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A study of transverse beam break up instability

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POWER REACTOR AND NUCLEAR FUEL DEVELOPMENT
CORPORATION
Brookhaven National Laboratory

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横方向ビーム不安定性の研究

野村昌弘*、高橋博**

要旨

現在、事業団では大強度CW電子線形加速器の開発を行っている。この加速器の利用としては、核変換の研究、自由電子レーザ、陽電子源等が考えられている。大電流ビームを加速するときの一番大きな問題の一つはbeam break up(BBU)である。特に、長いパルスビームを加速するときには Cumurative BBUが一番の問題となってくる。動燃の電子線形加速器では、加速周波数にLバンドを選び、各加速管の高次の周波数を変えることによりBBUを押さえている。

今回の報告書では、高次のWake Field まで含んだ計算を行いその影響について調べた。この高次のWake Fieldはそれほど大きくはないが、ビームの変位に与える影響は必ずしも小さくはない。計算の結果、この影響でビームの変位に非対象性が生じることが判明した。

更に今回の報告書では、BBUの制御についても検討した。その結果、動燃でも採用している、各加速管の高次の周波数を変えることによりBBUを制御できることがわかった。しかしこの場合も、高次のWake Field の影響のため、ビームの前後でビームの変位が変わってしまうことが判明した。これを補正するため、外部の収束システムが必要となる。

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A study of transverse beam break up instability

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Abstract

Presently, Power Reactor and Nuclear Fuel Development Corporation (PNC) is developing a high-power high-duty electron linac for various applications including the transmutation of fission products, the Free Electron Laser, a positron source, and so on. When the high intensity beam is accelerated, beam break up (BBU) instability is one of the biggest problems. Especially, in the case of long pulse acceleration, cumulative BBU instability is the big problem. In PNC electron linac, to increase the threshold current level of regenerative BBU instability, L-band frequency is selected for accelerating frequency. And to suppress the cumulative BBU, every accelerator section is designed with the same frequency of TM01 mode, but with the different frequency of TM11-like mode.

We study transverse BBU instability by numerical calculations which include the higher deflecting modes of transverse dipole wake fields. The purposes of this report is to investigate the effects of second and third deflecting mode for transverse BBU instability. The amplitude of the higher deflecting modes are not so large, but the effects by those modes for the transverse displacements is not small. The asymmetry in the transverse displacement is occurred by the higher deflecting modes.

We also study the control of the BBU growth. As a result, the frequency spreads of deflecting modes can reduce the BBU growth But the second and third deflection mode have a large effect for the transverse displacement even in the low Q value case so that the displacement of the trailing bunches is away from the displacement of leading bunches. This means that the beam position must be adjusted by the external focusing system.

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1. Introduction

Presently, Power Reactor and Nuclear Fuel Development Corporation (PNC) is developing a high-power high-duty electron linac for various applications including the transmutation of fission products, the Free Electron Laser, a positron source, and so on. When the high intensity beam is accelerated, beam break up (BBU) instability is one of the biggest problems. Especially, in the case of long pulse acceleration, cumulative BBU instability is the big problem. In PNC electron linac, to increase the threshold current level of regenerative BBU instability, L-band frequency is selected for accelerating frequency. And to suppress the cumulative BBU instability, every accelerator section is designed with the same frequency of TM01 mode, but with the different frequency of TM11-like mode1). This means the deflecting modes of each cavity have the frequency spreads. Main specification of the PNC high-power high-duty electron linac is shown in Table 1.

Energy	**10 MeV**
Pulse Length	**4 m sec**
Pulse repetition	**50 Hz**
Duty Factor	**20 %**
Accelerator	**traveling-wave (TWRR)**
Accelerating Frequency	**1249.135 MHz**
Accelerating Mode	**2π/3 mode**
Number of accelerator tube	**7**

Table.1. PNC linac main specification

In this report, we study transverse BBU instability by numerical calculations. The transverse dipole wake fields are responsible for the transverse BBU instability. The basic analysis of a wake field interaction in TWRR(Traveling Wave Resonant ring) for PNC electron linac is surveyed quantitatively by S.Toyama *et. al*2). In that report, the physical picture for the wake field, numerical analysis of wake potentials for accelerator structure for PNC electron linac and BBU instability starting current are mentioned.

Generally, wake fields have a lot of deflecting modes, but previous works3),4) treated only one main deflecting mode. The purposes of this report is to investigate the effects of second and third deflecting mode for transverse BBU instability. To include some deflecting modes, the displacement and angle of each electron bunch in each cavity are calculated by a transfer matrix method. By using this method, accelerating beam can be calculated exactly. The transverse dipole wake fields are calculated by ABCI code5),6).

We also study the control of BBU growth. This can be achieved by restricting the transverse motions of electron bunches through external focusing, (e.g., solenoidal, quadrupole focusing) or by modification of the deflecting modes, (e.g., lowering of the Q value of the deflecting modes and using stagger tuning3),4). It is reported that the stagger tuning is good for the reduction of BBU growth in a long pulse beam. In this report, the effects of stagger tuning are examined.

2. Equation of motion

We now consider tracking the electron bunches through accelerator. At first we consider two particle model. In this model, the leading bunch is unperturbed by its own transverse dipole wake field and the trailing bunch feels a deflecting wake field left behind by the leading bunch. The effect of deflecting wake field7) is

$$\frac{d^2x_2}{dz^2}=\frac{Nr_0W_1(z_{12})}{\gamma L}x_1, \tag{1}$$

where x_1 and x_2 are the transverse displacement of the leading and trailing bunch, respectively, N is the number of electrons in leading bunch, γ is the total electron energy, r_0 is the classical electron radius, z_{12} is the distance between the leading and trailing bunch, W_1 is the transverse dipole wake function for one cavity period, and L is the cavity period. The dimensionality of W_1 is [1/cm2] in cgs units.

Next, we consider many particle model. In many particle model, one bunch feels a deflecting wake fields by all previous bunches. Finally, the N-th bunch displacement and angle in the M-th cavity can be calculated by following equations8)

$$x_M{}^{N+1}=M_{11}x_M{}^N+M_{12}\frac{\gamma_N}{\gamma_{N+1}}\theta_M{}^N+M_{12}R, \tag{2}$$

$$\theta_M^{N+1} = M_{21} x_M^N + M_{22} \frac{\gamma_N}{\gamma_{N+1}} \theta_M^N + M_{22} R, \tag{3}$$

$$R = \sum_{k=1}^{M-1} \frac{N r_0 W_1 (z_{kM})}{\gamma_{N+1}} x_k^N, \tag{4}$$

$$W_1 (z_{kM}) = \sum_j \left(\frac{c R_j}{Q_j} \right) e^{\left(-\frac{\omega_j z_{kM}}{2 c Q_j} \right)} \sin \left(\frac{\omega_j z_{kM}}{c} \right), \tag{5}$$

$$M_{11} = \cos (k_b) + \alpha \sin (k_b), \tag{6}$$

$$M_{12} = \beta \sin (k_b), \tag{7}$$

$$M_{21} = -\gamma \sin (k_b), \tag{8}$$

$$M_{22} = \cos (k_b) - \alpha \sin (k_b), \tag{9}$$

where M is the transition matrix from cavity to cavity, α, β and γ are the twiss parameters, k_b is the betatron wave number, c is the light velocity, R_j, Q_j and w_j are the shunt impedance, Q value and angular frequency of the j-th deflecting mode of transverse dipole wake field, respectively. To take account of accelerating beam and some deflecting modes, bunch displacement and angle are calculated numerically by above equations.

