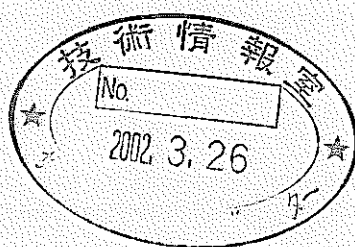


Study on Drag Coefficient for the Particle/Droplet with Vapor Film



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Study on Drag Coefficient for the Particle/Droplet with Vapor Film

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Abstract

The fuel coolant interaction (FCI) phenomena are characterized by a configuration that a hot particle/droplet surrounded with vapor film moves in coolant liquid. A drag correlation has not been developed for such a configuration. In this study, based on the basic conservative equations, the drag coefficients between a single hot particle/droplet and the surrounding coolant liquid under laminar and turbulent flow conditions are developed first. The coefficients are expressed as functions of Reynolds number, vapor/liquid viscosity and density ratios and the other two dimensionless numbers newly introduced in this study. The drag coefficients for a multi-particle/droplet system under laminar and turbulent flow conditions are then developed based on the above single hot particle/droplet model and the mixture viscosity concept. The proposed correlations are coupled into the SIMMER-III code and are used to simulate the QUEOS experiment. It is shown that the proposed correlations reasonably improve the agreement of the calculated results with the experimental data.

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蒸気膜で覆われている粒子の抵抗係数の研究 (研究報告)

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要 旨

燃料—冷却材相互作用(FCI:Fuel Coolant Interaction)では、冷却材中を運動している高温粒子・液滴が蒸気膜で覆われていることが特徴である。この高温粒子・液滴の抵抗係数が開発されていない。本研究では、基礎保存式から、層流と乱流の条件での単一の高温粒子・液滴に対する抵抗係数の無次元抵抗関係式を開発した。この抵抗係数は Re 数、蒸気と冷却材との密度比及び粘度比、そして本研究で新たに導入した2つの無次元数の関数として表現する。単一粒子モデルと混合粘性モデルから、層流と乱流の条件で、多粒子流体体系での高温粒子・液体の抵抗係数の無次元抵抗関係式を開発した。この関係式を SIMMER-III コードに組み込み、QUEOS 実験の解析に適用して、結果に実験結果との一致は適切に改善された。

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Contents

ABSTRACT	I
要 旨	I
CONTENTS	I
LIST OF FIGURES.....	II
1. INTRODUCTION	1
2. THE DRAG COEFFICIENT FOR A SINGLE HOT PARTICLE.....	4
2.1 THE DRAG COEFFICIENT EXPRESSION	4
2.2 THE DRAG CORRELATION UNDER LAMINAR FLOW CONDITION	5
2.3 THE DRAG CORRELATION UNDER TURBULENT FLOW CONDITION	10
3. DRAG COEFFICIENT FOR MULTI-PARTICLE SYSTEM.....	13
3.1 DRAG CORRELATION UNDER LAMINAR FLOW CONDITION.....	14
3.2 DRAG COEFFICIENT UNDER TURBULENT FLOW CONDITION	15
4. APPLICATION OF THE PROPOSED DRAG CORRELATIONS.....	16
5. CONCLUSION	17
ACKNOWLEDGEMENTS	18
NOMENCLATURE.....	18
REFERENCE	20
APPENDIX A: DRAG COEFFICIENT FOR A SINGLE PARTICLE UNDER LAMINAR FLOW CONDITION.....	35
APPENDIX B: DRAG COEFFICIENT FOR A SINGLE PARTICLE UNDER TURBULENT FLOW CONDITION.....	38

List of Figures

Fig. 1 Configuration of a single droplet moving in coolant in Nelson experiment (NUREG/CR-2718)	22
Fig. 2 Scheme of a hot particle moving in coolant liquid	23
Fig. 3 The drag coefficients obtained from the experimental data and those predicted by the fitted correlation under turbulent condition arranged in Reynolds number	24
Fig. 4 The drag coefficients obtained from the experimental data and those predicted by the fitted correlation under turbulent condition arranged in Ev number	25
Fig. 5 The relativity of the drag coefficient predicted by equation (29) and their experimental data	26
Fig. 6 The variation of the drag coefficient of the particles with vapor film under laminar condition with Reynolds number. (Ev number is 200, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles)	27
Fig. 7 The variation of the drag coefficient of the particles with vapor film under laminar condition with Ev number. (Reynolds number is 1000, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles)	28
Fig. 8 The variation of the drag coefficient of the particles with vapor film under laminar condition with the volume fraction of the particles. (Ev number is 200, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles)	29
Fig. 9 The variation of the drag coefficient of the particles with vapor film under turbulent condition with Reynolds number. (Ev number is 200,	

μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles).....	30
Fig. 10 The variation of the drag coefficient of the particles with vapor film under turbulent condition with Ev number. (Reynolds number is 10000, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles).....	31
Fig. 11 The variation of the drag coefficient of the particles with vapor film under turbulent condition with the volume fraction of the particles. (Ev number is 200, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles).....	32
Fig. 12 The advancements of the particle cloud in QUEOS-12, calculated by different drag coefficient correlations	33
Fig. 13 The advancements of the particle cloud in QUEOS-35, calculated by different drag coefficient correlations	34

1. Introduction

Fuel Coolant Interactions (FCIs) are the important phenomena in nuclear reactor severe accident analysis and have been extensively studied in recent years. Several mathematical models of multi-phase multi-component flow, based on a multi-field description, have been developed with the aim of describing the events during the Core Disruptive Accidents (CDAs) in Liquid Metal Cooled Fast Breeder Reactors (LMFBRs) or Core Melt Accidents (CMAs) in Light Water Reactors (LWRs). The SIMMER-III code, developed at JNC [1,2], is a two-dimensional, three-velocity-field, multi-phase, multi-component, Eulerian, fluid-dynamics computer code coupled with a space-dependent neutron kinetics model. The QUEOS experiment, which studied the premixing phase of a FCI with hot particles released into water [3], was simulated [4,5] by the code to verify the momentum exchange between the fuel and the coolant, in which Ishii's correlations [6] are employed. The simulated results showed disagreement of the front advancement of the particle cloud in the water with the experimental results. The difference was further enhanced with increasing the particle temperature [5]. This means that the capability to predict the drag between the hot particles/droplets and the coolant liquid in multi-particle system should be improved for better analyzing FCIs.

First let us review the drag laws currently employed in the simulation tools. The configuration, based on which these drag laws for a single particle/droplet were developed, is that a fluid particle moves in other continuous fluid and these two fluids directly contact each other. The drag force exerted on the particle/droplet under steady-state condition can be given in terms of the drag coefficient C_D based on the relative velocity between them as

$$F_D = -\frac{1}{2} A_d C_D \rho_c v_{cp} |v_{cp}|, \quad (1)$$

where F_D is the drag force exerted on the particle/droplet, A_d is the projected area of

the particle/droplet on a plane perpendicular to the velocity direction, C_D is the drag coefficient between the particle/droplet and the continuous fluid, v_{cp} is the relative velocity given by $v_{cp} = v_c - v_p$ and ρ_c is the density of the continuous fluid.

The drag coefficients of the single particle/droplet in an infinite medium has been studied extensively by many researchers and they are given by the empirical correlations [7, 8, 9]. Based on the single particle/droplet experimental data, Ishii [9] developed the correlations for the multi-particle system. In viscous regime the drag coefficient of the particle/droplet in multi-particle system is given by the correlation:

$$C_D = \frac{24}{Re} (1 + 0.1 Re^{0.75}) \quad (2)$$

where Re is the Reynolds number by using the mixture viscosity. In Newton's regime the drag coefficient of the particle/droplet in multi-particle system is given by the correlation:

$$C_D = 0.45 \left(\frac{1 + 17.67[f(\alpha_d)]^{6/7}}{18.67f(\alpha_d)} \right)^2 \quad (3)$$

where

$$f(\alpha_d) = (1 - \alpha_d)^{0.5} \left(2 - \frac{\alpha_d}{0.62} \right)^{1.55} \quad (4)$$

and α_d is the volume fraction of the particles. It is shown in equation (1) that the drag force is dominated by the drag coefficient and the surface area of the particles/droplets. If the surface area of the particles/droplets is fixed in the simulation of an experiment, the drag correlation can be estimated. In the previous study, the QUEOS premixing experiment was simulated [4,5], because in this experiment, the hot solid particles (the surface area of the particles is fixed) were released into water. The front advancement of the particle cloud moving in water (affected by the drag coefficient) was analyzed to estimate the drag correlations. The results show that the front advancement of the particle cloud in the simulation disagreed with that in the experiment and the difference was enhanced with increasing the temperature of the hot particle. These results suggest

that the influence of the temperature on the drag coefficient should be considered in the drag correlation in the case of FCI study. This influence is not considered in the currently employed drag correlations.

Then let us check the configuration of a hot particle/droplet moving in coolant liquid to understand what induced the difference of the front advancement of the particle cloud between the simulation and the experiment. In Nelson's experiment [10], a single droplet FCI, it is showed that when a hot particle/droplet (In this study, hot particle and droplet are the same. In the following the 'particle' is used) moves in coolant liquid, it is surrounded by a thin vapor film, as shown in Fig. 1. The hot particle does not contact directly with the coolant liquid. The boundary around the particle is vapor, which is continuously generated from the coolant liquid-vapor interface and flows inside the film from the lower part to the upper part around the particle. It is the vapor flowing inside the film that gives a resistance to the moving particle. The currently employed drag correlations may not be applicable in such a configuration, especially with high particle temperature.

In the multi-particle system, the effect of other particles on the original particle's drag force should be considered. Ishii [6] developed a method to consider the effect of multi-particle (without vapor film) on drag coefficient. In the viscous flow regime, the effect of other particles in the multi-particle system on the original particle's drag is considered by using the mixture viscosity. In his model it is considered that the force exerted on the original particle by other particles is in the form of viscous stress. In the case of the hot particle with vapor film, since the coolant liquid-vapor interfacial velocity is affected by other particles, the force exerted on the original particle can be expressed by viscous stress through the liquid-vapor interface. In this study, the mixture viscosity concept will be employed to develop the drag coefficient for multi-particle system under laminar and turbulent flow conditions.

Since the drag correlation with the effect of the vapor film for a single particle or multi-particle system has not been developed, in this study we will develop the drag

correlation of the particle with vapor film moving in coolant liquid. As the first step, the drag coefficients between a single particle with vapor film and the surrounding coolant liquid under laminar and turbulent flow conditions will be developed respectively. Then, based on the single particle model and the mixture viscosity concept, drag coefficients in multi-particle system under laminar and turbulent flow conditions will be developed respectively. Finally the proposed drag correlations will be employed to again simulate QUEOS experiment.

