JP9950294

JAERI-Research
99-030

STATIONARY PERIODIC AND SOLITARY WAVES INDUCED IN AN ISOTROPIC PLASMA BY A STRONG SHORT LASER PULSE

March 1999

Levan N. TSINTSADZE, Kyoji NISHIKAWA*, Toshiki TAJIMA* and J. T. MENDONCA*

日本原子力研究所
Japan Atomic Energy Research Institute

編集兼発行　　日本原子力研究所

JAERI-Research 99-030

Stationary Periodic and Solitary Waves Induced in an Isotropic Plasma by a Strong Short Laser Pulse

Levan N. TSINTSADZE※, Kyoji NISHIKAWA*, Toshiki TAJIMA*2 and J. T. MENDONCA*3

Advanced Photon Research Center
(Tokai Site)
Kansai Research Establishment
Japan Atomic Energy Research Institute
Tokai-mura, Naka-gun, Ibaraki-ken

(Received March 11, 1999)

A description of propagation of a relativistically intense short laser pulse into an isotropic plasma is presented. A kinetic equation for the spectral function of the electromagnetic (EM) waves is derived for an arbitrary amplitude pump wave, where the fully relativistic case is considered. The resulting kinetic equation of the spectral function is used along with the set of equations of the plasma to derive a general dispersion relation, and the importance of relativistic effects is pointed out. In the case of a superstrong short laser pulse novel Langmuir waves, with phase velocities larger than the speed of light, and waves of ion-sound type, which are damped only on ions, are found. In addition, for the case when the plasma density along with the mass of the electrons satisfies the "frozen-in" condition, stationary nonlinear new type of ion-sound waves are investigated. The mechanism of the emission of these waves is also discussed.

Keywords: Strong Short Laser, Novel Langmuir Wave, Ion-sound Type Wave, Stationary Periodic Wave, Solitary Wave, Isotropic Plasma, Kinetic Equation, General Dispersion Relation

---

※ JAERI Research Fellow
* Kinki University
*2 University of Texas
*3 Institute Superior Tecnico

JAERI-Research 99-030

高強度短パルスレーザーによって等方的プラズマ中に誘起される定常周期孤立波

日本原子力研究所関西研究所光量子科学センター
Levan N. TSINTSADZE※・西川　恭治*　・田島　俊樹*2・J. T. MENDONCA*3

（1999年3月11日受理）

相対論的高強度短パルスレーザーの等方的プラズマ中の伝播について報告する。完全に相対論的な場合の任意振幅のポンプ波に対する電磁波のスペクトル関数についての運動論的方程式を導いた。この方程式をプラズマに関する方程式とともに用いることにより一般分散式を得、相対論的効果の重要性を指摘した。超高強度短パルスレーザーの場合、位相速度が光速より速い新しいタイプのラングミュア波及びイオンによってのみ減衰するイオン音波型波動を見いだした。さらに、プラズマ密度と電子質量が「凍結」条件を満たす場合に、新しいタイプの定常非線形イオン音波について解析を行った。これらの波動放射の機構について論議した。

関西研究所（東海駐在）：〒319－1195　茨城県那珂郡東海村白方白根2－4
※ 原研リサーチフェロー
*　近畿大学
*2 テキサス大学
*3 高度技術研究所

Contents

目　　次

1. INTRODUCTION

The development of ultra-intense short pulse lasers allows exploration of fundamentally new parameter regimes for nonlinear laser-plasma interaction. In fact, a number of experiments have been carried out in which plasmas are irradiated by laser beams with intensities up to $10^{19} W/cm^2$. At such intensities the electron quiver velocity rapidly approaches the speed of light, and a host of new phenomena have been predicted [1-9]. Numerous works [9-33] have been devoted to the investigation of relativistically intense EM wave propagation into an isotropic plasma, for the radiation pressure being larger than the plasma pressure. The above treatments where restricted to the case of monochromatic EM waves. However the bandwith of an initially narrow spectrum may eventually broaden, either as a result of several kinds of instability processes, or as the result of other nonlinear wave-wave interaction processes. In order to study the interaction of spectrally broad relativistically intense EM waves with a plasma, it was necessary to derive a general equation for the EM spectral intensity. Such an investigation was done in Refs. [34,35]. This new picture of high-frequency EM processes in a plasma opens the way to the formulation of conceptually new problems in plasma electrodynamics.

In the present paper, we consider certain problems involving the interaction of relativistically intense nonmonochromatic radiation bunches with an isotropic plasma. The paper is organized as follows. First in Sec.II, starting from Maxwell's equations for the EM field in a relativistic plasma, we derive a general equation for the EM spectral intensity [34,35]. Then in Sec.III we derive the plasma wave dispersion relation in the presence of the relativistic ponderomotive force and discuss a new type of longitudinal plasma waves induced by a strong short pulse laser. In the same section it is shown, that the ratio of the plasma density to the mass of the electrons is conserved, or there is a "frozen-in" condition in the case of stationary waves. The stationary nonlinear ion-sound waves are discussed in Sec. IV, and the velocity of the waves and the maximum potential of the field are defined. The mechanism of the emission of a new type of ion-sound waves is given in Sec.V. Finally, a brief summary of our resulsts is given in the last section.

2. DERIVATION OF THE KINETIC EQUATION FOR THE PHOTON GAS

We start from Maxwell equations [34] for a circularly polarized EM wave,

$$\Delta p - \frac{\partial^2 p}{\partial t^2} = \frac{n}{\gamma} p \ , \tag{1}$$

where the following dimensionless quantities have been introduced:

$$p \to \frac{p}{m_\circ c}, \ \ t \to \omega_{pe} t, \ \ \mathbf{r} \to k_p \mathbf{r}, \ \ k_p = \frac{\omega_{pe}}{c},$$
$$n \to \frac{n}{n_\circ}, \quad \gamma = (1+p^2)^{1/2} \ .$$

ω_{pe} is the electron plasma frequency, associated in the usual way with the mean plasma density $n_\circ$, and $m_\circ$ is the electron rest mass.

