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Analysis of beam envelope by transverse space charge effect

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Analysis of beam envelope by transverse space charge effect

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Abstract

It is important for high current accelerators to estimate the contribution of the space charge effect to keep the beam off its beak up. The application of an envelope equation is examined in previous report in which the beam is just coasting beam(non accelerating). The analysis of space charge effect is necessary for the comparison in coming accelerator test in PNC.

In order to evaluate the beam behavior in high current, the beam dynamics and beam parameters which are input to the equation for the evaluation are developed and make it ready to estimate the beam transverse dynamics by the space charge. The estimate needs to have enough accuracy for advanced code calculation.

After the preparation of the analytic expression of transverse motion, the non-linear differential equation of beam dynamics is solved by a numerical method on a personal computer. The beam envelope from the equation is estimated by means of the beam emittance, current and energy.

The result from the analysis shows that the transverse beam broadening is scaresely small around the beam current value of PNC design. The contribution to the beam broadening of PNC linac comes from its beam emittance. The beam broadening in 100 MeV case is almost negligible in the view of transverse space charge effect.

Therefore, the electron beam is stable up to 10 A order in PNC linac design. Of course, the problem for RF supply is out of consideration here. It is important to estimate other longitudinal effect such as beam bunch effect which is lasting unevaluated.

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縦方向空間荷電効果によるビーム包絡線解析

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要旨

ビーム発散を防ぐため、大電流加速器では空間荷電効果を見積もることが重要である。前回のレポートでは、ビームの膨れ評価に非加速のビーム(コースティングビーム)包絡線方程式を用いた。これに替わって、より一般的な加速系での空間荷電効果を含むビームの評価は今後予定される大電流CW電子線形加速器の試験で重要である。

そのため、解析の基本式である包絡線方程式の加速ビームへの一般化、パラメータの定義を行う。これによる評価は、本格的なコード計算の準備や解の予想を十分な精度で確保できるものとする。

得られた非線形の放絡線方程式を、精度が確保できるソフトウエアを用いてパーソナルコンピュータ上で数値計算する。パラメータは、電子ビームエミッタンス、電流及びエネルギーである。

計算の結果により、ビームの横方向発散は、設計値では十分に小さいことが分った。ビームの拡がりは設計ビームエミッタンスの寄与が殆どである。100MeVでは、縦方向の空間荷電効果は無視できる。

よって、RF供給等の問題を別として、PNCの加速構造で10Aまで縦方向空間荷電効果は大きな問題にならない。今後は、残されているビームバンチ効果を評価することが重要である。

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1. Introduction

The one of the major development items for the accelerators used for the transmutation is to stabilize very high current beam in its acceleration [1-3]. The mishandling of the intense beam leads to the hazardous condition such as the contamination of the air in an accelerator room and the fatal destruction of accelerator structure itself at last. The stable supply of the beam from an irradiation system to target volume is a necessary condition as an industrial device. It is partly achieved as irradiation machines in the market, but its beam current and quality are just for low current operation.

It is important to calculate the beam dynamics when we analyze the high power current beam acceleration [4]. There are various ways to evaluate the beam characteristics. Some methods consist of calculation of the sets of the phase development of single particle like in PARMELA[5] code, while the others relates to collective dynamics like in TRANSPORT[6] code. Particles in a beam transport are essentially governed by the classical mechanics, special theory of relativity and quantum theory of radiation. Quick calculating the beam shape with finite beam emittance and current sometimes makes sense even when we execute simulation codes.

The purpose of this report is to analyze beam behavior by means of the envelope equation which includes the effect of not only a beam emittance but also a space charge effect. The beam envelope equation employed in this report is reconstructed so as to contain the beam acceleration dynamics unlike the usual state of the coasting beam. It is more important when the accelerator gradient is lower like PNC linac.

To begin with, the property of a relativistic particle is reviewed to understand the behavior in the phase space. The property of single particle and multi particle dynamics are succeedingly reviewed, which are all potentially checked and modified to be applicable for the evaluation in this report, after the explanation of the emittance and the beam envelope used for the actual evaluation. The beam envelope equation is finally established for the realistic dynamics in an accelerator structure. The results from numerical calculation of this nonlinear equation are discussed in the view of the beam current for the cases of the variation of the beam emittance. In appendix, beam loading characteristics in a linac is summarized in order to calculate the accelerator gradient of PNC linac.

2. Dynamics of relativistic particles

The equation of motion is derived from Lagrange equation which is originated by Hamilton's principle by means of Lagrangian. We here describe only some result from intricate covariant Lagrangian approach. The action integral for free particle is

$$S=\int_{t_1}^{t_2} L_{free}\,dt. \tag{1}$$

The Lorentz invariance demands to reformulate eq. (1) as follow:

$$S=\int_{\tau_1}^{\tau_2} \gamma L_{free}\,d\tau, \tag{2}$$

where a proper time τ is assumed simply increasing function. This means also that we can employ τ beside t. Actually, Lagrange equation here is as follows [10] :

$$\frac{d}{d\tau}\left(\frac{\partial\Lambda}{\partial p_\nu/m}-\frac{\partial\Lambda}{\partial q_\nu}\right)=0 \quad ,\Lambda=\gamma L. \tag{3}$$

The zero-th component of p_ν represents total energy divided by c. From eq. (2), taking into account to low velocity limit, the Lagrangian for free particle is

$$L=-\frac{1}{\gamma}mc^2=-mc^2\sqrt{1-\beta^2}, \tag{4}$$

where m is the rest mass of a particle. Then, we consider a particle in an external field. The mathematical form of the interaction Lagrangian is the scalar product of 4-momentum and 4-potential, because the non-relativistic limit means simple electrostatic interaction. That is

$$L_{int}=-\frac{e}{mc\gamma}p_\alpha A^\alpha, \tag{5}$$

where

$$p^\alpha=\left(\frac{E}{c},P\right), \tag{6}$$

$$A^\alpha=(\Phi,A). \tag{7}$$

We employ CGS unit system entire in this report. Equation (5) corresponds to the interaction

Lagrangian $-V$ because electromagnetic potential is independent of the velocity of a particle. Hence, total relativistic Lagrangian L in external field is

$$L=L_{free}+L_{int}=-\frac{1}{\gamma}[mc^2+\frac{e}{mc}p_\alpha A^\alpha]=-[mc^2\sqrt{1-\beta^2}+e\Phi-\frac{e}{c}u\cdot A], \tag{8}$$

where we remember the Lagrangian is originally a function of the coordinates, the velocity and time only. Now this means a relativistic Lagrangian is a function of 4-coordinate, 4-momentum and the proper time. It is notable that the Lagrangian accepts the addition constant and we are enough to care utmost quadrant order for a momentum vector because Euler-Lagrange equation originates from an action integral.

The Hamiltonian H (we here call H Hamiltonian even though working in classical mechanics) for relativistic particles is presented by Hamilton's principle from following:

$$H=\boldsymbol{P}\cdot u-L, \tag{9}$$

$$P^\alpha\equiv\frac{\partial L}{\partial u^\alpha}=p^\alpha+\frac{e}{c}A^\alpha, \tag{10}$$

where P is a canonical momentum beside P_0 component which means total energy of the particle. It is notable that P_0 and $\boldsymbol{P}$ is not independent. Total energy E is

$$E^2=p^2c^2+m^2c^4. \tag{11}$$

At last, total Hamiltonian H is

$$H=\sqrt{(c\boldsymbol{P}-e\boldsymbol{A})^2+m^2c^4}+e\Phi, \tag{12}$$

where $\boldsymbol{P}$ is a canonical 3-momentum.

The state of a particle is defined by its position q in generalized coordinate and momentum p in generalized momentum, which are implicitly related to canonical coordinate Q and momentum P. That is

$$q=f(Q,P,t), \tag{13}$$

$$p=g(Q,P,t). \tag{14}$$

From Hamilton's principle, a canonical transformation imposes on two Hamiltonian K and H that

$$\lambda(p_i\dot{q}_i-H)=P_i\dot{Q}_i-K+\frac{dF}{dt}. \tag{15}$$

where λ, F, H and K are a constant, an integrand which second derivative is continuous and Hamiltonians before and after the transformation, respectively. If F is independent of t, equation (15) leads a simple scale transformation

$$Q'=\mu q_i, \quad P'=\nu p_i. \tag{16}$$

With this relationship and Hamiltonian (12), we get following scalar canonical transformation:

$$\begin{aligned} Q' &= \frac{q}{\beta\gamma}, \\ P' &= p, \\ K &= mc^2, \end{aligned} \tag{17}$$

which means that a relativistic velocity normalized by $\beta\gamma$ is canonical transformation which transfers from an observer system to an on-mass system. This concerns a normalized beam emittance in a phase space.

3. Phase space

3.1 Poincare invariance in phase space

It is well known that two volume elements $d\zeta$ and $d\eta$ in two phase spaces which are each other connected by a canonical transformation reveals following relationship:

$$d\vec{\zeta}=\|M\|d\vec{\eta}, \tag{18}$$

where $\|M\|$ is Jacobian determinant of the transformation. If we take this sequentially

$$|M|^2|J|=|J|, \tag{19}$$

$$\vec{J}=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{20}$$

where J is a matrix which dimension is 6 N, which N is a number of particles. Equation (19) clearly means that the volume element under a canonical transformation is invariant. That is

$$J_n=\int\cdots\int d\vec{\eta}=invariant, \tag{21}$$

which is known as the integral invariance of Poincare.

3.2 Liouville's theorem

The invariance of Poincare integral shows us an elementary quantity in phase space approach. If the motion of multi-particle is in a canonical ensemble, there is clearly no incoming or outgoing particle beyond the volume boundary. Particle number inside the volume is conserved during the motion. Liouville's theorem summarize this into the proclaim that the density in a phase space is a constant under a canonical transformation, as shown in Fig. 1.

It is important to take a look into Poisson bracket in order to get further meaning,

$$\frac{dA}{dt}=[A,H]-\frac{\partial A}{\partial t}. \tag{22}$$

In this case, the total time derivative is zero, then

$$[A,H]=\frac{\partial A}{\partial t}, \tag{23}$$

When the state is in equilibrium,

$$[A, H]=0. \tag{24}$$

Hence, A is turned out to be constant of motion, like the density in this case. It means that Poisson bracket is potentially a strong tool to analyze the particle motion under the conservative force. Actually its versatility is beyond a classical mechanics.

It is also important to remember that above consideration is based on conservative force only [7]. If we have non-conservative force like a friction loss, Liouville's theorem is not perfectly right. The predominant nature of non-conservative system is seen in the electron circular accelerator or light source which produce radiation. It is indispensable to pay attention the canonicality for the motion which is on the spot.

3.3 Vlasov equation

The canonical transformation assures the Liouville's theorem in a phase space. Apart from this for a while, here let us consider the volume element for N particles.

$$N=\int\int\ldots\int\Psi dx_1 dx_2\ldots dx'_{n-1} dx'_n. \tag{25}$$

where Ψ is a density.

We consider the square like in Fig. 2 in which area A_P is $\Delta w \Delta p_w$. If we have the following time development for area A_Q:

$$\begin{aligned} \dot{w} &= f_w(w, P_w, t), \\ \dot{p}_w &= g_w(w, p_w, t). \end{aligned} \tag{26}$$

Because of the conservation of particles,

$$\Psi(w+f_w\Delta t, p_w+\Delta t, t+\Delta t)A_P=\Psi(w, p_w, t)A_Q. \tag{27}$$

Applying Taylor's expansion for A_Q, and changing into the expression for the phase volume, we get,

$$\Delta V_Q=\Delta r\Delta p[1+\nabla_r f\Delta t+\Delta_p g\Delta t]. \tag{28}$$

Also applying Taylor's expansion again to equation (27), finally we get

$$\frac{\partial\Psi}{\partial t}+f\nabla_r\Psi+g\nabla_p\Psi=-(\nabla_r f+\nabla_p g)\Psi. \tag{29}$$

This equation is called Vlasov equation which describes the time development of a phase volume. The right hand side is called a dumping term, then

$$\frac{\partial f_w}{\partial w}+\frac{\partial g_w}{p_w}=-\alpha_w,$$
$$\ddot{w}+2\alpha_w\dot{w}+w_0^2 w=0. \tag{30}$$

Therefore if there is no dumping in the system, we have

$$\frac{\partial f_w}{\partial w}+\frac{\partial g_w}{p_w}=0. \tag{31}$$

It is apparent that a phase volume is conserved if there is no dumping in the system. This is assured by the transformation (26) which may not be canonical but an enlarged transformation than that. It means that Vlasov equation involves a expanded transformation rather than Liouville's theorem does.