2. The drag coefficient for a single hot particle

2.1 The drag coefficient expression

From the photograph describing a single hot particle coolant interaction experimented by Nelson [10], as shown in Fig. 1, the configuration of the particle moving in coolant liquid can be figured out: when the particle is immersed into coolant, it is surrounded by a thin film of vapor, which is generated continuously from the interface of coolant liquid-vapor, at the front part of the particle. The vapor leaves the particle as a bubble from the top of it, forming a wake region. Based on the observation, the analytical model could be described as shown in Fig. 2.

The drag coefficient of the particle with vapor film moving in coolant liquid could be expressed as the following form:

$$C_{D-fb} = \frac{F_y}{\pi r_p^2 \frac{1}{2} \rho_c V_{cp}^2}, \quad (5)$$

where C_{D-fb} is the drag coefficient for the particle surrounded with vapor film moving in coolant liquid, V_{cp} is the velocity difference between the particle and coolant liquid, ρ_c is the density of coolant liquid, πr_p^2 is the characteristic area taken to be the area obtained by projecting the particle on a plane perpendicular to the velocity of the

coolant flow and F_y is the drag force in y direction exerted on the particle including the pressure and the viscous stress. Although the drag coefficient is defined based on the velocity difference between the particle and coolant liquid, the force exerted on the particle is given directly by the vapor flowing inside the vapor film.

As shown in Fig. 2, the vapor flow along the particle surface is divided into two parts, including the laminar (front) region (called region I) and the wake region (called region II). The forces exerted on the particle surface will be calculated on the two regions separately. The total force exerted on the surface is equal to the sum of the two regions,

$$F_y = F_y^I + F_y^{II} . \quad (6)$$

The forces exerted on the region I include pressure and friction. The total force is calculated as

$$F_y^I = \int_0^{\theta_s} (P \cos \theta + \tau_{s1} \sin \theta) 2\pi r_p^2 \sin \theta d\theta , \quad (7)$$

where P and τ_{s1} are the pressure and viscosity force exerted on the surface of the particle, r_p is the radius of the particle and θ_s is the separation point of the two regions. In region II wake region, the friction on the surface can be neglected and the pressure in the wake region is the same at the separation point. The force exerted on the region II can be calculated as

$$F_y^{II} = \int_{\theta_s}^{\pi} P_{\theta_s} 2\pi r_p^2 \sin \theta \cos \theta d\theta , \quad (8)$$

where P_{θ_s} is the pressure at the separation point. In the following the pressure distribution and viscous stress, which are related to the velocity profile of the vapor in the film, will be calculated according to that the flow condition of the coolant liquid is laminar or turbulent respectively.

2.2 The drag correlation under laminar flow condition

Since the thickness of the vapor film is thin comparing to the diameter of the

particle, the curvilinear orthogonal coordinate (x, y) and the angle θ ($\theta = \frac{x}{R}$) can be applied to solve this problem, as shown in Fig. 2. In this study, the two-dimensional system is employed to model the three-dimensional phenomena, because the main contribution to the force exerted on the particle is induced by the vapor flow in x direction. Comparing with the vapor flow on the x direction, the vapor flow on the circular direction is small, which can be neglected. The following assumptions are made to vapor film and coolant liquid respectively. To the vapor film, the vapor is assumed to form a continuous film layer around the particle, evaporated from the coolant liquid-vapor interface, which is smooth. The vapor film layer is laminar in region I. (It is reasonable since the calculation shows that local Reynolds number is about several hundreds.) The velocity of the vapor in y direction can be neglected. Evaporation rate is constant along the interface of coolant liquid-vapor. Steady state and steady flow conditions apply. Vapor physical properties are constant. To the coolant liquid, since the laminar flow is assumed in this case, a potential flow field exists in the coolant liquid and the Bernoulli equation is applied to the coolant flow.

Then the governing equations in the vapor film including mass and momentum equations are

$$\frac{\partial \rho u}{\partial x} = 0, \quad (9)$$

$$\frac{\partial \rho u^2}{\partial x} = -\frac{\partial P}{\partial x} - \rho g \sin \theta + \mu \frac{\partial^2 u}{\partial y^2}, \quad (10)$$

where u is the vapor velocity in the film in x direction, ρ is the vapor density, μ is the vapor viscosity and P is the pressure in the film. The variables without subscript are used to describe the vapor in the film (the same in the following and subscript c refers coolant liquid). The boundary conditions are

$$\begin{aligned} y=0; & \quad u=0, \\ y=\delta; & \quad u=u_i. \end{aligned}$$

The velocity of the liquid-vapor interface u_i is given by applying potential flow as

$$u_i = \frac{3}{2} V_{cp} \sin \theta. \quad (11)$$

The pressure at angle θ , $P(\theta)$ on the liquid-vapor interface can be expressed by applying Bernoulli equation in coolant liquid as

$$P(\theta) = P_0 + \rho_c g r_p \cos \theta + \rho_c \frac{V_{cp}^2}{2} - \frac{\rho_c}{2} \left(\frac{3}{2} V_{cp} \sin \theta \right)^2, \quad (12)$$

where P_0 is the pressure at the particle center. If the vapor film is taken as a whole channel, then the following mass equation can be written:

$$\frac{\partial \rho \bar{u} A}{\partial \theta} = \Gamma \frac{\partial S_1}{\partial \theta}, \quad (13)$$

where

$$A = \pi \sin \theta (\delta^2 + 2r_p \delta), \quad (14)$$

which is the cross area of the vapor film perpendicular to x direction in a control volume (CV) with length scale $r d\theta$ along the channel, δ is the thickness of the vapor film at θ and S_1 is the area on the particle surface,

$$\frac{\partial S_1}{\partial \theta} = 2\pi r_p^2 \sin \theta \quad (15)$$

Γ is the evaporation rate on the coolant liquid-vapor interface and

$$\bar{u} = \frac{1}{\delta} \int_0^\delta u dy, \quad (16)$$

which is the average velocity of the vapor in the vapor film channel in x direction.

According to the flow pattern of the vapor in the film, separation point between region I and II could be calculated by [12]

$$\left. \frac{\partial u}{\partial y} \right|_{y=0} = 0. \quad (17)$$

The velocity profile in the vapor film can be obtained by solving equations (9),

(10), (11), (12), (13), (14), (15) and (16). Then the viscous stress on the surface of the particle is calculated by

$$\tau_{s_i} = -\mu \left. \frac{\partial u}{\partial y} \right|_{y=0}. \quad (18)$$

Finally we can calculate the drag coefficient of the single particle with vapor film under laminar condition from equation (5), (the detailed calculation is presented in the Appendix A,) but the expression is too complicated to be used in simulation codes, which should be simplified. Here we use the following dimensionless numbers,

$$\text{Re} = \frac{V_{cp} D_p \rho_c}{\mu_c}, \quad (19)$$

$$\text{Ev} = \frac{\Gamma D_p}{\mu}, \quad (20)$$

$$g_0 = \frac{D_p^3 \rho_c^2 g}{\mu_c^2}, \quad (21)$$

$$\mu_0 = \frac{\mu}{\mu_c}, \quad (22)$$

$$\rho_0 = \frac{\rho}{\rho_c}, \quad (23)$$

where Ev and g_0 are newly introduced in this study. The Ev number is defined to describe that the vapor evaporated from the interface flows inside the vapor film. It is a ratio of the evaporation dynamic force to the vapor viscous force,

$$\text{Ev} = \frac{\text{dynamic_force}}{\text{viscous_force}}.$$

The dimensionless number g_0 is defined to describe the particle in coolant system with gravity g. It is the ratio of the gravity force to the coolant viscous force,

$$g_0 = \frac{\text{gravity_force}}{\text{viscous_force}}.$$

Then the drag coefficient of the single particle with vapor film under laminar condition

can be obtained as

$$\begin{aligned}
 C_{D-fb}^I = & \frac{9}{8} \left(1 + \left(\frac{2\delta}{D_p} \right) \right) (1 - \cos^2 \theta_s)^2 \\
 & + \frac{12\mu_0}{\text{Re}} \left(\frac{2\delta}{D_p} \right)^{-1} \left(\frac{1}{3} \cos^3 \theta_s - \cos \theta_s + \frac{2}{3} \right) \\
 & - \frac{2g_0}{\text{Re}^2} \left(\frac{1}{2} \cos \theta_s - \frac{1}{6} \cos^3 \theta_s + \frac{1}{3} \right) \\
 & + \frac{g_0}{\text{Re}^2} \left(\frac{2\delta}{D_p} \right) \left(\frac{1}{3} \cos^3 \theta_s - \cos \theta_s + \frac{2}{3} \right)
 \end{aligned} \quad (24)$$

where

$$\left(\frac{\delta}{D_p} \right)^3 + \frac{3\text{Re}\mu_0}{2g_0} \left(\frac{\delta}{D_p} \right) = \frac{Ev\mu_0^2}{g_0\rho_0}. \quad (25)$$

and

$$\cos \theta_s = -\frac{8\mu_0}{3\text{Re}} \left(\frac{\delta}{D_p} \right)^{-2} - \frac{2g_0}{9\text{Re}^2}. \quad (26)$$

By solving equations (24), (25) and (26) the drag coefficient of a particle with a vapor film moving in coolant liquid under laminar flow condition can be calculated. Due to the complexity and inconvenience of the expression (equation (24)) in applying to a computer code, a simplified drag coefficient correlation is arranged as

$$C_{D-fb}^I = a_0 + \frac{Ev\mu_0}{\text{Re}\rho_0} a_1 + \left(\frac{Ev\mu_0^2}{g_0\rho_0} \right)^{\frac{1}{3}} a_2 + \frac{g_0}{\text{Re}^2} a_3 + \frac{1}{Ev} a_4, \quad (27)$$

where $a_0=0.849$, $a_1=0.00205$, $a_2=3.47$, $a_3=0.0424$ and $a_4=-2.18$, which are obtained by fitting the expression (equation (24)) in the range of Reynolds number from 500 to 3000, Ev number from 60 to 600, g_0 from 10^5 to 10^9 , ρ_0 from 10^{-4} to 10^{-3} and μ_0 from 0.01 to 0.1 respectively. The equation (27) shows the drag coefficient of the single particle with vapor film moving in coolant liquid under laminar flow condition is the function of the Reynolds number, Ev number ($\Gamma D_p/\mu$), vapor/liquid viscosity and density ratios and dimensionless number $g_0(D^3\rho_c^2g/\mu_c^2)$.

