JNC TN9400 2001-007

Inelastic structural design approach using their relaxation locus

(Research Report)

August 2000

Japan Nuclear Cycle Development Institute
O-arai Engineering Center

KO

JNC TN9400 2001-007
August 2 0 0 0

Inelastic structural design approach using their relaxation locus

(Research Report)

Naoto Kasahara*

Abstract

Elevated temperature structural design codes pay attention to strain concentration at structural discontinuities due to creep and plasticity, since it causes to enlarge creep-fatigue damage of materials. One of the difficulties to predict strain concentration is its dependency on loading, constitutive equations, and relaxation time. This study investigated fundamental mechanism of strain concentration and its main factors. The results revealed that strain concentration was caused from strain redistribution between elastic and inelastic regions, which can be quantified by the characteristics of structural compliance. Characteristic of compliance is controlled by elastic region in structures and is insensitive to constitutive equations. It means that inelastic analysis is easily applied to get compliance characteristics. By utilizing this fact, simplified inelastic analysis method was proposed based on characteristics of compliance change for prediction of strain concentration.

* Structure and Material Research Group, System Engineering Division, OEC, JNC

JNC TN9400 2001-007
２ ０ ０ ０ 年 ８ 月

応力緩和軌跡を利用した非弾性設計法

（研究報告書）

笠原 直人*)

要　旨

高温低圧機器の構造不連続部では、ひずみ集中による強度低下を考慮した評価が必要となる。本研究では、弾塑性とクリープによる構造不連続部のひずみ集中挙動に関して以下を明らかにした。

ひずみ集中係数は、材料の構成方程式および応力緩和時間の各因子に依存して変化するが、ひずみ集中の要因であり応力とひずみの再配分は一本の応力緩和軌跡上で生じる。上記の応力緩和軌跡は構成方程式に鈍感であり、その理由は、応力ひずみの再配分特性がひずみ集中部を拘束する弾性領域のコンプライアンス特性によって規定されるためである。構造不連続部では、応力緩和の進展に伴う弾性領域の拘束力低下によって系のコンプライアンス特性が変化するため、応力緩和軌跡が曲線になる。

さらに、上記メカニズム分析結果に基づき、応力緩和軌跡の変化をモデル化した力学モデルを考案すると共に、本モデルを利用した非弾性設計アプローチを提案した。

尚、本内容は1999年9月から2000年8月までの期間にCEAカダラッシュ研究所にて実施した業務の一部である。

*) 大洗工学センター システム技術開発部 構造材料技術開発グループ

Contents

List of tables

List of figures

1 INTRODUCTION

In high temperature and low-pressure components such as Fast Breeder Reactors, initiation of creep-fatigue cracks by cyclic thermal stresses becomes one of main failure modes. Especially, consideration is paid to structural discontinuities, since strain concentration by plastic and creep deformation causes their strength degradation [1].

The elastic follow-up concept [2] can explain the qualitative mechanism of the strain concentration. When a high-rigid and low stress structure is connected to a low-rigid and high stress one and they are subjected to displacement load, creep deformation in the low-rigid portion unloads both portions. During relaxation process, elastic recovery of the high-rigid portion enhances displacement of the low-rigid one since total displacement is constant. As a result, the low-rigid part has a strain concentration, which does not occur in the elastic deformation.

For quantitative evaluation of strain concentration, the inelastic finite element method is expected because prediction of this behavior becomes a statically indeterminate problem in which stress and strain couple. However, there are some difficulties to apply it to design, such as dependency of their solutions on constitutive equations and load histories. Therefore, conventional designs were based the elastic analysis methods with help of conservative strain concentration factors [3], and their rationalization is requires from a demand on the plant economy.

In this paper, a quantitative mechanism of strain concentration is clarified through investigations of their sensitive factors by the inelastic finite element method. Finally, this study provides a realistic design approach based on the inelastic finite element method and a simplified method with elastic analysis.

2. STRAIN CONCENTRATION MECHANISM OF STRUCTURAL DISCONTINUITIES

2.1. PLASTIC BEHAVIOR

Elastic-plastic strain concentration behaviors in the Y-piece were examined, since the Y-piece can represent strain behaviors in structure discontinuities. Load conditions were assumed as quasi-static thermal transients with heating the inner surface of the cylinder to 550℃ while the skirt edge is kept 50℃.

The Y-piece is composed of a high-rigid portion (juncture) and a low-rigidity part (skirt part). Interaction of both portions causes a secondary stress and a peak stress is overlapped according to the local shape discontinuity of the juncture. The mechanism of stress generation is common among typical structures in plant components such as nozzles and tubesheets.

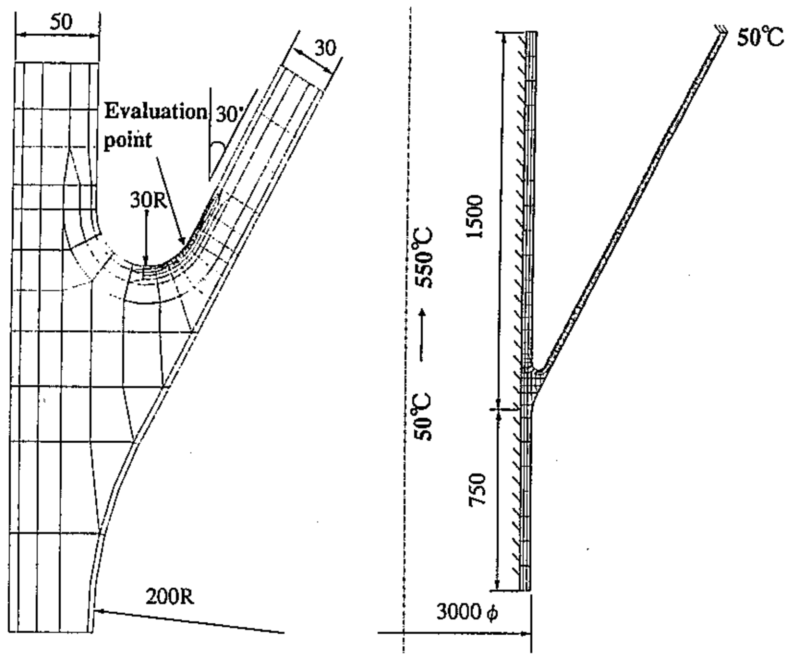


Fig.2.1 Y-piece structure due to thermal loading

The materials were assumed as 304SS and various constitutive equations are applied to investigate sensitivities of strain concentrations to them. Adopted equations are elastic one, Ramberg-osgood type that can simply express non-linearity of stress-strain relationship

$$\varepsilon = \sigma / E + A\sigma^{n} \quad (E=16200, A=1.0\times10^{-8}, n=3,5,7), \tag{2.1}$$

Ludwik type one that can consider yield stress

$$\varepsilon = \frac{\sigma}{E} + \left\{\frac{\sigma - \sigma_p}{K}\right\}^{n} \quad (\sigma > \sigma_p, \sigma_p=75, n=3,5,7), \tag{2.2}$$

perfectly plastic one and bi-linear equations with two different expected strain ranges. Temperature dependency of materials was ignored for clear understanding of phenomena. Strain behaviors with above constitutive equations were calculated according to the analysis cases in Table 2.1. The analysis code is a general-purpose nonlinear structural analysis system FINAS[4], and the mesh model is the axi-symmetric one of Fig.2.1.

Calculated stress-strain behaviors at the largest strain place (see Fig.2.1) were compared on the stress-strain chart of Fig.2.2[5]. These results exhibit dependency of strains on constitutive equations, and their strain concentration factor K_ε that is defined by ratios of elastic plastic strains ε_{in} to elastically calculated ones ε_e

$$K_\varepsilon = \varepsilon_{in} / \varepsilon_e \tag{2.3}$$

increases, when non-linearity of stress-strain relationship is stronger.

Table 2.1 Elastic-plastic analysis cases with various constitutive equations

Case	Analysis Type	Constitutive Equation	Power
TE	Elastic	Elastic	n=1
P3	Elastic-Plastic	Ramberg-Osgood	n=3
P5	Elastic-Plastic	Ramberg-Osgood	n=5
P7	Elastic-Plastic	Ramberg-Osgood	n=7
PL3	Elastic-Plastic	Ludwik	n=3
PL5	Elastic-Plastic	Ludwik	n=5
PL7	Elastic-Plastic	Ludwik	n=7
PP	Elastic-Plastic	Perfectly plastic	∞
PB5	Elastic-Plastic	Bi-linear	(Δt=0.5%)
PB10	Elastic-Plastic	Bi-linear	(Δt=1.0%)

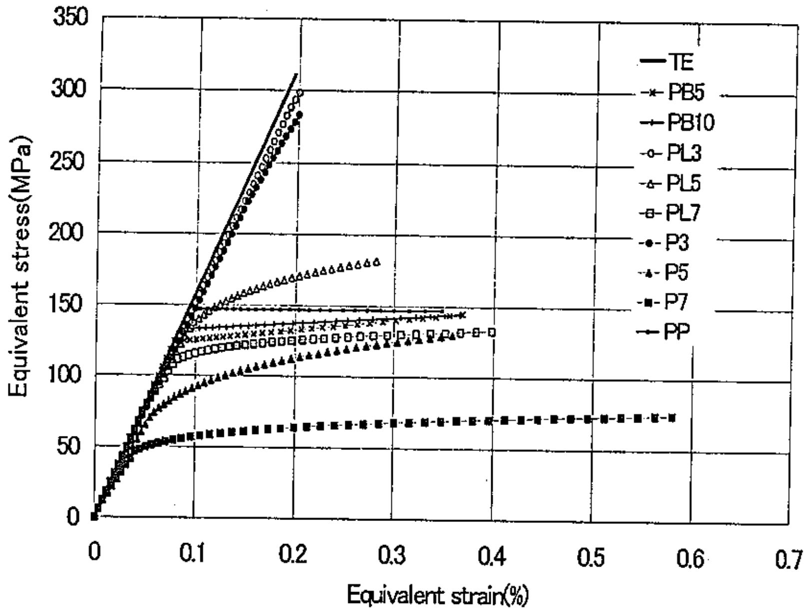


Fig.2.2 Elastic-plastic behaviors under various constitutive equations

Next, sensitivities to load levels were investigated. With the fixed constitutive equation as Ramberg-osgood type (n=5), strain behaviors were calculated for temperature increase to 217℃(1/3 σ max), to 383℃(2/3 σ max) and to 550℃(σ max) when temperature of the edge of the skirt was kept 50℃. The analysis cases are in Table 2.2 and obtained results are shown on the stress-strain chart of Fig.2.3[6].