3. Effects of higher order modes and frequency spread

At first, PNC accelerator tube structure and real part of transverse dipole impedance are shown in Fig. 1. and 2. respectively. In PNC electron linac, each accelerator tube has fifteen cavities. The real part of transverse dipole impedance is calculated by ABCI code. The electron bunch is assumed to be Gaussian bunch. The wake fields are described as a function of time after the passage of a δ-function beam. The wake fields and the impedance are related by Fourier transforms so that the impedance contains the information of the frequency components of deflecting modes. Fig. 2. shows many deflecting modes of transverse dipole wake fields. But when the transverse BBU instability was studied, previous works treated only one main deflecting mode.

Next, to study the effect of higher deflecting modes, each bunch displacement at the end of accelerator is calculated on two conditions, one includes only one main deflecting mode and the other includes second and third one. The calculation results are shown in Fig.3. and 4., respectively. In both Fig. 3. and 4., deflecting wake fields left behind by the leading bunches deflect the trailing bunches so that the displacement of the trailing bunches becomes larger and larger. But after cavity number about 180, bunches displacement becomes smaller and smaller, because wake fields have the damping factor $\exp(-\omega z/cQ)$. The position of the maximum bunches displacement in the bunch number is strongly depend on the Q value.

The amplitude of the second and third deflecting mode are not so large in Fig. 2., but the effects by those modes for the bunches displacement is not necessarily small. Because the wake field which the trailing bunches feel is not only depend on the amplitude but also depend on the frequency and bunch separation in time

(See eq.5.). Comparing Fig.3. and Fig. 4., the effects of the higher deflecting modes are understood. In Fig. 4., the asymmetry around bunch number 160 is occurred by the second and third deflecting mode.

We also study the frequency spread in the deflecting modes. In PNC electron linac, this stagger tuning is used for controlling the BBU growth. In previous works[3],[4], frequency spread can control the BBU growth. In this case, the higher deflecting modes have a large effect on bunch displacement, too. Each bunch displacement is calculated in two cases, all frequencies are not spread and each frequency increases linearly from 97.5% to 102.5%. Both cases include the second and third deflecting mode. Those results are shown in Fig.5 and 6, respectively. In these calculations, the high Q value were used so that the damping effect was excluded in order to investigate the pure frequency spread effects. In Fig. 5., large transverse BBU instability is observed. It is clear from Fig.5. and 6. that the frequency spreads can reduce the BBU growth. This result agree with Ref. 3). And it is worth noting that the asymmetry is occurred in Fig. 6. so that the displacement of the trailing bunches after bunch number 100 is away from the displacement of leading bunches. This phenomena is not reported in previous works. We calculated in the case of only one deflecting mode, too. This result is shown in Fig.7. In Fig. 7., the asymmetry isn't observed. It is clear from Fig. 6. and 7. that this phenomena is caused by the second and third deflecting mode. This means that frequency spread of deflecting modes can reduce the BBU growth but the displacements of the bunch in the steady state are away from the leading bunches so that the external focusing system is needed to adjust the beam position.

We also study the case of the low Q value. The Q value is approximately inverse proportional to the square of frequency so that the second and third deflecting mode became small faster than main one. The results are sown in Fig.

8. and 9. The Q-value of main, second and third deflecting mode is assumed to be 166, 141 and 116. In Fig. 8., the asymmetry is still observed and the displacement of the trailing bunches is away from the displacement of leading bunches. In short a word, even in the case of low Q, the second and third deflection mode still have a large effect.

4. Summary

We study transverse BBU instability by numerical calculations which include the higher deflecting modes of transverse dipole wake fields. The amplitude of the higher deflecting modes are not so large, but the effects by those modes for the transverse displacement is not small. The asymmetry in the transverse displacement is occurred by those higher deflecting modes.

We also study the control of the BBU growth. The calculation results show that the frequency spreads of deflecting modes can reduce the BBU growth The stagger tuning is good for the reduction of the BBU growth. But the second and third deflection mode have a large effect for the transverse displacement even in the low Q value case so that the displacement of the trailing bunches is away from the displacement of leading bunches. This means that the beam position must be adjusted by the external focusing system.

5. Acknowledgments

Authors are grateful to thank Dr. X. Chen for useful advice of bunch displacement calculations and wishes to thank Dr. An Yu who is the visiting researcher from China for useful discussions.