The Ev number is a ratio of the evaporation dynamic force to the vapor viscous force. The dimensionless number g_0 is the ratio of the gravity force to the coolant viscous force. The dimensionless number μ_0 is the ratio of the vapor viscosity to coolant viscosity and the dimensionless number ρ_0 is the ratio of the vapor density to the coolant density. All the five dimensionless number is employed to describe the correlation of the drag coefficient. In this study the definition of these dimensionless numbers is the same. In the next section, based on the same method, the drag coefficient of the single particle with vapor film under turbulent condition will be developed.

2.3 The drag correlation under turbulent flow condition

In the turbulent flow cases, the assumptions of the potential flow and Bernoulli equation could not be applied in coolant liquid. The energy loss and interfacial velocity change due to turbulence should be considered. Here it is assumed that the interfacial velocity can be written as

$$u_i = AV_{cp} \sin \theta \quad (28)$$

and the pressure equation can be written as the same formulation as in laminar flow case by adding one term:

$$P(\theta) = P_0 + \rho_c g R \cos \theta + \rho_c \frac{V_{cp}^2}{2} - B \rho_c \frac{(1 - A \sin \theta)^2 V_{cp}^2}{2} - \frac{\rho_c}{2} (AV_{cp} \sin \theta)^2, \quad (29)$$

where A and B are parameters to be used to consider the influence of turbulent flow on interfacial velocity and pressure distribution around the particle. They will be determined by employing experimental data. The assumptions to vapor are the same as in laminar flow case. Then we can use the equations (5) through (24), replacing (11) and (12) by (28) and (29), in turbulent flow case. By solving those equations, the drag coefficient of the particle with vapor film moving in coolant liquid under turbulent flow condition can be obtained from equation (5) with the undetermined parameters A and B as

$$\begin{aligned}
C'_{D-fb} = & \frac{A^2}{2} \left(1 + \frac{2\delta}{D_p} \right) (1 - \cos^2 \theta_s)^2 \\
& - 2B \left(1 + \frac{2\delta}{D_p} \right) \left(\frac{A}{3} \sin^3 \theta_s - \frac{A^2}{4} \sin^4 \theta_s \right) \\
& + \frac{2A\mu_0}{\text{Re}} \left(\frac{2\delta}{D_p} \right)^{-1} \left(\frac{1}{3} \cos^3 \theta_s - \cos \theta_s + \frac{2}{3} \right) \\
& - \frac{2g_0}{\text{Re}^2} \left(\frac{1}{2} \cos \theta_s - \frac{1}{6} \cos^3 \theta_s + \frac{1}{3} \right) \\
& + \frac{g_0}{\text{Re}^2} \left(\frac{2\delta}{D_p} \right) \left(\frac{1}{3} \cos^3 \theta_s - \cos \theta_s + \frac{2}{3} \right), \tag{30}
\end{aligned}$$

where θ_s satisfies

$$\left(\frac{A^2}{2} + \frac{BA^2}{2} \right) \cos \theta_s - \frac{AB}{2} \frac{\cos \theta_s}{\sin \theta_s} + \left(\frac{\mu_0 A}{2\text{Re}} \left(\frac{\delta}{D_p} \right)^{-2} + \frac{g_0}{2\text{Re}^2} \right) = 0. \tag{31}$$

and

$$\delta = \left(\frac{\alpha E \nu \mu_0}{A \rho_0 \text{Re}} + \beta \left(\frac{E \nu \mu_0^2}{g_0 \rho_0} \right)^{\frac{1}{3}} \right) D_p / 2. \tag{32}$$

The detailed calculation is presented in the Appendix B. In turbulent flow case, the separation point of vapor laminar flow is over $\pi/2$, but very near $\pi/2$, which is shown in the Nelson experiment [10] and previous study [12]. Here $\theta_s = \pi/2$ is employed. Then the drag coefficient of the single particle with vapor film under turbulent condition (equation (30)) can be written into the formula arranged in dimensionless number as the following:

$$C'_{D-fb} = w_0 + \frac{E \nu \mu_0}{\text{Re} \rho_0} w_1 + \left(\frac{E \nu \mu_0^2}{g_0 \rho_0} \right)^{\frac{1}{3}} w_2 + \frac{g_0}{\text{Re}^2} w_3, \tag{33}$$

where w_i is the constant. By employing the experimental data of Nelson [10] and QUEOS [3] to fit the equation (33), it is obtained that $w_0=0.0065$ $w_1=0.0689$ $w_2=0.0115$ and $w_3=5.11$ (the parameters A and B in equations (28) and (29) are included in these constants), in the range of Reynolds number from 4000 to 120000, Ev number from 50 to 300, g_0 from 10^5 to 10^9 . In these two experiments, because the coolant water was filled in the tank opening to the outside with the pressure of 1 bar, ρ_0 is about 0.0007 and μ_0 is about 0.05 respectively. The drag coefficients obtained from the experimental data and those predicted by the fitted correlation under turbulent condition arranged in Reynolds number and Ev number are plotted in Figs. 3 and 4. The relativity of the drag coefficients predicted by equation (33) and their experimental data is plotted in Fig. 5. It is showed that the fitting is acceptable. The equation (33) shows the drag coefficient of the particle with vapor film moving in coolant liquid under turbulent flow condition also can be described by the Reynolds number, Ev number ($\Gamma D_p / \mu$), vapor/liquid viscosity and density ratios and dimensionless number $g_0(D^3 \rho_c^2 g / \mu_c^2)$. In the calculation of Ev number, the film boiling heat transfer is calculated by using the formula in ref. [11]. It should be noted that the QUEOS experiment is a multi-particle premixing experiment, in which hot solid particles are released into water. The particle velocity at the leading edge of the particle cloud is estimated as the single particle velocity in the experiment, which are employed to fit the parameters in the correlation (equation (33)). At the leading edge, since the volume fraction of the particles is small, the effect of other particles can be neglected, which will be discussed in the next section.

3. Drag coefficient for multi-particle system

In the multi-particle system, it is assumed that the configuration of the original particle with vapor film is not deformed by other particles in the system and the force on the original particle by other particles is exerted through the interfacial velocity change, which could be expressed as a factor of viscous force. The drag coefficient of the original particle with vapor film in multi-particle system could be calculated by equation (5). The governing equations for vapor are the same as those in the single particle case. The effect of other particles on the original particle is considered by the interfacial velocity and pressure distribution equations. It is assumed that the Ishii's model [6], in which the mixture viscosity is used for particles in multi-particle system, can be applied in the case of the particle with vapor film in multi-particle system. The mixture viscosity of the coolant in multi-particle system is given by Ishii [6]

$$\frac{\mu_m}{\mu_c} = (1 - \alpha_p)^n \quad (34)$$

where α_p is the volume fraction of the particles in multi-particle system, μ_c is the coolant viscosity, μ_m is the mixture viscosity of the coolant in the multi-particle system and $n=1.75$ for particles [6]. On the coolant liquid-vapor interface, the viscous force in coolant liquid side can be written as

$$\mu_m \left. \frac{\partial u_c}{\partial y} \right|_{y=\delta^+} = \mu_c \left. \frac{\partial u_c (1 - \alpha_p)^n}{\partial y} \right|_{y=\delta^+}, \quad (35)$$

then the interfacial velocity can be written as

$$u_m \Big|_{y=\delta^+} = (1 - \alpha_p)^n u_c \Big|_{y=\delta^+} = (1 - \alpha_p)^n u_i \quad (36)$$

where $u_m \Big|_{y=\delta^+}$ is the interfacial velocity on the coolant liquid-vapor interface in multi-particle system considering the effect of other particles and u_i is the interfacial velocity in single particle system. Then the method developed for calculating the drag

coefficient of the single particle can be applied to the particle in multi-particle system. The drag correlations under laminar and turbulent flow conditions in multi-particle system will be developed respectively in the following according to the different interfacial velocity as in the single particle cases.

3.1 Drag correlation under laminar flow condition

The velocity of the liquid-vapor interface u_i for the original particle under laminar flow condition can be written as

$$u_i = \frac{3}{2} V_{cp} (1 - \alpha_p)^n \sin \theta . \quad (37)$$

The pressure at angle θ on the liquid-vapor interface in the coolant liquid could be expressed as

$$P(\theta) = P_0 + \rho_c g R \cos \theta + \rho_c \frac{V_{cp}^2}{2} - \frac{\rho_c}{2} \left(\frac{3}{2} V_{cp} (1 - \alpha_p)^n \sin \theta \right)^2 \quad (38)$$

Then the equations (5) through (24), replacing (11) and (12) by (37) and (38), can be employed to calculate the drag coefficient of the particle with vapor film in multi-particle system under laminar flow condition. By employing the same method in the previous section, the drag coefficient of the particle with vapor film under laminar condition in multi-particle system can be obtained as the formula expressed in dimensionless number:

$$C'_{D-p} = a_0 (1 - \alpha_p)^{2n} + \frac{Ev \mu_0}{Re \rho_0} (1 - \alpha_p)^{2n} a_1 + \left(\frac{Ev \mu_0^2}{g_0 \rho_0} \right)^{\frac{1}{3}} (1 - \alpha_p)^{2n} a_2 + \frac{g_0}{Re^2} a_3 + \frac{1}{Ev} (1 - \alpha_p)^{2n} \quad (39)$$

where the constant a_i and the ranges of Reynolds number, Ev number, g_0 , ρ_0 and μ_0 are the same value as in the single particle case, because, when α_p is 0, equation (39) is reduced to that for the single particle case expressed in equation (27).

The drag coefficients of the hot particle, predicted by equation (39), are plotted in Figs. 6, 7 and 8, which are arranged according to Reynolds number, Ev number and the volume fraction of the particles. As shown in Fig. 6, the drag coefficients decrease with

the increase of Reynolds number, but they are not changed too much when Reynolds number becomes larger. It is showed that the drag coefficient of the particle with vapor film is larger than that of the particle without vapor film given by Ishii [6] in laminar case, which can be explained by that in laminar case, the velocity of vapor flowing inside the film is larger than that of the particle moving in coolant liquid. Figure 7 shows that the drag coefficients increase slowly with the increase of the Ev number, which can be explained by the increase of the thickness of the vapor film with the increase of the Ev number. Comparing to the drag coefficient of the particle without vapor film, as shown in Fig. 8, the drag coefficients of the particle with vapor film increase with the increase of the volume fraction of the particles much faster than that of the particle without vapor film.