We shall consider Eq.(1) at two distinct points and instants of time. Following the procedure described in Ref.[34], we can derive an equation for the correlation function $< p(\mathbf{r}_1, t_1) p(\mathbf{r}_2, t_2) >= \Pi(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2)$, where $< ... >$ denotes ensemble averaging:

$$\big(\nabla_1^2 - \nabla_2^2\big)\Pi(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2) - \big(\frac{\partial^2}{\partial t_1^2} - \frac{\partial^2}{\partial t_2^2}\big)\Pi(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2) = (\rho_1 - \rho_2)\Pi(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2) \ , \tag{2}$$

where $\rho = n(\mathbf{r},t)/\gamma(\mathbf{r},t)$.

Introducing new variables,

$$\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \ \ \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \ \ t = \frac{1}{2}(t_1 + t_2), \ \ \tau = t_1 - t_2 \ , \tag{3}$$

Eq.(2) yields

$$\big(\nabla_{\mathbf{R}}\nabla_{\mathbf{r}} - \frac{\partial^2}{\partial t \partial \tau}\big)\Pi(\mathbf{R},\mathbf{r},t,\tau) = \frac{1}{2}(\rho_1 - \rho_2)\Pi(\mathbf{R},\mathbf{r},t,\tau) \ , \tag{4}$$

$$where \qquad \rho_1 - \rho_2 = \rho_1\big(\mathbf{R} + \frac{\mathbf{r}}{2}, t + \frac{\tau}{2}\big) - \rho_2\big(\mathbf{R} - \frac{\mathbf{r}}{2}, t - \frac{\tau}{2}\big) \ .$$

Performing a Fourier transformation of $\Pi(\mathbf{R},\mathbf{r},t,\tau)$ on the variables $(\mathbf{r},\tau)$ we can introduce the power spectral function $\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega)$:

$$\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega) = \int d\mathbf{r} \int d\tau \Pi(\mathbf{R},t,\mathbf{r},\tau) expi(\mathbf{kr} - \omega t) \ . \tag{5}$$

We can also write

$$\Pi(\mathbf{R},t) = < p^2(\mathbf{R},t) > = \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \mathbf{P}(\mathbf{R},t,\mathbf{k},\omega) \ . \tag{6}$$

Taking the double Fourier transformation of Eq.(4) and expanding $(\rho_1 - \rho_2)$ in Taylor series, we obtain after integration the following equation for the spectral function

$$\Big(\omega\frac{\partial}{\partial t} + \mathbf{k}\cdot\nabla_R\Big)\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega) = \sum_{l=0}^{\infty} \frac{(-1)^l}{(2l+1)!}\Big(\frac{1}{2}\frac{\partial^{2l+1}\rho}{\partial \mathbf{R}^{2l+1}} \cdot \frac{\partial^{2l+1}\mathbf{P}}{\partial \mathbf{k}^{2l+1}} - \frac{1}{2}\frac{\partial^{2l+1}\rho}{\partial t^{2l+1}} \cdot \frac{\partial^{2l+1}\mathbf{P}}{\partial \omega^{2l+1}}\Big) \ . \tag{7}$$

3. LINEAR LONGITUDINAL PLASMA WAVES. FROZEN-IN CONDITION

We now investigate the propagation of small perturbations in such a plasma. To this end, we linearize Eq.(7) with respect to the perturbations, which are represented as

$$\rho = \rho_\circ + \delta\rho expi(\mathbf{qr} - \Omega t), \quad \mathbf{P}(\mathbf{R},t,\mathbf{k},\omega) = \mathbf{P}_\circ(\mathbf{k},\omega) + \delta\mathbf{P} expi(\mathbf{qr} - \Omega t) \ . \tag{8}$$

The result is

$$(\mathbf{qk} - \Omega\omega)\delta\mathbf{P} = \delta\rho \sum_{l=0}^{\infty} \frac{1}{(2l+1)!}\frac{1}{2^{2l+1}}\Big(\mathbf{q}\nabla_k + \Omega\frac{\partial}{\partial\omega}\Big)^{2l+1} \cdot \mathbf{P}_\circ(\mathbf{k},\omega) \ , \tag{9}$$

or after summation we obtain the following relation

$$(\mathbf{qk} - \Omega\omega)\delta\mathbf{P} = \delta\rho\Big\{\mathbf{P}_\circ^+\Big(\mathbf{k} + \frac{\mathbf{q}}{2}, \omega + \frac{\Omega}{2}\Big) - \mathbf{P}_\circ^-\Big(\mathbf{k} - \frac{\mathbf{q}}{2}, \omega - \frac{\Omega}{2}\Big)\Big\} \ . \tag{10}$$

Then from Eq.(6) we have for the perturbation of Π,

$$\delta\Pi = \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \cdot \frac{\mathbf{P}_\circ^+ - \mathbf{P}_\circ^-}{\mathbf{qk} - \Omega\omega} \cdot \delta\rho \tag{11}$$

and for $\delta\rho$ we can write

$$\delta\rho = \frac{\delta n}{\gamma_\circ} - \frac{1}{2\gamma_\circ^3}\delta\Pi \ . \tag{12}$$

¿From Eqs.(11) and (12) follows the relation between $\delta\Pi$ and δn

$$\Big\{1 + \frac{1}{2\gamma_\circ^3}\int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{\mathbf{P}_\circ^+ - \mathbf{P}_\circ^-}{\mathbf{qk} - \Omega\omega}\Big\}\delta\Pi = \frac{\delta n}{\gamma_\circ} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{\mathbf{P}_\circ^+ - \mathbf{P}_\circ^-}{\mathbf{qk} - \Omega\omega} \ . \tag{13}$$

In the absence of the density perturbation δn we get from Eq.(13) the dispersion relation due to relativistic selfmodulation

$$1+\frac{1}{2\gamma_\circ^3}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{\mathbf{P}_\circ^+-\mathbf{P}_\circ^-}{\mathbf{qk}-\Omega\omega}=0\ . \tag{14}$$

Equation (14), as well as the case with $\delta n\neq 0$, has been studied in Ref.[9] for monochromatic waves.

