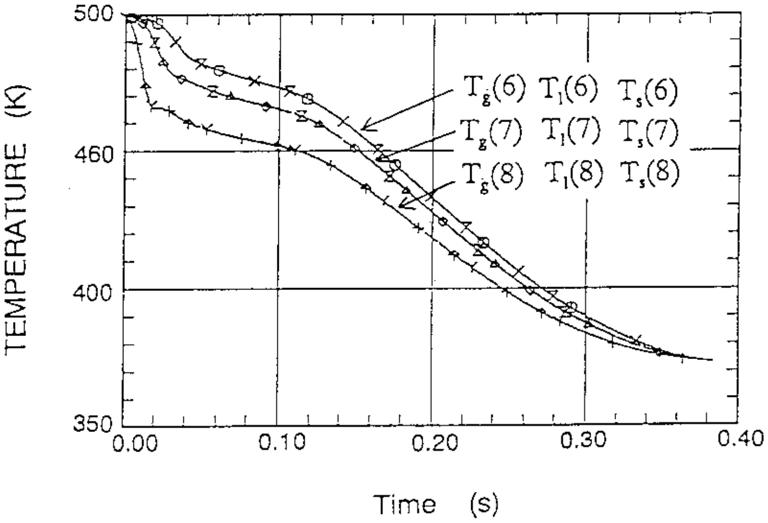


Fig.23 Comparison of temperatures in Volume 6~8 (α_0=0.3)

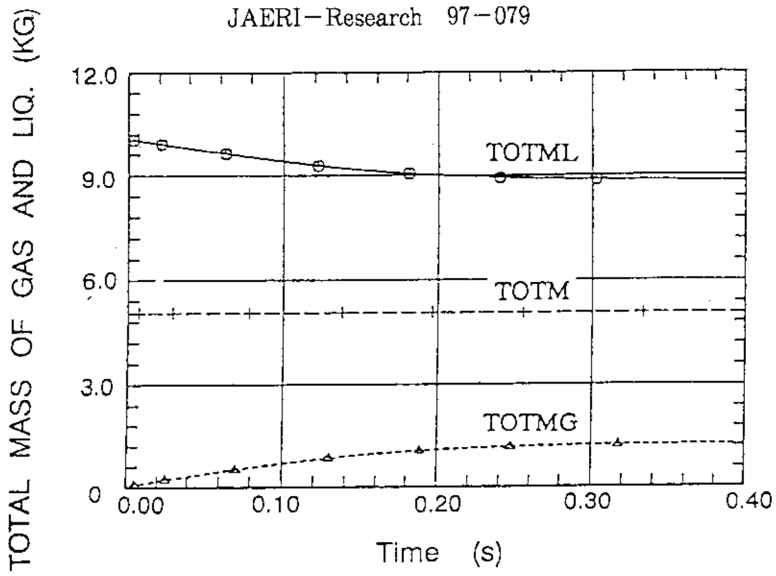


Fig.24 Total mass of gas and liquid including discharged one.

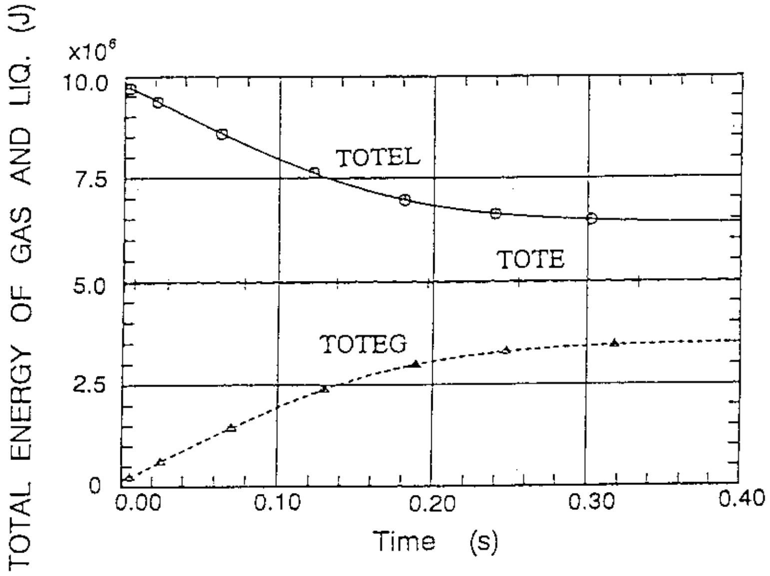


Fig.25 Total energy of gas and liquid including discharged one.

[APPENDIX I] Basic equations for horizontal flow in partial differential form

Basic equations for horizontal flow are expressed on reference to the previous report$^{(1)}$ as follows.

Mass conservation :

$$\frac{\partial}{\partial t}\left(\rho_g A_g\right)+\frac{\partial}{\partial Z}\left(\rho_g A_g u_g\right)-w_{gl}-w_{lg}=0 \tag{R1}$$

$$\frac{\partial}{\partial t}\left(\rho_l A_l\right)+\frac{\partial}{\partial Z}\left(\rho_l A_l u_l\right)+w_{gl}+w_{lg}=0 \tag{R2}$$

Momentum conservation :

$$\frac{\partial}{\partial t}\left(\rho_g A_g u_g\right)+\frac{\partial}{\partial Z}\left(\rho_g A_g u_g \cdot u_g\right)-u_l w_{gl}-u_g w_{lg}+\frac{\partial F_{Wg}}{\partial Z}+\frac{\partial F_{gl}}{\partial Z}+A_g\frac{\partial P}{\partial Z}=0 \tag{R3}$$

$$\frac{\partial}{\partial t}\left(\rho_l A_l u_l\right)+\frac{\partial}{\partial Z}\left(\rho_l A_l u_l \cdot u_l\right)+u_l w_{gl}+u_g w_{lg}+\frac{\partial F_{Wl}}{\partial Z}+\frac{\partial F_{lg}}{\partial Z}+A_l\frac{\partial P}{\partial Z}=0 \tag{R4}$$

Energy conservation (saturated two-phase flow) :

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho_g A_g\left(e_g+\frac{u_g^2}{2}\right)\right\}+\frac{\partial}{\partial z}\left\{\rho_g A_g u_g\left(h_g+\frac{u_g^2}{2}\right)\right\}+P\frac{\partial A_g}{\partial t}\\&\qquad=w_{gl}\left(h_g+\frac{u_l^2}{2}\right)+w_{lg}\left(h_l+\frac{u_g^2}{2}\right)+W_g\left(\frac{\partial q_{Eg}^0}{\partial Z}+\frac{\partial q_{Eg}'}{\partial Z}\right)\end{aligned} \tag{R5}$$

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho_l A_l\left(e_l+\frac{u_l^2}{2}\right)\right\}+\frac{\partial}{\partial Z}\left\{\rho_l A_l u_l\left(h_l+\frac{u_l^2}{2}\right)\right\}+P\frac{\partial A_l}{\partial t}\\&\qquad=-w_{gl}\left(h_g+\frac{u_l^2}{2}\right)-w_{lg}\left(h_l+\frac{u_g^2}{2}\right)+W_l\left(\frac{\partial q_{El}^0}{\partial Z}+\frac{\partial q_{El}'}{\partial Z}\right)\end{aligned} \tag{R6}$$

[APPENDIX II]

1. Physical meaning of the first law of thermodynamics.

The entropy change is expressed by the relationship between thermodynamics state

change as follows.