6. References

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2) S.Toyama, *et. al.*, “Study on Stability of High current electron CW Accelerator ”, PNC report 1996
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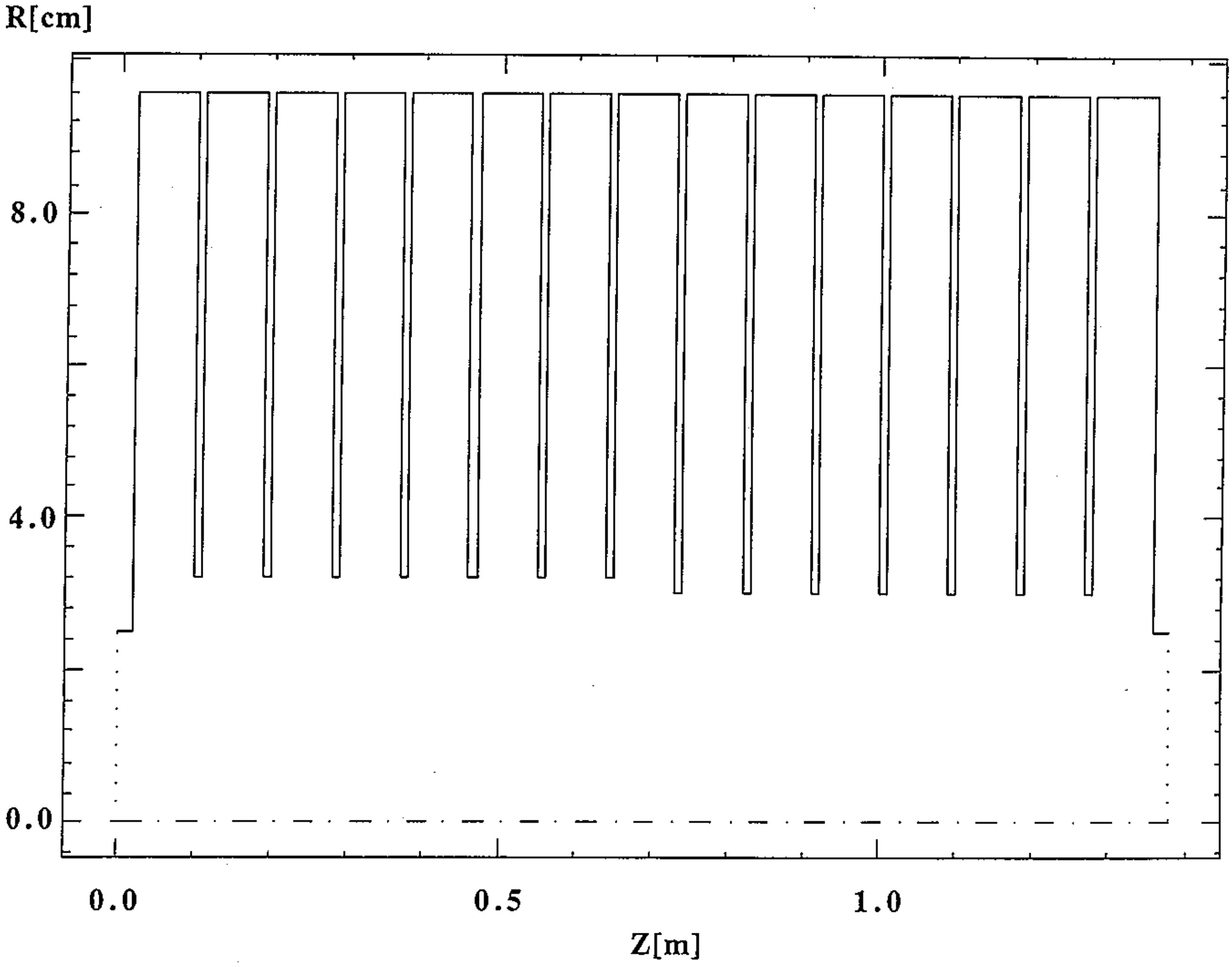

Fig. 1. PNC accelerator tube structure. Each accelerator tube is consist of 15 cavities.

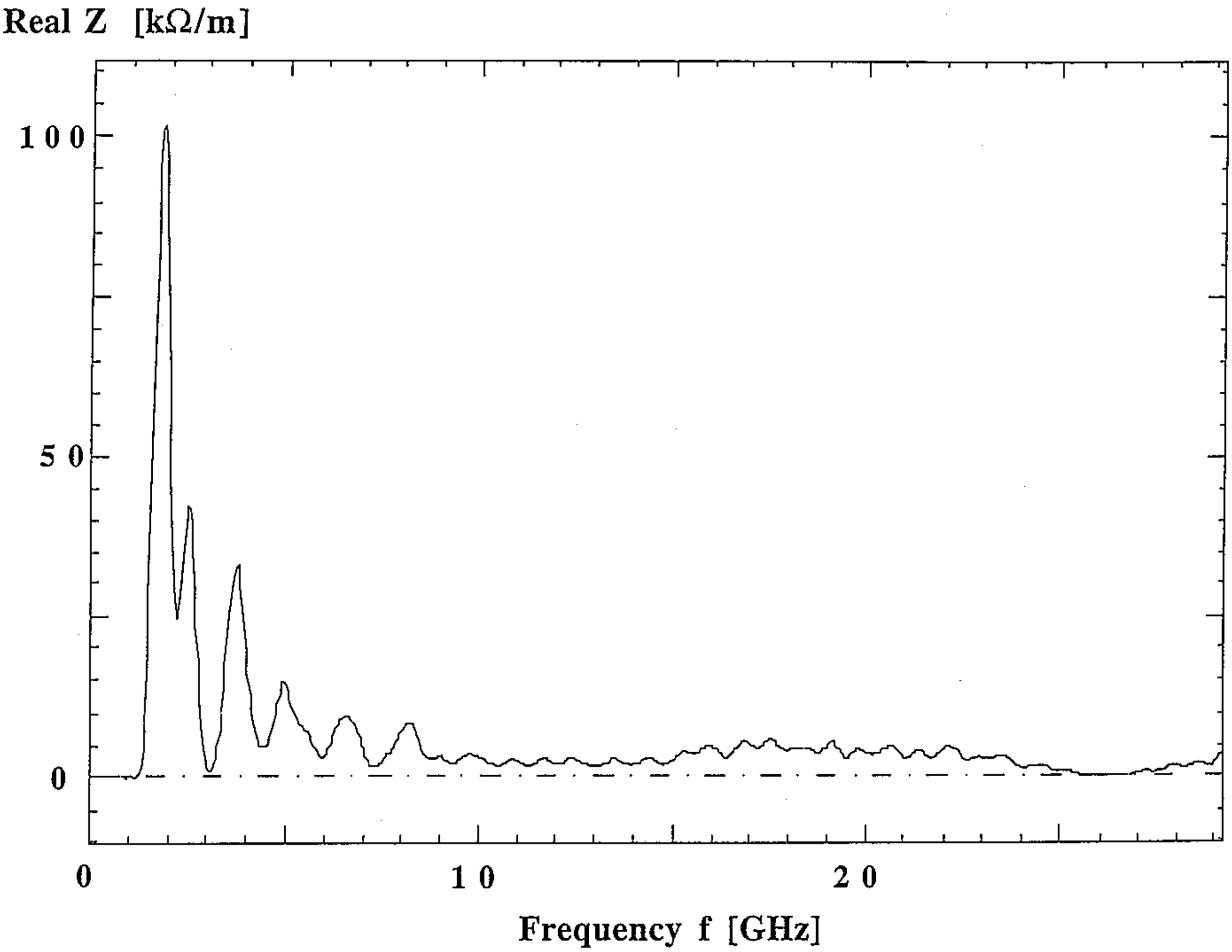

Fig. 2. Real part of transverse dipole impedance.
The frequency of the first, second and third deflection mode are 1.80270, 2.50505 and 3.72250 GHz, respectively. The spectrum is calculated by ABCI code. The electron bunch is assumed to be a Gaussian bunch. In this calculation, one standard deviation of bunch length is 4 mm.

