

PASSIVE ELECTROMAGNETIC NDE FOR MECHANICAL  
DAMAGE INSPECTION  
BY DETECTING LEAKAGE MAGNETIC FLUX

( I. Reconstruction of Magnetic Charges from Detected Field Signals )

July 1999

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**Passive Electromagnetic NDE for Mechanical Damage  
Inspection by Detecting Leakage Magnetic Flux  
(I. Reconstruction of Magnetic Charges from Detected Field Signals)**

Zhenmao Chen, Kazumi Aoto, and Syoichi Kato\*

**Abstract**

In this report, reconstruction of magnetic charges induced by mechanical damages in a test piece of SUS304 stainless steel is performed as a part of efforts to establish a passive nondestructive testing method on the basis of the inspection of leakage magnetic field. The approach for solving this typical ill-posed inverse problem is selected as a way in the least square method category. Concerning the ill-posedness of the system of equations, an iteration algorithm is adopted to its solving in which the designations of initial profile, the weight coefficients and the total number of iterations are taken as means of regularization. From examples using simulated input data, it is verified that the approach gives good reconstruction results in case of signals with a relative high S/N ratio. For improving the robustness of the proposed method, a Galerkin procedure with base functions chosen as the Daubechies' wavelet is also introduced for discretizing the governing equation. By comparing the reconstruction results of the least square method and those using wavelet discretization, it is found that the wavelet used approach is more feasible in the inversion of noise polluted signals. Reconstruction of 1-D and 2-D magnetic charges with the least square strategy and reconstruction of an 1-D problem with the wavelet used method are carried out from both simulated and measured magnetic field signals which are used as the validation of the proposed inversion strategy.

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自然磁束漏洩の計測による材料損傷の非破壊検査法に関する研究  
(第1報 磁束漏洩データから材料における磁荷分布の再構成)

(研究報告)

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概要

本報告書では、自然磁束漏洩からき裂・損傷を非破壊的に検査する研究の一環として、測定した磁束信号より材料における磁荷分布(損傷による)の再構成を行った。この代表的な非適切問題には最小自乗法に基づいた反復計算アルゴリズムを用いた。問題を適切化するために初期値、重み係数及び反復計算の回数を選び方を検討した。シミュレーション信号を用いた再構成結果より、本手法がノイズの少ない信号に対して有効であることを確認した。ノイズに対するロバスト性を向上するために、ウェーブレットをガラクシン法に適用した手法をシステム方程式の離散化に導入した。最小自乗法と比較した結果、ウェーブレットを用いた手法はS/N比の低い信号に対しても有効であることが判った。本報告書では最小自乗法に基づいた手法を1次元及び2次元の磁荷分布、ウェーブレットを用いた手法を1次元の磁荷の再構成に適用し、提案した手法の妥当性を実証した。

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## 1. Introduction

As well known, safety ensurance is an important issue in both the design and operation procedure of a large system of structure, especially in case of the machinery of a nuclear power plant concerning the severe consequences of even a small accident. For enhancing the reliability of such systems, NonDestructive Evaluation (NDE) is being routinely applied for huge key structural components. As the-state-of-the-art, the present techniques of NDE still have a necessity to be upgraded in order to meet the increasing requirements on a more efficient and reliable nondestructive evaluation. A typical example of such needs is the evaluation of damages or degradations caused by mechanical loadings and/or an irradiation source in an aged plant which is still in difficult for the conventional NDE approaches. Recently, a new concept of nondestructive testing is proposed aiming at solving such problems by applying an inspection of leakage magnetic flux density. The basis of the new approach is a phenomenon that the damages occurred in an object alternate the magnetic property of material and even cause magnetization without applying external field for some low alloy and austenitic steels<sup>[1]~[4]</sup>. However, the mechanism of the phenomenon and the correlation between the magnetic and damage parameters are still left as an attractive subject to be challenged.

In view of the background above, an experiment is being conducted for investigating the correlation between local plastic deformation and the corresponding magnetizations for the major structural material of Fast Breeding nuclear Reactor (FBR) and fusion reactor machines, namely, a stainless steel of type SUS304. As the magnetization in material is impossible to be directly measured, an indirect way in terms of a scanning of the leakage flux density surrounding the material and an inverse analysis for predicting the magnetizations was chosen. As a part of this study, this report presents a work dealing with the inversion of the measured signals of leakage magnetic flux.

In present report, the magnetic charges are selected as the unknown parameters to be reconstructed in view of the equivalence of the magnetizations and a distribution of magnetic charges in the explanation of related physical phenomena<sup>[5]</sup>. Another motivation to choose charge as unknowns is for improving the condition number of the corresponding ill-posed inverse problem. Moreover, as an approximation of the practical problem dealing with a test piece in plane stress state, the charges are assumed constants in its thickness direction, i.e., only reconstruction of the magnetic charges in a two-dimensional distribution is considered. Three ways are adopted in the inversion procedure for evaluating there feasibilities. From examples using simulated signals, the least square method using a direct solver of linear equations is found hard to give a satisfactory reconstruction results, while the predicted charge profiles by using an iteration algorithm (the steepest descent method or conjugate gradient method) agree with the true distributions well for signals with a low noise level. In case of signals containing a relative large noise, the iteration method also failed to give acceptable solutions. To enhance its robustness, wavelet transformation is introduced to the equations discretized with the moment method to weaken the constraint conditions. A

strategy to select the weight coefficients is also proposed based on the wavelet decomposition of the reconstructed functions. Numerical results show that the method using wavelet can give good reconstruction even for signals with a noise level over 50%, which demonstrated the validity of the strategy.

This report is arranged as follows, an outline of the experiment is presented in the next section. In the section 3, the basic procedures for reconstructing the charges are described respectively for the different methods. Section 4 gives simulation results obtained using iteration algorithm. The discussions on weight coefficients and convergence conditions are also given in this part. Finally, concluding remarks and prospects are summarized in the end of the report.



## 2. The preliminary experiment for detecting mechanical damages

### 2.1 Principle of the experiment

During fatigue or/and plastic deformation, dislocations and martensite transformation take place as a result of the introduced mechanical energy. Moreover, the application of external loading and the damages induced by the plastic deformation also lead to an anisotropy especially between the directions of the principle strain and the others. As the original phase of magnetic particles and those transformed during deformation exist in a structure of magnetic zone in which the spins of electron orientate in same direction, this anisotropic property may results in a passive magnetization in the plasticly deformed materials. Moreover, unlike the magnetic particles originally existed, e.g. ferrite phase in SUS304 austenite stainless steel, the martensite cells formed during the deformation do not exist randomly but mainly located in some special crystal directions. This may also be considered as an important source of the passive magnetization<sup>[6]</sup>. These considerations mean that changes in microstructure of a material, i.e. the state of damages which may lead to a micro-crack, is possibly predicted by measuring the local passive magnetizations. In practical, an indirect way is necessary in the measurement of magnetizations which reconstructs the magnetic sources from the measured leakage flux density by using an inverse analysis.

To evaluate the feasibility of the above strategy in the structural steel of a FBR nuclear power plant, we designed an experiment to investigate the correlation of the mechanical damages and the magnetization in the type SUS304 stainless steel. The basic procedure of the experiment is as follows: at first, plastic deformation (damages) is introduced to a demagnetized button type CCT test piece by a simple tension or a zero tension fatigue testing. The test pieces, then, are unloaded and the flux density at planes parallel to the planar surfaces are scanned in detail by using a Flux-Gate (FG) sensor in order to acquire the data necessary in the reconstruction of magnetization. The local plastic strains are also measured with an optical way (the moire method). The correlation of the plastic damages and the magnetization will be extracted finally by comparing the measured residual strains with the reconstructed magnetizations. As a part of this work, the emphasis of this report is placed upon the inverse analysis of the magnetic field signals.

### 2.2 Experimental setup

Fig.1 shows a schematic diagram of the testing system. The button type CCT test piece (Fig.2) of SUS304 stainless steel which is unloaded after a tensile or a fatigue testing is set at a x-y stager for measuring its distribution of leakage flux density with a 2-D scanning. An advanced flux-gate sensor<sup>[7]</sup> is employed for the measurement of the magnetic field which has a sensitivity over 0.001Gs and with a higher space resolution comparing with the conventional sensors. The detected magnetic signals and the corresponding coordinate information are stored in the Fixed-disk of a Personal Computer (PC) through a A/D converter which will be applied as the input data of the inverse analysis described in the

section 3 and 4.

Fig.3 is a typical example of the measured magnetic field where fatigue cracks were initiated at the ends of the central slit by a strain controlled loading of 0.5% strain range. The lift-off used in the testing was about 0.5 mm, and the scanning pitch was 0.5 mm. A region of  $30\text{mm} \times 30\text{mm}$  was selected for reconstructing the sources at the planar part of the test piece. The central slit is of 3 mm length and 0.3 mm width, and was manufactured into the test piece by the ElectroDischarge Machining (EDM) technique. Before fatigue testing, the test piece was demagnetized and a prestrain of 3% was introduced. The test piece was unloaded once the fatigue cracks initiated at the ends of the slit at about 1000 cycles of the load in strain range 0.5%. It needs to be point out that there is no external magnetic field was applied to the test piece besides the natural magnetic field of the earth. From these signals, it is clear that magnetizations has occurred, especially in the near-by region of the crack tips.