The solution which corresponds to each load of 1/3 σ max, 2/3 σ max and σ max by the elastic plastic analysis P5 were compared with ones by elastic analysis TE (indicated by symbol □). These results proved that strain concentration coefficient K ε increases when the load level is bigger.

Table 2. 2 Elastic-plastic analysis cases under different load level

Case	Analysis Type	Constitutive Equation	Power	Loading
TE	Elastic	Elastic	n=1	$1/3\sigma_{max}, 2/3\sigma_{max}, \sigma_{max}$
P5	Elastic-Plastic	Ramberg-Osgood	n=5	$1/3\sigma_{max}, 2/3\sigma_{max}, \sigma_{max}$

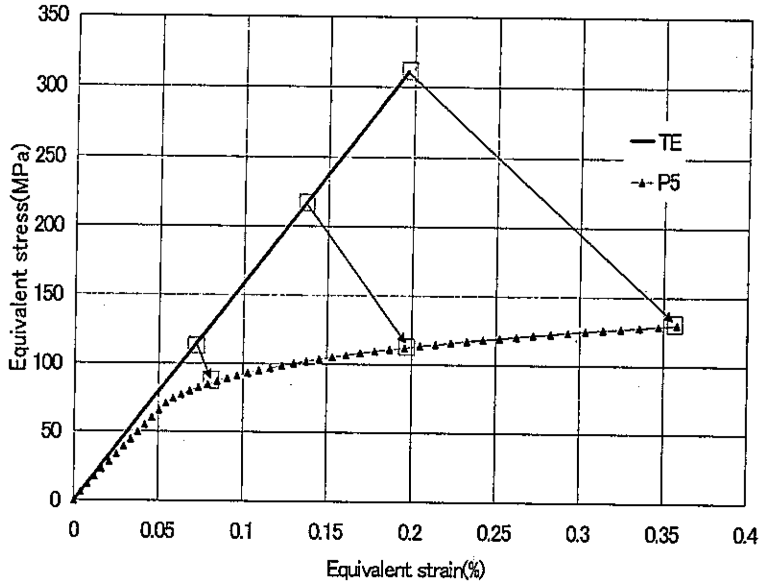


Fig.2. 3 Elastic-plastic behaviors under different load levels

2.2. CREEP BEHAVIOR

Strain behaviors at the maximum strain portion of the Y-piece were examined, during stress relaxation when surface of which is kept 550℃ and edge of the skirt is fixed at 50℃ as in Fig.2.1. For investigation of sensitivities to creep strain equations, various kinds of creep strain equations were utilized, such as Norton's law that can simply express non-linearity of materials

$$\varepsilon_c = B\sigma^m \quad (B=5.86\times10^{-15}, m=3,5,7), \tag{2.4}$$

and Blackburn type equation[7]

$$\varepsilon_c = C_1(1-e^{-r_1 t}) + C_2(1-e^{-r_2 t}) + \varepsilon_m \ t \tag{2.5}$$

which can describe detail characteristics.

The list of the analysis cases is in Table 2.3. The case of P5-C7 is the series of elastic-plastic analysis by a plastic equation P5 and elastic-creep one by a creep equation C7. Calculated stress relaxation behaviors for the 210,000 hours were compared on the stress-strain chart of Fig. 2.4[5]. Only a relaxation locus is shown for the case P5-P7. From this figure, strain concentration factor $K\varepsilon$ is increased with the progress for the stress relaxation hour, and it is proven that the size of strain concentration factor after the 210,000 hours depends on the creep strain equations[8].

In the meantime, all of the relaxation processes exist on the identical relaxation locus, regardless of the creep strain equations. Consequently, it was proven that structures have the unique characteristics that are insensitive to the creep strain equations.

Table 2.3 Creep analysis cases with various creep strain equations

Case	Analysis Type	Constitutive Equation	Power
TE	Elastic	Elastic	n=1
C3	Elastic-Creep	Norton's Law	m=3
C5	Elastic-Creep	Norton's Law	m=5
C7	Elastic-Creep	Norton's Law	m=7
P5-C7	Elastic-Plastic-Creep	Norton's Law Ramberg-Osgood	m=7 n=5
CBL	Elastic-Creep	Blackburn	—

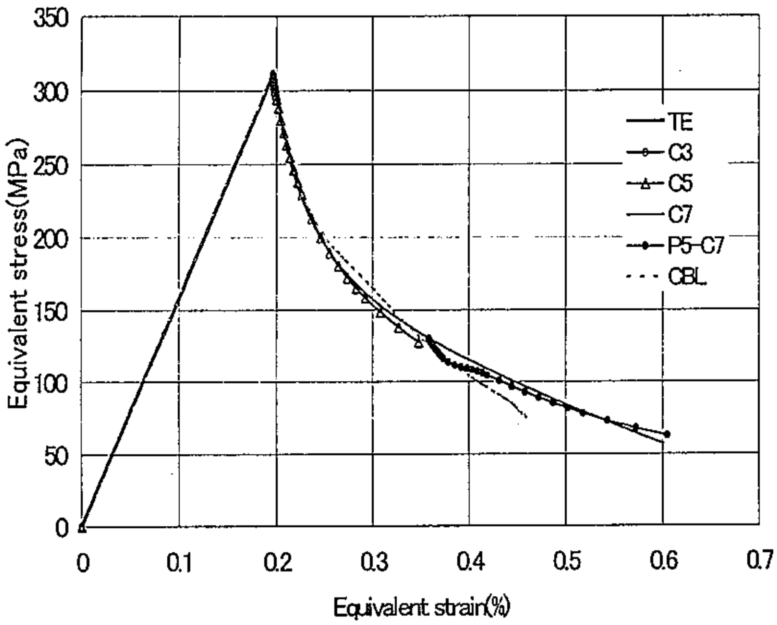


Fig.2.4 Stress relaxation behaviors with various creep strain equations

Next is the investigation of sensitivities to load levels. With the fixed constitutive equation as Norton's law (m=5), 21,000 hours stress relaxation behaviors were calculated when the inner surface of cylinder is 217℃(1/3 σ max), is 383℃(2/3 σ max) and is 550℃(σ max) under constant temperature 50℃ of the edge of the skirt. The analysis case is in Table 2.4, and calculated stress relaxation locus is shown in Fig.2.5[6].

From this figure, it is proven that stress relaxation quantity and strain concentration factors K ε are bigger, when the load level is higher.

Table 2. 4 Creep analysis cases under different load levels

Case	Analysis Type	Constitutive Equation	Power	Loading
TE	Elastic	Elastic	n=1	$1/3\sigma_{max}$, $2/3\sigma_{max}$, σ_{max}
C5-1/3	Elastic-Creep	Norton's Law	m=5	$1/3\sigma_{max}$
C5-2/3	Elastic-Creep	Norton's Law	m=5	$2/3\sigma_{max}$
C5	Elastic-Creep	Norton's Law	m=5	σ_{max}

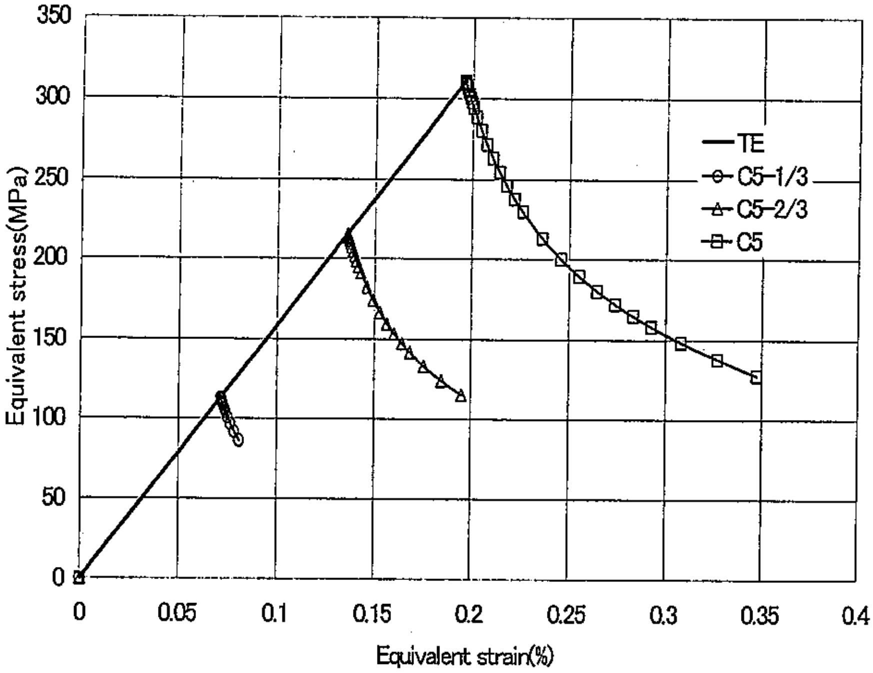


Fig.2. 5 Stress relaxation behaviors during 210,000 hours under different load revels

2.3. UNIFIED CHARACTERIZATION OF INELASTIC DEFORMATION BY RELAXATION LOCUS

To understand the basic mechanism of strain concentration, common factors are examined among influences of constitutive equation, of load level and of the stress relaxation period on the strain concentration behavior by plasticity and creep.

First, it is noticed that the stress relaxation processes of the same structures are not dependent on the creep strain equations and exist on the identical relaxation locus, as shown in Fig.2.4. Since the locus of P5-C7 case is equal to other cases calculated by creep analysis, there is the possibility of a common mechanism between plastic and creep behaviors. Then, try to plot elastic plastic stress-strain curves in Fig.2.2 with a stress relaxation curves of Fig.2.4. The result described in Fig.2.6 shows that the elastic plastic solutions exist on the almost equal stress relaxation locus. It can be estimated from these results that stress relaxation locus from a certain load means common strain concentration characteristics between plasticity and creep, which does not depend on both constitutive equations.