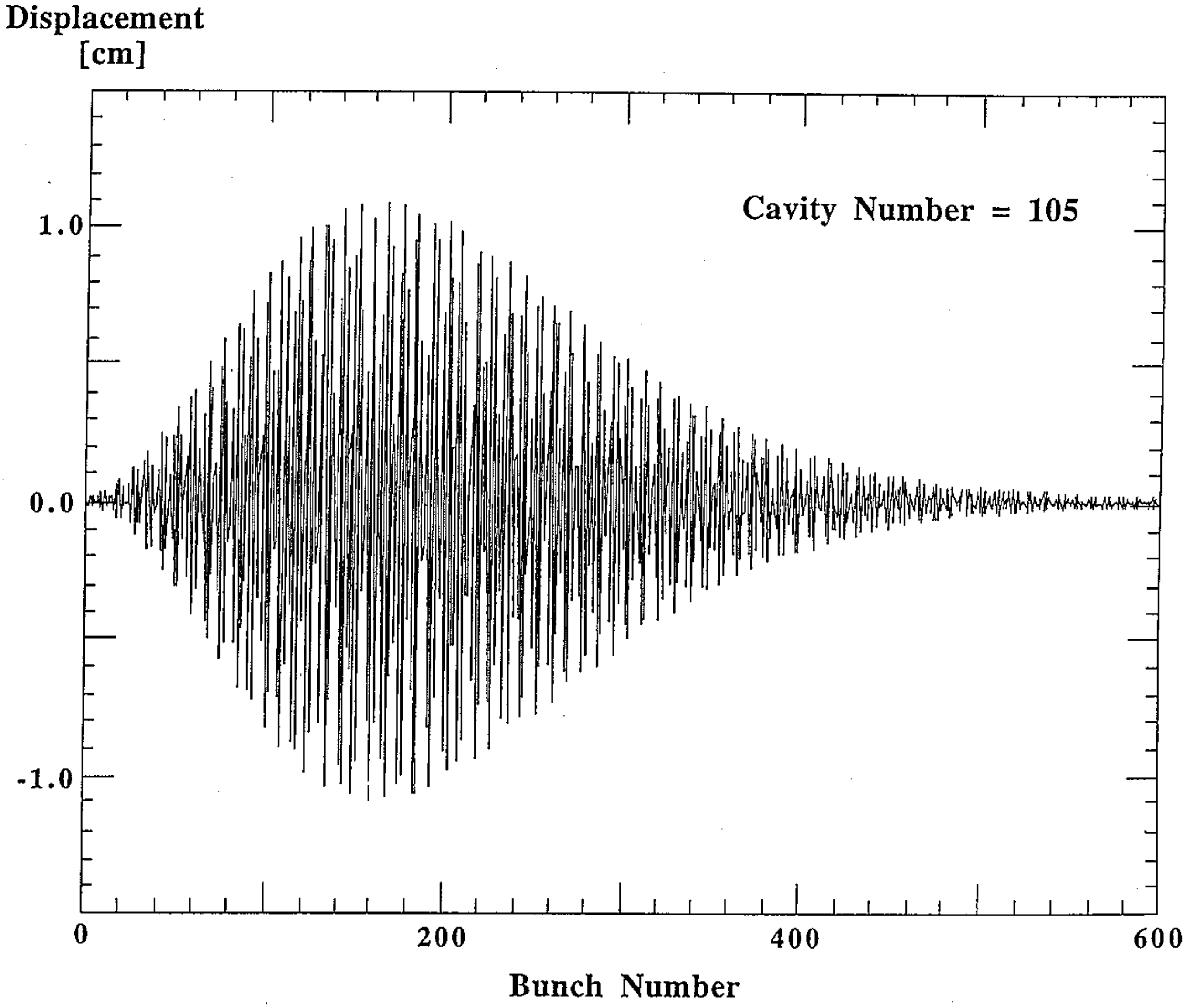

Fig.3. The displacement at the end of accelerator as a function of bunch number. This result includes only main deflecting mode of transverse dipole wake field which is calculated by ABCI code. The calculation conditions are shown below. The beam current is 5 [A], Q-value of main deflecting mode is assumed to be 166, betatron wave length is 10 [m], electrons are accelerated from 1.5 to 10 MeV through 105 cavities and all bunches are initially offset by 1 [mm].