$$\left.\begin{aligned} &Tds = dh - vdP \\ \text{or}\quad &Tds = de + Pdv \end{aligned}\right\} \qquad \text{(A1)}$$

Equation (A1) is the total differential form of entropy s.

$$\left.\begin{aligned} \text{Where}\quad &S = f(h, P) \\ \text{or}\quad &S = f(e, v) \end{aligned}\right\} \qquad \text{(A2)}$$

$$\left.\begin{aligned} \text{and}\quad &\left(\frac{\partial s}{\partial h}\right)_P = \frac{1}{T} \;,\; \left(\frac{\partial s}{\partial P}\right)_h = -\frac{v}{T} \\ \text{or}\quad &\left(\frac{\partial s}{\partial e}\right)_v = \frac{1}{T} \;,\; \left(\frac{\partial s}{\partial v}\right)_e = \frac{P}{T} \end{aligned}\right\} \qquad \text{(A3)}$$

These equations are originally based on the equations of the first law of thermodynamics and the definition of entropy for the reversible process of closed system, as follows.

$$\left.\begin{aligned} &dq = dh - vdP \\ &dq = de + Pdv \end{aligned}\right\} \qquad \text{(A4)}$$

$$\text{and}\qquad dq = Tds \qquad \text{(A5)}$$

Where, thermodynamics theory defines the entropy increase of working fluid receiving the heat dq from a heat source of temperature T in Eq.(A5).

However, the entropy increase in Eq.(A1) is valid for any change regardless of reversible or irreversible process because entropy s is determined if independent variables h and P or e and v are given in Eq.(A2). Moreover, temperature T is defined as that of working fluid. Consequently, Eqs.(A4) and (A5) can be used not only for the external heat dq in reversible process but also for all the heat dq contributive to the entropy increase in an irreversible process such that frictional heat takes place in a flow.

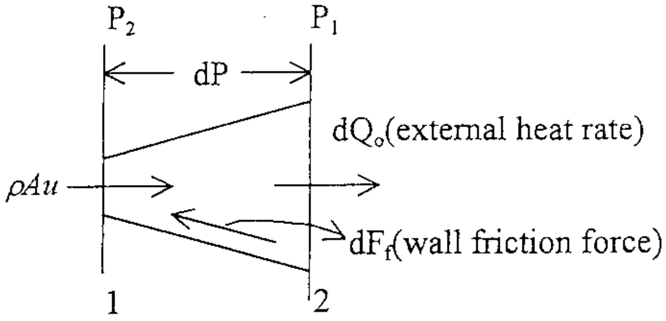
Therefore, we can generally express the equation of the first law of thermodynamics and the equation of the definition of entropy increase for any of reversible or irreversible process by considering all the heat dq' contributive to the state change of working fluid with temperature T as follows.

$$\left.\begin{aligned} dq' &= dh - vdP \\ dq' &= de + Pdv \end{aligned}\right\} \tag{A6}$$

$$dq' = Tds \tag{A7}$$

We investigate with the case of state change in a fluid flow.

We study for the case of one dimentional steady flow of single phase in a horizontal pipe for simplicity as shown in the figure below.



Where

$dP = P_2 - P_1$

dF_f : Wall friction force acting on the fluid between section 1 and 2.

dq_o : external heat added to the unit mass of fluid while passing through from section 1 to section 2.

$dQ_o = \rho Au \cdot dq_o$ External heat rate in the small test section.

Basic equations are :

Mass : $d(\rho Au) = 0$ (A10)

Momentum : $d(\rho udu + dP) + dF_f = 0$ (A11)

Energy[8] : $d\left\{\rho Au\left(h + \frac{u^2}{2}\right)\right\} = dQ_o$ (A12)

From Eq.(A10) and Eq.(A12), we get

$$d\left(h + \frac{u^2}{2}\right) = dq_o \tag{A13}$$

Equations (A13) and (A11) are respectively changed as follows

$$dh + udu = dq_o$$

$$udu = -\frac{1}{\rho}\left(dP + \frac{dF_f}{A}\right)$$

Combining these two equations, we get

$$dh - \frac{dP}{\rho} = dq_o + \frac{dF_f}{\rho A} \tag{A14}$$

Equation (A14) expresses the relation between thermodynamic state change, occurred in the flow from section 1 to section 2, and the heat contributive to the state change. Combining the Eq.(A14) with Eq.(A1), we get Eq.(A15) which expresses that the heat contributive to the entropy increase of fluid is equal to the heat contributive to the state change and consists of external heat dq_o and frictional heat $dF_f/\rho A$.

$$\left.\begin{array}{ll} & Tds = dq_o + \dfrac{dF_f}{\rho A} \\ \text{or} & (\rho A u)\,Tds = dQ_o + u\,dF_f \end{array}\right\} \tag{A15}$$

We can consider Eq.(A14) as the first law of thermodynamics concerning the state change in the flow. The equation of the first law of thermodynamics was derived using the basic equations for the flow (A10), (A11) and (A12). Accordingly, if three out of four equations above are given, the other can be derived.

We have used the first law of thermodynamics as a general expression of a thermodynamics state change in the derivation of basic equations of two-phase flow in the previous paper(1).

2. A few problems of energy conservation equations of two-phase flow.

Discussions are made for the energy equations widely used in the thermo-hydraulic analysis code for nuclear safety. We consider one dimensional horizontal flow with no external heat for simplicity. Taking the energy equations used in TRAC(2) code which is similar to RELAP(3) code, the basic equations are as follows.

Mass conservation :

$$\frac{\partial}{\partial t}(\alpha\rho)_g + \frac{\partial}{\partial Z}(\alpha\rho u)_g = \Gamma \tag{A16}$$

$$\frac{\partial}{\partial t}(\alpha\rho)_l + \frac{\partial}{\partial Z}(\alpha\rho u)_l = -\Gamma \tag{A17}$$

Momentum balance :

$$(\alpha\rho)_g\left(\frac{\partial u_g}{\partial t} + u_g\frac{\partial u_g}{\partial Z}\right) = -\alpha_g\frac{\partial P}{\partial Z} - \Gamma^+(u_g - u_l) - \frac{1}{A}\left(\frac{\partial F_{Wg}}{\partial Z} + \frac{\partial F_i}{\partial Z}\right) \tag{A18}$$

$$(\alpha\rho)_l\left(\frac{\partial u_l}{\partial t} + u_l\frac{\partial u_l}{\partial Z}\right) = -\alpha_l\frac{\partial P}{\partial Z} - \Gamma^-(u_g - u_l) - \frac{1}{A}\left(\frac{\partial F_{Wl}}{\partial Z} + \frac{\partial F_i}{\partial Z}\right) \tag{A19}$$

Energy conservation for gas phase :

$$\frac{\partial}{\partial t}(\alpha\rho e)_g+\frac{\partial}{\partial Z}(\alpha\rho u e)_g=-P\frac{\partial\alpha}{\partial t}-P\frac{\partial(\alpha u)_g}{\partial Z}+q_{ig}+\Gamma\cdot h_g \tag{A20}$$

Energy conservation for gas and liquid mixture :

$$\frac{\partial}{\partial t}(\alpha\rho e)_{g+l}+\frac{\partial}{\partial Z}(\alpha\rho u e)_{g+l}=-P\frac{\partial}{\partial Z}(\alpha u)_{g+l} \tag{A21}$$

We cannot read directly the physical meaning of these equations since the kinetic energy terms are eliminated from the original total energy conservation form. Therefore, we try to derive the same from of energy equations from the total energy conservation equation of two phase flow mixture which is obviously understandable of the physical meaning as presented in Eq.(A22).

$$\frac{\partial}{\partial t}\left\{\alpha\rho\left(e+\frac{u^2}{2}\right)\right\}_{g+l}+\frac{\partial}{\partial Z}\left\{\alpha\rho u\left(h+\frac{u^2}{2}\right)\right\}_{g+l}=0 \tag{A22}$$

This equation is formed only accounting income and outgo of total energy in a control volume and has no concern with the matter of occurrence inside the control volume$^{(8)}$.

