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Study of Heterogeneity Effects on the Sodium Void Coefficients

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Study of Heterogeneity Effects on the Sodium Void Coefficient*

Abstract

In order to study heterogeneity effects on the sodium void Coefficient, a formulation for a finite heterogeneous system has been obtained in terms of collision probability based on the one-group integral transport theory, and it has been applied to a prototype fast power reactor with blanket. The exact numerical calculation of collision probability in a hexagonal lattice has been carried out, and some approximate methods are given to compute collision probabilities for distant regions and for a reactor with radial blanket. It is concluded that the heterogeneity effect on the sodium void coefficient is significant and a further study is desirable.

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Akira SUGAWARA**
Yuji SEKI**
Hiroko IKEZAWA**

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* Work performed under contracts between Power Reactor and Nuclear Development Corporation and Mitsubishi Atomic Power Industries, Inc. (MAPI)

** MAPI.

Study of Heterogeneity Effects on Sodium Void Coefficient

C O N T E N T S

1. Introduction

The sodium void coefficient is a quantity highly significant for the safety of a fast power reactor. But in the normal design calculation, the multigroup diffusion theory has been used and there are involved a positive component deriving from the spectral hardening and a negative component resulting from a leakage. However, by the multi-group diffusion theory, it is impossible to take the spectral differences between the fuel and coolant and the effect of heterogeneity on the leakage into account. Furthermore, as the void coefficient is determined as the difference between the positive and the negative quantity, it is necessary, in order to ensure a further accuracy, to conduct a calculation of both components with high accuracy.

For that purpose, a formulation for a finite heterogeneous system has been obtained in terms of collision probability and applied to the prototype fast power reactor with blanket. In order to have a spectral effect, multigroup computations are required.

This time, however, as first step for the purpose of development of a collision probability calculation formula, evaluating the heterogeneity effect on the leakage one group analysis were undertaken. The extension of this one-group analysis to multi-group analysis will not be too difficult.

2. Heterogeneity Analysis Method by Means of Collision Probability

The reactor under consideration will be divided into M-number of concentric regions (hereinafter called "zone"), and a cell will be created for each one fuel rod in the zone. This cell also will be divided into - number of smaller sub-regions. (Fig. 2 − 1)

By assuming that all the cells belonging to the same one zone are equivalent, the total region numbers to be considered can be reduced to M × L-numbers. First, considering a bare reactor (Fig. 2-2 (A)), and defining the probability that a neutron born in region "i" from a uniform isotropic source will have next collision in region "j" as Pjj, then neutron balance equation is generally represented by :

$$Vi \ \Sigma ti \ \Phi i = \underset{k}{Sum} \ Pki \ (Vk \ \Sigma tk \ \Phi k \ Kk + Sk) \quad \cdots\cdots (2\text{-}1)$$

where Σt is total macroscopic cross section, Φ is neutron flux, V is volume, Kk is the number of 2ndary neutron at each collision, S is the source of external neutron, and the added letters indicate the zones. When there is no external neutron,

$$Kk = Ksk + \frac{1}{\lambda} Kfk \quad \cdots\cdots (2\text{-}2)$$

$$Ksk = \frac{\Sigma sk}{\Sigma tk}, \quad Kfk = \frac{\nu \Sigma tk}{\Sigma tk} \quad \cdots\cdots (2\text{-}3)$$

and can be reduced to the eigen-value problem relating to λ which corresponds to the effective multiplication factor. Thereafter, for the simplification of the symbols, let φi represent the number of collisions instead of neutron flux.

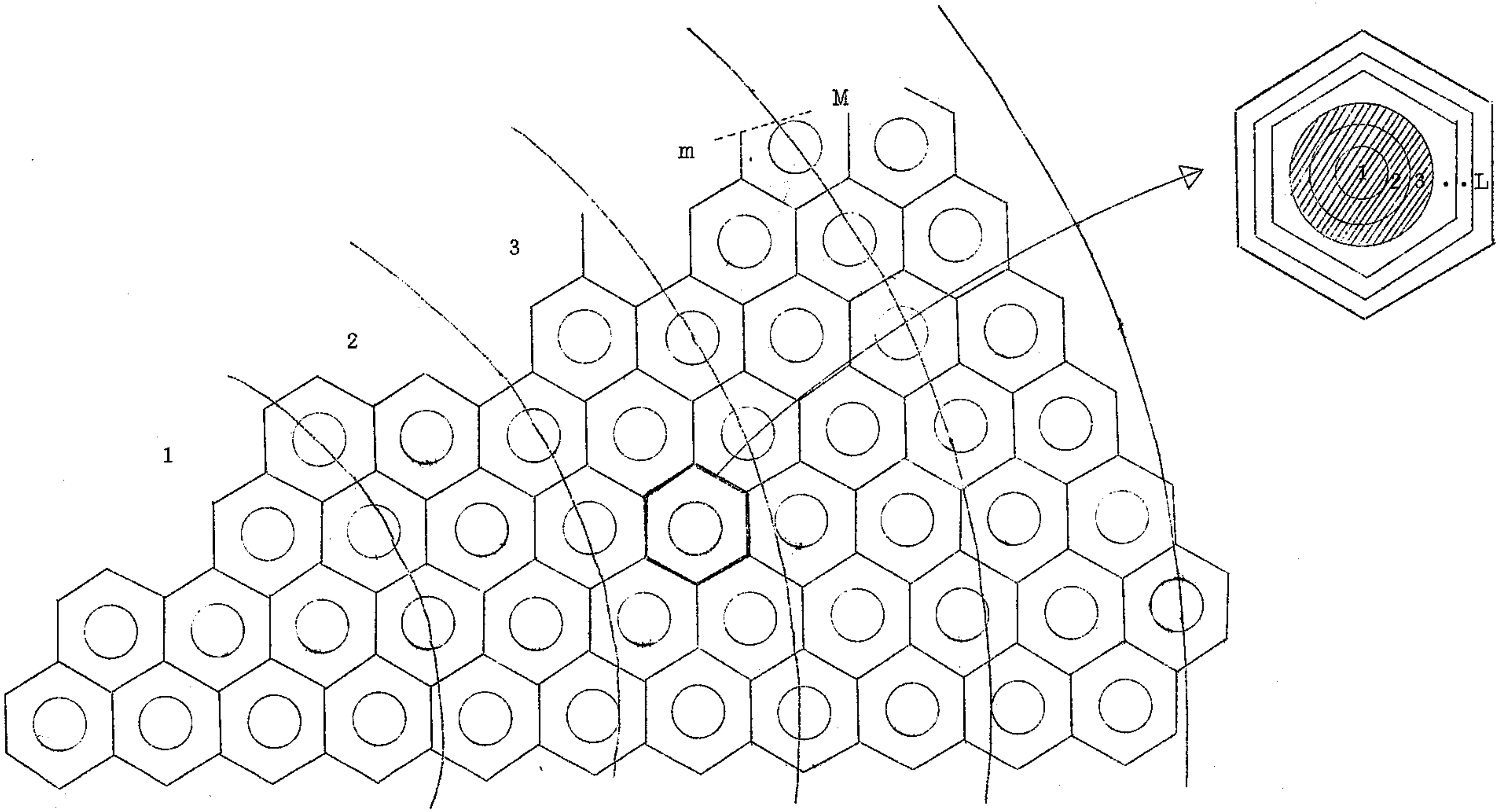


Fig. 2 — 1. Calulation model.

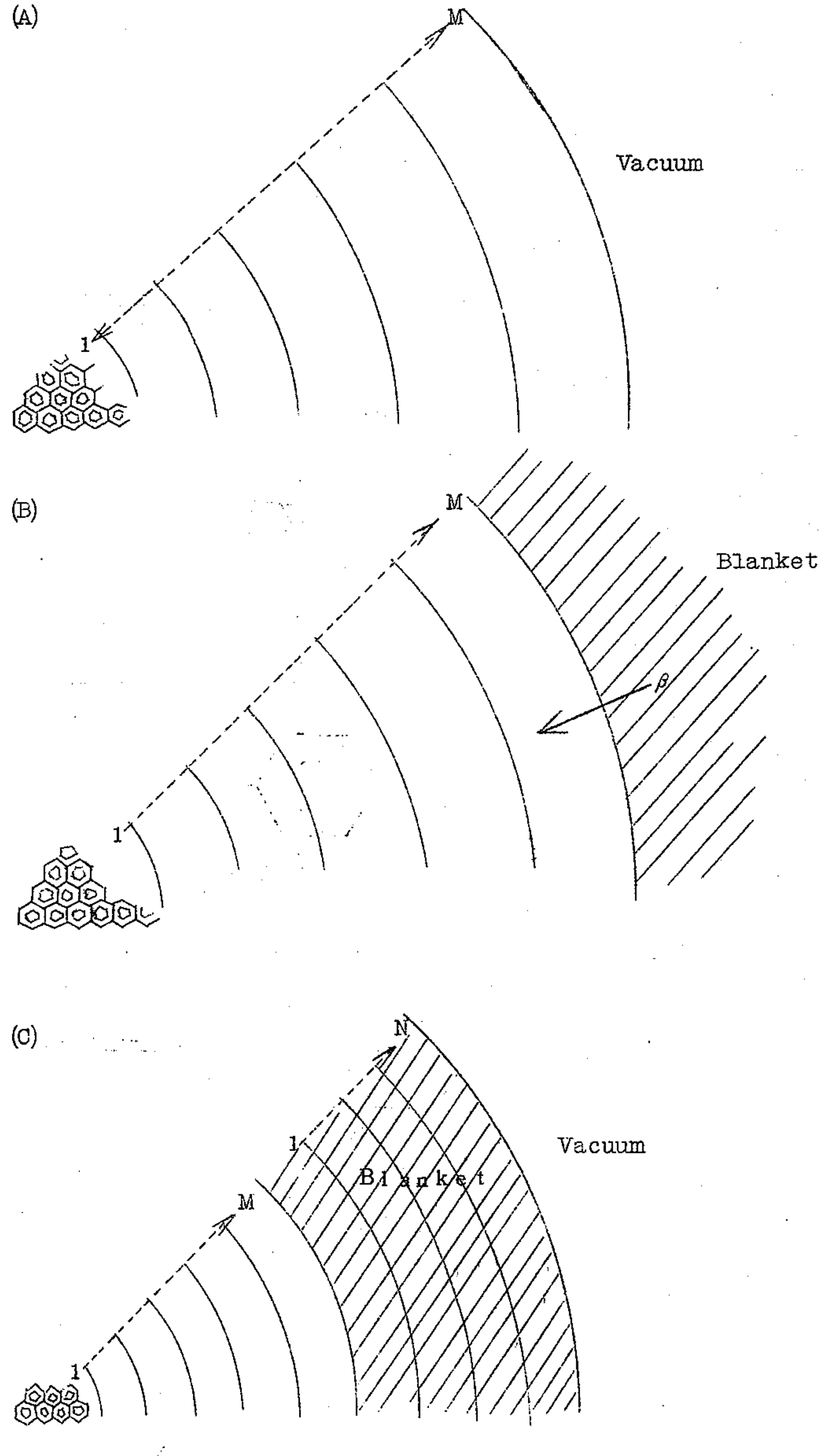


Fig. 2 — 2. Blanket handling

Namely,

$$\varphi i = Vi\, \Sigma ti\, \Phi i \quad \cdots\cdots (2\text{-}4)$$

With this, in the case of non-external neutron source,

$$\varphi i = \underset{k}{Sum}\ Pki\ \varphi k\ (Ksk + \frac{1}{\lambda} Kfk) \quad \cdots\cdots (2\text{-}5)$$

can be written.