We now write the relativistic expression for the ponderomotive force

$$\mathbf{F}=-\nabla\gamma=-\nabla(1+\Pi(\mathbf{R},t))^{1/2}\ . \tag{15}$$

After linearization with respect to the perturbation we have

$$\mathbf{F}=-\frac{1}{2\gamma_\circ}\nabla\delta\Pi expi(\mathbf{qr}-\Omega t)\ , \tag{16}$$

or using Eq.(13) we obtain

$$\mathbf{F}=-\frac{1}{2\gamma_\circ^2}\frac{\int d\mathbf{k}/(2\pi)^3\int(d\omega/2\pi)(\mathbf{P}_\circ^+-\mathbf{P}_\circ^-)/(\mathbf{qk}-\Omega\omega)}{1+(1/2\gamma_\circ^3)\int d\mathbf{k}/(2\pi)^3\int(d\omega/2\pi)(\mathbf{P}_\circ^+-\mathbf{P}_\circ^-)/(\mathbf{qk}-\Omega\omega)}\cdot\nabla\delta nexpi(\mathbf{qr}-\Omega t)\ . \tag{17}$$

Some interesting relativistic features follow from the expression of the ponderomotive force (17). First, in the case when the dominator goes to zero $\mathbf{F}$ increases, or $\delta n\to 0$. Second, when the integral in the dominator becomes much greater than unity, we have

$$\mathbf{F}=-\gamma_\circ\nabla\delta nexpi(\mathbf{qr}-\Omega t)\ . \tag{18}$$

This expression of the ponderomotive force coincides formally with the gasdynamic force, only instead of the temperature we have $m_\circ\gamma_\circ c^2$ in Eq.(18), and it exists only for the relativistic motion of the electrons in a superstrong short pulse laser.

Now, if we write kinetic equations for electrons and ions with the ponderomotive force (15) and linearize them, taking into account the relation (13), we obtain the general dispersion relation. The result is

$$\varepsilon\Big(1+\frac{1}{2\gamma_\circ^3}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{\mathbf{P}_\circ^+-\mathbf{P}_\circ^-}{\mathbf{qk}-\Omega\omega}\Big)+(1+\delta\varepsilon_i)\delta\varepsilon_e\frac{q^2c^2}{\omega_{pe}^2}\frac{1}{2\gamma_\circ^3}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{\mathbf{P}_\circ^+-\mathbf{P}_\circ^-}{\mathbf{qk}-\Omega\omega}=0\ , \tag{19}$$

where

$$\varepsilon = 1 + \delta\varepsilon_e + \delta\varepsilon_i, \quad \delta\varepsilon_\alpha = \frac{4\pi e^2}{q^2}\int \frac{(\mathbf{q}\partial f_{\circ\alpha}/\partial \mathbf{p})}{\Omega - \mathbf{qv}} d\mathbf{p} \ .$$

Equation (19) has several kinds of complex solutions for Ω, resulting in a different type of instability. But here we focus our attention on the case of the propagation of a stationary longitudinal wave in a plasma due to a strong laser pulse. Such a possibility exists, if

$$\frac{1}{2\gamma_\circ^3}\int \frac{d\mathbf{k}}{(2\pi)^3}\int \frac{d\omega}{2\pi}\frac{\mathbf{P}_\circ^+ - \mathbf{P}_\circ^-}{\mathbf{qk} - \Omega\omega} \gg 1 \tag{20}$$

and from Eq.(19), then we have

$$(1 + \delta\varepsilon_i)\Big(1 + \delta\varepsilon_e \frac{q^2c^2}{\omega_{pe}^2}\Big) + \delta\varepsilon_e = 0 \ . \tag{21}$$

The dispersion relation (21) describes the propagation of a stationary longitudinal wave in the presence of relativistically intense EM waves.

Let us now consider some special cases. First, in the case when only electrons participate in the oscillation, i.e. $\delta\varepsilon_i = 0$, for $\Omega \gg qv_{tre}$, where $v_{tre} = (T_e/m_\circ)^{1/2}$, we obtain from Eq.(21)

$$\Omega^2 = \omega_{pe}^2 + q^2c^2 \ . \tag{22}$$

This is a novel Langmuir wave due to strong relativistic effects. The physical interpretation for Eq. (22) is that the strong ponderomotive force not only leads to the separation of charge and creation of the longitudinal self-consistent field, but also generates the dispersion term q^2c^2, which is due to the strong coupling of EM waves with the electrons.

Next in the case, when $\delta n_i \neq 0$, two frequency ranges can be considered for Ω. One is $kv_{tri} \ll \Omega \ll kv_{tre}$, and the other is $kv_{tre} \ll \Omega \ll \omega_{pe}$. For both cases we obtain from Eq.(21) a new type of ion-sound solution

$$\Omega = \Big(\frac{m_\circ\gamma_\circ}{m_i}\Big)^{1/2}\frac{qc}{(1 + q^2c^2/\omega_{pe}^2)^{1/2}} = \frac{qc_s}{(1 + q^2c^2/\omega_{pe}^2)^{1/2}} \ . \tag{23}$$

It is clear that now the characteristic length of the inhomogeneity is comparable to c/ω_{pe}, but not to the electron Debye length as we have for the ion-sound wave without a laser pulse. As follows from Eq.(23) the Maximum value of the frequency is ω_{pi}. We specifically note here that $c_s = c(m_o\gamma_o/m_i)^{1/2}$ now depends not only on the mass of the particles, but also on the intensity of the laser pulse ($\gamma_o = (1 + \Pi_o)^{1/2}$). Therefore, it is possible to observe these waves in an experiment by changing the intensity of the laser pulse and the sort of gas.