Now, we take the case $\Gamma>0$ which means evaporation occurs in the two-phase flow mixture as a net phase change.

Changing Eq.(A22) and upon rearranging it, we get

$$\frac{\partial}{\partial t}(\alpha\rho e)_{g+l}+\frac{\partial}{\partial Z}(\alpha\rho u h)_{g+l}+\left\{\alpha\rho u\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial Z}\right)\right\}_{g+l}+\left[\frac{u^2}{2}\left\{\frac{\partial}{\partial t}(\alpha\rho)+\frac{\partial}{\partial Z}(\alpha\rho u)\right\}\right]_{g+l}=0 \tag{A22)'}$$

Substituting Eqs.(A16)~(A19) for Eq.(A22)' and if rearranged using h=e+P/ρ, we get

$$\begin{aligned}&\frac{\partial}{\partial t}(\alpha\rho e)_{g+l}+\frac{\partial}{\partial Z}(\alpha\rho u e)_{g+l}=-P\frac{\partial}{\partial Z}(\alpha u)_{g+l}\\&\underbrace{+\frac{u_g-u_l}{A}\cdot\frac{\partial F_i}{\partial Z}+\frac{u_g}{A}\frac{\partial F_{Wg}}{\partial Z}+\frac{u_l}{A}\frac{\partial F_{Wl}}{\partial Z}+\frac{(u_g-u_l)^2}{2}\Gamma}_{\Downarrow \\ q_F}\end{aligned} \tag{A22A}$$

Comparing Eq.(A22A) with Eq.(A21,) we find the frictional heat term q_F in Eq.(A22A) is missing in Eq.(A21).

Although the value of q_F may be small compared with the internal energy of fluid, the effect of frictional heat may not be negligible when pressure change is evaluated by the change of internal energy of fluid, because it is very sensitive for the saturated vapor. This is the first problem involved in the evaluation of pressure change.

Phase change rate Γ is obtained as follows.

Substituting e=h-p/ρ for the left hand side of Eq.(A22A) and after rearranging, we get

$$\text{Left hand side of Eq.(A22A)}=\left[h\left\{\frac{\partial}{\partial t}(\alpha\rho)+\frac{\partial}{\partial Z}\right\}\right]_{g+l}+\left(\alpha\rho\frac{\partial h}{\partial t}\right)_{g+l}+\left(\alpha\rho u\frac{\partial h}{\partial Z}\right)_{g+l}-\frac{\partial P}{\partial t}$$

$$-(\alpha u)_{g+l}\frac{\partial P}{\partial Z}-P\frac{\partial}{\partial Z}(\alpha u)_{g+l}$$

Substituting Eqs.(A16), (A17) for the first term of right hand side of the above and after rearranging, we get

$$\Gamma=\frac{-1}{h_g-h_l}\left\{(\alpha\rho)\left(\frac{\partial h}{\partial t}-\frac{1}{\rho}\frac{\partial P}{\partial t}\right)+(\alpha\rho u)\left(\frac{\partial h}{\partial Z}-\frac{1}{\rho}\frac{\partial P}{\partial Z}\right)-q_F\right\}_{g+l} \qquad \text{(A23)}$$

Equation (A23) represents the phase change rate in an unsteady two-phase flow of saturated condition with decreasing pressure expressed by the first law of thermodynamics of vapor and liquid mixture.

We can understand from Eq.(A23) that phase change occurs in each phase independently exchanging latent heat. Therefore, we can express as follows

$$\Gamma=\Gamma_{gl}+\Gamma_{lg}$$

$$q_F=q_{Fg}+q_{Fl}$$

So, we can get the equation of phase change rate in the each phase :

$$\Gamma_{gl}=\frac{-1}{r}\left\{(\alpha\rho)_l\left(\frac{\partial h_l}{\partial t}-\frac{1}{\rho_l}\frac{\partial P}{\partial t}\right)+(\alpha\rho u)_l\left(\frac{\partial h_l}{\partial Z}-\frac{1}{\rho_l}\frac{\partial P}{\partial Z}\right)-q_{Fl}\right\}_{sat} \qquad \text{(A24)}$$

$$\Gamma_{lg}=\frac{-1}{r}\left\{(\alpha\rho)_g\left(\frac{\partial h_g}{\partial t}-\frac{1}{\rho_g}\frac{\partial P}{\partial t}\right)+(\alpha\rho u)_g\left(\frac{\partial h_g}{\partial Z}-\frac{1}{\rho_g}\frac{\partial P}{\partial Z}\right)-q_{Fg}\right\}_{sat} \qquad \text{(A25)}$$

We can divide the term q_F in Eq.(A22A) into gas and liquid phase, as follows

$$q_{Fg}=\frac{u_g}{A}\cdot\frac{\partial F_{Wg}}{\partial Z}+\frac{1}{2}(u_g-u_l)^2\cdot\Gamma_{gl}+\eta\frac{u_g-u_l}{A}\frac{\partial F_i}{\partial Z}$$

$$q_{Fl}=\frac{u_l}{A}\cdot\frac{\partial F_{Wl}}{\partial Z}+\frac{1}{2}(u_g-u_l)^2\cdot\Gamma_{lg}+(1-\eta)\frac{u_g-u_l}{A}\frac{\partial F_i}{\partial Z}$$

The terms Γ_{gl} and Γ_{lg} are the evaporation rate from saturated liquid and condensation rate from saturated vapor, respectively, these equations are the same as the ones in the chapter 3 of reference [1].

Equations (A24) and (A25) are expressed by the first law of thermodynamics in the flow concerning the state change with time and space. The contributive heat (q_g, q_l) to the state change in each phase in Eq.(A24) and in Eq.(A25) is given as follows, respectively, as we have seen in Eq.(A14).

$$(\alpha\rho)_l\left(\frac{\partial q}{\partial t}+u\frac{\partial q}{\partial Z}\right)_l=-r\Gamma_{gl}+q_{Fl} \tag{A24A}$$

$$(\alpha\rho)_g\left(\frac{\partial q}{\partial t}+u\frac{\partial q}{\partial Z}\right)_g=-r\Gamma_{lg}+q_{Fg} \tag{A25A}$$

Further, the energy equation of each phase is derived as follows.
Substituting $e_l=h_l-p/\rho_l$ for the liquid phase of left hand side of Eq.(A22A) and after rearranging, we get
Left hand side of Eq.(A22A)

$$=\frac{\partial}{\partial t}(\alpha\rho e)_g+\frac{\partial}{\partial Z}(\alpha\rho ue)_g+\underbrace{h_l\left\{\frac{\partial}{\partial t}(\alpha\rho)+\frac{\partial}{\partial Z}(\alpha\rho u)\right\}}_{I_l}$$

$$+\underbrace{\left\{(\alpha\rho)\left(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial Z}\right)\right\}_\ell}_{I_2}-\frac{\partial}{\partial t}(\alpha P)_l-\frac{\partial}{\partial Z}(\alpha uP)_l \tag{A22B}$$