In a normal fast reactor, a lattice is provided with a blanket on its outside, and its handling requires some devices. Namely, as described in Section 3, when lattice consists of more than two different regions, pij will require an enormous amount of calculation. Henceforth, implicit treatment of the blanket by use of albed β (Ref. 1) or explicit treatment by some approximation methods (Fig. 2-2 (B) and (C)) can be considered. However, in an actual case, the former can be proved to be a special case of the latter (Ref. 2).

In the case of an albed method, when a neutron enters into the blanket from the core and a β particle returns to the core, the equation of neutron balance is :

$$\varphi i = \underset{k}{Sum}\ Pki^{\dagger}\ \varphi k\ (\ Ksk + \frac{1}{\lambda} Kfk\ ) \quad \cdots\cdots (2\text{-}6)$$

where

$$Pki^{\dagger} = Pki + \frac{\beta Poi\ \rho k}{1-\beta R} \quad \cdots\cdots (2\text{-}7)$$

$$\rho k = 1 - \underset{j}{Sum}\ Pkj \quad \cdots\cdots (2\text{-}8)$$

$$R = 1 - \underset{1}{Sum}\ Poi \quad (2\text{-}9)$$

and Poi is the probability that the incident neutron into the core from outside will have a collision in i-zone. In the case of an isotropic incidence, it can be given by Ref. 3 as follows:

$$Poi = \frac{4}{So} Vi \Sigma ti \{ 1 - \underset{j}{Sum} Pij \} \qquad (2\text{-}10)$$

As above pointed out, since it was found out that the formulation by albed was included in the B-method under Section 3-5, this method was not particularly adopted.

As a method to explicitly handle the presence of the blanket, it is to provide a number of zones of concentric circular form as shown in Fig. 2-2, treating the blanket as homogeneous, and calculate the collision probability with regard to the system including the core and the blanket by the method as mentioned in Section 3. The collision probability obtained is written as Pki^*, and the equation of the neutron balance will arrive at the formula of (2-6). That is:

$$\varphi i = \underset{k}{Sum} \overset{*}{P}ki \ \varphi k \ (Ksk + \frac{1}{\lambda} Kfk) \qquad (2\text{-}11)$$

This equation (2-11) after a simple manipulation can be transformed to:

$$A\varphi = \frac{1}{\lambda} B\varphi \qquad (2\text{-}12)$$

Here, φ is the vector of the order of the total number of zones, and A and B represent a square matrix:

$$\varphi = \begin{matrix} \varphi 1 \\ \varphi 2 \\ \vdots \end{matrix}$$

$$A = (Ai'i) \qquad Ai'i = \delta i'i - Ksi \, Pii'^*$$

$$B = (Bi'i) \qquad Bi'i = Kf,i \ P^*ii' \qquad (2\text{-}13)$$

The eigenvalue λ and the eigen vector can be computed by use of Wielandt method. Namely, if the assumed value of λe is given:

$$\lambda = \lambda e + \delta \lambda \quad \cdots\cdots (2\text{-}14)$$

In which, it will transform into the following eigenvalue problem and $1/\delta\lambda$ is sought:

$$M\varphi = -\frac{1}{\delta\lambda}\varphi$$

$$M = (B - \lambda e\ A)^{-1}A \quad \cdots\cdots (2\text{-}15)$$

In the computation of void coefficient, the answers will be sought by obtaining the eigenvalue differences between the normal time and the void occurrence time relating to the heterogeneity system, and repeating the same computation with regard to the homogeneous system. From the void coefficient so obtained, the heterogeneity effects was to be valuated.

3. Computation Method of Collision Probability

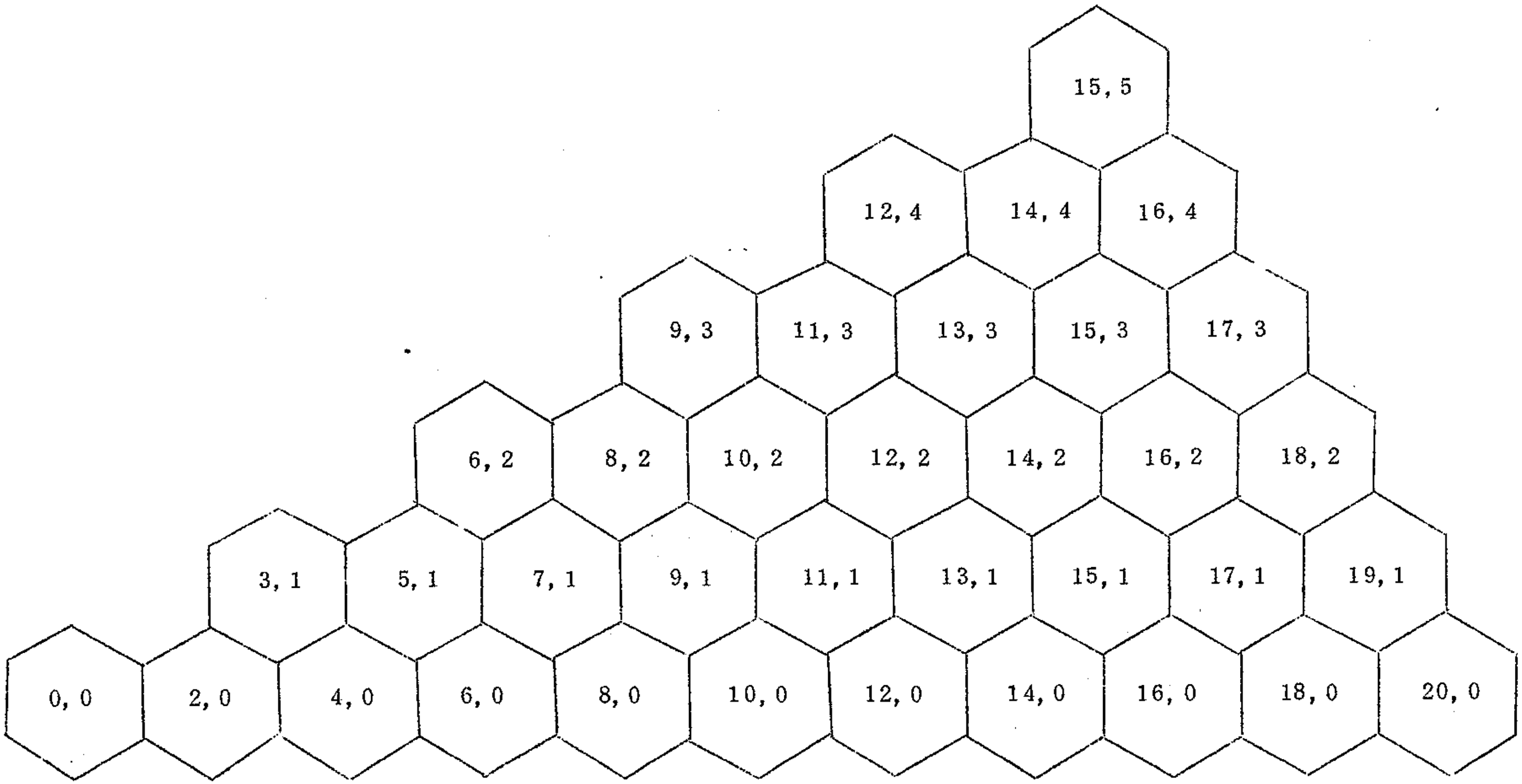
3-1. Computation Method of Collision Probability from Cell to Cell

The cell's index is provided as Fig. 3-1. As one-region core is considered, the collision probability in a random cell of the neutrons produced in a random cell is decided by the relative position of the two cells. For this reason, having calculated the probability that neutrons produced in the (0,0) cell as indicated in the figure will have next collision in the cell (I,J) (within the scope of the figure judging from its symmetrical nature), all the subsequent necessary collision probabilities may be safely computed. The cell is assumed as having two zones of fuel and coolant, while as to Z-axis, they are all assumed identical.

The collision probability Pij means by definition the probability that the neutrons produced in the region i from uniform isotropic neutron source will have next collision in the region-j.

Consequently, Pij can be obtained by averaging the probability that the neutrons produced from the line source in the region-i will have the next collision in the region-j with respect to the space and the angle within the region-i.

The probability of the neutrons which were born from the line source not to collide until to the point where the projection to the prependicular plane is given by τ , can be obtained by Ki2 (τ). The probability that the neutrons produced uniformly on a straight line as specified by (α, γ) in Fig. 2-2 will make collision in


15, 5
12, 4
14, 4
16, 4
9, 3
11, 3
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19, 1
0, 0
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Fig. 3 — 1.