3.4 Beam emittance

3.4.1 Particle distribution in phase space

Any periodic motions have closed orbits in phase space. The simplest example is basically harmonic motion which has a ellipse form in the phase space. Let us check some useful analytic characteristics of a motion in phase space.

An ellipse in phase space in x and x' is presented by

$$\gamma_0 x_0^2+2\alpha_0 x_0 x_0'+\beta_0 x_0'^2=\epsilon, \tag{32}$$

where area ϵ is called beam emittance, and each parameter has following geometrical relationship;

$$\beta\gamma-\alpha^2=1. \tag{33}$$

Frequently like here ϵ is normalized by π.

The geometrical picture is presented in Fig. 3. It is easy to remember the quantity in beam optics, if we consider the condition of each point in the figure. ϵ is invariant by Liouville's theorem if there is only conservation force and no inter-particle interaction. In the case of acceleration particles get to have relativistic effect. Therefore, it is better to use multiplied x' by $\beta\gamma$. We call the multiplied emittance ϵ_N in following expression normalized emittance:

$$\epsilon_N=\beta\gamma\epsilon. \tag{34}$$

Hence, unless there is transverse force in the accelerating beam line, observed emittance looks like smaller and smaller. This phenomena is called a adiabatic dumping.

As the conservation of emittance comes from Liouville's theorem, it is notable that we should

check if the motion is canonical.

3.4.2 Statistical emittance

In an actual case, beam phase space could not be expressed by the clear cut model mentioned in previous section. This leads to have a statistical treatment of the beam practically.

P.Lapostolle[8] proposed following statistical expression:

$$\tilde{\varepsilon} = 4\sqrt{\overline{x^2}\,\overline{x'^2} - (\overline{xx'})^2} = 4\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}, \tag{35}$$

which is called effective emittance or r.m.s. emittance.

The effective emittance is suitable to using in simulation codes and practical purposes. This equation means that the emittance comes from the sum of all $N(N\text{-}1)$ triangle areas which consist of any two points in phase space and the origin [9]. Therefore we should be careful of the contribution from far solitude particles and the case of an emittance filamentation.

We can deduce equation (35) by using standard deviations σ_x and $\sigma_{x'}$ as

$$\tilde{\varepsilon} = 4\sigma_x\sigma_{x'}\sqrt{1-r^2}, \tag{36}$$

where

$$r = \frac{\langle xx'\rangle}{\sqrt{\langle x^2\rangle\langle x'^2\rangle}} \tag{37}$$

is called a correlation constant. It means that if the total mapping of area is axis symmetric, r.m.s. emittance is equivalent to the emittance mentioned previous section beside factor 4.

It is expected that r.m.s. emittance is conserved under the some kind of conditions. This will be mentioned in the section of collective beam dynamics.

4. Single particle beam dynamics

In this section, the property for the beam dynamics including no particle-particle interaction is summarized for a practical purpose. Important formulation such as a transverse motion is seen in next chapter for its total convenience.

4.1 Phase stability

When a synchrotron is proposed [10,11] as a high energy accelerator for electrons or heavier ions, the phase stability is also studied [12] for an electron linear accelerator. Here, the dynamics in phase space for relativistic particle is described elementarily.

We consider the particles which synchronous velocity and rest velocity are v_0 and v'. Then we can have the following expansion without a space charge effect.

$$p=\frac{m_0 v_0}{\sqrt{1-\frac{v_0^2}{c^2}}}+\frac{m_0 v'}{\sqrt{1-\frac{v_0^2}{c^2}}}+\ldots$$
$$=p_0+m_1 v'+\ldots=p_0+p'+\ldots\ldots \tag{38}$$

Therefore the Hamiltonian in s coordinate is written down as

$$H=m_0 c^2\sqrt{1-\frac{v_0^2}{c^2}}-\frac{p'^2}{2m_1}-eE\frac{v_0}{\omega}\cos\frac{\omega s'}{v_0}, \tag{39}$$

where

$$m_1=\frac{m_0}{(1-\frac{v_0^2}{c^2})^{3/2}} \tag{40}$$

is the mass increased by the longitudinal motion. This Hamiltonian leads an equation of motion in the field $-eE\sin\omega s'/v_0$ which synchrotron frequency is,

$$\omega_0=\sqrt{\frac{e}{m_0}E\frac{\omega}{v_0}(1-\frac{v_0^2}{c^2})^{2/3}}. \tag{41}$$

From above equations, it is clear that the separatrix has considerable elongation in a velocity axis and that the synchrotron frequency goes quite small, which situation is seen in Fig. 4. This figure

shows that if the electron injected in the origin of the phase which energy is bigger than critical energy, it is quickly reaches the final fixed phase position. This is a one of the characteristics of the particle with speed of light. Of course, it is apparent that if electrons have high injection energy, their phases are fixed easily tight.

The dynamics of particles with graduating their speed is basically similar to the dynamics of synchrotrons. A space charge effect for a high current linac sometimes limits a actual beam current because of its divergent force against a phase stability. The phase stability relates the transverse defocusing as mentioned following section.

4.2 Beam defocusing in a linac

The accelerator theory and technology made a remarkable progress after the World War II. One of the greatest achievement among them is the principle of Alternate Gradient focusing[13] (AG focusing or strong focusing) discovered by Brookhaven National Laboratory and other laboratory. Since then, This idea is basically still alive as a lattice structure which is called separate function in all high energy circular accelerators.

AG focusing was quickly applied[14] in a linac in order to converge the beam radially. Up to now, this stems the basic focusing structure of modern linear accelerators.

Let consider traveling wave acceleration by TM_{01} mode for example. The more far from the reference axis is, the weaker the electric field is. This means that the electric field is a function of power series of inverse of the radial displacement r from the reference axis. Panofsky Wenzel theorem[15] requires that the transverse divergence of a longitudinal force is proportional to the derivative of a transverse force by the reference axis. This manifest impose that longitudinal force and transverse force have opposite sign each other. Therefore, in TM_{01} mode, it is turned out that there is a divergent force if beam has a phase stability.

Actually the transverse force has r dependence according to Panofsky Wenzel theorem, thereafter, it is clear that a quadrupole fields is enough to cancel the defocusing. It is interesting to notice that the quadrupole focusing system has the relationship with RF field which has r dependence. In realistic beam transport systems, higher multipoles are frequently used for canceling out the chromatic aberration in a beam transport line.

5. Collective beam dynamics

5.1 Transverse motion

In a linear beam transport theory[16], a particle trajectory u is described by following equation:

$$u''+k(s)u=0, \tag{42}$$

where $k(s)$ is a focusing function of the system. This equation was studied in last century for astromechanics and called Hill's equation. The general solution for $k(s) < 0$ is solvable by Lagrange's method of variation of integration constant. This is

$$u(s) = \sqrt{\epsilon}\sqrt{\beta}\cos[\phi(s)-\phi_0], \tag{43}$$

where ϵ, and β are a beam emittance and one of beta functions. The physical meaning of ε and β is obvious, if we consider the comparison with the derivative of $u(s)$.

Equation (42) governs the beam line with reference energy because there is no parameter for the statistics of the particle energy. The dispersion system, which we call system having energy distribution in a transport line, is described as

$$u''+k(s)u=\frac{\delta}{\rho(s)}=\eta, \tag{44}$$

where δ, ρ and η are quantities called the dispersion, and the bending radius, and the η function in a periodic system if exists, respectively. It should be noticed that equation (44) is mathematically equivalent to one which describes a motion with nothing for the focusing system but general external force.

The solution of equation (44) is composed of the particular solution from the homogenous equation (42), thereafter, by the usual method[17], then the solution is

$$u = u_1\left[c_1-\int\frac{\delta\, u_2}{\rho\Delta(u_1,u_2)}ds\right]+u_2\left[c_2-\int\frac{\delta\, u_1}{\rho\Delta(u_1,u_2)}ds\right], \tag{45}$$

where $\Delta(u_1,u_2)$ is a Wronskian consists of u_1 and u_2. c is a constant parameter. At last, the solution of equation (44) is, in popular form,

$$u(s) = aC(s)+bS(s)+\delta\int_0^s\frac{1}{\rho}[S(s)C(\tilde{s})-C(s)S(\tilde{s})]d\tilde{s}, \tag{46}$$

where *S(s)* and *C(s)* are particular solutions of equation (42), that is, the contantless solutions presented by equation (42). The last term in this equation is called $\delta D(s)$. Considering an initial condition,

$$u(s)=\sqrt{\frac{\beta}{\beta_0}}(\cos\phi+\alpha_0\sin\phi)\ u_o+\sqrt{\beta\beta_0}\sin\phi\ u_0'+\delta D(s) \tag{48}$$

where α_0 is one of beta functions in initial state. The actual shape of *D(s)* is determine after the definition of *k(s)*. Apart from the dispersion phenomena, equation (44) and its solution is sometimes useful for the analytic search of beam evolution. Equation (44) presents a single particle dynamics without a particle-particle interaction.

Remember that this method presented by equation (46) is only valid for linear motion like equation (44). If we the equation has a non linear term, there is no simple relationship between the homogenous and inhomogeneous equations mentioned here.

5.2 Envelope Equation for a linac

Generally, equation (42) can be modified in order to take into account other interaction term $\xi(\rho,s)$ which for example means a space charge effect or a beam pipe interaction,

$$u''_{x,y}+k_{x,y}(s)\,u_{x,y}+\xi_{x,y}(\rho,s)=0. \tag{49}$$

The transverse beam motion has both of *x* and *y* component because the focusing magnets essentially cause to have a x-y plane asymmetry. In order to consider space charge effect here, ξ is $4r_o\,I\,/\,b^2\,g^2\,(x+y)$ which is called space charge term, where *I* is the charge density of a beam.

We can write down the *x* component solution of equation (44) as following:

$$u(s)=a(s)\cos\phi_x(s). \tag{50}$$

Hence it produces next two equations:

$$a''-a\ (\phi_x')^2=-K_x(s)\,a-\frac{\xi(\rho,s)}{a+b}, \tag{51}$$

$$a\phi_x''+2a'\phi_x'=0. \tag{52}$$

The latter equation means

$$\phi_x' = \frac{\epsilon_x}{a(s)^2}. \tag{53}$$

Finally, we get following equation:

$$a'' + K_x a - \frac{\epsilon_x^2}{a^3} = \frac{\xi(\rho, s)}{a+b}. \tag{54}$$

This equation is called envelope equation.

Equation (53) is basically relativistic, but does not work for an accelerating system because longitudinal momentum P is assumed constant. In linear beam dynamics, it should be noticed that approximately we have

$P_x = p\,x' = mc\,\beta\gamma\,x'$,

where ' means taking the derivative by s as is said before, and that equations (50) has second order differential term only. $\beta\,\gamma$ here are the relativistic parameters this time. It leads the thought that the transformation

$$a \rightarrow \frac{a}{\sqrt{\beta\gamma}} \tag{55}$$

will basically keep good framework under calculus for solving the equation of motion. It is sometimes useful to keep the following in mind:

$$\frac{cp}{m} = \beta\gamma, \tag{56}$$

$$(\beta\gamma)' = \frac{\gamma'}{\beta}. \tag{57}$$

Using latter one, an adiabatic dumping presents

$$\frac{\gamma'}{\gamma} \ll \frac{\beta^2 a'}{a}. \tag{58}$$

Under the transformation (54), the envelope equation is turned to

$$a'' + \left(K_x + \frac{\alpha^2}{4\beta^2\gamma^2}\right) a - \frac{(\beta\gamma\epsilon_x)^2}{a^3} = \frac{\beta\gamma\xi(\rho, s)}{a+b}, \tag{59}$$

where $\alpha = (\beta\gamma)'$ = constant in the acceleration of relativistic particles. The emittance multiplied by $\beta\gamma$ is just called normalized emittance. $m_0c^2\,\gamma'$ is an accelerator gradient which is given in Appendix

in this report, and it is considerably low for the case of PNC linac.