3.2 Drag coefficient under turbulent flow condition

Based on the same method described in the previous section, the drag coefficient of the original particle with vapor film in multi-particle system under turbulent flow condition can be calculated. The velocity of the liquid-vapor interface u_i for the original particle is assumed as

$$u_i = AV_{cp} (1 - \alpha_p)^n \sin \theta. \quad (40)$$

The pressure at angle θ on the liquid-vapor interface in the coolant liquid could be expressed as

$$P(\theta) = P_0 + \rho_c g R \cos \theta + \rho_c \frac{V_{cp}^2}{2} - B \rho_c \frac{\left(1 - A(1 - \alpha_p)^n \sin \theta\right)^2 V_{cp}^2}{2} - \frac{\rho_c}{2} \left(A V_{cp} (1 - \alpha_p)^n \sin \theta\right)^2 \quad (41)$$

where α_p is the volume fraction of the particles. A and B are parameters to be used to consider the influence of turbulent flow on interfacial velocity and pressure distribution around the particle, which is the same as that in the single particle case.

Then the equations (5) through (24), replacing (11) and (12) by (40) and (41), can be employed to calculate the drag coefficient of the particle with vapor film in multi-particle system under turbulent flow condition. The drag coefficient can be

expressed as the formula in dimensionless number:

$$C'_{D-fb} = w_0(1-\alpha_p)^{2n} + \frac{Ev\mu_0}{Re\rho_0}w_1(1-\alpha_p)^n + \left(\frac{Ev\mu_0^2}{g_0\rho_0}\right)^{\frac{1}{3}}w_2(1-\alpha_p)^{2n} + \frac{g_0}{Re^2}w_3 \quad (42)$$

where the constant w_i and the ranges of Reynolds number, Ev number, g_0 , ρ_0 and μ_0 are the same value as in the single particle case, because, when α_p is 0, Eq. (42) is reduced to that for the single particle case expressed in equation (33).

The drag coefficients of the hot particle, predicted by equation (42), are plotted in Figs. 9, 10 and 11, which are arranged according to Reynolds number, Ev number and the volume fraction of the particles. As shown in Fig. 9, unlike the drag coefficient of the particle without vapor film in turbulent case, the drag coefficients of the particle with vapor film decrease with the increase of Reynolds number. In large Reynolds number cases the drag coefficients are smaller than that of the particle without vapor film, as shown in Fig. 9, which can be explained by that in turbulent case, the velocity of vapor flowing inside the film is less than that of the particle moving in coolant liquid. Figure 10 shows that the drag coefficients increase with the increase of the Ev number faster than that in laminar case, which can be explained by that in the turbulent flow cases, the thickness of the vapor film is thinner than that in laminar flow cases due to the large interfacial velocity. For the same increase of the evaporation, more increase of the resistance to the particle is given in the turbulent flow cases than in laminar flow cases. Figure 11 shows that the drag coefficients increase very fast with the increase of the volume fraction of the particles, which is similar to that of the particle without vapor film.

4. Application of the proposed drag correlations

The proposed drag correlations (equations (39) and (42)) are introduced into the SIMMER-III code and applied to simulate the QUEOS experiments (Q12 and Q35) [3],

the experiment for the premixing phase of FCIs, in which 6.9 and 10 kilograms of solid molybdenum spheres with the temperature of 2300K and 1800K are released into water, respectively. The QUEOS facility and the calculation system are described in refs. [3,4,5]. The front advancement of the particle cloud in water is analyzed to validate the proposed drag correlations. In the calculation of Ev number, the film boiling heat transfer is calculated by using the formula in ref. [11]. The front advancement of the particle cloud, moving in coolant water, obtained from the experimental data, predicted by employing the Ishii's correlations and the proposed correlations are plotted in Figs. 12 and 13, respectively. The results show the proposed correlations improve the agreement of the evolution of the location of the leading edge of the particle cloud with the experimental data. From these figures, the remaining difference is shown between the simulation with the proposed correlation and the experimental data. This is caused by the following reason. During the falling of the particle cloud in the vessel, its front part is mixed with water and the other part is mixed with vapor. The motion of the particle cloud moving in the vapor mixing region is affected by the momentum exchange between the particle and the vapor, including momentum exchange coefficient and vapor velocity. This then affects the contour shape of the particle clouds (as investigated in the previous study [5]). The overestimation of the momentum exchange between the particle and the vapor in vapor mixing region is thought to cause the above remaining difference.

5. Conclusion

The drag coefficient for a single particle surrounded by vapor film moving in coolant liquid under laminar and turbulent flow conditions are studied, based on the conservative equations and assumptions. The drag correlations can be described by the Reynolds number, vapor/liquid viscosity and density ratios and the two dimensionless

numbers, newly introduced in this study to suitably describe the influence of the vapor film on the drag coefficient. In the turbulent flow regime, the drag coefficient correlation is fitted by experimental data and is smaller than that of the particle without vapor film.

Based on the single particle model and the mixture viscosity concept, the drag correlations of the particle with vapor film in multi-particle system are proposed for the laminar and turbulent flow cases respectively. The proposed drag correlations are used to simulate the QUEOS experiment and the results show that the proposed correlations improve the agreement of the simulated results with the experimental data. Since the constants in the proposed drag correlations (33) and (42) under turbulent condition are fitted by experimental data in limited ranges of dimensionless parameters, not necessarily sufficient for covering the reactor conditions, more experimental data are required to verify the proposed correlations.

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Nomenclature

English Symbols

a_i	constant
A	area (m^2), undetermined parameter
B	undetermined parameter

- C_D drag coefficient
- C_{D-fb}^l drag coefficient of a hot particle/droplet with vapor film under laminar condition
- C_{D-fb}^t drag coefficient of a hot particle/droplet with vapor film under turbulent condition
- D diameter (m)
- Ev evaporation flow number, which is defined to describe the vapor flow inside the vapor film induced by the evaporation of coolant liquid on the coolant liquid/vapor interface $Ev = \frac{\text{dynamic_force}}{\text{viscous_force}}$, written as $(Ev = \frac{\Gamma D_p}{\mu})$
- F force (N)
- g gravity (m/s^2)
- g_0 dimensionless number which is defined to describe the particle in coolant system with gravity g , $g_0 = \frac{\text{gravity_force}}{\text{viscous_force}}$, written as $(g_0 = \frac{D_p^3 \rho_c^2 g}{\mu_c^2})$
- P pressure (Pa)
- r_p radius of particle (m)
- Re Reynolds number $(Re = \frac{v_{cp} \rho_c D_p}{\mu_c})$
- u vapor velocity in x direction (m/s).
- \bar{u} average vapor velocity along vapor film (m/s).
- V_{cp} velocity (m/s)
- w_i constant
- Greece Symbols
- α_p volume fraction of particle/droplet
- Γ evaporation rate ($kg/(m^2 \cdot s)$)
- δ thickness of vapor film (m)

θ angle

μ viscosity ($Pa \cdot s$).

μ_0 dimensionless number which is the ratio of the viscosity of the vapor to coolant

$$\text{liquid } (\mu_0 = \frac{\mu}{\mu_c})$$

ρ density (kg/m^3).

ρ_0 dimensionless number which is the ratio of the densities of the vapor to

$$\text{coolant liquid } (\rho_0 = \frac{\rho}{\rho_c})$$

τ tangential force

Subscripts:

c coolant, continuous phase

p particle/droplet

s surface, separation point

y y direction

θ x direction

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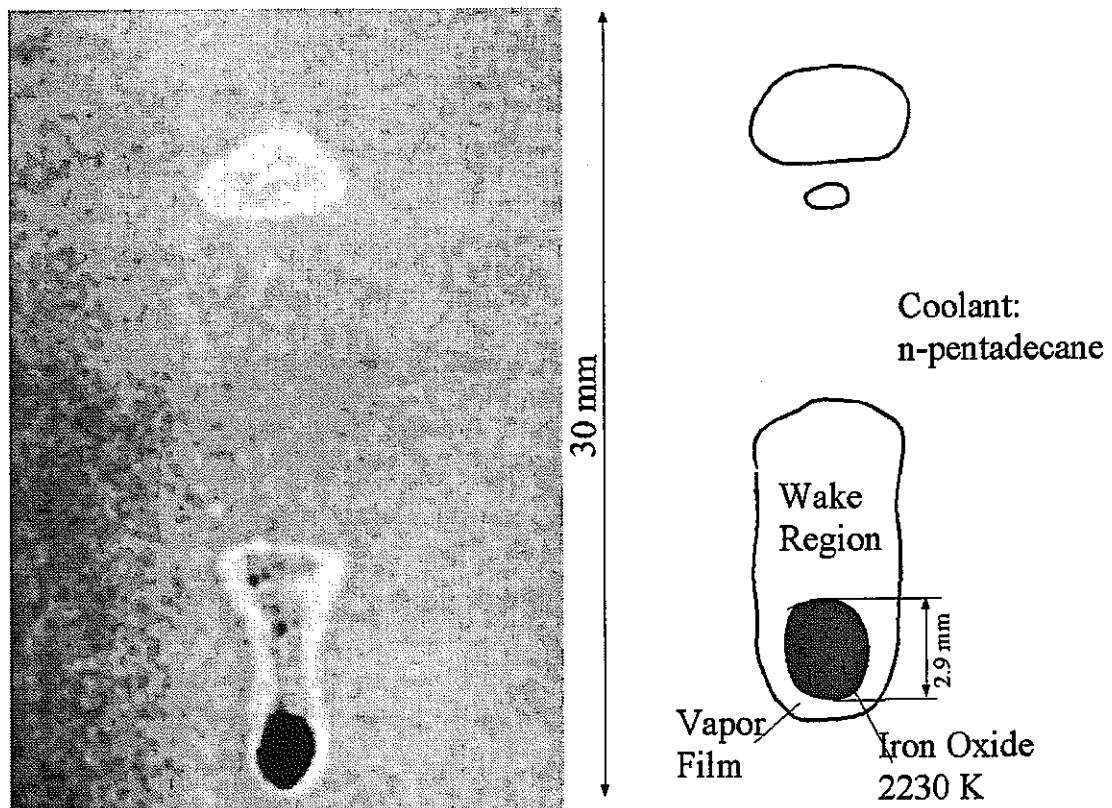


Fig. 1 Configuration of a single droplet moving in coolant in Nelson experiment (NUREG/CR-2718)