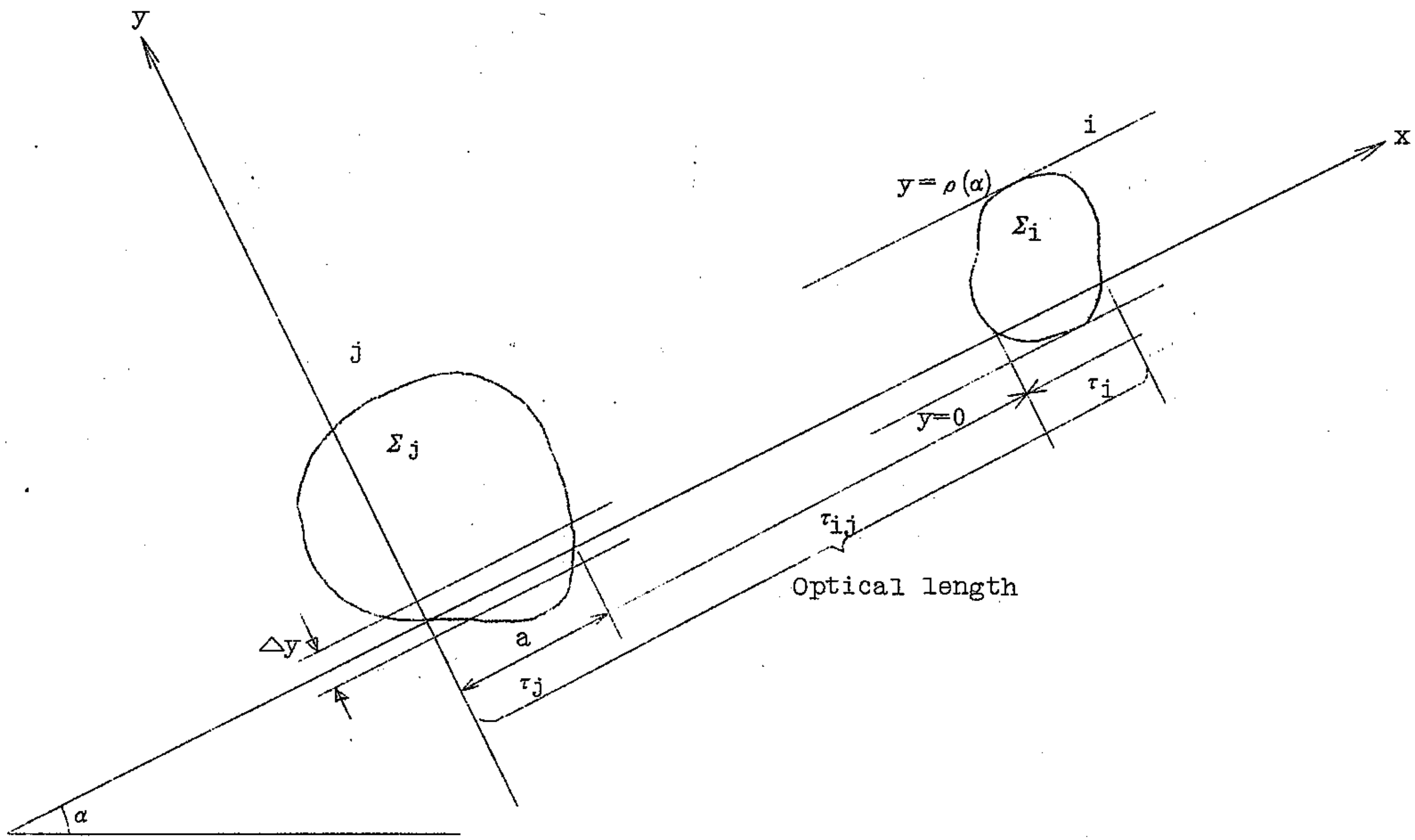


Fig. 3 - 2.

region-j may be given by the following formula:

$$Pij(\alpha, y) = \frac{1}{a}\int_0^a [Ki_2(\Sigma i(a-x)+\tau ij) - Ki_2(\Sigma i(a-x)+\tau ij+\tau j)]dx$$

$$= \frac{1}{\Sigma ia}[Ki_3(\tau ij) - Ki_3(\tau ij+\tau i) - Ki_3(\tau ij+\tau j) + Ki_3(\tau ij+\tau_i+\tau_j)] \cdots (3\text{-}1)$$

where, τ represents an optical length (cross section x length). Consequently, the collision probability can be obtained averaging this formula with respect to y and the angle:

$$Pij = \frac{1}{2\pi \Sigma i Vi}\int d\alpha \int dy [Ki_3(\tau ij) - Ki_3(\tau ij+\tau i) - Ki_3(\tau ij+\tau j) + Ki_3(\tau ij+\tau i+\tau j)] \cdots\cdots (3\text{-}2)$$

$$Pij = 1 - \frac{1}{2\pi \Sigma i Vi}\int d\alpha \int dy [Ki_3(0) - Ki_3(\tau i)] \cdots\cdots (3\text{-}3)$$

where the cell is composed of two regions, the fuel-region and the coolant region. The neutrons are born either in the fuel-region or in the coolant region of the cell (o,o), and the probability that the neutrons born uniformly in the coolant-region of the cell (o,o) will make collision in the fuel region of the cell (I,J) can be computed from the probability that the neutrons born uniformly in the fuel region will make a collision in the coolant region of the cell (I,J) by use of a reciptocity relation. For this reason, what is to be done is to calculate the three types of collision probability such as, from fuel-region to fuel-region, and from fuel-region to coolant region, and from coolant region to coolant region. The general calculation flow is shown in Fig.3-3. The explanation following the calculation flow is given below:

The path of neutron as shown in Fig. 3-4 is determined by (r, Ψ), (Psi), while the integration is done by adopting GauBian quadrature with regards to r and Ψ. The path determined by (r, Ψ) will satisfy the following linear equation:

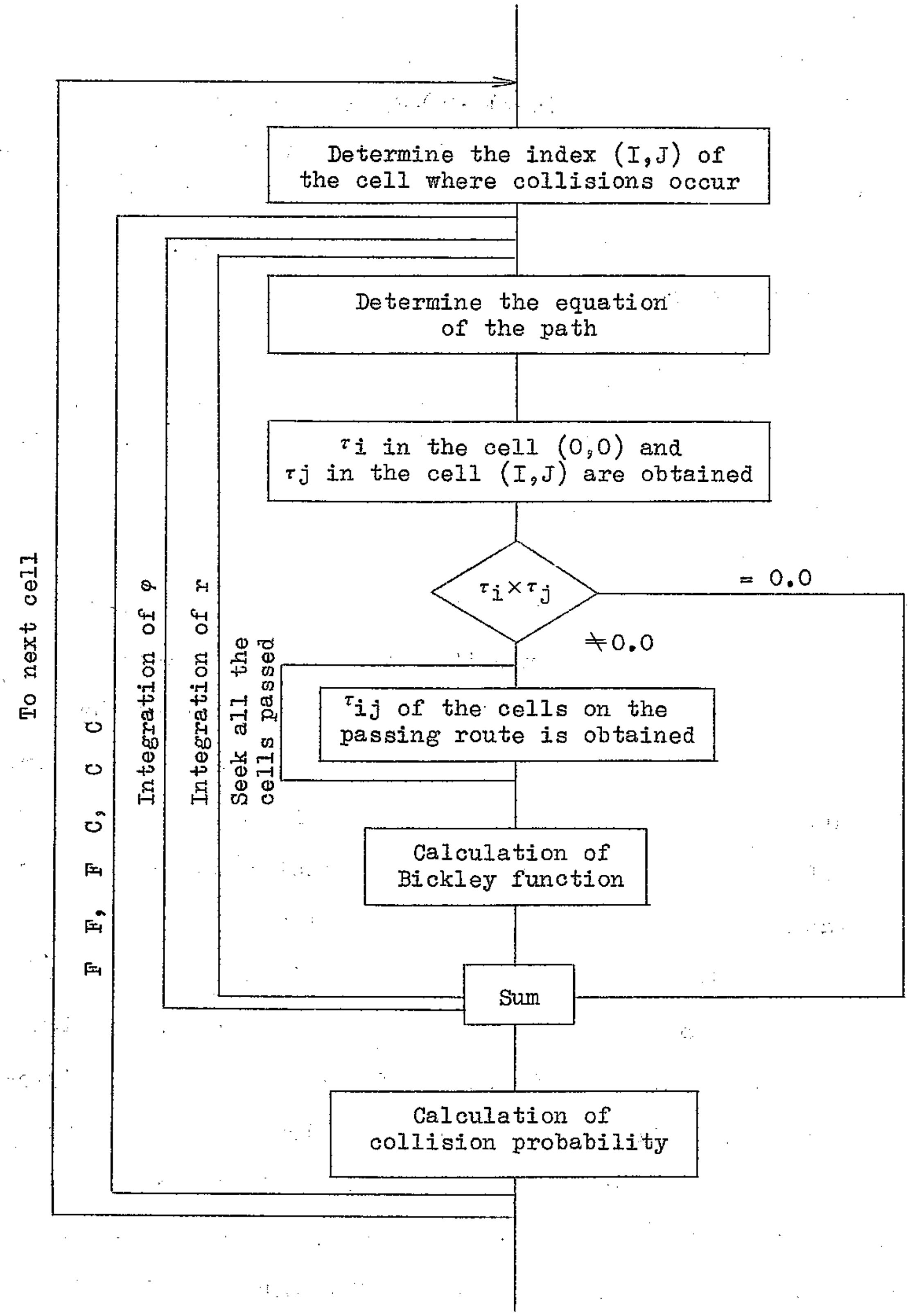

Determine the index (I,J) of the cell where collisions occur
Determine the equation of the path
τi in the cell (0,0) and τj in the cell (I,J) are obtained
τi × τj
= 0.0
≠0.0
τij of the cells on the passing route is obtained
Calculation of Bickley function
Sum
Calculation of collision probability
To next cell
F F, F C, C C
Integration of φ
Integration of r
Seek all the cells passed

Fig. 3-3.