We now try to understand physically existence of the solutions (22) and (23). First note that, for the stationary case when the laser pulse propagates with a constant velocity $\mathbf{v} = \mathbf{k}c^2/\omega$ ($\mathbf{P}(\mathbf{R}, t, \mathbf{k}, \omega) = \mathbf{P}(\mathbf{k}, \omega, R - vt)$), the result (18) for the ponderomotive force can be obtained without the linearization of equation (7). In this case , the left-hand side of Eq.(7) becomes zero and one of the solutions of equation (7) is

$$\rho = \frac{n(\mathbf{R}, t)}{\gamma(\mathbf{R}, t)} = constant \ . \tag{24}$$

Equation (24) can be expressed as $n/m_e(\gamma) = constant$, which shows that the plasma density and the mass of the electrons satisfy the "frozen-in" condition. This means that there is a localization of the energy of the laser pulse in the region of high density. In other words, the behaviour of the plasma-photon gas system is similar to a one component fluid. The solution identical with Eq.(24) was found in Ref.[27], considering the strong EM wave propagation in an electron-positron plasma. In the case when expression (24) is valid, we obtain the simple expression for the ponderomotive force from Eq.(15) for arbitrary variation of the density

$$\mathbf{F} = -m_o\gamma_o c^2 \nabla \frac{n}{n_o} \ . \tag{25}$$

One can simply show that if the "frozen-in" condition is fulfield, the hydrodynamic equations, the equation of motion, and the equation of continuity for electrons, become linear and for an arbitrary variation of the electron density we have the following linear equation

$$\Big(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - c^2\Delta\Big)\frac{n - n_o}{n_o} = 0 \tag{26}$$

This equation shows that for plane waves one can obtain the same dispersion relation as Eq.(22).

It is important to emphasize that Eq.(1) with condition (24) becomes a linear equation and EM wave momentum with arbitrary power will always spread out in a plasma.

4. STATIONARY PERIODIC AND SOLITARY WAVES

In this section, we consider the propagation of stationary nonlinear ion-sound waves, when the phase velocity of the new type of ion-sound waves is large compared with the electron thermal velocity and the plasma density along with the mass of the electrons satisfies the "frozen-in" condition (24). To describe the one-dimensional motion of the electrons and ions of such waves we employ hydrodynamic equations with a self consistent field:

$$\frac{\partial p_e}{\partial t} = e\frac{\partial \phi}{\partial x} - m_o c^2 \gamma_o \frac{\partial}{\partial x}\frac{n}{n_o} , \tag{27}$$

$$m_i\Big(\frac{\partial}{\partial t} + u_i\frac{\partial}{\partial x}\Big)u_i = -e\frac{\partial \phi}{\partial x} , \tag{28}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} n u_e = 0 , \tag{29}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} n_i u_i = 0 . \tag{30}$$

Here u_i, n_i are the ion velocity and density, respectively. ϕ is the electrostatic potential, which is coupled with the electron and ion densities through the Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n - n_i) . \tag{31}$$

Equations (27)-(31) are a closed set of equations describing the propagation of one-dimensional stationary waves. In this case all quantities depend on coordinates and time as $x - vt$, where v is constant. ¿From Eqs. (27)-(30) it is easy to obtain the following expressions for electron and ion densities

$$\frac{n}{n_o} = 1 + \frac{e\phi}{m_o \gamma_o c^2} , \tag{32}$$

$$\frac{n_i}{n_o} = \Big(1 - \frac{2e\phi}{m_i v^2}\Big)^{-1/2} . \tag{33}$$

Substituting these expressions for the densities into the Poisson equation, we get

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e n_o\Big(1 + \frac{e\phi}{m_o \gamma_o c^2} - \Big(1 - \frac{2e\phi}{m_i v^2}\Big)^{-1/2}\Big) . \tag{34}$$

Now we first consider the case when $e\phi \ll m_i v^2/2$, i.e., stationary waves with weak non-linearity. In this case, the last term in Eq.(34) can be expanded in a power series, and we obtain

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\omega_{pi}^2}{v^2}\Big(\frac{v^2}{c_s^2} - 1\Big)\phi - \frac{3}{2}\frac{\omega_{pi}^2}{v^2}\frac{e\phi^2}{m_i v^2} \ . \tag{35}$$

If we neglect the last term in Eq.(35), then there are two possibilities in the linear approximation. The first is the propagation of the ion-sound waves with the velocity given in Eq.(23), when $m_i v^2 < m_\circ \gamma_\circ c^2$. In the second case, when the oposite unequality is valid, we obtain a type of the Debye potential with the characteristic scale length

$$r_D = \frac{c}{\omega_{pe}(1 - c_s^2/v^2)^{1/2}} \ . \tag{36}$$

This expression shows that the effect of the Coulomb field of charge extends to a distance of the order of the "Debye length" r_D, beyond which it essentially vanishes.