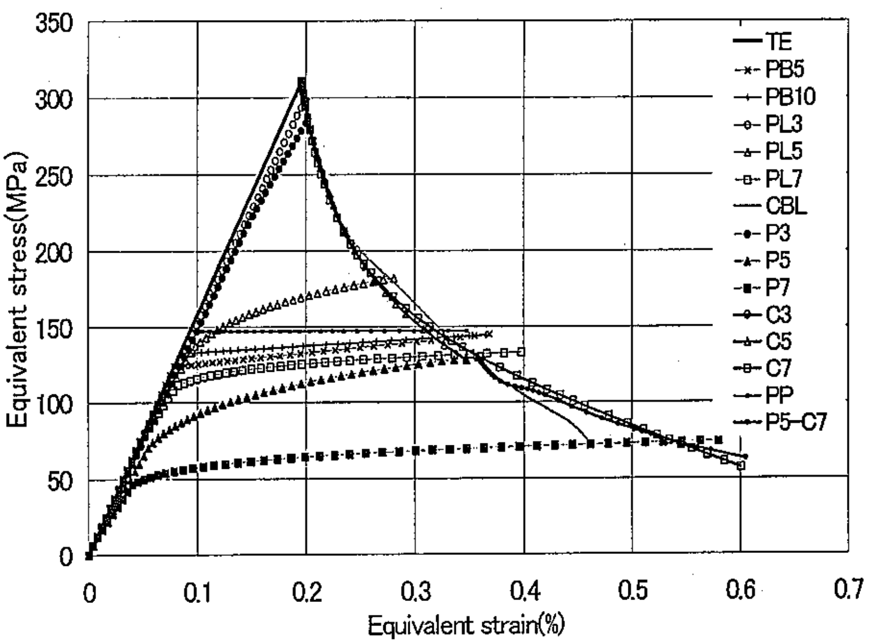


Fig.2. 6 Comparison of elastic plastic behaviors and creep ones with various constitutive equations

Next, the effect of the load level is examined. When over-plotting elastic plastic solutions for 3 kinds of load levels described in Fig.2.3 with ones for creep of Fig.2.5, results becomes Fig.2.7. Stress relaxation locus at the each load level crosses with the elastic plastic solution for the equal load level, and strain factor K_ε is identical for both plasticity and creep. Therefore, the load level seems to equivalently influence on elastic plastic and creep behaviors.

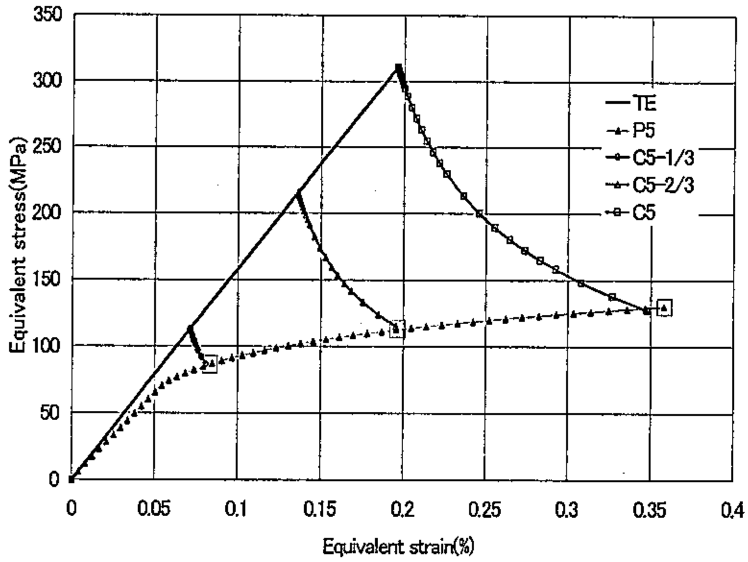


Fig.2. 7 Comparison of elastic plastic behaviors and creep ones under different load levels

2. 4. RELAXATION LOCUS REGULATED BY CHARACTERISTICS OF COMPLIANCE CHANGE

The reason why the stress relaxation locus exhibits the common strain concentration characteristics between plasticity and creep is investigated from a mechanical point of view. The elastic follow-up parameter q, defined by the gradient of the line which connects the elastic analysis result on the stress-strain chart of Fig.2.8 with the inelastic solution [7] is introduced here, in order to quantitatively handle the stress relaxation locus. Q=1 means that strain is constant and corresponds to a displacement-controlled condition. On the other hand, Q=∞ expresses a fixed stress and it is called a stress-controlled condition. Usual structures have intermediate conditions with q value between 1 and ∞, and this value becomes a parameter to quantify a relaxation locus.

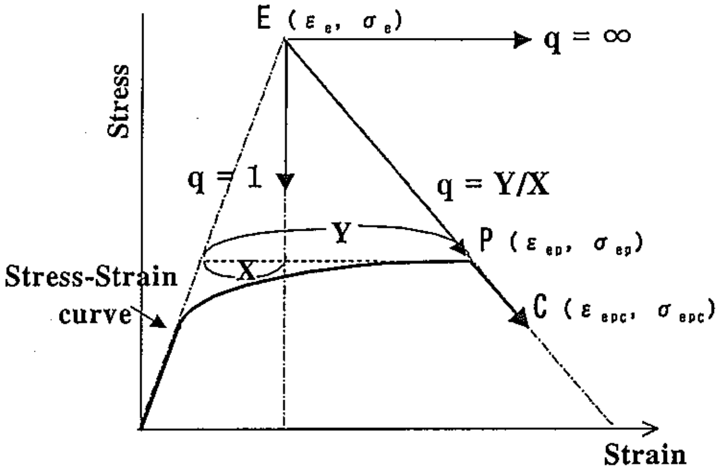


Fig.2. 8 Strain concentration behaviors described by elastic follow-up parameter q

Next, the characteristics of q value is examined on a two-bar model (Fig.2.9) composed of a low rigid high stress bar (i=1) and high-rigid low stress one (i=2), which simply explains elastic follow-up concept[2].

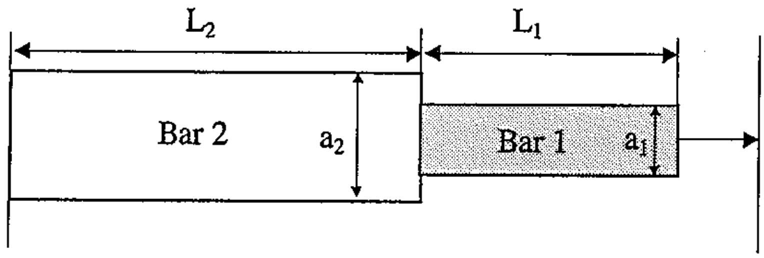


Fig.2. 9 Two bar model to explain elastic follow-up concept

When Norton' law (Eq.2.4) is assumed for a creep strain equation, the q value on the creep is deduced by an explicit equation as

$$q = \frac{1 + (l_2 / l_1)(a_1 / a_2)}{1 + (l_2 / l_1)(a_1 / a_2)^n} \text{ (creep)} \tag{2.6}$$

and it is proven to become the constant value, which depends on shape and non-linearity of the stress-strain relationship. Stress relaxation locus of the two-bar model becomes the straight line, because the q value means the gradient on the stress-strain chart.

In addition, the q value can be defined for elastic plastic behaviors as same way as creep (Fig.2.8). When assuming Ramberg-Osgood expression (Eq.2.4), we can obtain the similar equation to Eq.2.6 for plasticity as

$$q = \frac{1 + (l_2 / l_1)(a_1 / a_2)}{1 + (l_2 / l_1)(a_1 / a_2)^n} \text{ (Plasticity)} \tag{2.7}$$

In the case of actual structural design, the elastic region (elastic core) remains in wall thickness, since full section plasticity is not allowed. For this case, it is possible to consider the high-rigid low stress portion of Fig.2.9 as an elastic body. When Bar 2 is made of an elastic body Eq.(2.6) and Eq.(2.7) become the following same equation.

$$q = 1 + (l_2 / l_1)(a_1 / a_2) \tag{2.8}$$

Eq.(2.8) is a function only of the geometry which does not depend on constitutive equations. The q value has the physical meaning of the compliance characteristics of the system, because Eq.(2.8) can be described like the following by compliance Ci of Bar i, when Young's modulus of the material is E.

$$q = 1 + (l_2 / l_1)(Ea_1 / Ea_2) \\ = 1 + C_2 / C_1 \tag{2.9}$$

Next, the case of the Y-piece is examined. The progress of creep strain distribution was observed in the case which passed on the stress relaxation locus which follows m=7 Norton' law (C7) in Fig.2.10 with Time1～Time3.

The creep strain has been generated in low-rigidity high stress portion in the juncture of a skirt, as shown in the upper stage of Fig.2.11 and it is proven that the high-rigid low stress portion in the circumference remains in the elastic condition[8]. From this fact, the reason why the stress relaxation locus of the Y-piece is insensitive to constitutive equations seems to be for regulating the q value by the compliance characteristics, because the high-rigid low stress portion is the elasticity condition as well as the two-bar model.

When the stress relaxation progresses from Time 1 to Time 3, the creep strain region expands. Because redistribution of stress and strain can occur under the constant boundary condition since the Y-piece is the statically indeterminate structure[9]. This progression of creep regime leads to change of compliance ratio of the low-rigidity high stress portion (corresponds to C1 in Eq.(2.9)) to the high rigid low stress portion (corresponds to C2 in Eq.(2.9)). In other words, restraint force from the surrounded elastic region decreases.

Vary of compliance characteristic leads to change of q value that is a gradient on the stress-strain chart. As a result, stress relaxation locus would draw the curve.

When over plotting elastic plastic solutions (Cases P3, P5, P7 in Table 2.1) with relaxation locus of Case C7 on the stress-strain chart, P3,P5 and P7 stress-strain curves cross in each position of relaxation locus at Time1, Time2 and Time3 as in Fig.2.10. The lower stage of Fig.2.11 shows plastic strain distribution of P3,P5 and P7, and they are quite similar to the distribution of the creep strain at Time1, Time2 and Time3 on the stress relaxation locus. Surrounded portions are the elastic cores with the same configuration between plasticity and creep, which causes the same compliance in structures. From above observations, the reason why strain concentration characteristic is common between plasticity and creep and is insensitive to constitutive equations is concluded with that their q value is regulated by the compliance characteristics of the system, when the elastic region has been left in the structures.

In the meantime, the compliance of the Y-piece changes according to reduction of the elastic-core during relaxation, which can not be explained by the 2 bar model. Therefore, stress relaxation locus in the structural discontinuity becomes a curve, while relaxation locus of the 2 bar model with the fixed q value becomes a straight line like Fig.2.8.