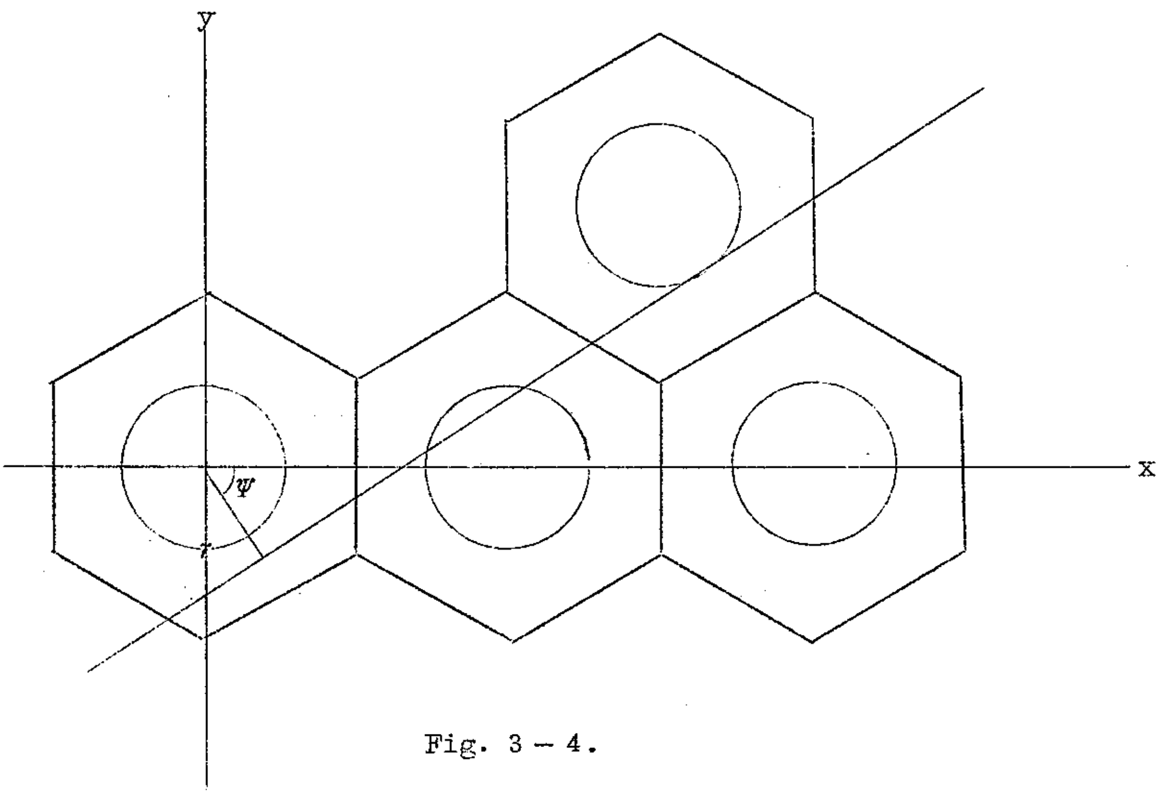


Fig. 3 – 4.

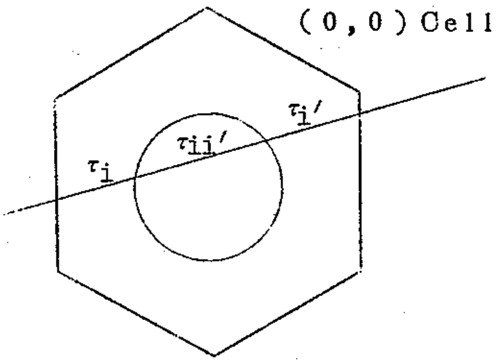


Fig. 3 – 5.

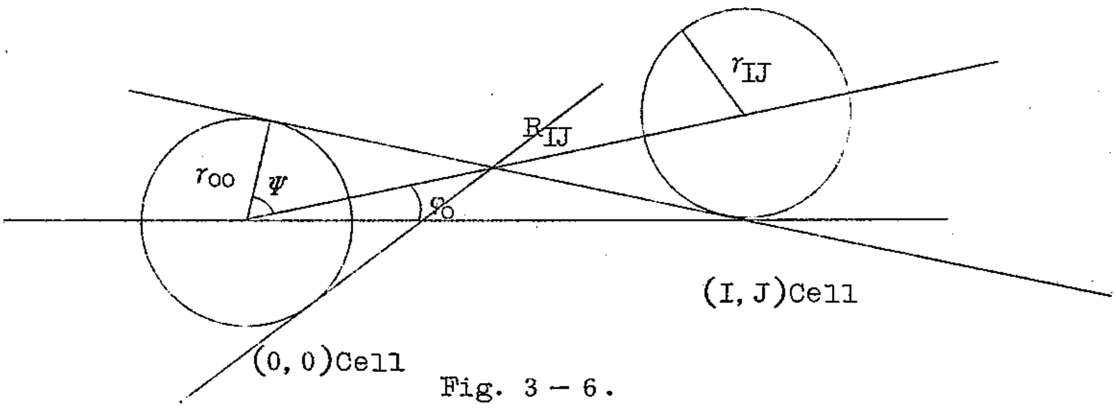


Fig. 3 – 6.

$$y = -\frac{\cos\Psi}{\sin\Psi}x + \frac{r}{\sin\Psi} \qquad (3\text{-}4)$$

Let the center coordinates of the cell (I,J) be (xo,yo), they are given by:

$$xo = (I + J) \times \frac{Pitch}{2}$$
$$yo = \sqrt{3} \times J \times \frac{Pitch}{2} \qquad (3\text{-}5)$$

The length of the foot of the perpendicular dropped from the center of the cell (I,J) to the path is given by

$$r_{IJ} = |\, xo \cos\Psi + yo \sin\Psi - r \,| \qquad (3\text{-}6)$$

Whether the range will pass through either the fuel-rod zone or the coolant zone of a random cell may be examined firstly by r_{IJ}. The optical length in the cell where the path passes through is obtained by first seeking algebraically the crossing point of the path and the circle which represents the respective sides of the hexagon which forms the cell as well as the fuel, and then by multiplying the length of the path passing through the respective zones by the cross section of the zones. The collision probability computation was made by first finding out all the cells which the neutrons born in the cell (o,o) had passed through until they actually collided in the cell (I,J). Then τ_{ij} was calculated. Thereafter, from τ_i of the initial zone and τ_j of the collision zone, the collision probability was computed by adoption of formula (3-2) and (3-3). Provided, however, the probability that the neutron born in the coolant in the cell (o,o) as shown in Fig.3-5 will make collision in the coolant in the cell (o,o) was computed by the following formula:

$$Pii = 1 - \frac{1}{2\pi \Sigma i \, Vi} \int a\varphi \int d\gamma \,[\,(Ki_3(0) - Ki_3(\tau i)) + (Ki_3(0) - Ki_3(\tau i')) + (-Ki_3(\tau ii') + Ki_3(\tau ii' + \tau i') + Ki_3(\tau ii' + \tau i) - Ki_3(\tau ii' + \tau i' + \tau i))\,]$$

The range of γ integration was taken to $o \rightarrow \gamma_F$ for the fuel-rod zone and $o \rightarrow \frac{2}{\sqrt{3}}$ pitch for the coolant zone.

Even replacing $\frac{2}{\sqrt{3}}$ pitch by an equivalent radius γ_e of the cell, there was seen no significant difference. The contribution from the path which did not pass the cell (o,o) was zero and made little effect on the results.

Assuming the range of the integration with respect to Ψ as $0 \rightarrow \pi$ for the cells (ŏ,o) and 2,o), and for the distant cells such as (I,J), the coordinates in the center of the cell (I,J) as X(I,J) Y(I,J), and the distance between the center of each cell (o,o) and (I,J) as R_{IJ}, the integration was performed up to $\Psi + \varphi_0 \rightarrow \pi - \Psi + \varphi_0$:

$$\Psi = \cos^{-1} \frac{\gamma oo + \gamma IJ}{RIJ} \qquad \varphi_0 = \tan^{-1} \frac{y(I,J)}{x(I,J)}$$

In this case, γ should be γ_F if the assumed zone is the fuel-rod zone, while, if it is the coolant zone, it should be γ_e. (Refer to Fig. 3-6).

In the numerical calculation, the Bickly functions were computed by the following approximation formula.

$$Ki_3(x) = \frac{0.5219798 + x\,(3.4150861 + x\,(2.8822975 + x\;0.49523037))}{0.66459895 + x\,(0.1979495 + x\,(2.7422822 + x\,(0.39516928)))} \times \frac{1}{e^x\sqrt{1+x}}$$

A calculation code FFCP was prepared for the operation of the above formula, and the subsequent analysis was performed. Also, the following study was undertaken to examine the method and the codes:

As the collision probability from fuel to fuel can be precisely calculated also by Fukai method as shown by Ref. 5, the direct comparison of the results has been made as per Table 3-1, which indicates the close agreement between the two. Although the collision probabilities from fuel-rod to coolant, and from coolant to coolant are not in any comparable relation, physically, the probability of the neutrons born either in the fuel-rod or in the coolant to collide in a certain cell must be 1. Viewing this value from the example of application to a prototype fast reactor as later mentioned, for the neutrons produced in the central cell, using 10-point Gauß to each of r and φ for those cells of which centers are within 4cm radius, and using 5-point Gauß to each r and φ for those cells within 23cm outside the 4cm radius, and using an approximation collision probability as later described for those further away from this 23cm distance, the probability of the neutrons produced in the central fuel-rod zone under normal condition to collide in the fuel-rod zone within the radius of 23cm is 0.61569, while, that of in the coolant zone is 0.37619, and the probability of collision in a certain zone is 0.99188, and the probability within a 2cm radius is 0.994. The probability of the neutrons produced in the central coolant zone to collide in a certain zone within a 2cm radius is 0.996. As later

Table 3−1. Comparison of collision probability by Fukai's method.