Now let us consider the structure of a solitary wave, for this it is necessary that $m_i v^2 > m_\circ \gamma_\circ c^2$, then the solution of equation (35) is

$$e\phi = \frac{m_i v^2 (v^2/c_s^2 - 1)}{ch^2(\omega_{pi}/v)(v^2/c_s^2 - 1)^{1/2}(x - vt)} \tag{37}$$

and

$$\frac{n}{n_\circ} = 1 + \frac{v^2}{c_s^2}\Big(\frac{v^2}{c_s^2} - 1\Big)\frac{1}{ch^2(\omega_{pi}/v)(v^2/c_s^2 - 1)^{1/2}(x - vt)} \tag{38}$$

Since we have supposed that $e\phi \ll m_i v^2/2$, from Eq.(37) it is clear that $\mid v^2/c_s^2 - 1 \mid \ll 1$. The relation between the propagation velocity v and the maximum amplitude ϕ_{max} of the wave, can be obtained from Eq.(37),

$$v^2 = c_s^2 + \frac{e\phi_{max}}{m_i} \tag{39}$$

and now for ϕ we have

$$\phi = \frac{\phi_{max}}{ch^2(\omega_{pi}/c_s^2)(e\phi_{max}/m_i)^{1/2}(x - vt)} \ . \tag{40}$$

We see that $n > n_\circ$ and $n_i > n_\circ$, since $\phi > 0$. A solitary wave in a quasi-equilibrium plasma is therefore always a compressional wave.

Turning now to the study of the equation (34), we integrate it once to obtain

$$E^2(\phi) = \left(\frac{\partial\phi}{\partial x}\right)^2 = 4\pi e n_\circ\left\{2\phi + \frac{e\phi^2}{m_\circ\gamma_\circ c^2} + \frac{2}{e}m_i v^2\left(1 - \frac{2e\phi}{m_i v^2}\right)^{1/2}\right\} + A \ . \tag{41}$$

Various periodic waves can be now found depending on the choice of the integration constant A. In the case when ϕ and $\partial\phi/\partial x \to 0$ at $\mid x - vt \mid \to \infty$, we have $A = -8\pi n_\circ m_i v^2$. This case corresponds to a solitary wave, and we easily find the equation which determines the potential ϕ as a function of the coordinates and time

$$x - vt = \pm\int\frac{d\phi}{\left(E^2(\phi)\right)^{1/2}} \ . \tag{42}$$

The velocity of propagation of this wave v, as a function of the maximum amplitude of the wave ϕ_{max}, is found from Eq.(41) by writing $\partial\phi/\partial x = 0$ at $\phi = \phi_{max}$, i.e.

$$2e\phi_{max} + \frac{e^2\phi^2_{max}}{m_\circ\gamma_\circ c^2} + 2m_i v^2\left\{\left(1 - \frac{2e\phi_{max}}{m_i v^2}\right)^{1/2} - 1\right\} = 0 \ . \tag{43}$$

Finally, the relation between the wave amplitude and its propagation velocity is

$$v = c_s\left(1 + \frac{e\phi_{max}}{2m_\circ\gamma_\circ c^2}\right) \ . \tag{44}$$

We note here that equation (43) has a solution only provided ϕ_{max} is not too large. ¿From Eq.(43) it follows, that the maximum possible value of the amplitude of the ion-sound wave can be determined from the relation $m_i v^2/2 = e\phi_{max}$, because ions can no longer get across the potential barrier. Solving this equation together with equation

(44) we obtain $e\phi_{max} = 2m_\circ\gamma_\circ c^2$ and for the velocity of the stationary ion-sound solitary wave, $v = 2c_s$.

5. EMISSION OF A NEW TYPE OF ION-SOUND WAVES BY ACCELERATION OF A SOLITON

It has been shown in Refs.[15,32] that a soliton moving in an inhomogeneous plasma with acceleration can, like a particle, emit different kind of waves.

In this section we include in our consideration the plasma inhomogeneity, which leads to laser pulse acceleration, defining the acceleration by a. In the following we derive the equation for the perturbation of acceleration, using Eq.(33) of Ref.[34]:

$$\frac{d}{dt} < \mathbf{v} >= -\frac{Uc^2}{2n_\gamma}\nabla\rho - \frac{1}{n_\gamma}\nabla P_\gamma + \frac{U}{n_\gamma} < \mathbf{v} > \frac{\partial \rho}{\partial t} , \tag{45}$$

where

$$n_\gamma = 2\int \frac{d\mathbf{k}}{(2\pi)^3} N_k$$

is the density of the photon gas, N_k is the occupation number in the $\mathbf{K}$th state,

$$U = 2\omega_{pe}^2 \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{N_k}{\omega^2(k)} = \frac{\omega_{pe}^2}{< \omega_k^2 >} n_\gamma ,$$

$$P_\gamma = \frac{2}{3}\int \frac{d\mathbf{k}}{(2\pi)^3}\Big(\frac{\mathbf{k}c^2}{\omega} - < \mathbf{v} >\Big)^2$$

is the pressure of the photon gas, and $< \mathbf{v} >$ is the photon mean velocity. The factor 2 comes in n_γ, U, P_γ and $< \mathbf{v} >$ due to the existence of two independent photon polarization states.

Now the expression for ρ is

$$\rho = \frac{n + \Delta n(\mathbf{r})}{n_\circ \gamma} = \frac{1}{\gamma_\circ} + \frac{\Delta n(\mathbf{r})}{n_\circ \gamma} = \rho_\circ + \rho\prime\Big(x - x\prime(t)\Big) ,$$

here $\Delta n(\mathbf{r})$ is due to an inhomogeneity. In the absence of inhomogeneity, i.e. $\Delta n(\mathbf{r}) = 0$, $< \mathbf{v} >$ is a constant.

We now suppose that the first term on the right hand-side of Eq.(45) is large than the last two terms, as $< \mathbf{v} >^2 /c^2$ and $< (\delta \mathbf{v})^2 > /c^2$, where $< (\delta \mathbf{v})^2 >$ is the average of the square of the thermal velocity of photons.