From Eq.(A17), we get $I_1=-h_l\Gamma$

Using Eq.(A24), we get $I_2=-r\Gamma_{gl}+\alpha_l\frac{\partial P}{\partial t}+(\alpha u)_l\frac{\partial P}{\partial Z}+q_{Fl}$

Substituting these equations for the left hand side of Eq.(A22A), we get the energy equation of gas phase, as follows.

$$\frac{\partial}{\partial t}(\alpha\rho e)_g+\frac{\partial}{\partial Z}(\alpha\rho ue)_g=h_l\Gamma_{lg}+h_g\Gamma_{gl}-P\frac{\partial\alpha_g}{\partial t}-P\frac{\partial}{\partial Z}(\alpha u)_g+q_{Fg} \tag{A26}$$

If we change the internal energy of the second term of left hand side to enthalpy, we get

$$\frac{\partial}{\partial t}(\alpha\rho e)_g+\frac{\partial}{\partial Z}(\alpha\rho uh)_g=h_l\Gamma_{lg}+h_g\Gamma_{gl}-P\frac{\partial\alpha_g}{\partial t}+(\alpha u)_g\frac{\partial P}{\partial Z}+q_{Fg} \tag{A26'}$$

Similarly, substituting $e_g=h_g-p/\rho_g$ for the gas phase of Eq.(A22A) and using Eq.(A16) and Eq.(A25), we can get the energy equation of liquid phase.

$$\frac{\partial}{\partial t}(\alpha\rho e)_l+\frac{\partial}{\partial Z}(\alpha\rho ue)_l=-h_l\Gamma_{lg}-h_g\Gamma_{gl}+P\frac{\partial\alpha_g}{\partial t}-P\frac{\partial}{\partial Z}(\alpha u)_l+q_{Fl} \tag{A27}$$

or,

$$\frac{\partial}{\partial t}(\alpha\rho e)_l+\frac{\partial}{\partial Z}(\alpha\rho u h)_l=-h_l\Gamma_{lg}-h_g\Gamma_{gl}+P\frac{\partial\alpha_g}{\partial t}+(\alpha u)_l\frac{\partial P}{\partial Z}+q_{Fl} \qquad \text{(A27)'}$$

Equations (A26) and (A27) are derived considering the phase change occurs in each phase independently ; evaporation from saturated liquid phase and condensation from saturated vapor phase.

But, the phase change equation used in the TRAC code is as follows.

$$\Gamma=\frac{-q_{ig}-q_{il}}{h_g-h_l} \qquad \text{(A28)}$$

This equation evaluate only the net phase change rate in the mixture even though the interfacial heat transfer to gas phase and to liquid phase are independently considered as follows.

$$q_{ig}=h_{ig}\cdot A_i\frac{T_s-T_g}{Vol} \qquad \text{(A29)}$$

$$q_{il}=h_{il}\cdot A_i\frac{T_s-T_l}{Vol} \qquad \text{(A30)}$$

Here terms A_i and h_i are an interfacial area and heat transfer coefficients, respectively, and T_s is the saturation temperature.

The energy equation of gas phase is derived as follows if we follow this idea of phase change model. In this idea, the contributive heat to state change of liquid phase is q_{il}+q_{Fl}. So, the term I_2 in Eq.(A22B) is expressed as follows, by the first law of thermodynamics.

$$I_2=\left\{(\alpha\rho)\left(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial Z}\right)\right\}_l=\left\{(\alpha\rho)\left(\frac{\partial q}{\partial t}+u\frac{\partial q}{\partial Z}\right)\right\}_l+\alpha_l\left(\frac{\partial P}{\partial t}+u_l\frac{\partial P}{\partial Z}\right)$$

Where $(\alpha\rho)_l\left(\frac{\partial q}{\partial t}+u\frac{\partial q}{\partial Z}\right)_l=q_{il}+q_{Fl}$ (A31)'

The term I_1 in Eq.(A22B) is given from Eq.(A28) as ;

$$I_1=h_l\Gamma=q_{ig}+q_{il}+h_g\Gamma$$

Substituting I_1 and I_2 for Eq.(A22B) and combining with Eq.(A22A), after rearranging we get the energy equation of gas phase.

$$\frac{\partial}{\partial t}(\alpha\rho e)_g+\frac{\partial}{\partial Z}(\alpha\rho u e)_g=-P\frac{\partial}{\partial Z}(\alpha u)_g-P\frac{\partial\alpha}{\partial t}+h_g\Gamma+q_{ig}+q_{Fg} \qquad \text{(A31)}$$

Comparing Eq.(A31) with Eq.(A20) which is given in the basic equations of TRAC code, the frictional heat term q_{fg} in Eq.(A31) is missing in Eq.(A20).

Similarly, the energy equation of liquid phase on the basis of phase change model expressed by Eqs.(A28)~(A30) is obtained as follows.

$$\frac{\partial}{\partial t}(\alpha\rho e)_l+\frac{\partial}{\partial Z}(\alpha\rho u e)_l=-P\frac{\partial}{\partial t}(\alpha u)_l+P\frac{\partial\alpha}{\partial t}-h_l\Gamma+q_{il}+q_{Fl} \qquad \text{(A32)}$$

Where, contributive heat to the state change of gas phase is as follows.

$$(\alpha\rho)_g\left(\frac{\partial q}{\partial t}+u\frac{\partial q}{\partial Z}\right)_g=q_{ig}+q_{Fg} \qquad \text{(A32)'}$$

Comparing Eq.(A31) and Eq.(A32), we can get the energy equation of two-phase mixture flow as Eq.(A22A)

Comparing the contributive heat terms to the state change between two phase change models; that is between Eq.(A24A) and Eq.(A31)' , and between Eq.(A25A) and Eq.(A32)', we can know the following relations between Γ_{lg} and q_{ig}, and between Γ_{gl} and q_{il}, respectively.

$$\left.\begin{aligned}\Gamma_{lg}&=\frac{-q_{ig}}{h_g-h_l}\\ \Gamma_{gl}&=\frac{-q_{il}}{h_g-h_l}\end{aligned}\right\} \qquad \text{(A34)}$$

Accordingly, we can see the following form in Eq.(A28).

$$\Gamma=\Gamma_{gl}+\Gamma_{lg}$$

In the Calculation of phase change rate caused by the pressure decrease of saturated two-phase flow using Eqs.(A28)~(A30), temperatures T_g, T_l vary around T_s keeping $T_g< T_s< T_l$ and making heat transfer to gas phase $q_{ig}>0$ and to liquid phase $q_{il}<0$ resulting like $\Gamma_{gl}<0$ and $\Gamma_{lg}>0$.

These situations of heat exchange and phase change are similar to those given by phase change equations (A24) and (A25).

But, in the case of non-equilibrium state between gas and liquid phase ; that is super heated gas and subcooled liquid, physical matter becomes different.