Equation (58) describes the general behavior of a beam envelope. Each terms in equation (58) have physical meaning which comes from the order of a. For example, the two components in second term mean an external focusing function mostly fitted to quadrupole focusing, and an intrinsic defocusing action happened to be caused from by a beam acceleration. The latter component will be considerably neglected if acceleration is relativistic enough. It is also notable to have emittance term which has inverse of the third power of the envelope. We must be careful that the a dependence of a space charge effect is not explicit here. Equation (58) is generally solved by means of a numerical approach because of its highly nonlinearity.

We can also define r.m.s. envelope

$$\tilde{x}=\langle x^2\rangle^{1/2}. \tag{60}$$

Then, considering following formulae in combination with equation (44):

$$\begin{aligned}
&\frac{d}{ds}\langle x^2\rangle=2\langle xx'\rangle,\\
&\frac{d}{ds}\langle x'^2\rangle=2\langle x'x''\rangle\\
&=2k(s)\langle xx''\rangle+2\langle x'\xi\rangle,\\
&\frac{d}{ds}\langle xx'\rangle=\langle x'^2\rangle+\langle xx''\rangle\\
&=\langle x'^2\rangle-k(s)\langle x^2\rangle+\langle x\xi\rangle.
\end{aligned} \tag{61}$$

The identity $p = -k(s)\,x + \xi$ is used here. Finally we get the r.m.s. envelope equation as;

$$\frac{d^2}{ds^2}\tilde{x}+k(s)\,\tilde{x}-\frac{\tilde{e}^2}{\tilde{x}^3}-\frac{\langle x\xi\rangle}{\tilde{x}}=0. \tag{62}$$

This equation has completely the same form as the envelope equation (58). It means that in many simulation codes we can r.m.s. envelopes as actual envelope.

From the development of the square of r.m.s. emittance defined in equation (35) and relation (60), in the case of no external focusing, we get

$$\frac{d}{ds}\tilde{e}^2=2\eta\,[\langle x^2\rangle\langle x'\xi\rangle-\langle xx'\rangle\langle x\xi\rangle]. \tag{63}$$

It leads an important suggestion that r.m.s. emittance is constant if interaction forces between the particles are linear. Generally, r.m.s. emittance is not conserved under the presence of an external force. These strange situations come from the fact that r.m.s. emittance does not satisfy a canonical transformation.

6. Beam broadening in PNC linac by space charge effect

Eq. (58) for a constant acceleration for a linac instantly makes it possible for PNC linac to estimate the behavior of the beam by the space charge effect in addition to a general property given by a mathematical structure of non-linear differential equations. The electron beam in CW linac is not eventually DC beam, but it is good approximation in relativistic particles.

Let us consider eq. (58) in the case of no focusing $k(s) = 0$ for x and y components. Then we write eq. (58) as

$$a'' + \Delta a - \frac{(\epsilon_{Nx})^2}{a^3} = \frac{\xi_N(\rho, s)}{2}, \tag{64}$$

$$\Delta = \frac{\alpha^2}{4\beta^2\gamma^2}, \quad \epsilon_N = \beta\gamma\epsilon, \quad \xi_N = \beta\gamma\xi, \tag{65}$$

where ε_N is called normalized emittance. ε and ξ are $10\,\pi\,10^{-5}$ cm rad and $2.665\ 10^{-12}$ for 1 mm beam radius, respectively in CGS unit for PNC linac specification. The electron beam is assumed an annular beam which radius is 1 mm.

Δ is not apparently negligible small though the accelerating gradient is small in PNC linac, because g is at most 20 and the accelerator gradient is low seen in Appendix. Let us assume the right hand side and the last term of the left hand side in eq. (63) are zero, and it is clear that the envelope does not diverge but oscillate with the periodicy by Δ term. The contribution of this term is vanishing when the periodic length gets considerably long in relativistic particles. Hence, Δ term shall be negligible in order to keep the additional phenomena in eq. (63) off the beam broadening. Fig. 5 shows the contribution of Δ term to Eq. (63), in which Δ is 10^{-7}, 10^{-8} and 10^{-10} and the emittance is $10\,\pi$ mm mmrad with no space charge effect. a_0 is beam x radius in the waste point. The shape of beam envelope is divergent, parallel and vibrating by the critical value $\Delta = \varepsilon_N$.

The beam dynamical dependence of accelerating electron beam in PNC linac is surveyed by means of a numerical analysis of equation (63) for some cases differing the beam emittance, the beam current and the beam energy. The results for 10 and 100 MeV followed by 3 MeV are summarized including parametrization of the beam emittance and current. All the calculation is achieved by NeXT STEP Mathematica ver.3.00 on Pentium processor (133 MHZ). All the cases to be calculated are summarized in Table 1.

Fig. 6 shows the beam envelope of 10 MeV acceleration with 0.1 and 0 A electron beam with 0.1 π mm mrad emittance. The dashed line corresponds to no space charge effect. Flight paths are normalized by initial waste radius a_0 and multiplied by 10^5 here. The beam envelopes for π mm mrad are shown in Fig. 7, where the two solid lines for 0.1 and 10 A beam are presented and 0 current line is essentially in accordance with 0.1 A line. There are no actual beam broadening and no difference between each beam currents at normalized s is 0.1 (100 m in PNC linac). The total length of PNC linac is about 20 m. Of course, the beam loading gives small gradient change in linac structure, but

beam passes though well in the linac if the beam line center is precise enough.

The calculation for 100 MeV is summarized in Fig. 8 and 9. The difference by beam current gets so scarce more than the case of 10 MeV that the dashed line presenting no space charge effect almost over-wraps the solid line presenting 0.1 A electron beam in Fig. 8. The effect of space charge gets quite weak in larger beam emittance at high energy acceleration, as seen in Fig 9. It means that the beam size broadening by space charge for PNC linac eventually does not appears strongly and that the transported beam in the regular section may looks stable after the making beam waste by magnetic lenses in the electron gun system, if the beam passes through exact transport center. This situation will be held in the case of 10 A current and 10 mm mrad emittace. It is naturally notable that the wake field effect should be estimated when beam bunch is coming short.

Fig. 10 shows the beam broadening by space charge for 3 MeV 0.1 A PNC linac. The emittance is π mm mrad here in order to make the difference clear, although design value is 10 π in the electron gun. The beam broadening at 0.1 normalized distance (10 m in actual length) is less then 10 %. It corresponds to the broadening around AC1 in PNC linac, while 200 keV beam line has focusing system by solenoid magnets to keep the beam waste radius.

In the analysis in this report, the broadening from the acceleration is estimated larger than the result from non- accelerating calculation, because initial space charge parameter is stronger in a beam acceleration. This confusing situation comes from the fact that in the calculation some energies are used final or initial values arbitrary. It is important to calculate with the correct equation like the envelope equation derived in this report in order to get enough precision.

The right hand side almost looks vanishing in PNC case, which solution is an analytic parabolic function. It meas that the beam emittance in the electron gun is so large that beam is scarcely affected from beam charge. The increase of emission density of thermionic cathode is encouraged because the beam broadening is still small when the emission density is 10 times higher.

7. Summary

The effect of transverse space charge is surveyed. The beam characteristics in acceleration field is examined. The longitudinal space charge gets weak in the relativistic region. The beam envelope equation is formulated including the beam acceleration. The beam broadening by the space charge effect is controllable up to 10 A, while the beam emittace for PNC linac design is dominant for the beam broadening. The beam broadening of present PNC linac mainly comes from its beam emittance. It is notable that the emittance of a thermionic cathode is determined by the area of the cathode and high brilliance cathode with small size may cause hard beam broadening by a space charge when very smaller beam emittance is necessary. It is mandatory for detailed survey to analyze other beam instability such as a wake field and the longitudinal space charge in CW beam which is neglected in this report, in order to handle extremely brilliant beam.

8. Acknowledgment

The analysis in this report is achieved by the helpful discussion with the people in Brookhaven National Laboratory in USA. Especially, the author of this report would like to thank Dr. H. Takahashi who gave me a lot of suggestion and continuous encouragement in the analysis in this report.

References

[1] S. Tōyama and S. Tani, "Development of High Power CW Electron Linac," PNC TN 9410 93-215, 1993.
[2] Y.L. Wang, I. Sato, S. Tōyama, and Y. Himeno, "Design of High Power Electron Linac in PNC," Jou. Nucl. Sci. and Tech., **30**, p 1261, 1993.
[3] T. Emoto and M. Nomura, "Development of High Power CW Electron Linac Injector," PNC TN 9410 95-271, 1996.
[4] R.B. Miller, "An introduction to the physics of intense charged particle beams," Plenum Press, 1985.
[5] F.L. Krawczyk, J. Billen, R. Ryne, H. Takeda and L. M. Young, "THE LOS ALAMOS ACCELERATOR CODE GROUP," Proceedings of the 1995 Particle Accelerator Conference and International conference on High Energy Accelerators, 1995.
[6] K.L. Brown, D.C. Carey, Ch. Iselin and F. Rothacker, "TRANSPORT," CERN 80-04, 1980
[7] H.Goldstein, "Classical Mechanics," Addison-Wesley Pub. Co., p 326, 1980.
[8] P.M.Lapostolle, "POSSIBLE EMITTANCE INCREASE THROUGH FILAMENTATION DUE TO SPACE CHARGE IN CONTINUOUS BEAM," IEEE Trans. Nucl. Sci. NS-18, p 1101, 1971.
[9] J.Buon, "BEAM PHASE SPACE AND EMITTANCE," CERN 94-1, p 89, 1994.
[10] V.I.Veksler, "A New Method of Acceleration of Relativistic Particles," J.Phys. USSR, **9** ,p 153, 1945.
[11] E.M.McMillan, "The Synchroton--A Proposed High Energy Particle Accelerator," Phys. Rev. **68**, p 143, 1945.
[12] J.C.Slater, "The Design of Linear Accelerators," Rev. Mod. Phys. 20, p 473, 1948.
[13] E.D. Courant, M. Stanley Livingston, and H.S.Snyder, "The Strong-Focusing Synchroton--A New High Energy Accelerator," Phys. Rev. 88, p 1190, 1952.
[14] J.P. Blewett, "Radial Focusing in the Linear Accelerator," Phys. Rev. 88, p 1197, 1952.
[15] W.K.H. Panofsky and W.A. Wenzel, "Some Considerations Concerning the Transverse Deflection of Charged Particle in Radio-Frequency Fields," Rev. Sci. Instrum. p 967, 1957.
[16] H.Wiedemann, "Particle Accelerator Physics," Springer-Verlag, p 104, 1993.
[17] P.M.Morse and H.Heshbach, "METHODS OF THEORETICAL PHYSICS," McGraw-Hill Inc. ,p 529, 1953.

Appendix : Beam acceleration and beam loading

In this appendix, the formula of accelerating structure for not only traveling wave type but also TWRR type is derived. The formula help deduce realistic parameters for an efficient calculation for the beam.

There are several types of accelerators of interest for generating a high beam current. The most simple and straightforward is an induction linac beside an electrostatic accelerator. The electric charge in one burst is very high; it accelerates more than a few tens of kA in peak current with energy of a few tens of kW[1]. The efficiency of an induction linac is relatively high because particles are accelerated by electromagnetic induction so that a high-power RF generator is not essential. However, the duty factor is still low. Achieving a longer pulse duration and a high repetition rate needs revolutionary research on materials for the ferrite core that is used as a transformer[2].