Let us consider the case of a weak inhomogeneity along the x direction, using the second Hamilton equation $dx\prime(t)/dt =< v_x >=< kc^2/\omega >$, and hereby we denote $dx\prime(t)/dt$ as $\dot{x}\prime(t)$.

¿From Eq.(45) for the coordinate of the center of the pulse (or soliton) $x\prime(t)$ we obtain equation of the acceleration

$$\frac{d^2x\prime(t)}{dt^2} = -\frac{1}{2}\frac{\omega_{pe}^2}{<\omega_k^2>}\frac{\partial}{\partial x}\rho\prime \ . \tag{46}$$

In the following, we shall assume that the density of the inhomogeneous plasma changes over a distance of the order of the width of the pulse. Then we can limit ourselves in the expansion

$$\rho\prime = \rho\prime\Big(x\prime(t)\Big) + \Big(x - x\prime(t)\Big)\frac{\partial\rho\prime}{\partial x}\,|_{x=x\prime(t)} + \frac{\Big(x - x\prime(t)\Big)^2}{2}\frac{\partial^2\rho\prime}{\partial x^2}\,|_{x=x\prime(t)} \ . \tag{47}$$

Substitution of Eq.(47) into Eq.(46) yields the following equation of the acceleration

$$\frac{d^2x\prime(t)}{dt^2} = -\frac{1}{2}\frac{\omega_{pe}^2}{<\omega_k^2>}\Big(\frac{\partial}{\partial x}\rho\prime\Big)_{x=x\prime(t)} \ . \tag{48}$$

We note here that equation (48) has been derived in Ref.[15] from the set of Maxwell equations for the nonrelativistic case.

Let us now consider the emission of the new type of ion-sound waves in a weak inhomogeneous plasma. To this end, we linearize the set of equations (27)-(30), taking into account the condition of quasineutrality ($\delta n \simeq \delta n_i$), to obtain

$$\Big(\frac{\partial^2}{\partial t^2} - c_s^2\frac{\partial^2}{\partial x^2}\Big)\frac{\delta n}{n_\circ} = c_s^2\frac{\partial^2}{\partial x^2}\frac{\gamma}{\gamma_\circ} \ . \tag{49}$$

In equation (49) for δn we isolate that part of the density perturbation which is concentrated within the pulse:

$$\frac{\delta n}{n_\circ} = -\frac{\gamma\Big(x - x\prime(t)\Big)}{1 - \dot{x}\prime^2/c_s^2} + N_R \ , \tag{50}$$

The quantity N_R characterizes the perturbation of the density by the ion sound outside the pulse (or soliton, e.g. Eq.(37)).

Now we shall assume that the velocity of the sourse of radiation of the pulse is much less than the sound velocity, i.e.,

$$\dot{x}\prime(t) \ll c_s \ , \tag{51}$$

and the change in the velocity of the pulse within the time that the ion sound passes through a distance of the order of the width of the pulse is small in comparision with the ion-sound velocity, i.e. ,

$$\ddot{x}\prime(t) \cdot d \ll c_s^2 \ , \tag{52}$$

where d is the width of the pulse (or soliton).

Taking into account inequalities (51) and (52), and substituting expression (50) into Eq.(49) we obtain for N_R the following equation

$$\Big(\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial x^2}\Big) N_R = -\ddot{x}\prime \frac{\partial}{\partial x} \gamma\big(x - x\prime(t)\big) \ . \tag{53}$$

¿From this equation we can find the explicit form of the density distribution in the emitted ion sound. We introduce the natural initial condition: at the initial instant of time, $t = 0$, let the emission be absent, $N_R = 0$. This means that from the moment $t = 0$ the pulse (soliton) begins to move with acceleration. Then we have

$$N_R = -\frac{1}{2} \int_0^t dt\prime \ddot{x}\prime(t\prime) \int_{x - x\prime(t\prime) - c_s(t - t\prime)}^{x - x\prime(t\prime) + c_s(t - t\prime)} dz \frac{\partial}{\partial z} \gamma(z - z\prime(t\prime)) \ . \tag{54}$$

The analyses of the integrals in Eq.(54) are given explicitly in Ref.[15].

If we want to analyze the radiation field, it is necessary to indicate the profile of the equilibrium density. For a linear profile, $\triangle n(x) = n_o x/L$, the motion of the center of the pulse (soliton) is described, according to Eq.(48), by the equation

$$x\prime(t) = x\prime(0) + vt - \frac{1}{2} a t^2 \ , \tag{55}$$

where

$$a = \frac{\omega_{pe}^2 c^2}{2 < \omega_k^2 >} \frac{1}{L} \ ,$$

and L is the characteristic lenght of inhomogeneity.

6. SUMMARY

We have investigated the propagation of a relativistically intense short laser pulse into an isotropic plasma. Starting from the fully relativistic equations, we have derived a general kinetic equation for the photon gas. This is valid for waves with a large spectral width. The relativistic expression for the ponderomotive force is also derived and some interesting relativistic features are discussed. The kinetic equation was used to derive the plasma wave dispersion relation and the propagation of stationary longitudinal waves in the presence of relativistically intense EM waves is studied. Due to strong relativistic effects a novel Langmuir waves, with phase velocities larger than the speed of light, and waves of ion-sound type, which are damped only on ions, are found. In addition, for the case when the plasma density along with the mass of the electrons satisfies the "frozen-in" condition, stationary periodic and solitary waves are studied. The relation between the wave amplitude and its propagation velocity is derived. Finally, the mechanism of the emission of a new type of ion-sound waves is discussed.

Acknowledgments

Authors gratefully acknowledge Prof. Nodar L.Tsintsadze, Dr. Y. Kishimoto, Dr. T. Arisawa, and Dr. J. Koga for useful comments and discussions.