In this case, direct condensation of vapor to liquid phase takes place due to temperature difference and all energy of condensing vapor moves into liquid simultaneously. The physical condition of the phase change above is completely different from the case of the saturated two-phase mixture with decreasing pressure. Therefore, only one type of phase change model expressed by Eqs.(A28)~(A30) is not able to deal with correctly other type of phase change phenomena such as condensation due to temperature difference between vapor and liquid.

Further, the most important difference is in the physical meaning between these equations. Equations (A24) and (A25) are theoretically derived by the first law of thermodynamics concerning state change in a flow. On the contrary, Eqs.(A28)~(A30) are empirically established. In the theoretical equation, phase change rate is therefore expressed as a function of pressure change, and in the empirical equation, phase change rate is expressed as a function of two kinds of temperature difference ; T_s-T_g and T_s-T_l (normally $T_g < T_s$ and $T_l > T_s$ in decreasing pressure), where temperature of each phase is unknown variable. The different physical meaning between the theoretical and the empirical equation yields the difference not only in the number of unknowns but also in the numerical stability. In the case of theoretical equation, phase change rate is directly calculated with pressure change. But in the case of empirical equation, it has to be calculated by the iterative method with temperature change or implicit method to avoid numerical instability.

3. Problems in the derivation of difference equations for numerical calculation in the volume-junction model.

In the numerical calculation with the volume-junction model (Fig.1), flow change is determined at the junction by evaluating the pressure difference between volumes and momentum change from volume to junction, and the thermodynamic state change is determined in the volume by evaluating income and outgo of mass and energy. The thermodynamic state in the volume is assumed to be uniform. However both of the state change from volume to volume and the flow change from junction to junction are not small because the length of volume is not small. Moreover, the flow rate in the volume is also different not only from the junction of volume inlet but also from that of volume outlet.

The energy equation of two-phase mixture (A22A) obtained from the total energy Eq.(A22) by eliminating the term of kinetic energy cannot be directly converted into the difference equation. Because the elimination of the velocity term is made considering that velocity in partial differential terms by t and by Z are the same in the partial differential equations system. But, these velocities are different in the partial difference equations system for the volume-junction model. Thermodynamic state terms are also in the same situation in deriving the energy equation of each phase (A26), (A27) or (A31), (A32) which are changed considering that thermodynamic state in partial differential terms by t and by Z are the same.

Before going to the elimination of kinetic energy term from the total energy conservation equation of each phase with the volume-junction model, we check the physical meaning of momentum equations (39) and (40) which are derived in the chapter 2.4.

The momentum equation for the flow in the volume of each phase is derived using the conservation equations of total energy and mass, and the first law of thermodynamics for each phase.

However, the physical meaning of the equation as a momentum equation is difficult to be understood because each term comes from energy convection term of the energy conservation equation in which phase change terms in the junction flow are contained. Thereupon, if we consider the case of a small pressure difference in the junction flow and the flow quality $0<<x<<1$, the phase change rate can be assumed to be very small compared to the flow rate in the junction flow.

So, the terms, in Eq.(41) of this paper become

$$B_1 \fallingdotseq 1,\quad B_4 \fallingdotseq 1,\quad B_2 \fallingdotseq \dot{M}_{ZE,J-1},$$

$$B_6 \fallingdotseq \dot{M}_{ZE,J-1},\quad B_3 = \dot{M}_{ZC,J-1},\quad B_5 = \dot{M}_{ZC,J-1}$$

Then, the momentum Eq.(39) of gas phase in the Vol. K becomes

$$\begin{aligned}\left(\Delta Z\frac{\Delta W_g}{\Delta t}\right)_K &= W_{g,J-1}\cdot u_{g,I} - W_{g,J}\cdot u_{g,K} + \left(\dot{M}_{tE}\cdot u_l\right)_K - \left(\dot{M}_{tC}\cdot u_g\right)_K \\ &\quad + \dot{M}_{ZE,J-1}\cdot u_{l,I} - \dot{M}_{ZC,J-i}\cdot u_{g,I} - \left(\Delta F_{wg} + \Delta F_{gl}\right)_{J-1}\cdot\frac{u_{g,J-1}}{u_{g,K}} \\ &\quad - \frac{\left(u_g A_g\right)_{J-1}}{u_{g,K}}\cdot\left(P_K - P_I\right) \end{aligned} \tag{39A}$$

Eq.(39)A is the same form as the momentum Eq.(5) in the junction flow showing the clear physical meaning.

Elimination of kinetic energy term of total energy conservation equation in the difference equation with the volume-junction model is made as follows. The total energy conservation equation of gas phase in Vol. K is presented in Eq.(3). In Eq.(3) the differential term of kinetic energy in Vol. K is changed to Eq.(57) and the first term of Eq.(57), the differential of flow rate in Vol. K is substituted by Eq.(39). Then, we get the internal energy change rate in Vol. K as follows.

$$\begin{aligned}\frac{\Delta}{\Delta t}\left(M_g e_g\right)_K &= W_{g,J-1}\cdot h_{g,I} - W_{g,J}\cdot h_{g,K} + \dot{M}_{tE,K}\cdot h_{g,K} - \dot{M}_{tC,K}\cdot h_{l,K} \\ &\quad - \left(PV\frac{\Delta\alpha}{\Delta\quad t}\right)_K + h_{g,K}\cdot\dot{M}_{ZE,J-1} - h_{g,K}\cdot\dot{M}_{ZC,J-1} \\ &\quad + W_{g,J-1}\sum_{i=1}^{n}\left(r\frac{\Delta\dot{M}_{ZC}}{W_g}\right)_{J-1}^{(i)} + W_{g,J-1}\sum_{i=1}^{n}\left(\frac{\Delta P}{\rho_g}\right)_{J-1}^{(i)} + E_{loss}\end{aligned} \tag{A35}$$

Where

$$\begin{aligned}E_{loss} &= \left(W_{g,J-1}\cdot B_1 - B_3\right)\cdot\frac{1}{2}\left(u_{g,I} - u_{g,K}\right)^2 + \dot{M}_{tE,K}\frac{1}{2}\left(u_g - u_l\right)_K^2 \\ &\quad + B_2\frac{1}{2}\left(u_{g,K} - u_{l,I}\right)^2 + B_1\left(\Delta F_{wg} + \Delta F_{gl}\right)_{J-1}\cdot u_{g,J-1} + B_1\cdot\Delta Q'_{Eg}\end{aligned}$$

Equation (A35) could become nearly the same difference equation by using Eq.(39) as the one directly converted from Eq.(A26)'. But it is not the same.

Further, we change the left hand side of Eq.(A35) as follows.

$$\Delta\left(M_g e_g\right) = M_g\Delta e_g + h_g\Delta M_g - Pv_g\cdot\Delta M_g$$

Substituting mass conservation Eq.(1) of gas phase for the second term of this equation, $\Delta M_g = \Delta(\alpha V/v_g)$ for the third term and the enthalpy change equation (38) in the flow from Vol. I to Vol. K for the right hand side of Eq.(A35), we can get

$$\left(M_g \frac{\Delta e_g}{\Delta t}\right)_K = \left(r \cdot \dot{M}_{tC}\right)_K + \frac{1}{2}\left(u_g - u_l\right)_K^2 \cdot \dot{M}_{tE,K} - \left(M_g P \frac{\Delta v_g}{\Delta t}\right)_K \qquad \text{(A36)}$$

If we convert the internal energy term into the enthalpy term in the left hand side, we get

$$\left(M_g \frac{\Delta h_g}{\Delta t}\right)_K = \left(r \cdot \dot{M}_{tC}\right)_K + \frac{1}{2}\left(u_g - u_l\right)_K^2 \cdot \dot{M}_{tE,K} + \left(M_g v_g \frac{\Delta P}{\Delta t}\right)_K \qquad \text{(A37)}$$

Equation (A37) is identical with Eq.(34) which is obtained by changing Eq.(25A). Equation (A37) is expressed by the first law of thermodynamics concerning the relation between the contributed heat and the thermodynamic state change in Vol. K. The partial differential equation (25A) of the first law of thermodynamics does not contain any partial differentials with respect to Z, and hence Eq.(A37) should be identical with Eq.(34).