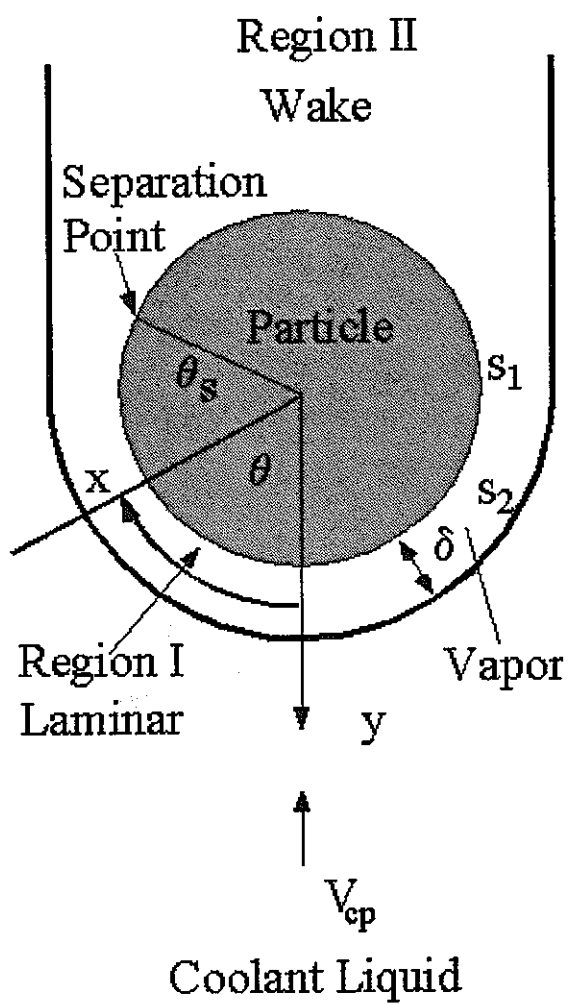


Fig. 2 Scheme of a hot particle moving in coolant liquid

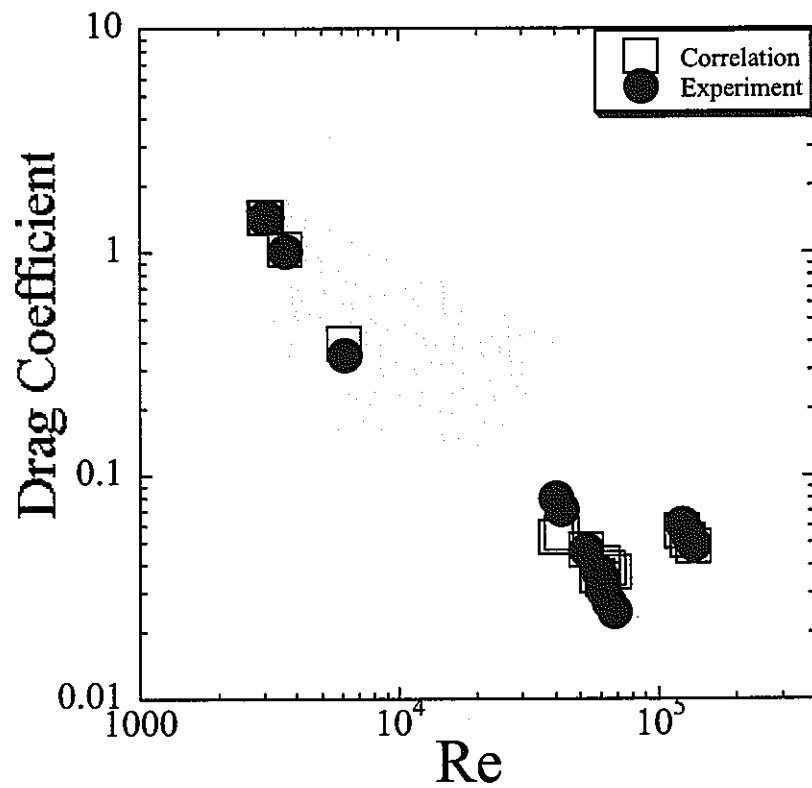


Fig. 3 The drag coefficients obtained from the experimental data and those predicted by the fitted correlation under turbulent condition arranged in Reynolds number

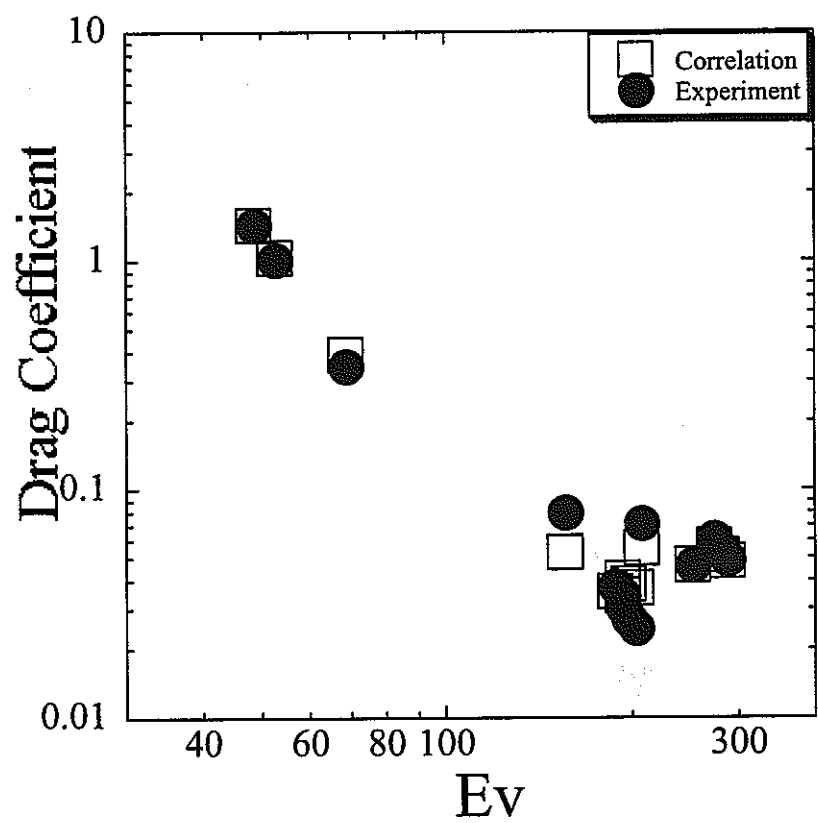


Fig. 4 The drag coefficients obtained from the experimental data and those predicted by the fitted correlation under turbulent condition arranged in Ev number

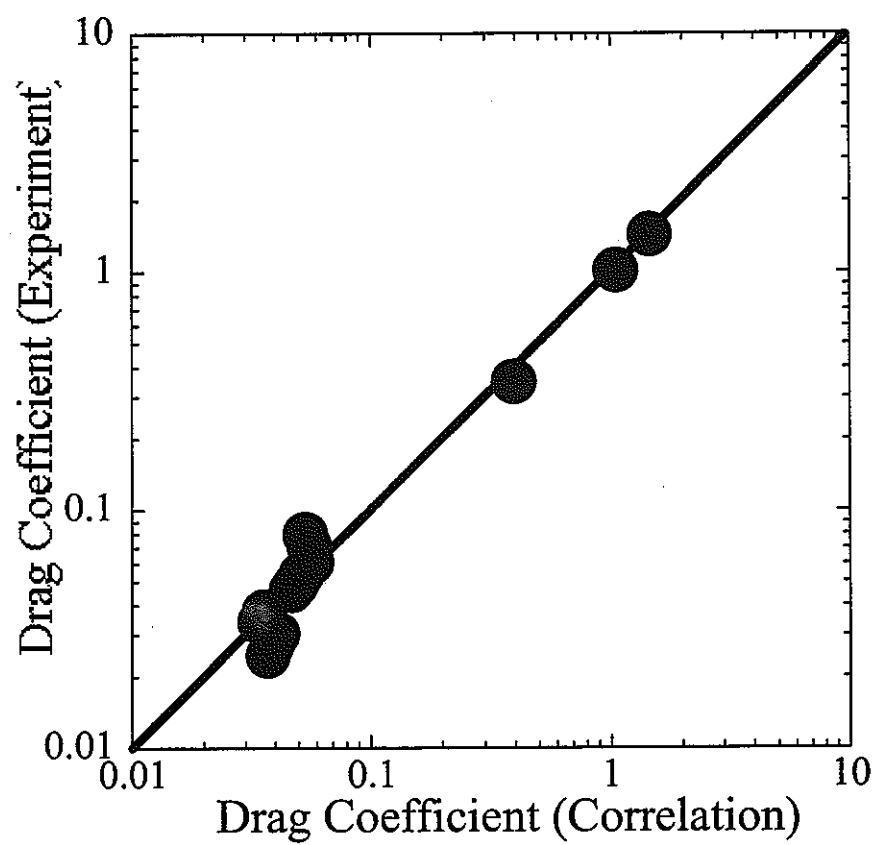


Fig. 5 The relativity of the drag coefficient predicted by equation (29) and their experimental data

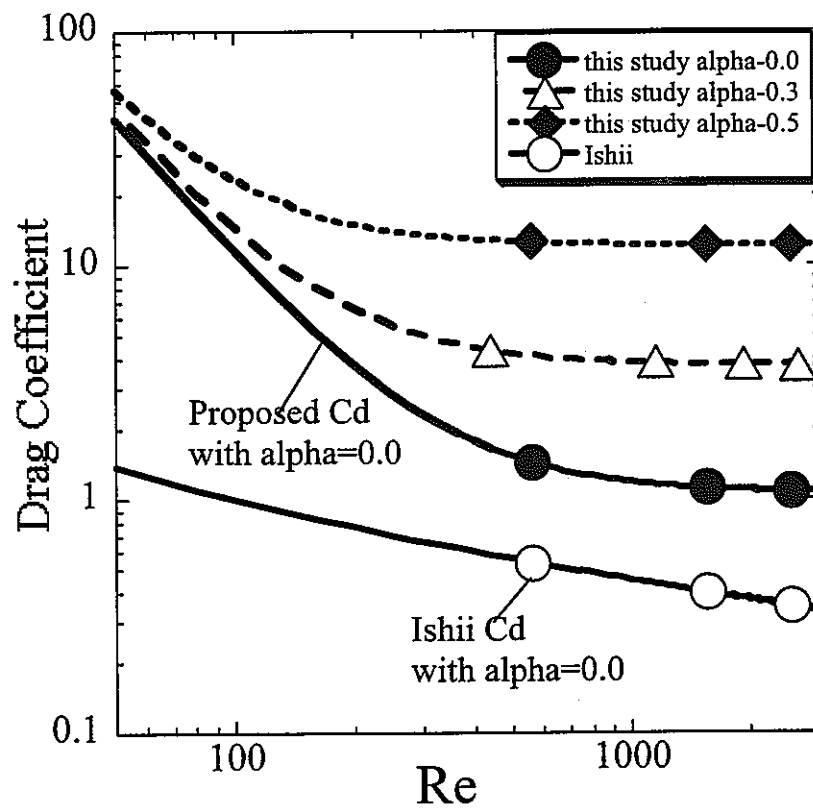


Fig. 6 The variation of the drag coefficient of the particles with vapor film under laminar condition with Reynolds number. (Ev number is 200, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles)

