# An Adjusted Cross Section Library for DFBR

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#### An Adjusted Cross Section Library for DFBR

Peter J.Collins\*

#### Abstract

While in the Reactor Physics Research Section of the Advanced Technology Division at OEC, I participated in the project to construct a data library for the demonstration fast breeder reactor (DFBR). This library would be produced using a combination of evaluated differential cross sections together with integral experimental data for fast reactors, so as to assure sufficiently accurate calculations for the DFBR designs. I had much experience of the design and use of experiments for the large-size cores at ZPPR under the title JUPITER which was performed under the USDOE/PNC joint agreement. My contribution here was mainly in extension of the experimental database to include the very-hard spectrum fast criticals from the Los Alamos National Laboratory (LANL). The data for these cores are described.

Our work at ANLW with the GMADJ code, which is similar in effect to the ABLE code that we use at PNC, showed why many experiments are important in this project as well as those in the more obvious Pu/U oxide conventional cores which are of current interest for the DFBR. This point was not appreciated at PNC and is discussed here.

The data from the fast spectrum critical experiments made at Los Alamos are described together with information that I have been able to find concerning the uncertainties. The main interest is these experiments has been for prediction of criticality. Consequently, the full covariance information that we would like, has not been published. However, the uncertainty in the fuel content is, by far, the major contributor to the uncertainty. The LANL experiments have been a principal leg of the data testing for fast reactors for all versions of ENDF/B in the US. For our work, they provide measurements at Mev energies which are not available from the experiments in the softer-spectrum of the LMFBR.

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## 高速実証炉のための 炉定数調整に関する研究

ピーター [ コリンズ\*

## 要旨

大洗工学センターの基盤技術開発部炉心技術開発室に滞在している間、私は高速増殖 実証炉(DFBR)のために炉定数ライブラリの開発を行うプロジェクトに参加した。こ のライブラリは、核断面積(微分データ)を高速炉に関する炉心実験(積分データ)と 結びつけ、DFBRの設計に充分な精度を確保しようとするものである。私は、USDOEと PNCの協定に基づきZPPR装置において実施された大型高速炉心臨界実験JUPITERの企 画とそのデータ利用に関して、豊富な経験を有している。大洗での私の役割は主に、過 去にロスアラモス国立研究所(LANL)で実施された非常に硬いスペクトルでの高速炉 臨界実験を実験データベースに加えることであった。これらの実験炉心に関するデータ は本報告書に記載されている。

我々はANL-Westで、動燃のABLEコードと同じ機能を持つGMADJコードを使って行った研究により、現在のDFBRが対象としている伝統的なMOX炉心だけではなく、このプロジェクトではより多くの実験を用いる必要があることを明らかにした。動燃ではこの点への認識がこれまで為されていなかったので、本報告書で議論している。

LANLで行われた高速スペクトルの臨界実験データは、私が収集した誤差評価に関する情報とともにまとめた。これらの実験の最大の目的は臨界性データを得ることであったので、炉定数調整で必要となる誤差の相関(共分散)情報は報告されていないが、実験誤差の最大の要因は燃料組成誤差であることが分かっている。このLANL実験は、米国でENDF/Bライブラリの各バージョンを開発する際には、その積分テストに重要な役割を果たしてきた。DFBRのための炉定数開発に対しては、これらのLANLデータは、通常の軟らかいスペクトルを持つLMFBRでは得られないMeV領域の情報を提供できる。

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#### Chapter 1 INTRODUCTION

In the first years of reactors, it was necessary to build an experimental mock-up of the design because calculations were crude and nuclear data were not well known. This situation has now changed with improved measurements of cross sections, more sophisticated codes and increases in the power of computers. A mock-up of a design is needed only if it falls outside of the range of existing data. A full-scale mock-up may be too expensive, is usually limited in some areas and cheaper experiments will often suffice.

However, for a fast reactor it is recognised that the present nuclear data does not permit calculations with sufficiently low uncertainties for many of the parameters required. One solution is to use a combination of differential and integral data where the "a priori" calculated parameters are augmented with data from integral measurements, making use of sensitivity coefficients. These are just the fractional change in a parameter divided by the fractional change in a cross section. Sophisticated computer codes have been written to accomplish the calculation of sensitivity coefficients. A large database of measurements for fast reactors has been compiled over the past decades.

A convenient method of synthesis of differental and integral data is basically the method of least squares fitting as defined by Gauss some 200 years ago and generalised by Aitken, this provides a method of incorporating additional experimental data (the integral measurements with their sensitivity coefficients) to an existing library (the evaluated data file). This should produce an unbiassed estimator with minimum variance. Many codes have been written to accomplish this synthesis. The ANLW code is called GMADJ. At PNC we are use the ABLE code based on the Baysean approach which accomplishes the same end and in France, the recent work for production of the new adjusted library, "ERALIB" calls its basis "information theory" but again minimises the chi-square for the difference between calculation and measurement relative to the uncertainties.

Around 1970, adjusted data files were created in the UK (FGL4/5) and in France (CARNAVAL3/4). These used integral data from measurements in relatively small cores and their uncertainties were not thoroughly assessed. ANL hosted an International Comparison of Calulations for a 1200 MWe fast reactor in 1978. It was found that the adjusted libraries performed very poorly for predictions of spatially-dependent properties because experiments from small fast reactors only were used in the database. In our project to create an adjusted library for the DFBR, we now have the advantage of the JUPITER experiments which were made in intermediate-size cores.

I show first, why the larger size fast reactors exhibit a high spatial sensitivity to reactivity perturbations and the poor performance of experimental data from the smaller and intermediate size cores (similar to Monju). Fortunately, the difficulties in the larger fast reactors stem from "decoupling" between different core regions and the decoupling can be increased in radially-heterogeneous designs such as we have in the USDOE/PNC Jupiter series.

The broad shape of the core spectrum is defined by the fission spectrum and absorption and inelastic scattering in the actinides. For the DFBR, the effects of neccessary diluent materials, such as oxygen, sodium and steel can be readily seen in the spectrum both by measurement and by calculation.