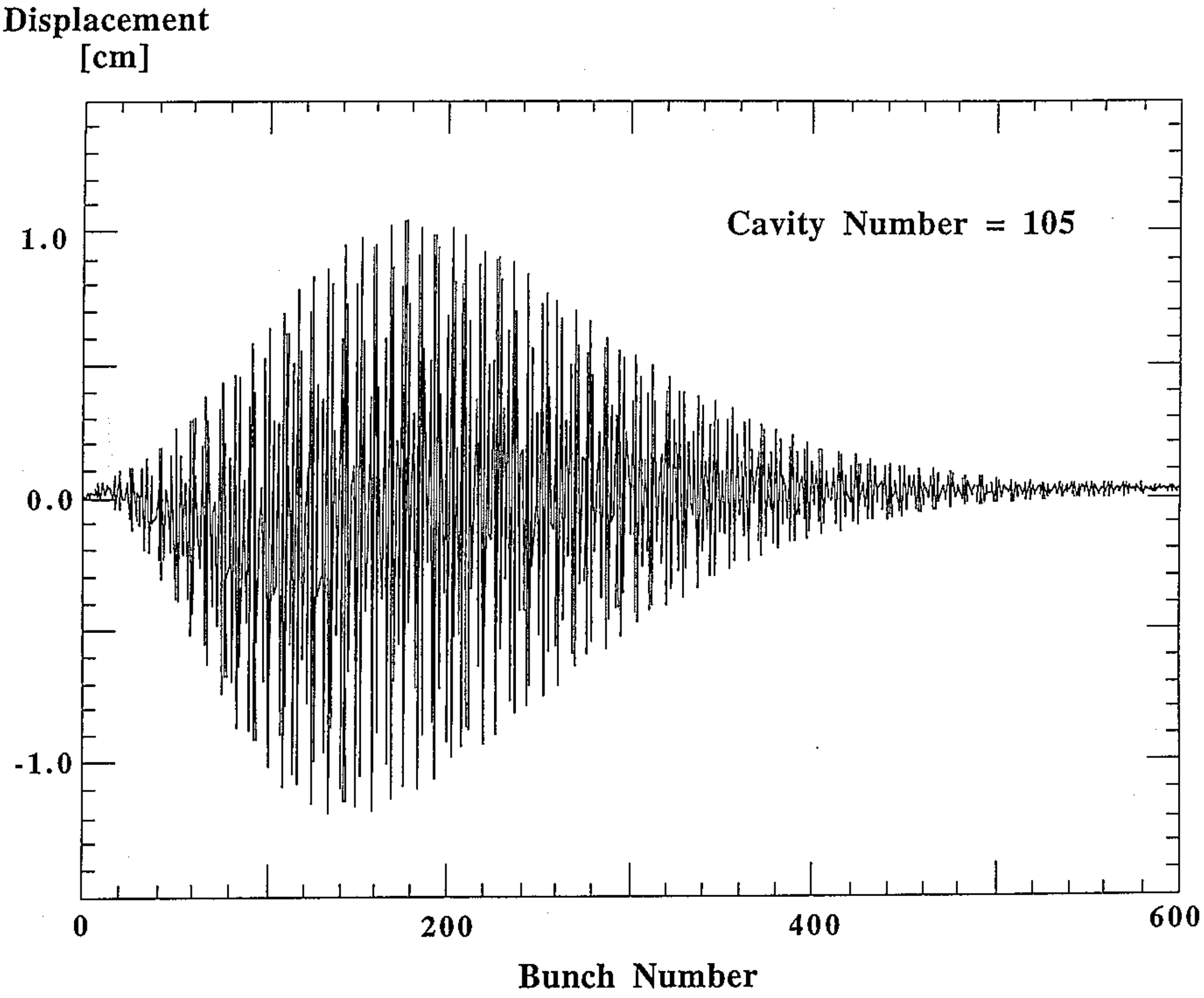

Fig.4. The displacement at the end of accelerator as a function of bunch number. This result includes not only main deflecting mode but also second and third one. The Q-value of main, second and third one is 166, 141 and 116, respectively. The Q value is assumed to be inverse proportional to the square of frequency. The other calculation conditions are same as Fig. 3. The asymmetry is occurred by the higher deflecting modes.

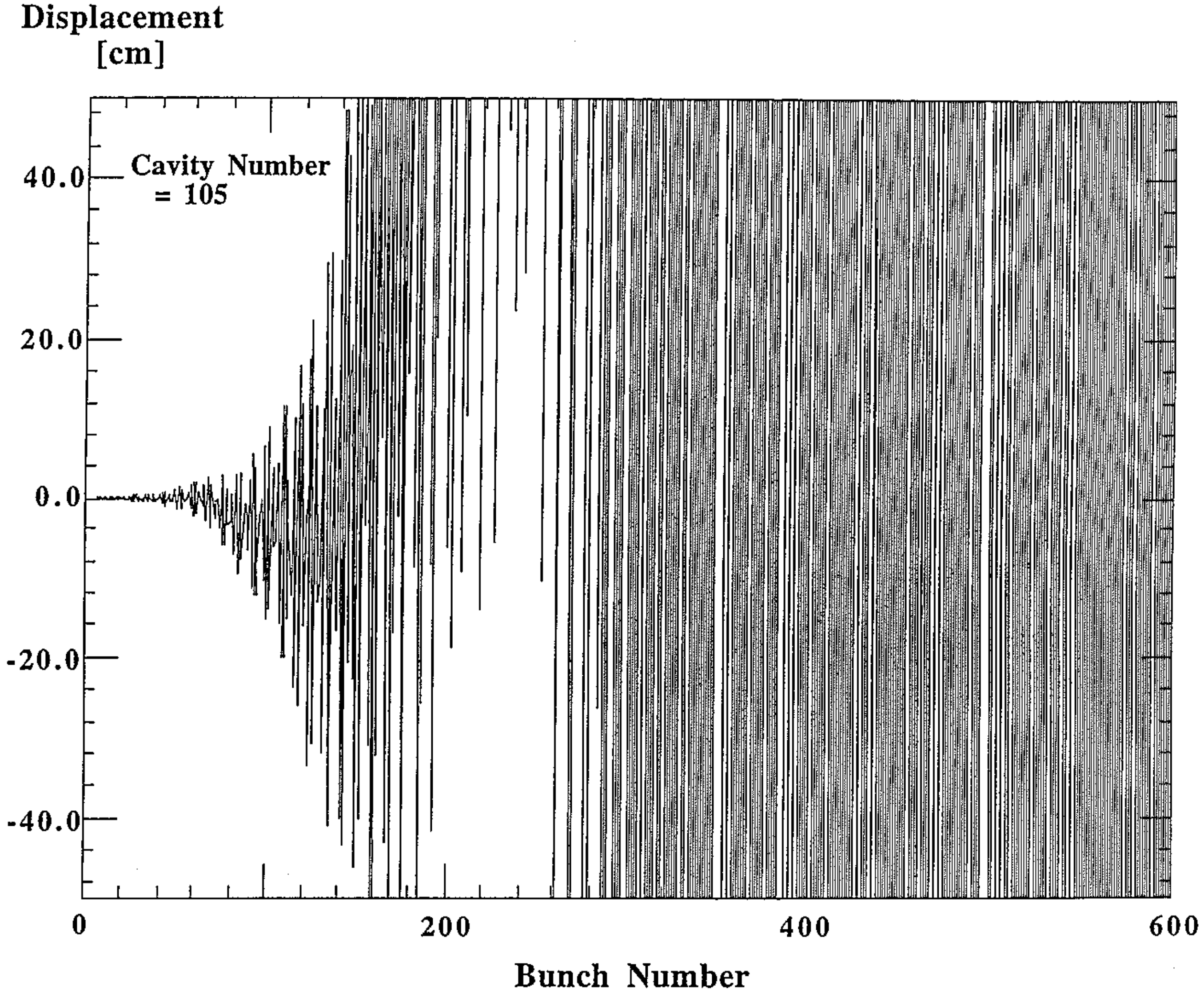

Fig.5. The displacement at the end of accelerator as a function of bunch number. This result includes main, second and third deflecting mode. To study the effect of frequency spread of the deflecting mode, high Q-value is used. The Q-value of main, second and third one is 8324, 7062 and 5793, respectively. In this result, each deflecting mode dose not have frequency spreads. The calculation conditions are shown below. The beam current is 5 [A], betatron wave length is 10 [m], electrons are accelerated from 1.5 to 10 MeV through 105 cavities and all bunches are initially offset by 1 [mm].