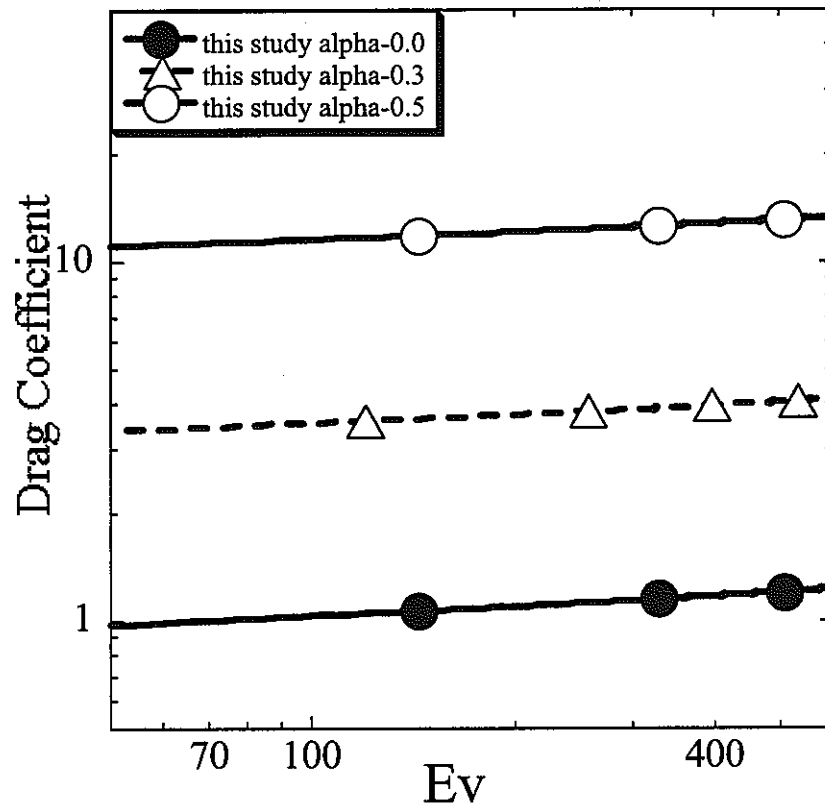


Fig. 7 The variation of the drag coefficient of the particles with vapor film under laminar condition with Ev number. (Reynolds number is 1000, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles)

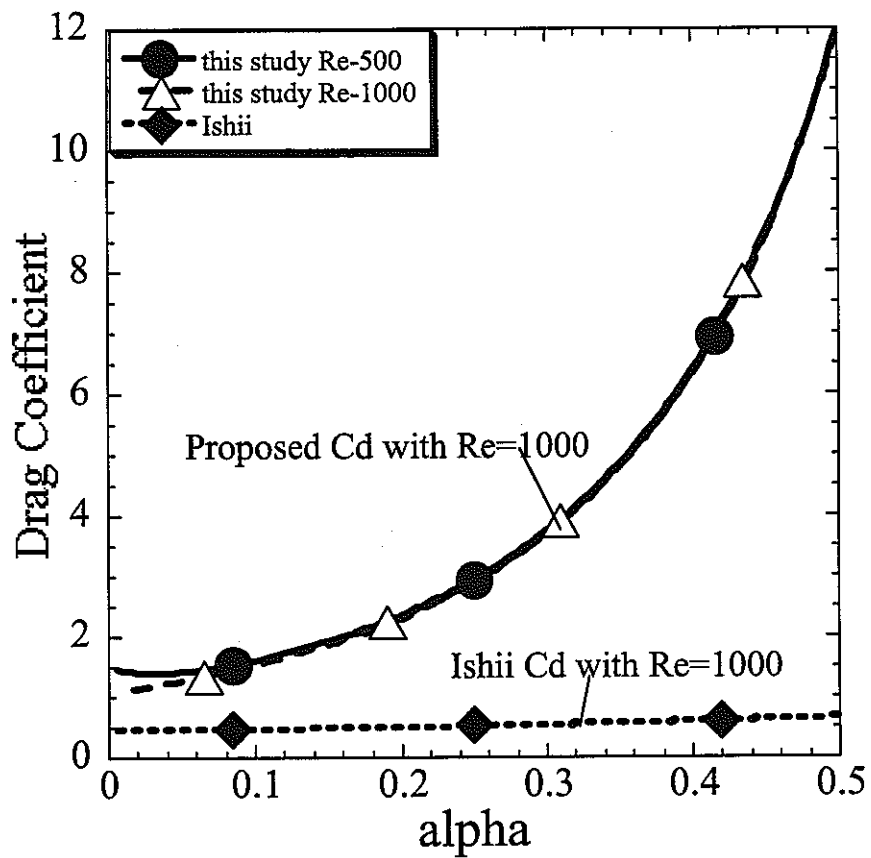


Fig. 8 The variation of the drag coefficient of the particles with vapor film under laminar condition with the volume fraction of the particles. (Ev number is 200, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The α is the volume fraction of the particles)

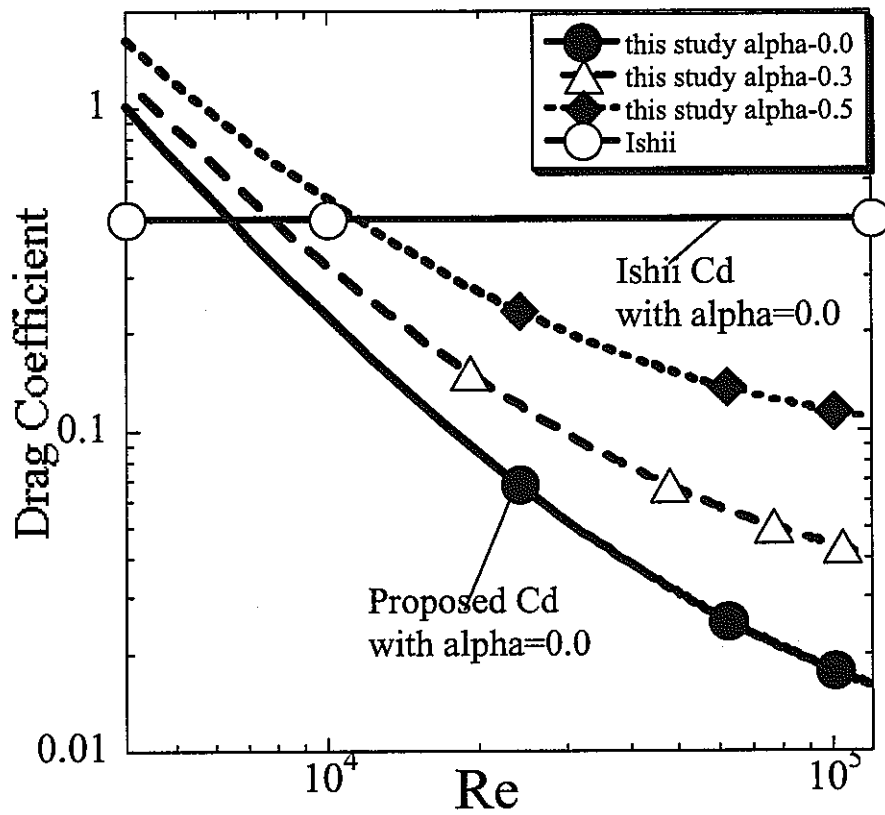


Fig. 9 The variation of the drag coefficient of the particles with vapor film under turbulent condition with Reynolds number. (Ev number is 200, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles)

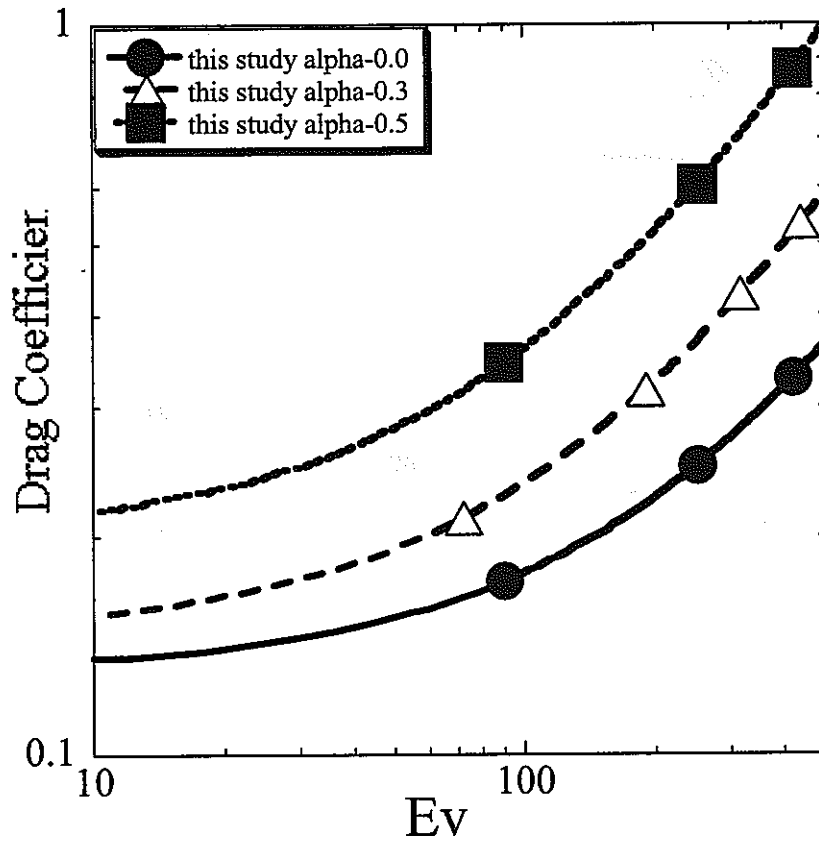


Fig. 10 The variation of the drag coefficient of the particles with vapor film under turbulent condition with Ev number. (Reynolds number is 10000, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles)