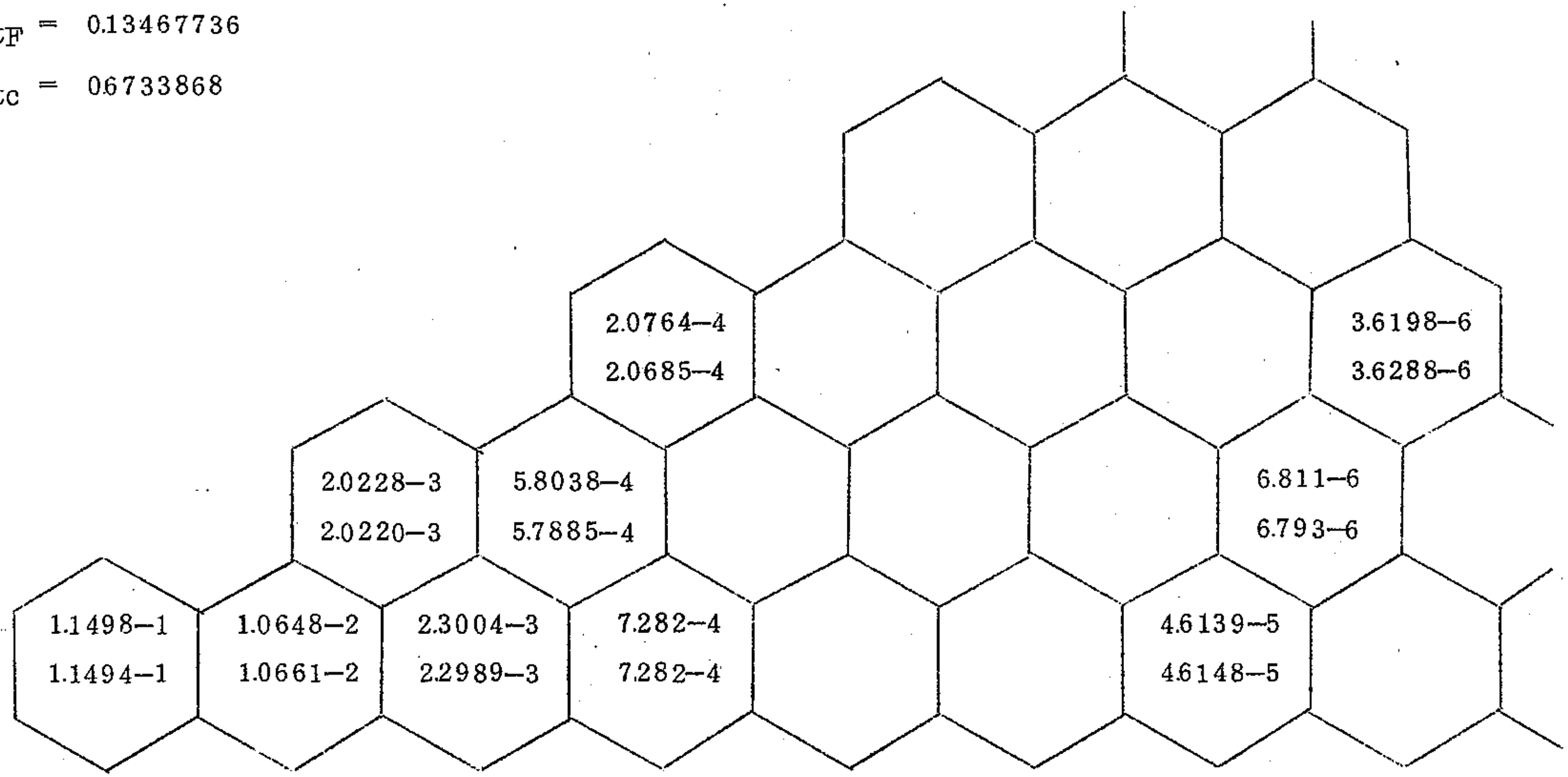
1. Upper stage Fukai 20−point Gauβ (Ref. 6)
2. Lower stage This Method 16−point Gauβ

r_F = 0.7425153 cm

Pitch= 2.0 cm

Σ_{tF} = 0.13467736

Σ_{tc} = 06733868



described, the error of this standardization is the result of the error of the approximation performed to those in the 23cm distance. Therefore, with an improvement in this sector, the error can be maintained within 0.1%. For this the collision probability within 23cm which is the major collision occurrence area may be considered to have a sufficient accuracy.

3-2. Collision Probability from Zone to Zone

As a uniform core is considered, the probability of the neutrons which are produced in a random α cell to collide in a random β cell is dependent upon the relative position of the two cells, and is independent from their positions in the core. Consequently, if a table is prepared for the probability of the neutrons born in the cell (0,0) as in Fig. 3-1 to collide in the cell (I,J), it may be possible to compute the collision probability from zone to zone. The probability of the neutrons born in ℓ'-sub-zone of zone-i to collide in ℓ-sub-zone of zone-i can be given by:

$P_{i'\ell',i\ell}$ = Mean Sum (Probability of the neutrons born in ℓ'-zone of α-cell to collide in ℓ-zone of β-cell)

For Mean and Sum, a calculation code was prepared using the symmetry for the process of these two.

3-3. Approximation Method

In order to seek the collision probability Pij in the core, it is necessary to consider all the cells in the zone to which i-region belongs as well as all the cells in the zone to which j-region belongs as described in the preceding chapter. This will require a considerable amount of calculation. For this reason, an adequate appxorimation method was applied for the purpose of simplification of computation of the collision probabilities between the regions to which the zones beyond a certain distance are belonging.

By homogenizing the intermediate medium calculated in Section 3-2, and retaining the cells heterogeneous on both end, the probability of the heterogenized cells on both ends is represented by the $f(d)$. Here, d represents the distance from center to center of the two cells, and $f(d)$ may, according to the case, correspond to any one of the probabilities from fuel-rod to fuel-rod, fuel-rod to coolant, or from coolant to coolant. First, relating to d, a table of f(d) is prepared. The collision probability fij from the zone-i to the zone-j is approximated by an integral form and not by sum as indicated below:

$$f_{ij} = \frac{2\pi N_j}{S_i S_j} \int_{r_{i-1}}^{r_i} dr_i' \int_{r_{j-1}}^{r_j} dr_j' \int_0^{2\pi} d\varphi f\left(\sqrt{r_i'^2 + r_j'^2 - 2r_i' r_j' \mathrm{Coo}\,\varphi}\right) \cdots\cdots (3\text{-}7)$$

Then, a numerical integration is performed by using long-linear interpolation of f. Here, N_j is the number of cells in the zone-j, while S represents the area of the zone.

In the actual calculation, i is fixed, and j is computed from the point where overlaps with the last point of the zone which is already precisely calculated under Section 3-2, and in this zone, the subsequent f_{ij} is restandardized so that it will correspond to the precisely analyzed values.

3-4. Calculation Method of P_{i*j}

Now that all the collision probabilities in the core have been sought as described in the preceding pages, as the next step, the calculation method of the collision probabilities P_{i*j} in the system including the blanket is described as follows:

Representing the fuel-rod zone and the coolant zone in the zone-m of the core by m_f and m_c respectively;

$$i = \begin{cases} 2\ m-1 & \text{.......... Fuel-rod zone} \\ 2\ m & \text{............ Coolant zone} \end{cases} \qquad (3\text{-}8)$$

This way, m_f and m_c can be aligned in a numerical order from one without adding any subscript (for example, $P_{3f,5c}$ becomes $P_{5,10}$). Furthermore, if the homogenized blanket is divided into a number of concentric circles and put into an alignment, the whole region can be expressed by one index, and the collision probabilities between these zones shall be indicated by P_{i*j}.

3-4-1. A-Method (Normalization Method)

First, by homogenizing the blanket and the core respectively, divide the core region into the zones same as described in Section 3-2, and the blanket is also divided into circular zones as Fig. 2-2.C. In such circular system, the probability P_{ij}^{H} that the neutrons produced in the zone-i to collide in the zone-j can be accurately obtained. Considering also the B-method, the black body function is defined as follows:

$$B\ (r-r_o) \equiv \begin{cases} 1 & r_o < r \\ 0 & o < r < r_o \end{cases}$$

Here, $r<r_o$ is completely full of a black body, and if there is no black body, $r_o = 0$ can be considered. When this is used, the collision probability from zone to zone is obtained by:

$$P_{ii}^H = \frac{2}{\Sigma_i V_i}\int_0^{r_i} d\gamma [2\{\tau_i - K_{i3}(0) + K_{i3}(\tau_i)\} + B(\gamma-\gamma_o)\{K_{i3}(2\tau_i+\tau_{ii}) + K_{i3}(\tau_{ii}) - K_{i3}(\tau_{ii}+\tau_i) - K_{i3}(\tau_{ii}+\tau_i)\}] \quad (3-9)$$

$$P_{ij}^H = \frac{2}{\Sigma_i V_i}\int_0^{r_i} d\gamma [K_{i3}(\tau_{ij}) + K_{i3}(\tau_i+\tau_{ij}+\tau_j) - K_{i3}(\tau_i+\tau_{ij}) - K_{i3}(\tau_j+\tau_{ij}) + B(\gamma-\gamma_o)\{K_{i3}(\tau_{ii}+\tau_{ij}+\tau_i) + K_{i3}(2\tau_i+\tau_{ii}+\tau_{ij}+\tau_j) - K_{i3}(\tau_{ii}+2\tau_i+\tau_{ij}) - K_{i3}(\tau_{ii}+\tau_i+\tau_{ij}+\tau_j)\}] \quad (3-10)$$

$$P_{i,\,vacuum}^H = \frac{2}{\Sigma_i V_i}\int_0^{r_i} d\gamma [K_{i3}(\tau_{ij}) - K_{i3}(\tau_i+\tau_{ij}) + B(\gamma-\gamma_o)\{K_{i3}(\tau_{ii}+\tau_{ij}+\tau_i) - K_{i3}(\tau_{ii}+2\tau_i+\tau_{ij})\}] \quad (3-11)$$

Where, τ is optical path length, and the meaning of each subscript is the same as indicated in Fig. 3-7.

By A-method, $P_i{}^H{}_j$ is obtained by putting $r_o = 0$ and $P_i{}^*{}_j$ is constructed by the following method: In this case, the case of $i \leq j$ is sought, and then by a reciprocity theorem, what is replaced between i and j is obtained. Therefore, firstly, i is determined, and then j can start from j = i. Whereas, the method to obtain it.