REFERENCES

1. A.I.Akhieser and R.V.Polovin, Sov. Phys. JETP **3**, 696 (1956).

2. N.L.Tsintsadze, Sov. Phys. JETP **32**, 684 (1971).

3. J.F.Drake, Y.C.Lee, K.Nishikawa, and N.L.Tsintsadze, Phys. Rev. Lett. **36**, 196 (1976).

4. A.G.Litvak, Sov. Phys. JETP **30**, 344 (1970).

5. N.L.Tsintsadze, Phys. Lett. **50**A, 33 (1974).

6. P.Kaw and J.Dawson, Phys. Fluids **13**, 472 (1970).

7. N.L.Tsintsadze and D.D.Tskhakaya, Sov. Phys. JETP **45**, 252 (1977).

8. D.P.Garuchava, N.L.Tsintsadze, D.D.Tskhakaya, Sov. Phys. JETP **71**(5), 873 (1990).

9. L.N.Tsintsadze, Sov. J. Plasma Phys. **17**, 872 (1991).

10. M.N.Rosenbluth and C.S.Liu, Phys. Rev. Lett. **29**, 702 (1972).

11. C.E.Max, J.Arons and A.Bruce Langdon, Phys. Rev. Lett. **33**,209 (1974).

12. J.F.Drake, Y.C.Lee, and N.L.Tsintsadze, Phys. Rev. Lett. **36**, 31 (1976).

13. T.Tajima and J.M.Dawson, Phys. Rev. Lett. **43**, 267 (1979).

14. V.I.Berezhiani, N.L.Tsintsadze, D.D.Tskhakaya, J. Plasma Phys. **24**, 15 (1980).

15. P.Kaw, N.L.Tsintsadze, D.D.Tskhakaya, Sov. Phys. JETP **55**, 839 (1982).

16. P.K.Shukla, M.Y.Yu and N.L.Tsintsadze, Phys. Fluids **27**, 327 (1984).

17. T.Tajima, Laser. Part. Beams **3**, 351 (1985).

18. P.Chen, J.M.Dawson, R.W.Huff and T.Katsouleas, Phys. Rev. Lett. **54**, 693 (1985).

19. P.K.Shukla, N.N.Rao, M.Y.Yu and N.L.Tsintsadze, Phys. Rep. **138**, 1-49 (1986).

20. D.P.Garuchava, Z.I.Rostomashvili and N.L.Tsintsadze, Sov. J. Plasma Phys. **12**, 776 (1986); D.P.Garuchava, I.G.Murusidze, G.I.Suramlishvili, N.L.Tsintsadze, D.D.Tskhakaya, Sov. J.Plasma Phys. Rep. **22**, 841 (1996).

21. P.K.Shukla, R.Bharuthram and N.L.Tsintsadze, Phys. Rev. A **35**, 4889 (1987).

22. P.Sprangle, E.Esaray, A.Ting and G.Joyce, Appl. Phys. Lett. **53**, 2746 (1988).

23. W.B.Mori, C.Joshi, J.Dawson, D.W.Forslund and J.M.Kindel, Phys. Rev. Lett. **60**, 1295 (1988).

24. N.L.Tsintsadze, Phys. Scr. **41**, 267 (1990).

25. T.Katsouleas and W.Mori, Phys. Rev. Lett. **61**, 90 (1990).

26. L.N.Tsintsadze, Proc. *International Topical Conf. on Research Trends in Coherent Radiation Generation and Particle Accelerators* (La Jolla, 1991), ed. by J.M.Buzzi, P.Sprangle, K.Wille (AIP, New York, 1992), 474.

27. V.I.Berezhiani, L.N.Tsintsadze, P.K.Shukla, J. Plasma Phys. **48**, 139 (1992).

28. M.Tabak et al, Phys. Plasmas **1**, 1626 (1994).

29. N.L.Tsintsadze, Proc. *International Conference on Plasma Physics* (Foz Do Iguacu, 1994), ed. by P.H.Sakanaka (AIP, New York, 1995), 290.

30. L.N.Tsintsadze, K.Kusano, and K.Nishikawa, Phys. Plasmas **4**(4), 911 (1997); L.N.Tsintsadze and K.Nishikawa, Phys. Plasmas **3**, 511 (1996).

31. J.C.Adam, A.Heron, S.Guerin, G.Laval, P.Mora, and B.Quesnel, Phys. Rev. Lett. **78**, 4765 (1997); S.Guerin, P.Mora and G.Laval, Phys. Plasmas **5**, 376 (1998).

32. N.L.Tsintsadze, J.T.Mendonca, L.Silva, Phys. Rev. E **58**, 4890 (1998).

33. L.N.Tsintsadze, K.Mima and K.Nishikawa, Plasma Phys. Control. Fusion **40**, 1933 (1998).

34. N.L.Tsintsadze and J.T.Mendonca, Phys. Plasmas **5**, 3609 (1998).

35. L.N.Tsintsadze and N.L.Tsintsadze, Proc. *International Conference on Superstrong Fields in Plasmas* (Varenna, 1997), edited by M.Lontano (AIP, New York, 1998), 170.