On the other hand, a CW technique for RF linac lately has shown remarkable progress as an accelerating structure. There has been distinctive progress in the research and development of a high-power CW klystrons. We can get the klystrons with high efficiencies in lower frequency RF. Therefore, we designed a CW high-power electron linac for our basic study of how to generate a high current beam with favorable quality. The CW beam has an advantage in the point that it is not only possible to get high average current but also possible to reduce the long-range wake force and space charge effect in beam bunches. Considerable progress has been made in developing of a superconducting accelerator structure also, but at the first stage, we have employed a room-temperature accelerating structure.

There are some applications to employ TWRRs in resonators or accelerators[3-7].

A.1 A traveling wave structure

A.1.1 Constant impedance accelerator

First, let us consider beam acceleration in a constant impedance(CI) traveling wave structure because of the simplicity of the mathematical treatment. The constant gradient (CG) structure actually can be broken down into a structure which has an exact CI structure. The energy conservation in an accelerating structure is presented by following equation:

$$\frac{\partial W}{\partial t}+\frac{\partial P}{\partial s}+P_w+iE=0, \qquad (1)$$

where, W, P, P_w, I, S and E are the stored energy per unit length, the power flow along s, the wall loss per unit length, the elastance, the beam current and the beam accelerating field, respectively. r_s and α are the shunt impedance and the attenuation per unit length. Each quantities have the following physical relationship:

$$r_s=\frac{E^2}{P_w}, \tag{2}$$

$$Q=\frac{W}{P_w/\omega}, \tag{3}$$

$$v_g=\frac{\omega}{2\alpha}Q, \tag{4}$$

$$P=v_gW=\frac{v_gE^2}{s}=\frac{E^2}{2\alpha r_s}, \tag{5}$$

$$s=\frac{E^2}{W}, \tag{6}$$

where r_s, Q, and v_g are the shunt impedance, the quality factor and the group velocity in each. These three quantities ideally are constant anywhere in a CI structure. In equilibrium, equation (1) becomes:

$$\frac{\partial P}{\partial s}+P_w+iE=0. \tag{7}$$

From this, and from equations (4), (5), and (6), we get the following diffusion equation:

$$\frac{\partial P}{\partial s}+aP+ib\sqrt{P}=0, \tag{8}$$

where a and b are the constants which are mentioned bellow. This equation can be solved easily setting $P(s) = x\ (s)^2$ which $x\ (s)^2$ is a function of s. Finally, we get the power flow $P(s)$, the beam acceleration $E_b(s)$, and the gain $V_b(L)$, as follows:

$$P(s)=[\ (\sqrt{P_0}+\frac{ib}{a})\ exp(-\frac{a}{2}s)-\frac{ib}{a}]^2, \tag{9}$$

$$E_b(s)=\sqrt{ar_s}\left(\sqrt{P_0}+\frac{ib}{a}\right)exp\left(-\frac{a}{2}s\right)-\frac{i\sqrt{r_s}b}{\sqrt{a}}, \tag{10}$$

where, P_0 and L are the input RF power and the length of structure, respectively. We notice for a and b:

$$a=\frac{\omega}{v_gQ}=2\alpha, \tag{11}$$

$$b=\sqrt{\frac{r_s\omega}{v_gQ}}=\sqrt{2\alpha r_s}, \tag{12}$$

$$\alpha L=\tau, \tag{13}$$

or,

$$E(s)_{CI}=\sqrt{2\alpha r_sP_0}e^{-\alpha s}-\frac{ir_s}{\sqrt{2\alpha}}(1-e^{-\alpha s}), \tag{14}$$

where ω, and τ are the angular frequency and the total attenuation, respectively. Remember the attenuation constant is proportional to RF frequency, which means that power dissipation rises at higher RF frequencies. Equation (14) shows that higher frequencies can generate higher accelerating gains. Physically, this occurs because the lengths of high frequency waves in the accelerating structure become shorter.

Finally, from equation (9) and (10), and using the total attenuation τ, thus the gain V_{CI}, the conversion efficiency from RF to beam power η_{CI} (hereafter we call this transmission efficiency) and the power fed out into the dummy load of the accelerating structure $P(L)_{CI}$, are as follows:

$$V_{CI}=\sqrt{2r_sP_0L\tau}\frac{1-exp(-\tau)}{\tau}-ir_sL\left(1-\frac{1-exp(-\tau)}{\tau}\right), \tag{15}$$

$$\eta_{CI}=i\sqrt{\frac{2\tau r_sL}{P_0}}\frac{1-exp(-\tau)}{\tau}-i^2\frac{r_sL}{P_0}\left(1-\frac{1-exp(-\tau)}{\tau}\right), \tag{16}$$

$$P_{CI}(L)=[\sqrt{P_0}exp(-\tau)-i\sqrt{\frac{r_sL}{2\tau}}(1-e^{-\tau})]^2. \tag{17}$$

Transmission efficiency increases when the attenuation constant decreases, i.e. when the RF frequency decreases. Transmission efficiency depends also on the Q value and the shunt impedance. Hence, it is an attractive method for high transmission efficiency to employ a superconducting accelerating structure, if it were provided that the operation can easily produce a high beam current.

A.1.2 Constant gradient accelerator

We can deduce the formula for CG with equation (7) like the case of CI. It is assumed here that the electric field is constant with no beam loading. In general, from equation (5),

$$\frac{dP}{ds}=\frac{E}{\alpha r_s}(\frac{dE}{ds}-\frac{E}{2\alpha}\frac{d\alpha}{ds}). \tag{18}$$

Hence, with equation (7)

$$\frac{dP}{ds}=-iE-2\alpha P=-iE-\frac{E^2}{r_s}=\frac{E}{\alpha r_s}(\frac{dE}{ds}-\frac{E}{2\alpha}\frac{d\alpha}{ds}). \tag{19}$$

The electric field and the attenuation for CI and CG are determined by the above equations using the condition without beam loading. That is,

$$\frac{dE}{ds}=-\alpha E,\quad \frac{d\alpha}{ds}=0,\quad (CI) \tag{20}$$

$$\frac{dE}{ds}=0,\quad \frac{d\alpha}{ds}=2\alpha^2,\quad (CG). \tag{21}$$

Remember the elementary formula,

$$P=\frac{E^2}{2\alpha r_s},\quad \frac{dP}{ds}=\frac{dP}{d\alpha}\frac{d\alpha}{ds}. \tag{22}$$

The attenuation of CG is derived via the output power ratio to the input:

$$\frac{P(L)}{P(0)}=\frac{E_L^2/2\alpha_L r_s}{E_0^2/2\alpha_0 r_s}=\frac{\alpha_0}{\alpha_L}=e^{-2\tau}. \tag{23}$$

Finally for the attenuation of CG,

$$\alpha=\frac{\alpha_0}{1-2\alpha_0 s}, \tag{24}$$

$$\alpha_0=\frac{1-e^{-2\tau}}{2L}. \tag{25}$$

The power in the CG is derived from equation (18) associated with equations (22) and (23), under the assumption that the shunt impedance is approximately constant. The equation (18) is solved by the method used for the case of CI, and the basic start equation is as bellow:

$$\frac{dP}{ds}+\frac{2\alpha_0}{1-2\alpha_0}P+i\frac{\sqrt{2r_s\alpha_0}}{\sqrt{1-2\alpha_0 s}}\sqrt{P}=0. \tag{26}$$

Hence the power flow $P_{CG}(s)$, the electric field $E_{CG}(s)$ and the gain V_{CG} are

$$P_{CG}(s)=[\sqrt{P_0}(1-2\alpha_0 s)^{1/2}+\frac{i}{2}\sqrt{\frac{r_s}{2\alpha_0}}(1-2\alpha_0 s)\,Log(1-2\alpha_0 s)]^2, \tag{27}$$

$$E_{CG}(s)=\sqrt{\frac{2r_sP_0}{2L}}\sqrt{1-e^{-2\tau}}+\frac{ir_s}{2}Log(1-2\alpha_0 S), \tag{28}$$

$$V_{CG}=\sqrt{r_sP_0L}(1-e^{-2\tau})^{1/2}-\frac{ir_sL}{2}(1-\frac{2\tau e^{-2\tau}}{1-e^{-2\tau}}), \tag{29}$$

where we can realize that each equation consists of the generator part and the beam loading part. The transmission efficiency η_{CG} and the power into the load $P_{CG}(L)$ are

$$\eta_{CG}=i\sqrt{\frac{r_s L}{P_0}}(1-e^{-2\tau})^{1/2}-\frac{i^2 r_s L}{2P_0}\left(1-\frac{2\tau e^{-2\tau}}{1-e^{-2\tau}}\right), \tag{30}$$

$$P_{CG}(L)=\frac{e^{-2\tau}}{2\alpha_0 r_s}\left[\sqrt{2\alpha_0 r_s P_0}-i r_s \tau\right]^2, \tag{31}$$

respectively.

The current dependence of the gradient on CI structure is shown in Fig. 1 by means of a normalized current $I\,(r_s\,L/P_0)^{1/2}$ and a normalized voltage $V/(r_s\,L/P_0)^{1/2}$, using the variation of the attenuation parameters. The dotted line in this figure corresponds to the dependence on a CG structure. A CG structure does not make great difference with a CI case in the beam loading (Fig. 1). A lower attenuation is suitable to high current acceleration without the change of the considerable energy gain.

Fig. 2 gives the transmission efficiency for both a CI and a CG in the variation of the attenuation. Apparently, both types have almost the same magnitude of the efficiency because of the equal attenuation of accelerator guides. The structure which has low attenuation has the maximum in higher normalized current. The transmission efficiency in τ=0.05 is about 97%. But, zero current energy reduces about one third. It is notable that the efficiency depends not only on RF frequency but also on the length of the accelerator guide. A long guide favors high energy efficiency, meanwhile, however, a short guide is favorable in preventing a cumulative Beam Break-Up (BBU) which determines the current limit.

A.2 TWRR in high power CW linac

When there is no reflection in TWRR, the power build up A_n and the power into dummy load N_n in TWRR in resonance for n times RF round trip are

$$A_n=\frac{(1-(T\sqrt{1-C^2})^n)C}{1-T\sqrt{1-C^2}}A_0, \tag{32}$$

$$N_n=\sqrt{1-C^2}A_0-\frac{1-T\sqrt{(1-C^2)}^n}{1-T\sqrt{1-C^2}}C^2A_0, \tag{33}$$

where T, C, and A_0 are the voltage propagation constant, the coupling of the directional coupler and the RF amplitude, respectively. In the optimum coupling, after sufficient round trips, the voltage multiplication factor M is represented by the following:

$$M=\frac{A_\infty}{A_0}=\frac{C}{1-T\sqrt{1-C^2}} \quad , C=\sqrt{1-T^2}, \tag{34}$$

while $N_n = 0$.

In principle, the power dissipation in TWRR is caused only by wall loss when the coupling C is optimum. The characteristics of the field multiplication versus the coupling are shown in Fig. 3, employing the parameters for PNC linac.

Considering the amplitude inside the TWRR in equilibrium, and by means of the coupling constant C, the field multiplication factor M_R and energy gain V_R for TWRR in the CI and CG structure are[13] as follows:

$$M_{RCI}=\frac{E_0C-ir_s(1-e^{-(\tau+\tau_{W1}+\tau_{W2})})\sqrt{1-C^2}}{E_0(1-e^{-(\tau+\tau_{W1}+\tau_{W2})}\sqrt{1-C^2})}, \tag{35}$$

$$V_{RCI}=\frac{E_0C-ir_s(1-e^{-\tau})\sqrt{1-C^2}}{1-e^{-\tau}\sqrt{1-C^2}}\frac{L(1-e^{-\tau})}{\tau}-ir_sL(1-\frac{1-e^{-\tau}}{\tau}), \tag{36}$$

$$M_{RCG}=\frac{E_0C-ir_s\tau e^{-(\tau+\tau_{W1}+\tau_{W2})}\sqrt{1-C^2}}{E_0(1-e^{-(\tau+\tau_{W1}+\tau_{W2})}\sqrt{1-C_2})}, \tag{37}$$

$$V_{RCG}=\frac{E_0C-ir_s\tau e^{-\tau}\sqrt{1-C^2}}{1-e^{-\tau}\sqrt{1-C_2}}-\frac{ir_sL}{2}(1-\frac{2\tau e^{-2\tau}}{1-e^{-2\tau}}), \tag{38}$$

and

$$E_0=\sqrt{2\tau r_sP_0/L}, \tag{39}$$

where E_0 is the electric field just in the first cavity. Moreover τ_{W1} and τ_{W2} are the attenuation of upstream and downstream in the recirculator, which are neglected in the presentation of the voltage term.