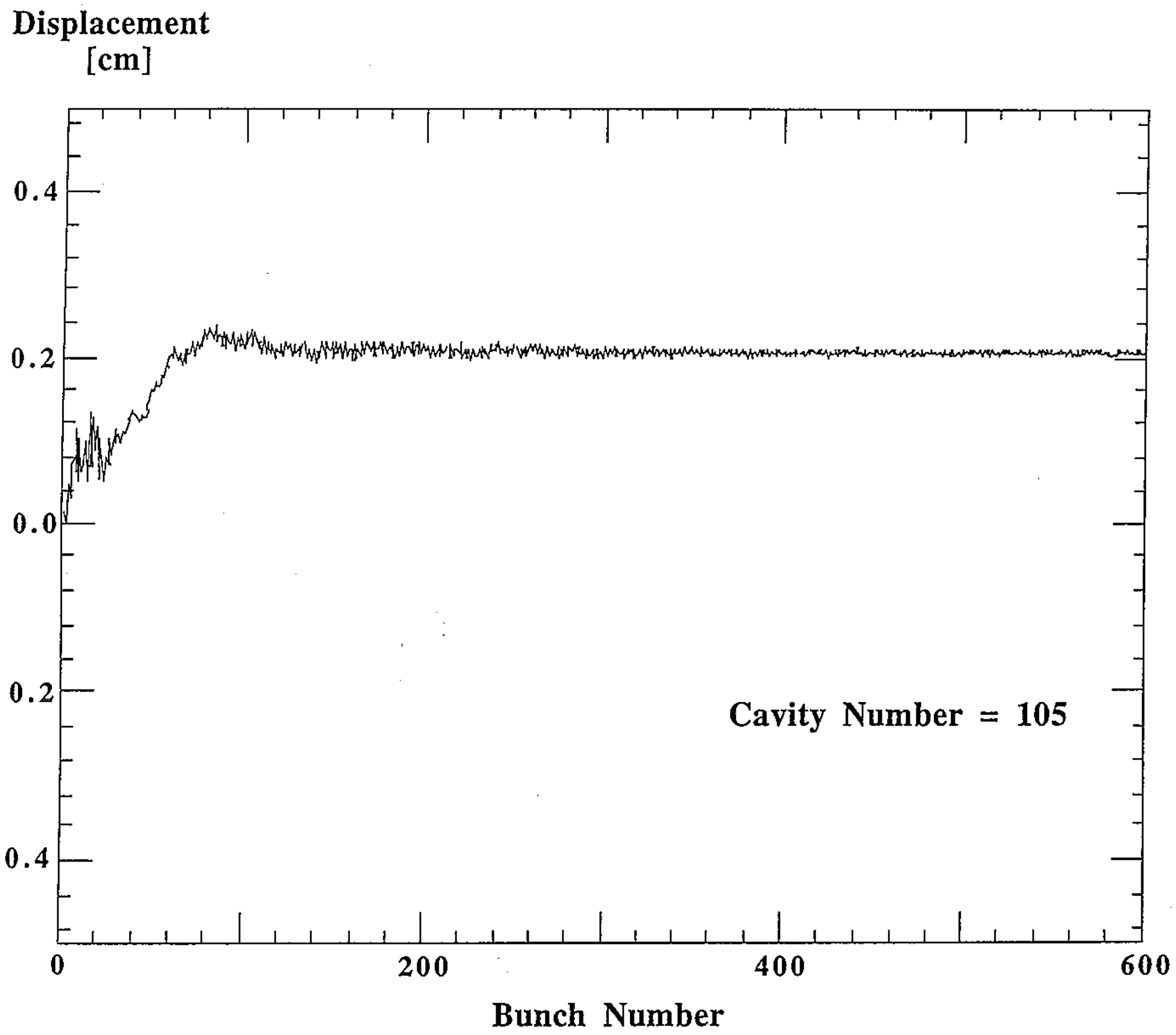

Fig.6. The displacement at the end of accelerator as a function of bunch number. This result includes main, second and third deflecting mode. In this result, the frequency of each deflecting mode increases linearly from 97.5% to 102.5%. It is clear that the frequency spread can reduce the BBU growth and the asymmetry is strongly occurred by higher deflecting mode. The other calculation conditions are same as Fig. 5.

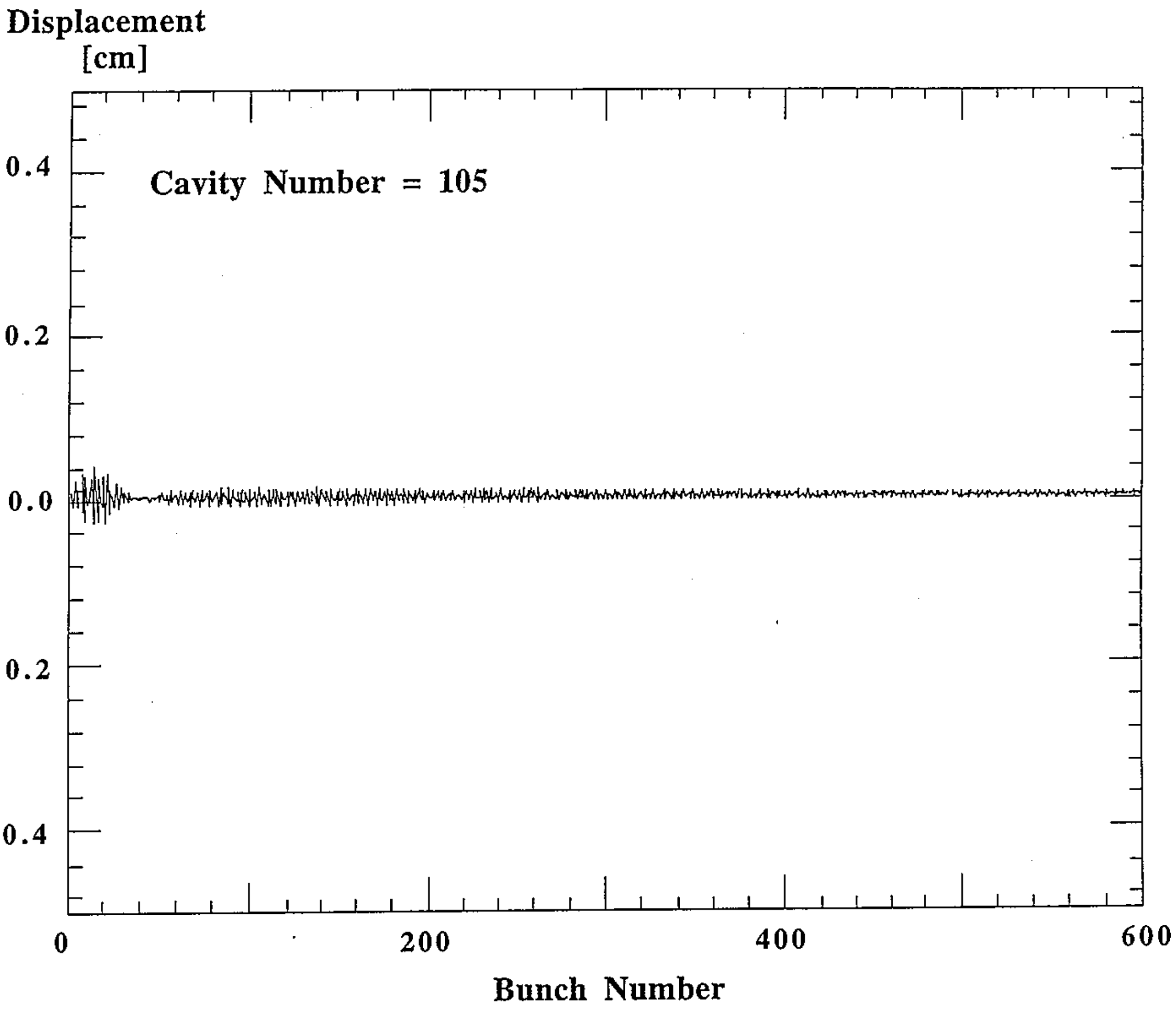

Fig.7. The displacement at the end of accelerator as a function of bunch number. This result includes only main deflecting mode. In this result, the frequency of each deflecting mode increases linearly from 97.5% to 102.5%. The frequency spread can reduce the BBU growth. The asymmetry isn't observed. The other calculation conditions are same as Fig. 5.