The critical measurements from Los Alamos provide a crucial test of the data at the high energies. They have simple geometry with few materials. Thus they constrain any adjustments to the data in the MeV range. These cores have been benchmarks for testing of all evaluations of ENDF/B. They are important for us because the put constraints on adjustment of the data at high energies where fluxes are relatively low and the data from JUPITER show low sensitivities. When these measurements were done, the principal issue was criticality. Hence, data has not been reported in the degree of detail that we would like at this time. My assessment of uncertainties in these data and preliminary results with them in the ABLE code are given in chapter 5.

#### Chapter 2. THE NEED FOR A NEW ADJUSTED DATA LIBRARY

The first generation of fast reactors required an experimenal mock-up because reactor calculation codes were in their infancy, computers were at the early stage of development and the nuclear data were not sufficiently-well known. A selection of power reactors and their mock-up experiments is shown in Table 1. This table was constructed from my memory and certainly does not give a complete list.

The earlier small cores; EBR-II, JOYO and FFTF, required only limited materials and small-size facilities. The PFR and PHENIX, reactors of less than 300MWe required larger-size facilities and the experimental reactors ZEBRA and MASURCA were built. It was possible to test the initial design for MONJU in the ZEBRA/MOZART series. The ZPPR reactor was built to permit full-scale experiments for the ill-fated CRBRP. The ZPPR "matrix" was later expanded in size. The ANLE reactors in Chicago, ZPR-6 and ZPR-9, had served their purpose and were shut down. The materials were moved to ZPPR which was fortunate because it enabled measurements to be made in the intermediate- and larger-size cores with over 4 tonnes of fissile material in the expanded "matrix".

New problems were not anticipated at this stage but it proved fortunate that the need for experiments in a full-scale mock-up were still felt to be necessary.

Following President Carter's decision to terminate CRBRP, the large-size fast reactors were still considered to be the most-assured energy source in the US for the near future. Therefore, the research continued at ZPPR under the PNC/USDOE cooperative agreement with the interest from the US industry who had been making paper studies of the LSPB - the large size prototype breeder!

These experiments were termed JUPITER. ZPPR was the only assembly with sufficient resources in which to make the full-scale mock-up assemblies. But basic studies continued at MASURCA and a collaborative series of experiments, known as BIZET was done at ZEBRA. One of these cores, BZD/3 included a large central blanket zone which "decoupled" the reactor core.

But, by-and-large, it was felt that sufficient knowlege had been accumulated for fast reactors and it was not necessary to go to the expense of obtaining the large quantities of materials that would be necessary. The superphenix(SPX) core of about 1200MWe did not have a full-scale mock-up but special experiments were done in Masurca. When problems were uncovered in calculation for SPX flux distributions, the results of critical experiments in ZPPR-13A -which had a similar eigenvalue separation, were used as independent verification of these measurements (1).

Table 1. Some Power Reactors and Experiments

Reactor	Mock-up	Size (MWe)*
EBR-II	ZPR-3	25
JOYO*	FCA	100 MWt
FFTF*	ZPR-9	400 MWt
PFR (UK)	ZEBRA	250
PHENIX(FR)	MASURCA	230
MONJU(JP)	ZEBRA	280
CRBR(US)**	ZPPR	300
BN800(IPPE)***	BFS-2	800
DFBR(JP)	ZPPR	600
SPX ****	(MASURCA)	e a
	(Z-13)	1200

\* Approx size. JOYO and FFTF did not produce electricity.

<sup>\*\*</sup> Designed, experiments, some construction, never built

<sup>\*\*\*</sup> Experiments done. Construction?

<sup>\*\*\*\*</sup> Superphenix(SPX) was a homogeneous core. ZPPR-13 was not a mockup but was a smaller decoupled core with a similar sensitivity and was used to verify results from SPX. Ref. Salvatores NEA/NSC/DOC(93)

#### 2.1 The early adjusted data libraries for fast reactors

The early adjusted libraries were developed from experiments in the smaller cores of ZEBRA and MASURCA using the generalised least squares fitting (GLS) method that we are currently employing for DFBR.

However, I would now consider that the uncertainties in the data were not sufficiently-well treated. This is easy to say in hindsight but it took much work and experience with experiments on the large core before this became sufficiently-well known.

In 1978. Argonne hosted a meeting on the benchmark calulations for a large LMFBR reactor (2) of 1200MWe size. This demonstrated several things:

\*\*\* The radial power distributions calculated with the adjusted libraries were remarkably different to those calculated with the evaluated libraries (ENDF/B-IV, for example) but the results with FGL and CARNAVAL were not consistent between themselves.

This meant that the fluxes and adjoint fluxes obtained with these data showed little evidence, if any, of improvement with two libraries that each been adjusted to fit the available experiments.

Some other points arising out of this exercise were

- (1) Solutions using the same basic data library were not consistent. This highlights the importance of the data processing codes which produced the multigroup libraies.
- (2) A simple sensitivity study in one energy group showed that experimental results from ZPPR-9 (an intermediate-size core) could account for 70% or more of the difference in power shapes found in the 1200MWe benchmark.
- (3) The adjusted libraries emphasised the importance of the U238 capture cross section which highlighted the long-standing discrepancy that ANL had had between integral measurements and calculations with ENDF/B cross sections.

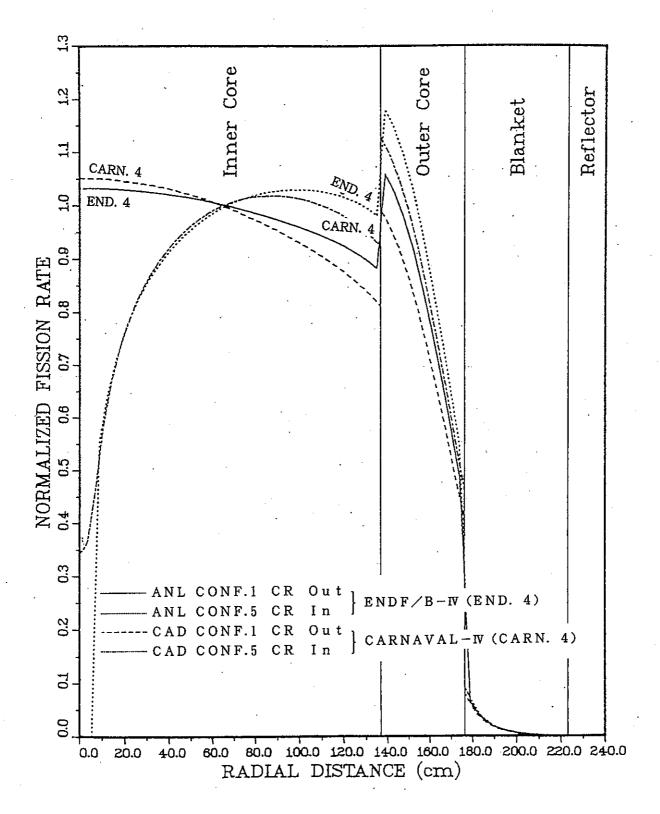
The results of this benchmark destroyed the credibility of the use of adjusted data at Argonne. The method of using least squares fitting to get good agreement with measurement was regarded as mere "data diddling" with no scientific basis. However, the method became respectable with Poenitz's introduction of the GMA code for simultaneous evaluation of basic nuclear data measurements. This method removed the arbitrariness that had occured hitherto in nuclear data evaluations for reactors.