However, Eq.(A35) represents some different terms from the difference equation obtained by directly converting the differential Eq.(A26)' to difference equation. These terms are the seventh, eighth and the first term of E_{loss}. The seventh and eighth terms in Eq.(A35) correspond to the second term of right hand side of Eq.(A26)'. These two terms appear because phase change in the junction flow is considered. The first term of E_{loss} appears because energy dissipation occurs in the volume due to mixing with the fluid of the upper volume, which is peculiar to the difference equation with the volume-junction model.

Thereupon, if we consider the case that P_I-P_K is small and the junction flow quality $0 << x << 1$, we have following approximations in Eq.(41) of this paper.

$$B_1 \fallingdotseq 1, \quad B_4 \fallingdotseq 1, \quad B_2 \fallingdotseq \dot{M}_{ZE,J-1}, \quad B_6 \fallingdotseq \dot{M}_{ZE,J-1}, \quad B_3 \fallingdotseq \dot{M}_{ZC,J-1}, \quad B_5 \fallingdotseq \dot{M}_{ZC,J-1}$$

Further, in the right hand side of Eq.(35)

the eighth term $\fallingdotseq \left(h_g - h_l\right)_K \cdot \dot{M}_{ZC,J-1}$

the ninth term $\fallingdotseq \left(\dfrac{W_g}{\rho_g}\right)_{J-1} \cdot \left(P_K - P_I\right) = \left(\alpha A u_g\right)_{J-1} \cdot \left(P_K - P_I\right)$

Accordingly, Eq.(A35) becomes as follow in this case.

$$\frac{\Delta}{\Delta t}\left(M_g e_g\right)_K = W_{g,J-1}\cdot h_{g,I} - W_{g,J}\cdot h_{g,K} + \dot{M}_{tE,K}\cdot h_{g,K} - \dot{M}_{tC,K}\cdot h_{l,K} + \dot{M}_{ZE,J-1}\cdot h_{g,K}$$
$$- \dot{M}_{ZC,J-1}\cdot h_{l,K} - \left(PV\frac{\Delta\alpha}{\Delta t}\right)_K + \left(\alpha A u_g\right)_{J-1}\cdot\left(P_K - P_I\right) + E_{loss} \qquad \text{(A35)A}$$

Equation (A35)A is almost equal to the equation which is obtained by directly changing the difference Eq.(A26)' to the difference equation except the dissipation term (the first term of E_{loss}) which arises due to mixing with the fluid of upper Volume. Attention must be paid that Eq.(A35)A is obtained by using momentum balance Eq.(A39) for the volume flow and by considering the case that P_I-P_K is small.

Here, we study about another way of thinking for the numerical method with the volume-junction model. In that way, phase change is considered only in the volume and is not considered in the junction flow. That is, the thermodynamic state does not change in the junction flow and the flow from the upstream volume mixes in the downstream volume keeping the state of the upstream volume. This method is used in the codes of TRAC and RELAP etc. The equations of phase change, momentum balance and energy conservation in the volume in which kinetic energy term is eliminated are derived as follows by using this method.

We consider for the gas phase in Vol. K. As for the contributing heat $\dot{Q}_g$ to the thermodynamic state change in the volume, all heat which were considered in the junction flow so far has to be included. Accordingly, referring to the chapter 3 in this paper, we get

$$\dot{Q}_{g,K} = W_{g,J-1}\left(h_{g,I} - h_{g,K}\right) + W_{g,J-1}\frac{1}{2}\left(u_{g,I} - u_{g,K}\right)^2 + \dot{M}_{tE,K}\frac{1}{2}\left(u_g - u_l\right)_K^2$$
$$+ \left(\Delta F_{wg} + \Delta F_{gl}\right)_{J-1}\cdot u_{g,J-1} + \Delta\dot{Q}'_{Eg,j-1} \qquad \text{(A38)}$$

Phase change rate arisen from the gas phase in Vol. K becomes as follows from Eqs.(25A) and (27).

$$\dot{M}_{tC,K} = \left[\frac{M_g}{r}\left\{\left(\frac{dh_g}{dP}\right)_S - \frac{1}{\rho_g}\right\}\frac{\Delta P}{\Delta t} - \frac{\dot{Q}_g}{r}\right]_K \qquad \text{(A39)}$$

We can get mass and energy conservation equations as follows excluding the phase change term in the junction flow from Eqs.(1) and (3).

Mass : $$\left(\frac{\Delta M_g}{\Delta t}\right)_K = W_{g,J-1} - W_{g,J} + \left(\dot{M}_{tE} - \dot{M}_{tC}\right)_K \tag{A40}$$

Energy : $$\frac{\Delta}{\Delta t}\left\{M_g\left(e_g + \frac{u_g^2}{2}\right)\right\}_K = W_{g,J-1}\left(h_g + \frac{u_g^2}{2}\right)_I - W_{g,J}\left(h_g + \frac{u_g^2}{2}\right)_K$$

$$+ \dot{M}_{tE,K}\left(h_g + \frac{u_l^2}{2}\right)_K - \dot{M}_{tC,K}\left(h_l + \frac{u_g^2}{2}\right)_K - \left(PV\frac{\Delta\alpha}{\Delta t}\right)_K + \Delta Q'_{Eg,J-1} \tag{A41}$$

Momentum balance equation in Vol. K is derived as follows by using these equations above. Change the left hand side of Eq.(A41) as follows.

$$\frac{\Delta}{\Delta t}\left\{M_g\left(e_g + \frac{u_g^2}{2}\right)\right\}_K$$

$$= \left\{h_g\frac{\Delta M_g}{\Delta t} + M_g\frac{\Delta h_g}{\Delta t} - V\frac{\Delta}{\Delta t}\left(\alpha_g P\right) + \left(\Delta Z \cdot u_g\right)\frac{\Delta W_g}{\Delta t} - \frac{u_g^2}{2}\frac{\Delta M_g}{\Delta t}\right\}_K \tag{A41)'$$

Change Eq.(A39) as follows.

$$\left(M_g\frac{\Delta h_g}{\Delta t}\right)_K = \left(r \cdot \dot{M}_{tC} + \alpha_g V\frac{\Delta P}{\Delta t} + \dot{Q}_g\right)_K \tag{A39)'$$

After substituting Eqs.(A40) and (A39)' for Eq.(A41)', rearrange Eq.(A41)' using Eq.(A41) and Eq.(A38), then we get

$$\left(\Delta Z \cdot u_g\frac{\Delta W_g}{\Delta t}\right)_K = W_{g,J-1} \cdot u_{g,I} \cdot u_{g,K} - W_{g,J} \cdot u_{g,K}^2 + \dot{M}_{tE,K}\left(u_g \cdot u_l\right)_K$$

$$- \dot{M}_{tC,K} \cdot u_{g,K}^2 - \left(\Delta F_{Wg} + \Delta F_{gl}\right)_{J-1} \cdot u_{g,J-1} \tag{A42}$$