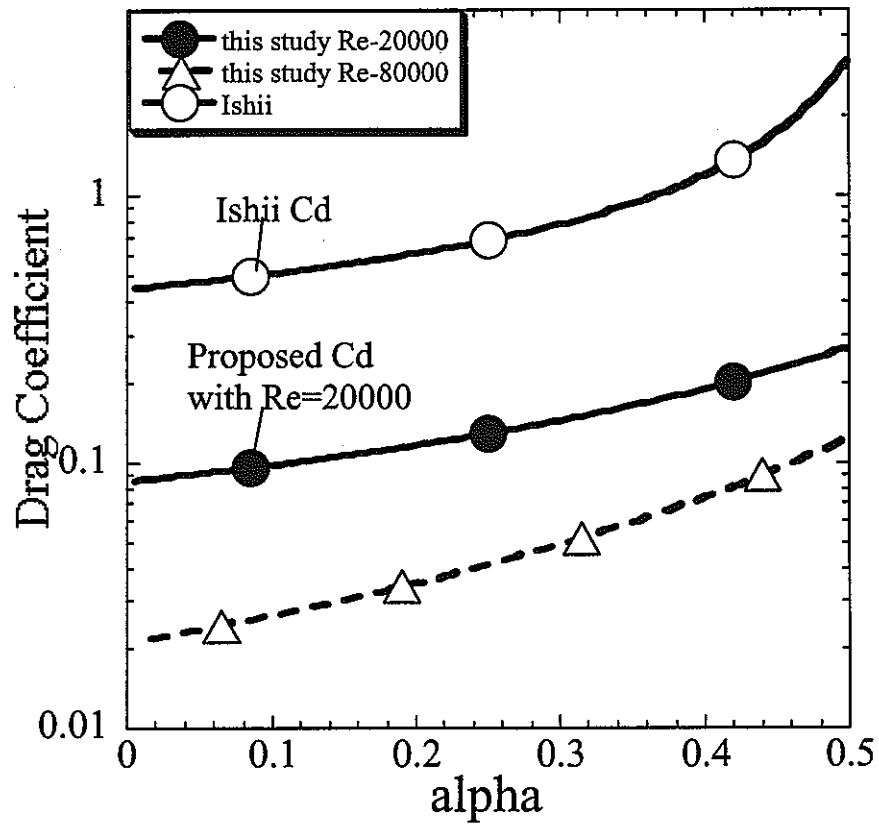


Fig. 11 The variation of the drag coefficient of the particles with vapor film under turbulent condition with the volume fraction of the particles. (Ev number is 200, μ_0 is 0.12, ρ_0 is 0.00056 and g_0 is 8×10^6 . The alpha is the volume fraction of the particles)

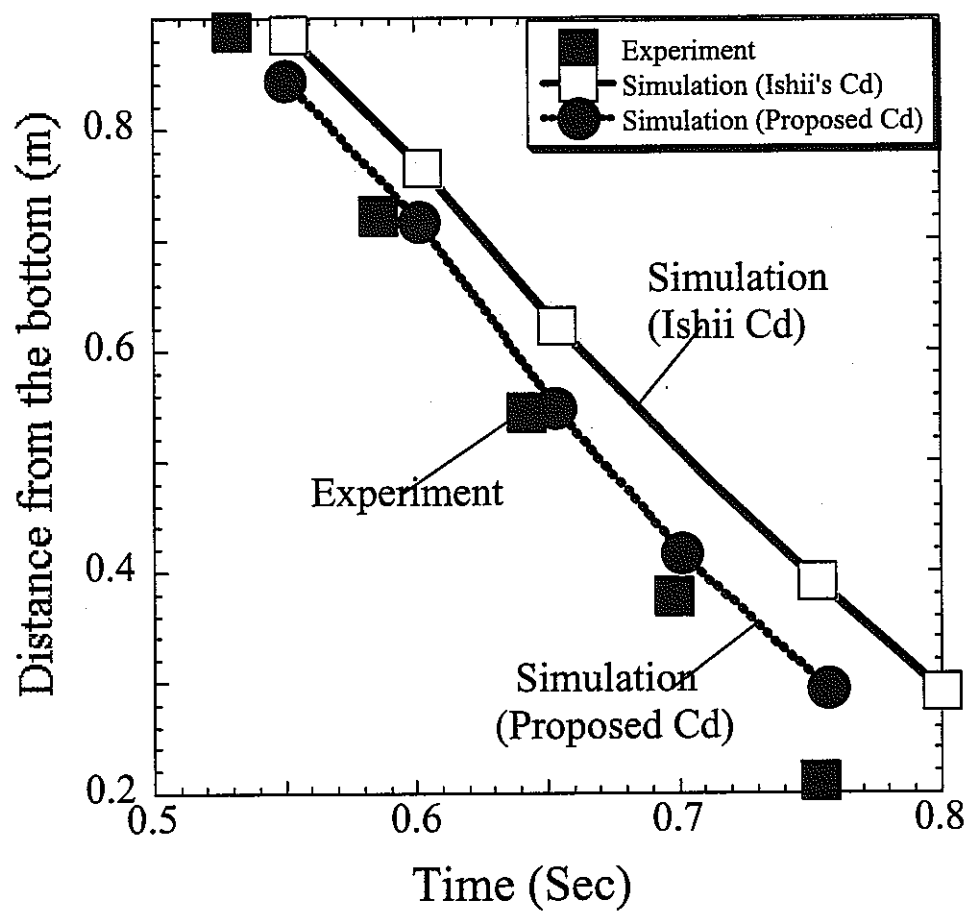


Fig. 12 The advancements of the particle cloud in QUEOS-12, calculated by different drag coefficient correlations

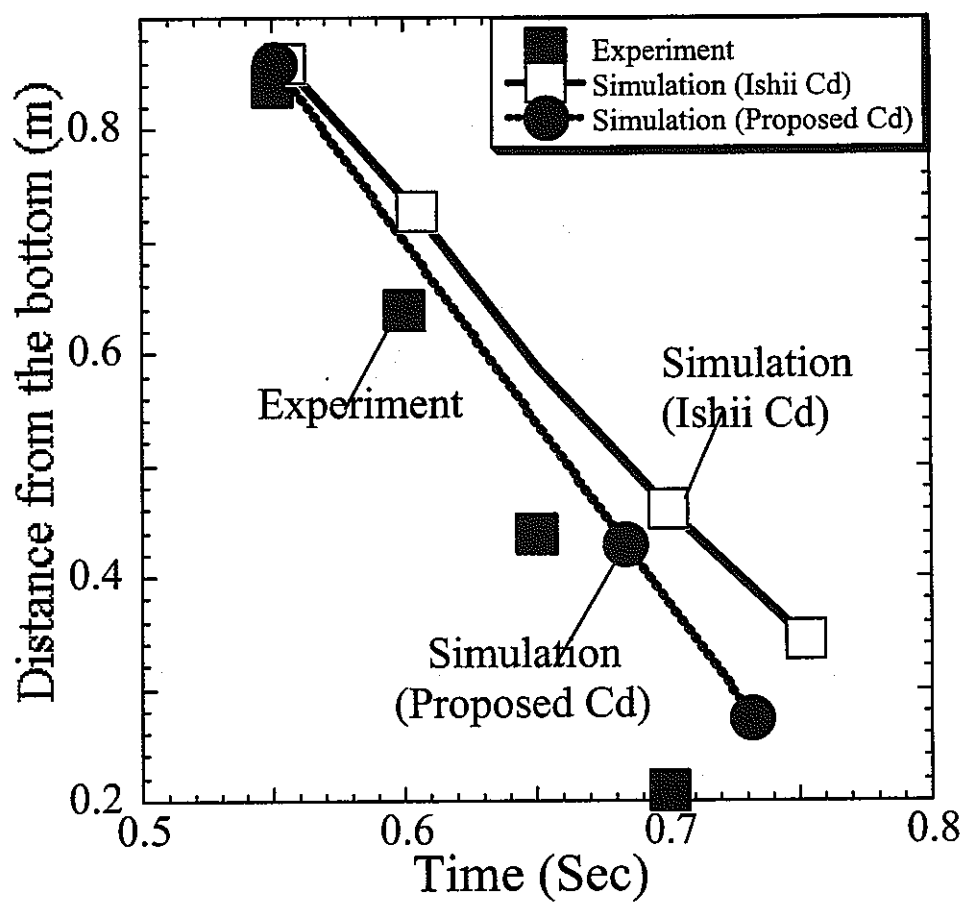


Fig. 13 The advancements of the particle cloud in QUEOS-35, calculated by different drag coefficient correlations

Appendix A: Drag coefficient for a single particle under laminar flow condition

The governing equations for the vapor in the film are

$$\frac{\partial \rho u}{\partial x} = 0, \quad (1)$$

$$\frac{\partial \rho u^2}{\partial x} = -\frac{\partial P}{\partial x} - \rho g \sin \theta + \mu \frac{\partial^2 u}{\partial y^2}. \quad (2)$$

The boundary conditions are

$$y=0; \quad u=0,$$

$$y=\delta; \quad u=u_i.$$

The interfacial velocity is

$$u_i = \frac{3}{2} V_{cp} \sin \theta \quad (3)$$

From equations (1) and (2) we can obtain

$$\frac{\partial P}{\partial x} = -\rho_c g \sin \theta + \mu \frac{\partial^2 u}{\partial y^2} \quad (4)$$

The pressure distribution can be written as

$$P(\theta) = P_0 + \rho_c g r_p \cos \theta + \rho_c \frac{V_{cp}^2}{2} - \frac{\rho_c}{2} \left(\frac{3}{2} V_{cp} \sin \theta \right)^2 \quad (5)$$

Then

$$\frac{\partial P}{\partial x} = -\rho_c g \sin \theta - \frac{1}{2r_p} \rho_c V_{cp}^2 \sin 2\theta \quad (6)$$

By combining the equations (4) and (6), we can get

$$\mu \frac{\partial^2 u}{\partial y^2} = -(\rho_c - \rho) g \sin \theta - \frac{1}{2r_p} \rho_c V_{cp}^2 \sin 2\theta \quad (7)$$

Then the velocity distribution can be obtained

$$u = \frac{3y}{2\delta} V_{cp} \sin \theta - \left(\frac{(\rho_c - \rho)g}{2\mu} \sin \theta + \frac{9\rho_c V_{cp}^2}{16\mu r_{cp}} \sin 2\theta \right) (y^2 - y\delta) \quad (8)$$

The viscous force on the particle surface can be written as

$$\begin{aligned} \tau_{s_1} &= \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} \\ &= \frac{3\mu}{2\delta} V_{cp} \sin \theta + \left(\frac{(\rho_c - \rho)g}{2} \sin \theta + \frac{9\rho_c V_{cp}^2}{16r_{cp}} \sin 2\theta \right) \delta \end{aligned} \quad (9)$$