We neglect the attenuation from the recirculator, and assume that the accelerator guide in TWRR is CI, because there is an approximate accordance in the RF characteristics of CI and CG when attenuation is low just as it is our case.

The power into the dummy terminator in TWRR for CI and CG is derived by the same manner in an equilibrium. Then,

$$N_{tCI}=\frac{A_t}{A_{RF}}=\frac{1}{1-e^{-\tau}}[\sqrt{1-C^2}-e^{-\tau}+i\sqrt{r_sL/P_0}C\frac{1-e^{-\tau}}{\sqrt{2\tau}}] \tag{40}$$

$$N_{tCG}=\frac{A_t}{A_{RF}}=\frac{1}{1-e^{-\tau}\sqrt{1-C^2}}[\sqrt{1-C^2}-e^{-\tau}+i\sqrt{r_sL/P_0}C\tau e^{-\tau} \tag{41}$$

When there is no beam loading, those two equations is reduced to optimum coupling relation which is seen in before. If we have a beam loading, the exact relationship between the coupling C and the attenuation τ is given by solving those equations. It is notable that the field multiplication factor has maximum value when there is no power flow into the dummy terminator and that the transmission does not have the maximum with the condition where there is no power in the terminator.

Fig. 4 shows the energy gain dependence due to normalized current mentioned in Section A.1. Around unit current area, the gain is increasing compared with ones from the ordinary traveling wave. Fig. 5 shows the transmission efficiency of TWRR by normalized current and voltage, where the

curves for both a CI and CG are depicted. When there is actually a RF reflection in TWRR, the multiplication factor is reduced by cancellation by the backward wave.

The actual accelerator parameters are summarized in Table A1 which includes not only achieved value but also design value. The total beam loading characteristics is shown in Fig. A6, in which both of 200 kW and 1 MW RF inputs into TWRR are displayed. The two RF powers are produced by 1 MW (CW mode) and 4.1 MW (short pulse mode) operation of klystrons.

References of Appendix

[1] D.Keefe, "INDUCTION LINACS," LBL-21528, 1986.
[2] W.Barletta, A.Faitens, E.Henestroza et al,"High Current Induction Linacs," LBL-35960, 1994.
[3] S.J.Miller, "The TRAVELING WAVE RESONATOR and HIGH POWER MICROWAVE TESTING," Microwave Journal, p.50, 1960.
[4] J.L. Kirchgessner and M.Tigner, "OPERATION OF A HIGH GRADIENT RECIRCULATING RF ACCELERATING CAVITY," Proceedings of Particle Accelerator Conference, p.928, 1983.
[5] Y.L.Wang, "RESEARCH ON ELECTRON LINEAR ACCELERATORS WITH TRAVELING WAVE RESONATORS (2)," IEEE Trans. Nucl. Sci. NS-30, p 3024, 1983.
[6] T.Oogoe, A.Enomoto and I.Sato, "High-gradient acceleration test using resonant ring," KEK Preprint 94-133, 1994.
[7] S.Yamaguchi, A.Enomoto, I.sato and Igarashi, "High-Power Test of a Traveling Wave Type RF pulse compressor," KEK Preprint 95-38, 1995.

Table 1 Cases of the envelope calculation

Case	Beam energy	Normarized emittance	Beam current
1	10 (MeV)	0.1π (mm mrad) π	0 (A) 0.1 10
2	100	0.1 π π	0 0.1 10
3	3.0	π	0 0.1 10

Table A1 Design and achieved parameters of TWRR accelerator structure

	B	AC1	AC2	AC3	AC4	AC5	AC6	AC7
τ (D)*	0.0408	0.0320	0.0352	0.0389	0.0432	0.0482	0.0540	0.0608
(A)**	0.0392	0.0331	0.0330	0.0396	0.0435	0.0440	0.0499	0.0594
Q (D)	18407	20195	20181	20166	20153	20140	20130	20118
(A)	19300	19400	21400	19700	19900	21900	21600	20400
C (D)	0.4472	0.4472	0.4693	0.4930	0.5175	0.5439	0.5720	0.6025
(A)	0.448	0.448	0.470	0.494	0.521	0.542	0.575	0.599
r_s (D)	28.57	34.07	34.75	35.45	36.14	36.84	37.54	38.25
(A)	-- ***	--	--	--	--	--	--	--

* Designed value

** Achieved value

*** not measured R / Q

The every designed value is an arithmetic sum of individual cavities in the structure. Units are τ (Nep.), r_s (M Ω), in each.

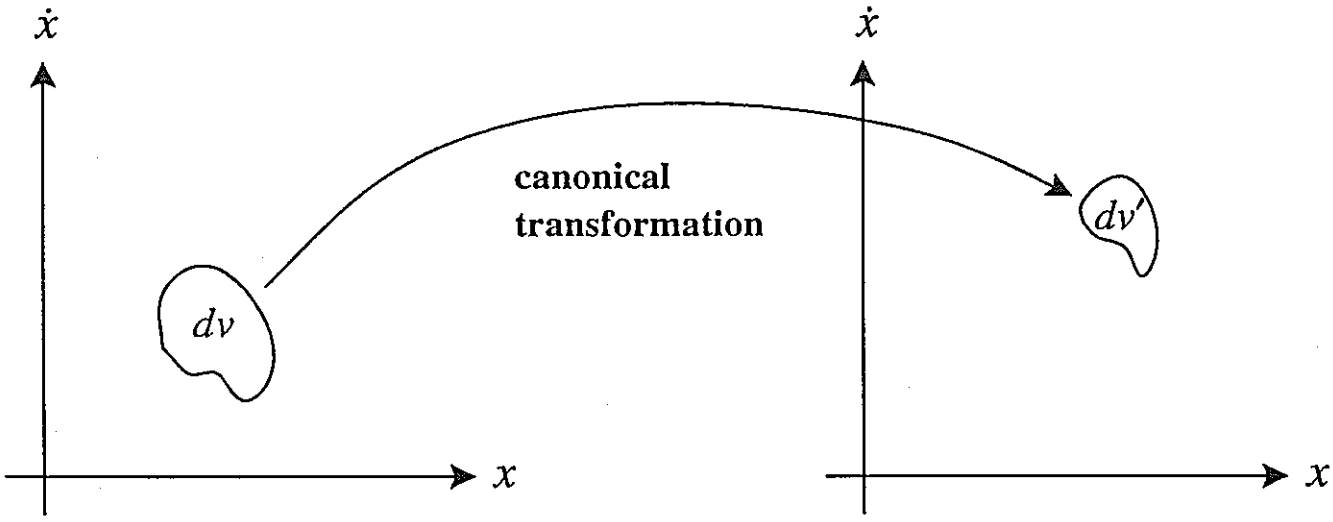

Fig. 1 Conceptual figure of a canonical transformation. The areas volume elements dV and dV' are conserved under canonical transformation.

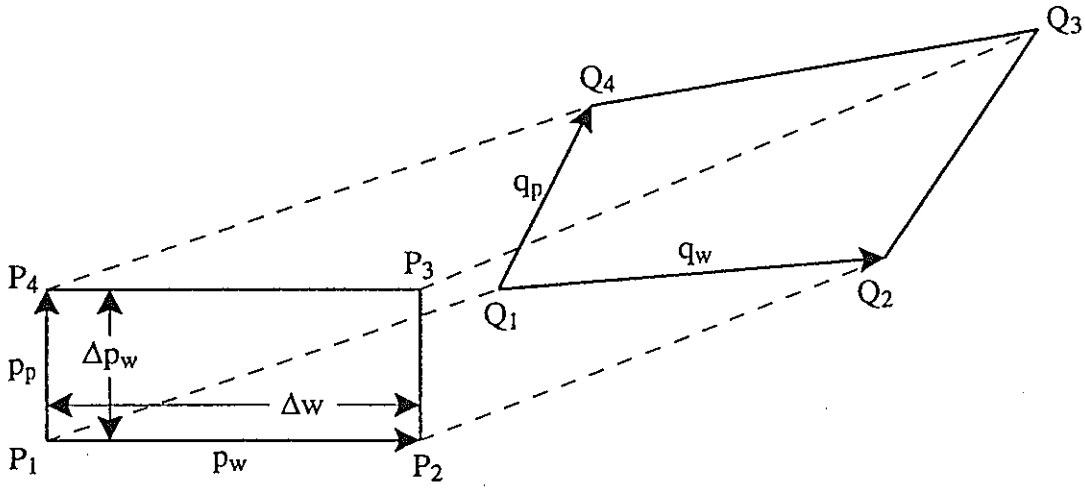

Fig. 2 Transformation of square element by the developing of infinitesimal time dt in phase space.

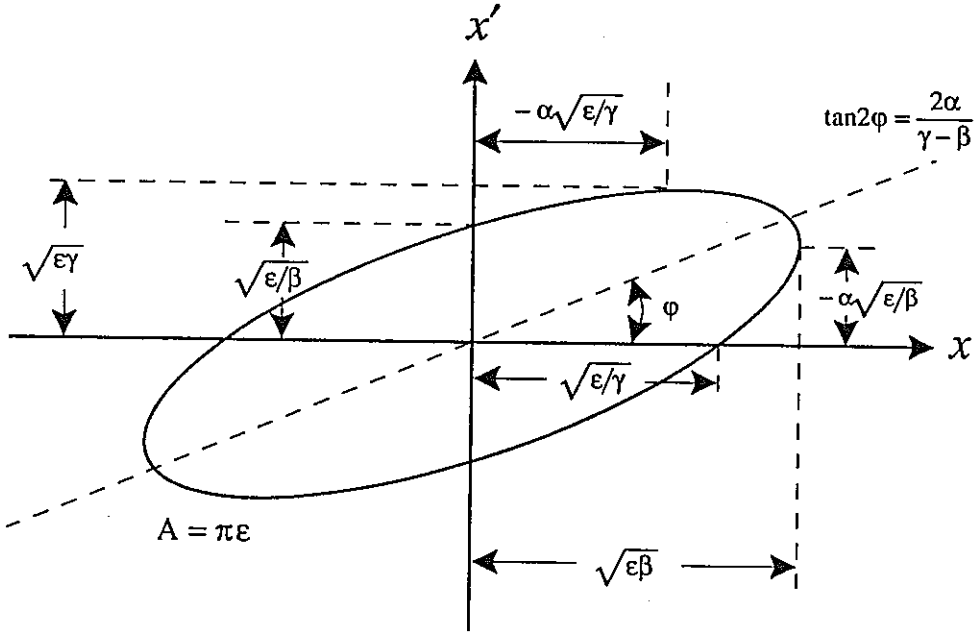

Fig. 3 Graphical representation of beta functions α, β and γ with the beam envelope in phase space. The cross section of the beam envelope is assumed πε.

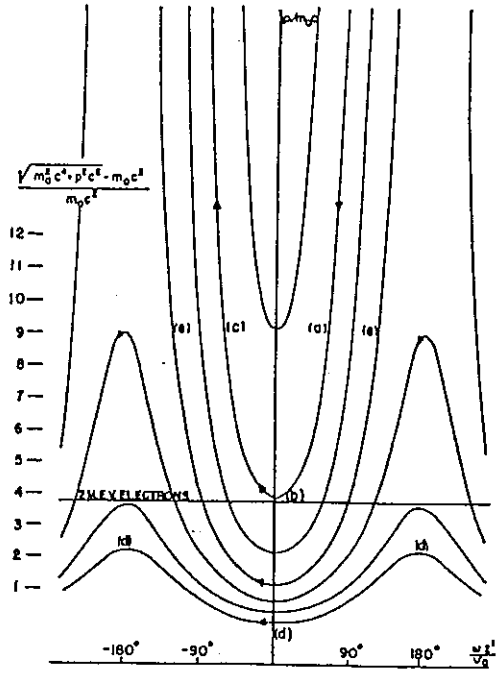

Fig. 4 Deformation of beam orbit for phase space in the case of relativistic particle. This picture is referred from [5].