FIGURE 1

P2

RADIAL FISSION RATE DISTRIBUTION

FOR ANL AND CADARACHE



Adjusted Data Libraries were constructed around 1970 in the United Kingdom for PFR and in France for Phenix:

UK (PFR)

FGL4/5

FRANCE (Phenix)

CARNAVAL -III/IV

These performed badly for spatially-varying parameters in the 1200MWe LMFBR benchmark set up by ANL in 1978 Ref 2. An example of the fission distributions with a central control rod inserted is shown in Figure 1. This shows a comparison between CARNAVAL-4 data and ENDF/B data. The reactivity worths of a central control rod are compared below:

Relative to ENDF/B-4

FGL5

+9%

CARNAVAL-IV

+14%

The important point is that measurements in small cores show predictions accurate to about 5% with ENDF/B, and with FGL5 and with CARNAVAL-IV.

The 1200MWe core showed large differences in power shapes whereas measurements in small cores showed agreement with experiment to within 1% which was within the <u>relative</u> experimental uncertainty.

\*\* The UK and French libraries were adjusted with data from small cores but do not agree between themselves in the large core.

It would seem that the covariances (Uncertainties) in the libraries were not thoroughly-enough assessed. These would have been produced during the production of the libraries but no estimate of the uncertainty was provided with the benchmark solutions.

The reason for the poor performance of the early libraries is important for us in the work for a DFBR library. It would seem that:

- (i) They were developed only from experiments in smaller cores
- (ii) Not enough attention was given to uncertainties in C and in E!
- (iii) The effects of decoupling in large LMFBR's was not known.

#### 2.2 Large LMFBRs

- \* These cores contain many critical masses, for example,
  - -- FLATTOP-Pu

6 kg Pu

-- JUPITER

2500 kg Pu to 4000 kg Fissile

Of course, the fuel enrichments must be reduced and diluents such as steel and sodium must be used in the power- producing fast reactor. To maintain criticality, this means that

\* Core size becomes large compared with neutron mean free path

Core Radius range 100cm to 200cm

Diffusion length for fast neutrons around 20cm

The result is that

\* Regions of core are "loosely coupled"

That is, there is a high probability of neutron absorption before producing fission in another region. As a consequence

- \* The neutron balance becomes sensitive to ratio of leakage/absorption between regions, which means that
- \* The core has high sensitivity to reactivity perturbations

As an example, for the measurements in the Jupiter series:

- -- a 1mm midplane gap over the 4000 mm diameter of the assembly, produced measurable changes in fission rates (percents) and changes in worths of about twice that in reaction rates (delta-sigma flux times adjoint flux)
  - -- small local Pu mass changes of order1% give change in worths of symmetric CRs of 5 %
- \* Point kinetics is not accurate
  - -- space-dependent kinetics must be used
- \* Coupling of regions is decreased markedly by internal blankets and CRs

For the DFBR, the sensitivities lead both to both useful features in the use of experiments for the large LMFBRs and to difficulties which must be taken account in the design of the power reactor. I have attempted to summarise both aspects in Table 2.

The useful feature is that we have the Jupiter measurements available. The "decoupling" is enhanced in the heterogeneous cores although these may not be representative of a conventional design. A measure of the sensitivity to flux "tilts" is given by the eigenvalue separation\* as has been shown by several authors, Wade and Rydin (3) and, following the Jupiter experiments, by Hashimoto (4). This is useful for experiments. Because of the decoupling, these can be made in with much less fuel than in the full-scale power reactor.

<sup>\*</sup> Assuming that an expansion of the actual reactor flux can be made as a sum over the harmonic modes, the eigenvalue separation k(0) - k(i) is the difference between the highest eigenvalue and the eigenvalue of the harmonic i. See references 3 and 4.

#### Table 2 Experiments with large LMFBRs

#### The good news

- --- Experiments for DFBR are available to PNC in the USDOE/PNC JUPITER Collaboration
- --- Sensitivity of experiments increased by decoupling an advantage in nuclear data testing
- --- Representative experiments possible with less fuel
   A "radially-heterogeneous" core can have the sensitivity of a much larger "conventional-type" core
- --- Nuclear data errors excite the "higher harmonics" in calculation and produce bad agreement with experiment
- --- A measure of the sensitivity is given by the lowest eigenvalue separation
  - K.Shirakata Physor 96, Hashimoto, Wade and Rydin

#### The bad news

- --- A large-size LMFBR will be sensitive to certain reactivity perturbations
- --- Operation of a fast power reactor will require taking into account the higher-mode eigenfunctions
- --- This feature must be taken into account in design K.Shirakata Physor 96
- --- Results from Superphenix - Nuc.Sci.Eng. <u>106</u> 1990

The very useful feature for our work in construction of an adjusted library for the larger-size cores is that:

- \* Errors in nuclear data excite the higher harmonics in a calculation
- \* whereas the critical core is in the fundamental mode. The result is
- \* discrepancies between measurement and calculation.

This means that errors may be seen in spatially-dependent parameters such as reaction rate distributions and reactivity worths can provide useful independent tests of the nuclear data. These may be particularly effective because

- (i) The reaction rate distribution and rhe reactivity distribution depend only on the relative uncertainties and not the absolute calibration which may be more uncertain.
- (ii) Errors in reactivity predictions can be about twice as large as those in reaction rates because the adjoint flux distribution can have an error similar to that of the real flux distribution. Thus, errors in the relative reactivities can be more accurate than in reaction rates.
- (iii) If one adjusts a library so that it will give a good account of reactivity distributions, then it will also give good predictions for reaction rates.