### 3. Reconstruction of magnetic charges

The magnetic charge is a concept introduced referring to the definition of the electric charge. Though experimental results do not support the practical existence, the magnetic charge still keeps as an important physical quantity in view of its equivalence with the current dipole model for describing many physical phenomena. Concerning the bad condition number of the ill-posed inverse problem in the reconstruction of the magnetization vector, choosing the amplitude of magnetic charges as unknown parameters is much more reasonable in order to image the magnetizations inside the object. Moreover, such a treatment can also result in a significantly reduced number of unknowns in the inverse analysis, or in other word, can save much computer burden.

At a field point (denoting as  $\mathbf{r}$ ) outside the volume  $V$  of the magnetized object, the magnetic scalar potential induced by the magnetization at a source point (denoting as  $\mathbf{r}'$ ) is equivalent to that induced by a related magnetic dipole. Referring to formulae for magnetic dipoles, the magnetic scalar potential due to magnetizations can be expressed as<sup>[8],[9]</sup>

$$\phi(\mathbf{r}) = \frac{1}{4\pi\mu_0} \int_V \mathbf{m}(\mathbf{r}') \cdot \nabla' \frac{1}{|\mathbf{r} - \mathbf{r}'|} dv' = \frac{1}{4\pi\mu_0} \left\{ \oint_{\Omega} \mathbf{m}(\mathbf{r}') \cdot \mathbf{n} \frac{1}{|\mathbf{r} - \mathbf{r}'|} ds' + \int_V \nabla' \cdot \mathbf{m}(\mathbf{r}') \frac{1}{|\mathbf{r} - \mathbf{r}'|} dv' \right\}, \quad (1)$$

where,  $\phi(\mathbf{r})$  is the magnetic scalar potential at the field point  $\mathbf{r}$  in air region,  $\mathbf{m}(\mathbf{r}')$  is the magnetization vector at the source point  $\mathbf{r}'$ ,  $q(\mathbf{r}') = \nabla' \cdot \mathbf{m}(\mathbf{r}')$  is magnetic charge density at point  $\mathbf{r}'$  according to its definition, and  $\mathbf{m}(\mathbf{r}') \cdot \mathbf{n}$  is the surface charge density at a surface point with  $\mathbf{n}$  being its normal unit vector.

From Eq.(1), it is not difficult to find that the effect of the magnetization is equivalent to that caused by both the volume and the surface charges. For a button type CCT test piece under a loading of plane stress state, the surface magnetic charges at the planar and side surfaces are zero considering the symmetry condition and the vanishing property of the total charge. Moreover, for a problem dealing with magnetizations nearby the tips of cracks initiated at the central part of a test piece, the magnetic charges at the other surfaces can also be neglected. Under such conditions, the formula for the flux density induced by the charges can be obtained by taking gradient operation at the two sides of Eq.(1) but with the term of surface integral omitted. Eq.(2) shows the correlation of the flux density and the volume distributed magnetic charges which will be used as the basic governing equation in the following part,

$$\mathbf{B}(\mathbf{r}) = \frac{1}{4\pi} \int q(\mathbf{r}') \nabla \frac{1}{|\mathbf{r} - \mathbf{r}'|} dv', \quad (2)$$

where,  $\mathbf{B}(\mathbf{r})$  is the leakage flux density vector at the field point  $\mathbf{r}$ .

#### 3.1 Inversion with the least square method<sup>[11],[12],[13]</sup>

In the follows, we consider the magnetizations localized in a limited volume  $V$  as described in the previous sections. By subdividing  $V$  into  $\bar{n}'$  cells and taking the charge density as a constant value in each cell, the magnetic charges in any source point  $\mathbf{r}'$  can be

approximated as,

$$q(\mathbf{r}') = \sum_j^{\bar{n}'} q_j \xi_j(\mathbf{r}') \quad (3)$$

where,  $\xi_j(\mathbf{r}')$  is a box function with a zero value outside and a unit value inside the  $j$ -th cell.

Substituting the discretization equation into Eq.(2), some calculation shows that the leakage flux density  $\mathbf{b}(\mathbf{r}_i)$  at an arbitrary field point  $\mathbf{r}_i$  can be obtained from the discretized charge parameters with the following formula,

$$\mathbf{b}(\mathbf{r}_i) = \sum_j^{\bar{n}'} \frac{1}{4\pi} v_j \frac{(\mathbf{r}_i - \mathbf{r}_j')}{|\mathbf{r}_i - \mathbf{r}_j'|^3} q_j. \quad (4)$$

Similarly, a component of the leakage flux density vector (for instance, the  $x$  component  $\{q_x\}$ ) at  $\bar{n}$  inspection points connects to the charge parameters under governing of the following system of equations,

$$\{b_x\} = [K]\{q\} \quad (5)$$

with the coefficient  $k_{ij} = \frac{1}{4\pi} v_j (x_i - x_j') / |\mathbf{r}_i - \mathbf{r}_j'|^3$ , and  $i = 1, 2, \dots, \bar{n}, j = 1, 2, \dots, \bar{n}'$ .

Theoretically, the charge parameters can be obtained by solving Eq.(5) with  $\{b\}$  using as a component of the measured flux density vector. Of course, Eq.(5) can not be solved directly as it is usually an over defined or a less defined problem, in other word,  $\bar{n} \neq \bar{n}'$ . To find a reasonable solution of such a system of equations, the least square method is an usual way which solves the system of equations from a point of view of a minimum square residual. The system of linear equations derived with the least square method is as follows,

$$[K]^T \{b\} = \{[K]^T [K]\} \{q\}. \quad (6)$$

Unfortunately, matrix  $[A] = [K]^T [K]$  usually is in a bad condition number, i.e, the solution of Eq.(6) is not unique from a rigorous point of view. One can only obtain a severely vibrated solution if a normal elimination algorithm is employed in its solving. In other word, the solution is easy to fall into a local minimum in the residual function because of the ill-posedness of the inverse problem. Fig.4 gives an example of the solution for an 1-D distribution of magnetic charges. From this result, it is not difficult to find that only a very small difference existed between the magnetic fields produced by the reconstructed and true charges while the charges themselves have a much different distribution. Therefore, the ill-posed problem need to be regularized somehow in order to obtain an acceptable solution.

In practical, magnetic charges in a severe oscillated distribution usually is not realistic. Imposing the solutions satisfy constraints based on such a priori knowledges is the basic idea to regularize the ill-posed problem. In next subsection, a method of iteration using gradient will be introduced for this purpose.

### 3.2 Inversion with the steepest descent method<sup>[14],[15],[16]</sup>

To avoid falling into the local minimum in solving the system of equations, an iteration method can give a better solution when an appropriate initial profile can be determined somehow. Actually, once the boundary condition of the parameters to be reconstructed was known, such an initial condition is not too difficult to be selected, e.g., we can choose zero distribution if we know that the values at the boundary is vanished.

An iteration method using gradient has a superior convergence speed than a zero order algorithm. In addition with the considerations previously mentioned, we choose the algorithm of the steepest descent method or its accelerated version (the conjugate gradient method) to solve the Eq.(6). The basic procedure of the method is as follows:

By choosing the objective function as the square residual of the magnetic field, Eq.(6) can be transformed into an optimization problem which minimizes the objective function subject to a prior knowledges on the charge distribution. Denoting  $\varepsilon$  as the residual function, it can be defined as,

$$\varepsilon_m = \sum_i^i w_i [b_i^{obs} - b_i(\{q\}^m)]^2, \quad (7)$$

where,  $w_i$  is one of the weight coefficients,  $b_i^{obs}$  is an observed data of magnetic flux density and  $b_i(\{q\}^m)$  is a field data due to an estimated charge distribution at  $m - th$  iteration step, say,  $\{b(\{q\}^m)\} = [K]\{\{q\}^m\}$ .

The iteration formula for solving Eq.(7) reads,

$$\{q\}^m = \{q\}^{m-1} + a_n \{f\}^m, \quad (8)$$

where,  $\{f\}^m = \{\partial\varepsilon_{m-1}/\partial q_i\} + G_m \{f\}^{m-1}$ , and  $\{\partial\varepsilon_{m-1}/\partial q_j\}$  is a gradient of the objective function at the direction along  $i - th$  coordinate vector which can be calculated from the coefficient matrix  $[K]$  as,

$$\frac{\partial\varepsilon_{m-1}}{\partial q_j} = 2 \sum_{i=1}^{\bar{n}} k_{ij} [b_i^{obs} - b_i(\{q\})], \quad j = 1, 2, \dots, \bar{n}' \quad (9)$$

The updating step length  $a_n$  in Eq.(8) is usually chosen as  $a_m = P/Q$ , and with

$$P = \sum_{i=1}^{\bar{n}} (b_i^{m-1} - b_i^{obs}) \frac{\partial b_i^{m-1}}{\partial a_m}, \quad Q = \sum_{i=1}^{\bar{n}} \left| \frac{\partial b_i^{m-1}}{\partial a_m} \right|^2. \quad (10)$$

and,

$$\frac{\partial b_i^{m-1}}{\partial a_m} = \sum_{j=1}^{\bar{n}} k_{ij} \frac{\partial \varepsilon^{m-1}}{\partial q_j} \quad (11)$$

Using the algorithm just conducted, a code for reconstructing an 1-D or a 2-D distribution of charge has been developed and reconstruction with use of simulated input signals has been successfully carried out. The details of the numerical simulation will be presented in the next section.