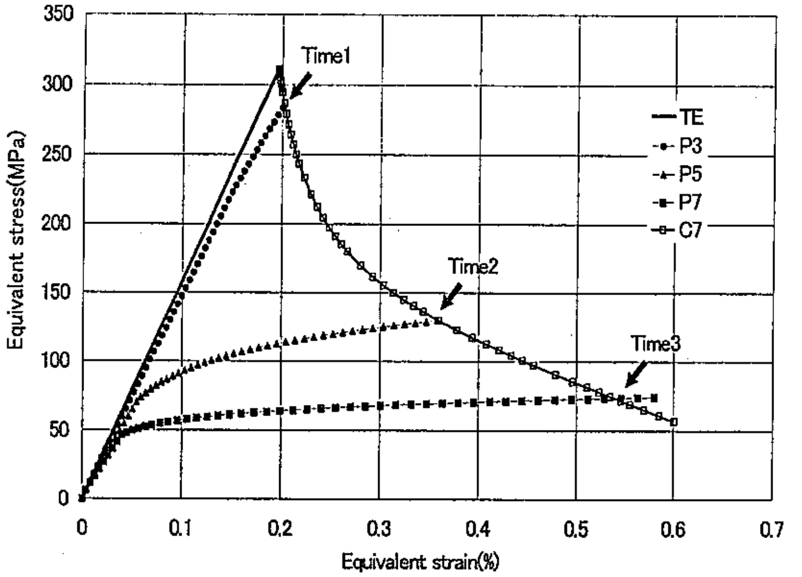


Fig.2. 10 Stress relaxation locus and elastic plastic curves

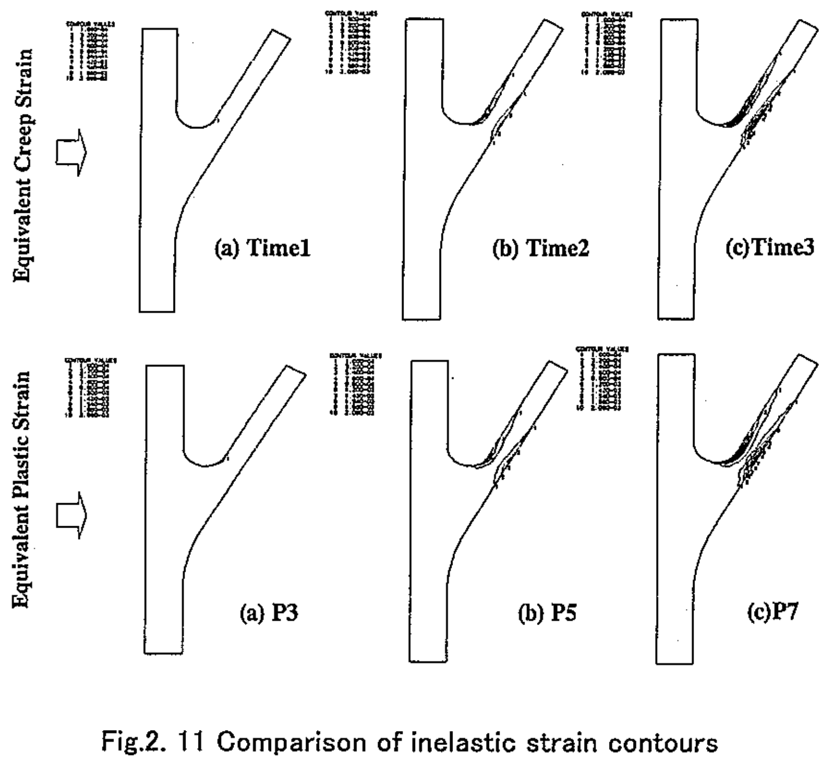


Fig.2. 11 Comparison of inelastic strain contours

The load level has also influence on the size of elastic core, which regulates the compliance characteristics. The effect of load level for strain concentration factor $K\varepsilon$ is analyzed from this viewpoint.

It tried to rewrite stress-strain curves and stress relaxation locus of Fig.2.7 for the relationship between inelastic strain and q value as in Fig.2.12, because a size of elastic core in high-rigid low stress portion is proportional to inelastic strain in the low-rigid high stress portion under the monotonic loading. In this figure, the reason why the result of P5 fluctuates in small strain region is high sensitivity of q value to calculate error in this regime. The q values of both plasticity and creep increase with enlargement of the load level or progress for the stress relaxation period. Quantitatively, the relation between q value and elastic-plastic strain is similar to one of creep strain, and characteristics after the 210,000 hours relaxation of the every load level agree with characteristics of the elastic plastic for the identical load level.

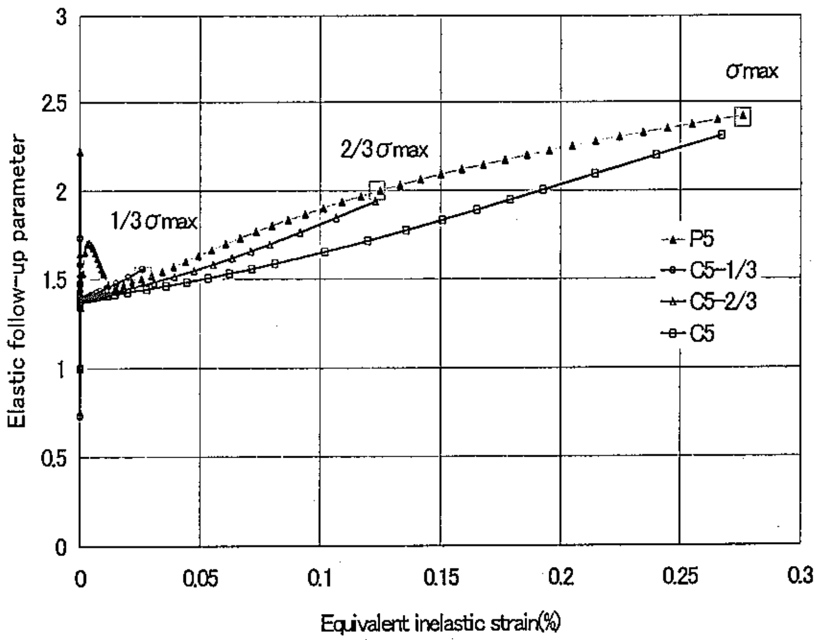


Fig.2. 12 Histories of elastic follow-up parameters

Then, creep strain distributions after the 210,000 hours stress relaxation from three level initial stresses were compared with the corresponding plastic strain distributions as in Fig.2.13.

In Fig.2.13, proportions of creep strain region and plastic strain one are almost the same, and it is proven that these tendencies are similar to Fig.2.11 which shows the change of the inelastic region according to constitutive equations and relaxation time.

From above study, it can be explained that the change of strain concentration factor by the load level is caused from variation of compliance ration between high and low rigid portions. Characteristics of compliance change are determined by reduction of the elastic core in structures, as well as effects from constitutive equations and relaxation time.

These characteristics can be evaluated by relaxation locus from the each load level and by elastic plastic analysis with the same inelastic strain as one after relaxation.

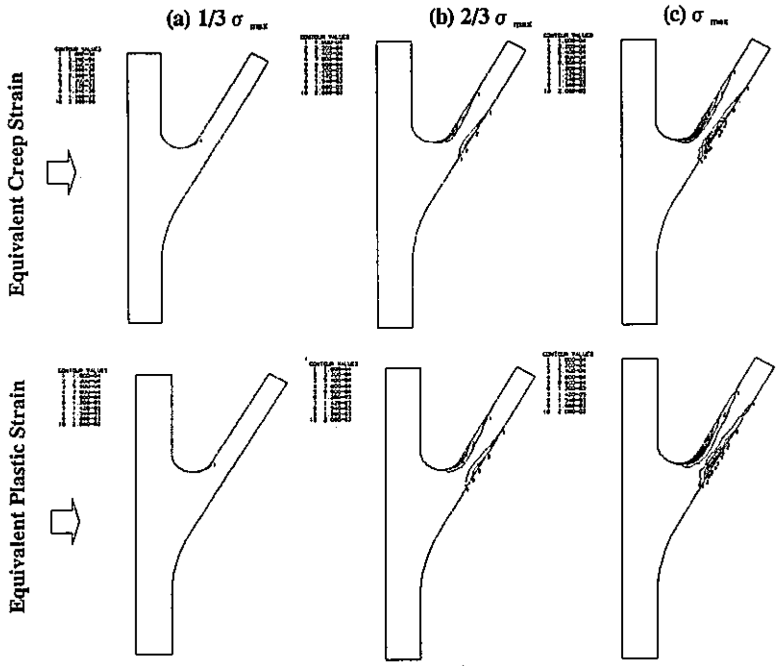


Fig.2. 13 Equivalent creep and plastic strain contour

3. INELASTIC DESIGN APPROACH USING RELAXATION LOCUS

3.1. DESIGN PROCEDURE WITH RELAXATION LOCUS

When the elasticity region (elastic core) has been left in the structure and inelastic regime is limited, the stress relaxation locus shows the characteristics of compliance change, which are insensitive to constitutive equations.

Therefore, any people can obtain the same relaxation locus (characteristics of compliance change) by stress relaxation analysis of one time using the simple creep strain equations like Norton's law.

And, it is also possible to estimate rough characteristics from the elastic analysis results, if it returns in regulating the compliance by the elasticity region[10].

However, attention is necessary on full inelastic deformation without elastic core, the q values of which are characterized by Eq.(2.6) or Eq.(2.7) and strongly depend on constitutive equations.

It is possible to predict the strain concentration behavior using characteristics of compliance change obtained by the next graphical solution (Fig.3.1), when the elasticity nucleus has been left.

(Step 1)

When the load level is regulated, calculate the stress relaxation locus (characteristics of compliance change) by relaxation creep analysis from this load.

If the load level is undecided, evaluate characteristics of the every load level by connecting relaxation locus from multiple initial stress or multiple elastic plastic solution for the identical load level.

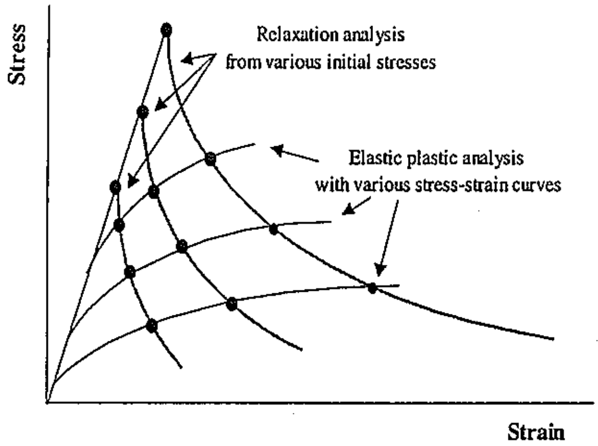
(Step 2)

Obtain the inelastic solution by the intersection point between stress relaxation locus, which shows compliance characteristics and material stress-strain curve.