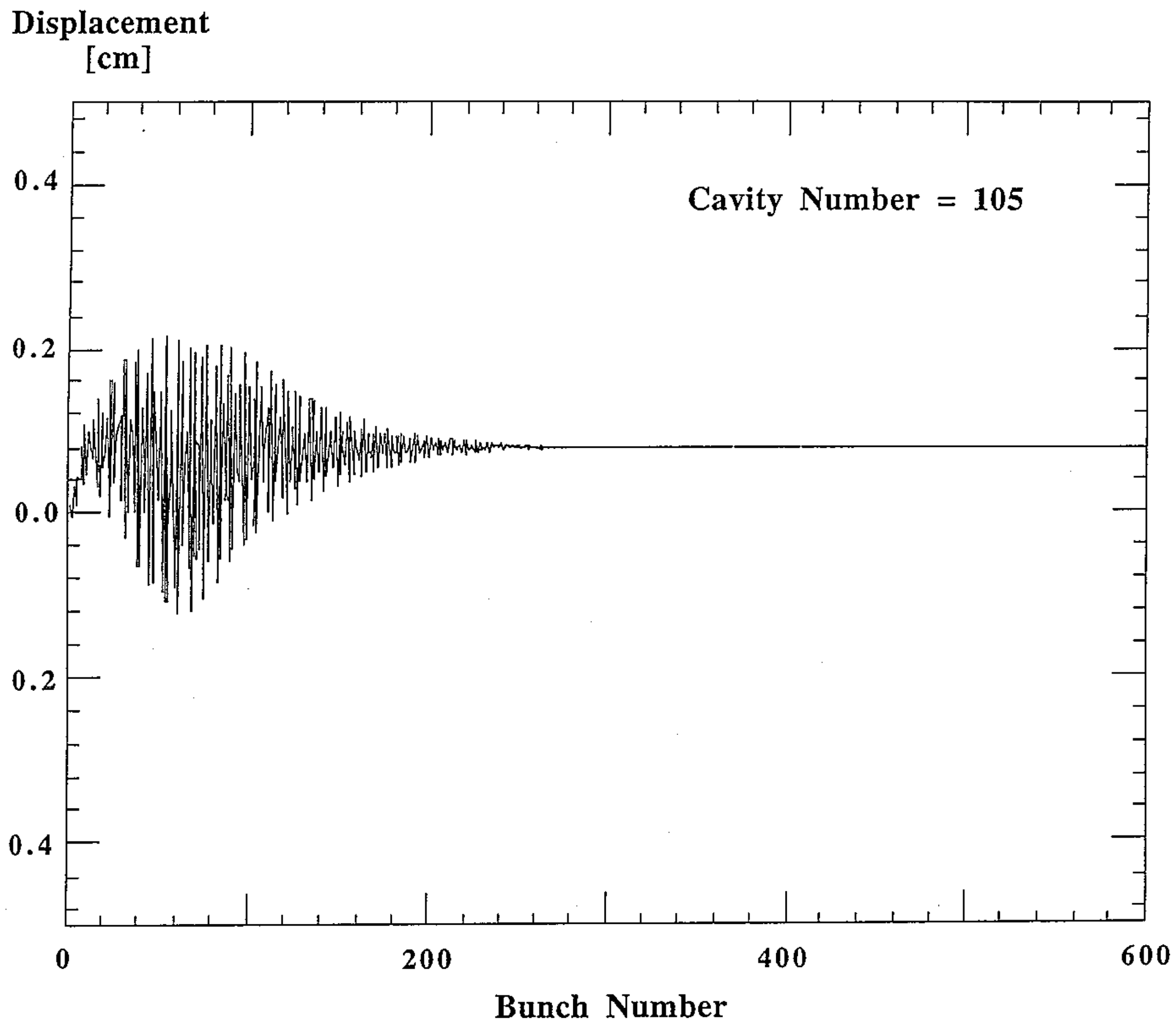

Fig.8. The displacement at the end of accelerator as a function of bunch number. This result includes main, second and third deflecting mode. The Q-value of main, second and third one is 166, 141 and 116, respectively. In the case of the low Q-value, the asymmetry is still observed. The other calculation conditions are same as Fig. 6.