The forces exerted on the part I include pressure and viscous force

$$\begin{aligned} F_y^I &= \int_0^{\theta_s} (P \cos \theta + \tau_{s_1} \sin \theta) 2\pi r_p^2 \sin \theta d\theta \\ &= 2\pi r_p^2 \left[\left(P_0 + \rho_c \frac{V_{cp}^2}{2} \right) \frac{\sin^2 \theta_s}{2} - \frac{\rho_c g r_p}{3} (\cos^3 \theta_s - 1) - \frac{9\rho_c V_{cp}^2}{32} \sin^4 \theta_s \right] \\ &\quad + 2\pi r_p^2 \left[\left(\frac{3\mu V_{cp}}{2\delta} + \frac{\rho_c g \delta}{2} \right) \left(\frac{1}{3} \cos^3 \theta_s - \cos \theta_s + \frac{2}{3} \right) + \frac{9\rho_c \delta V_{cp}^2}{32r_p} \sin^4 \theta_s \right] \end{aligned} \quad (10)$$

The force exerted on the part II can be calculated as

$$\begin{aligned} F_y^{II} &= \int_{\theta_s}^{\pi} P_{\theta_s} 2\pi r_p^2 \sin \theta \cos \theta d\theta \\ &= 2\pi r_p^2 \left[- \left(P_0 + \rho_c \frac{V_{cp}^2}{2} \right) \frac{\sin^2 \theta_s}{2} - \frac{\rho_c g r_p}{2} \cos \theta_s \sin^2 \theta_s + \frac{9\rho_c V_{cp}^2}{16} \sin^4 \theta_s \right] \end{aligned} \quad (11)$$

The total force is

$$F_y = F_y^I + F_y^{II} \quad (12)$$

The drag coefficient can be written as

$$\begin{aligned} C_{D-fb} &= \frac{F_y}{\pi r_p^2 \frac{1}{2} \rho_c V_{cp}^2} \\ &= \frac{9}{8} \left(1 + \left(\frac{2\delta}{D_p} \right) \right) (1 - \cos^2 \theta_s)^2 \end{aligned}$$

$$\begin{aligned}
& + \frac{12\mu}{V_{cp}D_p\rho_c} \left(\frac{2\delta}{D_p} \right)^{-1} \left(\frac{1}{3} \cos^3 \theta_s - \cos \theta_s + \frac{2}{3} \right) \\
& - \frac{2D_pg}{V_{cp}^2} \left(\frac{1}{2} \cos \theta_s - \frac{1}{6} \cos^3 \theta_s + \frac{1}{3} \right) \\
& + \frac{D_pg}{V_{cp}^2} \left(\frac{2\delta}{D_p} \right) \left(\frac{1}{3} \cos^3 \theta_s - \cos \theta_s + \frac{2}{3} \right)
\end{aligned} \tag{13}$$

with

$$\left(\frac{\delta}{D_p} \right)^3 + \frac{3\mu V_{cp}}{2D_p^2 \rho_c g} \left(\frac{\delta}{D_p} \right) = \frac{\Gamma \mu_0}{D_p^2 \rho_c^2 g \rho}. \tag{14}$$

and

$$\cos \theta_s = -\frac{8\mu}{3D_p V_{cp} \rho c} \left(\frac{\delta}{D_p} \right)^{-2} - \frac{2D_pg}{9V_{cp}^2}. \tag{15}$$

Appendix B: Drag coefficient for a single particle under turbulent flow condition

The governing equations for the vapor in the film are

$$\frac{\partial \rho u}{\partial x} = 0, \quad (1)$$

$$\frac{\partial \rho u^2}{\partial x} = -\frac{\partial P}{\partial x} - \rho g \sin \theta + \mu \frac{\partial^2 u}{\partial y^2}. \quad (2)$$

The boundary conditions are

$$\begin{aligned} y=0; \quad u=0, \\ y=\delta; \quad u=u_i. \end{aligned}$$

The interfacial velocity is

$$u_i = AV_{cp} \sin \theta. \quad (3)$$

From equations (1) and (2) we can obtain

$$\frac{\partial P}{\partial x} = -\rho_c g \sin \theta + \mu \frac{\partial^2 u}{\partial y^2}. \quad (4)$$

The pressure distribution is

$$P(\theta) = P_0 + \rho_c g r_p \cos \theta + \rho_c \frac{V_{cp}^2}{2} - B \rho_c \frac{(1 - A \sin \theta)^2}{2} V_{cp}^2 - \frac{\rho_c}{2} (AV_{cp} \sin \theta)^2. \quad (5)$$

Then

$$\frac{\partial P}{\partial x} = -\rho_c g \sin \theta - \frac{1}{2r_p} A^2 \rho_c V_{cp}^2 \sin 2\theta + \frac{1}{r_p} AB \rho_c (1 - A \sin \theta) \cos \theta V_{cp}^2. \quad (6)$$

By combining the equations (3) and (5), we can get

$$\mu \frac{\partial^2 u}{\partial y^2} = -(\rho_c - \rho)g \sin \theta - \frac{1}{2r_p} \rho_c V_{cp}^2 \sin 2\theta + \frac{1}{r_p} AB \rho_c (1 - A \sin \theta) \cos \theta V_{cp}^2. \quad (7)$$

Then the velocity distribution can be obtained

$$u = \frac{y}{\delta} AV_{cp} \sin \theta - \left(\frac{(\rho_c - \rho)g}{2\mu} \sin \theta + \frac{\rho_c V_{cp}^2}{4\mu r_p} A^2 \sin 2\theta \right) (y^2 - y\delta)$$

$$+\frac{1}{2\mu r_p} B\rho_c (1-A\sin\theta) A\cos\theta V_{cp}^2 (y^2 - y\delta) \quad (8)$$

The viscous force on the particle surface can be written as

$$\begin{aligned} \tau_{s_1} &= \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} \\ &= \frac{\mu}{\delta} AV_{cp} \sin\theta + \left(\frac{(\rho_c - \rho)g}{2} \sin\theta + \frac{\rho_c V_{cp}^2}{4 r_{cp}} A^2 \sin 2\theta \right) \delta \\ &\quad - \frac{1}{2r_p} B\rho_c (1-A\sin\theta) A\cos\theta V_{cp}^2 \delta \end{aligned} \quad (9)$$

The forces exerted on the part I include pressure and viscous force

$$\begin{aligned} F_y^I &= \int_0^{\theta_s} (P \cos\theta + \tau_{s_1} \sin\theta) 2\pi r_p^2 \sin\theta d\theta \\ &= 2\pi r_p^2 \left[\left(P_0 + \rho_c \frac{V_{cp}^2}{2} \right) \frac{\sin^2 \theta_s}{2} - \frac{\rho_c g r_p}{3} (\cos^3 \theta_s - 1) - \frac{A^2 \rho_c V_{cp}^2}{8} \sin^4 \theta_s \right] \\ &\quad - 2\pi r_p^2 \left[\frac{B\rho_c V_{cp}^2}{2} \left(\frac{1}{2} \sin^2 \theta_s - \frac{2A}{3} \sin^3 \theta_s + \frac{A^2}{4} \sin^4 \theta_s \right) \right] \\ &\quad + 2\pi r_p^2 \left[\left(\frac{\mu V_{cp}}{\delta} + \frac{\rho_c g \delta}{2} \right) \left(\frac{1}{3} \cos^3 \theta_s - \cos \theta_s + \frac{2}{3} \right) + \frac{\rho_c \delta V_{cp}^2}{8 r_p} A^2 \sin^4 \theta_s \right] \\ &\quad - 2\pi r_p^2 \left[\frac{B\rho_c V_{cp}^2 \delta A}{2 r_{cp}} \left(\frac{1}{3} \sin^3 \theta_s - \frac{A}{4} \sin^4 \theta_s \right) \right] \end{aligned} \quad (10)$$

The force exerted on the part II can be calculated as

$$\begin{aligned} F_y^{II} &= \int_{\theta_s}^{\pi} P_{\theta_s} 2\pi r_p^2 \sin\theta \cos\theta d\theta \\ &= 2\pi r_p^2 \left[- \left(P_0 + \rho_c \frac{V_{cp}^2}{2} \right) \frac{\sin^2 \theta_s}{2} - \frac{\rho_c g r_p}{2} \cos \theta_s \sin^2 \theta_s + \frac{\rho_c V_{cp}^2}{4} \sin^4 \theta_s \right] \\ &\quad + 2\pi r_p^2 \left[\frac{B\rho_c V_{cp}^2}{4} (1 - A\sin\theta_s)^2 \sin^2 \theta_s \right] \end{aligned} \quad (11)$$

The total force is

$$F_y = F_y^I + F_y^{II}. \quad (12)$$

The drag coefficient can be written as

$$\begin{aligned} C_{D-fb} &= \frac{F_y}{\pi r_p^2 \frac{1}{2} \rho_c V_{cp}^2} \\ &= \frac{A^2}{2} \left(1 + \left(\frac{2\delta}{D_p} \right) \right) \left(1 - \cos^2 \theta_s \right)^2 + \frac{4A\mu}{V_{cp} D_p \rho_c} \left(\frac{2\delta}{D_p} \right)^{-1} \left(\frac{1}{3} \cos^3 \theta_s - \cos \theta_s + \frac{2}{3} \right) \\ &\quad - \frac{2D_p g}{V_{cp}^2} \left(\frac{1}{2} \cos \theta_s - \frac{1}{6} \cos^3 \theta_s + \frac{1}{3} \right) + \frac{D_p g}{V_{cp}^2} \left(\frac{2\delta}{D_p} \right) \left(\frac{1}{3} \cos^3 \theta_s - \cos \theta_s + \frac{2}{3} \right) \\ &\quad - \left[2B \left(\frac{1}{3} A \sin^3 \theta_s - \frac{1}{4} A^2 \sin^4 \theta_s \right) + \frac{2AB\delta}{r_p} \left(\frac{1}{3} \sin^3 \theta_s - \frac{1}{4} A \sin^4 \theta_s \right) \right] \end{aligned} \quad (13)$$

where θ_s satisfies

$$\left(\frac{A^2 V_{cp}}{2} + \frac{BA^2 V_{cp}}{2} \right) \cos \theta_s - \frac{AB V_{cp}}{2} \frac{\cos \theta_s}{\sin \theta_s} + \left(\frac{AD_p \mu}{2\delta^2 \rho_c} + \frac{g D_p}{4V_{cp}} \right) = 0 \quad (14)$$

and

$$6\delta(AV_{cp}\mu + \rho_c g \delta^2) = \frac{6\Gamma\mu D_p^2}{\rho} \quad (15)$$