The elastic plastic solution of certain temperature is obtained here, if monotonic curve of the same temperature is used as stress-strain curve. And it is possible to evaluate strain range and initial stress of relaxation required for the creep-fatigue evaluation, if relationship of stress range - strain range would be utilized.

Furthermore, they may be able to simply predict the time history of stress relaxation by using isochronous stress-strain curves [11], which are made from the creep test results of various stresses under constant temperature and fixed time by the procedure of Fig.3.2.

Step 1 : Evaluate characteristics of compliance change



Step 2 : Get cross point of relaxation locus and stress-strain curves

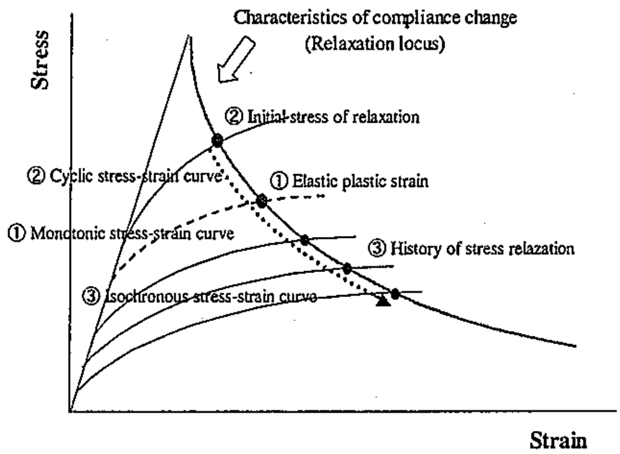


Fig.3. 1 Simplified inelastic analysis based on compliance characteristics

It is possible to widely foresee the effect of load condition and material property in this method, even if analytic work is not repeated. Still, characteristics of compliance change explain structural stability from local strain concentration to gross secondary stress redistribution [12] by them. It can be utilized for stress categorization [13] and for optimum inelastic design by restricting structural compliance (Fig.3.3)[14].

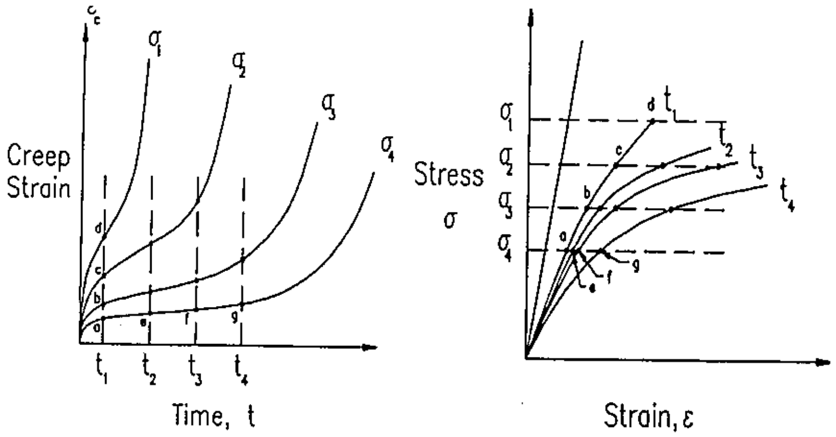


Fig.3. 2 Isochronous stress-strain curves [11]

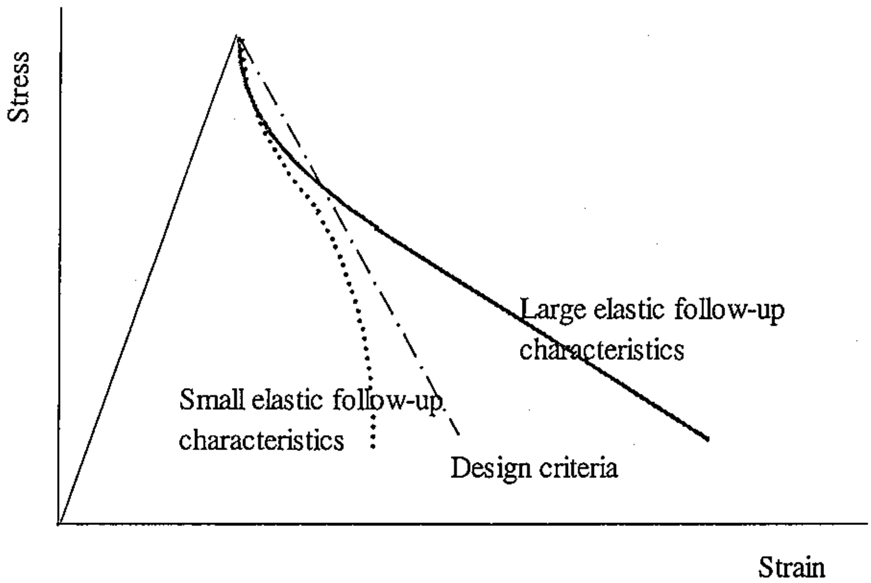


Fig.3. 3 Optimum design considering inelastic behaviors

3.2. QUANTIFICATION OF RELAXATION LOCUS BY INELASTIC ANALYSIS

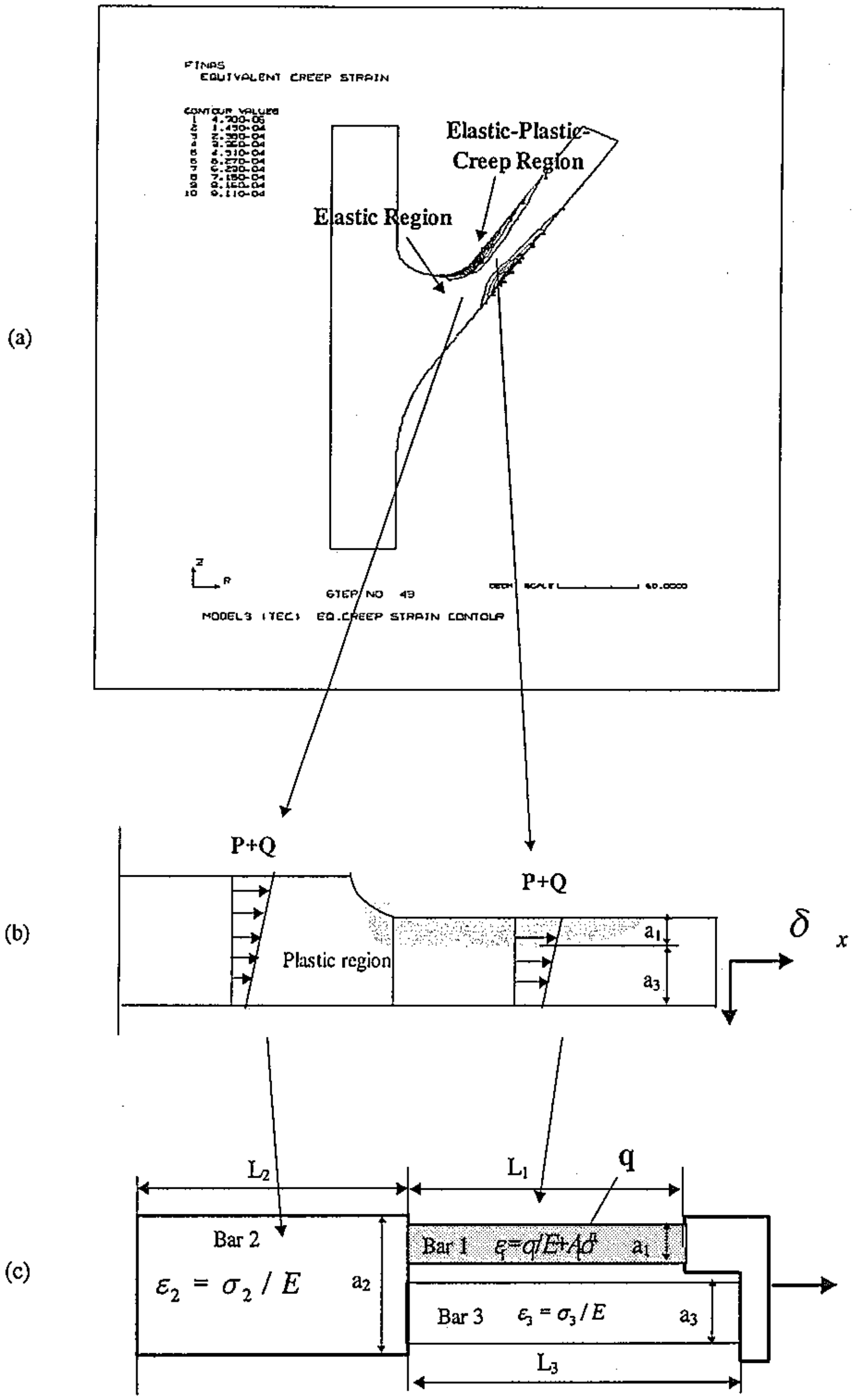
When the load level is regulated, one of the simplest way to get relazation locus is relaxation analysis with Norton'law. Power of eauation is required to be larger than actual materials with operation temperature to get locuses with sufficient ranges. Relaxation time is also required to be longer than service period.

3.3. QUANTIFICATION OF RELAXATION LOCUS BY ELASTIC FOLLOW-UP MODEL

Elastic core in the structural wall acts important roll on compriance characteristics which regulate the elastic-follow factor q. So that 2 bar model shown in Fig.2.9 was extended for considering the elastic core in the wall[10].

When observing strain distribution in the Y-piece (Fig.3.4), the inelastic region coexists with the elastic core which constraints the former in the same wall section. Some part of structural wall yielfs earyler than remaing parts, since there exist a bending stress and peak stress caused by local strain concentration.