(i) i, j, e Core

$$P_i{}^*{}_j = P_{ij} \quad (3-12)$$

(ii) i e Core, j e blanket

$$P_i{}^*{}_j = P^H_{[\frac{i+1}{2}]j} \times \left\{ \frac{\underset{k\,e\,\text{blanket+vacuum}}{\text{Sum}} P^H_{[\frac{i+1}{2}]k}}{1 - \underset{k\,e\,\text{Core}}{\text{Sum}} P^*_{ik}} \right\} \quad (3-13)$$

Here, [x] stands for Gaubian symbol and expresses the maximum integar not exceeding x.

(iii) i, j e blanket

$$P_i{}^*{}_j = P^H_{ij} \times \left\{ \frac{\sum_{k=j}^{vacuum} P^H_{ik}}{1 - \sum_{k=j}^{j-1} P^*_{ik}} \right\} \quad (3-14)$$

This method is based on an idea that the probability that the neutrons getting out of a certain zone will make collision in the

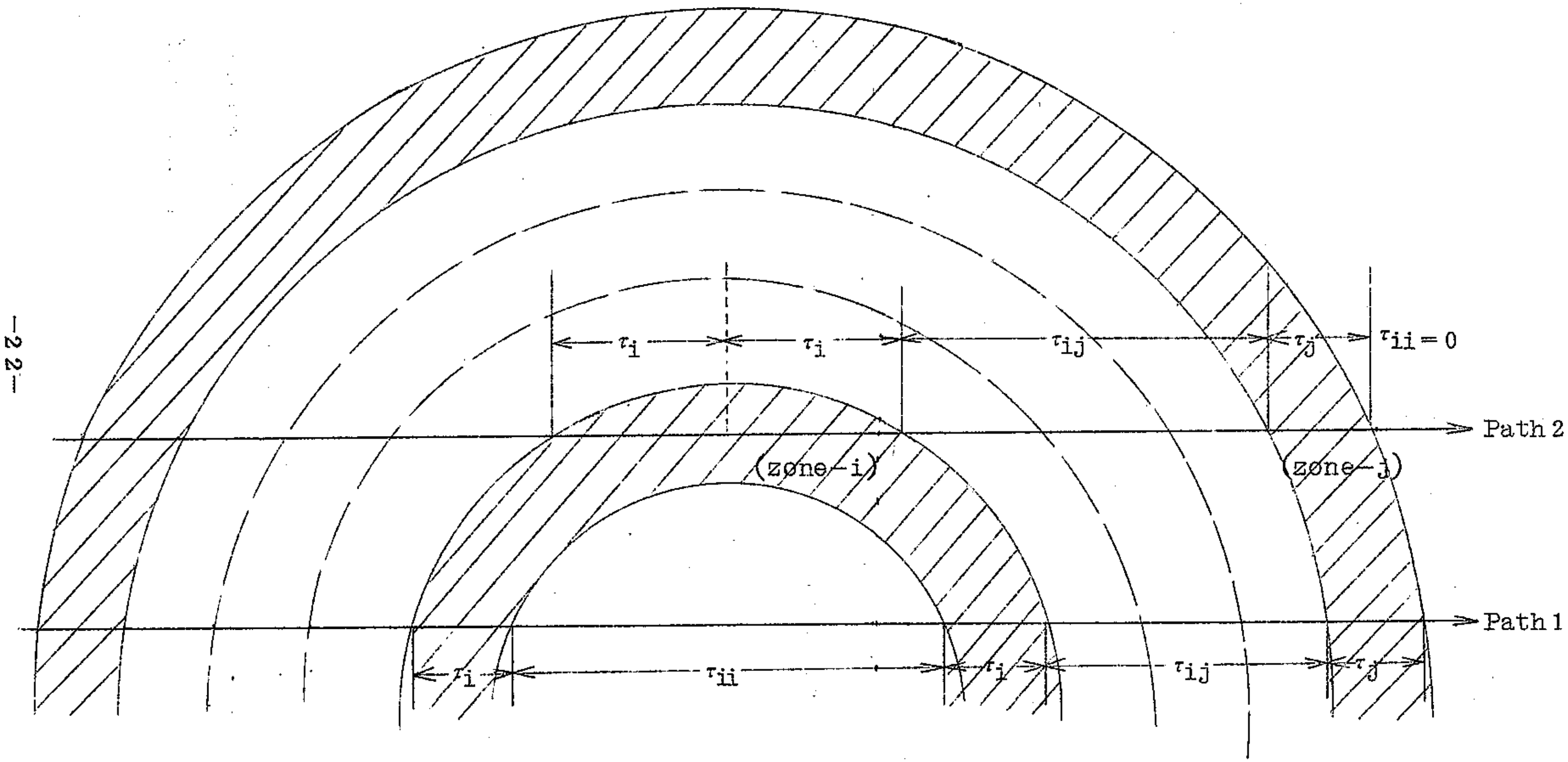


Fig. 3 – 7. Collision probability calculation method in homogeneous zone.

blanket is in proportion to the probability of the neutrons from the zone belonging to the initial departing region to collide in the blanket.

3-4-2. B-Method

This method is to divide the core and the blanket into a number of homogenized circular zones the same as described under Section 3-4-1, and make $r_o = r_c$ (r_c is the core's radius). If the probability calculated by the formulas (3-9) to (3-11) is expressed by P_{ij}, (i,j e blanket). This represents the probability of the neutrons departed from the zone-i in the blanket to collide in the zone-j without passing the core. $P_i{}^*{}_j$ from P_{ij} and $P_i{}'{}_j$ is obtained by the following approximation:

(i) i,j e Core

$$P_i{}^*{}_j = P_{ij} \tag{3-15}$$

(ii) i e Core, j e blanket

$$P_i{}^*{}_j = P_{io} P_{ij} \tag{3-16}$$

(iii) i, j e blanket

$$P_i{}^*{}_j = P_i{}'{}_j + P_{iI} \{ 1 - \underset{k \text{ e Core}}{\text{Sum}} P_{ok} \} P_{Ij} \tag{3-17}$$

Here, P_{io} is the probability of the neutrons, which departed from the zone-i in the core, to cross the boundary between core and blanket, P_{oi} is the probability of the neutrons, which entered into the core, to collide in the zone-i of the core. P_{jI} is the probability of the neutrons, which departed from the zone-j in the blanket, to cross the boundary between core and blanket, and

P_{Ij} represents the probability of the neutrons, which entered into the blanket by traversing the blanket boundary, to collide in the zone-j. Here, if the entering neutrons at the boundary are assumed all isotropic, the following equations may be introduced according to Ref. (3):

$$P_{jI} = 1 - \sum_{k \in \text{blanket + vacuum}} P_{jk} \qquad (3\text{-}18)$$

$$P_{jo} = 1 - \sum_{k \in \text{core}} P_{jk} \qquad (3\text{-}19)$$

$$P_{Ij} = \frac{4V_i}{S} \Sigma_{t,i} \, P_{iI} \qquad (3\text{-}20)$$

$$P_{oi} = \frac{4V_i}{S} \Sigma_{t,i} \, P_{io} \qquad (3\text{-}21)$$

Here, S is the surface area of the boundary between core and blanket. The $P_i{}^*{}_j$ which is obtained this way automatically satisfies the conservation of probability and the reciprocity theorem.

In order to make sure of the accuracy of this approximation, the core with 70.002cm radius was divided into 30 zones, and the blanket with a wall thickness of 29.998cm into 12 zones, and under a homogenized system, the exact results analysis and approximate results were compared. The probabilities from zone to zone of homogeneous concentric circular system were all obtained as $r_o = 0$ by application of the formulas (3-9) through (3-11). Next, $P_i{}'{}_j$ was calculated as $r_o = 70,002$, and the collision probability in the entire system were calculated from $P_i{}'{}_j$ by application of the formulas (3-5) - (3-21). The zone number 30 is the outermost core zone, and the blanket zone begins from No.31. The results are as shown in Table 3-2. As the contributions at and around the in-core blanket boundaries are considerably small, it is thought that an approximation to this level will satisfy the purpose.

Table 3-2. Comparison between B-method and exact analysis

Zone	Zone	Approximation	Exact analysis
27	31	0.2365	0.2534
	32	0.0678	0.0591
	33	0.0242	0.0196
	34	0.0093	0.0073
28	31	0.1312	0.1312
	32	0.0376	0.0380
	33	0.0134	0.0133
	34	0.0052	0.0050
29	31	0.0810	0.0779
	32	0.0232	0.0250
	33	0.0083	0.0091
	34	0.0033	0.0035
30	31	0.0528	0.0494
	32	0.0151	0.0169
	33	0.0054	0.0063
	34	0.0021	0.0025

4. Application to Prototype Fast Power Reactor

4-1. Calculation

Voluntarily determining a prototype fast power reactor with a core of 58.596cm radius and a blanket of 26.404cm thickness as the subject reactor, an analysis of void coefficient was performed. It is of one-region core, and one energy group. This prototype reactor is consisted of about 20,000 fuel-rods. Disregarding the wrapper-tube and the control-rod the core is of hexagonal lattice of same enrichment. The fuel and the coolant has been homogenized in the blanket. The various elements and the one group constants which are used as well as the zone division are as shown in Table 4-1 and 4-2 respectively.

A one group micro constants was obtained by group reduction in terms of the mean spectrum of the core and blanket which is calculated with 26 groups diffusion calculation. Then, a macro constant was simply obtained by use of an atomic number density. Namely, any of the effect arising from the changes in a group of constants affected by the spectrum change by occurrence of void and the self-shielding effect were excluded from the study of this time.

The number of in-core zones is 21, and the cell has two zones, one for the fuel rod and the other for the coolant, while the blanket is divided into 11 homogeneous, concentric circular zones.