Another very useful feature, shown in Table 2 is that:

- \* A Useful Measure of the "sensitivity" of a given core to a
- \* perturbation (which may be caused by nuclear data errors )
- \* is the "eigenvalue seperation".

This means that experiments in a "radially-heterogeneous" core which generally is more-decoupled and hence more-sensitive can be valuable be extremely useful for the data fitting work although the core may appear very different in appearance to the DFBR.

An example of this is ZPPR-13A. I was ordered to provide a benchmark for calculating the control rod worths in ZPPR-13A to verify the problems that had been found in the prediction of reaction rate distributions in SUPERPHENIX (Ref 5)\*.

<sup>\*</sup> The benchmark provided a simple model for calculation with complete corrections for cell-asymmetry and a full uncertainty analysis of the experiments. Unfortunately, it does not appear to have been distributed by the NEACRP as intended but was used by CEA.

The first radial harmonic eigenvalue separations for the two cores are as follows:

--- ZPPR-13A 8% Reference 5
--- SUPERPHENIX 7% Reference 6

\* It should be noted that the first harmonic (the one with the smallest eigenvalue separation) is actually an azimuthal harmonic but that this <u>eliminated</u> in these cores by <u>symmetry</u>. Because of this symmetry, the discrepancies found in predictions of fission rates and conrol worths are most sensitive to the second harmonic, which is a radial harmonic.

However, for some experiments in the "radially-heterogeneous cores ZPPR-15 and ZPPR-19 of the JUPITER series, the azimuthal harmonic is dominant and these can lead to very useful tests of the nuclear data in the "most-sensitive" sinution.

The results above mean that as far as nuclear data are concerned, the prediction of conrol rod worths in ZPPR-13A contains a wealth of information about predictions for SPX reaction rates although the first is a radially-heterogeneous core and the second is a larger conventional core..

#### Chapter 3. THE DATA ADJUSTMENT METHOD

I have found that many reactor engineers are "turned-off" by the least-squares-fitting/data-adjustment approach because the papers in the literature plunge straight in on the full mathematical treatment, which necessarily involves much matrix algebra. But, I consider it necessary at the present stage of development in order to take full advantage of the many experimental results that have been obtained over the past fifty years. The method is basically very simple, as I have attempted to show in figure 2. The complete treatment is given in many references. Figure 3 shows the matrix formulation as used in the GMADJ code which is described in reference 7.

Basically, there are just a few simple points which are discussed below:

- (1) A given cross section library, such as JENDL3.2, gives a unique result for any calculated reactor parameter. Any uncertainty due to modelling in the codes such as mesh/transport corrections is trivial in principal but provides food at the trough for many reactor engineers.
- (2) The real problem, at this time, is the lack of knowledge of the nuclear parameters (by "parameters" we mean cross sections, fission yields, delayed neutron yields etc.) particularly for much data in the fast-reactor range of KeVs to MeVs. Although many improvements in measurements and in theory have been achieved, it cannot be foreseen at the present that sufficiently-accurate data required for fast reactors can be achieved in many cases.
- (3) The most promising solution, at present, is the combination of differential and integral data using the generalised least squares (GLS) fitting procedure. Each has its own strengths the differential measurements provide data over a limited range the integral measurements give data of high accuracy (in many cases) but each item is limited to the spectrum in which it is measured. The synthesis, of what were originally indepent specialities, is achieved by generation of the nuclear data sensitivities. The combined set of data is now "overdetermined" and minimisation of the chi-square value by the method of GLS, or its equivalent as in our ABLE code, provides an unbiassed estimator of minimum variance for the set of data which are included.

#### Figure 2



#### Basis of Least Squares Fitting Method Applied to Integral Data

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- Evaluated cross section library gives a unique result for any parameter
- An integral measurement provides additional information System is overdetermined Improved data by least squares fitting

Minimize

$$\mathbf{M} = \Sigma \left( \frac{\mathbf{\sigma} - \mathbf{\sigma}'}{\Delta \mathbf{\sigma}} \right)^{2} + \Sigma \left( \frac{\mathbf{C}' - \mathbf{E}}{\Delta \mathbf{C} \mathbf{E}} \right)^{2}$$

Calculated parameters C related to cross sections  $\sigma_{j}$  by sensitivity coefficient

$$S_{ij} = \frac{\sigma_j}{C_i} \frac{\partial C_i}{\partial \sigma_j} \qquad \Delta C_i = S_{ij} \Delta \sigma_j$$

• Generalize to include covariances of cross sectios covariances of integral parameters

6/13/90

- Uncertainties in integral measurements are much smaller than in cross section measurements
  - Fitting is largely controlled by integral (C-E/ $\Delta$ CE)
- Information from fit is contained in adjusted covariance matrix
  - Uncertainties in calculations are reduced
  - Usually little improvement in cross section data



## Rigorous Uncertainty Evaluation and Improvements on Parameters and Derived Quantities

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$$C_Q = S C_p S^T$$

$$S_{ij} = \frac{\mathbf{p}_i}{\mathbf{Q}_i} \frac{\partial \mathbf{Q}_i}{\partial \mathbf{p}_i}$$

$$\delta = C_p S^T W^{-1} E$$

$$W = C_Q + C_E$$

$$Q_i = Q(1 + \sum_i S_{ij} \delta_i)$$

$$C_p' = C_p - C_p S^T W^{-1} S C_p$$

$$C'_Q = SC'_pS^T$$

- (4) The integral data control the minimisation process because of their lower uncertainties. However, the much improved prediction of an integral value is achieved by correlation of of the cross sections as permitted by their covariances. See, for example, Ishikawa in reference 8 where it is shown that the improvements are obtained by the net result ov both positive and negative changes in cross sections consistent with their uncertainties.
- (5) The new cross sections become tightly correlated and give improved calculations for a target core if it is in "range".
- (6) Individual parameters are only a little improved in the process unless there is a major sensitivity to a particular item. But, they must be improved to some extent, however minor it may be, because of the additional information that comes from any measurement with low uncertainty