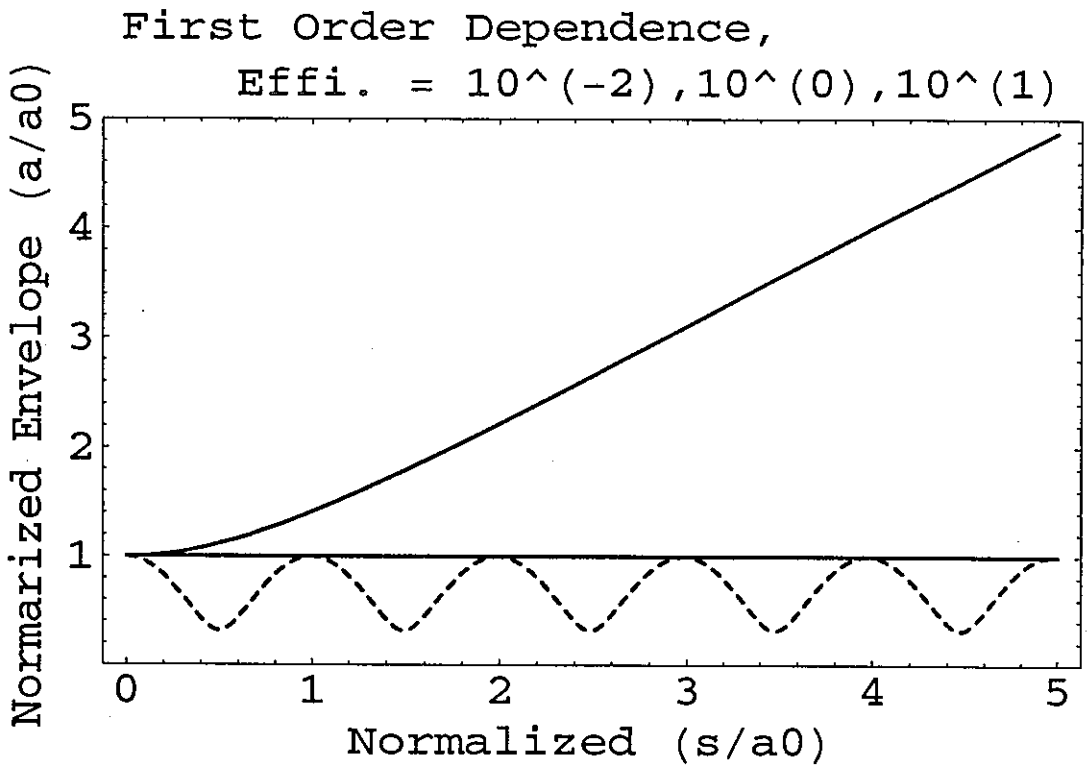

Fig. 5 Contribution of Δ in envelope equation in accelerating beam. Δ is 10^{-7} (dashed line), 10^{-8} (Parallel) and 10^{-10} (diverging) and the emittance is 10 π mm mrad with no space charge effect. Both s and envelopes are normalized by a_0 .

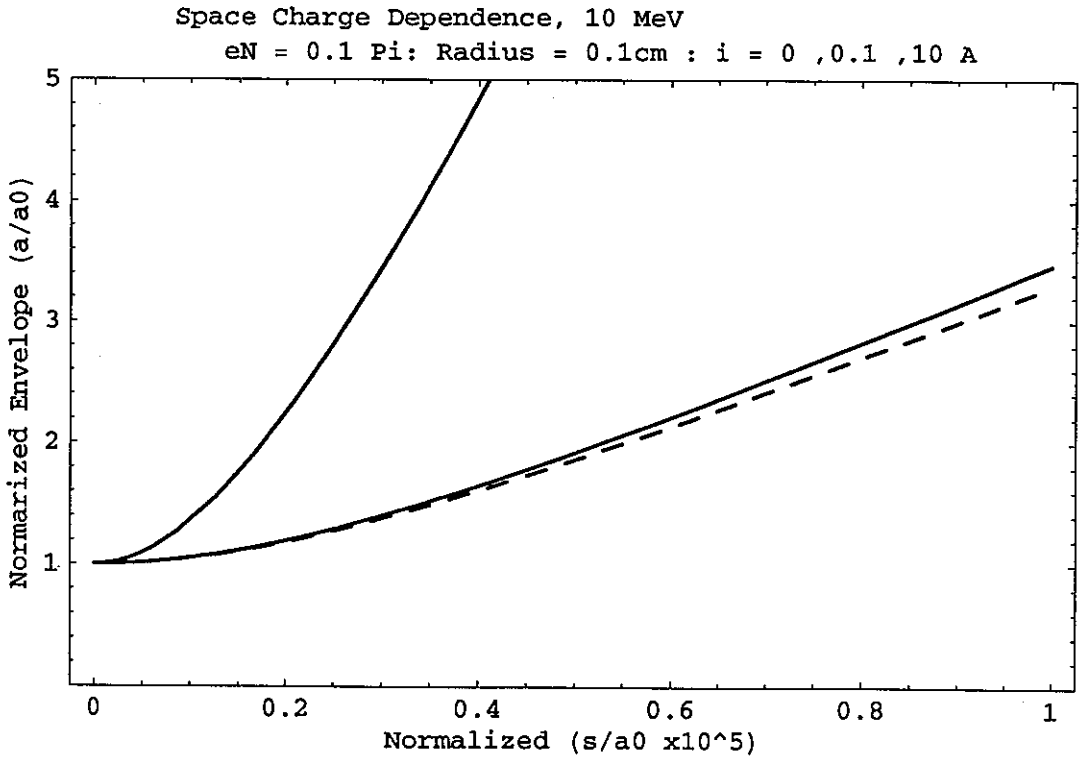

Fig. 6 Beam envelopes of 10 MeV acceleration with 0.1 and 10 A electron beam with the beam emittance 0.1 π mm mrad. The dashed line corresponds to no space charge effect. Flight paths are normalized by initial waste radius a_0 and multiplied by 10^5 here.

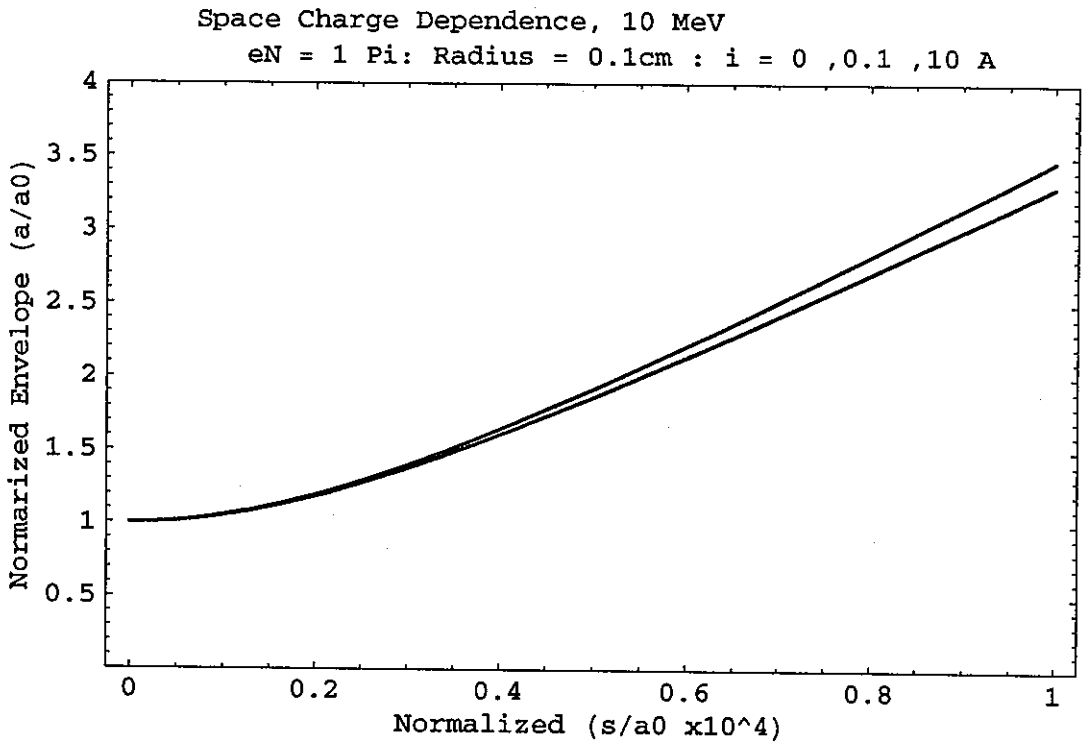

Fig. 7 Beam envelopes for 10 MeV acceleration and π mm mrad emittance. The solid lines for 0.1 (lower) and 10 A (upper) beam are presented and the line for 0 current is almost in accordance with 0.1 A line. Flight paths are normalized by 10^4 in this figure.

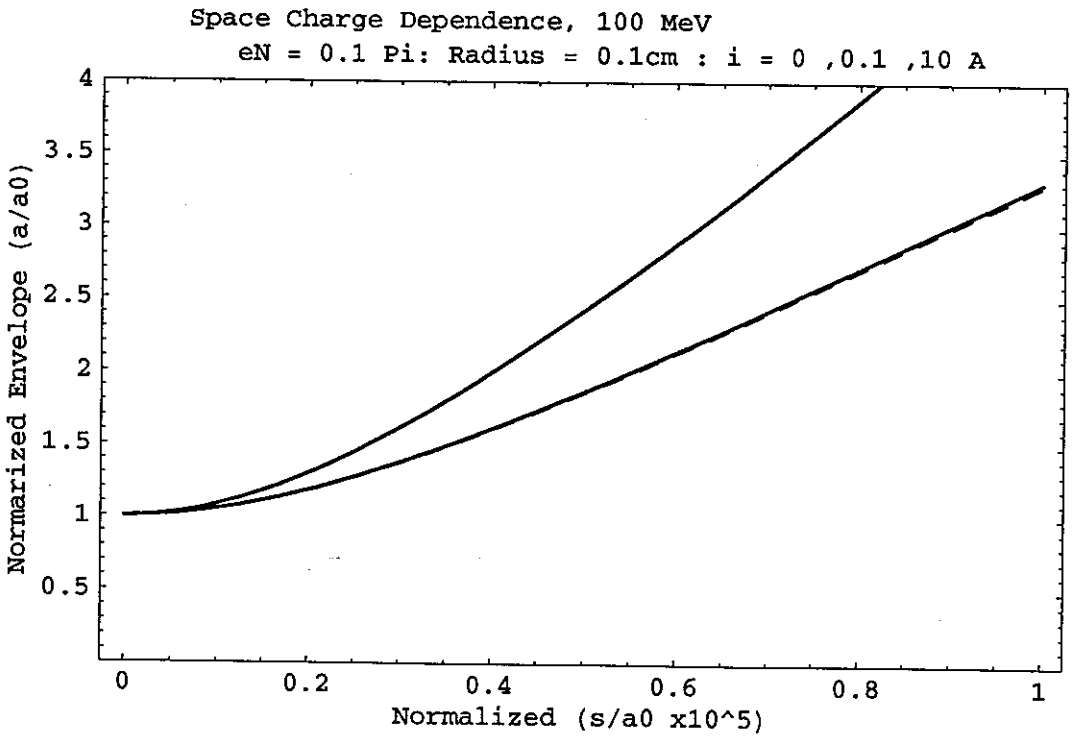

Fig. 8 Beam envelopes for 100 MeV acceleration and π mm mrad emittance. The solid lines for 0.1 (lower) and 10 A (upper) beam are presented and the line for 0 current is naerly in accordance with 0.1 A line. Flight paths are normalized by 10^5 in this figure.