For the first order approximation, the beam model is assumed for considering bending stress as in Fig.3.4. Here, a consideration of peak stress is a future discussion. In the second step, the beam model is replaced by the 3 bar model shown in Fig.3.4, which can consider both membrane and bending stresses. In this model, bar 1 corresponds to the inelastic region, bar 2 is a region where all of the section is elastic and bar 3 expresses the elastic core in the same wall section as bar 1.



$$q = \frac{1 + \dfrac{G_1 + G_3}{G_2}}{1 + \dfrac{G_3}{G_2}}$$

$$G_i = \frac{E_i a_i}{L_i}$$

Fig.3. 4 Elastic follow-up model for bar 1 considering elastic core

Elastic follow-up parameter q of bar 1 to both bar 2 and bar 3 is evaluated. In Fig.3.4, equations of equilibrium and compatibility are

$$\delta = L_1\varepsilon_1 + L_2\varepsilon_2. \tag{3.1}$$

$$\delta = L_2\varepsilon_2 + L_3\varepsilon_3 \text{ and} \tag{3.2}$$

$$a_1\sigma_1 + a_3\sigma_3 = a_2\sigma_2. \tag{3.3}$$

Since bar1 corresponds to plastic region and bar 2 and bar 3 are elastic, their constitutive equations are

$$\varepsilon_1 = \sigma_1 / E_1 + A_1\sigma_1^n , \tag{3.4}$$

$$\varepsilon_2 = \sigma_2 / E_2 \text{ and} \tag{3.5}$$

$$\varepsilon_3 = \sigma_3 / E_3. \tag{3.6}$$

From Eqs.(3.1) ~ (3.6), the elastic follow-up parameter q of bar 1 can be introduced as

$$q = \frac{\varepsilon_1 - \sigma_1 / E_1}{\varepsilon_{1e} - \sigma_1 / E_1} = \frac{1+\left(\frac{E_1 a_1}{L_1}\right)\Big/\left(\frac{E_2 a_2}{L_2}\right)+\left(\frac{E_3 a_3}{L_3}\right)\Big/\left(\frac{E_2 a_2}{L_2}\right)}{1+\left(\frac{E_3 a_3}{L_3}\right)\Big/\left(\frac{E_2 a_2}{L_2}\right)} = \frac{1+\frac{G_1+G_3}{G_2}}{1+\frac{G_3}{G_2}} \tag{3.7}$$

where $G_i = \frac{E_i a_i}{L_i}$ is stiffness of bar i.

Here, there is the next relation between bar 1 and bar 3 under elastic condition.

$$L_1\sigma_{1e} = L_3\sigma_{3e} \tag{3.8}$$

If low rigid part become full plastic condition, Eq.(3.6) becomes

$$\varepsilon_3 = \sigma_3 / E_3 + A_3\sigma_3^n \tag{3.9}$$

and Elastic follow-up parameter q of bar 1 is

$$q_G = 1 + \frac{G_1 + G_3}{G_2} \tag{3.10}$$

that is the special case of Eq.(3.7).

Since usual structures have a constant Young's modulus and $E_1=E_2=E_3=E$, Eq.(3.7) becomes,

$$q_G = \frac{1 + \left(\frac{L_2}{L_1}\right)\left(\frac{a_1}{a_2}\right) + \left(\frac{L_2}{L_3}\right)\left(\frac{a_3}{a_2}\right)}{1 + \left(\frac{L_2}{L_3}\right)\left(\frac{a_3}{a_2}\right)} \tag{3.11}$$

Next, characteristics of Eq.(3.11) is investigated. Fundamental parameters of this equation are such stiffness ratios as $(G_1+G_3)/G_2$ and G_1/G_2.

Relation of q to G_1/G_2 under the constant $(G_1+G_3)/G_2$ was evaluated and is shown in the next figure. q increases when G1/G2 becomes larger. It means that the elastic follow-up parameter increases when the plastic regime expands in the wall thickness.

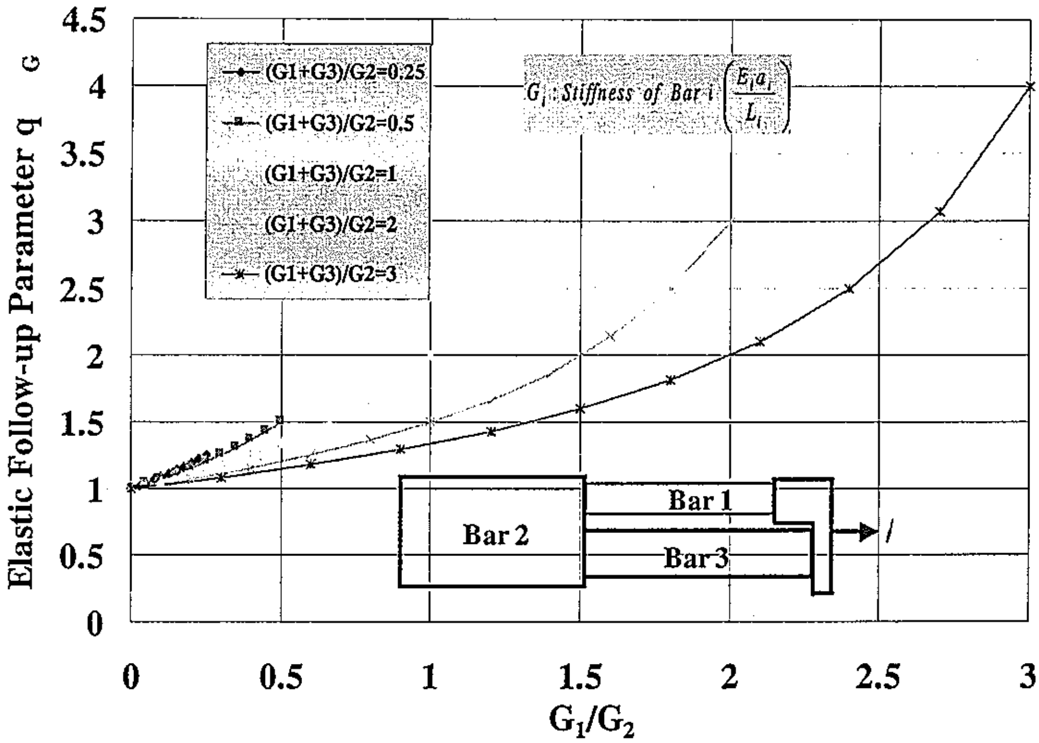


Fig.3. 5 Elastic follow-up parameter for redistribution of secondary stress (1)

Relation of q to $(G_1+G_3)/G_2$ under the constant G_1/G_2 was evaluated and is shown in the next figure. q decreases when $(G_1+G_3)/G_2$ becomes larger.

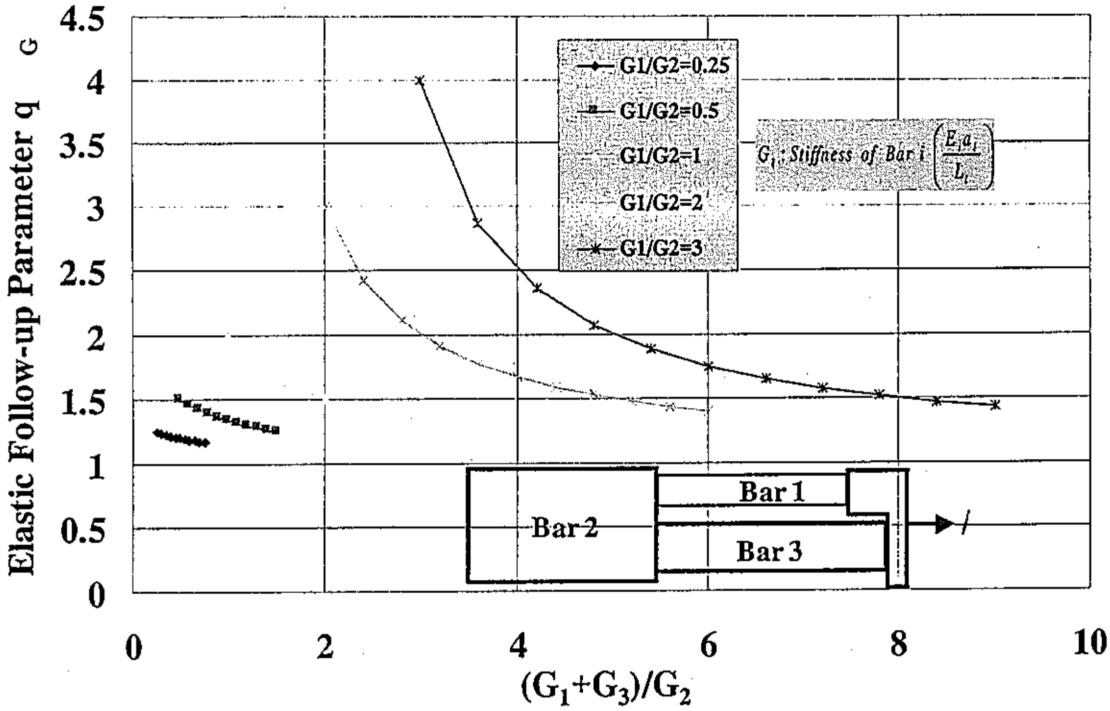


Fig.3. 6 Elastic follow-up parameter for redistribution of secondary stress (2)

Inelastic and elastic regions can be roughly distinguished by elastic analysis. Some ideas have been proposed, which are based on yield stress [3][10].

4. CONCLUSIONS

On strain concentration of the structural discontinuities by the plasticity and creep, the followings were clarified.

(1) Though a strain concentration factor is dependent on each factor of constitutive equation, load level and of the stress relaxation time, stress and strain exist on the unique relaxation locus of the structure.

(2) The stress relaxation locus is insensitive to the constitutive equations, since compliance characteristics of the elastic region restrict stress and strain redistribution.

(3) The compliance characteristics of the system changes by reduction of the elastic region according to progress of the stress relaxation. As a result, the stress relaxation locus in the structure discontinuity becomes a curve.

(4) By utilizing insensitivity of compliance characteristics to constitutive equations, the evaluation method of strain concentration was proposed using its characteristics and stress-strain curves of materials. In this method, the difference of the solution by the evaluation person is small, and it is possible to easily evaluate the effect of load and material.

5. DISCUSSIONS ON APPLICATION TO PIPING DESIGN

5.1. JNC FEASIBILITY STUDY ON COMMERCIALIZED FBR

JNC has cooperated with electric companies and other related organizations to study feasibility on commercialized fast reactor cycle systems from 1999 [15]. In the Phase I activity, candidate reactors to achieve requirements were selected. One of them is sodium cooling loop type reactor with a three dimensional seismic isolation system and two main loops. Furthermore, there are two variations in this reactor as a top entry type (Fig.5.1) and a side entry type (Fig.5.2).