Typical improvements in accuracy that have been achieved by the GLS fitting are:

Critical mass
Reaction Rate Ratios

Reaction Rate Distributions

factor of 10 factor of 3

within experimental unceretainty

#### Chapter 4. OMISSIONS FROM THE PRESENT DATABASE

Our library which is being constructed for DFBR, is based mainly on the Jupiter experiments which include experiments for the correct size and type of core. It is accepted that getting good agreement with experiment is not sufficient on its own (Ishikawa in reference 8). Therefore the experimental database is being extended to include indpendent measurements. This is a powerful way of guarding against "systematic errors". Presently, data from experiments in the MOZART experiments in ZEBRA, the MASURCA experiments and our own FCA are being introduced. In addition, the measurements in power reactors such as JOYO, Monju and Superphenix are being added. The power reactor data are necessary for proof-testing because they come fro the "real thing" but it should be recognised that they will have little weight in the fitting process because it is rarely practicable to obtain specifications at the same level of accuracy as in critical experiments..

From our work with GMADJ I see several glaring omissions which are discussed below. Those listed are available to PNC:

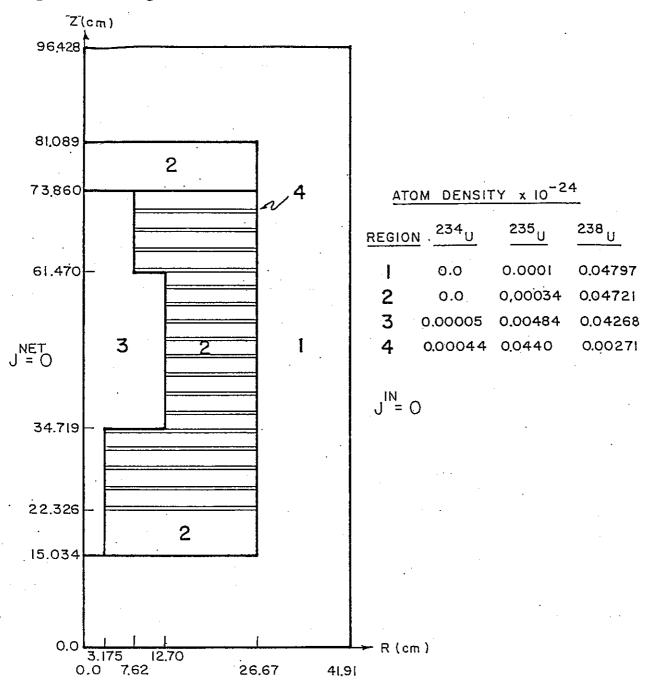
#### (i) LANL BIG-10

This is an important core for our work on a DFBR Library for many reasons. The core is described in reference 17.

- (a) It had a softer spectrum than the other LANL cores which is more like that in an LMFBR The central zone is made of just homogeneous uranium of 10% enrichment.
- (b) It helped us to find the solution to the longstanding "central worth discrepancy", while others just bypassed it by normalising woths to the fuel worth or choosing values for the delayed neutron fraction that gave better results.
- (c) The U238 capture ratio was here measured by several laboratories in the US in a homogeneous region, so compare both with measurements in the plate or pin cells and with different techniques.
- (d) Many measurements are available for other reaction rates, for the HAFM (helium accumulation fluence monitor), for worths and for beta-effective.

A detailed calculation model for BIG-10 is given in figure 4.

Figure 4 Big-10



BIG - 10 DETAILED RZ MODEL

Fig. 1

#### (ii) ZPPR-12

This core was build at the request of Westinghouse Advanced Energy Systems Division (WAESD) to test calculation methods that had been used for the CRBRP. It was a small "leaky" core and we were able to make several useful measurements - such as

- (a) Criticality of the cores loaded with sodium and cores completely voided of sodium
- (b) Reaction rates, covering the complete core region were made in each case
- (c) Comparison of the plate-loaded and pin-loaded cores similar to the experiments made in CADENZA and Winfrith
- (d) Comparison of cores with a Pu/U oxide blanket and with a U metal blanket

#### (iii) ZPPR-15C

- --- Correlation between zone-reactivity worths and reaction rate"tilts" with different zones such as
- (a) replacement of the 11% Pu240 fuel by 20% Pu240 fuel
- (b) substitution of normal-sodium cells with voided cells
- (c) substitution of Pu fuel by U235 fuel
- (d) substitution of "plate-fuel" by "pin-fuel"

Measurements of these effects are available elsewhere but the important thing is that the measurement of reaction rate "tilts" provides data that are independent of the reactivity measurement (although there are correlations due to the fuel )

--- Use of reaction rates and control rod worths at the x-axis and at the y-axis which are sensitive to the azimuthal harmonic to improve the adjusted data

#### (iii) ZPPR-19

--- Difference between reaction rates and worths along the x-axis and along the y-axis to compare the difference in predictions with U235 data and with Pu239 data

More importantly, because of their independence to ZPPR, I would like to have the detailed isotopic compositions and experimental uncertainties for

#### (iv) MASURCA

(v) The SPX benchmark

Without this information the data are completely useless.

#### 4.1 Use of chi-square

We have arrived at agreement that the value of chi-square provides a vital test of the data file before the adjustment. The chi-square distribution is the most common test for a normal distribution. In our case, we must allow for "systematic" errors in the measurements and compute the "generalised" chi-square. If the covariances are correctly assigned, the value of chi-square from the tables should show consistency of the hypothesis that the measurements follow a "normal" distribution. That is, the measurement results are randomly ditributed. If the chi-square is out of range, then it indicates that something is not correct.

If n measurements, x(i) are independently normally distributed with a common expected value, m, and a common standard deviation, s, then

$$(1/s^{**}2)$$
 Sum  $(x(i) - m)$  with sum from 1 to n

has a chi-square distribution with n - 1 degrees of freedom (14).

For 60 degrees of freedom, for example, the table below gives the probability that the measurements are randomly distributed as

In our case we have some 84 degrees of freedom. If then, the reduced chi-square is 0.5, there is less than a 1% probability that the measurements are normally distributed. The probability is less than 1% when chi-square/n is greater than 1.47.