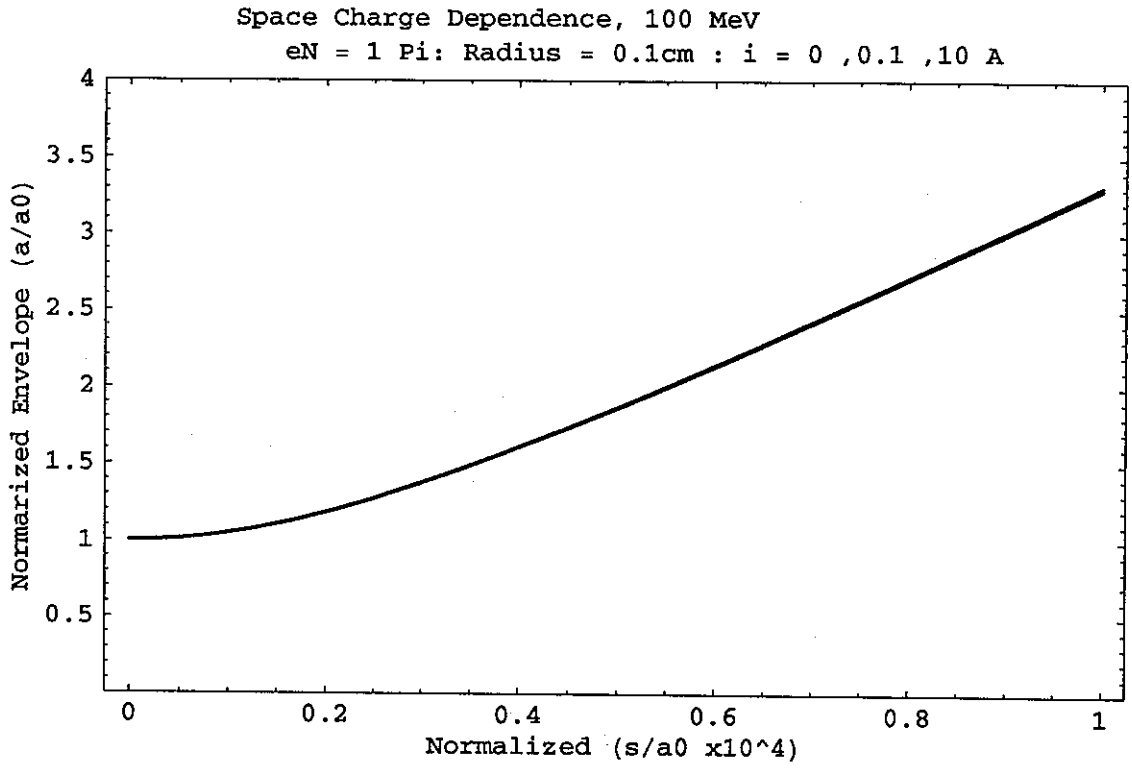

Fig. 9 Beam envelopes for 100 MeV acceleration and π mm mrad emittance. All lines for 0, 0.1 and 10 A beam are closing together. Flight paths are normalized by 10^4 in this figure.

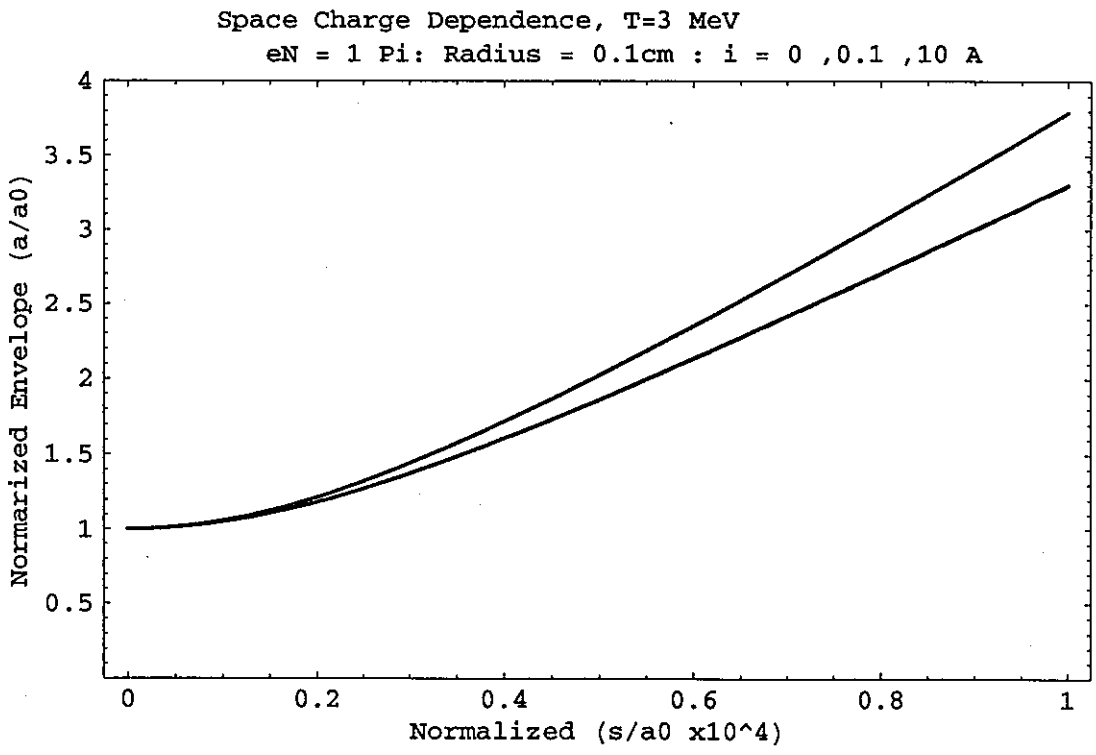

Fig.10 Beam broadening by space charge for 3 MeV injector of PNC linac. The emittance is π mm mrad here in order to make the differense clear, although the designed value is 10 π for the electron gun.

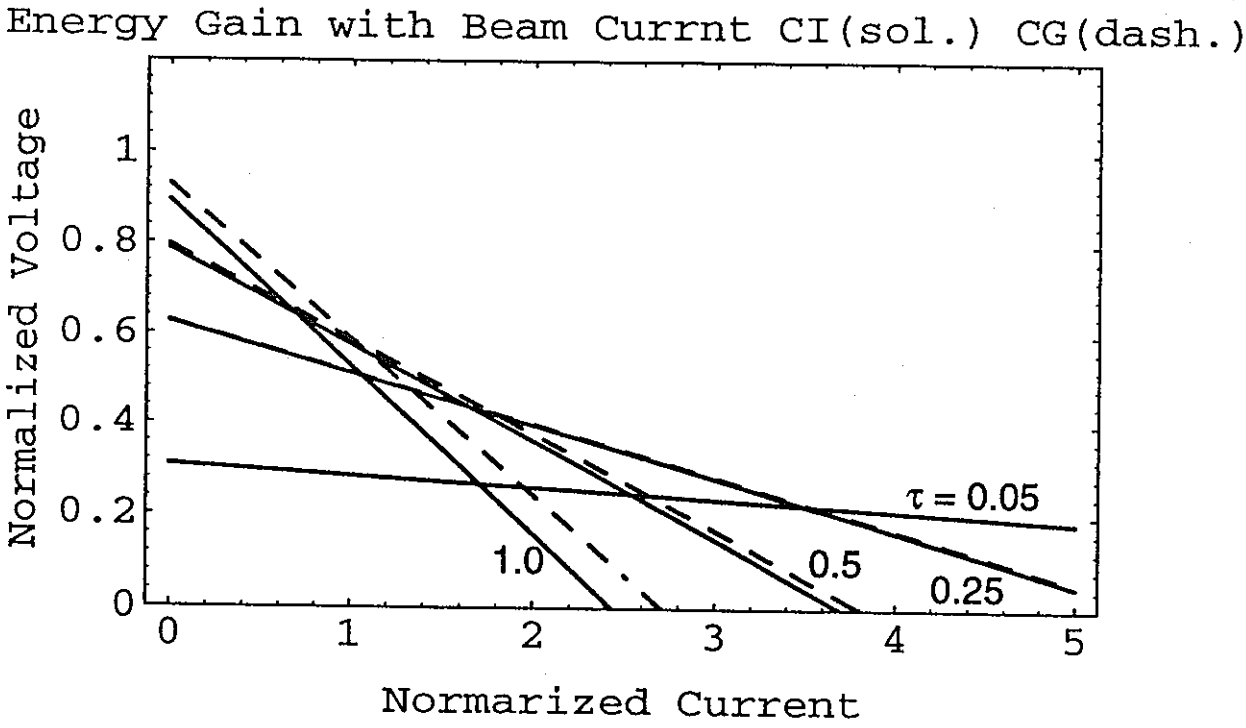

Fig.A1 Current dependence of gradient on traveling wave CI and CGstructure. A normalized current $I(r_sL/P_0)^{1/2}$ and a normalized voltage $V/(r_sL/P_0)^{1/2}$ are used with the variation of the attenuation parameters.

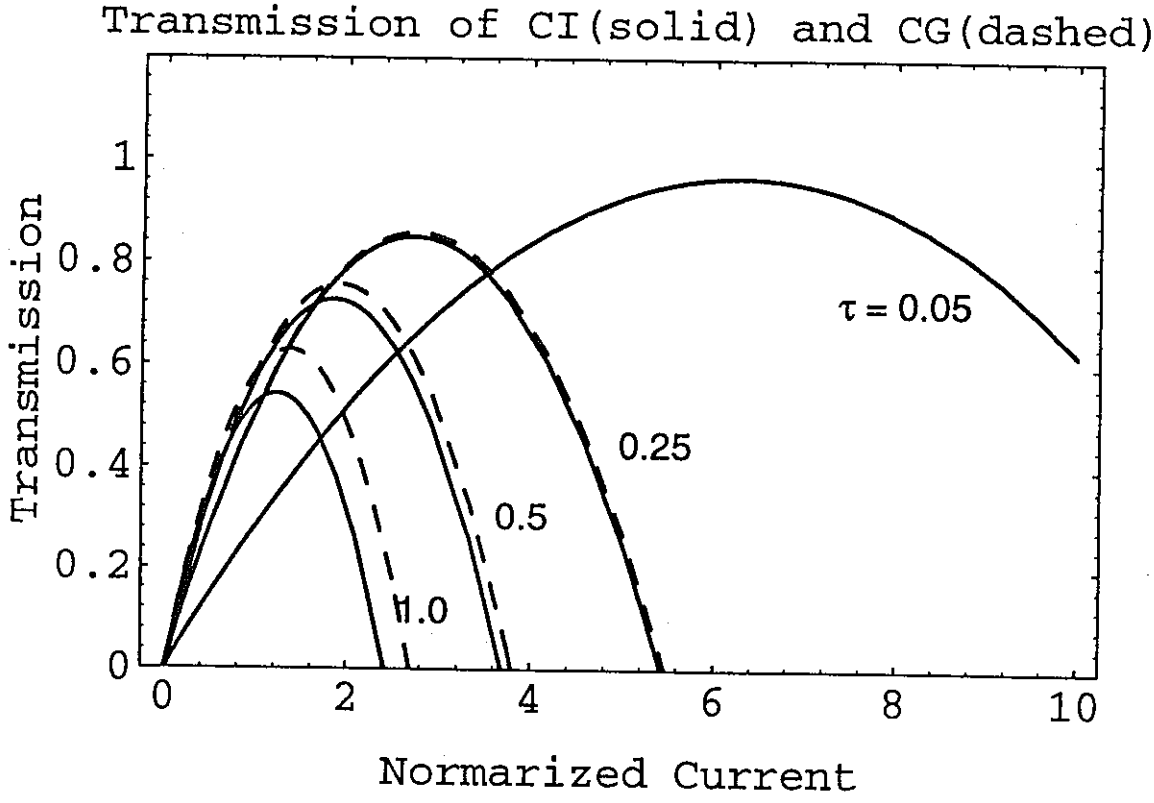

Fig. A2 Transmission efficiency for both traveling wave CI and CG in variation of the attenuation. Both types have almost the same magnitude of the efficiency.