### 3.3 Application of wavelet to the inversion analysis<sup>[17],[18],[19],[20]</sup>

Imposing Eq.(6) to be exactly satisfied at each selected measuring points is a strong constraint on the unknowns which may worsen the ill-posedness of the problem. An approach to weaken such a constraint is to use the moment method, i.e., only ask the equations to be satisfied from a point of view of weighted average. In this category, the Galerkin approximation is usually applied which chooses the weight function as the bases used in the discretization of the unknown variable. Hereafter, this strategy will be selected to establish the system equation of the inverse problem given in section 3.2.

As the magnetic charges possibly exist with a complicated distribution in the material to be inspected, discretizing it on basis of the wavelet functions is an attractive idea considering the merits of the wavelet transformation and wavelet decomposition for treating signals with abrupt change. Taking an 1-D problem as example, the Galerkin approximation using wavelet can be described as follows:

At first, we expand the charge function and the Green function  $g(x, x') (\equiv (x - x')/|r - r'|^3$  for  $x$  component) in Eq.(2) on the Daubechies' wavelet bases as,

$$q(x') = \sum_j c_{n,j} \phi_{n,j}(x) \quad (12)$$

$$g(x, x') = \sum_j \alpha_{n,j}(x) \phi_{n,j}(x'). \quad (13)$$

where,  $n$  is the resolution level,  $j$  is the shifting parameter,  $\phi_{n,j}(x) = 1/2^{n/2} \phi(2^n x - j)$  with  $\phi(x)$  being the mother wavelet of the Dabechies basis which is supported at  $(0, 2N - 1)$  and  $N$  is the index of the mother wavelet.  $\alpha_{n,j}$  can be calculated by numerically calculating the integral  $\alpha_{n,j}(x') = \int g(x, x') \phi_{n,j}(x) dx$ .

Substituting Eq.(12),(13) into Eq.(2), the  $x$  component of the flux density can be expressed as,

$$b_x(x) = \int_{\Omega} \sum_i \alpha_{n,i}(x) \phi_{n,i}(x') \sum_j c_{n,j} \phi_{n,j}(x') dx'. \quad (14)$$

Assuming that the supports of the wavelet bases are contained in the integral region  $\Omega$ , we can obtain the following equation by applying the orthogonal property of the wavelet bases,

$$b_x(x) = \sum_{i=2-2N}^{2^n-1} \alpha_{n,i}(x) c_{n,i}. \quad (15)$$

Choosing the bases for wavelet expansion as the weight functions, i.e., multiplying  $\phi_{n,i}(x)$  at the two side of Eq.(15) and taking integration over its support, we can obtain the system of equations for the unknown wavelet coefficient as,

$$\beta_{n,j} = \sum_{i=2-2N}^{2^n-1} h_{n,ji} c_{n,i}, j = 2 - 2N, \dots, 2^n - 1. \quad (16)$$

or written in a matrix form,

$$\{\beta\} = [H]\{c\}, \quad (17)$$

where,  $\beta_{n,j} = \int b_x(x)\phi_{n,j}(x)dx$  is the wavelet coefficient of the detected magnetic field data, and  $h_{n,ji} = \iint g(x, x')\phi_{n,j}(x)\phi_{n,i}(x')dxdx'$ .

From the reconstruction results, the parameters at a lower resolution level can be obtained by using the following Mallat's decomposition equation,

$$c_{n-1,k} = \frac{1}{2} \sum_j c_{n,j} p_{j-2k}, \quad (18)$$

with  $p_k (k = 0, 1, \dots, 2N - 1)$  being the filtering coefficients of the Daubechies' wavelet.

To solve Eq.(17), the gradient method given in the preceding subsection is applicable without much modification. Moreover, as the coefficient matrix  $[H]$  is independent of the charge parameters, they can be precalculated and stored in data-bases for some selected resolution levels and lift-off values. This procedure makes the matrix  $[H]$  possibly to be formed by just reading in the precalculated data-bases. Therefore, most of the computational burden can be reduced in a practical reconstruction especially for a 2-D or 3-D problem.

In addition to the capability to improve the ill-posedness of the inverse problem, the application of wavelet as the expansion bases and weight functions can also reduce the affect of noises because of the filtering effect of wavelet transformation. This means a slowly changed charges can be reconstructed well from a noisy signal with use of a low resolution level. Even for charges with abrupt spatial change, the reconstruction can be improved by designating the charge distribution reconstructed at a low resolution level as initial values for the high level reconstruction.

A general way to select the weight coefficients used in the weighted square residual is still an unclarified problem. Usually, the coefficient for a signal of relative small signal has to be selected as a small value concerning the relative small S/N ratio there. However, these coefficients are difficult to be directly determined from the measured data in case of a high noise level. Bearing in mind these considerations, we propose a strategy to determine the weight function by using the features of the wavelet function. The basic idea of the method is to apply the reconstructed results at a low resolution level or decomposed coefficients from a result at high resolution level. As the affect of noise is small in a low resolution level, taking the corresponding normalized magnetic field as the weight coefficients can reduce the affect of noise further, i.e.

$$\{w_i\} = [K]\{q\}_n, \quad (19)$$

where  $\{q\}_n$  is the reconstructed charge parameters at resolution level  $n$  with  $q_{n,i} = \sum c_{n,j}\phi_{n,j}(x_i)$ .

#### 4. Numerical results

The formulae of the steepest descent method were implemented for both an 1-D and a 2-D problem while those using Galerkin approximation and wavelet was implemented for a 1-D problem only. For the 1-D problem, charges located at a line in 20 mm length is reconstructed from magnetic field measured along a line with 1 mm or 5 mm lift-off (Fig.5 a). The scanning region is selected as in 30 mm length just over the line of source. It is also assumed that the charges at the boundary and outside the region are zero. The 2-D problem is an extension of the 1-D case (Fig.5 b). In this case, the charges are assumed located in a square region of size  $20 \times 20$  mm. The field data are of either a horizontal component or the normal component of the flux-density vector which were acquired from a 2-D scanning in a  $30 \times 30$  mm region  $h_0$  over the source plane. The simulated magnetic field as well as those measured from the experiment are adopted in the reconstruction. Artificial white noises are added into the input data for investigating the robustness of the analysis method. In the follows, the numerical results will be presented respectively for the direct and wavelet used method.

##### 4.1 Results of the least square method

###### a). 1-D problem

Fig.6 depicts a comparison of the reconstructed and the true distribution of the magnetic charges. In this case the initial values of the charges are selected as zero and no noise was included in the input data. Unlike the result using Gauss elimination method, very good convergence was realized in a short CPU time. In this case, both the conjugate gradient method and the steepest descent method give excellent results though the conjugate gradient method can converge in a much less iteration number.

Once noise was added into the input data, the convergence of the iteration was very slow and even become unstable for the conjugate gradient method. For the steepest descent method, as a result of the random noise the residual value can not decreased to a level designated previously, and the square residual between the predicted charges and the true values do not decrease with the reducing residual of the magnetic field. From lot of reconstruction testings for various of input data and noise level, we find that after about  $50 \sim 100$  iterations, a relative good reconstruction of the charges can be realized. More iterations may not improve the charge distribution significantly even make worse. Therefore, we decide to take the reconstructed result at 100 iteration as the final values in practical applications. Fig.7 shows the evolution of the square residual during iteration when different noise was added into the input signals. We can find that 60 iteration is an appropriate number to stop the iteration.

Fig.8 shows a simulated results for signals with 20% and 50% noise. We can find that it is difficult to distinguish the true charges from the result of 50% noise. Therefore, more efforts are needed to improve the robustness of the method.



## b). 2-D problem

Fig.9 gives results for a two dimensional problem with the initial profile of the charges selected as zero. The applied 2-D model corresponds to the experiment described in the section 2 with an assumption that the charges do not change too much in the thickness direction. The result shown in Fig.9 is a comparison of the reconstructed and the true distribution of the charges when the simulated signals with zero noise was applied. The corresponding comparison of magnetic field is shown in Fig.10. We can find that even for a two dimensional problem, the gradient method can gives excellent reconstruction results.

For investigating the robustness for a 2-D problem, the reconstruction was also performed with signals including artificial noise. Fig.11 (a) is a reconstruction result for signal with 20% noise. Fig.11 (b) gives the corresponding magnetic field. We can find that a relative good reconstruction was obtained again. For a magnetic charges with a more complicated distribution, it can also be reconstructed in a short CPU time. Fig.12 depicts results of a typical example for such a problem.

## 4.2 Results of the wavelet used Galerkin method

Fig.13 shows a reconstruction result using the wavelet method with the resolution level chosen as 6 and the lift-off as 1.0 mm. The other conditions are the same as that described in section 4.1. The initial values were also used as zero. Though there are some errors in the location where the charges change abruptly, satisfactory reconstruction results is obtained. The iteration number of this result is taken as 100.

For comparison of the results of the gradient method and that of the method using wavelet, several reconstruction were performed for signals including artificial noise. Fig.14 gives an example of 1-D problem with the same condition as described in the preceding subsection. A noise level of 50% was applied and the iteration number was chosen as 100 for both methods. One can find that the reconstruction result using wavelet has a relatively better precision.