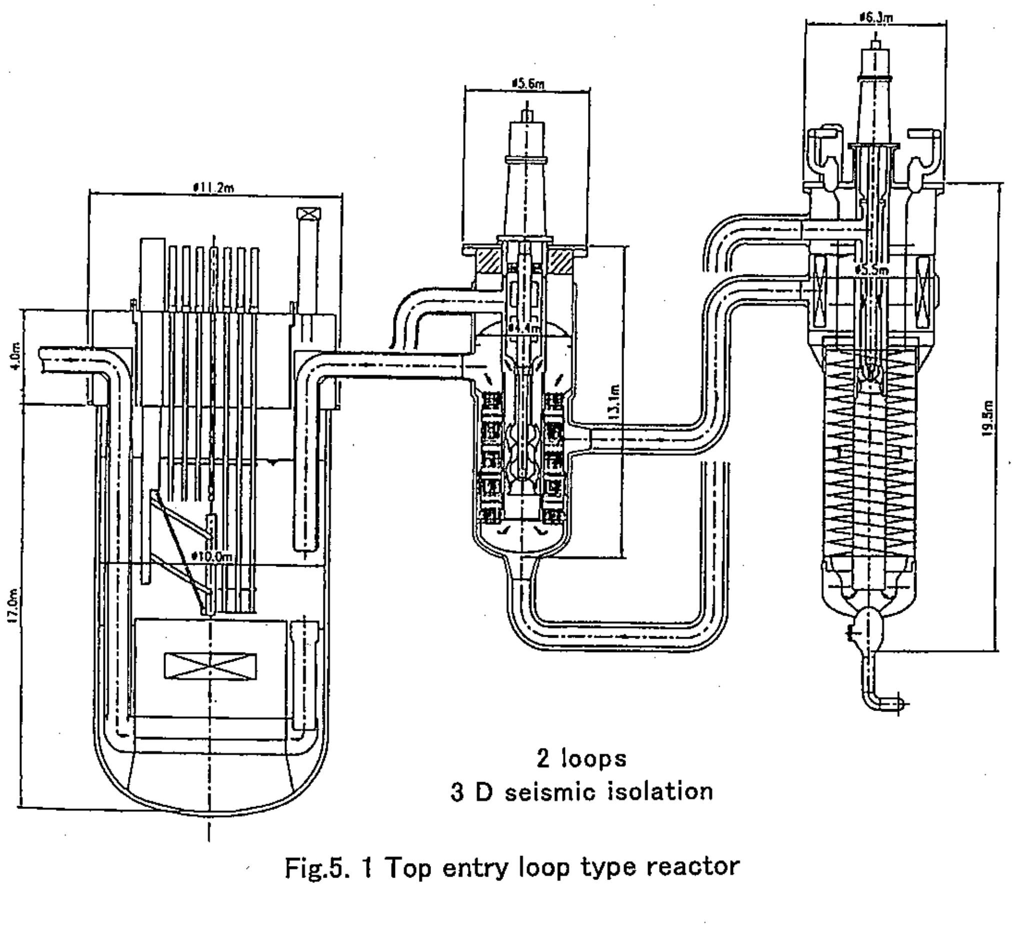


Fig.5. 1 Top entry loop type reactor

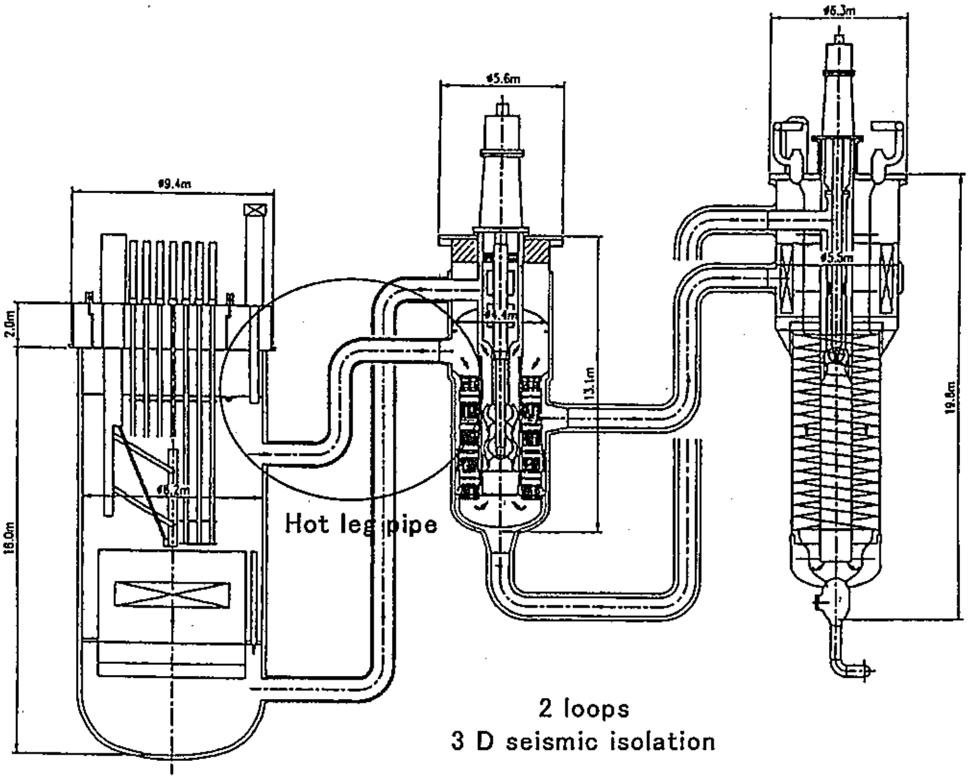


Fig.5. 2 Side entry loop type reactor

The top entry type was proposed to avoid thermal expansion stress of primary pipes and new R&Ds are necessary. Side entry type can achieve the simplest reactor vessel and requires the minimum R&Ds since it is the same type as Monju.

The difficulty of the side entry type is the structural integrity of hot leg pipe which can not satisfy the conventional creep-fatigue limit.

5.2. STRATEGY TO OPTIMIZE PIPING SYSTEMS AGAINST SECONDARY STRESS

Adoption of three-dimensional isolation system allows free designs from the primary stress and the main load become secondary stress caused from thermal loads. Here, strategy of optimum design with utilizing characteristics of secondary stress is discussed to achieve a simple hot leg piping system for the side entry reactor.

Against secondary stress, strain limit criteria may be suitable than stress one and efforts are made to incorporate it in design codes [16]. If the strain limit is introduced, allowable design area will be remarkably extended for displacement-controlled structures (stable structures) like the next figure.

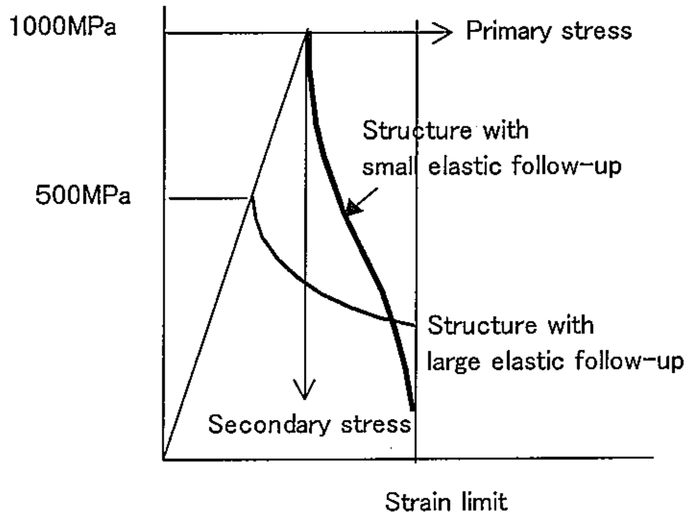


Fig.5. 3 Allowable design area for secondary stress with strain limit

Conventional design parameters of the hot leg pipe system are pipe length and flexibility of 2 elbows. If we can ignore the primary stress, other design options are possible such as 2 flexible nozzles and continuous flexible pipes (organic design) as in the next figure.