Either of these situations indicate that something may be wrong. Of course, this could be in the integral measurement, the differential data file, the sensitivities or in the covariances.

In GMADJ we have found the chi-square test extremely useful in detecting errors in measurements or calculations. (Ref 7)

#### 4.2 Integral Data Correlations

In the GMADJ code, we found the integral data correlations to be very useful in showing the effectiveness of a given measurement by its correlation with other independent measurements (15). The ABLE code produces the integral data correlations, but only after the least squares fitting.

\* I regard these as not much use because the integral data itself has already been used in the fitting process. The correlations in the data induced by the least squares fitting process, itself, ensure that the integral data will be tightly correlated provided that any discrepant measurements have been identified and either corrected or rejected on the basis of the chi-square test.

GMADJ also provides the correlations after the fitting. But the major difference is that some items of integral data have already been excluded on the basis of unsatisfactory values for chi-square (7).

## Chapter 5. DATA FOR THE LANL CRITICALS AND RESULTS WITH JENDL-3.2 AND ADJUSTED DATA

The critical experiments from Los Alamos National Laboratory have been a foundation stone for testing nuclear data since the earliest days of reactors (9). The have been benchmarks for testing all versions of the ENDF/B files. The latest data are available in Japan through participation in the Cross Section Evaluation Working Group (CSEWG). The most recent specifications Ref 10, are shown in appendix A.

The data are important for all types of nuclear reactor because neutrons from fission are created at the high, MeV, energies. For fast reactors the primary mechanism for moderation of neutrons is inelatic scattering with some effects from (n,2n) reactions and competition with neutron capture. The bare cores of Godiva and Jezebel test these data for U235 and for Pu239. The uranium-reflected corses, the "Flattops", in conjunction with these, provide important tests of U238 inelastic scattering data and fission important because most reactors contain large quantities of U238. The assembles make good approximations to bare spheres and reflected spheres with only relatively-small corrections. As such, they are easilily calculated by high-order transport codes or by Monte Carlo codes. To summarise, they provide:

- (a) simple spheres of homogeneous composition.
- (b) they are easily calculated
- (c) they contain only few isotopes
- (d) a test nuclear data at high energies
- (e) definition of the fast reactor spectrum
- (f) high accuracy data
- (g) limit the "adjustments" in LMFBR range
- (h) important for calculating high-energy spectra ("threshold" reactions, actinide burning)

<u>Jezebel</u>	Flattop-Pu	<u>Jezebel-Pu</u>
95% Pu239	U238 reflector	20% Pu240
17 kg Pu	6 kg Pu	19.5 kg Pu
Godiva 94% U235 52. kg U	Flattop-25 U238 reflector 18 kg U	

#### 5.1 Experimental uncertainties

It is important to realise that these cores were used for establishment of the critical masses and other purposes, but not for the data adjustment work. The criticality folks do have such stringent requirements and need to validate their codes and data in a complex arrangement. Generous uncertainties are usually taken. The data given in reference 10 are somewhat confusing with their uncertainty evaluation, Therefore, I have gone back to reference 11 for the uncertainty information. These are the data used in GMADJ which follow from discussions with Hansen. Insufficient information is given for construction of a full covariance file. However, as will be seen, the uncertainties in knowledge of the material masses dominates the total uncertainty by far. Rather than guess at the correlations, I have preferred to admit them altogether. However some obious correlations are noted.

I find some confusion in the uncertainties which apply to the results from the LANL cores:

- (1) Their main interest is in the prediction of critical mass rather than of k-eff. Therefore, uncertainties are given for critical mass. It is interesting that in reference 11,page 7 states that, for Jezebel, the "precision" of 0.2% in reactivity worth is equivalent to a precision of 0.4% in critical mass. Thus the relation of a factor of two that we used in the Jupiter experiments (derived easily by differentiation of the one-group neutron balance) would seem to apply to the small cores from LANL. This could be checked easily by making percent variations in spherical transport calculations.
- (2) The uncertainties for Los Alamos data are quoted as "probable errors" And quote them as +/-. This means that, using h as a measure of confidence, then 50% of all sampled events ought to produce a value for a parameter which lie between h units of the mean value (below or above). See, for example reference 12 page 64. In terms of the standard deviation s, The confidence that any single measurement x will produce a value for the mean m in the range

m-s < x < m+sIs approximately 54.74 % -- See ref 12 page 65.

Therefore, because of the approximate nature of the critical mass uncertainties quoted I take the one standard deviation to be sufficiently close to the probable error and the uncertainty in k-eff to be one-half of that in the critical mass. More exact evaluation would require more details from LANL but I use the uncertainties that we devoped for GMADJ which followed after discussions with George Hansen.

The JEZEBEL core is the most important for our work with the LMFBR library. Two versions of the idealised spherical assembly, with 4.5% Pu240 and another with 20% Pu240 (JEZEBEL-Pu) are available to us. I show in figure 5, a drawing of the JEZEBEL assembly. It will be apparent from the figure that, although the masses were measured very accurately, corrections must be applied for asphericity, the supporting structure, imprecise fiting-together of the parts and effects of reflection from the cell (or kiva as it was known). Effects, not obvious from the figure are due to homogenisation, internal cavities, trace impurities, and temperature. These are discussed in reference 11. As an example, from this reference, I show a list of corrections made to JEZEBEL and JEZEBEL-Pu in Table 3.

Table 3. Corrections to Idealised Spheres

		,
	JEZEBEL	_JEZEBEL-Pu
Critical mass ,kg a	16.784	19.173
Density, g/cu.cm	15.60	15.73
Corrections ,kg:		•
Asphericity	-0.047	-0.063
Homog. and internal Ni	0.033	0.062
Equatorial band	0.045 b	0.058
Polar supports	0.117	0.145
External Ni	0.074	0.070
Framework	0.002	0.002
Kiva reflection	0.010	0.012
Air reflection	0.004	0.005
Impurities c	-0.001	-0.001
Temperature	-0.007	-0.007
Critical mass ,kg	7.02	19.46
Density, g/cu.cm	15.61	15.73