Up to now, the results presented are reconstructed with the weight function used as 1 or in other word, the weight function was not introduced. By applying the weight function proposed in the section 3.3, it is not difficult to find that the reconstruction results can be improved. Fig.15 shows an example with an 1-D problem for signals with 20% noise. From this figure we can find that the application of the weight coefficients can improve the results of charge distribution significantly.

## 4.3 Reconstruction with measured data

The reconstruction results given in preceding subsections are obtained by using simulated data of magnetic field. Though these results are important for evaluating the feasibility of proposed methods, more validation is necessary for practical flux density data in order that the method possibly be applied to real problems. For this purpose, magnetic field data shown in Fig.3 was applied to the reconstruction using the 2-D code of the gradient method. Fig.16 depicts a reconstructed distribution of charges in the planar region of the test-piece.

The magnetic field produced by the reconstructed charges is shown in Fig.17. The magnetic field due to the predicted charges shows a good agreement with the measured data. Comparing the reconstructed distribution of charges with the position of fatigue cracks which has initiated at the two ends of the central slit, it also can find that the reconstructed charges is in a reasonable distribution if one pays attention at the peaks corresponding to the crack tips. This result supports the expectation that the proposed method is suitable in the correlation analysis of magnetic and damage parameters.

#### 4.4 Some discussions

As described in the preceding sections, some treatments on the selection of weight coefficients, the maximum iteration number and the component of magnetic flux density are introduced. As a highlighting, we summarize these considerations as follows:

To reconstruct the magnetic charges, one component of the flux density vector is enough. This is a fortunate fact as it is impossible to use more components in the inversion because the 3 components of flux density are not independent each other considering the vanishing divergence and rotation in the vacancy area. Usually, the normal component of the flux density vector is applied in the reconstructions concerning the affect of lift-off change. However, there is no much difference between the reconstruction results using different component for simulated data. This fact has been demonstrated when we try to find the affect of the different component. In case of our experiment, the lift-off do not varies much because of the application of x-y stager. On the other hand, the lift-off of sensor for the vertical is bigger than that of the horizontal one. Therefore, we decide to use the horizontal component in the reconstruction from the point of view of a high space resolution.

Once noise presented in the input data, selection of the threshold condition for convergence is of difficult. As described in section 4.1, we choose a given max. step number as the condition for different problems. This treatment can be explained by considering the property of the iteration method. In fact, at the iterations of low step number, the method try to find a solution which approaches to the measuring data from a globe point of view. After this procedure, the local distribution of the data is reconstructed in detail which may lead to a severe vibration in the solution because of the affect of noise. Therefore, using a maximum iteration number designated by experience is valid for our problems.

The application of weight coefficients can focus the inverter to the signals with a high S/N ratio. However, as we do not know the distribution of sources, designation of such coefficients is not easy. The method we proposed in the section 3.3 is to overcome this difficulty. The basic idea of this method is to choose the coefficient as magnetic field induced by an approximately reconstructed source. As the reconstruction at a low resolution level is very fast, this procedure does not need much computations. By designating a small value at the position of small field, the high S/N signals can make a full play in the inversion which, in turn, can improve the precision of the reconstruction results.

## 5. Concluding remarks

Based on the work described in the preceding sections, we come to the following concluding remarks.

1. The reconstruction of the magnetic charges from measured magnetic field is a typical ill-posed inverse problem. Solving of the system of equations of the least square method with an eliminate algorithm is impossible to give an acceptable solution.
2. In spite of the local minimum problem of an iteration algorithm using gradient method, it can give a satisfactory reconstruction with use of a zero initial values. Though the noise level in the measured signals maybe amplified in the reconstructed charges, an acceptable charge distribution can be obtained by select an appropriate iteration number.
3. The equation of Galerkin method using the Daubechies' wavelet can give a better reconstruction that using the equation of the least square method directly. It is also verified that applying the reconstructed magnetic field at a low resolution level as the weight coefficient in the objective function can improve the robustness of the proposed inversion method.
4. Reconstruction of the detected magnetic field at a plane parallel to the surface of the test piece, the charges is reconstructed by using the direct gradient method. The result indicates that charges are mainly localized near by the cracks lines.
5. As the prospect work, the validation of the wavelet used method for a 2-D problem and the reconstruction of a 3-D distributed charges is necessary.

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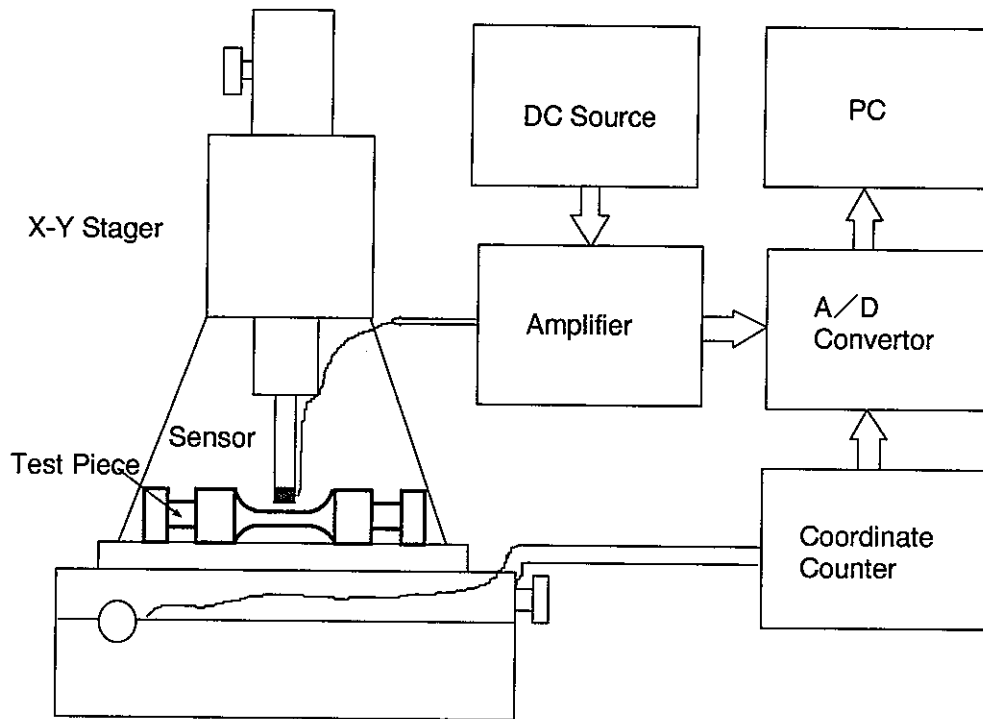


Fig.1. Flow-chart of experiment for detecting 2-D distribution of leakage flux density