Dividing both sides of Eq.(A42) by $u_{g,K}$, we get the momentum balance equation as follows.

$$\left(\Delta Z\frac{\Delta W_g}{\Delta t}\right)_K = W_{g,J-1} \cdot u_{g,I} - W_{g,J} \cdot u_{g,K} + \left(\dot{M}_{tE} \cdot u_l\right)_K - \left(\dot{M}_{tC} \cdot u_g\right)_K$$

$$- \left(\Delta F_{Wg} + \Delta F_{gl}\right)_{J-1} \cdot \frac{u_{g,J-1}}{u_{g,K}} \tag{A42)'$$

Eq.(A42)' is very simple in comparison with Eq.(A39), because of disappearance of the term originated from phase change in the junction flow. As the result, Eq.(A42)' is similar to Eq.(39)A which is available only for the situation that phase change rate is small compared with flow rate in the junction. However, the form of Eq.(A42)' looks strange for a momentum balance equation since the pressure difference term is missing which plays an important role for governing the motion of fluid.

However, the terms $\dot{M}_{tE}$, $\dot{M}_{tC}$ in Eq.(A42)' include heat exchange terms between phases which should be originally considered in the junction flow. Those terms are expressed by $\dot{Q}_{g,K}$ which is given as a contributing heat to $\dot{M}_{tC,K}$ by Eq.(A38). Therefore, we separate the phase change terms $\dot{M}_{tE}$, $\dot{M}_{tC}$ into $\dot{M}_{tE}^0$, $\dot{M}_{tC}^0$ and $\dot{M}_{ZE}^0$, $\dot{M}_{ZC}^0$ which are originally considered in the volume and the junction, respectively.

The original phase change terms in Vol. K are expressed as follows by combining Eqs.(24A), (25A), (27), (28) and (29).

$$\left.\begin{aligned}\left(r\cdot\dot{M}_{tE}^0\right)_K &= -M_l\left(\frac{dh_l}{dP}-\frac{1}{\rho_l}\right)\frac{\Delta P}{\Delta t}+\dot{M}_{tC,K}^0\cdot\frac{1}{2}\left(u_g-u_l\right)_K^2\\ \left(r\cdot\dot{M}_{tC}\right)_K &= M_g\left(\frac{dh_g}{dP}-\frac{1}{\rho_g}\right)\frac{\Delta P}{\Delta t}-\dot{M}_{tE,K}^0\cdot\frac{1}{2}\left(u_g-u_l\right)_K^2\end{aligned}\right\}\qquad\text{(A43)}$$

Combining Eq.(A43) and Eq.(A39) for gas, and Eq.(A43) and Eq.(24A) for liquid, respectively, we get

$$\left.\begin{aligned}\left(r\cdot\dot{M}_{tE}\right)_K &= \left(r\cdot\dot{M}_{tE}^0\right)_K+\dot{Q}_{l,K}-\dot{M}_{tC,K}^0\cdot\frac{1}{2}\left(u_g-u_l\right)_K^2\\ \left(r\cdot\dot{M}_{tC}\right)_K &= \left(r\cdot\dot{M}_{tC}^0\right)_K-\dot{Q}_{g,K}+\dot{M}_{tC,K}^0\cdot\frac{1}{2}\left(u_g-u_l\right)_K^2\end{aligned}\right\}\qquad\text{(A44)}$$

Where, similar to Eq.(A38)

$$\begin{aligned}\dot{Q}_{l,K} &= W_{l,J-1}\left(h_{l,J}-h_{l,K}\right)+W_{l,J-1}\frac{1}{2}\left(u_{l,J}-u_{l,K}\right)^2+\dot{M}_{tC,K}\frac{1}{2}\left(u_g-u_l\right)_K^2\\ &\quad+\left(\Delta F_{Wl}+\Delta F_{lg}\right)_{J-1}\cdot u_{l,J-1}+\Delta\dot{Q}_{El,J-1}\end{aligned}\qquad\text{(A45)}$$

As for the phase change terms in Junction J-1, we consider the case of the very small pressure decrease in the junction flow to make easy to understand the physical meaning of expressions. Then, from the Eqs.(7A) and (A8), we can get

$$\left.\begin{aligned}\left(r\cdot\dot{M}_{ZE}^0\right)_{J-1} &= -W_{l,J-1}\cdot\left(h_{l,K}-h_{l,J}\right)+A_lu_l\left(P_K-P_l\right)+\left(W_l\cdot\Delta q_{TPl}\right)_{J-1}\\ \left(r\cdot\dot{M}_{ZC}^0\right)_{J-1} &= W_{g,J-1}\cdot\left(h_{g,K}-h_{g,J}\right)-A_gu_g\left(P_K-P_l\right)-\left(W_g\cdot\Delta q_{TPg}\right)_{J-1}\end{aligned}\right\}\qquad\text{(A46)}$$

Where, the contributive heat to phase change in the junction flow is expressed as follows, for each phase.

$$\left.\begin{aligned}
\left(W_l\cdot\Delta q_{TPl}\right)_{J-1} &= \left\{\left(\Delta F_{Wl}+\Delta F_{lg}\right)\cdot u_l+\Delta Q'_{El}+\left(W_l-\dot{M}^0_{ZE}\right)\frac{1}{2}\left(u_{l,I}-u_{l,K}\right)^2\right.\\
&\qquad\left.+\dot{M}^0_{ZC}\frac{1}{2}\left(u_{g,I}-u_{l,K}\right)^2\right\}_{J-1}\\
\left(W_g\cdot\Delta q_{TPg}\right)_{J-1} &= \left\{\left(\Delta F_{Wg}+\Delta F_{gl}\right)\cdot u_g+\Delta Q'_{Eg}+\left(W_g-\dot{M}^0_{ZC}\right)\frac{1}{2}\left(u_{g,I}-u_{g,K}\right)^2\right.\\
&\qquad\left.+\dot{M}^0_{ZE}\frac{1}{2}\left(u_{l,I}-u_{g,K}\right)^2\right\}_{J-1}
\end{aligned}\right\}\qquad\text{(A47)}$$

After changing the first term of Eq.(A38) and Eq.(A45) by using Eqs.(A46) and (A47), respectively, substitute Eqs.(A38) and (A45) for Eq.(A44), then we can get the following equations.

$$\left.\begin{aligned}
\dot{M}_{tE,K} &= \dot{M}^0_{tE,K}+\dot{M}^0_{ZE,J-1}-\frac{\left(A_l u_l\right)_K}{r_{J-1}}\left(P_K-P_I\right)+\frac{H_E}{r_{J-1}}\\
\dot{M}_{tC,K} &= \dot{M}^0_{tC,K}+\dot{M}^0_{ZC,J-1}+\frac{\left(A_g u_g\right)_K}{r_{J-1}}\left(P_K-P_I\right)+\frac{H_C}{r_{J-1}}
\end{aligned}\right\}\qquad\text{(A48)}$$