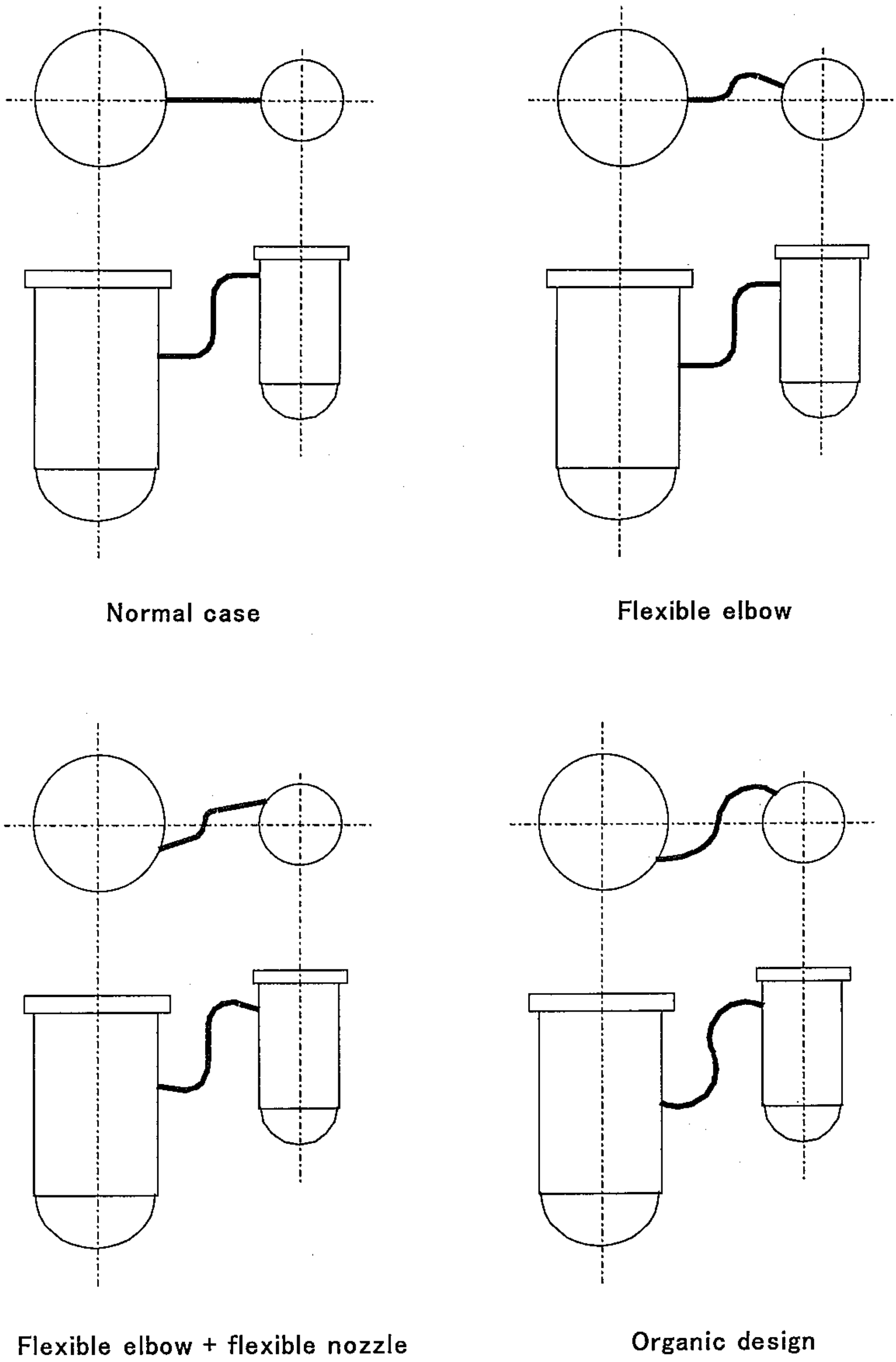


Fig.5. 4 Variations of piping layouts

Evaluation methods of elastic follow-up in pipes have been surveyed by Boyle and Nakamura [17][18]. As a typical method, Kachanov's one is based on the assumption that the local stresses will relax in proportion from their initial elastic values ; that is, all stresses will relax in proportion from their initial elastic values [19]. Dhalla proposed the other method based on reduced modulus. This method focuses on the weakest part in a system and evaluates its strain by reducing elastic modulus of this part [20][21].

France has developed class1 piping design rule for RCC-MR code with applying Kachanov's method, and they have the policy that the elastic follow-up is minimized [22][23][24]. Japan has proposed design procedure of piping by simplified inelastic analysis [25], which has not the sense of optimization.

Above methods can work well when other parts than the weakest part remain in the elastic condition. If more than two parts become inelastic conditions, interaction occurs among them. For example, the hot leg pipe with 2 elbows and 2 nozzles has different interaction modes as in the next figure.

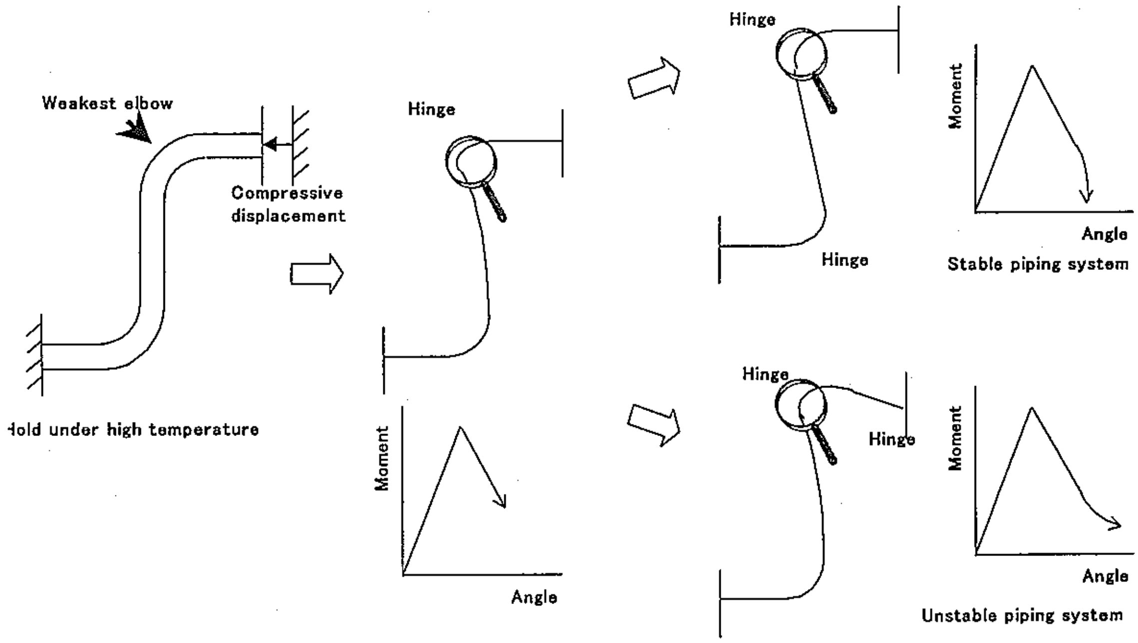


Fig.5. 5 Optimum design with considering interaction among devices

Dhalla's method considering one inelastic portion is obviously unconservative for the below case in Fig.5.5. On the other hand, if we can positively utilize this interaction and achieve the above case. This becomes the optimum design considering the characteristics of secondary stress. When both stress and characteristics of compliance change described by the relaxation locus will be taken into account, there is a possibly to get optimum design considering secondary stress.

ACKNOWLEDGEMENT

On approach with elastic follow-up concept and the stress categorization of the structures, greeting is deeply said in receiving guidance from emeritus professor Yasuhide Asada of Univ. of Tokyo. Discussion of simplified a failure analysis method with Dr. Robert Jetter of ASME Elevated Temperature Design Committee, Dr. Aspi Dhalla and Douglas L.Marriott of Pressure Vessel Research Council Elevated Temperature Design Committee are greatly acknowledged. Author is indebted to Dr. Koji Iwata of Japan Nuclear Cycle development institute for advice on dealing compliance concept. In addition, it is thankful in carrying out many finite element analyses to Mr. Hideki Takasho of the Jyoyo industry Co.Ltd. The author is pleased to acknowledge Dr. Marie Noel Berton of Commissariat a l'energie atomique for her comments to application of elastic-follow-up concept and correction of documentation.

REFERENCES

[1] Dhalla,A.K. et al.,'Simplified Methods, in Reccommennded Practice in Elevated Temperature Design: A Compendium of Breeder Reactor Experiences, Vol.II - Preliminaly Design and Simplified Methods',WRC Bulletin 362, WRC, New York,(1991)

[2] Robinson, E.L.,'Strain piping design to minimise creep concentration', Trans.of ASME,77, pp1147/1162,(1955)

[3] Marriott, D.L. et al.,'Simplified Analysis Methods for Elevated Temperature Life Assessment', Technology for 90's, Chapter10,ASME,(1994)

[4] PNC,"FINAS Version 12.0 User's Manual', PNC ZN9520 95-013, (1995)

[5] Kasahara,N, 'Strain concentration at structural discontinuities and quantification by elastic follow-up parameter (2) Sensitivities to material properties' , JSME, Proc.of M&M 98, in Japanese, No.98-5,Vol.B, 1256, pp421/422 (1998)

[6] Kasahara,N, 'Strain concentration at structural discontinuities and quantification by elastic follow-up parameter (1) Influence from load conditions' , JSME, Proc.of Annual Meeting(I) 539,, in Japanese,No.98-3, pp499/500, (1998)

[7] Iida, K. et al., 'Simplified analysis and design for elevated temperature components of MONJU', NED 98, pp305/317, (1987)

[8] Kasahara,N, 'Strain concentration at structural discontinuities and quantification by elastic follow-up parameter (3) Effect of relaxation time to strain concentration' , JSME, Proc.of Annual Meeting(Ⅲ),2903, No.99-1,pp399/400, (1999)

[9] Kasahara,N.,Nagata,T.,Iwata,K.,and Negishi,H., 'Advanced Creep-Fatigue Evaluation Rule for Fast Breeder Reactor Components : Generalization of Elastic Follow-up Model', NED 155, pp499/518, (1995)

[10] Kasahara,N、' Study on prediction of inelastic behaviors based on elastic follow-up concept' , University of Tokyo, Thesis, (1998)

[11] ASME, 'Criteria of the ASME Boiler and Pressure Vessel CODE for Design by Analysis in Secton Ⅲ and Section Ⅷ, Division 2', ASME, New York,(1964)

[12] Negishi, H. et al., 'A Method for Creep-Fatigue Damage Evaluation at Structural Discontinuity Portions (1): Stress Analysis of Fundamental Models,' Trans.of 11th SMiRT, Vol.E,(1991)

[13] Kasahara,N and Takasho,H., 'Stress categorization of three dimensional structures based on elastic follow-up concept (1) Stress categorization of a plate with single hole', JSME, Proc.of Annual Meeting(Ⅲ), 2915,in Japanese, No.99-1,pp423/424, (1999)

[14] Kasahara,N.,'Strain concentration at structural discontinuities and its prediction based on characteristics of compliance change in structures', JSME, Trans., Vol.66, No.643, pp224/231,(2000)

[15] Noda, H., 'Current Status of Feasibility Studies on Commercialized Fast Reactor Cycle System', J. of AESJ, Vol.42,No.7 (2000)

[16] Hayashi,M., et. al., 'Recent design improvements of elevated temperature structural design guide for DFBR in Japan, 15th SMiRT, Div.F, (1999)

[17] Boyle, J.T. et al.(1987),'The assessment of Elastic Follow-up in High Temperature Piping Systems-Overall Survey and Theoretical Aspects', Int.J.of Pres. Ves. & Pipng, 29, pp167/194

[18] Nakamura, K. et al.(1987),'The assessment of Elastic Follow-up in High Temperature Piping Systems-Some Example Problems', Int.J.of Pres. Ves. & Pipng, 29, pp249/273

[19] Kachanov,L.M., 'The theory of creep', Transl. of Russian Text, British Lending Library, (1967)

[20] Dhalla,A.K.(1986),'Verification of an Elastic Procedure to Estimate Elastic Follow-Up',Trans.of ASME, J. Pres. Ves. Tech., Vol.108

[21] Dhalla,A.K.(1986),'Numerical Estimate of Elastic Follow-up in Piping: Inelastic Evaluation', J. Pres. Ves. Tech., Vol.108

[22] Roche,R.L., 'Elastic Follow Up in Piping Systems How to Specify the Creep Use-Fraction Factor', 8th SMiRT, E6/8, (1985)

[23] Berton,M.N. et al, 'RCC-MR Design Rules for Class 1 Piping Components', Post SMiRT Conference on Construction Codes and Engineering Mechanics, Tokyo,(1991)

[24] Sperandio M., et al, 'RCC-MR code : presentation emphasizing recent design improvements', 14th SMiRT, FW/3, pp285/292,(1997)

[25] Kaguchi,T., et. al., 'Design Procedure of Piping under Thermal Expansion at Elevated Temperature', ASME/JSME PVP, San Diego, (1998)