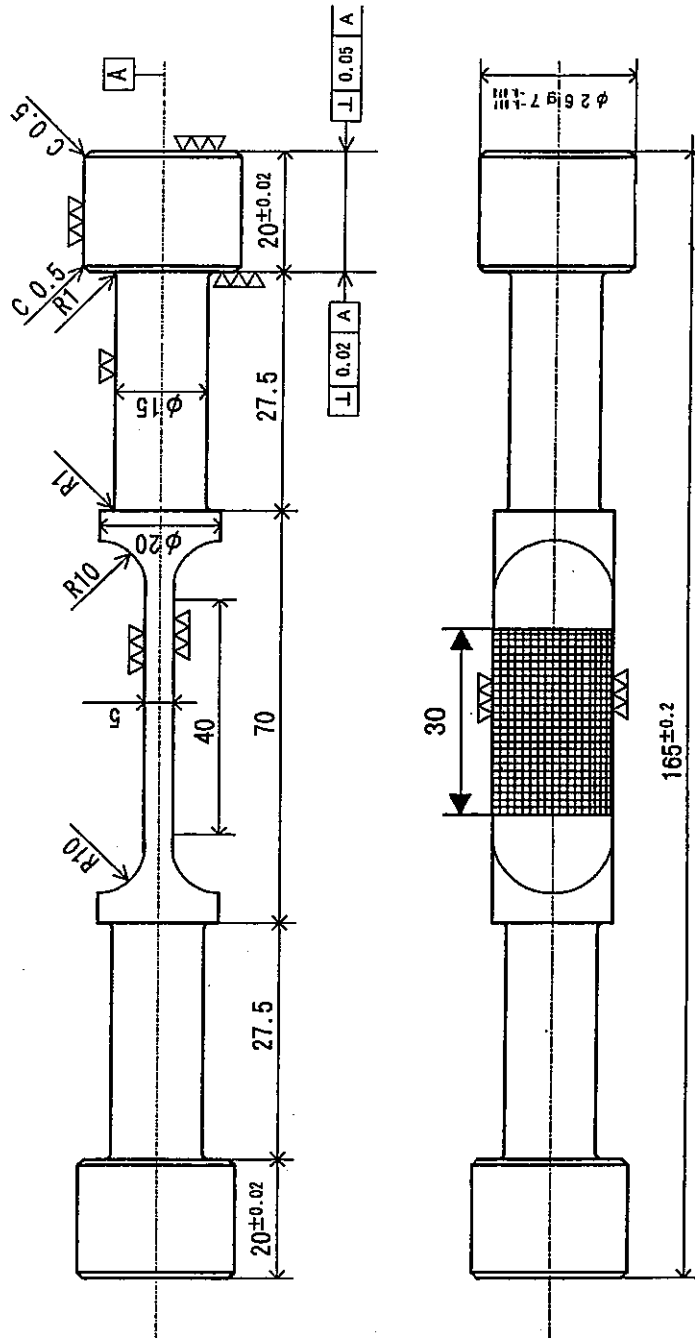


Fig.2 Button type CCT test piece for tension and fatigue testing

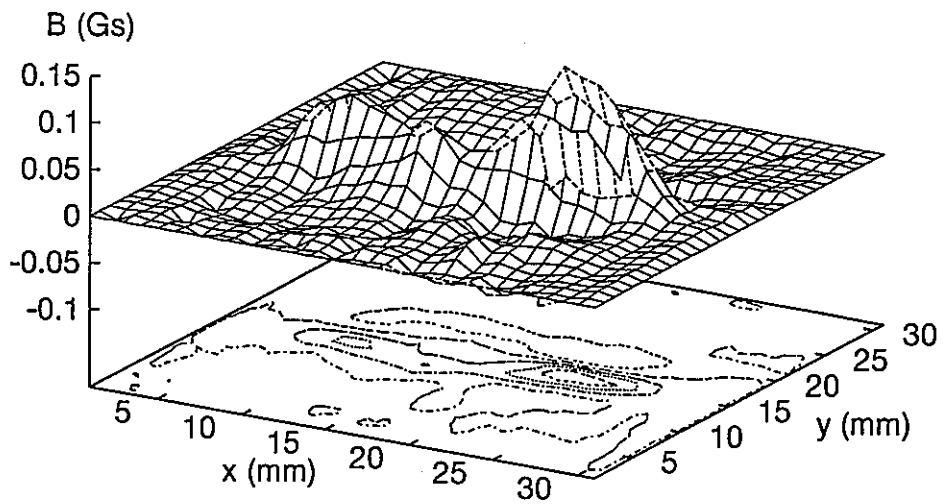
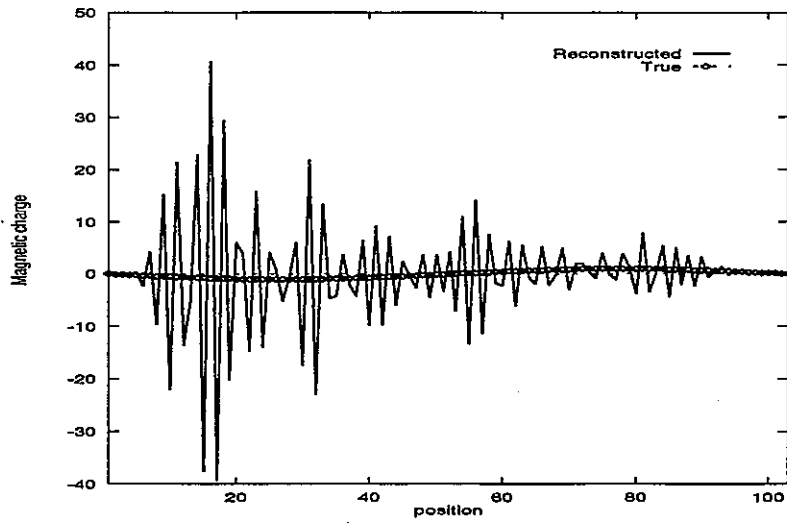
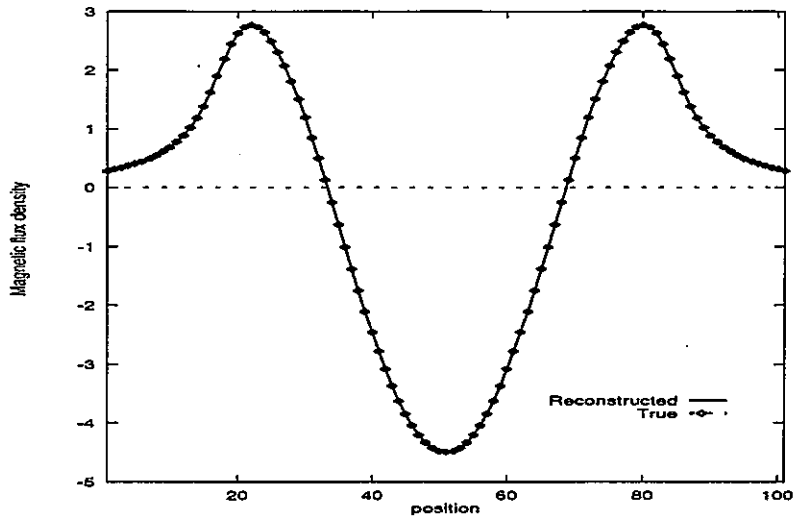


Fig.3 An example of measured leakage flux density  
(scanning results of a test piece after fatigue crack initiated)



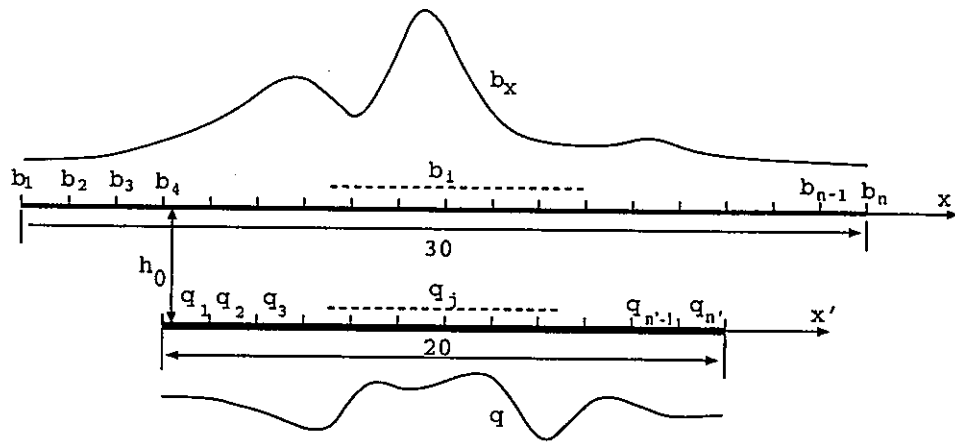


(a) magnetic charges

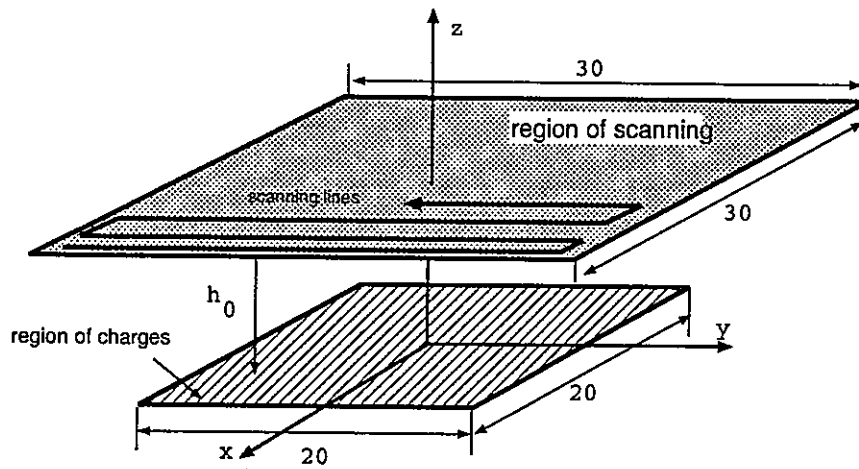


(b) Comparison of magnetic flux density

Fig.4 An example of result using elimination algorithm for equations of the least square method



a) model of the 1-D problem



b) Model of the 2-D problem

Fig.5 Concept diagram of the problems

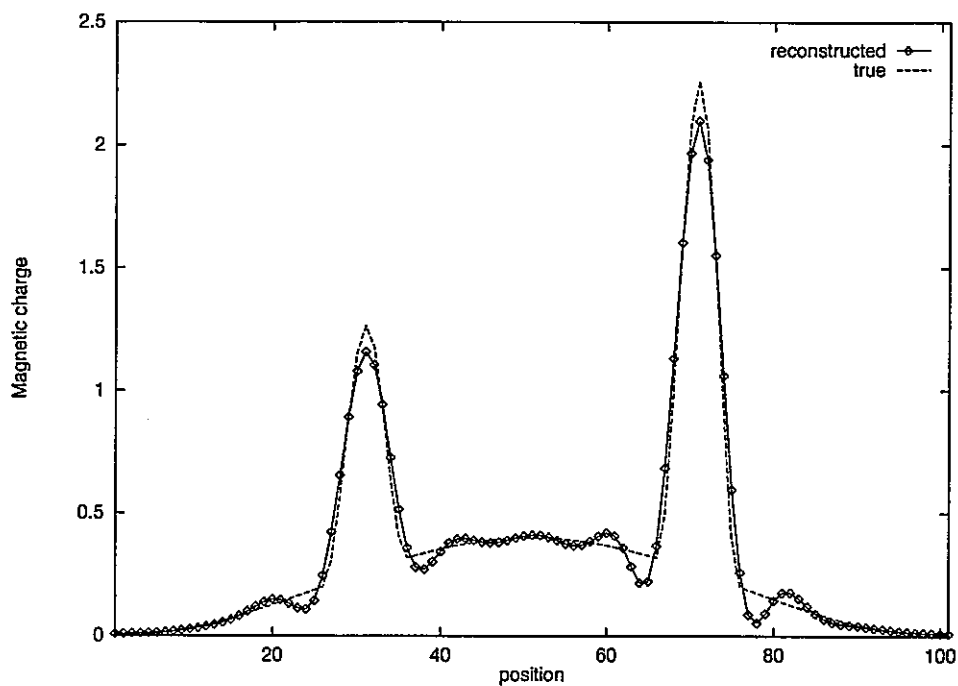


Fig.6 Comparison of reconstructed and the true distributions of 1-D charges from noise free signals by using gradient method and equations of the least square method