a major cavities removed

b includes correction to density 15.61 g/cu.cm

c Pu impurities 170 ppm C230 ppm 0.0115 ppm Fe

Figure 5

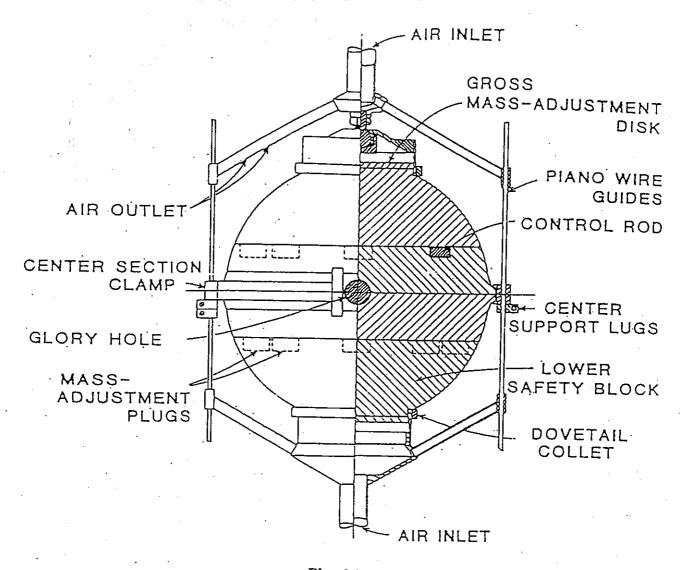


Fig. 14.

Jezebel under operating conditions. The nearly spherical assembly is supported by lightweight clamps and guides.

Radius 6.3cm

Mass 17kg Pu

LA-9685-H Unclassified

The random uncertainties for the LANL cores from knowledge of the fuel masses are dominant and represent about 99%. Clearly, some correlated components could be identified but more knowledge of the details would be needed. For example, it is possible that the FLATTOP cores had exactly the same fuel as their bare-core counterparts. This is not known to me at present I propose to take the total uncertainty as random as we do in GMADJ rather that make guesses about correlations. This is permissible because the experimental uncertainties are much lower than the uncertainties due to nuclear data.

It is also worth noing that the results from Los Alamos were independently confirmed in criticality experiments in ZPPR-21 (Reference 13.

I thus recommend that the uncertainties for the k-effs of these cores be taken as random with the following values:

1.	JEZEBEL	1 SD	0.0018	uncorrelated
2.	FLATTOP-Pu		0.0014	***
3.	JEZEBEL-Pu		0.0018	Ħ
4.	GODIVA		0.0010	tt
5.	FLATTOP-25		0.0010	п

The data for the LANL cores have been succintly summarised by N.Shirai. I reproduce his data here as Figure and 7. It is important to note that in this table I have corrected the density of U235 in the core of BIGTEN. In reference 10, this is mistyped as 0.0484 atoms/barn-cm where as it was actualy 0.00484 atoms/barn-cm. The corrected value comes from my original evaluation and makes perfect sense because it gives an enrichment of 10%.

The subject of BIGTEN is discussed in section4.

27

GodivaExpData.xls

Figure 7

Applicability Check Of Monte Caro Gode MVP to Criticality Evaluation Of Dry Reprocessing Plant
( Data from Paper of Dr. P.J.Collins ) IRG

Ýbbuc	ability Check	Of Motifie Os	210 0000 14141	to orresouncy	( Data from Pap	er of Dr. P.J.C	ollins)	IRG N. Shirai
Name of Facility	Godiva	Jezebel	Jezebeł-23	Jezebel-Pu	Flattop-25	Flattop-Pu	Flattop-23	Bigten
Driver Fuel Fissile %	U−235 93.8 a/o	Pu 95.5 a/o Ga stabilized	U−233 98.1 a/o	20.1 a/o Pu-240 Pu 79.5 a/o Ga stabilized	U-235 93.3 a/o	Pu 95.2 a∕o Ga stabilized	U−233 98.2 a/o	interleaved plate EU and DU eff. U-235 10.2%
Geometry	bare sphere	bare sphere	bare sphere	bare sphere	refl. sphere	refl. sphere	refl. sphere	~cyl.⇒equ.sph.
Core Radius Reflector Radius Reflector Saving	8.741 cm - -	6.385 cm - -	5.983 cm - -	6.860 om - 	6.116 cm 24.13 cm 2.6 cm	4.533 cm 24.13 cm 1.9 cm	(0.218 cm gap) 4.610 cm 24.13 cm 1.4 cm	( equ. radius ) 30.48 cm 45.72 cm not available
Critical Mass	49.1 Kg(25)	16.8 Kg(Pu)	16.2 Kg(23)	19.3 Kg(Pu)	16.8 Kg(25)	6.0 Kg(Pu)	7.4 Kg(23)	
Number Density Core N(23) N(24) N(25) N(28) N(49) N(40) N(41) N(42)	0.000492+24 0.04500 +24 0.002498+24	0.037050+24 0.001751+24 0.000117+24	0.04671+24 0.00059+24 0.00001+24 0.00029+24	0.029946+24 0.007887+24 0.001203+24 0.000145+24	0.00049+24 0.04449+24 0.00270+24	0.03674+24 0.00186+24 0.00012+24	0.04871+24 0.00059+24 0.00001+24 0.00028+24	0.00005+24 0.00484+24 0.04268+24
N(Ga) Ref. N(25)		0.001375+24		0.001372+24	0.00034+24 0.04774+24	0.00138+24 0.00034+24 0.04774+24	0.00034+24 0.04774+24	0.00010+24 0.04797+24
N(28)  Keff: C/E  Stanndard Dev.)	1.0027	0.9967	1.0131	1.0015	:	0.9923		

#### 5.2 Initial tests with the JENDL-3.2.data

The small cores have a very-high fraction of leakage/absorption so must be calculated with SnPn codes or by Monte Carlo codes. at ANL, our data processing method did not treat the anisotropy in inelastic scattering which was important for these cores. For that reason the GMADJ data were generated from calculations with the VIM Monte Carlo code. (Data processed through the NJOY route and used in the TWODANT code would also have been adequate )

My values were calculated by N.Higano using JENDL-3.2 data with the continuous-energy Monte-Carlo code MVP with the data of Table 4.

First I show a comparison of results for the LANL cores between JENDL-3.2 data, ENDF/B5.2 data and JEFF-2 data. All three of these cases used the "continuous-energy" Monte Carlo codes but which version of MCNP was used with JEFF-2 is not known to me..