国際単位系（SI）と換算表

表1　SI基本単位および補助単位

量	名　称	記　号
長　　さ	メートル	m
質　　量	キログラム	kg
時　　間	秒	s
電　　流	アンペア	A
熱力学温度	ケルビン	K
物　質　量	モ　ル	mol
光　　度	カンデラ	cd
平　面　角	ラジアン	rad
立　体　角	ステラジアン	sr

表3　固有の名称をもつSI組立単位

量	名　称	記号	他のSI単位による表現
周　波　数	ヘルツ	Hz	s^{-1}
力	ニュートン	N	$m \cdot kg/s^2$
圧力，応力	パスカル	Pa	N/m^2
エネルギー,仕事,熱量	ジュール	J	$N \cdot m$
工率，放射束	ワット	W	J/s
電気量，電荷	クーロン	C	$A \cdot s$
電位，電圧，起電力	ボルト	V	W/A
静電容量	ファラド	F	C/V
電気抵抗	オーム	Ω	V/A
コンダクタンス	ジーメンス	S	A/V
磁　　束	ウェーバ	Wb	$V \cdot s$
磁束密度	テスラ	T	Wb/m^2
インダクタンス	ヘンリー	H	Wb/A
セルシウス温度	セルシウス度	℃	
光　　束	ルーメン	lm	$cd \cdot sr$
照　　度	ルクス	lx	lm/m^2
放　射　能	ベクレル	Bq	s^{-1}
吸収線量	グレイ	Gy	J/kg
線量等量	シーベルト	Sv	J/kg

表2　SIと併用される単位

名　称	記　号
分，時，日	min, h, d
度，分，秒	°, ′, ″
リットル	l, L
トン	t
電子ボルト	eV
原子質量単位	u

$1\ eV = 1.60218 \times 10^{-19} J$

$1\ u = 1.66054 \times 10^{-27} kg$

表4　SIと共に暫定的に維持される単位

名　称	記　号
オングストローム	Å
バーン	b
バール	bar
ガル	Gal
キュリー	Ci
レントゲン	R
ラド	rad
レム	rem

$1\ Å = 0.1 nm = 10^{-10} m$

$1\ b = 100 fm^2 = 10^{-28} m^2$

$1\ bar = 0.1 MPa = 10^5 Pa$

$1\ Gal = 1 cm/s^2 = 10^{-2} m/s^2$

$1\ Ci = 3.7 \times 10^{10} Bq$

$1\ R = 2.58 \times 10^{-4} C/kg$

$1\ rad = 1 cGy = 10^{-2} Gy$

$1\ rem = 1 cSv = 10^{-2} Sv$

表5　SI接頭語

倍数	接頭語	記　号
10^{18}	エクサ	E
10^{15}	ペタ	P
10^{12}	テラ	T
10^{9}	ギガ	G
10^{6}	メガ	M
10^{3}	キロ	k
10^{2}	ヘクト	h
10^{1}	デカ	da
10^{-1}	デシ	d
10^{-2}	センチ	c
10^{-3}	ミリ	m
10^{-6}	マイクロ	μ
10^{-9}	ナノ	n
10^{-12}	ピコ	p
10^{-15}	フェムト	f
10^{-18}	アト	a

(注)

1. 表1－5は「国際単位系」第5版，国際度量衡局1985年刊行による。ただし，1 eVおよび1 uの値はCODATAの1986年推奨値によった。
2. 表4には海里，ノット，アール，ヘクタールも含まれているが日常の単位なのでここでは省略した。
3. barは，JISでは流体の圧力を表わす場合に限り表2のカテゴリーに分類されている。
4. EC閣僚理事会指令ではbar，barnおよび「血圧の単位」mmHgを表2のカテゴリーに入れている。

換　算　表

力	$N (=10^5 dyn)$	kgf	lbf
	1	0.101972	0.224809
	9.80665	1	2.20462
	4.44822	0.453592	1

粘　度　$1\ Pa \cdot s (N \cdot s/m^2) = 10\ P$(ポアズ)$(g/(cm \cdot s))$

動粘度　$1 m^2/s = 10^4 St$(ストークス)(cm^2/s)

圧力	MPa(=10bar)	kgf/cm^2	atm	mmHg(Torr)	lbf/in^2(psi)
	1	10.1972	9.86923	7.50062×10^3	145.038
	0.0980665	1	0.967841	735.559	14.2233
	0.101325	1.03323	1	760	14.6959
	1.33322×10^{-4}	1.35951×10^{-3}	1.31579×10^{-3}	1	1.93368×10^{-2}
	6.89476×10^{-3}	7.03070×10^{-2}	6.80460×10^{-2}	51.7149	1

エネルギー・仕事・熱量	$J (=10^7 erg)$	$kgf \cdot m$	$kW \cdot h$	cal(計量法)	Btu	$ft \cdot lbf$	eV
	1	0.101972	2.77778×10^{-7}	0.238889	9.47813×10^{-4}	0.737562	6.24150×10^{18}
	9.80665	1	2.72407×10^{-6}	2.34270	9.29487×10^{-3}	7.23301	6.12082×10^{19}
	3.6×10^6	3.67098×10^5	1	8.59999×10^5	3412.13	2.65522×10^6	2.24694×10^{25}
	4.18605	0.426858	1.16279×10^{-6}	1	3.96759×10^{-3}	3.08747	2.61272×10^{19}
	1055.06	107.586	2.93072×10^{-4}	252.042	1	778.172	6.58515×10^{21}
	1.35582	0.138255	3.76616×10^{-7}	0.323890	1.28506×10^{-3}	1	8.46233×10^{18}
	1.60218×10^{-19}	1.63377×10^{-20}	4.45050×10^{-26}	3.82743×10^{-20}	1.51857×10^{-22}	1.18171×10^{-19}	1

1 cal = 4.18605J（計量法）
= 4.184J（熱化学）
= 4.1855J（15℃）
= 4.1868J（国際蒸気表）

仕事率　1 PS(仏馬力)
= 75 kgf・m/s
= 735.499W

放射能	Bq	Ci
	1	2.70270×10^{-11}
	3.7×10^{10}	1

吸収線量	Gy	rad
	1	100
	0.01	1

照射線量	C/kg	R
	1	3876
	2.58×10^{-4}	1

線量当量	Sv	rem
	1	100
	0.01	1

（86年12月26日現在）

STATIONARY PERIODIC AND SOLITARY WAVES INDUCED IN AN ISOTROPIC PLASMA BY A STRONG SHORT LASER PULSE