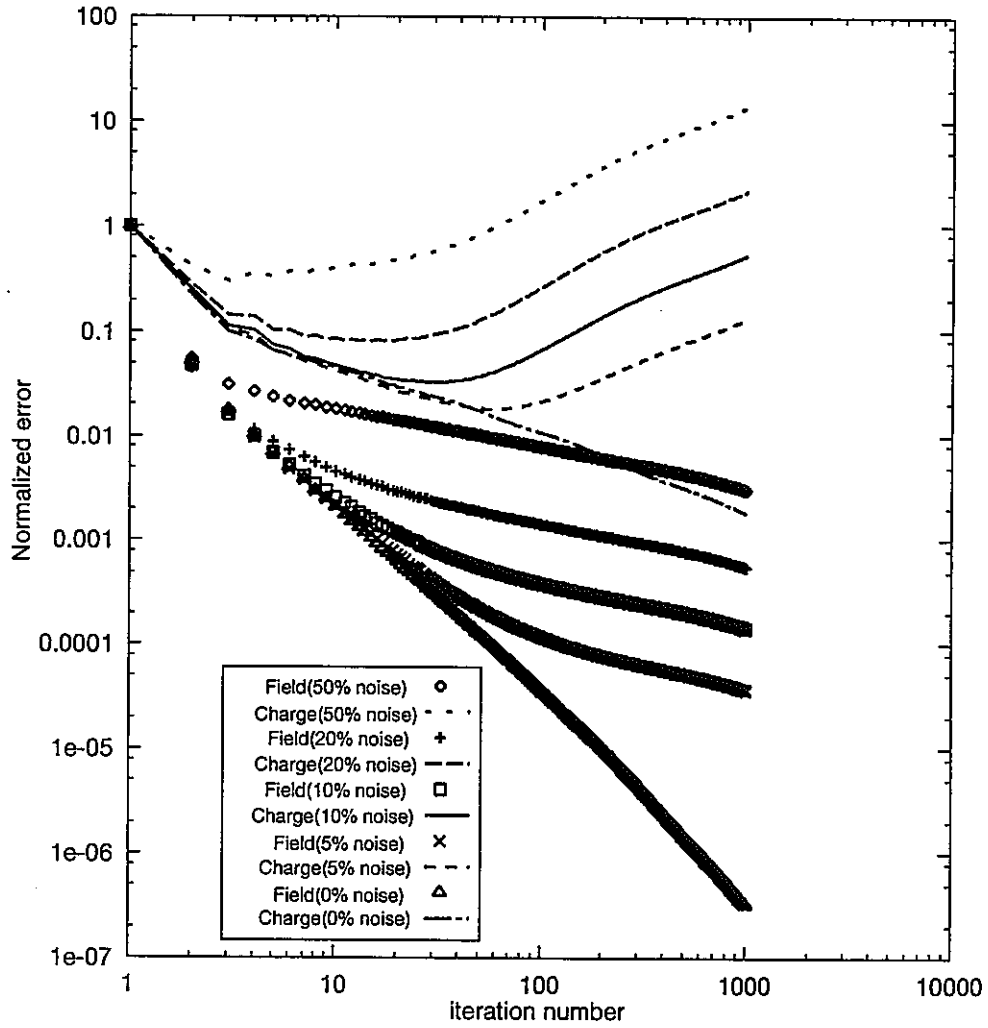
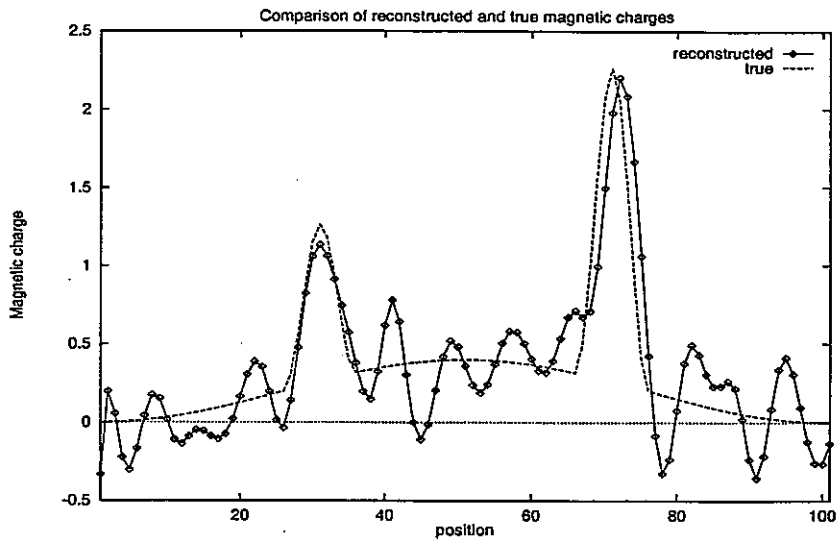
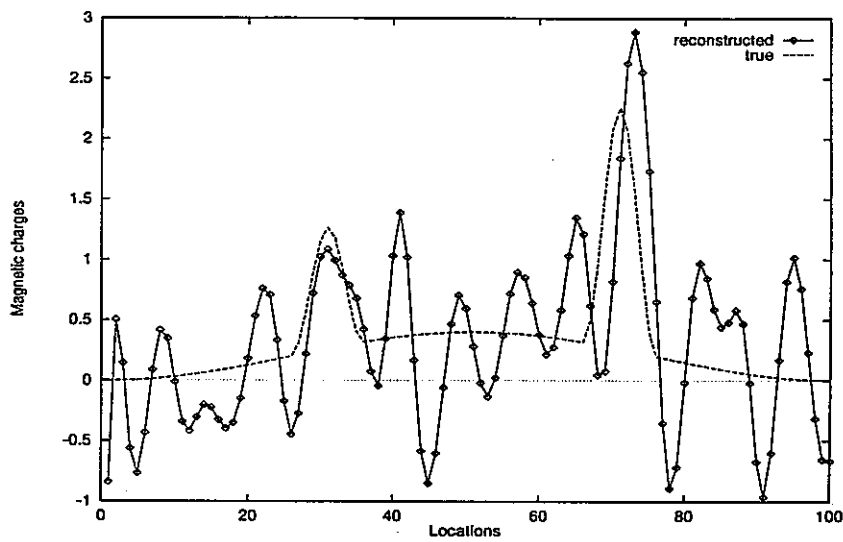


Fig.7 Evolution the residuals of the reconstructed field and charges for different noise level

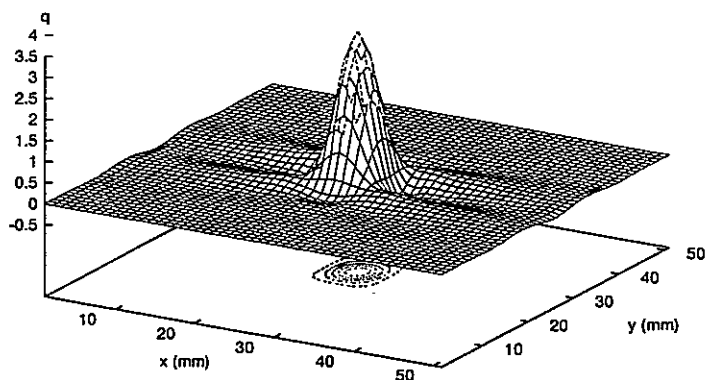


a). Result for signal with 20% noise

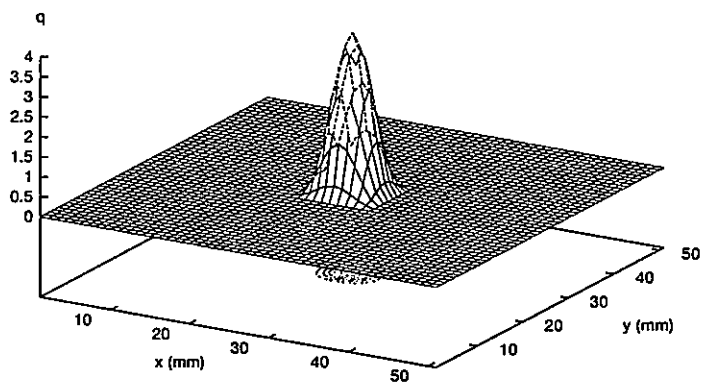


b). Result of signal with 50% noise

Fig.8 Reconstruction results of 1-D charges for signals with noise included by gradient method