In assessing the performance of the evaluated cross section libraries, I choose to use what I call the 'discrepancy", which to me is the

Difference between measurement and calculation Relative to the combined uncertainties in C and E

This is the quantity that is minimised in GMADJ. However, in the ABLE code, the covariances of the basic cross section data are also included in the calculational uncertainties.

Table 4. Discrepancies with Three Evaluated Libraries

Core	JENDL-3.2	ENDF/B5.2	JEFF-2.2
Jezebel	-1.46	-0.8	4.6
Flattop-Pu	- <b>5.22</b>	<b>4.0</b>	7.3
Jezebel-Pu	0.74	-0.1	2.3
Godiva	2.50	-2.3	5.1
Flattop-25	-1.49	2.5	7.0
Code	MVP	VIM	MCNP

Experimental uncertainties 0.1% to 0.2%. See results above

Note that all three groups used what is "conceptually" the best calculation possible for these small cores. In contrast, LANL were more interested in the performance of the high-order SnPn methods and have reported their results in Appendix A. At PNC, our reference method for DFBR is diffusion theory but transport corrections are applied where necessary.

#### 5.3 Results of the GLS Fit for LANL cores

A basic problem of including the LANL cores in our data-fitting work is that sensitivities from spherical transport are required, whereas we do not have this option. However Sato san (Ref 16) calculated with our SAGEP code in rz geometry but with cross sections produced according to our "equivalent P-3 prescription. Futhermore, for JEZEBEL, he made a direct check of the cross sections adjusted with these sensitivities combining results from rz transport/diffusion with spherical diffusion. The result was that the sensitivities agreed to about 10% whith the corrected calculation. Therefore all the sensitivities were generated in this way for the LANL cores.

I found this result truly remarkable while corections of -5% were found for cylindrical geometry to sherical geometry by diffusion theory and corrections of -8% to -9% were found for diffusion to transport in 70 groups and in 8 groups. It is a fact that our methods produced agreement with MVP to 5% for JEZEBEL, but there is a concern that this may not be generally true.

Table 5. Uncertainty with the adjusted library

#### SD = standard deviation

	PNC/ABLE		ANL/GMADJ
Core	Initial SD	LSFIT SD	LSFIT SD
Jezebel	1.53	0.16	0.23
Flattop-Pu	2.53	0.14	0.17
Jezebel-Pu	1.37	0.15	0.22
Godiva	1.49	0.11	0.14
Flattop-25	2.26	0.10	0.14
Data used			
Number	8	9	250
Cores	Jup	iter	A few Jupiter
	LÂNL		LANL
	Being	extended	NBS
	MOZART,MONJU		ZEBRA8,SCHERZO
	FCAJOYO		ANLE, ANLW
1, v	Mas	surca	Metal Fuel IFR
			Space reactors

Effect of LANL Data - Very small improvement for DFBR
Criticality
Reaction Rate Ratios
Reaction Rate Distributions
Control Rod Worth
Sodium Void Worth

Major improvement in MeV fluxes

Change in U238(,n.f)/Pu239(n,f) contribution by isotope/reaction to threshold fission rate

Table 6. The largest percent contributions to the LS Fit

Parameter	Jupiter alone	J + LANL	Difference
Pu239 chi	14.8	6.6	-8.2
O16 mu	1.0	2.2	+1.2
Na(n,n')	44.9	46.8	+2.1
Fe(n,n')	-34.2	-19.7	+14.5
Na mu	0.6	1.3	+0.7
U238(n,g)	-26.8	-20.3	+6.5
U238(n,f)	-119.3	-84.0	+35.3
U238(n,n')	237.2	185.0	-52.2
U238 mu	2.7	-1.7	-3.0
Pu239(n,g)	51.3	50.5	-0.8
Pu239(n,f)	-68.8	-59.3	9.5

Total Error*	-1.5 %	-1.1 %	0.4%
Uncertainty	2.08%	2.06%	

Uncertainty before LS fit 6.06%

Errors in calculation of F8/F9 - Ratios of Calc./Exp. for ZPPR-9, which is representative of the DFBR design are as follows:

JENDL 3.2 0.970

LS Fit Jupiter alone 0.985

LS Fit Jupiter plus LANL 0.989

These are due to large changes in cross sections which are both positive and negative. The total improvement due to LANL data is small. But,

\*\* The LANL data is sensitive to Mev fluxes while Jupiter is not hence the improvement in uncertainties of these fluxes is important.

<sup>\*</sup> Total includes many other parameters having

a smaller effect

#### Chapter 6 CONCLUSIONS

The inclusion of the LANL data in addition to that from JUPITER has a very small total effect for a core similar to that of the DFBR design (ZPPR-9). However the LANL data place important constraints on the data adjustments at high energies. Exactly where the fluxes are low from JUPITER. Therefore, the LANL data make an important addition to our library. The changes in the data are very large for some isotopes/reactions while the differences are both positive and negative, giving a similar improvement in the total. I claim that the LANL data are an important addition because they are sensitive to the high-energy data while the JUPITER data are not.

#### Proposals for the future

- (1) Use LANL Reaction Rate Ratio data
- (2) Use LANLs Big-10 for several reasons:
  - Softer spectrum
  - -- Simple homogeneous central region
  - -- Many reaction rate measurements
  - -- Sample worths and beta-eff
  - -- Solution to Jupiter "central worth discrepancy"
  - -- Mass spectrometry (HAFMs) for boron capture
- (3) Scherzo-556
- -- It is very simple, requiring only definition of uranium densities for the infinite homogeneous medium.
  - -- it uses independent measurements from UK, France, FRG
- (4) ZPPR-12
  - -- Supplied to PNC under Jupiter II
  - -- Plate core, Na-voided core, pin zones
  - -- Heterogeneity calculation -- ZEBRA CADENZA
- (5) Get spectrum uncertainties for DFBR
  - -- Actinide burning accuracy Am241, Am243
  - -- Steel dpa
  - -- Steel activation
    - -- Improvements on 'spectrum-unfolding" codes
- (6) Uncertainties for parameters which are difficult to measure
- (7) Continual testing against independent measurements
  -- Get definitive uncertainties for
  ZEBRA, FCA, MASURCA
  JOYO, MONJU

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