Where

$$\left.\begin{aligned}
H_E &= \dot{M}^0_{ZE,J-1}\frac{1}{2}\left(u_{l,I}-u_{l,K}\right)^2-\dot{M}^0_{ZC,J-1}\frac{1}{2}\left(u_{g,I}-u_{l,K}\right)^2+\left(\dot{M}_{tC}-\dot{M}^0_{tC}\right)_K\frac{1}{2}\left(u_g-u_l\right)^2_K\\
H_C &= \dot{M}^0_{ZC,J-1}\frac{1}{2}\left(u_{g,I}-u_{g,K}\right)^2-M^0_{ZE,J-1}\frac{1}{2}\left(u_{l,I}-u_{g,K}\right)^2+\left(\dot{M}_{tE}-\dot{M}^0_{tE}\right)_K\frac{1}{2}\left(u_g-u_l\right)^2_K
\end{aligned}\right\}$$

(A49)

Substituting Eqs.(A48) and (A49) for Eq.(A42)' and after rearranging we get

$$\begin{aligned}
\left(\Delta Z\frac{\Delta W_g}{\Delta t}\right)_K &= W_{g,J-1}\cdot u_{g,I}-W_{g,J}\cdot u_{g,K}+\left(\dot{M}^0_{tE}\cdot u_l\right)_K-\left(\dot{M}^0_{tC}\cdot u_g\right)+\dot{M}^0_{ZE,J-1}\cdot u_{l,K}\\
&\quad-\dot{M}^0_{ZC,J-1}\cdot u_{g,K}+\frac{1}{r_{J-1}}\left(H_E\cdot u_{l,K}+H_C\cdot u_{g,K}\right)-\frac{1}{r_{J-1}}\left(A_g u_g^2+A_l u_l^2\right)_K\cdot\left(P_K-P_I\right)\\
&\quad-\left(\Delta F_{Wg}+\Delta F_{g\ l}\right)_{J-1}\cdot\frac{u_{g,J-1}}{u_{g,K}}
\end{aligned}\qquad\text{(A 50)}$$

Comparing Eq.(A50) with Eq.(39)A (APP.II.3) which is obtained by assuming $(P_K - P_I)$ is very small in the momentum Eq.(39) which is derived considering the phase change in the junction flow, we can find differences as follows. The velocity multiplied by $\dot{M}^0_{ZE}$ or $\dot{M}^0_{ZC}$, the sectional area multiplied by $(P_K - P_I)$ and appearance of the terms H_E and H_C in Eq.(A50) are different between Eq.(A50) and Eq.(39)A. In these differences, the sectional area will be the most effective to the flow change in the volume. Obviously, whereas the sectional area in Eq.(39)A is reasonable as the term defining the force acting on the fluid, the sectional area in Eq.(50) is strange. The cause of the unreasonable momentum equation is originated from that the phase change in the flow from the upstream volume is considered after flowing into the downstream volume keeping the state of the upstream volume. For obtaining the correct form of the momentum equation in the volume, it is necessary that the phase change in the flow from the upstream volume should be considered in the junction flow where pressure decreases to the pressure of the downstream volume.

Elimination of kinetic energy term from the total energy conservation Eq.(A41) is made as follows. The kinetic energy term of this equation is expressed by Eq.(57) in this paper. For the first and second terms of Eq.(57) are substituted by Eq.(A42) and by Eq.(A40), respectively.

Finally, we can get the internal energy conservation Eq.(A51) for gas phase in Vol. K.

$$\begin{aligned}\frac{\Delta}{\Delta t}\left(M_g e_g\right)_K &= W_{g,J-1}\cdot h_{g,J} - W_{g,J}\cdot h_{g,K} + \dot{M}_{tE,K}\cdot h_{g,K} - \dot{M}_{tC,K}\cdot h_{l,K} \\ &\quad -\left(PV\frac{\Delta\alpha}{\Delta t}\right)_K + E_{loss1}\end{aligned} \tag{A51}$$

Where

$$\begin{aligned}E_{loss1} &= W_{g,J-1}\frac{1}{2}\left(u_{g,J}-u_{g,K}\right)^2 + \left(\Delta F_{Wg}+\Delta F_{gl}\right)_{J-1}\cdot u_{g,J-1} \\ &\quad + \dot{M}_{tE,K}\frac{1}{2}\left(u_g-u_l\right)_K^2 + \Delta Q'_{Eg,J-1}\end{aligned} \tag{A52}$$

Comparing Eq.(A51) with the equation obtained by directly converting the differential Eq. (A26)' to the difference equation, the term of $(\alpha Au)_g \frac{\partial P}{\partial Z}$ is missing in Eq.(A51).

If we change the left hand side of Eq.(A51), we get

$$\Delta\left(M_g e_g\right) = M_g \Delta e_g + h_g \Delta M_g - P v_g \Delta M_g$$

Substitute Eq.(A40) for the second term and $\Delta M_g = \Delta(\alpha V/v_g)$ for the third term of the right hand side of this equation, respectively, and after rearranging using Eq.(A38), we get

$$\left(M_g \frac{\Delta e_g}{\Delta t}\right)_K = \left(r \cdot \dot{M}_{tC}\right)_k + \dot{Q}_{g,K} - M_g P \frac{\Delta v_g}{\Delta t} \tag{A53}$$

If we use enthalpy for the left hand side of Eq.(A53), we get

$$\left(M_g \frac{\Delta h_g}{\Delta t}\right)_K = \left(r \cdot \dot{M}_{tC}\right)_k + \dot{Q}_{g,K} + \left(M_g v_g \frac{\Delta P}{\Delta t}\right)_K \tag{A54}$$

Equations (A53) and (A54) are basically the same form as Eqs.(A36) and (A37), respectively, which are obtained considering state change in the junction flow. But the calculation method of the thermodynamic state in Vol. K is different between Eq.(A53) or Eq.(A54) and Eq.(A36) or Eq.(A37), respectively. The former equations are obtained assuming that the flow from the upstream volume mixes in the downstream volume keeping the state of the upstream volume. On the other hand, the latter ones are obtained considering the state change and the phase change in the junction flow caused by pressure decrease and by the heat added to the fluids including the frictional heat. Therefore, the flow from the upstream volume mixes in the downstream volume after the state changed in the junction flow.

Accordingly, if we use the first law of thermodynamics for the evaluation of the thermodynamical state change in the volume, difference equation is obtained in the same from for both cases. Simply, the contributive heat to the state change varies and so does the phase change rate in the volume according as the state change in the junction flow is considered or not. However, the problem is if we evaluate the state change in the volume assuming that the flow the upstream volume mixes in the downstream volume keeping the state of upstream volume, the momentum balance equations for the volume flow become unreasonable as shown in Eq.(A42)' or in Eq.(50) from the viewpoint of the physical law of motion.

The flow in the volume should be derived correctly so as to conform to the physical law because the flow in the junction is evaluated by using the momentum change from volume to junction.

From the above mentioned, it is concluded as follows.

The difference equations for use in the volume-junction numerical model cannot be directly converted from the partial differential equations which are changed by substituting other differential equations under the condition of continuous flow change along the flow path. For the model in which discrete flow change is assumed, the difference equations have to be converted from the partial differential equations which are constituted by the terms of clear physical meaning for the control volume, so that the physical meaning of the original equations is preserved.

Further, in order to derive the rigorous momentum balance equations for the volume flow such that they conform to the physical law, it is necessary to consider the thermodynamic state change in the junction flow.

As for the energy equations to determine the thermodynamic state change in the volume, the equations of the first law of thermodynamics should be the best because they are simple and able to calculate most precisely even for a small change.