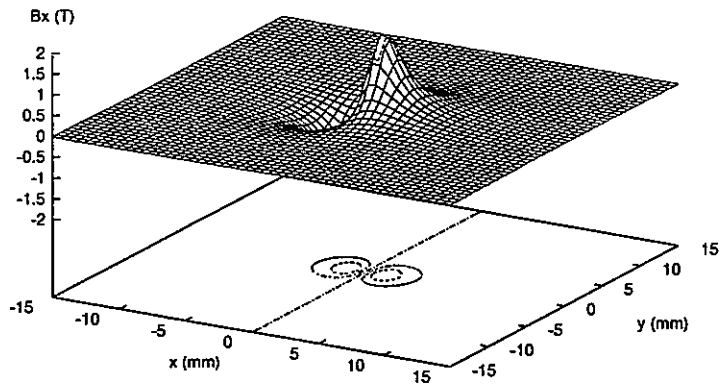


a). Reconstructed result

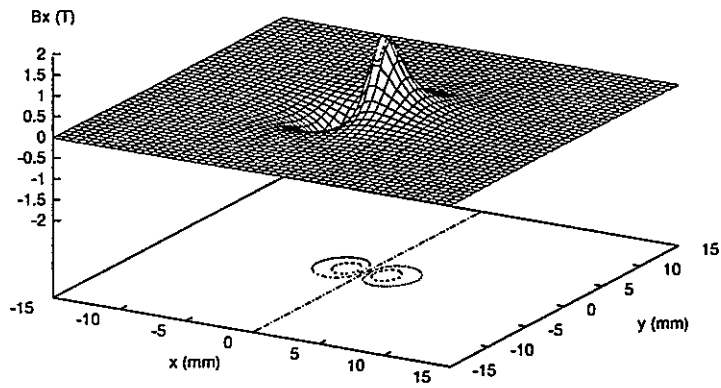


b). True distribution

Fig.9 Comparison of reconstructed and true distributions of 2-D magnetic charges by gradient method and the least square equations.

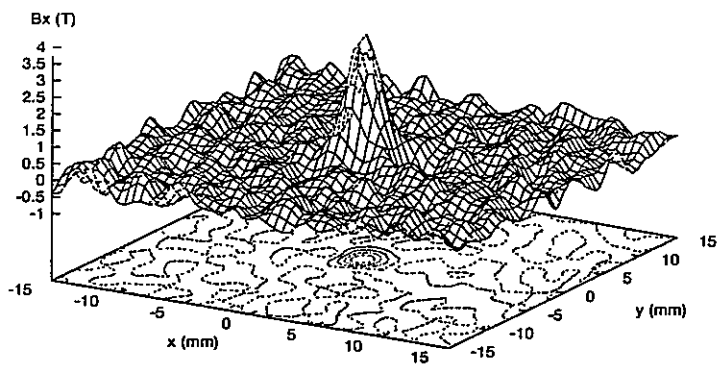


a). Reconstructed magnetic field

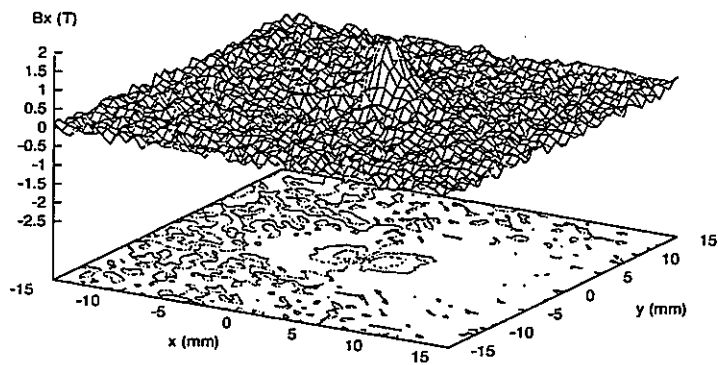


b). The true magnetic field

Fig.10 Comparison of reconstructed and true magnetic field



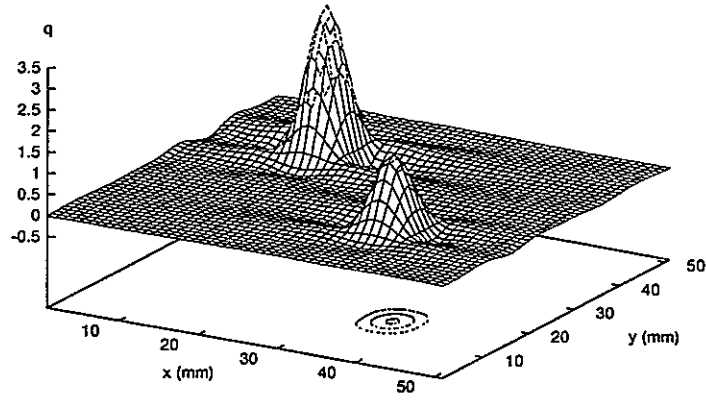
a). reconstructed charges



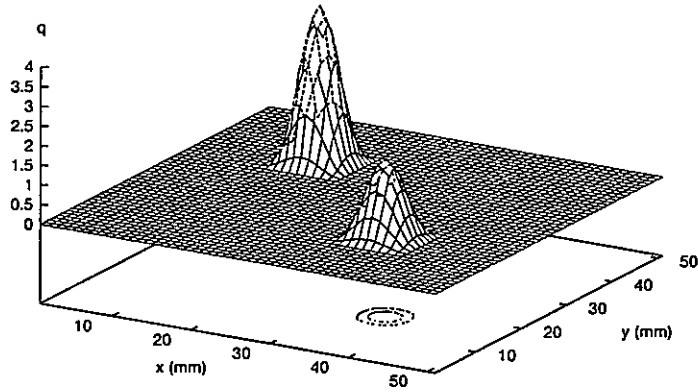
b). Mangetic field due to reconstructed charges

Fig.11 Reconstructed results for signals with 20% noise distribution of leakage flux density



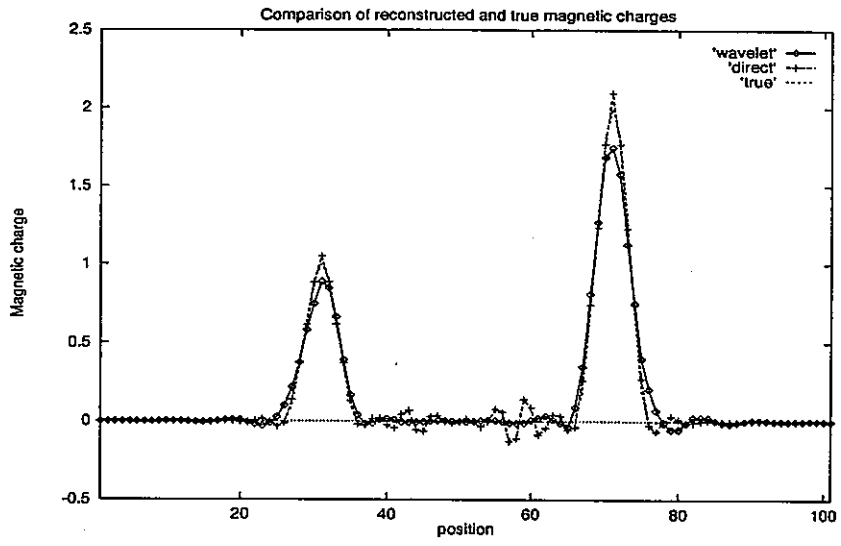


a). Reconstructed magnetic charges

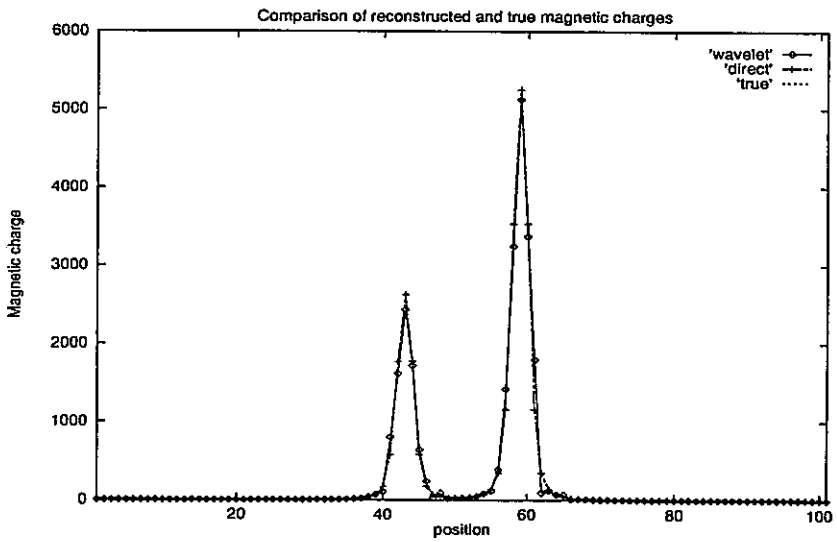


b). True distribution of magnetic charges

Fig.12 Reconstructed results for a more complicated distribution of 2-D magnetic charges distribution of leakage flux density



a). Comparison of reconstructed charges



b). Comparison of reconstructed magnetic field

Fig.13 Comparison of reconstructed results of the wavelet based Galerkin method and of the least square method