The computation of collision probabilities was performed by the following method:

As it will take a too long time to make a sufficiently accurate

Table 4-1. Constants and various elements

	Core						Blacket
	Fuel rod		Coolant		Homogenization		
	Normal	Void	Normal	Void	Normal	Void	
Σ_t (cm^{-1})	0.3342	0.3342	0.1719	0.08852	0.24324	0.19651	0.30183
Σ_a (cm^{-1})	0.01473	0.01473	0.0006181	0.0005875	0.0068212	0.00680412	0.004966
$\nu\Sigma_f$ (cm^{-1})	0.02218	0.02218	0.0	0.0	0.0097497	0.0097497	0.001096
Leakage (cm^{-1})	1.039×10^{-3}						4.730×10^{-4}

Fuel pin diameter	0.55cm	Fuel volume ratio	0.4396
Fuel pin pitch	0.79cm	Coolant volume ratio	0.5604
Core radius	58.596cm	Blanket thickness	26.404cm

Table 4-2. Zone division

	Zone	Outer radius	Number of fuel pins
Core	1	12.818	955
	2	18.094	948
	3	22.100	936
	4	25.586	966
	5	28.596	948
	6	31.301	942
	7	33.822	954
	8	36.081	918
	9	38.221	924
	10	40.413	1002
	11	42.322	918
	12	44.312	1002
	13	46.104	942
	14	47.765	906
	15	49.547	1008
	16	51.116	918
	17	52.677	942
	18	54.232	966
	19	55.724	954
	20	57.232	990
	21	58.596	
Blanket	22	60.0	
	23	62.5	
	24	65.0	
	25	67.5	
	26	70.0	
	27	72.5	
	28	75.0	
	29	77.5	
	30	80.0	
	31	82.5	
	32	85.0	

calculation of all the collision probabilities from cell to cell by the method given in Section 3-1, here I,J which was the definition given for the positions of cells as indicated in Fig. 3-1 are used to express:

(i) For those cells of $I + J \leq 10$, an integration of the formula of (3-2) and (3-3) with respect to r and φ has been done with 10-point Gaußian quadrature.

(ii) For those cells of $10 < I + J \leq 56$, the 5-point Gaußian quadrature was applied.

(iii) For those cells of $56 < I + J$, the values calculated by homogenizing the intermediate region were inter-polated with the distance from center to center of the cells, and then, it was renormalized so that the total probability became 1.

From this, the probability of the neutrons, which depart from the ℓ-region in the zone-i to collide in the ℓ'-region in the zone-i', can be obtained by the method given in Section 3-2. But as this method will take a considerable amount of calculation time, it was applied only to $/i - i'/ \leq 5$, while the method given in Section 3-3 was applied to $/i - i/ > 5$.

The leakage occurring in the axial direction due to the finite length of the Z-axial direction is considered as having an absorption equivalent to DB^2 under the normal diffusion equation. As the calculation of collision probability normally takes a substantial amount of time, $P_i{}^*{}_j$ was obtained without giving consideration to it, and in the solution of the neutron balance equation as in the form of 2-11, the case was added where the leakage term as shown in Table 4-1 were considered as "absorption" for Ks.Kf.

The study was undertaken with respect to the following three cases when leakage was not considered, and each void coefficient was calculated:

(1) In the case where one-group diffusion equation was applied based on the diffusion coefficient sought by a separate contraction.

(2) In the case where the collision probability method was applied for the heterogeneous system.

(3) In the case where the collision probability method was applied fot the homogeneous system.

In this case, the same zoning as in the case of (2) was taken, and used the same homogeneous constant for the fuel region and the coolant region (Fig. 4-1).

Also, when the leakage is considered, the void coefficient was calculated for the above cases (2) and (3). Further, with respect to (2) and (3), the calculation with leakage and without leakage were both performed by use of A-method under Section 3-4-1 and B-method under Section 3-4-2 respectively. The results are given in Table 4-3. Table 4-4 represents the number of collisions in each zone under normal time and under void occurrence time in both the homogeneous and the heterogeneous systems, while Table 4-5 shows the mean neutron flux.

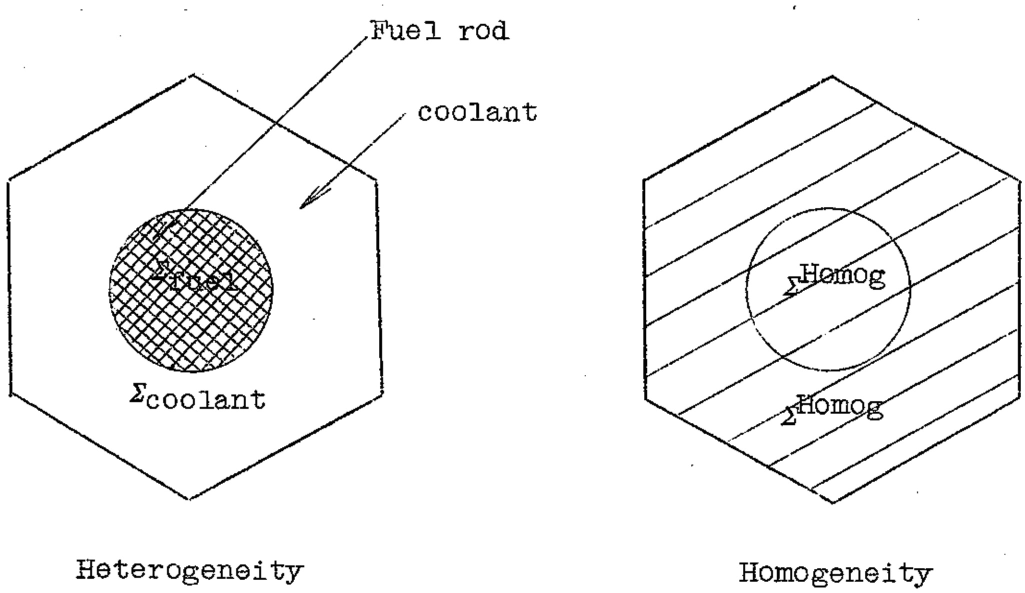


Fig. 4 − 1. Handling of cells.

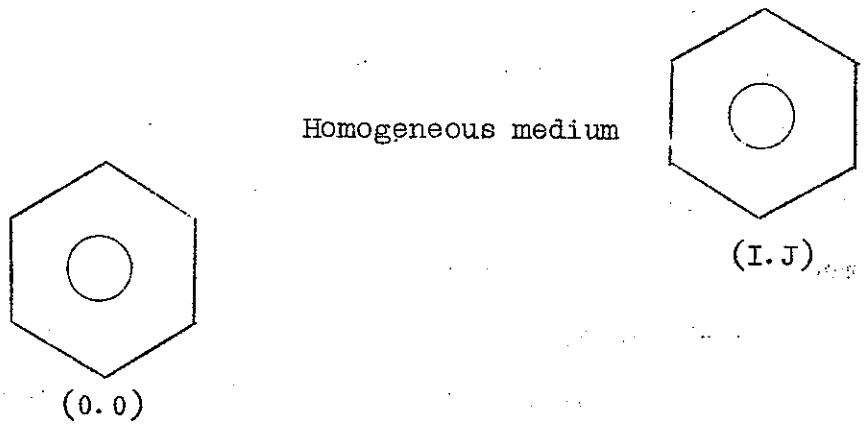


Fig. 4 − 2. System with homogenized path from cell(0. 0)to cell(I. J).

Table 4−3 Calculation results

	No leakage					With leak				
	1−group diffusion	A−method		B−method		1−group diffusion	A−method		B−method	
		Hetrog-eneity	Homoge-neity	Hetrog-eneity	Homoge-neity		Hetrog-eneity	Homoge-neity	Hetrog-eneity	Homoge-neity
Normal	1.16366	1.1602	1.1589	1.1634	1.1622	1.03311	1.0301	1.0285	1.0326	1.0311
Void	1.14494	1.1494	1.1414	1.1527	1.1452	099366	099797	0.98977	1.0005	099271
△ k	001872	0.0108	0.0175	0.0107	0.0170	0.03945	00321	0.0387	0.0321	0.0384
△k/k	001609	0.009309	001510	0.009197	0.01463	0.03819	0.03116	0.03763	0.03109	0.03724

Table 4-4. Total number of collisions (arbitrary unit) with no leak (B-method).

Region	Zone	Classification	Homogeneity		Heterogeneity	
			Normal	Void	Normal	Void
1	1	F	2.6304	3.1847	6.1097	4.9577
2		C	3.3555	4.0633	4.0047	1.6730
3	2	F	2.5382	3.0749	5.8954	4.7862
4		C	3.2389	3.9247	3.8649	1.6153
5	3	F	2.4234	2.9411	5.6287	4.5753
6		C	3.0927	3.7543	3.6902	1.5441
7	4	F	2.4129	2.9358	5.6039	4.5631
8		C	3.0793	3.7474	3.6740	1.5400
9	5	F	2.2792	2.7815	5.2926	4.3188
10		C	2.9086	3.5503	3.4698	1.4515
11	6	F	2.1772	2.6663	5.0546	4.1353
12		C	2.7782	3.4030	3.3137	1.3956
13	7	F	2.1172	2.6030	4.9143	4.0327
14		C	2.7018	3.3224	3.2218	1.3610
15	8	F	1.9538	2.4113	4.5324	3.7317
16		C	2.4926	3.0781	2.9716	1.2594
17	9	F	1.8840	2.3367	4.3714	3.6120
18		C	2.4048	2.9832	2.8663	1.2191
19	10	F	1.9515	2.4322	4.5269	3.7548
20		C	2.4910	3.1054	2.9683	1.2672
21	11	F	1.7065	2.1380	3.9577	3.2957
22		C	2.1784	2.7298	2.5951	1.1123
23	12	F	1.7748	2.2361	4.1150	3.4413
24		C	2.2656	2.8550	2.6982	1.1615
25	13	F	1.5862	2.0104	3.6764	3.0885
26		C	2.0247	2.5669	2.4106	1.0424
27	14	F	1.4520	1.8518	3.3642	2.8395
28		C	1.8533	2.3642	2.2058	0.95835
29	15	F	1.5315	1.9666	3.5471	3.0093
30		C	1.9547	2.5106	2.3257	1.0156
31	16	F	1.3197	1.7073	3.0553	2.6067
32		C	1.6844	2.1794	2.0032	0.87973
33	17	F	1.2802	1.6692	2.9624	2.5425
34		C	1.6337	2.1305	1.9422	0.85807
35	18	F	1.2375	1.6274	2.8621	2.4726
36		C	1.5791	2.0769	1.8763	0.83444
37	19	F	1.1463	1.5217	2.6497	2.3056
38		C	1.4625	1.9417	1.7370	0.77807
39	20	F	1.1103	1.4894	2.5651	2.2500
40		C	1.4167	1.9006	1.6815	0.75920
41	21	F	0.95288	1.2931	2.2004	1.9476
42		C	1.2159	1.6504	1.4420	0.65674
43	22	B	2.6605	4.5029	4.4610	3.9415
44	23	B	4.2034	7.1154	7.0478	6.2279
45	24	B	3.6516	6.1825	6.1226	5.4109
46	25	B	3.1542	5.3413	5.2885	4.6743
47	26	B	2.7001	4.5731	4.5272	4.0018
48	27	B	2.2818	3.8651	3.8257	3.3820
49	28	B	1.8928	3.2066	3.1734	2.8056
50	29	B	1.5267	2.5868	2.5598	2.2632
51	30	B	1.1774	1.9951	1.9740	1.7455
52	31	B	0.83826	1.4205	1.4054	1.2427
53	32	B	0.49568	0.8400	0.83105	0.73487

Classifications: F = fuel rod C = coolant B = blanket

Table 4-5. Mean neutron flux (arbitrary unit) with no leak (B - method).