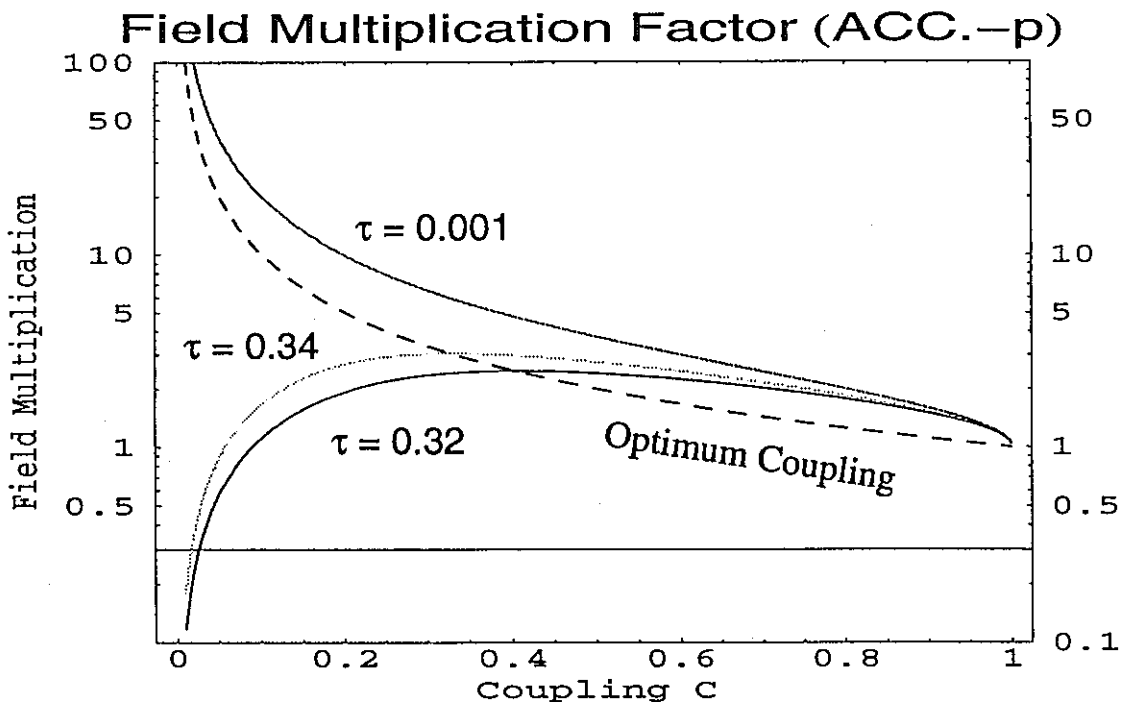

Fig. A3 Characteristics of field multiplication versus coupling, employing the parameters for PNC linac TWRR.

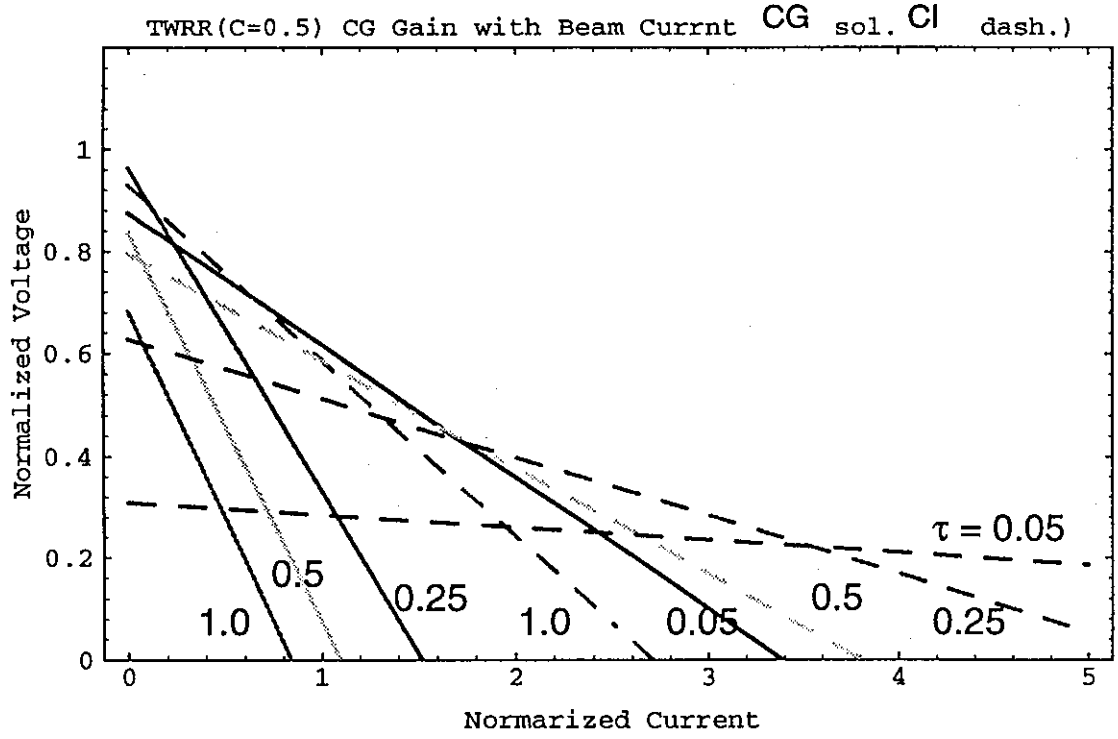

Fig. A4 Energy gain dependence of TWRR due to normalized current mentioned. Around unit current area, the gain is increased compare with ones from the ordinary traveling wave.

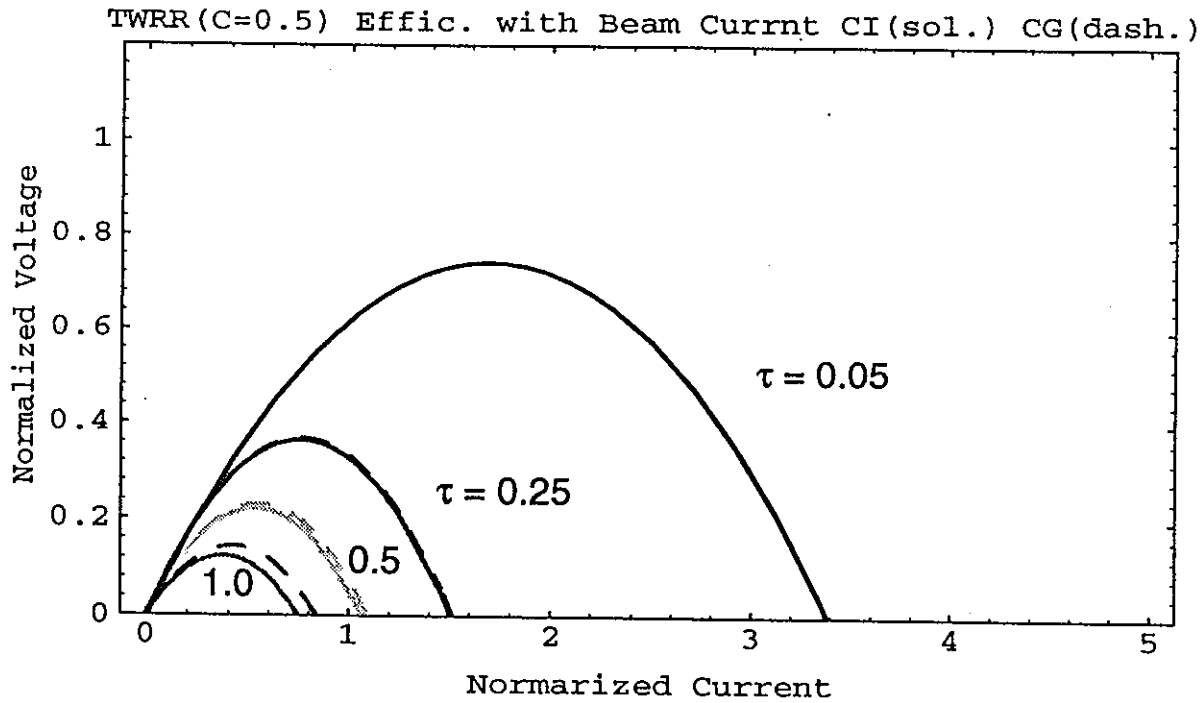

Fig. A5 Transmission efficiency of TWRR which is using normalized current and voltage, where the curves for both a CI and CG are depicted.

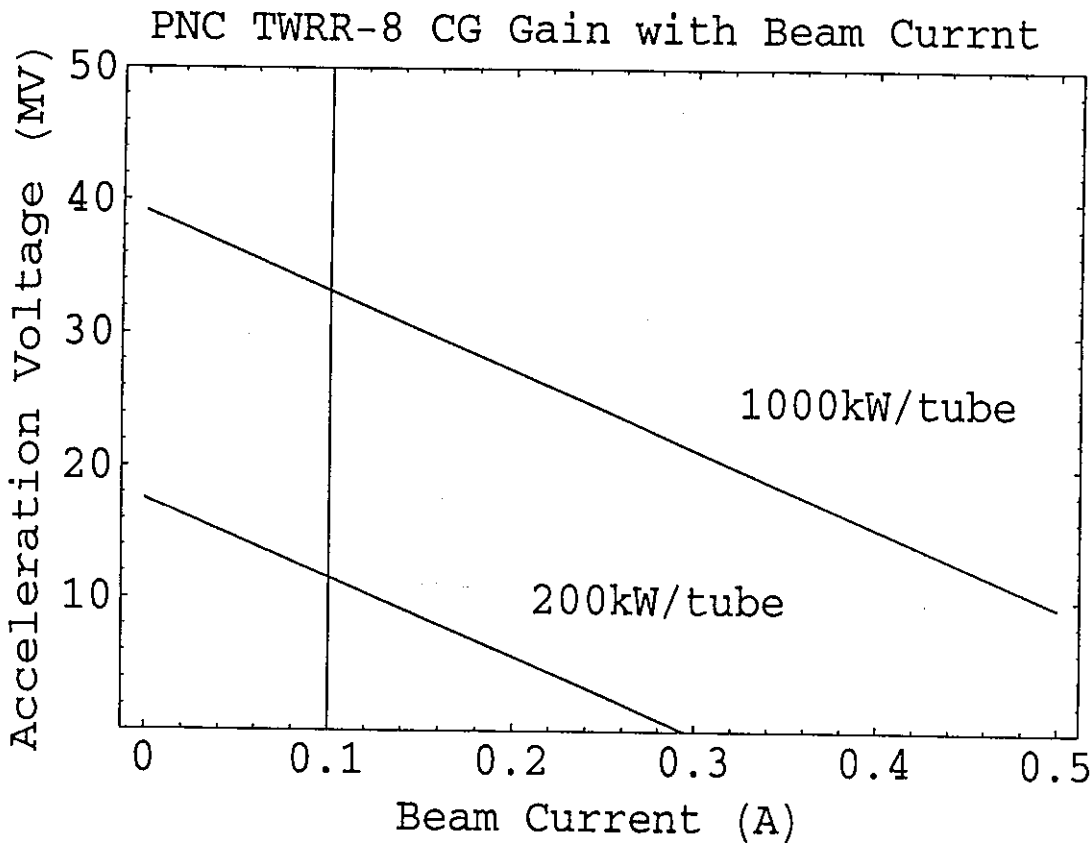

Fig. A6 Total beam loading characteristics for 200 kW and 1 MW RF inputs into each TWRR units in PNC. Vertical line inside means 0.1 A.