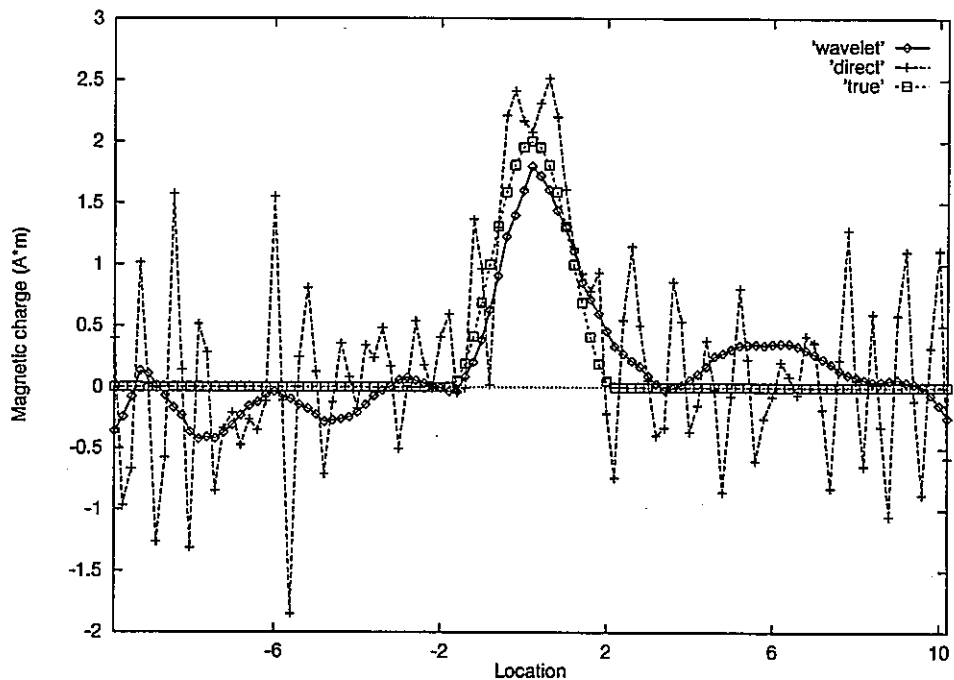
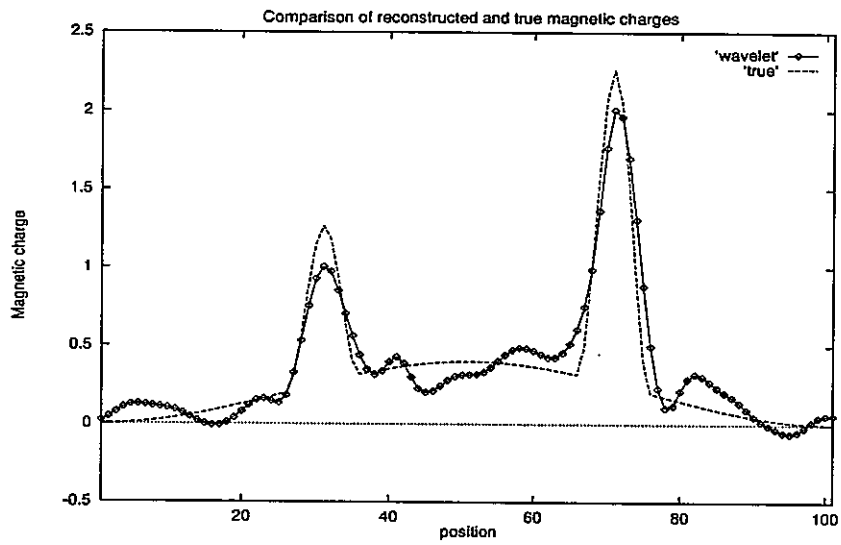
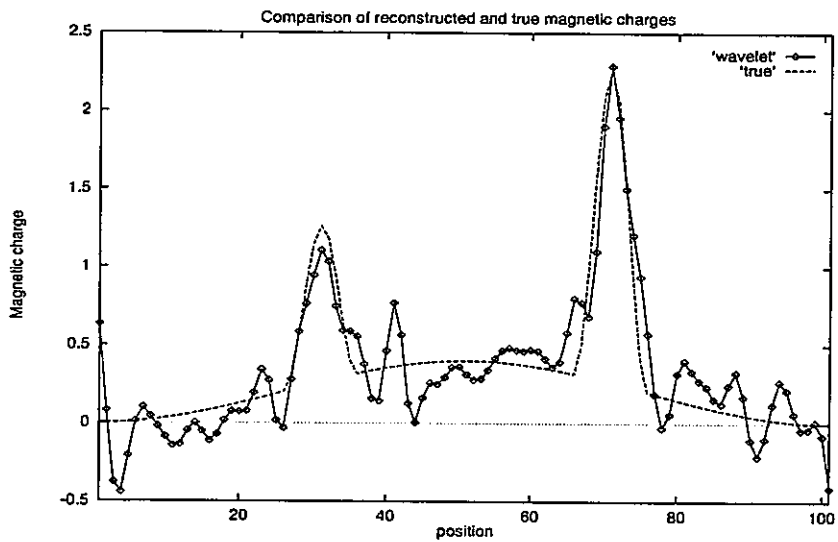


Fig.14 Comparison of reconstructed results of for signals with noise



a). Reconstructed result with use of new weight coefficients  
(wavelet, 1-D, 20% noise)



b). Reconstruction result without using weight coefficients  
(wavelet, 1-D, 20% noise)

Fig.15 Effect of proposed new weight coefficients

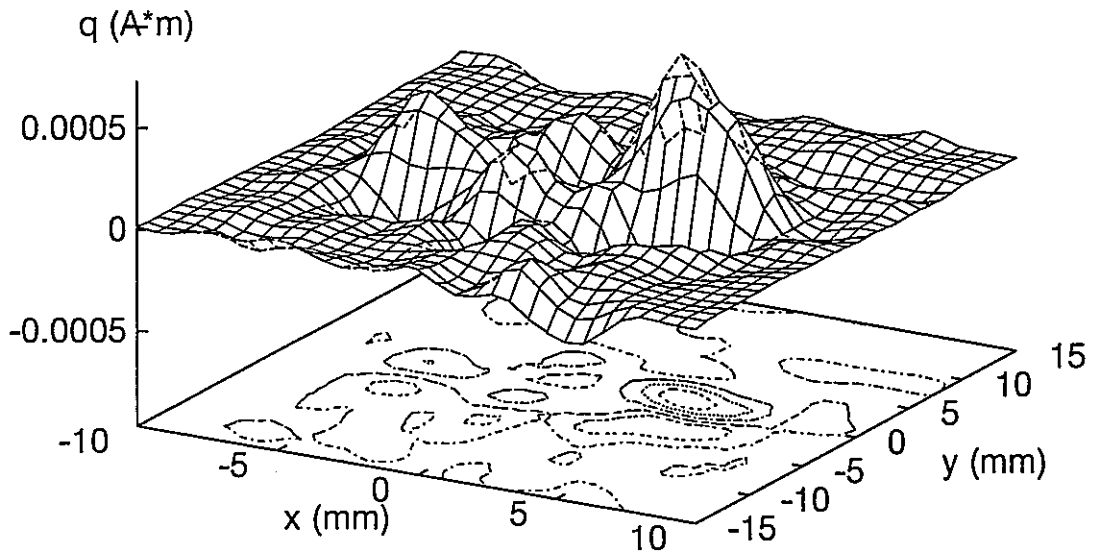


Fig.16 Reconstructed result from the measured magnetic field data

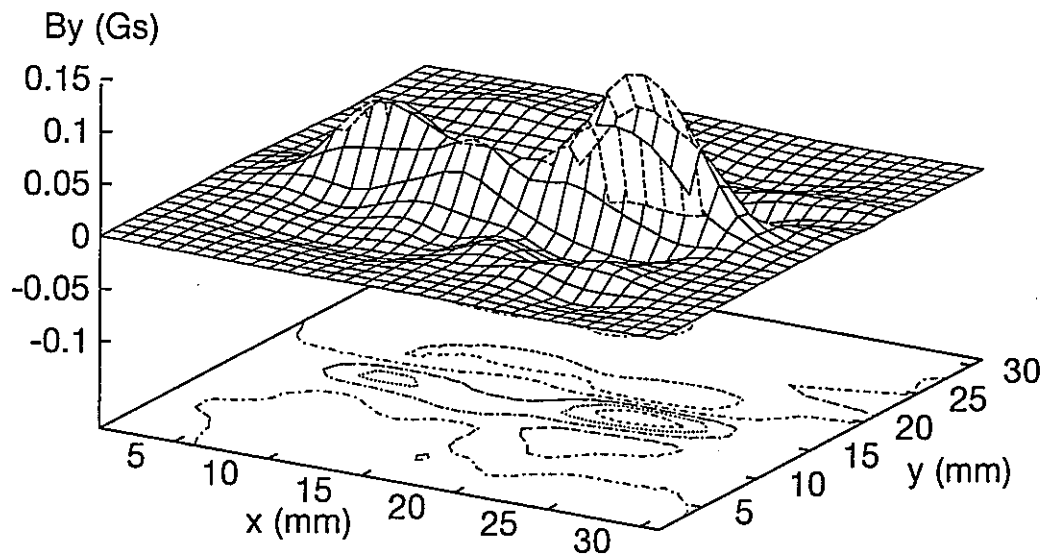


Fig.17 Comparison of the measured and the reconstructed distributions of the leakage flux density