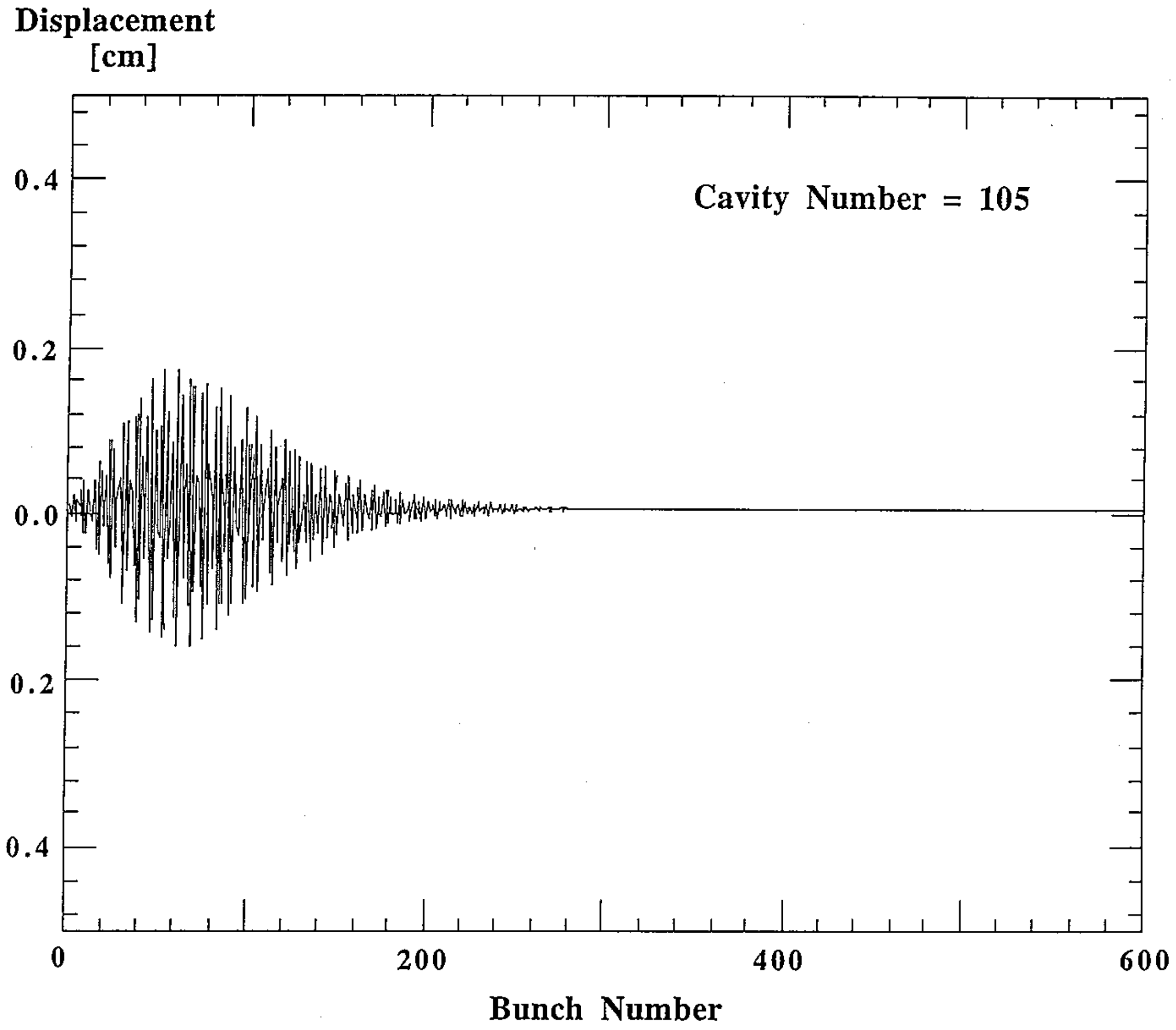

Fig. 9. The displacement at the end of accelerator as a function of bunch number. This result includes only main deflecting mode. The Q-value of main is 166. The asymmetry is not observed. The other calculation conditions are same as Fig. 6.

Appendix, Program List

```
c
c   wake4.for  for cheking 8/23/1996
c   kb is Changed 9/20/1996
c   Checking progrm
c
    implicit real*8 (a-h,o-z)
c
    parameter(maxnc=125,maxnb=20000)
    real*8 x(0:maxnc,maxnb),th(0:maxnc,maxnb)
    real*8 gamma(0:maxnc),wz(0:3),zq(0:3),f(0:3)
    real*8 tm(2,2),r(0:3),omega(0:3),mc2
c
c   electron classical radius e^2/(mc2) [cm]
    parameter(r0=2.813d-13,mc2=0.511d0,pi=3.14159)
    parameter(C=2.997d+10)
    real*8 ne,kb
c   cavity_length [cm]
    parameter(cavity_length=1.0d0)
    integer cavity_number
    parameter(cavity_number=125)
    parameter(xfact=2.0d0)
c
     open(99,file='bunch.top',status='unknown')
    write(99,*) 'set device tektronix "tek4010"'
    write(99,*) 'set limit y -3. to 3.'
    write(99,*) 'set limit x 0. to 2000.'
c
     open(98,file='cavity.top',status='unknown')
    write(98,*) 'set device tektronix "tek4010"'
    write(98,*) 'set limit y -5. to 5.'
    write(98,*) 'set limit x 0. to 125.'
c
c   Transition matrix
c
    alpha=0.0
c     wave length = 1000[cm]=10[m]
c     kb;[1/cavity],beta;[cm]
    kb=2.0*pi/(20.0d0/cavity_length)
c
    call tmatrix(alpha,kb,cavity_length,tm)
c
c   wz(0)=[(V/q)/cm1]/cavity=[1/cm2]/cavity
c
    wz(0)=0.3d+5
c   ne:number of electrons in one bunche
```

```
      ne=5.0d+10
c
      Eemax=10000.d0
      Eemin=10000.d0
      dEe=Eemax-Eemin
      Eeav=(Eemax+Eemin)/2.0d0
      do nc=0,cavity_number
       gamma(nc)=Eeav/mc2
      end do
c
c     initial values:
c
      nc=0
      do nb=1,maxnb
          x(nc,nb)=1.0d0
          th(nc,nb)=0.0d0
      end do
c
c     nc:number of cavities
      nnn=4
      do nc=0,cavity_number-1
cc        write(*,*) 'Cavity Number=',nc+1
c
c     nb :number of bunches
c     ncb:number of calculated bunches
       ncb=2
          do nb=1,ncb
        call zero(factx,factt)
           if(nb.gt.1) then
c
c     m:previous bunch
         do m=1,nb-1
c
c     nw: frequency of mode
             do nw=0,0
      factx=factx+(tm(1,2)*ne*r0*wz(nw)/gamma(nc+1))
     &      *x(nc,m)
c
      factt=factt+(tm(2,2)*ne*r0*wz(nw)/gamma(nc+1))
     &      *x(nc,m)
c
             end do
         end do
           else
          factx=0.0d0
            factt=0.0d0
           end if
          x(nc+1,nb)=tm(1,1)*x(nc,nb)
     &     +tm(1,2)*th(nc,nb)
     &     +factx
          th(nc+1,nb)=tm(2,1)*x(nc,nb)
```

```
     &      +tm(2,2)*th(nc,nb)
     &      +factt
cc         write(*,*) real(nc),x(nc,nb),th(nc,nb)
           end do
      end do
c
c     ************ Output.data ******************
c
c     output x=nb(bunch number), y=x(nc,nb)
c     nc=cavity number
c
      do nc=cavity_number,cavity_number
          do nb=1,ncb,1
            write(99,*) real(nb),x(nc,nb)
          end do
      end do
       write(99,*) 'join'
c
c
c     output x=s;position of z, y=x(nc,nb)
c     nb=bunch number
c
      do nb=1,2
       do nc=1,cavity_number
           s=real(nc)*cavity_length
         write(98,*) s,x(nc,nb)
       end do
       write(98,*) 'join'
      end do
c
      close(99)
      close(98)
c
      stop
      end

c
c     **********************************************
c
      subroutine tmatrix(alpha,kb,cavity_length,tm)
c
      implicit real*8 (a-h,o-z)
      real*8 kb,tm(2,2)
c
c     kb=[1/cavity]
c      tm(1,1)=[0]   , tm(1,2)=[L]
c      tm(2,1)=[1/L] , tm(2,2)=[0]
c
      beta=1.0/kb
      rmu=kb
      cosmu=cos(rmu)
```

```
      sinmu=sin(rmu)
      tgamma=(1.0d0+alpha**2)/beta
c
      tm(1,1)=cosmu+alpha*sinmu
      tm(1,2)=beta*sinmu*cavity_length
      tm(2,1)=-tgamma*sinmu/cavity_length
      tm(2,2)=cosmu-alpha*sinmu
c
      return
      end
c
c     **********************************************
c
      subroutine zero(x,y)
c
      real*8 x,y
c
      x=0.0d0
      y=0.0d0
c
      return
      end
```