Region	Zone	Classi-fication	Homogeneity		Heterogeneity	
			Normal	Void	Normal	Void
1	1	F	0.21576	0.20991	0.21618	0.21189
2		C	0.21588	0.21008	0.21608	0.21174
3	2	F	0.20974	0.20418	0.21015	0.20608
4		C	0.20993	0.20441	0.21008	0.20595
5	3	F	0.20284	0.19781	0.20323	0.19954
6		C	0.20303	0.19806	0.20317	0.19941
7	4	F	0.19562	0.19127	0.19598	0.19275
8		C	0.19583	0.19150	0.19593	0.19264
9	5	F	0.18835	0.18471	0.18867	0.18596
10		C	0.18853	0.18491	0.18861	0.18584
11	6	F	0.18110	0.17823	0.18137	0.17922
12		C	0.18126	0.17842	0.18131	0.17911
13	7	F	0.17381	0.17171	0.17402	0.17249
14		C	0.17397	0.17190	0.17397	0.17238
15	8	F	0.16668	0.16538	0.16686	0.16594
16		C	0.16686	0.16558	0.16682	0.16584
17	9	F	0.15969	0.15915	0.15983	0.15951
18		C	0.15987	0.15938	0.15980	0.15943
19	10	F	0.15260	0.15282	0.15269	0.15297
20		C	0.15276	0.15304	0.15266	0.15288
21	11	F	0.14561	0.14661	0.14568	0.14653
22		C	0.14579	0.14681	0.14566	0.14644
23	12	F	0.13876	0.14046	0.13876	0.14017
24		C	0.13889	0.14067	0.13875	0.14009
25	13	F	0.13195	0.13438	0.13191	0.13386
26		C	0.13209	0.13458	0.13190	0.13378
27	14	F	0.12551	0.12862	0.12544	0.12789
28		C	0.12565	0.12881	0.12541	0.12781
29	15	F	0.11903	0.12283	0.11891	0.12186
30		C	0.11917	0.12297	0.11889	0.12178
31	16	F	0.11262	0.11707	0.11246	0.11590
32		C	0.11273	0.11722	0.11244	0.11582
33	17	F	0.10648	0.11157	0.10630	0.11020
34		C	0.10660	0.11169	0.10627	0.11013
35	18	F	0.10032	0.10602	0.10009	0.10444
36		C	0.10039	0.10613	0.10005	0.10437
37	19	F	0.094162	0.10046	0.09390	0.09869
38		C	0.094232	0.10054	0.09386	0.09862
39	20	F	0.087842	0.094702	0.08755	0.09276
40		C	0.087911	0.094785	0.08751	0.09268
41	21	F	0.081289	0.088639	0.08097	0.08657
42		C	0.081359	0.088742	0.08091	0.08644
43	22	B	0.076288	0.083817	0.07581	0.08090
44	23	B	0.065522	0.072020	0.06512	0.06950
45	24	B	0.054687	0.060120	0.05435	0.05802
46	25	B	0.045472	0.049980	0.04517	0.04823
47	26	B	0.037508	0.041225	0.03727	0.03979
48	27	B	0.030585	0.033630	0.03038	0.03245
49	28	B	0.024496	0.026946	0.02435	0.02600
50	29	B	0.019124	0.021027	0.01900	0.02029
51	30	B	0.014285	0.015708	0.01419	0.01515
52	31	B	0.009863	0.010844	0.00979	0.01045
53	32	B	0.00569	0.006209	0.00562	0.00600

Classifications: F = fuel rod ; C = coolant B = blanket

4-2. Discussion of the Results

Looking at the results, it can be seen that the effect of heterogeneity is large reaching up to 30 - 40%. In the case of homogeneity, the calculation results by either this method or by the diffusion theory were quite similar, which, therefore, indicates that so far as homogeneous handling of a prototype fast reactor system is concerned, even the diffusion theory has proved to demonstrate a sufficient accuracy for the calculation of the void coefficient. Nevertheless, since the heterogeneity effect has proved to be considerably great, it is thought that its effect should not be ignored when an accurate result is desired.

Resultwise, the neutron flux in the coolant zone and the neutron flux in the fuel zone in any of the areas were in substantially a good concordance, and it has been determined that the heterogeneity effect was not due to the fine structure of the neutron flux and that, consequently, the cause was from the heterogeneity effect upon the leakage.

In order to examine more in detail the heterogeneity effect upon the neutron leakage, a study was undertaken to see how the collision probability was influenced by the heterogeneity effect. The collision probability in the case of homogenized cells on the intermediate medium as shown by Fig.4-2 and the collision probability in the heterogeneous system are compared in Table 4-6 and 4-7. From these tables, it can be seen that $F \rightarrow F$, $F \rightarrow C$, when homogenized, have large values as going further away, while on the contrary, C → C, when heterogeneous, shows larger values as going further away, of which trend becomes stronger at the time of void occurrence. The collision probability values given in the above referred tables are the values taken of the cells which are aligned sidewise. Whereas, as there are existing a substantial number of neutrons which have passed none at all through the fuel zone as against C-C, as a trend, it is considered reasonable. But in other direction, the trend may possibly turn around the other way.

As can be seen from Table 4-4, the scattering neutron source in the fuel zone being larger than the one in the coolant zone, there remains some vagueness with respect to the trend of C → C. But the neutrons produced at a random place may well be considered to reach a longer distance in a homogeneous system before their first collision takes place. This trend becomes stronger at the time of void occurrence.

As the leakage from the system is the monotonous function of the distance where the neutrons reaches in its movement before neutrons are absorbed, the leakage in the monogeneous system may be said larger than in the heterogeneous system.

These situations qualitatively support the calculation results of the void coefficient, and also are considered as supporting the fact that the heterogeneity effect is considerably large.

Since all the studies were performed with one group, there may be a number of phenomena overlooked. Due to time limitation for calculation, various types of approximation were applied to collision probability computation of which results, therefore, may not

Table 4-6. Effect on collision probability of homogenization of intermediate medicium under normal condition

I, J	F ⟶ F	F ⟶ C	C ⟶ C	Distance between the center to center (cm)
0.0	1.05536 -1 1.05536 -1	3.44900 -2 3.44900 -2	7.26302 -2 7.26302 -2	0
2.0	1.75513 -2 1.75484 -2	1.13370 -2 1.13299 -2	1.59709 -2 1.58236 -2	0.79
4.0	5.92674 -3 6.15131 -3	3.81552 -3 3.91244 -3	4.89989 -3 4.87592 -3	1.58
6.0	2.87795 -3 3.08048 -3	1.85259 -3 1.93432 -3	2.40226 -3 2.38271 -3	2.37
8.0	1.61015 -3 1.77261 -3	1.04829 -3 1.11291 -3	1.37177 -3 1.35273 -3	3.16
16.0	2.75212 -4 3.35417 -4	1.87361 -4 2.10186 -4	2.67038 -4 2.55644 -4	6.32
22.0	9.41033 -5 1.23145 -4	6.59039 -5 7.69031 -5	1.00941 -4 9.37033 -5	8.25
30.0	2.61099 -5 3.7419 -5	1.91509 -5 2.3320 -5	3.24229 -5 2.8476 -5	11.85
36.0	1.06782 -5 1.6343 -5	8.08702 -6 1.0177 -5	1.4907 -5 1.2442 -5	14.22
40.0	6.0162 -6 9.6102 -6	4.6620 -6 5.9818 -6	9.1176 -6 7.3182 -6	15.8
50.0	1.51799 -6 2.6882 -6	1.24950 -6 1.6721 -6	2.85995 -6 2.0483 -6	19.75

Upper section: Heterogeneity
Lower section: Intermediate medium homogenized

Table 4-7. Effect on collision probability of homogenization of intermediate medium under void occurrence

I, J	F ⟶ F	F ⟶ C	C ⟶ C	V
26.0	7.1851 -5 1.1513 -4	2.8964 -5 3.7198 -5	5.9162 -5 4.5761 -5	10.27
30.0	4.0728 -5 6.9573 -5	1.7113 -5 2.2458 -5	3.8462 -5 2.7652 -5	11.85
36.0	1.81925 -5 3.4058 -5	8.0655 -6 1.0983 -5	2.1417 -5 1.3538 -5	14.22
40.0	1.0887 -5 2.1600 -5	5.0150 -6 6.9627 -6	1.4970 -5 8.5874 -6	15.8
46.0	5.1866 -6 1.1173 -5	2.5414 -6 3.5997 -6	9.0950 -6 4.4429 -6	18.17
50.0	3.2155 -6 7.2929 -6	1.6382 -6 2.3490 -6	6.6719 -6 2.9004 -6	19.75
54.0	2.01636 -6 4.8009 -6	1.07158 -6 1.5460 -6	4.9741 -6 1.9096 -6	21.33
56.0	1.6030 -6 3.9062 -6	8.6915 -7 1.2578 -6	4.3150 -6 1.5539 -6	22.12

Upper section : Heterogeneity
Lower section : Intermediate medium homogenized

be expected to be precisely accurate if not considered to be too inaccurate. It is, therefore, hard to say that the results of the study have necessarily led to an accurate calculation of the heterogeneity effect upon the void coefficient. However, as the final conclusion, it may be said that there indeed exists far larger heterogeneity effect than is generally considered.

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